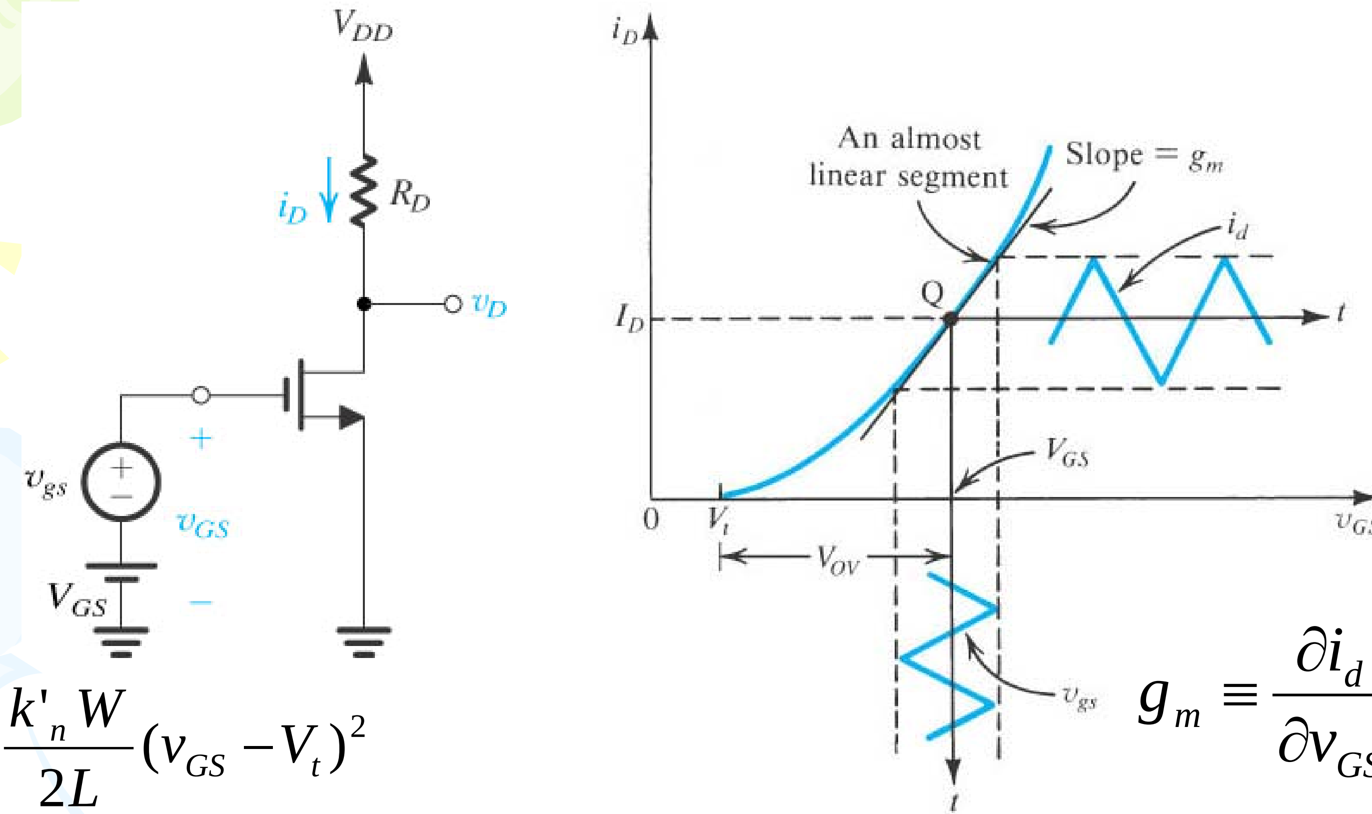
# Lecture 09

# MOSFET Circuits

# Topic

- Small signal equivalent circuit model
  - $\pi$ model
  - T model
  - Body effect
- Signal stage MOSFET Amplifier
  - CS Amplifier
  - CD Amplifier
  - CG Amplifier

# Small-signal equivalent-circuit model




$$g_m \equiv \left.\frac{\partial i_d}{\partial v_{GS}}\right|_{v_{GS}=V_{GS}}$$

$$i_D = \frac{k'_n W}{2L}(v_{GS} - V_t)^2$$

$$i_D = \frac{k'_n W}{2L}(V_{GS} + v_{gs} - V_t)^2$$

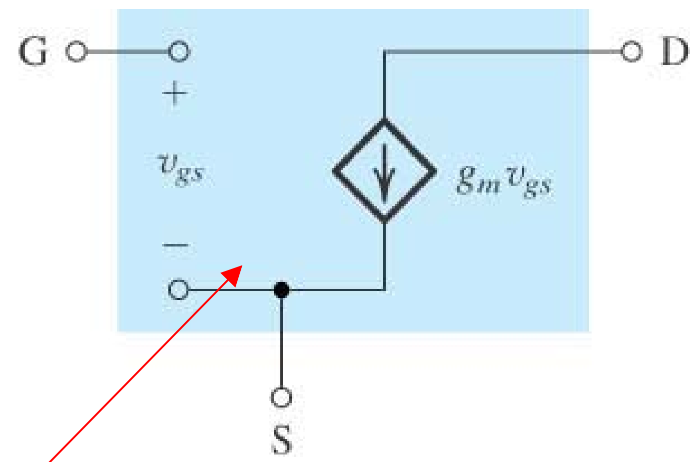
$$i_d \quad if \quad v_{gs} << 2(V_{GS} - V_t)$$

$$i_D = \boxed{\frac{k'_n W}{2L}(V_{GS} - V_t)^2} + \boxed{\frac{k'_n W}{L}(V_{GS} - V_t)v_{gs}} + \frac{k'_n W}{2L}v_{gs}^{\ 2}$$

$$I_D$$

$$i_d = \frac{k'_n W}{L}(V_{GS} - V_t)v_{gs}$$

$$\Rightarrow \boxed{g_m \equiv \frac{i_d}{v_{gs}} = \frac{k'_n W}{L}(V_{GS} - V_t)}$$




(a)

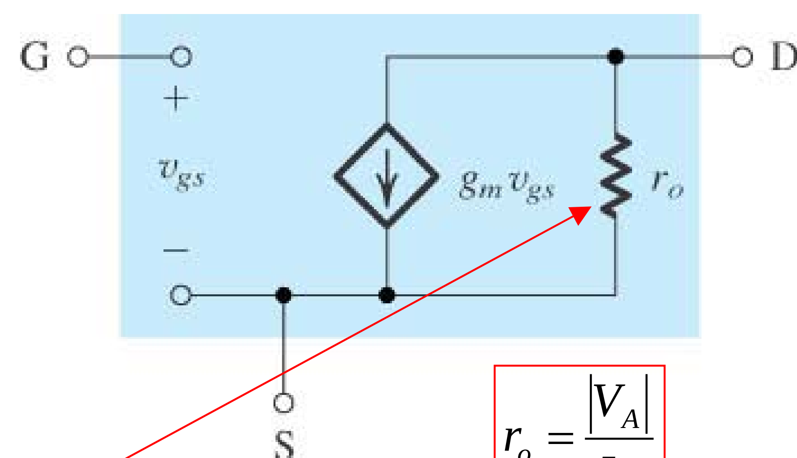
$$v_D = V_{DD} - R_D i_D$$

$$v_D = V_{DD} - R_D (I_D + i_d)$$

$$v_D = V_{DD} - R_D I_D - R_D i_d = V_D - R_D i_d$$

$$v_d = -R_D i_d = -R_D g_m v_{gs}$$

$$A_v \equiv \frac{v_d}{v_{gs}} = -g_m R_D$$



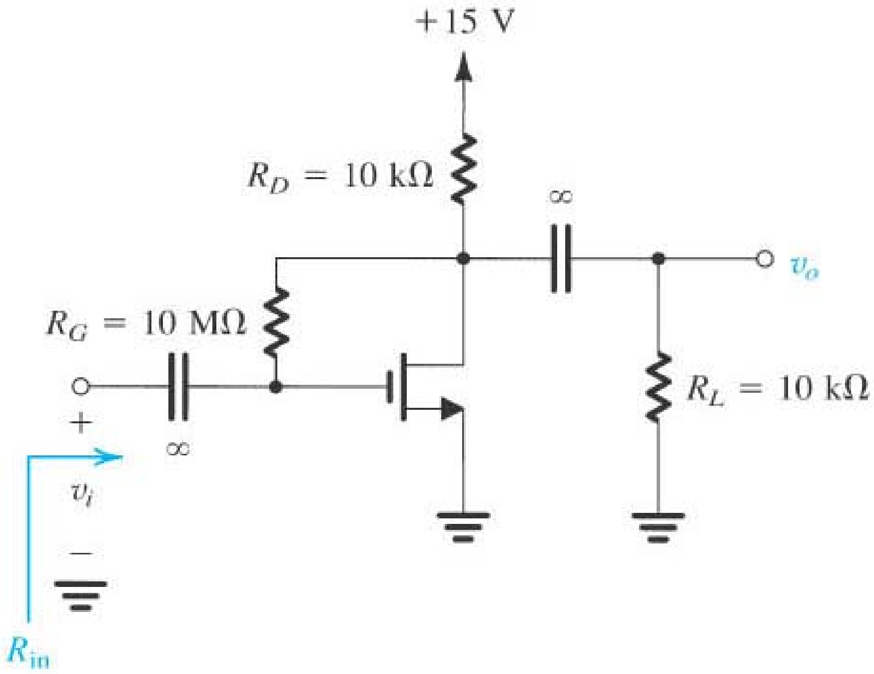

$$\boxed{r_o = \frac{|V_A|}{I_D}}$$

(b)

$$A_v \equiv \frac{v_d}{v_{gs}} = -g_m (R_D // r_o)$$

For the above circuit

Example 4.10



(a)

## 1. DC analysis

$$I_D = \frac{k'_n W}{2L}(V_{GS} - V_t)^2$$

$$= \tfrac{1}{2} \times 0.25m(V_{GS} - 1.5)^2$$

$$V_{GS} = V_D$$

$$V_D = V_{DD} - I_D R_D = 15 - I_D \times 10k$$

$$\Rightarrow \begin{cases} I_D = 1.06mA \\ V_D = 4.4V \end{cases}$$

$$V_t = 1.5V$$

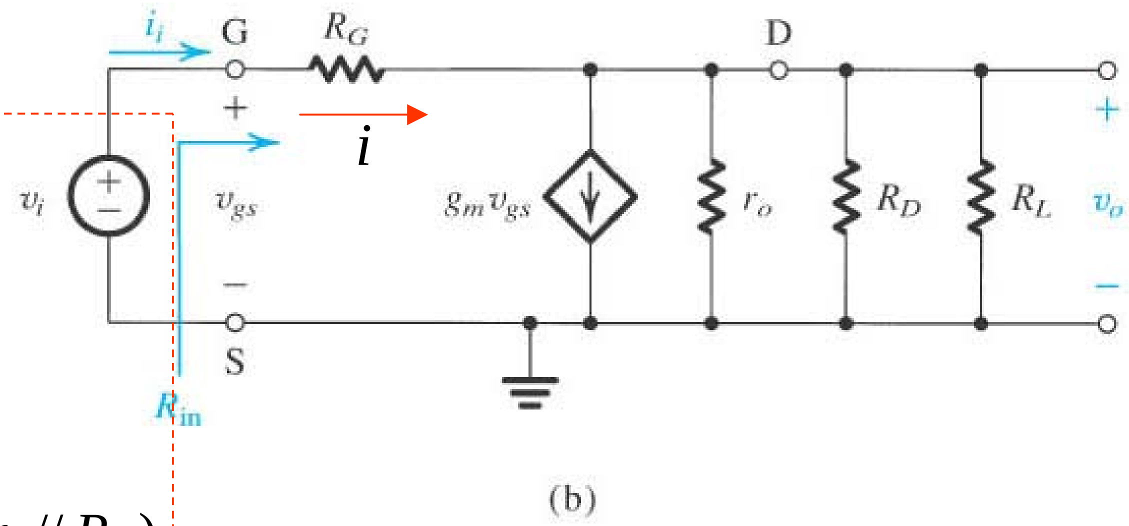
$$k_n'(W/L) = 0.25mA/V^2$$

$$V_A = 50V$$

$$g_m = \frac{k'_n W}{L}(V_{GS} - V_t) = 0.725mA/V$$

$$r_o = \frac{|V_A|}{I_D} = 47k$$

## 2. Small-signal analysis




$$v_o = (i - g_m v_{gs})(r_o // R_D)$$

$$v_i = v_{gs}$$

$$v_{gs} = v_o + iR_G \Rightarrow i = \frac{v_{gs} - v_o}{R_G}$$

$$A_v = \frac{v_o}{v_i}\bigg|_{R_L \to \infty} = \frac{\frac{r_o // R_D}{R_G} - g_m (r_o // R_D)}{(1 + \frac{r_o // R_D}{R_G})}$$

Best voltage gain

$$R_o\big|_{v_i=0} = R_G // r_o // R_D (\because v_{gs} = 0 \therefore g_m v_{gs} = 0)$$

$$i_o\big|_{R_L=0} = \frac{v_{gs}}{R_G} - g_m v_{gs}$$

$$i_i = \frac{v_{gs}}{R_G}$$

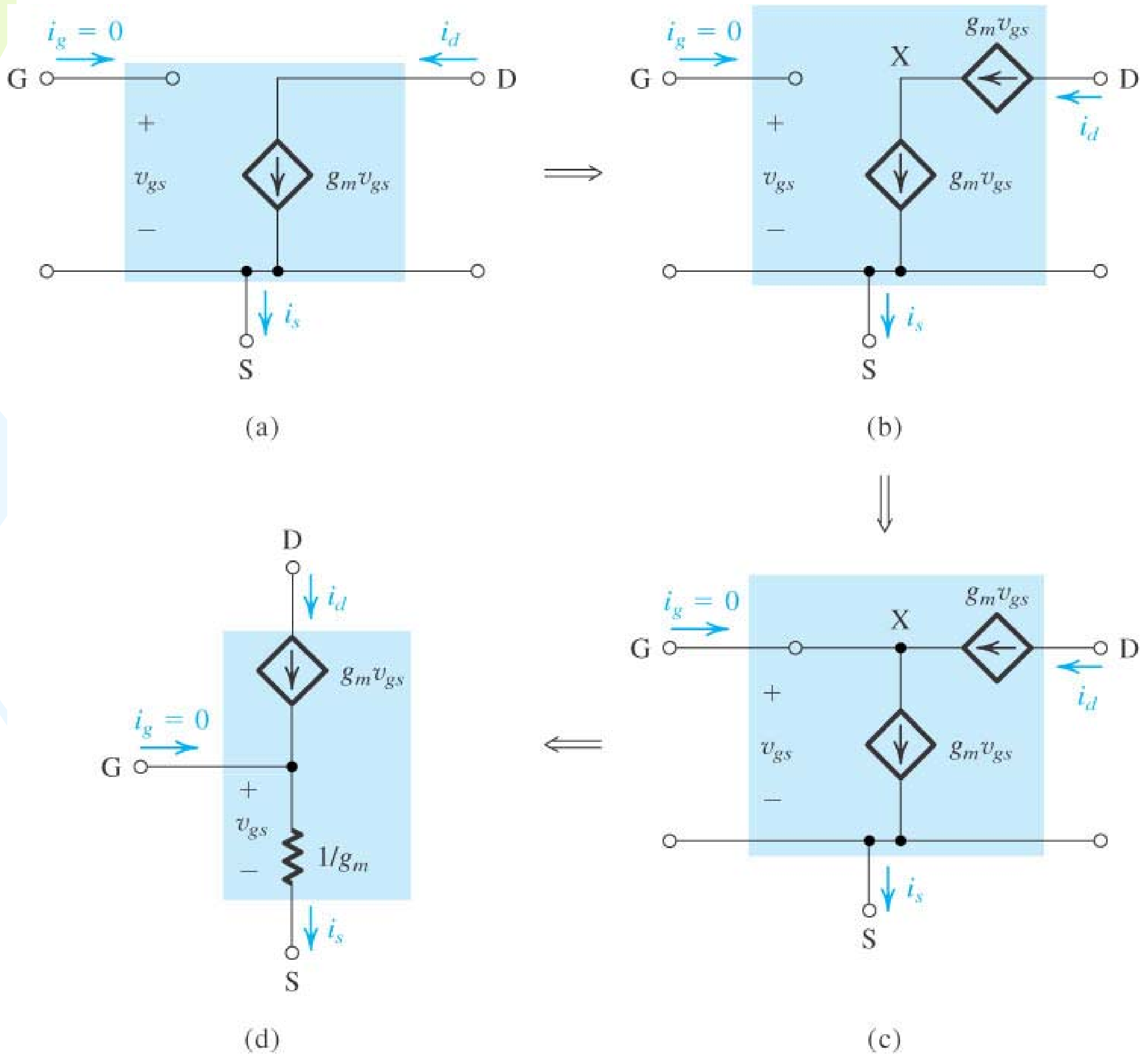
$$A_{is} = \frac{i_o}{i_s}\bigg|_{R_L=0} = 1 - g_m R_G$$

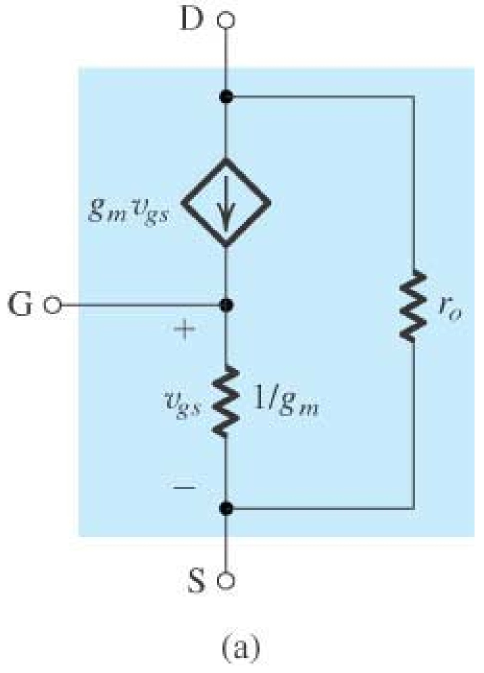
Best current gain

$$V = IR_G + v_o = IR_G + (I - g_m V)(r_o // R_D)$$

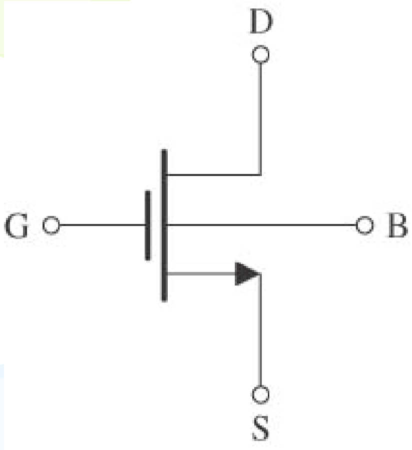
$$R_{in} = \frac{R_G + (r_o // R_D)}{1 + g_m (r_o // R_D)}$$

# T-model

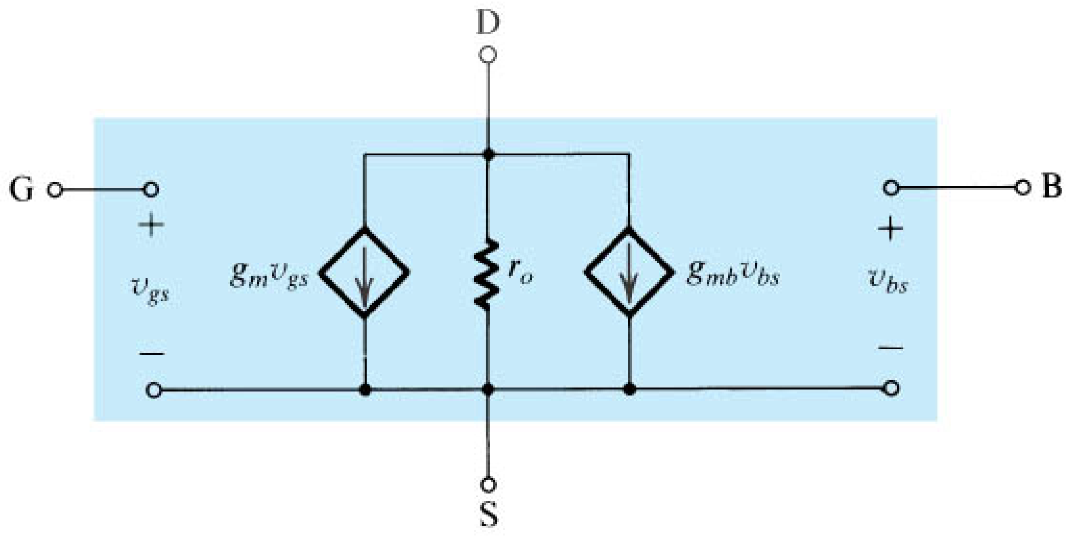



D
$g_m v_{gs}$
G
+
$v_{gs}$
$1/g_m$
−
$r_o$
S
(a)
D
$1 \times i$
G
$i$
$1/g_m$
$r_o$
S
(b)

# Modeling body effect



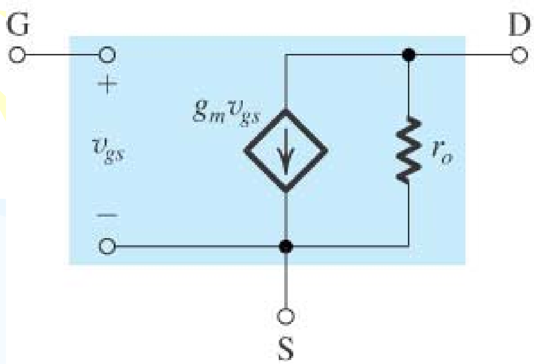

(a)

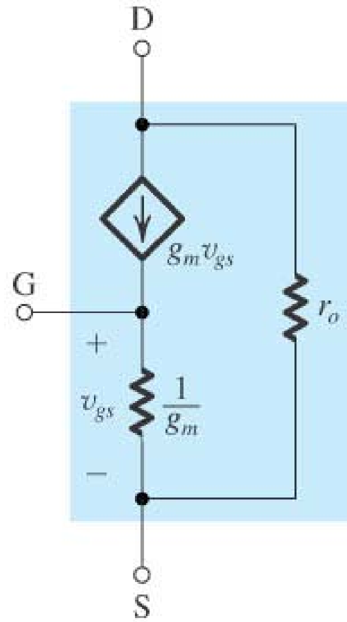


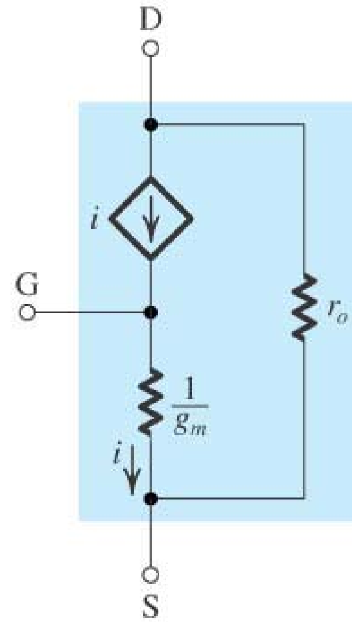

(b)

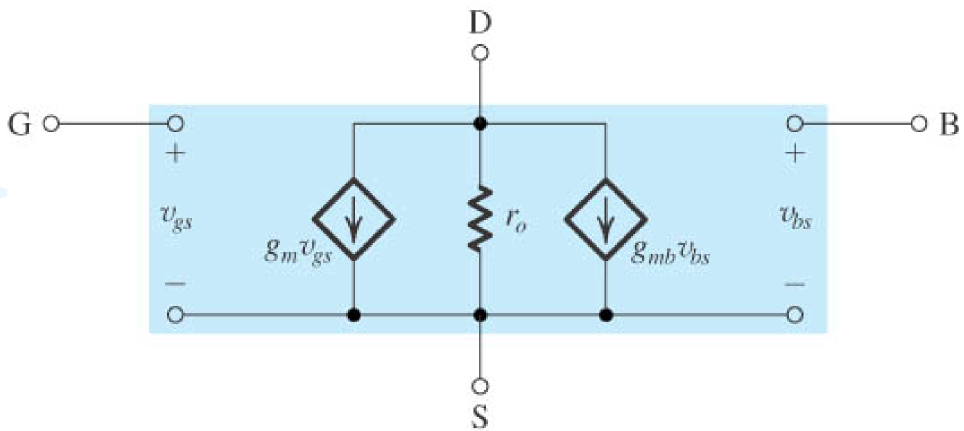
$$v_{gs} > 0, \quad and \quad v_{BS} > 0 \Rightarrow V_t \uparrow \Rightarrow i_D \downarrow$$

$$v_{gs} < 0, \quad and \quad v_{BS} < 0 \Rightarrow V_t \downarrow \Rightarrow i_D \uparrow$$

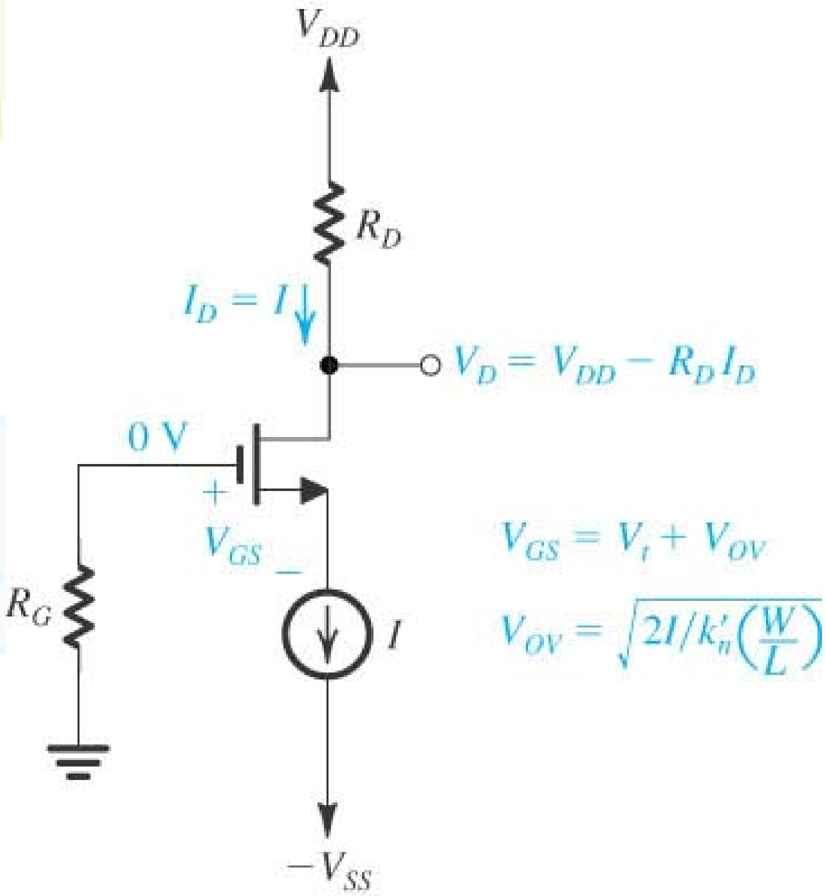

G
D
+
$v_{gs}$
−
$g_m v_{gs}$
$r_o$
S


D
G
$g_m v_{gs}$
$r_o$
+
$v_{gs}$
$\frac{1}{g_m}$
−
S


D
G
$i$
$r_o$
$\frac{1}{g_m}$
$i$
S


D
G
B
+
$v_{gs}$
−
$g_m v_{gs}$
$r_o$
$g_{mb} v_{bs}$
+
$v_{bs}$
−
S

# Fixed current Bias



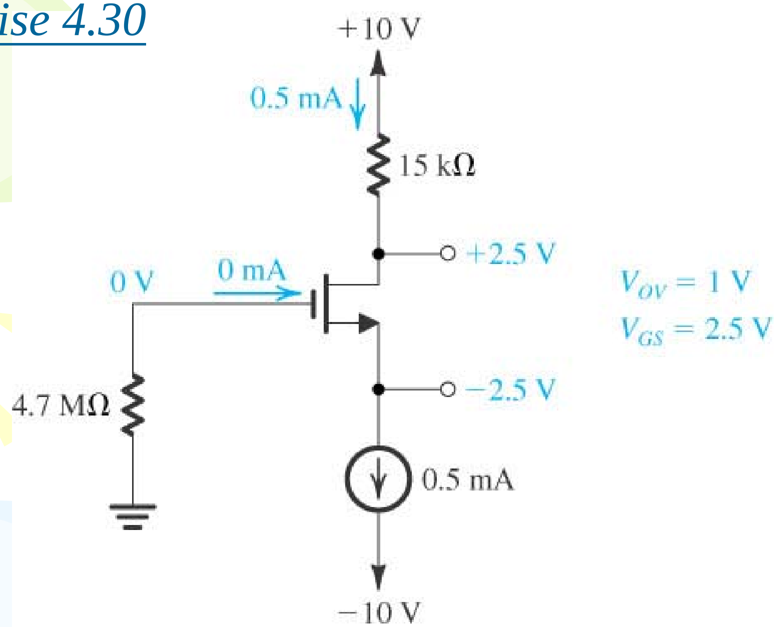

$$g_m = \frac{k'_n W}{L}(V_{GS} - V_t)$$

$$r_o = \frac{|V_A|}{I_D}$$

$$I = I_D = \frac{k'_n W}{2L}(V_{GS} - V_t)^2$$

$$V_{GS} - V_t = \sqrt{\frac{2L \times I}{k'_n W}}$$

## Exercise 4.30



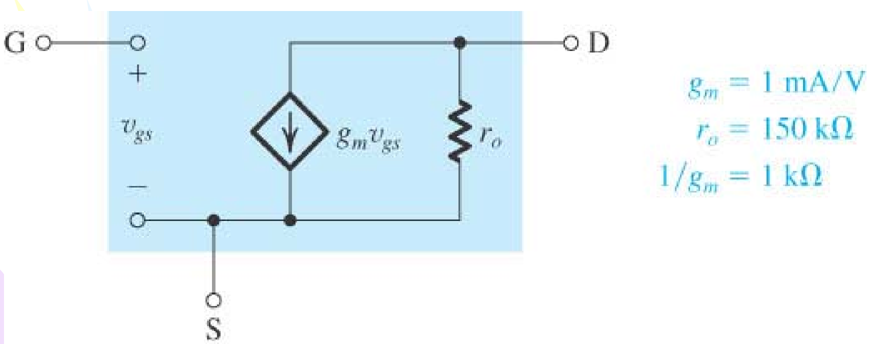

(a)

$$I = I_D = \frac{k'_n W}{2L}(V_{GS} - V_t)^2$$

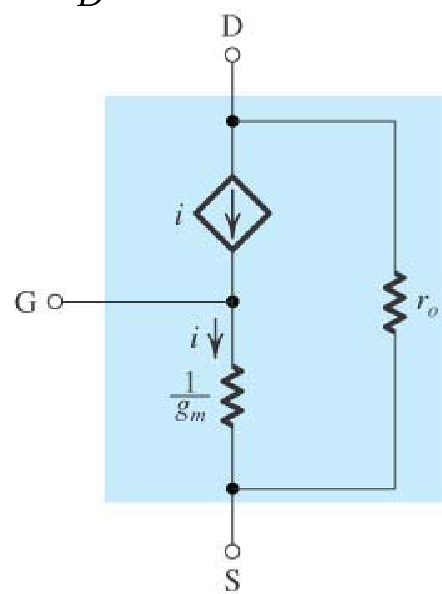
$$V_{GS} - V_t = \sqrt{\frac{2L \times I}{k'_n W}} = \sqrt{2 \times 1 \times 0.5} = 1$$

$$g_m = \frac{k'_n W}{L}(V_{GS} - V_t) = 1 \times 1 = 1$$

$$r_o = \frac{|V_A|}{I_D} = 75 \big/ 0.5m = 150k$$



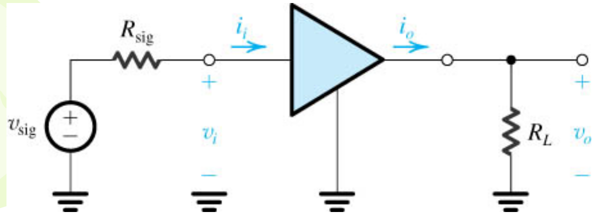

(b)



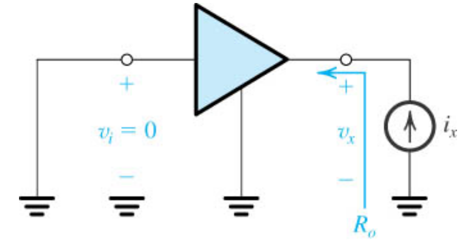

(c)

# Single stage MOS amplifiers

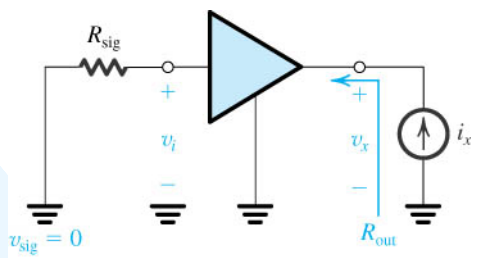
- Common source Amplifier
- Common source Amplifier + Source resistance
- Common Drain Amplifier
- Common Gate Amplifier



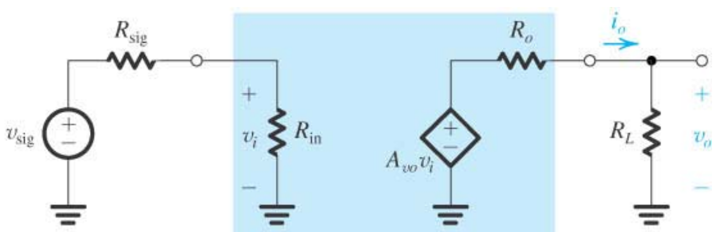
$R_{sig}$
$i_i$
$i_o$
$v_{sig}$
$v_i$
$R_L$
$v_o$

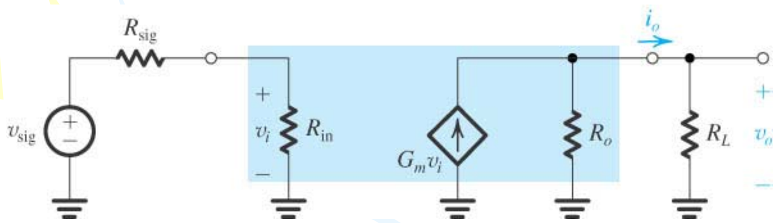
$v_i = 0$
$v_x$
$i_x$
$R_o$

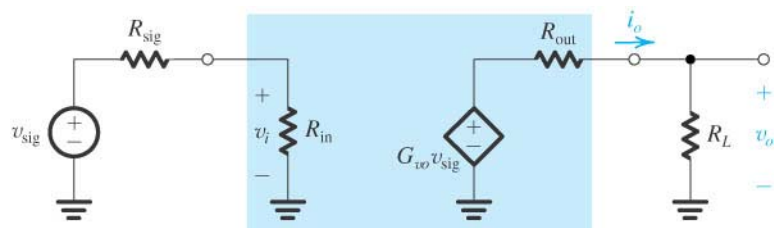
$R_{sig}$
$v_i$
$v_x$
$i_x$
$v_{sig} = 0$
$R_{out}$

$R_{sig}$
$R_o$
$i_o$
$v_{sig}$
$v_i$
$R_{in}$
$A_{vo}v_i$
$R_L$
$v_o$

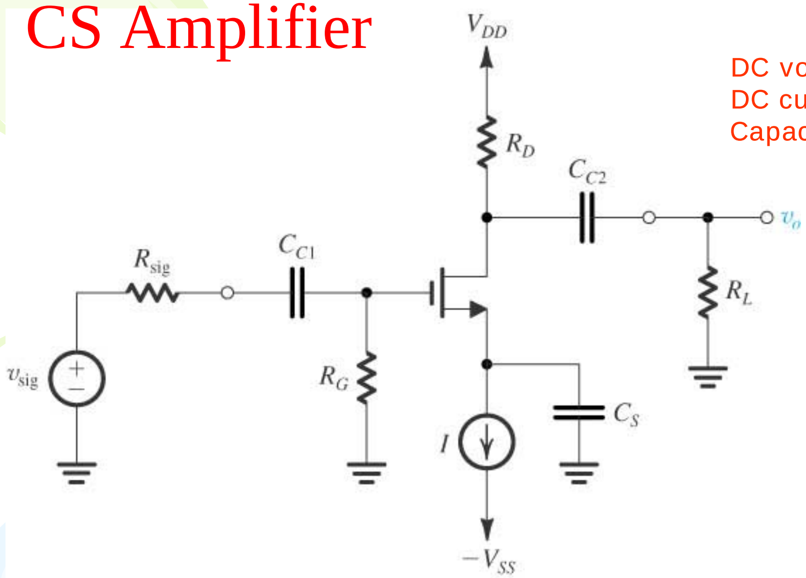



$R_{sig}$
$i_o$
$v_{sig}$
$v_i$
$R_{in}$
$G_m v_i$
$R_o$
$R_L$
$v_o$

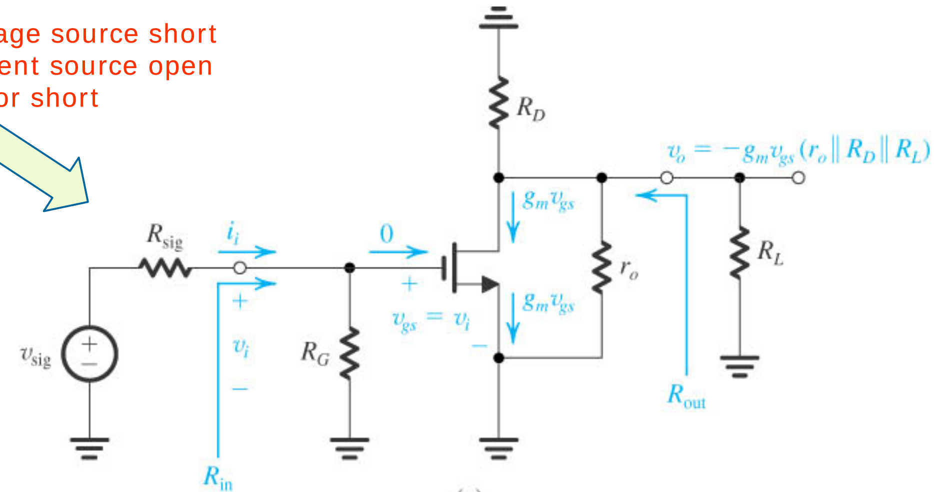
$R_{sig}$
$R_{out}$
$i_o$
$v_{sig}$
$v_i$
$R_{in}$
$G_{vo}v_{sig}$
$R_L$
$v_o$

# CS Amplifier



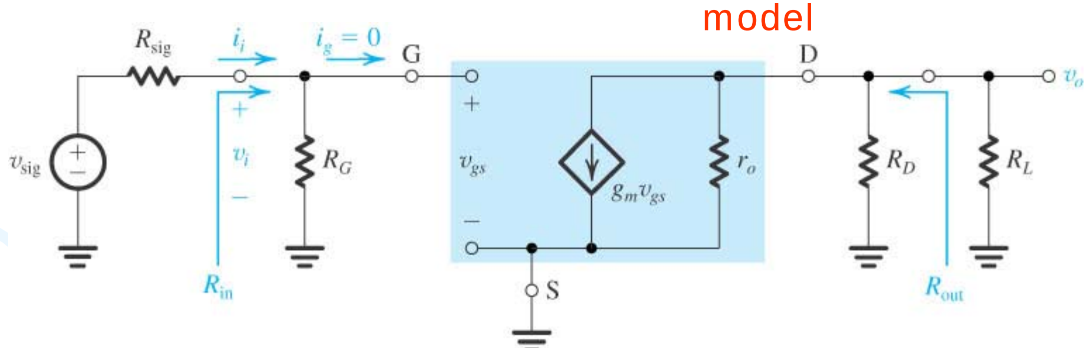

(a)

DC voltage source short
DC current source open
Capacitor short

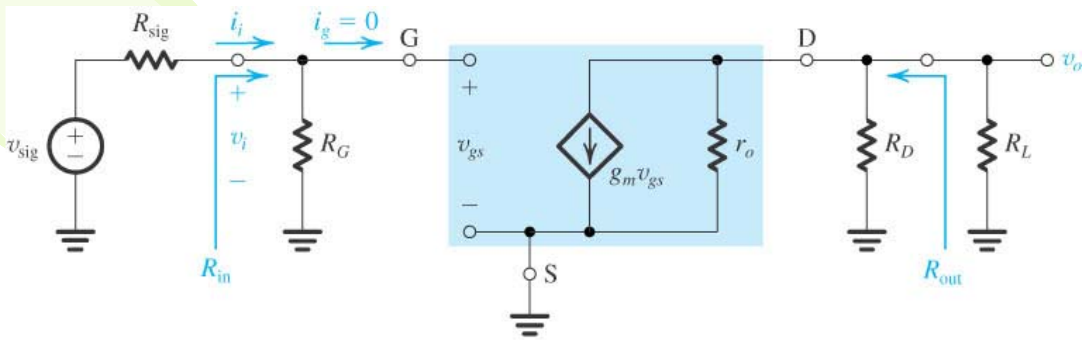



(c)

Replace transistor by small signal equivalent model




(b)




(b)

$$v_o\Big|_{R_L=\infty} = -g_m v_{gs}(r_o // R_D)$$

$$v_o = -g_m v_{gs}(r_o // R_D // R_L)$$

$$v_i = v_{gs}$$

$$A_v = -g_m(r_o // R_D // R_L)$$

$$i_o\Big|_{R_L=0} = -g_m v_{gs}$$

$$i_i = {v_{gs}}/{R_G}$$

$$A_{is} = -g_m R_G$$

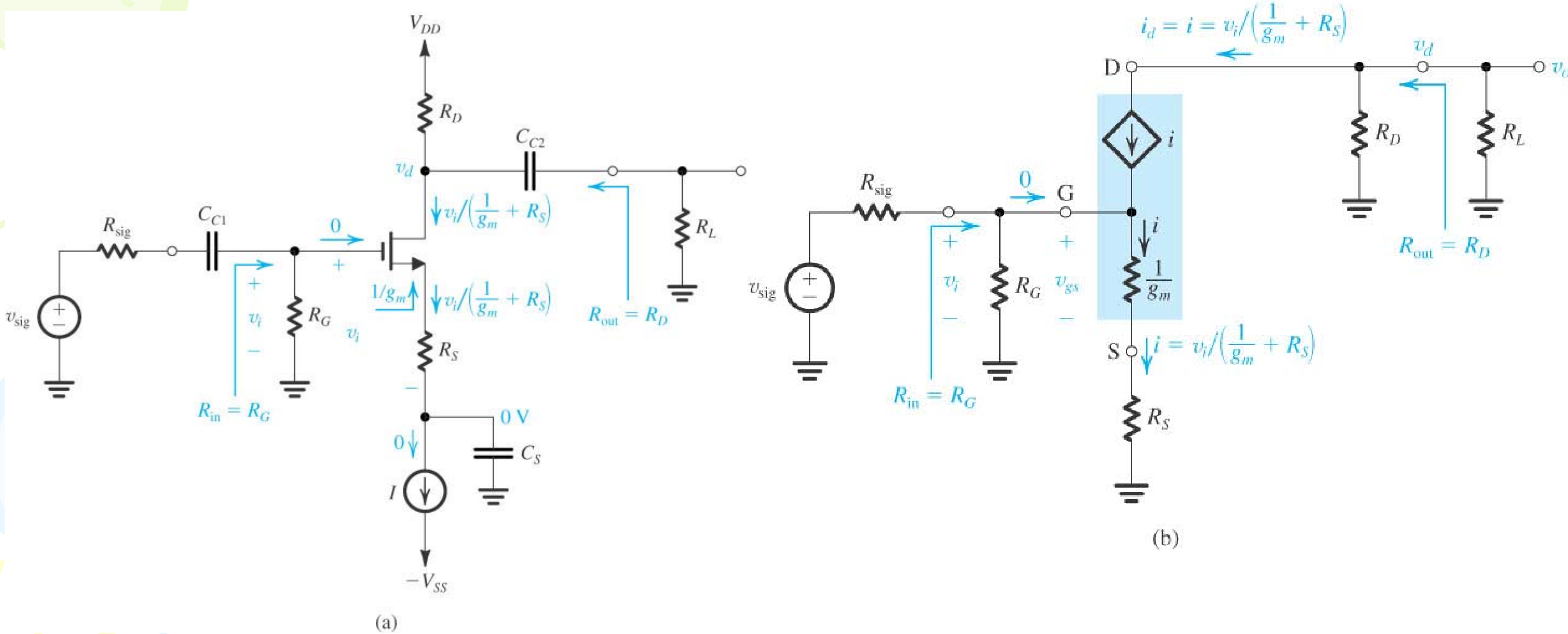
$$R_{out}\Big|_{v_s=0} = (r_o // R_D)$$

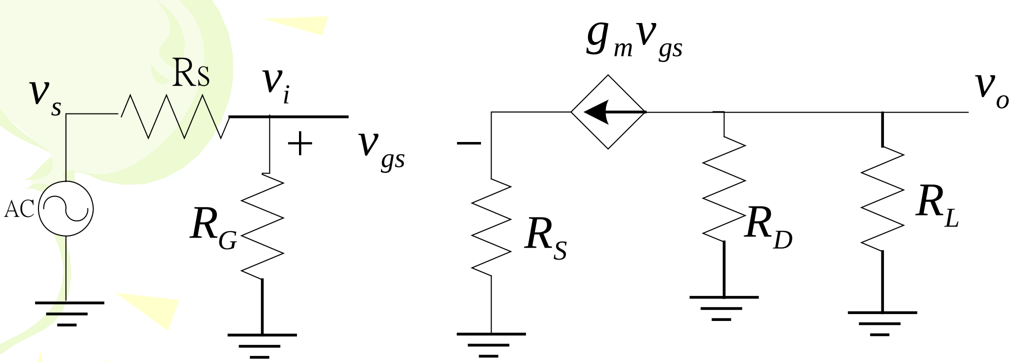
$$V = v_{gs}$$

$$I = {v_{gs}}/{R_G}$$

$$R_{in} = R_G$$

# CS Amplifier with source resistance



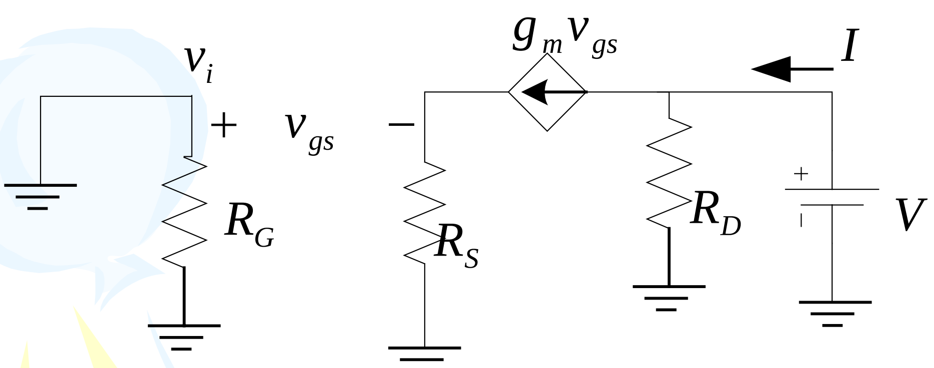



$$v_o\Big|_{R_L=\infty} = -g_m v_{gs} R_D$$

$$v_i = v_{gs} + v_s = v_{gs} + g_m v_{gs} R_S$$

$$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

$$i_o\Big|_{R_L=0} = -g_m v_{gs}$$




$$i_i = \frac{v_i}{R_G} = \frac{v_{gs}(1 + g_m R_s)}{R_G}$$

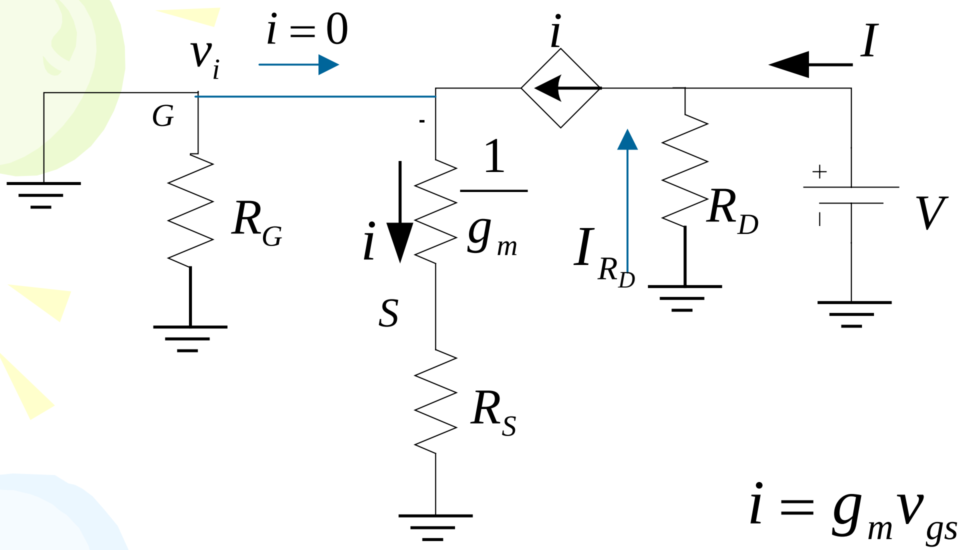
$$A_{is} = \frac{-g_m R_G}{(1 + g_m R_s)}$$

$$v_{gs} = -g_m v_{gs} R_s$$

$$\because -g_m R_s \neq 0 \ \therefore v_{gs} = 0$$

$$R_o\Big|_{v_i=0} = R_D$$

$$R_i = R_G$$




$$i = g_m v_{gs} = I + I_{R_D}$$

$$v_{gs} = -iR_s = -(I + I_{R_D})R_S$$

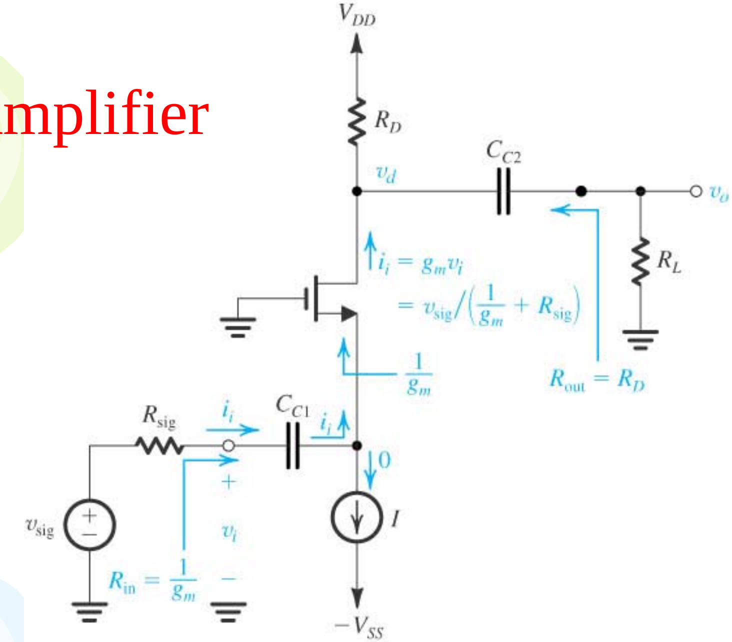
$$I + I_{R_D} = g_m v_{gs} = -g_m(I + I_{R_D})R_S$$

$$\Rightarrow (1 + g_m R_S)I = -(1 + g_m R_S)I_{R_D}$$

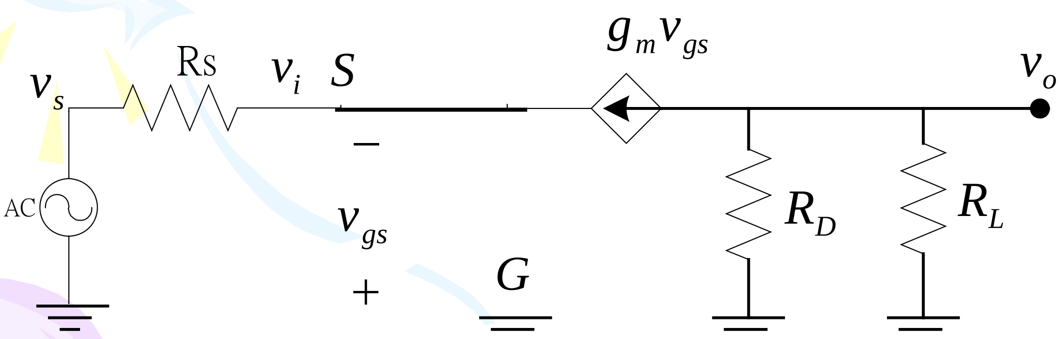
$$\Rightarrow I = -I_{R_D}$$

$$R_o = \frac{V}{I} = R_D$$

# CG Amplifier



(a)




$$v_o\Big|_{R_L=\infty} = -g_m v_{gs} R_D$$

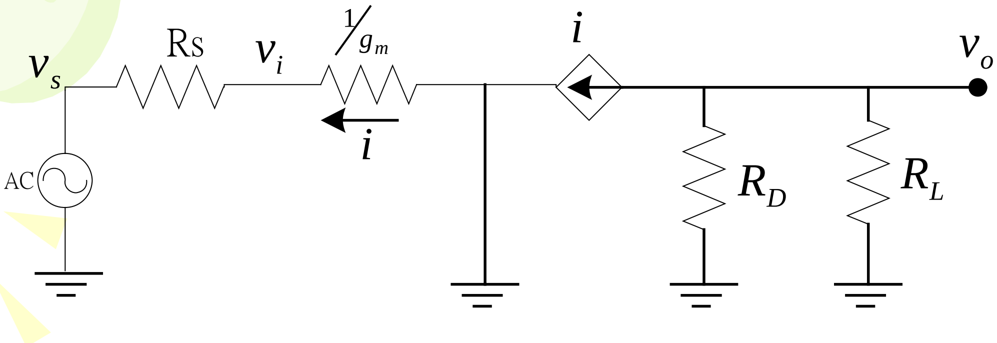
$$v_i = -v_{gs}$$

$$A_v = g_m R_D$$

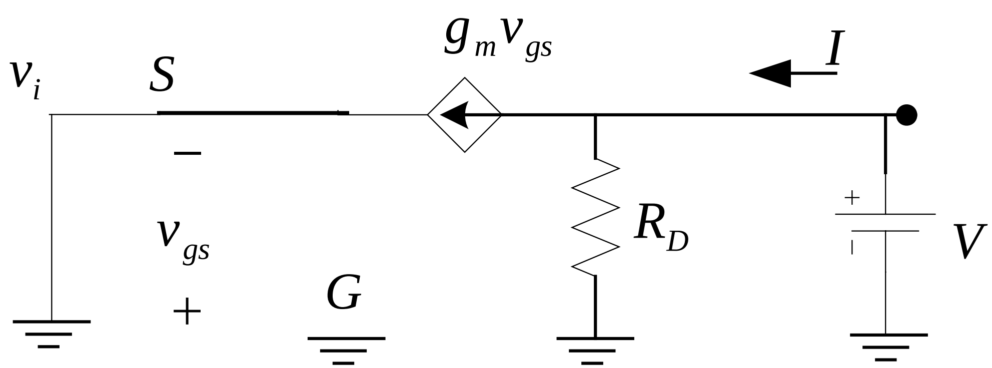
$$i_o\Big|_{R_L=0} = -g_m v_{gs}$$

$$i_i = -g_m v_{gs}$$

$$A_{is} = 1$$

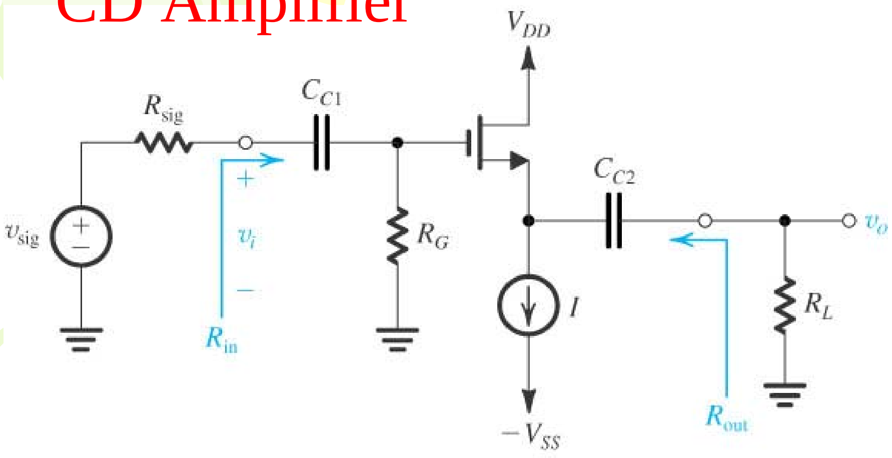



$$R_i = \frac{1}{g_m}$$

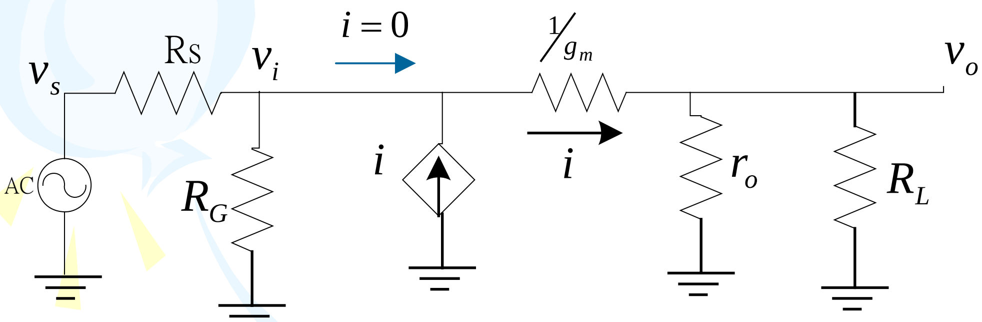



$$v_i = 0 \rightarrow v_{gs} = 0 \qquad R_o\Big|_{v_i=0} = R_D$$

# CD Amplifier




(a)



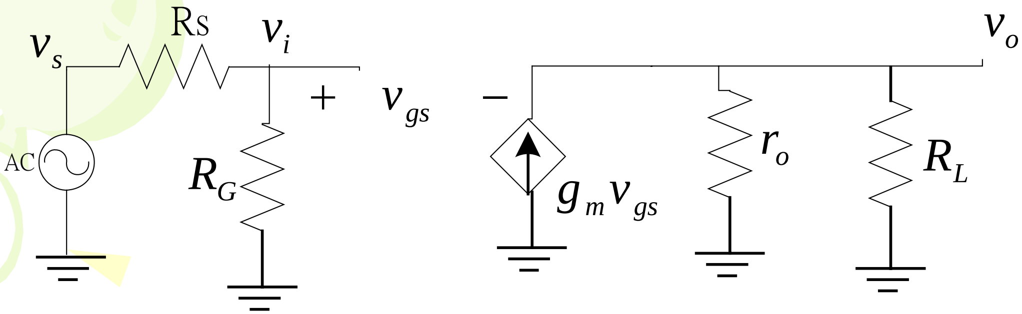

$$v_o\Big|_{R_L \to \infty} = \frac{r_o}{r_o + \dfrac{1}{g_m}} v_i$$

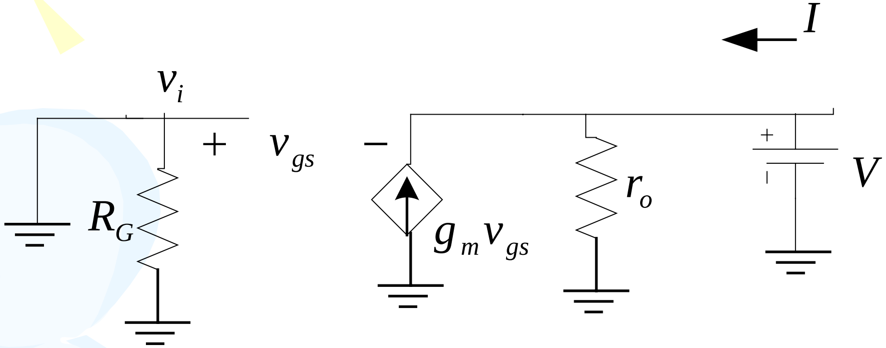
$$A_v = \frac{r_o}{r_o + \dfrac{1}{g_m}}$$

$$i_o\Big|_{R_L = 0} = i$$

$$i_i = \frac{v_i}{R_G} = \frac{v_g}{R_G} = \frac{i[\dfrac{1}{g_m}]}{R_G}$$

$$A_{is} = \frac{R_G}{\dfrac{1}{g_m}} = R_G g_m$$




$$v_o\big|_{R_L\to\infty} = r_o g_m v_{gs}$$

$$v_i = v_{gs} + v_o = v_{gs} + r_o g_m v_{gs}$$

$$A_v = \frac{r_o g_m}{1 + r_o g_m}$$

$$i_o\big|_{R_L=0} = g_m v_{gs}$$

$$i_i = \frac{v_i}{R_G} = \frac{v_{gs}}{R_G}$$

$$A_{is} = R_G g_m$$

$$I = \frac{V}{r_o} - g_m v_{gs} \rightarrow \because v_{gs} = -V$$

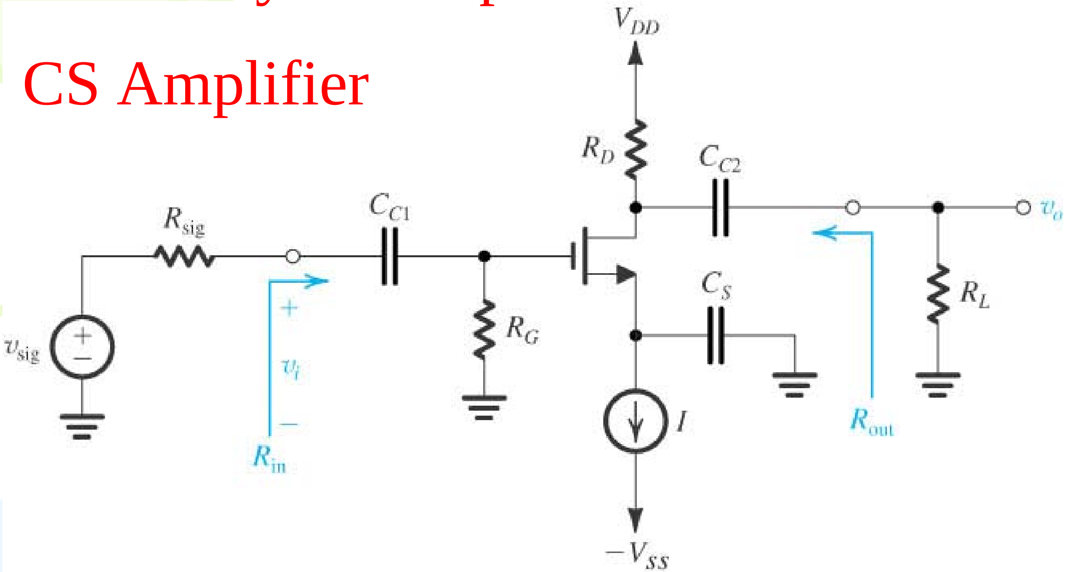
$$\rightarrow I = \frac{V}{r_o} + g_m V$$

$$R_o\big|_{v_i=0} = (r_o // \frac{1}{g_m})$$

$$R_i = R_G$$

# Summery & comparisons

## CS Amplifier

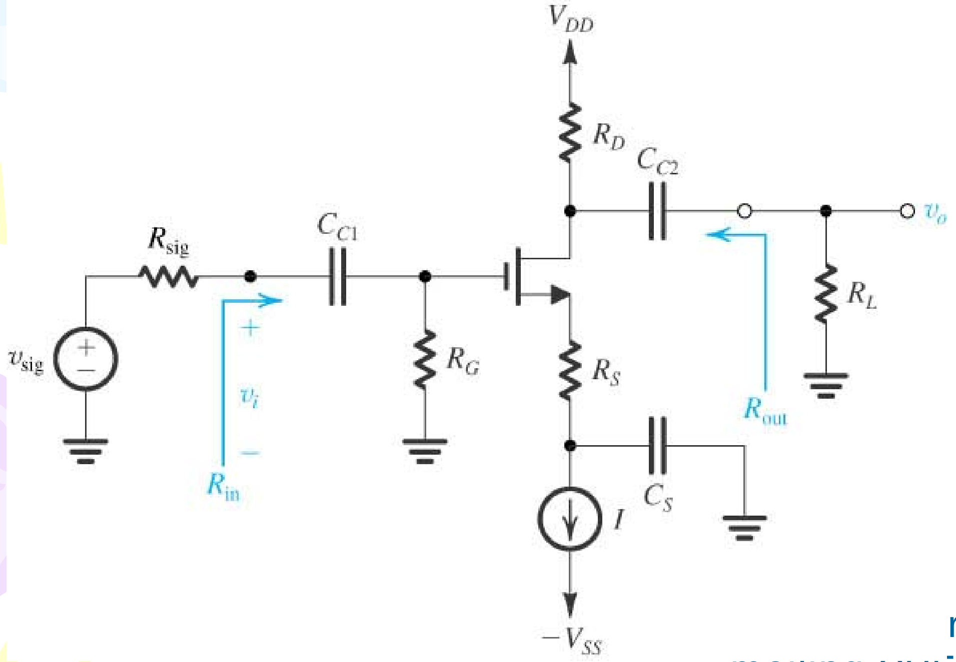



$$A_{vo} = -g_m(r_o // R_D)$$

$$A_{is} = -g_m R_G$$

$$R_{out}\Big|_{v_s=0} = (r_o // R_D)$$

$$R_{in} = R_G$$



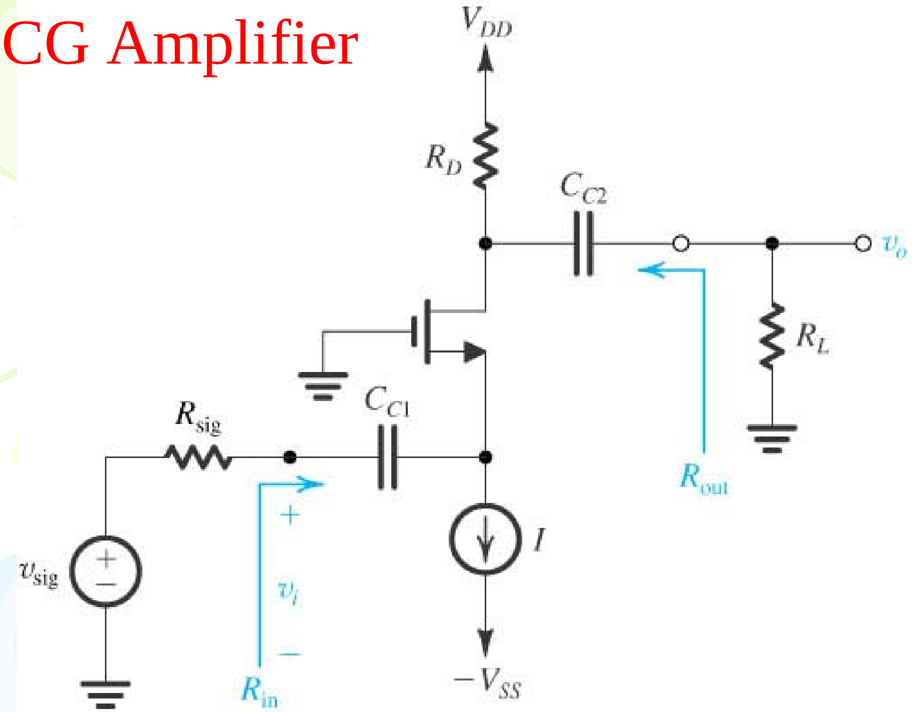

$$A_{vo} = \frac{-g_m R_D}{1 + g_m R_S}$$

$$A_{is} = \frac{-g_m R_G}{(1 + g_m R_s)}$$

$$R_o\Big|_{v_i=0} = R_D$$

$$R_i = R_G$$

# CG Amplifier



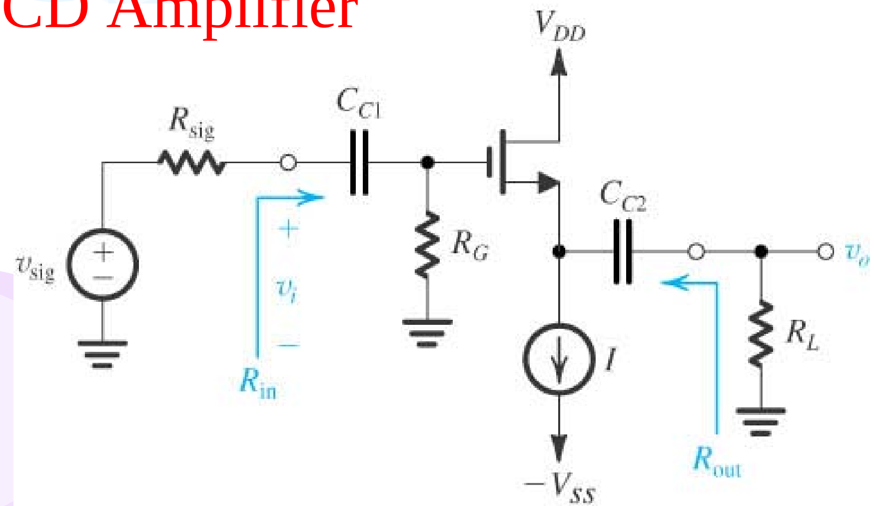

$$A_{vo} = g_m R_D$$

$$A_{is} = 1$$

$$R_o\Big|_{v_i=0} = R_D$$

$$R_i = \frac{1}{g_m}$$

# CD Amplifier




$$A_{vo} = \frac{r_o g_m}{1 + r_o g_m}$$

$$A_{is} = R_G g_m$$

$$R_o\Big|_{v_i=0} = (r_o // \frac{1}{g_m})$$

$$R_i = R_D$$