$$0 = \psi_2 \frac{d^2\psi_1}{dx^2} + \frac{d\psi_2}{dx}\frac{d\psi_1}{dx} - \frac{d\psi_2}{dx}\frac{d\psi_1}{dx} - \frac{d^2\psi_2}{dx^2}\psi_1$$

$$\psi_2 \frac{d^2\psi_1}{dy^2} + \frac{d\psi_2}{dy}\frac{d\psi_1}{dy} - \frac{d\psi_2}{dy}\frac{d\psi_1}{dy} - \frac{d^2\psi_2}{dy^2}\psi_1$$

$$0 = \frac{d}{dx}\underbrace{\left[\psi_2 \frac{d\psi_1}{dx} - \frac{d\psi_2}{dx}\psi_1\right]}_{\Downarrow\ \text{const}} + \frac{d}{dy}\left[\psi_2 \frac{d\psi_1}{dy} - \frac{d\psi_2}{dy}\psi_1\right]$$

$$\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} = 0$$

$$\Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \boxed{\frac{\hbar^2 l(l+1)}{r^2} + V(r)} \leftarrow \tilde{V}(r)$$

$$\Delta\psi(r) = E\psi(r)$$

$$E\psi_1 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\psi_1 + \tilde{V}\psi_1$$

$$E\psi_2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\psi_2 + \tilde{V}\psi_2$$

$$0 = \psi_2 \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\psi_1 - \psi_1\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\psi_2$$

$$0 = \psi_2\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\psi_1\right) - \psi_1\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\psi_2\right)$$

$$\frac{\partial\psi_2}{\partial r}\cdot r^2\frac{\partial}{\partial r}\psi_1 - \frac{\partial\psi_2}{\partial r}\, r^2\frac{\partial\psi_1}{\partial r} = \frac{\partial\psi_1}{\partial r}\cdot r^2\frac{\partial\psi_2}{\partial r}$$

$$\frac{\partial}{\partial r}\left\{\psi_2\cdot r^2\frac{\partial}{\partial r}\psi_1\right\} - \frac{\partial}{\partial r}\left\{\psi_1\cdot r^2\frac{\partial}{\partial r}\psi_2\right\}$$

$$= \frac{\partial}{\partial r}\left\{\psi_2 r^2\frac{\partial}{\partial r}\psi_1 - \psi_1 r^2\frac{\partial}{\partial r}\psi_2\right\} = 0$$

$$\int|\psi|^2 r^2 dr < \infty$$

$$\psi_2 r^2\frac{\partial}{\partial r}\psi_1 - \psi_1 r^2\frac{\partial}{\partial r}\psi_2 = \text{const}$$

$$\uparrow\ 0$$

$$\frac{\psi_2}{\psi_1} = \frac{\psi_2'}{\psi_1'}$$

$$\psi_2 r^2\frac{\partial}{\partial r}\psi_1 = \psi_1 r^2\frac{\partial}{\partial r}\psi_2$$

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$$\hat{L}^2 = \epsilon_{ijk} x^j p^k \epsilon_{ilm} x^l p^m \qquad \sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{ilm} = \{ \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl} \}$$

$$= (\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}) x^j p^k x^l p^m \qquad p^k x^l = x^l p^k + [p^k, x^l]$$

$$= x^l p^k - i\hbar \delta^{kl}$$

$$= (\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}) x^j (x^l p^k - i\hbar \delta^{kl}) p^m$$

$$x^l p^k p^m = p^m x^l p^k + [x^l, p^m] p^k$$

$$x^j x^l p^k p^m - i\hbar \delta^{kl} x^j p^m$$

$$= p^m x^l p^k + i\hbar \delta^{lm} p^k$$

$$= x^j x^j \cdot p^k p^k - (x \cdot p)^2 - i\hbar \delta_{jm}\delta_{kl} x^j \delta^{lm} p^k$$

$$- (\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}) \delta^{kl} x^j p^m$$

$$\delta_{kl}\delta^{kl} = \sum_{k=1}^{3} \delta_{kk}\delta^{kk} = \sum_{k=1}^{3} 1 \cdot 1 = 3$$

$$= x^2 p^2 - (x \cdot p)^2 - i\hbar (x\cdot p) - i\hbar (x \cdot p) + i\hbar \cdot 3 (x \cdot p)$$

$$= x^2 \cdot p^2 - (x \cdot p)^2 + i\hbar (x \cdot p)$$

$$x \cdot p = r \frac{\partial}{\partial r}$$

$$L^2 = r^2 p^2 - \left(r \cdot \frac{\partial}{\partial r}\right)^2 + i\hbar \left(r \cdot \frac{\partial}{\partial r}\right) \longrightarrow = -\hbar^2 r^2 \Delta - \left(r \frac{\partial}{\partial r}\right)^2 + i\hbar \left(r \cdot \frac{\partial}{\partial r}\right)$$

$$-\hbar^2 \Delta - \frac{\hat{L}^2}{r^2} = -\hbar^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) - \frac{L^2}{r^2}$$

$$= \underbrace{\left\{ -\hbar^2 \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{L^2}{r^2} \right\}}_{-\hbar^2 \Delta} - \frac{L^2}{r^2} = -\hbar^2 \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$$

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$$\int_{-1}^{1} dx\, (1-x^2)^n = \frac{2^{2n+1} (n!)^2}{(2n+1)!} \qquad n = 0 \quad 2 = 2 \text{ ok.}$$

$$\int_{-1}^{1} dx\, (1-x^2)^k = \frac{2^{2k+1} (k!)^2}{(2n+1)!}$$

$$x = \cos\theta \qquad 1 - x^2 = \sin^2\theta.$$

$$dx = d\cos\theta = -\sin\theta \cdot d\theta = -\sqrt{1-x^2}\, d\theta$$

$$\int_{-1}^{1} dx\, (1-x^2)^{k+1} = -\int_{-1}^{1} \sqrt{1-x^2}\, d\theta \cdot (1-x^2)^{k+1}$$

$$= -\int_{\pi}^{0} \sin\theta \cdot \sin^{2(k+1)}\theta \cdot d\theta$$