Lecture 16

FINITE VOLUME METHOD: METHODOLOGY AND GRIDS

16.1 FINITE VOLUME METHOD

The finite volume method (FVM) is based on the approximate solution of the integral form of the conservation equations. The problem domain is divided into a set of non-overlapping control volumes (called *finite volumes*). The conservation equations are applied to each finite volume. The integrals occurring in the conservation equations are evaluated using function values at computational nodes (which are usually taken as centroids of finite volumes). This process involves use of approximate integral formulae and interpolation methods (to obtain the values of variables at surfaces of the CVs).

The FVM can accommodate any type of grid, and hence, it is naturally suitable for complex geometries. This explains its popularity for commercial CFD packages, which must cater to problems in arbitrarily complex geometries. This method has immensely benefited from the unstructured grid generation methods developed for finite element method.

16.2 FINITE VOLUME SOLUTION PROCESS

Finite volume method employs the integral form of the conservation equations as the starting point. The integral form can be obtained using Reynolds transport theorem, and for a generic scalar, its transport equation is given by

$$\frac{\partial}{\partial t}\int_{CV} \rho\phi d\Omega + \int_{S} \rho\phi \mathbf{v}\cdot \mathrm{d}\mathbf{A} = \int_{S} \Gamma\nabla\phi \cdot \mathrm{d}\mathbf{A} + \int_{\Omega} S_{\phi} d\Omega \qquad (16.1)$$

An alternative approach could be to derive (16.1) using the conservative form of differential scalar transport equation given by

$$\frac{\partial(\rho\phi)}{dt} + div(\rho\phi u) = div(\Gamma grad\,\phi) + S_{\phi} \qquad (16.2)$$

To obtain the integral form, integrate (16.2) over a three-dimensional control volume (CV) yielding

$$\int_{CV} \frac{\partial(\rho\phi)}{dt}\,\mathrm{d}V + \int_{CV} div(\rho\phi u)\,\mathrm{d}V = \int_{CV} div(\Gamma grad\,\phi)\,\mathrm{d}V + \int_{CV} S_{\phi}\mathrm{d}V$$

Using Gauss' divergence theorem, above equation can be written as

$$\frac{\partial}{\partial t}\left(\int_{CV} \rho\phi\,\mathrm{d}V\right) + \int_{A} n.(\rho\phi u)\,\mathrm{d}A = \int_{A} n.(\Gamma grad\,\phi)\,\mathrm{d}A + \int_{CV} S_{\phi}\,\mathrm{d}V \qquad (16.3)$$

Equations (16.1) and (16.3) are identical, and hence, either approach can be employed in finite volume formulation.

Solution domain is sub-divided into a finite number of small control volume (CVs) by a grid. The grid defines the CV boundaries, not the computational modes. The integral conservation equation (16.1) applies to each CV as well as to the solution domain as a whole. Thus, global conservation is built into the method. To obtain an algebraic equation for each CV, the surface and volume integrals are approximated using quadrature formulae. Thus, the overall finite volume solution process involves the following steps:

- Discretize the solution domain by a grid (i.e. a set of finite volumes), and define the computational nodes at which variables are to be evaluated.
- Apply the integral form of conservation law to each control volume. Approximate the surface and volume integrals using appropriate quadrature formulae in terms of the function values at computational nodes.
- Collect the algebraic equations for all the finite volumes to obtain a system of algebraic equations in terms of unknown values of the variable at computational nodes.
- Solve the resulting system of algebraic equations to obtain values of the variable at each computational node.

16.3 TYPES OF FINITE VOLUME GRIDS

There are two common approaches to finite volume discretization: (a) cell-centred and (b) face centred grids (Figure 16.1).

- **Cell-centred approach**: CVs are defined by a suitable grid (structured or unstructured) and computational nodes are assigned at the CV centre.
- **Face-centred approach**: Nodal locations are defined first, and CVs are then constructed around them so that CV faces lie midway between the nodes. It can be used only with structured grids.

In view of its versatility, the first approach is most commonly employed.

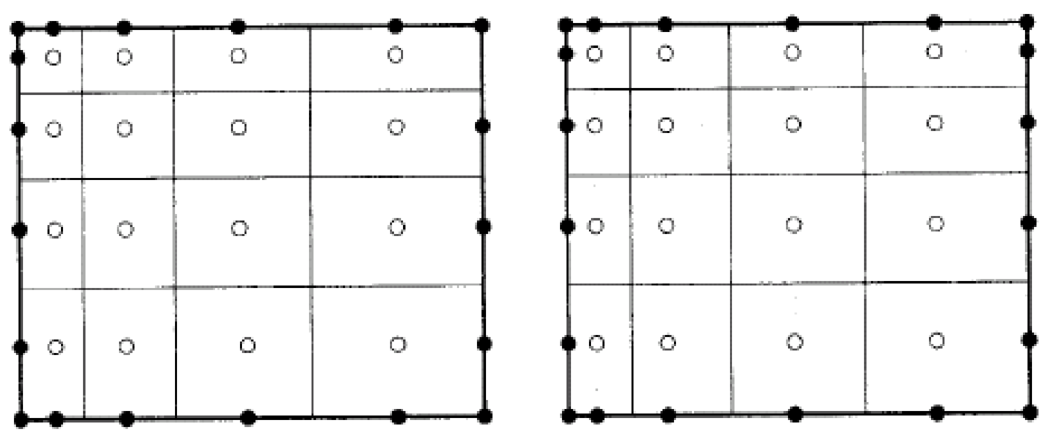

Figure 16.1 Types of FV grids: nodes centred in CVs (left) and CV faces centred between nodes (right) (Ferziger and Peric, 2003)

16.4 COMPASS NOTATION

For structured Cartesian grids, compass notation is employed for computational nodes as shown in Figures 16.2 and 16.3 for 2-D and 3-D respectively. CV surfaces can be subdivided into 4 (in 2-D) or 6 (in 3-D) plane faces denoted by lower case letters corresponding to their direction (e, w, s, n, t and b) with respects to the central node P.

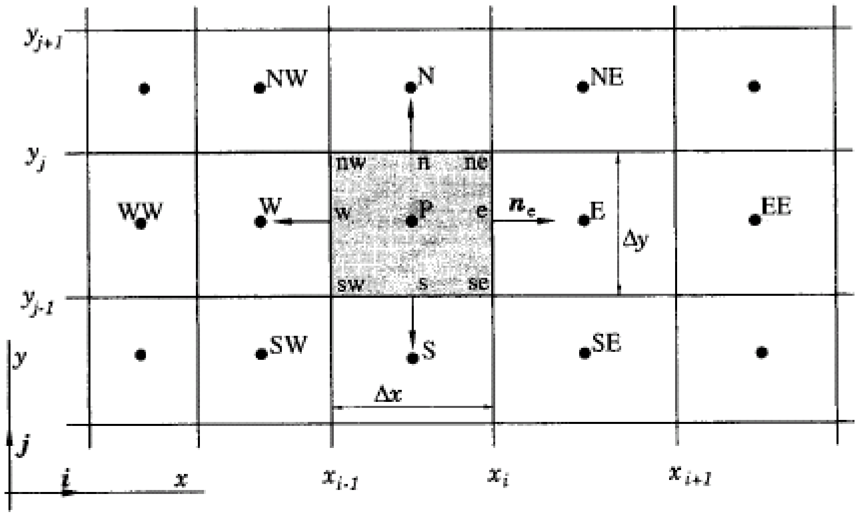

Figure 16.7 A typical CV and notation used for 2-D Cartesian finite volume grid

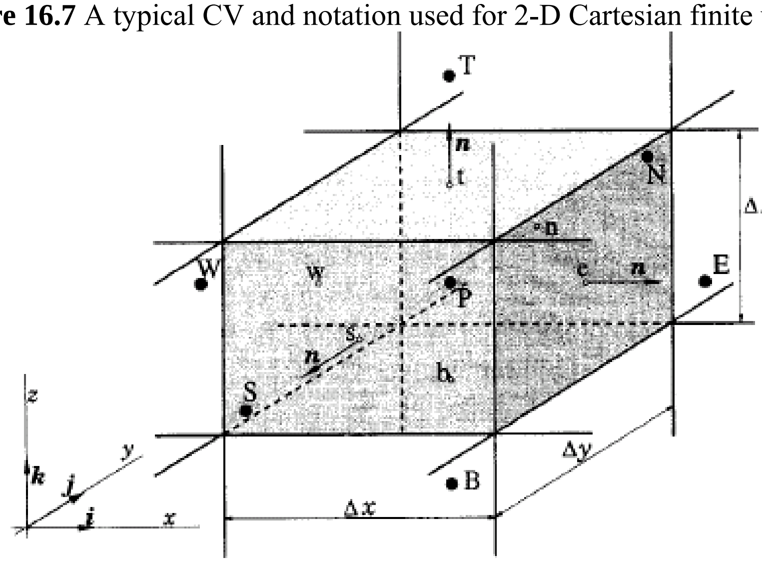

Figure 16.8 A typical CV and notation used for 3-D Cartesian finite volume grid

REFERENCES/FURTHER READING

Chung, T. J. (2010). *Computational Fluid Dynamics*. 2nd Ed., Cambridge University Press, Cambridge, UK.

Ferziger, J. H. And Perić, M. (2003). *Computational Methods for Fluid Dynamics*. Springer.

Versteeg, H. K. and Malalasekera, W. M. G. (2007). *Introduction to Computational Fluid Dynamics: The Finite Volume Method*. Second Edition (Indian Reprint) Pearson Education.