Lecture 37

TURBULENT FLOWS: REYNOLDS AVERAGING

37.1 REYNOLDS DECOMPOSITION

Turbulent flows involve randomly fluctuating flow variables (velocity, pressure, temperature, etc.). Osborne Reynolds suggested that a flow variable at a given spatial point at a given instant can be represented as the sum of a mean value and a random fluctuation about this mean value. Such decomposition is referred to as *Reynolds decomposition* and the process of obtaining the average (or mean) value is referred to as *Reynolds averaging*. Thus, for any flow variable ϕ, its spatio-temporal variation can be expressed as

$$\phi(x_i,t)=\bar{\phi}(x_i)+\phi'(x,t) \tag{37.1}$$

where $\bar{\phi}$ is the mean value and ϕ' represents the fluctuating component. For statistically steady turbulent flows, $\bar{\phi}$ is the time average defined as

$$\bar{\phi}(x_i)=\lim_{T\to\infty}\frac{1}{T}\int_0^T\phi(x_i,t) \tag{37.2}$$

where T represents the averaging interval which must be large compared to the typical time scales of fluctuations. For unsteady flows, $\bar{\phi}$ represents ensemble averaging defined as

$$\bar{\phi}(x_i,t)=\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\phi_n(x_i,t) \tag{37.3}$$

where N is the number of identical experiments. The Reynolds decomposition (37.1) and averaging procedure (37.2) or (37.3) represent linear operators. Hence, the following properties hold for algebra of averages and fluctuations of any two flow variables ϕ and ψ:

$$\begin{aligned}
&\overline{\phi'}=0=\overline{\psi'}, \qquad \overline{\left(\bar{\phi}\right)}=\bar{\phi}, \qquad \overline{\frac{\partial\phi}{\partial s}}=\frac{\partial\bar{\phi}}{\partial s}, \qquad \overline{\int\phi\,\mathrm{d}s}=\int\bar{\phi}\,\mathrm{d}s\\
&\overline{\phi+\psi}=\bar{\phi}+\bar{\psi}, \qquad \overline{\phi\psi}=\bar{\phi}\bar{\psi}+\overline{\phi'\psi'}, \qquad \overline{\phi\bar{\psi}}=\bar{\phi}\bar{\psi}, \qquad \overline{\phi'\bar{\psi}}=0
\end{aligned} \tag{37.4}$$

37.2 REYNOLDS AVERAGED NAVIER-STOKES EQUATIONS

We can apply Reynolds averaging to the continuity and momentum equations for an incompressible flow and use properties (37.4) to obtain the following set of equations (usually called Reynolds averaged Navier-Stokes equations):

$$\text{Continuity:} \quad \frac{\partial(\rho\bar{v}_i)}{\partial x_i}=0 \tag{37.5}$$

$$\text{Momentum:} \quad \frac{\partial(\rho\bar{v}_i)}{\partial x_i}+\frac{\partial(\rho\bar{v}_i\bar{v}_j)}{\partial x_j}=-\frac{\partial\bar{p}}{\partial x_i}+\frac{\partial\left(\bar{\tau}_{ij}-\tau_{ij}^R\right)}{\partial x_j} \tag{37.6}$$

where $\tau_{ij}^R=-\rho\overline{v_i'v_j'}$ is called the Reynolds stress tensor. Similarly, Reynolds averaged transport equation for a scalar ϕ can be obtained as

Scalar transport: $$\frac{\partial\left(\rho\bar{\phi}\right)}{\partial t}+\frac{\partial\left(\rho\bar{v}_j\bar{\phi}\right)}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\Gamma\frac{\partial\left(\rho\bar{\phi}\right)}{\partial x_j}+q_j^R\right) \qquad (37.7)$$

where $q_R^j=-\rho\overline{v_j'\phi'}$ is called the turbulent flux. Presence of Reynolds stress and turbulent flux terms in conservation equations requires their modelling in term of averaged quantities to ensure closure.

FURTHER READING

Ferziger, J. H. And Perić, M. (2003). *Computational Methods for Fluid Dynamics*. Springer.

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