$$L_x = -i\hbar\left(-\frac{\cos\theta}{\sin\theta}\cos\varphi\frac{\partial}{\partial\varphi} - \sin\varphi\frac{\partial}{\partial\theta}\right)$$

$$L_y = -i\hbar\left(-\frac{\cos\theta}{\sin\theta}\sin\varphi\frac{\partial}{\partial\varphi} + \cos\varphi\frac{\partial}{\partial\theta}\right)$$

$$L_+ = L_x \pm iL_y$$

$$= -i\hbar\left\{-\frac{\cos\theta}{\sin\theta}\underbrace{(\cos\varphi \pm i\sin\varphi)}_{e^{\pm i\varphi}}\frac{\partial}{\partial\varphi} - \underbrace{(\sin\varphi \mp i\cos\varphi)}_{\mp i(\cos\varphi \pm i\sin\varphi)}\frac{\partial}{\partial\theta}\right\}$$

$$= -i\hbar\left\{-\cot\theta\, e^{\pm i\varphi}\frac{\partial}{\partial\varphi} \mp i e^{\pm i\varphi}\frac{\partial}{\partial\theta}\right\}$$

$$= -i\hbar\, e^{\pm i\varphi}\left\{-\cot\theta\frac{\partial}{\partial\varphi} \pm i e^{\pm i\varphi}\frac{\partial}{\partial\theta}\right\}$$

$$= \pm\hbar e^{\pm i\varphi}\left\{\frac{\partial}{\partial\theta} \pm i\cot\theta\cdot\frac{\partial}{\partial\varphi}\right\}$$

$$L_-\left(e^{im\varphi}\,\Theta_l^m(\theta)\right)$$

$$= \boxed{-\hbar e^{-i\varphi}\left(\frac{d}{d\theta} + m\cot\theta\right)}\left(e^{im\varphi}\,\Theta_l^m(\theta)\right)$$

$$\frac{d}{d\theta}\left(\sin^m\theta\cdot\Theta_l^m\right) = m\sin^{m-1}\theta\cos\theta\,\Theta_l^m + \sin^m\theta\frac{d}{d\theta}\Theta_l^m$$

$$\frac{d\cos\theta}{d\theta} = -\sin\theta \qquad \frac{d}{d\cos\theta} = \frac{d\theta}{d\cos\theta}\frac{d}{d\theta} = -\frac{1}{\sin\theta}\frac{d}{d\theta}$$

$$e^{i(m-1)\varphi}\sin^{-(m-1)}\theta\left(-\frac{1}{\sin\theta}\frac{d}{d\theta}\right)\left(\sin^m\theta\cdot\Theta_l^m\right)$$

$$= (=)\left\{(-)\frac{1}{\sin\theta}\right\}\left\{m\sin^{m-1}\theta\cos\theta\,\Theta_l^m + \sin^m\theta\cdot\frac{d}{d\theta}\Theta_l^m\right\}$$

$$= -e^{i(m-1)\varphi}\sin^{-m}\theta\{=\}$$

$$= e^{-i(m-1)\varphi}\left\{m\cot\theta\,\Theta_l^m + \frac{d}{d\theta}\Theta_l^m\right\}$$

$$= e^{-i(m-1)\varphi}\sin^m\theta\frac{d}{d\theta}\left(\sin^m\theta\,\Theta_l^m\right)$$

$$= -e^{-i(m-1)\varphi}\sin^{-(m-1)}\theta\cdot\frac{d}{d\cos\theta}\left(\sin^m\theta\,\Theta_l^m\right)$$

$$\propto e^{-i(m-1)\varphi}\,\Theta_l^{m-1}(\theta)$$

$$\sin^{m-1}\theta\,\Theta_l^{m-1} \propto \frac{d}{d\cos\theta}\left(\sin^m\theta\,\Theta_l^m\right) \qquad (12.92)$$

$$1-x^2 \equiv t \qquad \frac{dt}{dx} = -2x \qquad \frac{d}{dx} = \frac{dt}{dx}\cdot\frac{d}{dt} = -2x\cdot\frac{d}{dt}$$

(12.96)

$$-2x^{-\frac{m}{2}}\sqrt{1-t}\cdot\underbrace{\frac{d^{l-m}}{dt^{l-m}}t^l}_{\propto t^m} \qquad x^2 = 1-t \quad x = \sqrt{1-t}$$

$$\propto t^{\frac{m}{2}}\sqrt{1-t}$$