VECTOR IDENTITIES

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{C}\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\vec{C}\times\vec{A})$$
$$\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}\cdot\vec{C})-\vec{C}(\vec{A}\cdot\vec{B})$$

$$\nabla(\Phi+\Psi)=\nabla\Phi+\nabla\Psi$$
$$\nabla\cdot(\vec{A}+\vec{B})=\nabla\cdot\vec{A}+\nabla\cdot\vec{B}$$
$$\nabla\times(\vec{A}+\vec{B})=\nabla\times\vec{A}+\nabla\times\vec{B}$$
$$\nabla(\Phi\Psi)=\Phi\nabla\Psi+\Psi\nabla\Phi$$
$$\nabla\left(\frac{\Phi}{\Psi}\right)=\frac{\Psi\nabla\Phi-\Phi\nabla\Psi}{\Psi^2}$$
$$\nabla\Phi^n=n\Phi^{n-1}\nabla\Phi$$
$$\nabla\cdot(\Phi\vec{A})=\vec{A}\cdot\nabla\Phi+\Phi\nabla\cdot\vec{A}$$
$$\nabla\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot\nabla\times\vec{A}-\vec{A}\cdot\nabla\times\vec{B}$$
$$\nabla\times(\Phi\vec{A})=\nabla\Phi\times\vec{A}+\Phi\nabla\times\vec{A}$$
$$\nabla\times(\vec{A}\times\vec{B})=\vec{A}\nabla\cdot\vec{B}-\vec{B}\nabla\cdot\vec{A}+(\vec{B}\cdot\nabla)\vec{A}-(\vec{A}\cdot\nabla)\vec{B}$$
$$\nabla(\vec{A}\cdot\vec{B})=\vec{A}\times(\nabla\times\vec{B})+\vec{B}\times(\nabla\times\vec{A})+(\vec{A}\cdot\nabla)\vec{B}+(\vec{B}\cdot\nabla)\vec{A}$$

$$\nabla\cdot\nabla\Phi=\nabla^2\Phi$$
$$\nabla\cdot\nabla\times\vec{A}=0$$
$$\nabla\times\nabla\Phi=0$$
$$\nabla\times\nabla\times\vec{A}=\nabla\nabla\cdot\vec{A}-\nabla^2\vec{A}$$

VECTOR INTEGRAL THEOREMS

$$\iiint_v(\nabla\cdot\vec{A})dv=\oiint_{S_{[v]}}\vec{A}\cdot d\vec{s}\quad\text{(Divergence theorem, Gauss identity)}$$
$$\iint_S(\nabla\times\vec{A})\cdot d\vec{s}=\oint_{C_{[S]}}\vec{A}\cdot d\vec{l}\quad\text{(Curl theorem 1, Stokes' theorem)}$$
$$\iiint_v(\nabla\times\vec{A})dv=\oiint_{S_{[v]}}d\vec{s}\times\vec{A}\equiv\oiint_{S_{[v]}}(\hat{n}\times\vec{A})ds\quad\text{(Curl theorem 2)}$$

SOME INTEGRALS OFTEN MET IN EM PROBLEMS

$$\int\frac{1}{x}dx=\ln|x|+C$$
$$\int\frac{1}{(a^2\pm x^2)^{3/2}}dx=\pm\frac{x}{a^2\sqrt{a^2\pm x^2}}+C$$
$$\int\frac{x}{(a^2+x^2)^{3/2}}dx=-\frac{1}{\sqrt{a^2+x^2}}+C$$
$$\int\frac{x^2}{(a^2+x^2)^{3/2}}dx=-\frac{x}{\sqrt{a^2+x^2}}+\ln\left(x+\sqrt{a^2+x^2}\right)+C$$
$$\int\frac{1}{a^2+x^2}dx=\frac{1}{a}\arctan\frac{x}{a}+C$$
$$\int\frac{1}{x^2-a^2}dx=\begin{cases}\frac{1}{2a}\ln\left(\frac{a-x}{a+x}\right)=-\frac{1}{a}\operatorname{arctanh}\left(\frac{x}{a}\right), & |x|<a\\ -\frac{1}{a}\operatorname{arccoth}\left(\frac{x}{a}\right) & ,|x|>a\end{cases}$$
$$\int\frac{x}{a^2+x^2}dx=\frac{1}{2}\ln\left(a^2+x^2\right)+C$$
$$\int\frac{x}{\sqrt{a^2+x^2}}dx=\sqrt{a^2+x^2}+C$$
$$\int\frac{1}{\sqrt{a^2+x^2}}dx=\ln(x+\sqrt{a^2+x^2})+C$$
$$\int\frac{1}{x\sqrt{a^2+x^2}}dx=-\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2+x^2}}{x}\right)+C$$
$$\int x\sin(ax)dx=\frac{1}{a^2}[\sin(ax)-ax\cos(ax)]$$
$$\int x\cos(ax)dx=\frac{1}{a^2}[\cos(ax)+ax\sin(ax)]$$

$$\int \frac{dx}{(ax^2+b)\sqrt{fx^2+g}} = \frac{1}{\sqrt{b}\sqrt{ag-bf}} \arctan\left(\frac{x\sqrt{ag-bf}}{\sqrt{b}\sqrt{fx^2+g}}\right), (ag > bf)$$

$$\int \tan x dx = -\ln|\cos x| + C, x \neq (2k+1)\frac{\pi}{2}$$

$$\int \cot x dx = \ln|\sin x| + C, x \neq 2k\pi$$

$$\int \frac{1}{\sin x} dx = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C$$

$$\int \frac{1}{\cos x} dx = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C$$

SOME USEFUL DEFINITE INTGERALS

$$\int_0^{2\pi} \sin mx \cdot \sin nx \; dx = \begin{cases} 0 \quad , \quad m \neq n \\ \pi, \quad m = n \neq 0 \end{cases}$$

$$\int_0^{2\pi} \cos mx \cdot \cos nx \; dx = \begin{cases} 0 \quad , \quad m \neq n \\ \pi, \quad m = n \neq 0 \end{cases}$$

$$\int_0^{2\pi} \sin mx \cdot \cos nx \; dx = 0$$

$$\int_0^{\pi} \sin mx \cdot \sin nx \; dx = \begin{cases} 0 \quad , \quad m \neq n \\ \pi/2, \quad m = n \neq 0 \end{cases}$$

$$\int_0^{\pi} \cos mx \cdot \cos nx \; dx = \begin{cases} 0 \quad , \quad m \neq n \\ \pi/2, \quad m = n \neq 0 \end{cases}$$

$$\int_0^{\pi} \sin mx \cdot \cos nx \; dx = \begin{cases} 0 \quad , \quad m+n = \text{even number} \\ \dfrac{2m}{m^2-n^2}, \quad m+n = \text{odd number} \end{cases}$$

$$\int_0^{\pi} \frac{(a-b\cos x)}{(a^2+b^2-2ab\cos x)} dx = \begin{cases} \dfrac{\pi}{a}, \quad a > b > 0 \\ 0, \quad b > a > 0 \end{cases}$$

COORDINATE TRANSFORMATIONS

Rectangular ↔ Cylindrical

$$\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \\ z &= z \end{aligned} \qquad \begin{aligned} r &= \sqrt{x^2+y^2} \\ \phi &= \arctan\left(\frac{y}{x}\right) \\ z &= z \end{aligned}$$

Rectangular ↔ Spherical

$$\begin{aligned} x &= R\sin\theta\cos\phi \\ y &= R\sin\theta\sin\phi \\ z &= R\cos\theta \end{aligned} \qquad \begin{aligned} R &= \sqrt{x^2+y^2+z^2} \\ \theta &= \arccos\left(z/\sqrt{x^2+y^2+z^2}\right) \\ \phi &= \arctan(y/x) \end{aligned}$$

Cylindrical ↔ Spherical

$$\begin{aligned} r &= R\sin\theta \\ \phi &= \phi \\ z &= R\cos\theta \end{aligned} \qquad \begin{aligned} R &= \sqrt{r^2+z^2} \\ \phi &= \phi \\ \theta &= \arccos\left(z/\sqrt{r^2+z^2}\right) \end{aligned}$$

VECTOR TRANSFORMATIONS

Rectangular Components ↔ Cylindrical Components

$$\begin{aligned} a_x &= a_r\cos\phi - a_\phi\sin\phi \\ a_y &= a_r\sin\phi + a_\phi\cos\phi \\ a_z &= a_z \end{aligned} \qquad \begin{aligned} a_r &= a_x\cos\phi + a_y\sin\phi \\ a_\phi &= -a_x\sin\phi + a_y\cos\phi \\ a_z &= a_z \end{aligned}$$

Note: ϕ is the position angle of the point at which the vector exists.

Rectangular Components ↔ Spherical Components

$$\begin{aligned} a_x &= a_R\sin\theta\cos\phi + a_\theta\cos\theta\cos\phi - a_\phi\sin\phi \\ a_y &= a_R\sin\theta\sin\phi + a_\theta\cos\theta\sin\phi + a_\phi\cos\phi \\ a_z &= a_R\cos\theta - a_\theta\sin\theta \end{aligned}$$

$$\begin{aligned} a_R &= a_x\sin\theta\cos\phi + a_y\sin\theta\sin\phi + a_z\cos\theta \\ a_\theta &= a_x\cos\theta\cos\phi + a_y\cos\theta\sin\phi - a_z\cos\theta \\ a_\phi &= -a_x\sin\phi + a_y\cos\phi \end{aligned}$$

Note: ϕ and θ are the position angles of the point at which the vector exists.

Cylindrical Components↔ Spherical Components

$$\begin{vmatrix} a_r = a_R \sin\theta + a_\theta \cos\theta \\ a_\phi = a_\phi \\ a_z = a_R \cos\theta - a_\theta \sin\theta \end{vmatrix} \qquad \begin{vmatrix} a_R = a_r \sin\theta + a_z \cos\theta \\ a_\theta = a_r \cos\theta - a_z \sin\theta \\ a_\phi = a_\phi \end{vmatrix}$$

Note: θ is the position angle of the point at which the vector exists.

DERIVATIVES OF ELEMENTARY FUNCTIONS

$(const.)' = 0$

$(x)' = 1$

$(x^k)' = kx^{k-1}$

$(e^x)' = e^x$

$(a^x)' = a^x \ln a$

$(\ln x)' = \frac{1}{x}$

$(\log_a x)' = \frac{1}{x \ln a}, a \neq 1, x > 0$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \frac{1}{\cos^2 x}, x \neq (2k+1)\pi$

$(\cot x)' = -\frac{1}{\sin^2 x}, x \neq k\pi$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, |x| < 1$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}, |x| < 1$

$(\arctan x)' = \frac{1}{1+x^2}$

$(\operatorname{arc\,cot} x)' = -\frac{1}{1+x^2}$

$(\sinh x)' = \cosh x$

$(\cosh x)' = \sinh x$

$(\tanh x)' = \frac{1}{\cosh^2 x} = 1 - \tanh^2 x$

$(\coth x)' = -\frac{1}{\sinh^2 x} = 1 - \coth^2 x$

$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{1+x^2}}$

$(\operatorname{arccosh} x)' = \pm\frac{1}{\sqrt{x^2-1}}, x > 1$

$(\operatorname{arctanh} x)' = \frac{1}{1-x^2}, |x| < 1$

$(\operatorname{arccoth} x)' = \frac{1}{1-x^2}, |x| > 1$

DIFFERENTIAL OPERATORS

Rectangular Coordinates

$$\nabla\Phi = \hat{x}\frac{\partial\Phi}{\partial x} + \hat{y}\frac{\partial\Phi}{\partial y} + \hat{z}\frac{\partial\Phi}{\partial z}$$

$$\nabla\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla\times\vec{F} = \hat{x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{y}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$\nabla\cdot(\nabla\Phi) \equiv \nabla^2\Phi \equiv \Delta\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}$$

$$\nabla^2\vec{F} = \hat{x}\nabla^2 F_x + \hat{y}\nabla^2 F_y + \hat{z}\nabla^2 F_z$$

Cylindrical Coordinates

$$\nabla\Phi = \frac{\partial\Phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\hat{\varphi} + \frac{\partial\Phi}{\partial z}\hat{z}$$

$$\nabla\cdot\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\varphi}{\partial\varphi} + \frac{\partial F_z}{\partial z}$$

$$\nabla\times\vec{F} = \hat{r}\left(\frac{1}{r}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right) + \hat{z}\left(\frac{1}{r}\frac{\partial(rF_\phi)}{\partial r} - \frac{1}{r}\frac{\partial F_r}{\partial\phi}\right)$$

$$\nabla\cdot(\nabla\Phi) \equiv \nabla^2\Phi \equiv \Delta\Phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$$

$$\nabla^2\vec{A} = \hat{r}\left(\frac{\partial^2 A_r}{\partial r^2} + \frac{1}{r}\frac{\partial A_r}{\partial r} - \frac{A_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 A_r}{\partial\phi^2} - \frac{2}{r^2}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial^2 A_r}{\partial z^2}\right) +$$
$$\hat{\phi}\left(\frac{\partial^2 A_\phi}{\partial r^2} + \frac{1}{r}\frac{\partial A_\phi}{\partial r} - \frac{A_\phi}{r^2} + \frac{1}{r^2}\frac{\partial^2 A_\phi}{\partial\phi^2} + \frac{2}{r^2}\frac{\partial A_r}{\partial\phi} + \frac{\partial^2 A_\phi}{\partial z^2}\right) +$$
$$\hat{z}\left(\frac{\partial^2 A_z}{dr^2} + \frac{1}{r}\frac{\partial A_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 A_z}{\partial\phi^2} + \frac{\partial^2 A_z}{\partial z^2}\right)$$

Spherical Coordinates

$$\nabla\Phi = \frac{\partial\Phi}{\partial R}\hat{R} + \frac{1}{R}\frac{\partial\Phi}{\partial\theta}\hat{\theta} + \frac{1}{R\sin\theta}\frac{\partial\Phi}{\partial\varphi}\hat{\varphi}$$

$$\nabla\cdot\vec{F} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 F_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(F_\theta\sin\theta) + \frac{1}{R\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}$$

$$\nabla\times\vec{A}=\hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\varphi\sin\theta)-\frac{\partial A_\theta}{\partial\varphi}\right]+$$
$$\hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\varphi}-\frac{\partial}{\partial R}(RA_\varphi)\right]+$$
$$\hat{\varphi}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta)-\frac{\partial A_R}{\partial\theta}\right]$$

$$\nabla^2\Phi=\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial\Phi}{\partial R}\right)+\frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right)+\frac{1}{R^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2}$$

$$\nabla^2\vec{A}=\hat{R}\left(\frac{\partial^2A_R}{\partial R^2}+\frac{2}{R}\frac{\partial A_R}{\partial R}-\frac{2}{R^2}A_R+\frac{1}{R^2}\frac{\partial^2A_R}{\partial\theta^2}+\frac{\cot\theta}{R^2}\frac{\partial A_R}{\partial\theta}+\right.$$
$$\left.\frac{1}{R^2\sin^2\theta}\frac{\partial^2A_R}{\partial\varphi^2}-\frac{2}{R^2}\frac{\partial A_\theta}{\partial\theta}-\frac{2\cot\theta}{R^2}A_\theta-\frac{2}{R^2\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}\right)+$$
$$\hat{\theta}\left(\frac{\partial^2A_\theta}{\partial R^2}+\frac{2}{R}\frac{\partial A_\theta}{\partial R}-\frac{A_\theta}{R^2\sin^2\theta}+\frac{1}{R^2}\frac{\partial^2A_\theta}{\partial\theta^2}+\frac{\cot\theta}{R^2}\frac{\partial A_\theta}{\partial\theta}+\right.$$
$$\left.\frac{1}{R^2\sin^2\theta}\frac{\partial^2A_\theta}{\partial\varphi^2}+\frac{2}{R^2}\frac{\partial A_R}{\partial\theta}-\frac{2\cot\theta}{R^2\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}\right)+$$
$$\hat{\varphi}\left(\frac{\partial^2A_\varphi}{\partial R^2}+\frac{2}{R}\frac{\partial A_\varphi}{\partial R}-\frac{A_\varphi}{R^2\sin^2\theta}+\frac{1}{R^2}\frac{\partial^2A_\varphi}{\partial\theta^2}+\frac{\cot\theta}{R^2}\frac{\partial A_\varphi}{\partial\theta}+\right.$$
$$\left.\frac{1}{R^2\sin^2\theta}\frac{\partial^2A_\varphi}{\partial\varphi^2}+\frac{2}{R^2\sin\theta}\frac{\partial A_R}{\partial\varphi}+\frac{2\cot\theta}{R^2\sin\theta}\frac{\partial A_\theta}{\partial\varphi}\right)$$

DIFFERENTIAL ELEMENTS

Cartesian coordinates

$d\vec{l}=\hat{x}dx+\hat{y}dy+\hat{z}dz$; $d\vec{s}=\hat{x}dydz+\hat{y}dxdz+\hat{z}dxdy$; $dv=dxdydz$

Cylindrical coordinates

$d\vec{l}=\hat{r}dr+\hat{\varphi}rd\varphi+\hat{z}dz$; $d\vec{s}=\hat{r}rd\varphi dz+\hat{\varphi}drdz+\hat{z}rdrd\varphi$; $dv=rdrd\varphi dz$

Spherical coordinates

$d\vec{l}=\hat{R}dR+\hat{\theta}Rd\theta+\hat{\varphi}R\sin\theta d\varphi$;

$d\vec{s}=\hat{R}R^2\sin\theta d\theta d\varphi+\hat{\theta}R\sin\theta dRd\varphi+\hat{\varphi}RdRd\theta$;

$dv=R^2\sin\theta dRd\theta d\varphi$

ELECTROMAGNETIC EQUATIONS

Coaxial line

$C_1=\dfrac{2\pi\varepsilon}{\ln(b/a)}$, F/m; $L_1=\dfrac{\mu_0}{2\pi}\ln\left(\dfrac{b}{a}\right)+\dfrac{\mu_0}{8\pi}$, H/m

Twin-lead line

$C_1=\dfrac{\pi\varepsilon}{\ln\left(\dfrac{h}{r}+\sqrt{\left(\dfrac{h}{r}\right)^2-1}\right)}$ F/m; $L_1=\dfrac{\mu}{\pi}\ln\left(\dfrac{h}{r}+\sqrt{\left(\dfrac{h}{r}\right)^2-1}\right)$ H/m