7th Australian Algebra Conference

22–24 November 2023

Monash University

The 7th Australian Algebra Conference is sponsored by the School of Mathematics at Monash University and the Australian Algebra Group.

General Information

Goal and Topics

The main aim of the annual Australian Algebra Conference is to foster communication between algebraists in Australia. We interpret algebra quite broadly, including areas such as topological algebra, algebraic logic, graph theory and coding theory. The conference is run by the Australian Algebra Group, which is a special interest group of the Australian Mathematical Society.

Invited Speakers

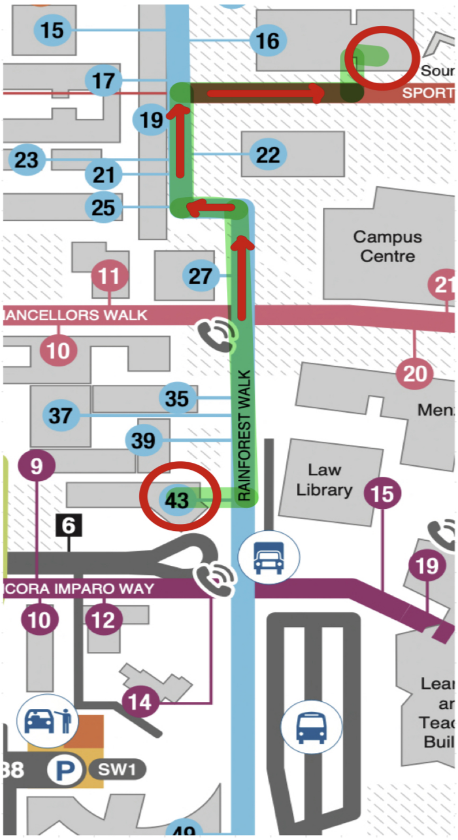
Eamonn O'Brien, *University of Auckland, New Zealand.*
Colva Roney-Dougal, *University of St. Andrews, UK.*
Geertrui Van de Voorde, *University of Canterbury, New Zealand.*

Organising Committee

Santiago Barrera Acevedo, *Monash Unversity.*
Heiko Dietrich, *Monash Unversity.*
Melissa Lee, *Monash Unversity.*
Tomasz Popiel, *Monash Unversity.*

Local Information

The conference will be held at Monash University, Clayton Campus, Melbourne. On Wednesday and Friday, talks will be held in Lecture Theatre South 1 (43 Rainforest Walk), close to the bus loop. Talks on Thursday will be held in Lecture Theatre South 1 (43 Rainforest Walk) and Lecture Theatre S12 (16 Rainforest Walk). It takes about 5 minutes to walk between these venues; a route is shown below.



Please see the Clayton Campus Map for details. (Click here for a static version of the map.)

We acknowledge and pay respect to the Traditional Owners and Elders—past, present and emerging—of the lands on which Monash University operates.

Internet Access

Eduroam is available throughout campus.

Coffee and Food :)

We will offer some simple morning and afternoon teas (as listed in the schedule), and we have organised a BBQ lunch for Wednesday. For all other meals, Monash Clayton has a large variety of coffee shops and food venues. Most of these are located in the campus centre and surroundings.

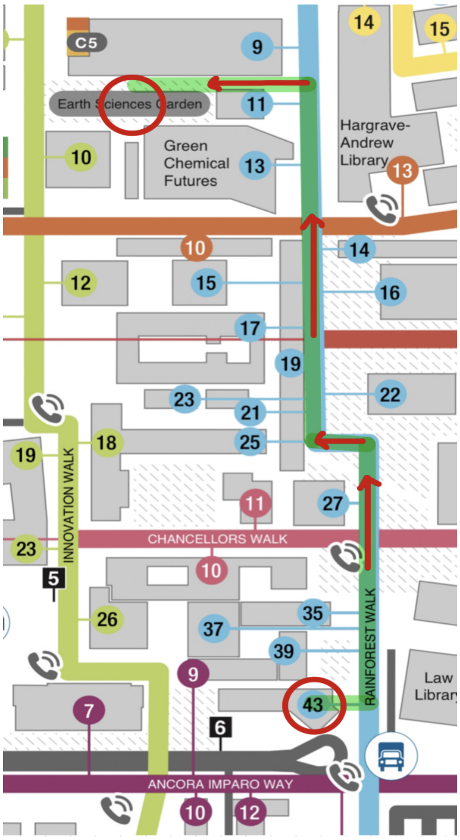
Please click here to navigate an interactive food map, or scan the QR code below (make sure you open the map in full screen to see names of vendors).

Medical Centre and Pharmacy

These are are both located in the campus centre.

BBQ

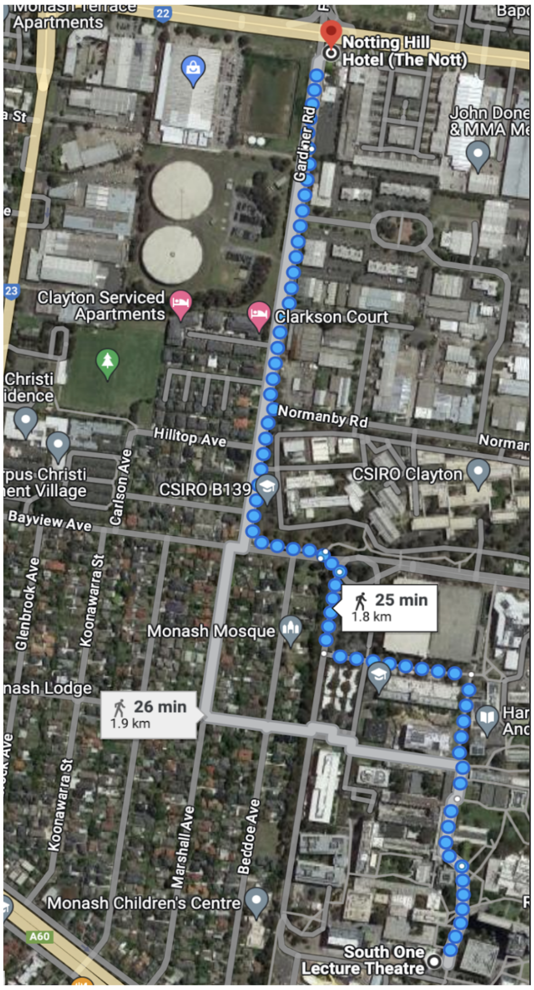
The Monash School of Mathematics is shouting a BBQ on Wednesday 22 November from 12:00–2:00 pm at the Earth Science (Rock) Garden, located in front of the School of Mathematics building, 9 Rainforest Walk.



Conference Dinner

The (self-funded) conference dinner will take place at the Notting Hill Hotel at 6:00 pm on Thursday 23rd of November. Please check their menu here.

It takes about 25 minutes to walk from Lecture Theatre South 1 (43 Rainforest Walk) to The Nott. A suggested route is shown below.



Australian Women of Mathematics Exhibit

The School of Mathematics is currently hosting the Australian Women of Mathematics Exhibit, which was put together by Sylvie Paycha and Pierre Portal. The exhibit is located on the ground floor of the School of Mathematics building at 9 Rainforest Walk, near the Mathematics Learning Centre. Please visit the exhibit if you have some spare time after lunch.

Exploring Monash

Dayton

Corner of Research Way & Rainforest Walk, Clayton Campus

Dayton by Ronnie van Hout is a humanoid robot that playfully encourages us to think about the increasingly human-like nature of robots, and our corresponding and intensifying reliance on technology. Dayton also plays with the relationship between humanity and 'pure' or abstract form by riffing off minimalist sculpture. From certain views Dayton appears to be an arrangement of rectangular and square forms, resolving as a figure as one passes by, before dissolving into abstraction again.

Kenneth Hunt Garden

The Kenneth Hunt garden was named in honour of Monash University's first Dean of Engineering, Professor Kenneth Hunt. It is largely an English-styled garden, planted with exotic species and features Newton's Apple tree – a clone from the original tree from which Sir Isaac Newton reportedly observed an apple fall, prompting him to formulate his theories of motion and gravity. In 2012, the Kenneth Hunt garden underwent significant redevelopment and expansion to provide more seating areas and better access.

West Gate Garden

As part of an ongoing initiative to provide students with many varied places to gather, the West Gate garden was created in 1994. The central focus of the garden is the ephemeral pond, or "billabong" which acts as an infiltration basin and water source for birds. The garden features plenty of places to sit amongst a selection of plant species which tolerate both wet and dry conditions including collections of Sheoaks (Casuarina sp.), Paperbarks (Melaleuca sp.) and Emu Bushes (Eremophila sp.). Did you know? Housed in the West Gate garden are fragments from the collapsed section of Victoria's West Gate Bridge. Following the 1970 collapse of the West Gate Bridge between piers 10 and 11, Monash University was asked to participate in the investigation into the cause. It is said that these twisted fragments were placed in the West Gate garden to remind future engineers of the consequences of errors.

Public Art Walk

The MUMA Public Art Walk brochure has been developed as a way of introducing and highlighting the many public artworks that are located on the Clayton and Caulfield campuses and are in the Monash University Collection; it has been designed as an open loop that can be joined at any point along the path.

List of Participants

Abdul Basit, Monash University
Adam Piggott, Australian National University
Anton Baykalov, The University of Western Australia
Ari Markowitz, University of Auckland,
Azeef Muhammed Parayil Ajmal, Western Sydney University
Bailey Mclellan, University of Melbourne
Barry Gardner, University of Tasmania
Brian Davey, La Trobe University
Brian Ng, Monash University
Brice Arrigo, Monash University
Colva Roney-Dougal, University of St Andrews
Daniel Glasson, RMIT University
Darius Young, University of Auckland
David Jefferies, Monash University
Deborah Jackson, La Trobe University
Dion Nikolic, UNSW Canberra
Don Taylor, University of Sydney,
Đorđe Mitrović, University of Auckland
Eamonn O'Brien, University of Auckland
Eli Hazel, Macquarie University
Gabriel Verret, University of Auckland,
Geertrui Van de Voorde, University of Canterbury
Graham Farr, Monash University
Guillermo Badia, University of Queensland
Heiko Dietrich, Monash University
Ian Wanless, Monash University
Jackson Ryder, UNSW
James Carr, University of Queensland
James East, Western Sydney University
Jean-Simon Pacaud Lemay, Macquarie University
Jeroen Schillewaert, University of Auckland
Jerry Shen, University of Technology Sydney
Joe Gurr, UNSW
John Power, Macquarie University
Joshua Howie, Monash University
Joshua Zhang, The University of Melbourne

Lawrence Reeves, The University of Melbourne
Lei Chen, The University of Western Australia
Luke Morgan, The University of Western Australia
Marcel Jackson, La Trobe University
Marcus Chijoff, The University of Newcastle
Marston Conder, University of Auckland
Matthew Conder, University of Auckland
Matthias Fresacher, Western Sydney University
Meizheng (Steven) Fu, University of Auckland
Melissa Lee, Monash University
Michael Wynne, Monash University
Murray Elder, University of Technology Sydney
Nicola Di Vittorio, Macquarie University
Norman Do, Monash University
Padraig Ó Catháin, Dublin City University
Peter McNamara, University of Melbourne
Santiago Barrera Acevedo, Monash University
Saul Freedman, The University of Western Australia
Sean Gardiner, UNSW
Seethalakshmi Kayanattath, The Australian National University
Soichiro Fujii, Macquarie University
Stephen Glasby, The University of Western Australia
Tabish Ali Rather, Swinburne University of Technology
Tomasz Popiel, Monash University
Tumadhir F Alsulami, La Trobe University
Vandit Trivedi, Australian National University
Victor Fagundes, Monash University
Wanbao Zhang, The University of Melbourne
Yusuf Hafidh, The University of Melbourne
Yuxuan Li, The University of Melbourne
Zhaochen Ding, University of Auckland

Conference Program

On Wednesday and Friday, all talks will be held in Lecture Theatre South 1 (43 Rainforest Walk), close to the bus loop. On Thursday, after the 9am invited talk, there will be two parallel sessions: one in Lecture Theatre South 1 (43 Rainforest Walk), and the other in Lecture Theatre S12 (16 Rainforest Walk).

	Wednesday	Thursday		Friday
8:50 - 9:00	Welcome			
9:00 - 10:00	Eamonn O'Brien	Colva Roney-Dougal		Geertrui Van de Voorde
10:00 - 10:30	Break	Break		Break
		Stream 1: South 1 LT	Stream 2: S12 LT	
10:30 - 11:00	James East	Jeroen Schillewaert	Soichiro Fujii	Stephen Glasby
11:00 - 11:30	Matthias Fresacher	Vandit Trivedi	Marcel Jackson	Marston Conder
11:30 - 12:00	Adam Piggott	Marcus Chijoff	Barry Gardner	Don Taylor
12:00 - 12:30	BBQ/Photo	Jean-Simon Pacaud Lemay	Norm Do	Murray Elder
12:30 - 2:00	BBQ	Lunch (& AAG meeting 12:45 - 1:45)		Conclusion & Lunch
2:00 - 2:30	Peter McNamara	Guillermo Badia	Yusuf Hafidh	
2:30 - 3:00	Joshua Zhang	Azeef Parayil Ajmal	James Carr	
3:00 - 3:30	Jerry Shen	Ian Wanless	Tumadhir Alsulami	
3:30 - 4:00	Break	Break		
4:00 - 4:30	Padraig Ó Catháin	Saul Freedman	Eli Hazel	
4:30 - 5:00	Zhaochen Ding	Anton Baykalov	Lei Chen	
5:00 -5:30	Đorđe Mitrović			
5:30 - 6:00				
6:00 - 9:00		Conference dinner at The Nott		

The three invited talks (9am each day) are 50 minutes each, with 5 minutes for questions. All other talks are 20 minutes each, with 5 minutes for questions and 5 minutes for swapping speakers.

The BBQ will take place at the Rock Garden, located in front of the School of Mathematics building, 9 Rainforest Walk.

The Australian Algebra Group meeting will be held on Thursday 23rd of November from 12:45 pm to 1:45 pm in Room 105 in the Faculty Office, Building 14.

The (self-funded) conference dinner will take place at the Notting Hill Hotel at 6:00 pm on Thursday 23rd of November. Please check their menu here.

Book of Abstracts

Wednesday 22 November

Conjugacy in finite groups

Eamonn O'Brien
University of Auckland

We report on the successful outcome of a project to provide complete theoretical and practical solutions to conjugacy problems in finite classical groups. In particular we can list all classes; construct centralisers; and decide constructively conjugacy between elements of all classical groups. We discuss the implications of this work for the identical problems in finite groups. This is joint work with Giovanni de Franchesci and Martin Liebeck.

Heights of congruence lattices of semigroups, acts and algebras

James East
Western Sydney University

Lattices of one-sided congruences of a semigroup can be extremely complicated. For example, these lattices are not understood even for finite full transformation semigroups. Nevertheless, one can still sometimes compute certain combinatorial invariants of such lattices. This talk will focus on the height of the lattice, ie the supremum of the sizes of chains. One of our results is an explicit height formula in a class of semigroups containing many families of transformation and diagram monoids. The key tool is to consider a semigroup S and its one-sided ideals as S-acts, and this in turn leads us to broaden our scope to wider classes of (universal) algebras. This is all joint work with Matthew Brookes, Craig Miller, James Mitchell and Nik Ruškuc.

Congruence Lattices of Finite Twisted Brauer Monoids

Matthias Fresacher
Western Sydney University

In 2022, East and Ruškuc published the congruence lattice of the infinite twisted partition monoid. As a by product, they established the congruence lattices of the finite d-twisted partition monoids. This talk is a first step in adapting the work of East and Ruškuc to the setting of the Brauer monoid. Specifically, it presents the newly established congruence lattice of the 0-twisted Brauer monoid. With simple to grasp visual multiplication and applications in theoretical physics and representation theory, the family of partition monoids are of particular interest to a number of fields as well are of stand alone interest.

Geodetic groups are virtually free

Adam Piggott
Australian National University

A graph is geodetic if there is a unique shortest path between each pair of vertices. A group is geodetic if it admits a locally-finite geodetic Cayley graph. In this talk we report on a collaboration with Murray Elder, Giles Gardam, Davide Spriano and Kane Townsend in which we prove that all geodetic groups are virtually-free

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The Spin Brauer Category

Peter McNamara
University of Melbourne

The Brauer category is a tool used to study the representation theory of the (special) orthogonal Lie group/Lie algebra. A drawback is it doesn't see the spin representations of the special orthogonal Lie algebras. In joint work with Alistair Savage, we introduce and study a spin version - the Spin Brauer category, which sees the entire representation theory of the type B/D Lie algebras.

Card shuffle groups

Joshua Zhang
The University of Melbourne

For positive integers k and n, the shuffle group $G_{k,kn}$ is generated by the $k!$ permutations of a deck of kn cards performed by cutting the deck into k piles with n cards in each pile, and then perfectly interleaving these cards following certain order of the k piles. For $k = 2$, the shuffle group $G_{2,2n}$ was determined by Diaconis, Graham and Kantor in 1983. The Shuffle Group Conjecture states that, for general k, the shuffle group $G_{k,kn}$ contains A_{kn} whenever $k \notin \{2, 4\}$ and n is not a power of k. In particular, the conjecture in the case $k = 3$ was posed by Medvedoff and Morrison in 1987. The only values of k for which the Shuffle Group Conjecture was confirmed up to 2022 are powers of 2, due to work of Amarra, Morgan and Praeger based on Classification of Finite Simple Groups. In this talk, I will introduce our approach to a complete solution of the Shuffle Group Conjecture, which involves applying results on 2-transitive groups and elements of large fixed point ratio in primitive groups. Joint work with Binzhou Xia, Junyang Zhang and Wenying Zhu.

Complexity of epimorphism testing with virtually abelian targets

Jerry Shen
University of Technology Sydney

The epimorphism problem for virtually abelian targets is a decision problem which asks: given a finite presentation for a group, if an epimorphism to a virtually abelian group exists? The problem is shown to decidable to certain subclasses of virtually abelian targets by Friedl and Löh (2021, Confl. Math.), we discuss the complexity of said classes and additional subclasses.

On the Hadamard maximal determinant problem

Padraig Ó Catháin
Dublin City University

In a celebrated paper of 1893, Hadamard proved the maximal determinant theorem, which establishes an upper bound on the determinant of a matrix with complex entries of norm at most 1. His paper concludes with the suggestion that mathematicians study the maximum value of the determinant of an $n \times n$ matrix with entries in $\{\pm 1\}$. This is the Hadamard maximal determinant problem. In this talk we will give an algebraic proof of the determinant inequality due originally to Fischer, and explore stronger bounds for the maximal determinant problem in odd dimensions. We explore how these results extend to certain cyclotomic fields and sketch connections to finite fields and character sums. We will finish with some open problems.

Compatible groups and inverse limits

Zhaochen Ding
The University of Auckland

Two groups L_1 and L_2 are called compatible if there is a group G with two isomorphic normal subgroups N_1 and N_2 such that $G/N_1 \cong L_1$ and $G/N_2 \cong L_2$. In this talk, we will discuss some recent work (joint with Gabriel Verret) on compatibility of groups, including a new construction based on inverse limits.

Graph growth of transitive permutation groups

Đorđe Mitrović
University of Auckland

Let Γ be a connected graph and let G be a vertex-transitive group of automorphisms of Γ. The pair (Γ, G) is locally-L if the group induced by the action of the stabiliser G_v on the neighbourhood of a vertex v is permutation isomorphic to L. Using this language, a classical theorem of Tutte states that for locally-A_3 and locally-S_3 pairs, $|G|$ grows linearly with $|V(\Gamma)|$. More generally, given a transitive permutation group L, we are interested in determining how $|G|$ grows with respect to $|V(\Gamma)|$ for locally-L pairs (Γ, G). We present new results on this topic and highlight an exciting connection with the study of eigenspaces of graphs over finite fields.

Thursday 23 November

Base size and relational complexity

Colva Roney-Dougal
University of St Andrews

The relational complexity of a permutation group was introduced in model theory, and is a measure of the extent to which partial information about the action of a group element determines the existence of the element. A base of a permutation group G is a sequence of points whose pointwise stabiliser is trivial. There are intimate connections between relational complexity and various types of bases. This talk will give a survey of recent activity in this rapidly-developing area.

Session 1 — South 1 Lecture Theatre

Braid groups, elliptic curves and resolving the quartic

Jeroen Schillewaert
University of Auckland

In 1540 the 18-year old Lodovico Ferrari discovered a method of solving the quartic equation, which involves first solving a closely related cubic equation. This "resolution of the quartic" corresponds to a beautiful holomorphic map $R : Poly_4 - > Poly_3$, where $Poly_n$ denotes the space of monic, degree n, square free polynomials over the complex numbers. I will discuss recent joint work with Peter Huxford that extends Chen and Salter's classification of the holomorphic maps between many of the $Poly_n$, and progress towards a conjecture of Benson Farb which characterizes the resolving quartic map R. We also classify the holomorphic families of elliptic curves over complex configuration spaces. To do this we classify homomorphisms between braid groups with few strands and the modular group, then apply powerful results from complex analysis and Teichmüller theory. Furthermore, we prove a conjecture of Castel about the equivalence classes of endomorphisms of the braid group with three strands.

Explicit computations in Aut(F_2)

Vandit Trivedi
Australian National University

Much is known about the automorphism group of the free group of rank 2. In particular, it is known that Aut(F_2) has quadratic isoperimetric inequality and is biautomatic. However, certain details are not explicitly known. In this talk, we discuss some of these details.

Classifying Discrete (P)-closed Groups Using Local Action Diagrams

Marcus Chijoff
The University of Newcastle

We prove necessary and sufficient conditions for (P)-closed groups to be discrete. These conditions are proven using local action diagrams: combinatorial structures that are in a one-to-one correspondence with (P)-closed groups. The conditions we prove are phrased entirely in terms of local action diagrams.

Zinbiel Algebras and the Fundamental Theorems of Calculus

Jean-Simon Pacaud Lemay
Macquarie University

Derivations and integrations (also called O-operators or Rota-Baxter operators) generalize the notions of the derivative and integral from calculus. Zinbiel algebras are a special kind of commutative non-unital associative algebras that capture the notion of riffle shuffle permutations. Zinbiel algebras and integrations are closely related: every integration induces a Zinbiel algebra, and every Zinbiel algebra induces an integration. However, these constructions are not inverses! Indeed, while every Zinbiel algebra is constructed from an integration, not every integration is constructed from a Zinbiel algebra. In this talk, I will explain how an integration is constructed from a Zinbiel algebra precisely when said integration comes equipped with a derivation, and that together they satisfy the two fundamental theorems of calculus. Thus we obtain an equivalent characterization of Zinbiel algebras in terms of the fundamental theorems of calculus, and also a nice equivalence of categories. I will also explain how this story generalizes nicely in a differential category, and how Zinbiel algebras can be used to construct antiderivatives in a differential category.

Asymptotic truth-value laws in many-valued logics

Guillermo Badia
University of Queensland

This paper studies which truth-values are most likely to be taken on finite models by arbitrary sentences of a many-valued predicate logic. We obtain generalizations of Fagin's classical zero-one law for any logic with values in a finite lattice-ordered algebra, and for some infinitely valued logics, including Łukasiewicz logic. The finitely valued case is reduced to the classical one through a uniform translation and Oberschelp's generalization of Fagin's result. Moreover, it is shown that the complexity of determining the almost sure value of a given sentence is PSPACE-complete, and for some logics we may describe completely the set of truth-values that can be taken by sentences almost surely.

Free (idempotent- and projection-generated) regular $\star$-semigroups

Azeef Muhammed Parayil Ajmal
Western Sydney University

Regular $\star$-semigroups is a class of semigroups which occupies something of a 'sweet spot' between the important classes of inverse and regular semigroups, and contains many natural examples. Some of the most significant families include the partition, Brauer and Temperley-Lieb monoids, among other diagram monoids. In a recent work, by focussing on the *projection algebra* structure of the semigroup, it was shown that the category **RSS** of regular $\star$-semigroups is isomorphic to the category of so-called *chained projection groupoids*. This generalisation of the celebrated ESN (Ehresmann-Schein-Nambooripad) Theorem also leads to the construction of certain free (idempotent- and projection-generated) regular $\star$-semigroups, which we shall call as *chain semigroups*. Free idempotent generated semigroups is a hot topic in the area of algebraic theory of semigroups. In this talk, we shall begin with a general overview of free idempotent generated semigroups and regular $\star$-semigroups. We shall discuss the construction of chain semigroups and show how they form free objects in the category **RSS**. To be precise, we show that the forgetful functor $\mathtt{P}$ from the category **RSS** to the category **PA** of projection algebras, has a left adjoint $\mathtt{F}$. At the object level, the free functor $\mathtt{F}\colon \mathbf{PA} \to \mathbf{RSS}$ maps a projection algebra P to the chain semigroup $\mathcal{C}_P$. This construction involves the discovery of the notion of *linked pairs*, which are the regular $\star$-analogues of Nambooripad's singular squares in biordered sets. Using the free functor $\mathtt{F}$, we can identify **PA** with a coreflective subcategory of **RSS**. This result has an analogue in inverse semigroups, wherein it can be seen that the category of semilattices is coreflective in the category of inverse semigroups. This is a recent joint work with James East.

Automorphisms of quadratic quasigroups

Ian Wanless
Monash University

Let $\mathbb{F}_q$ be a finite field of odd order q. Let $\square$ denote the set of squares in $\mathbb{F}_q$. Suppose $a, b \in \mathbb{F}_q$ are such that $ab, (a-1)(b-1) \in \square \setminus \{0\}$. We define $Q_{a,b}$ to be the *quadratic quasigroup* $(\mathbb{F}_q, *)$ defined by

$$x * y = \begin{cases} x + a(y-x) & \text{if } y - x \in \square, \\ x + b(y-x) & \text{otherwise.} \end{cases}$$

We will briefly survey the algebraic and combinatorial applications of quadratic quasigroups. We will then report on new work which answers the following questions.

(1) What is the automorphism group of $Q_{a,b}$?
(2) When is $Q_{a,b}$ isomorphic to $Q_{c,d}$?
(3) What are the minimal subquasigroups of $Q_{a,b}$?
(4) When is $Q_{a,b}$ isotopic to some finite group?

Joint work with Aleš Drápal, Charles University, Prague.

Spreading primitive groups of diagonal type do not exist

Saul Freedman
The University of Western Australia

The synchronisation hierarchy of finite permutation groups, introduced by Araújo, Cameron and Steinberg in 2017, consists of classes of groups lying between 2-transitive groups and primitive groups. This includes the class of spreading groups, which are defined in terms of sets and multisets of permuted points. Araújo et al. proved that all spreading groups are primitive of almost simple, affine or diagonal type. In addition, Bray, Cai, Cameron, Spiga and Zhang showed in 2020 that a spreading group of diagonal type must have socle $T \times T$ for some non-abelian finite simple group T. In this talk, we prove that in fact no spreading group of diagonal type exists, by considering transitive actions (and several character tables) of the non-abelian finite simple groups. This is joint work with John Bamberg and Michael Giudici.

Rank three innately transitive permutation groups

Anton Baykalov
The University of Western Australia

In this talk, I will discuss a recent joint work with Alice Devillers and Cheryl Praeger where we extend classifications of finite primitive and quasiprimitive groups of rank 3 to a classification for the finite innately transitive groups. Apart from being an interesting group-theoretical problem, the classification is motivated by the search of highly symmetrical partial linear spaces with imprimitive automorphism group.

<u>Session 2 — Lecture Theatre S12</u>

Ordered semirings and subadditive morphisms

Soichiro Fujii
Macquarie University

An ordered semiring is a commutative semiring equipped with a compatible pre-order. The notion of ordered semiring generalises those of distributive lattice and of commutative ring, and enables us to unify certain aspects of lattice theory and ring theory. The ideals of an ordered semiring A form an integral commutative quantale Idl(A), and similarly, the radical ideals of A form a (spatial) frame Rad(A). We characterise Idl and Rad as the left adjoints of the forgetful functors from the categories of integral commutative quantales and of frames to that of ordered semirings and subadditive morphisms between them. The (sober) topological space pt(Rad(A)) = Spec(A) corresponding to Rad(A) is a space of prime ideals of A.

On limit varieties of additively idempotent semirings

Marcel Jackson

La Trobe University

A variety, in the sense of Garret Birkhoff, is an equationally defined class of algebras. An application of Zorn's Lemma shows that every nonfinitely axiomatisable equational class contains a minimally nonfinitely axiomatisable subclass: a so-called *limit variety*. The explicit description of a limit variety of groups remains one of the last longstanding open problems in the theory of group varieties, though continuum many examples are known to exist. In this talk we present a number of limit varieties in the theory of additively idempotent semirings. We are able to explicitly identify (and even classify) infinitely many examples built from finite groups, continuum many built from infinite groups (existence proof only), as well as several further ad hoc examples using word and hypergraph constructions and from the max-plus algebra. For one of the ad hoc examples we are able to identify an explicit generator, though the others are existence proofs only. This is joint work with Miaomiao Ren, Donglin Lei and Xianzhong Zhao (Northwest University, Xi'an, China)

Radicals of some generalized monoid rings

Barry Gardner

University of Tasmania

In 1956 Amitsur proved an important theorem relating the Jacobson radical of a polynomial ring and the nil radical of the coefficient ring. This has since been generalized to various kinds of skew polynomial rings. Polynomial rings are examples of monoid rings and correspondingly skew polynomial rings are examples of generalized monoid rings defined by D-structures. We have begun an investigation of results for radicals of generalized monoid rings and the extent of their resemblance to analogous results for ordinary monoid rings. This is joint work with Elena Cojuhari.

Integration on Grassmannians and deformed monotone Hurwitz numbers

Norm Do

Monash University

Integrals on the space U(N) of unitary matrices have a large N expansion whose coefficients count factorisations of permutations into "monotone" sequences of transpositions. We will show how this classical story can be adapted to integrals on the complex Grassmannian Gr(M,N), which leads to a 1-parameter deformation of the aforementioned enumeration. The resulting polynomials obey remarkable properties, some known and some conjectural. (This is joint work with Xavier Coulter and Ellena Moskovsky.)

Perfect codes in Cayley graphs on finite good abelian groups

Yusuf Hafidh
The University of Melbourne

A set of vertices C on a connected graph G is called a *perfect code* if every vertex not in C is a neighbor of exactly one vertex in C. A group is called *good* if in every direct sum factorization, there is a periodic factor. In this talk, we will discuss the characterization of Cayley graph on finite good abelian groups that contain a perfect code. We will also discuss all of the perfect codes in that Cayley graph.

Finite Homomorphism Preservation for Many-Valued Logics

James Carr
University of Queensland

A canonical result in model theory is the homomorphism preservation theorem (h.p.t) which states that a first-order formula is preserved under homomorphisms on all structures if and only if it is equivalent to an existential-positive formula, standardly proved via a compactness argument. Rossman (2008) established that the h.p.t. remains valid when restricted to finite structures. This is a significant result in the field of finite model theory, as it stands in contrast to other results proved via compactness where the failure of the latter also results in the failure of the former. It is also a result of interest to the field of constraint satisfaction due to the equivalence of existential-positive formulas and unions of conjunctive queries. At the same time Dellunde and Vidal (2019) established that a version of the (infinite) h.p.t. holds for a collection of many-valued predicate logics, namely those whose structures are defined over a fixed finite MTL-chain.

In this talk we unite these two strands. We show how one can extend Rossman's proof of a finite h.p.t. to a very wide collection of many-valued predicate logics. In doing so we establish a finite variant to Dellunde and Vidal's result which not only applies to structures defined over more general algebras than MTL-chains but where we allow the algebra our structures are defined over to vary within a given class. In doing so we highlight that the validity of such a finite h.p.t (and indeed its more usual infinite variant) for a given many-valued logic is determined by properties of its algebraic semantics. This is due to the identification of the fairly minimal critical features of Boolean algebras that enable Rossman's proof from a model theoretic point of view, and demonstrate how any variety of algebras satisfying them will inherit an appropriate finite h.p.t for models whose truth values are defined over its members. One requirement of this is a generalisation of back-and-forth equivalence for many-valued logics first presented by Dellune et al (2018). The investigation provides a starting point in a wider development of finite model theory for non-classical logics and just as the classical finite h.p.t has implications for constraint satisfaction the many-valued finite h.p.t has implications for valued constrain satisfaction problems.

Decidability and axiomatizability questions for the algebra of positive arithmetic and combinatorial operations

Tumadhir F Alsulami

La Trobe University

The possible decidability of the first order logic of the real exponential field R_{exp} is one of the outstanding problems in model theory. The equational fragment was shown to be decidable by Macintyre in 1981, and moreover in the positive case (no subtraction) was shown to coincide with the equational fragment of the algebra of natural numbers N with exponentiation (and addition, multiplication). A second proof of decidability was found by Gurevich in 1985, who also proved the decidability of the logical entailment relation between valid equations on N. Later, Gurevich proved that no finite axiomatisation for the equational theory is possible. We present the start of a new explorations in this style for other elementary arithmetic operations on the natural numbers such binomial coefficients, factorial and restricted forms of exponentiation. We derive a new proof of the decidability for the equational theory of exponentiation, based on the 1980 work of Wilkie, and extend Gurevich's arguments for the logical entailment relation to include the new operations. Finally we explore the equational theory of arithmetic with binomial coefficients and for fixed base exponentiation, finding a finite axiomatisation to various extents.

Non-commutative Stone duality: a monadic approach

Eli Hazel

Macquarie University

Classical Stone duality is the dual equivalence of categories between Boolean algebras and totally disconnected compact Hausdorff spaces, or Stone spaces. In this talk, we discuss our work recovering a number of Stone-type dualities by way of monads, illuminating the algebraic nature of the dual categories of spaces. Boolean algebras are a subvariety of *skew Boolean algebras*, in which $\wedge$ and $\vee$ are not required to be commutative. A duality between skew Boolean algebras and a category of sheaves over Stone spaces was given by Kudryavtseva in 2012 [1].

We build on derivation of classical Stone duality by way of the ultrafilter monad, which may be induced by an adjunction between the category of sets and Boolean algebras. The category of algebras for the ultrafilter monad is the category of compact Hausdorff spaces, originally proved by Manes [2]. The category of Stone spaces is a subcategory of compact Hausdorff spaces, and Stone duality may be recaptured by restricting the canonical comparison functor (contravariant) from Boolean algebras to compact Hausdorff spaces to its essential image [3].

We extend this result by considering an adjunction between the category of skew Boolean algebras and polynomial functors on the category of sets, viewed as a category of 'sheaves over sets', or sheaves over spaces with the discrete topology. It turns out that the monad induced here is essentially the same as the *ultrasheaf*

monad, which was described in [4]. Algebras are sheaves over compact Hausdorff spaces and free algebras admit an interesting description as families of ultraproducts indexed by a space of ultrafilters. Applying our machinery we obtain a version of the non-commutative Stone duality of [1]. This talk is based on joint work with Richard Garner.

[1] Kudryavtseva, G. A refinement of Stone duality to skew Boolean algebras. *Algebra universalis.* 67(4):397–416, 2012.

[2] Manes, E. A triple theoretic construction of compact algebras. In Eckmann, B. editor, *Sem. on Triples and Categorical Homology Theory (ETH, Zurich, 1966/67)*, volume 80 of *Lecture Notes in Mathematics*, 91–118, Springer, Berlin, Heidelberg, 1969.

[3] Petrisan, DL. *Investigations into algebra and topology over nominal sets.* PhD thesis, University of Leicester, 2012.

[4] Kennison, JF. Triples and compact sheaf representation. *Journal of Pure and Applied Algebra.* 20:13–38, 1981.

S-arc-transitivity of vertex-primitive digraph

Lei Chen

The University of Western Australia

The property of s-arc-transitivity has been studied for decades. In 1947, Tutte showed that a cubic graph can be at most 5- arc-transitive. Weiss proved that finite undirected graphs of valency at least 3 that are not cycles can be at most 7-arc-transitive. In stark contrast with the situation in undirected graphs, Praeger showed that for each s and d there are infinitely many finite s-arc-transitive digraphs of valency d that are not (s + 1)-arc-transitive.However, once we add the constraint of primitivity, things can get quite different. The vertex-primitive s-arc-transitive digraphs for large s seem rare. Although extensive attempts had been made to construct a vertex-primitive s- arc-transitive digraph for s ≥ 2, no such examples were found until Giudici, Li, and Xia, found infinite families of vertex-primitive 2-arc-transitive examples, and Giudici and Xia also ask the following question: Question 1: if there exists an upper bound on s for vertex-primitive s-arc- transitive digraphs that are not directed cycles. Giudici and Xia managed to show that to determine the value of the upper bound in Question 1 it suffices to look at the case where the vertex- primitive automorphism group is almost simple. My Ph.D. project investigates the upper bound on s for Sp(2n,q), G2(q), 3D4(q), 2G2(q), 2F4(q), 2B2(q), and certain types of PSU(n,q). It turns out that s is at most 2 for those groups.

Friday 24 November

The product of field elements with prescribed trace

Geertrui Van de Voorde
University of Canterbury, New Zealand

Many problems in finite geometry deal with constructions, characterisations and classifications of certain point sets within a finite projective space. While some of those can be tackled in a purely combinatorial way, many of them require tools that are more algebraic in nature. One particular question from finite geometry led us to the following problem: *Is it possible to write every element of a (finite) field as the product of two elements with prescribed trace?* The exact same problem pops up in the study of various seemingly unrelated topics, such as PN-functions and polynomials with prescribed coefficients. I will talk how these problems are interrelated and how some tools from algebraic geometry can be used to give a (partial) answer to our question. (Joint work with J. Sheekey and J.F. Voloch).

Maximizing weighted sums of binomial coefficients using generalized continued fractions

Stephen Glasby
The University of Western Australia

How does one accurately approximate the partial sum $s_m(r) := \sum_{i=0}^{r} \binom{m}{i}$ of binomial coefficients where $m, r \in \mathbb{Z}$, $0 \leqslant r \leqslant m$, and r is not close to $0, \frac{m}{2}, m$? We stumbled across an elementary method to find excellent upper and lower bounds to $s_m(r)$ using (generalized) continued fractions. However, our original motivation was different: namely to locate the maximum of a unimodal sequence $(\omega^{-r} s_m(r))_{0 \leqslant r \leqslant m}$ where $\omega \geqslant 1$ is a fixed real number. We proved that the maximum occurs at $\lfloor \frac{m+2}{\omega+1} \rfloor$ or at $\lfloor \frac{m+2}{\omega+1} \rfloor + 1$ if $\omega \geqslant \sqrt{3}$, and at $\lfloor \frac{m+2}{\omega+1} \rfloor$ if $\omega \in \{3, 4, \dots\}$. Our original motivation stemmed from coding theory, but approximating $\sum_{i=0}^{r} \binom{m}{i}$ is a spin off with many applications. This is joint work with G. R. Paseman, Shepherd Systems, UC Berkeley.

[1] S. P. Glasby and G. R. Paseman. On the maximum of the weighted binomial sum $2^{-r} \sum_{i=0}^{r} \binom{m}{i}$, *Electron. J. Combin.* **29**(2) (2022), article P2.5.

[2] S. P. Glasby and G. R. Paseman. Maximizing weighted sums of binomial coefficients using generalized continued fractions. arXiv:2310.12517

Some observations about normal quotients of symmetric graphs

Marston Conder
University of Auckland

In her abstract for a talk at an online conference in August, Cheryl Praeger mentioned a suggestion made by Caiheng Li in a 2001 paper (in the *Bulletin of the London Math. Society*) that 'non-basic' 2-arc-transitive graphs of prime-power order that occur as normal covers of smaller 2-arc-transitive graphs might be rare and difficult to construct. This seemed suspicious to me, so I set myself the challenge of finding examples, which turned out to be remarkably easy (despite initial scepticism by Cheryl). In this talk I'll briefly explain what is meant by s-arc-transitivity, normal quotients and normal covers, and then describe three infinite families of graphs that refute Caiheng's suggestion. One obvious family is that of the cycles of proper prime-power order $q > 7$, while another is the (infinite) family of all 2-arc-regular 3-valent graphs of 2-power-order that admit arc-reversing automorphisms of order 2, and the third is the family of all n-cube graphs Q_n where $n > 2$ is even. Also I'll explain how the family of n-cube graphs Q_n where $n > 3$ is odd contradicts one of the main theorems in Caiheng's 2001 paper. (Some of this is joint work with Primož Potočnik (Ljubljana)).

Lang's Theorem, bilinear forms, trilinear forms and conjugacy

Don Taylor
University of Sydney

Lang's Theorem (from 1956) is an effective tool to deduce results about finite groups from linear algebraic groups. A. M. Cohen and S. H. Murray published an efficient algorithm for Lang's Theorem for connected reductive groups over fields of characteristic greater than 3. I will report on a version of this algorithm for a group preserving a bilinear or trilinear form (without restriction on the characteristic). This work was carried out with Eamonn O'Brien as a step towards the effective computation of the conjugacy classes of exceptional groups of types F_4 and G_2.

Finite groups with geodetic Cayley graphs

Murray Elder

University of Technology Sydney

A connected graph is called *geodetic* if for every pair of vertices there is a unique shortest path connecting them. For example, an odd length cycle, a complete graph, and a tree. For finite groups, the only geodetic Cayley graphs which we know of are odd cycles (for cyclic groups of odd order with a single generator), and complete graphs (any group taking the entire group (minus the identity) as a generating set). We conjecture that this is all there is. We devised a computer search to verify our conjecture for all groups (with all possible generating sets!) of order up to 512. To make it that far without breaking the computer, we proved several results which reduce the search space, most notably: a bound on the size of a possible generating set for a geodetic Cayley graph that is not the complete graph; and a result which eliminates group with even order centre. Joint work with Adam Piggott, Florian Stober and Armin Weiß.
