Lecture 31

GRID GENERATION TECHNIQUES

31.1 GENERATION OF STRUCTURED GRIDS

Structured grid generation techniques map a block with curved boundary surfaces into a rectangular block. Thus, the grid generation process essentially involves computation of physical coordinates of grid points which correspond to Cartesian grid points in the transformed rectangular domain.

Structured grid generation techniques can be broadly classified into the following categories:

1. Algebraic techniques
2. Conformal mapping
3. PDE methods

Of these, conformal mapping is restricted to two dimensional geometries. Other two can be used for 2-D or 3-D grid generation. Algebraic techniques are usually fast and simple to program, but may not provide a very good quality mesh. Thus, these are often used to provide initial guess to mesh generation methods based on PDEs.

Algebraic Methods

Algebraic methods generate geometric data of the Cartesian coordinates in the interior of a domain from the values specified at boundaries through interpolations or specific functions of the curvilinear coordinates. Of these, transfinite interpolation method is the most popular technique.

Partial Differential Equation (PDEs) Methods

These methods solve partial differential equations in which the dependent and independent variables are the physical domain coordinate and transformed computational coordinates, respectively. These PDEs may be of elliptic, hyperbolic or parabolic form. These techniques provide a good control over grid smoothness, orthogonality and grid spacing.

Elliptic grid generation methods employ either a Poisson or a Laplace equation, and are used for bounded domains. Hyperbolic grid generation methods are employed to generate orthogonal grids on unbounded domains.

For full details of these methods, see Chung (2010), Muralidhar and Sundarajan (2003) or Thompson et al. (1999).

31.2 GENERATION OF UN-STRUCTURED GRIDS

Generation of unstructured grids involves a systematic subdivision of the problem domain (or block) into cells of desired size and shape (triangle/quadrilateral in 2-D or tetrahedron/hexahedron in 3-D). Many of these methods employ decomposition of the geometry in triangular elements, and hence, these techniques are also referred as *triangulation techniques*. Some of the most-popular methods are:

- **Delaunay-Voronoi Methods**: These methods systematically decompose the problem geometry in a set of packed convex polygons/polyhedra based on Delauny tessellation/Vornoi polygons**.**

- **Advancing Front Method:** This method starts with a boundary grid of edges (in two dimensions) or faces (in three dimensions) which is referred as a front. New points are created at a certain distance from the set of grid-points on this front to construct an element. This process of point creation is repeated till a new layer of element based on the current front is formed. The edge/face formed by newly created nodes is taken as the new front and the process is repeated till the entire domain has been decomposed into desired elements.

- **Quad-tree (2D) / Octree Methods (3D):** These methods decompose the domain in a regular partition of quadrilaterals/cubes and construct the mesh based on this regular decomposition.

For full details of these unstructured grid generation algorithms, see the references cited below.

REFERENCES/FURTHER READING

Chung, T. J. (2010). *Computational Fluid Dynamics*. 2nd Ed., Cambridge University Press.

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