Lecture 18
FINITE VOLUME INTERPOLATION SCHEMES

18.1 INTERPOLATION SCHEMES

The approximation of surface and volume integrals may require values of the variable at locations other than the computational nodes of the CV. Values at these locations are obtained using interpolation formulae. There are various possibilities, and we mention only a few of these in this lecture. For further details on various interpolation schemes, especially the total variation diminishing (TVD) schemes used for convection dominated problems, see Versteeg and Malalasekera (2007) and Chung (2010).

18.2 UPWIND INTERPOLATION

The upwind interpolation (UDS) for approximating the value of a variable at the east face of a control volume is given by

$$f_\mathrm{e} = \begin{cases} f_\mathrm{P} & \text{if } (\mathbf{v}\cdot\mathbf{n})_e > 0 \\ f_\mathrm{E} & \text{if } (\mathbf{v}\cdot\mathbf{n})_e < 0 \end{cases} \tag{18.1}$$

This interpolation scheme is equivalent to using a backward or forward finite difference approximation (depending on the flow direction). It is first order accurate, and is numerically diffusive with a coefficient of numerical diffusion $\Gamma_e^{\mathrm{num}} = (\rho u)_e \Delta x / 2$.

18.3 LINEAR INTERPOLATION (CDS)

We can approximate the value of the variable at CV face centre by linear interpolation of the values at two nearest computational nodes. Thus, at location 'e' on a Cartesian grid, the variable value is approximated by

$$f_\mathrm{e} = f_\mathrm{E}\lambda_\mathrm{e} + f_\mathrm{P}(1-\lambda_\mathrm{e}), \qquad \lambda_\mathrm{e} = \frac{x_\mathrm{e} - x_\mathrm{P}}{x_\mathrm{E} - x_\mathrm{P}} \tag{18.2}$$

The linear interpolation is equivalent to the use of central difference formula of the first order derivative, and hence, this scheme is also termed as central difference scheme (CDS). This scheme is second order accurate, and may produce oscillatory solutions (Ferziger and Peric, 2003).

18.4 QUADRATIC UPWIND INTERPOLATION (QUICK)

A quadratic upwind interpolation (QUICK) scheme can be derived using polynomial fitting. The QUICK interpolation on a uniform Cartesian grid is given by

$$\phi_\mathrm{e} = \frac{6}{8}\phi_\mathrm{U} + \frac{3}{8}\phi_\mathrm{D} - \frac{1}{8}\phi_\mathrm{UU} \tag{18.3}$$

where D, U and UU denote the downstream, the first upstream and the second upstream node respectively (E, P, and W or P, E and EE depending on the flow direction). QUICK scheme is third order accurate, but prone to oscillations.

REFERENCES/FURTHER READING

Chung, T. J. (2010). *Computational Fluid Dynamics*. 2nd Ed., Cambridge University Press, Cambridge, UK.

Ferziger, J. H. And Perić, M. (2003). *Computational Methods for Fluid Dynamics*. Springer.

Versteeg, H. K. and Malalasekera, W. M. G. (2007). *Introduction to Computational Fluid Dynamics: The Finite Volume Method*. Second Edition (Indian Reprint) Pearson Education.