Lecture 3

APPROXIMATE SOLUTION TECHNIQUES

3.1 INTRODUCTION

Numerous approximate solution techniques have been developed for different types of problems in CFD. These methods can be classified into two categories:

- Mesh-based methods which require discretization of the problem domain into a mesh (or grid), e.g. finite difference, finite element, and finite volume methods.
- Mesh-free methods which primarily use a collection of nodes with no apparent connectivity, e.g. smooth particle hydrodynamics (SPH), mesh-less Petrov-Galerkin (MLPG), lattice Boltzmann methods.

Of the preceding two types, mesh-based methods are more popular in CFD. Of these, finite volume method has been the most popular due to its simplicity and ease of application for problems in complex geometries. In fact, majority of commercial CFD packages (e.g. Fluent, StarCD, etc.) are based on finite volume method. In this lecture, we will have a brief overview of finite difference, finite element and finite volume methods.

3.2 FINITE DIFFERENCE METHOD (FDM)

The FDM is the oldest method for numerical solution of partial differential equations. This method is also the easiest method to formulate and program for problems on simple geometries. In FDM, the solution domain is discretized using a structured (usually Cartesian) grid. The conservation equations in differential form are approximated at each grid point by replacing the partial derivatives by finite difference approximations in terms of nodal values of the unknown variables. This process results in an algebraic equation for each node. These algebraic equations are collected for all the grid points and resulting system of discrete equations are solved to yield the approximate solution of the problem at the grid nodes.

The main disadvantage of the finite difference method is its restriction to simple geometries (although immersed boundary techniques do remove this restriction). We provide a detailed description of this method in the following section.

3.3 FINITE ELEMENT METHOD (FEM)

The finite element method is based on the division of the problem domain into a set of finite elements which are generally unstructured. The elements are usually triangles or quadrilaterals in two dimensions, and tetrahedra or hexahedra in three dimensions. Starting point of the method is conservation equation in differential form. The unknown variable is approximated using an interpolation procedure in terms of nodal values and a set of known functions (called shape functions). This approximation is substituted into the differential equation. The resulting residual (error) is minimized in an average sense using a weighted residual procedure. The weighted integral statement leads to a system of discrete equations in terms of unknown nodal values, which is solved to obtain the solution of the problem.

FEM is ideally suited to problems on complex geometries, and hence, this method has been very popular in computational solid mechanics. There is an extensive literature available on all aspects of this method: type of elements, shape functions, mesh generation, applications to different type of problems, etc. For detailed study of FEM, interested reader can refer to books by Zienkiewicz et al. (2005a, 2005b), Reddy (2005), Reddy and Gartling (2010) amongst others.

3.4 FINITE VOLUME METHOD (FVM)

The finite volume method is based on the integral form of conservation equations. The problem domain is divided into a set of non-overlapping control volumes (called finite volumes). The conservation equations are applied to each finite volume. The integrals occurring in the conservation equations are evaluated using function values at computational nodes (which are usually taken as centroids of finite volumes). This process involves use of approximate integral formulae and interpolation methods (to obtain the values of variables at surfaces of the CVs).

The FVM can accommodate any type of grid, and hence, it is naturally suitable for complex geometries. This explains its popularity for commercial CFD packages, which must cater to problems in arbitrarily complex geometries. This method has immensely benefited from the unstructured grid generation methods developed for the finite element method.

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