Lecture 39

TURBULENT FLOWS: LARGE EDDY SIMULATION

39.1 LARGE EDDY SIMULATION (LES)

Large scales of motion (large eddies) are generally much more energetic than small scale once. These larger eddies are the most effective transporters of conserved quantities (mass, momentum and energy). Smaller eddies are usually much weaker, and hence, have very limited role in mass, momentum and energy exchange. Furthermore, these smaller scales of motion show universal behaviour in turbulent flows irrespective of the context and geometry of the flow. Hence, it should be a lot easier to capture the effect of smaller eddies through a model. These observations lead to the basic philosophy of LES: treat the large eddies of the flow exactly, and model the more universal small scale eddies. Large eddy simulations are inherently time dependent and three-dimensional simulations. These are less costly than DNS but at lot more expensive than RANS for the same flow. LES is the preferred method for obtaining accurate time history for high Reynolds number and complex geometry flows (for such flows, DNS is still not feasible owing to its astronomical computing requirements, and RANS simulations are not very accurate).

39.2 CONCEPTUAL STEPS IN LES

LES requires computation of large scales of motion. For this purpose, it employs a spatial filtering operation to separate the larger and smaller eddies. The filtered or resolved scale field is the one which is simulated (i.e. computed numerically) using a suitable Navier-Stoke solver discussed in earlier lectures. Conceptual steps involved in LES can be summarized as follows:

- Choose an appropriate spatial filter to decompose velocity field $v_i\left(\mathbf{x},t\right)$ into the sum of a filtered or resolved component $\overline{v}_i\left(\mathbf{x},t\right)$ and a residual (or subgrid scale SGS) component $v_i'\left(\mathbf{x},t\right)$. $\overline{v}_i\left(\mathbf{x},t\right)$ represents the motion of large eddies.

- Obtain filtered unsteady Navier-Stokes equation. These equations are very similar to original Navier-Stokes equation except for an additional residual stress (subgrid) term arising from filtering.

- Model the subgrid term using a subgrid model.

- Obtain the filtered velocity, pressure and temperature field using an appropriate Navier-Stokes solver.

39.3 SPATIAL FILTERING OF NAVIER-STOKES EQUATIONS

In large eddy simulation, the spatial filtering operation for any transported field is defined by a using filter function as follows:

$$\bar{\phi}(\mathbf{x},t) = \int G(\mathbf{x},\mathbf{x}',\Delta)\ \phi(\mathbf{x}',t)\,\mathbf{dx}' \qquad (39.1)$$

where $G(\mathbf{x},\mathbf{x}',\Delta)$ is the filter kernel with a cut-off width Δ. In LES, different type of filter kernels have been used, which include a Gaussian filter, a top-hat or box filter, and a spectral cut-off filter (for spectral simulations). In this section, an over-bar indicates spatial filtering (not the time averaging used in RANS). Filtering operation roughly implies that eddies of size larger than Δ are *large eddies* (represented by resolved or filtered field) while eddies of size smaller Δ are *small eddies* which must be modelled. The cut-off width Δ can, in principle, have any size. However, a value smaller than typical mesh-size in FEM/FVM/FDM simulations is meaningless, and most common value of Δ for a structured mesh is chosen as

$$\Delta = \sqrt[3]{\Delta_x \Delta_y \Delta_z} \qquad (39.2)$$

where Δ_x, Δ_y, Δ_z represent length, width and height respectively of a typical hexahedral element.

Using definition (39.1), the filtered or resolved velocity is given by

$$\bar{v}_i(\mathbf{x},t) = \int G(\mathbf{x},\mathbf{x}',\Delta)\ v_i(\mathbf{x}',t)\,\mathbf{dx}' \qquad (39.3)$$

Application of the filtering operation to the continuity equation yields the LES continuity equation for incompressible flow given by

$$\frac{\partial(\rho\bar{v}_i)}{\partial x_i} = 0 \qquad (39.4)$$

Further, filtered momentum equation takes the form

$$\frac{\partial(\rho\bar{v}_i)}{\partial x_i} + \frac{\partial\left(\rho\overline{v_i v_j}\right)}{\partial x_j} = -\frac{\partial\bar{p}}{\partial x_i} + \frac{\partial\bar{\tau}_{ij}}{\partial x_j} \qquad (39.5)$$

where $\bar{\tau}_{ij} = \mu\left(\frac{\partial\bar{v}_i}{\partial x_j} + \frac{\partial\bar{v}_j}{\partial x_i}\right)$. The convective term in the preceding equation cannot be computed in terms of the resolved velocity field. To get an approximation for this term, let us introduce the so-called sub-grid stress (or sub-grid scale Reynolds stress) tensor defined as

$$\tau_{ij}^S = -\rho\left(\overline{v_i v_j} - \bar{v}_i\bar{v}_j\right) \qquad (39.6)$$

and re-write Eq. (39.5) as

$$\frac{\partial(\rho\bar{v}_i)}{\partial x_i} + \frac{\partial\left(\rho\bar{v}_i\bar{v}_j\right)}{\partial x_j} = -\frac{\partial\bar{p}}{\partial x_i} + \frac{\partial\left(\bar{\tau}_{ij} + \tau_{ij}^S\right)}{\partial x_j} \qquad (39.7)$$

SGS Reynolds stress τ_{ij}^S represents large scale momentum flux caused by small or unresolved scales, and it must be modelled to ensure closure.

39.4 SUBGRID SCALE MODELS

Many subgrid scale models have been proposed in the literature. Some of the most popular models are Smagorinsky model (Smagorinsky, 1963), scale similarity model (Bardina et al. 1980), and dynamic SGS model (Germano et al. 1991). We provide a brief summary of the first two models in this lecture. For further details, see Versteeg and Malalasekera (2007) and Lesieur (2008) and references therein.

Smagorinsky Model

Smagorinsky model is an eddy viscosity model in which the local subgrid stress τ_{ij}^{S} is taken to be proportional to the local rate of strain of the resolved flow, i.e.

$$\tau_{ij}^{s} = 2\mu_t \overline{S}_{ij} + \frac{1}{3}\tau_{kk}^{s}\delta_{ij} \tag{39.8}$$

where μ_t is the SGS eddy viscosity and $\overline{S}_{ij}$ is the filtered (or resolved) strain rate given by

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{v}_i}{\partial x_j} + \frac{\partial \overline{v}_j}{\partial x_i}\right) \tag{39.9}$$

The SGS eddy viscosity is assumed to be proportional to the sub-grid length scale Δ and a characteristic turbulent velocity $v_t = \Delta\left|\overline{S}\right|$ (where $\left|\overline{S}\right| = \sqrt{\overline{S}_{ij}\overline{S}_{ij}}$). Thus, it can be expressed as

$$\mu_t = \rho C_s^2 \Delta^2 \left|\overline{S}\right| \tag{39.10}$$

where C_s is a model parameter. Value of C_s is not a universal constant; it depends on the type of flow. $C_s \approx 0.2$ for isotropic turbulence. For channel flows, a lower value of $C_s \approx 0.06$ is usually recommended. For regions close to the wall, this value is reduced even further using van Driest damping given by

$$C_S \approx C_{S0}\left(1 - e^{-n^+/A^+}\right)^2 \tag{39.11}$$

where n^+ is distance from wall in viscous units (i.e. $n^+ = nu_\tau/\nu$ in which $u_\tau = \sqrt{\tau_w/\rho}$ is the shear velocity, τ_w being the shear stress at the wall), and A^+ is a constant usually taken to be approximately 25.

Scale Similarity Model

Bardina et al. (1980) argue that important interactions between the resolved and unresolved scales involve the smallest resolved eddies and largest eddies of the unresolved scales. Thus, there exists a similarity between the smallest resolved scales and still smaller unresolved scales, and this leads to the scale similarity model given by

$$\tau_{ij}^{S} = -\rho\left(\overline{\overline{v}_i\overline{v}_j} - \overline{\overline{v}}_i\overline{\overline{v}}_j\right) \tag{39.12}$$

where the double overbar indicates a quantity that has been filtered twice. This model correlates very well with actual SGS Reynolds stress, but hardly dissipates any energy. Hence, to stabilize the computations, a damping term in the form of Smagorinsky model is added leading to the mixed model:

$$\tau_{ij}^{S} = -\rho\left(\overline{\overline{v}_i\overline{v}_j} - \overline{\overline{v}}_i\overline{\overline{v}}_j\right) + 2\rho C_s^2\Delta^2\left|\overline{S}\right|\overline{S}_{ij} \tag{39.13}$$

REFERENCES/FURTHER READING

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