Lecture 12

BJT's Differential Pair

topics

- Ideal characteristics of differential amplifier
  - Input differential resistance
  - Input common-mode resistance
  - Differential voltage gain
  - CMRR
- Non-ideal characteristics of differential amplifier
  - Input offset voltage
  - Input biasing and offset current
- Differential Amplifier with active load

Differential pair

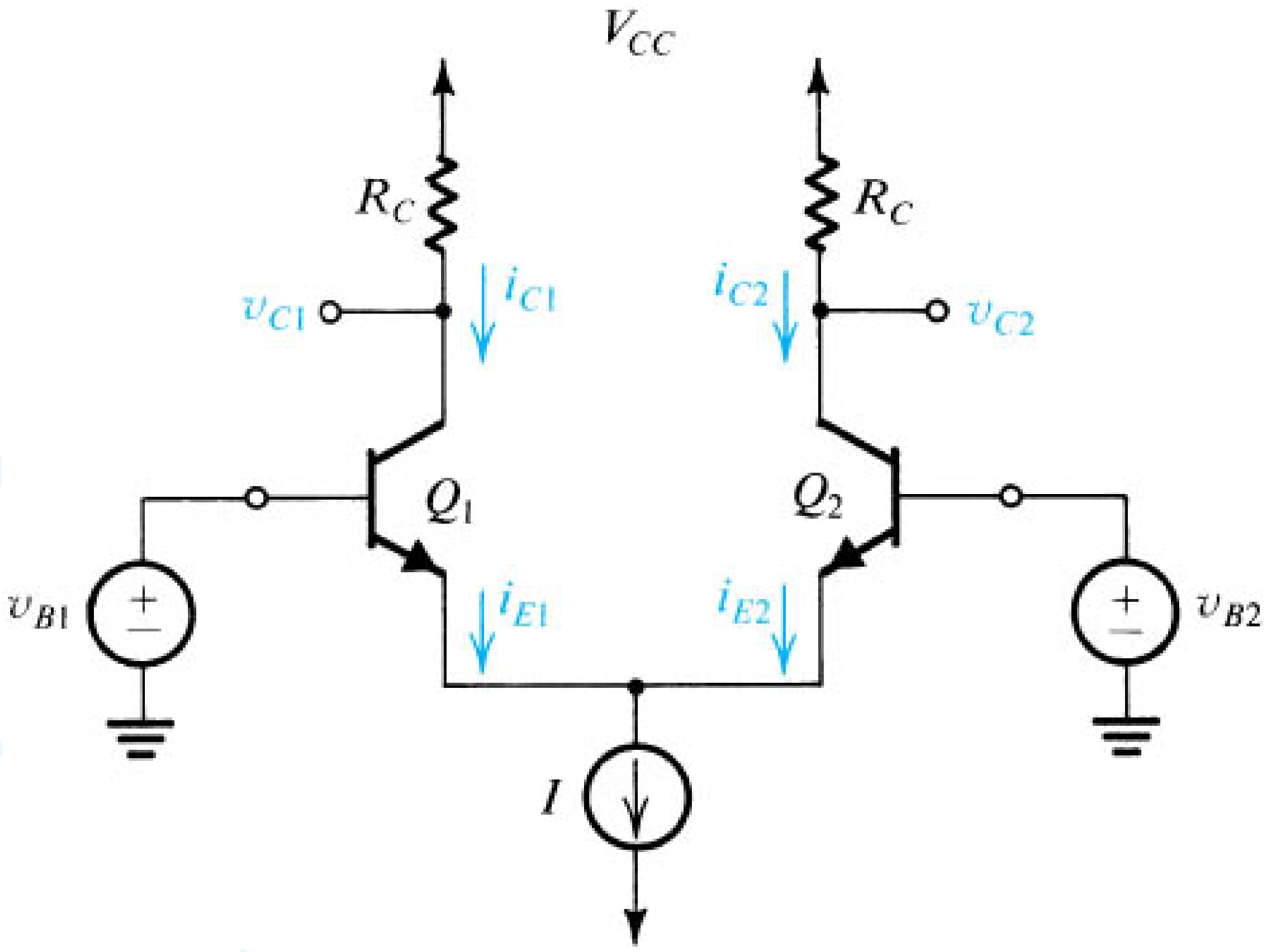
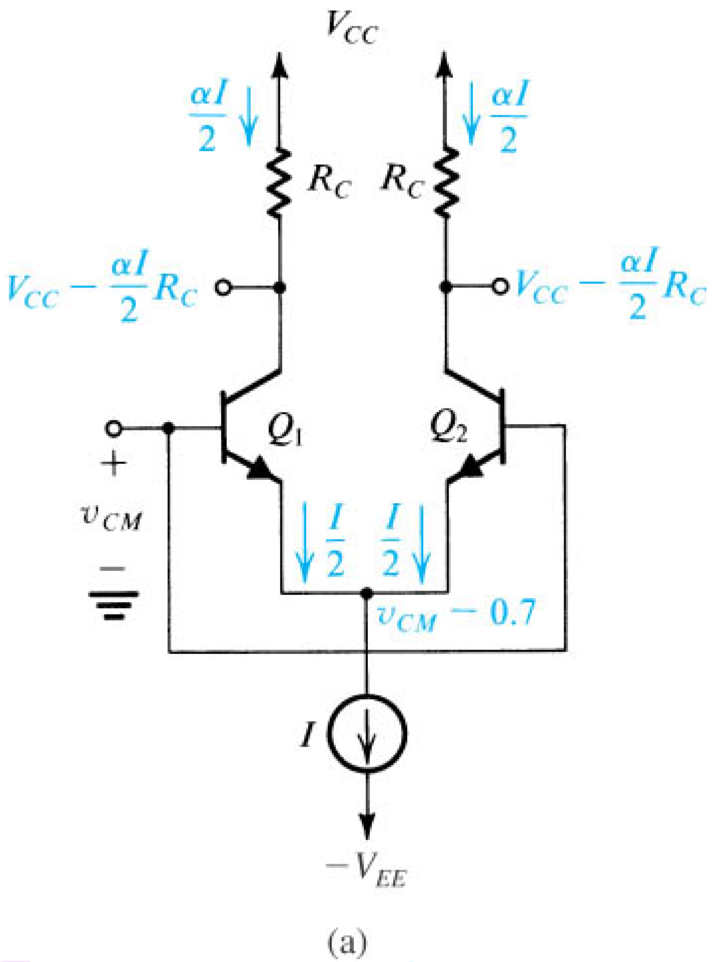


Figure 7.12 The basic BJT differential-pair configuration.

Common mode operation



$$\because Q_1 = Q_2$$

$$\therefore v_{B1} = v_{B2} = v_{CM}$$

$$\Rightarrow i_{E_1} = i_{E2} = \frac{I}{2}$$

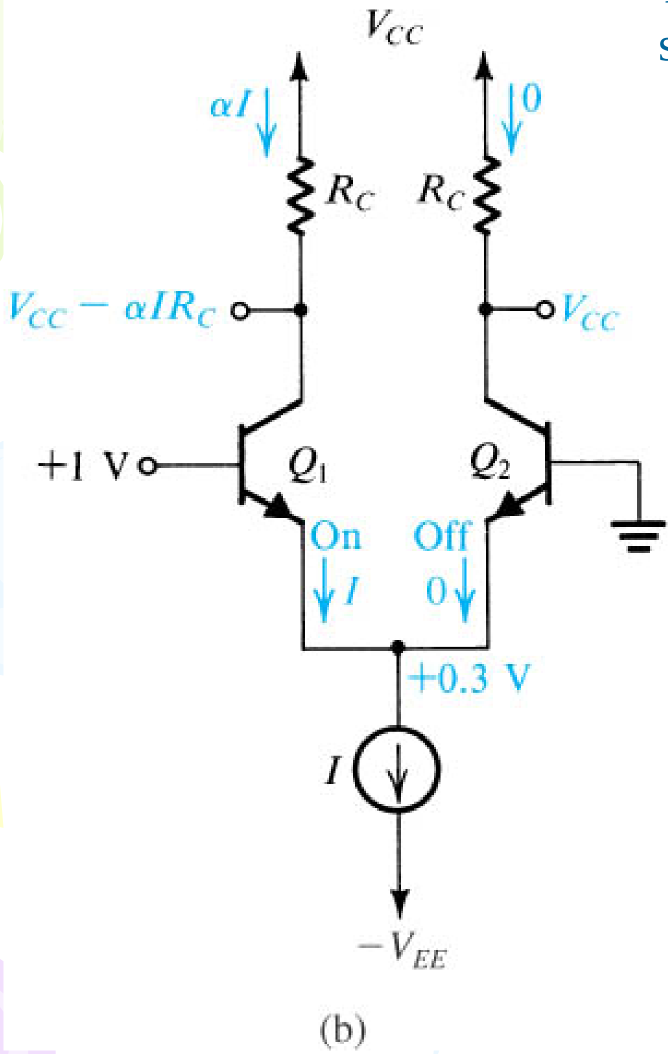
$$\Rightarrow v_{C1} = v_{C2} = V_{CC} - \alpha \frac{I}{2} R_C$$

$$v_{C1} - v_{C2} = 0$$

Reject common mode input

Figure 7.13 Different modes of operation of the BJT differential pair: **(a)** The differential pair with a common-mode input signal v_{CM}.

The differential pair with a "large" differential input signal



$$(1) V_{B1} >> V_{B2}$$

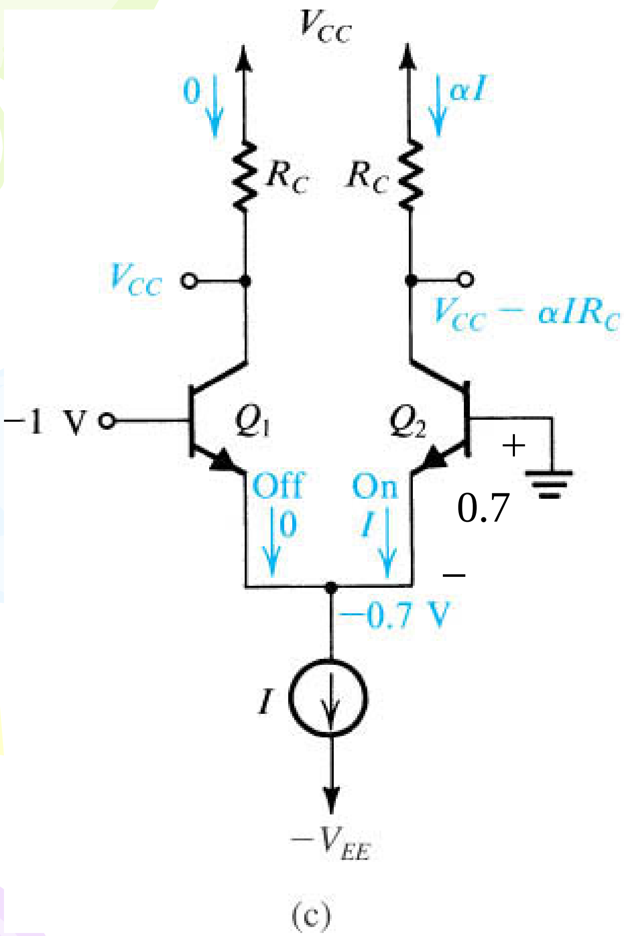
$$V_{B1} = +1V, V_{B2} = 0$$

$$Q_1 \quad on \rightarrow V_{E1} = 0.3V = V_{E2} \rightarrow Q_2 \quad off$$

$$V_{C1} = V_{CC} - \alpha I R_C, V_{C2} = V_{CC}$$

$$V_{C1} - V_{C2} = -\alpha I R_C$$

Figure 7.13 Different modes of operation of the BJT differential pair:. **(b)** The differential pair with a "large" differential input signal.



$$(2) V_{B1} << V_{B2}$$

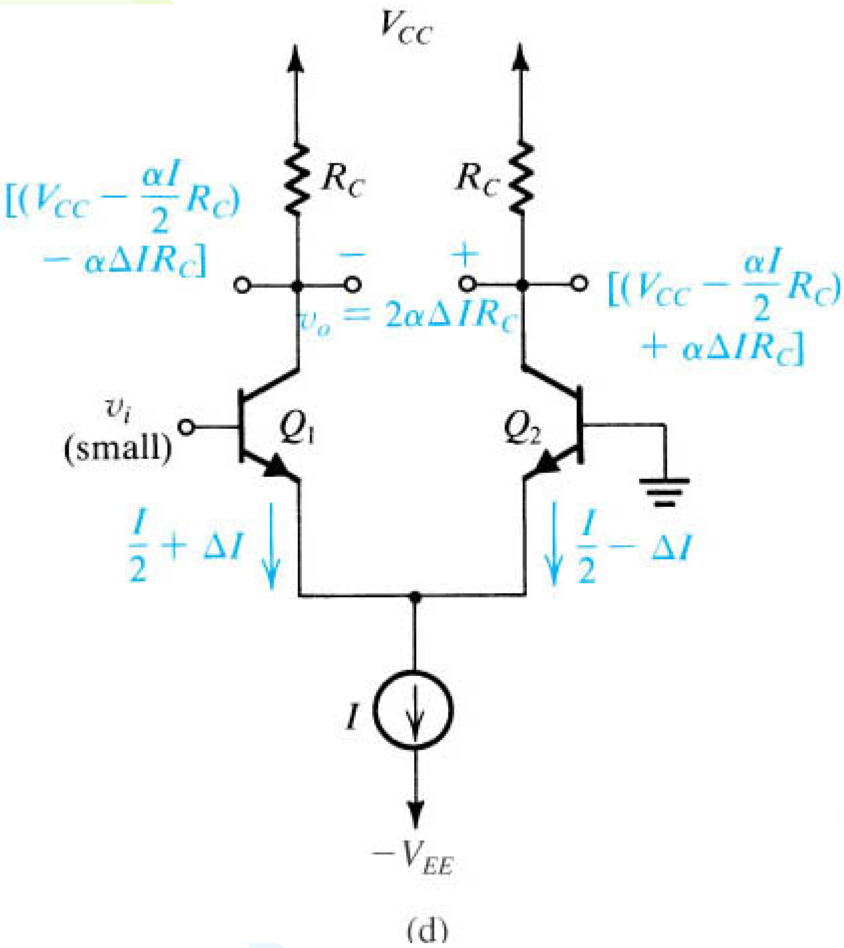
$$V_{B1} = -1V, V_{B2} = 0$$

$$Q_2 \quad on \rightarrow V_{E2} = -0.7V = V_{E2} \rightarrow Q_1 \quad off$$

$$V_{C2} = V_{CC} - \alpha I R_C, V_{C1} = V_{CC}$$

$$V_{C1} - V_{C2} = \alpha I R_C$$

Figure 7.13 *(Continued)* **(c)** The differential pair with a large differential input signal of polarity opposite to that in .



$$(3) V_{B1} > V_{B2}$$

$$V_{B1} = small, V_{B2} = 0$$

$$I_{E1} = \frac{I}{2} + \Delta I, I_{E2} = \frac{I}{2} - \Delta I$$

$$V_{C1} = V_{CC} - \alpha \frac{I}{2} R_C - \alpha \Delta I R_C$$

$$V_{C2} = V_{CC} - \alpha \frac{I}{2} R_C + \alpha \Delta I R_C$$

$$V_{C1} - V_{C2} = v_i = -2\alpha \Delta I R_C$$

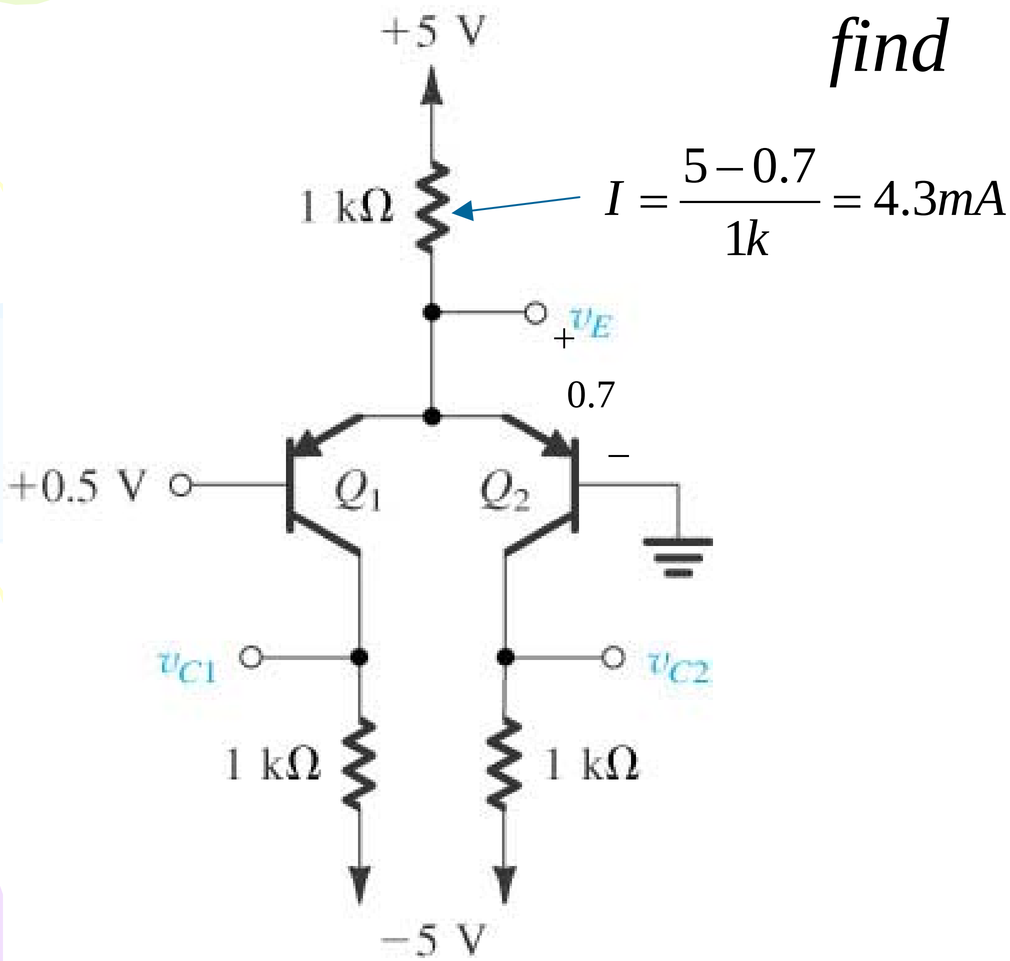
$$\Rightarrow \boxed{v_i = f(\Delta I)}$$

Figure 7.13 *(Continued)* **(d)** The differential pair with a small differential input signal v_i. Note that we have assumed the bias current source I to be ideal (i.e., it has an infinite output resistance) and thus I remains constant with the change in v_{CM}.

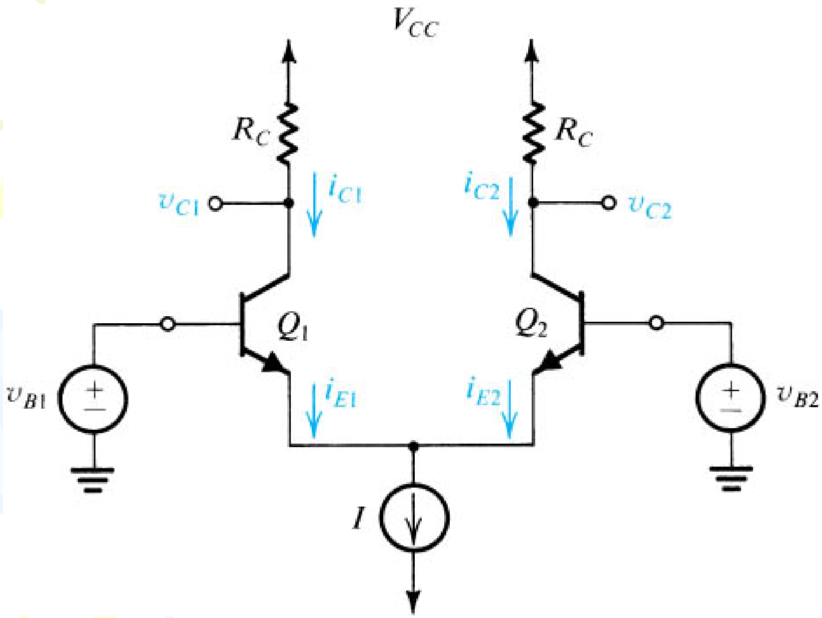
Exercise 7.7

$$let \quad \alpha \approx 1, v_{BE} = 0.7V$$

$$find \quad v_E, v_{C1}\, and \quad v_{C2}$$



Large signal operation



$$i_{E1} = \frac{I_S}{\alpha} e^{(v_{B1}-v_E)/V_T}$$

$$i_{E2} = \frac{I_S}{\alpha} e^{(v_{B2}-v_E)/V_T}$$

$$\frac{i_{E1}}{i_{E2}} = e^{(v_{B1}-v_{B2})/V_T}$$

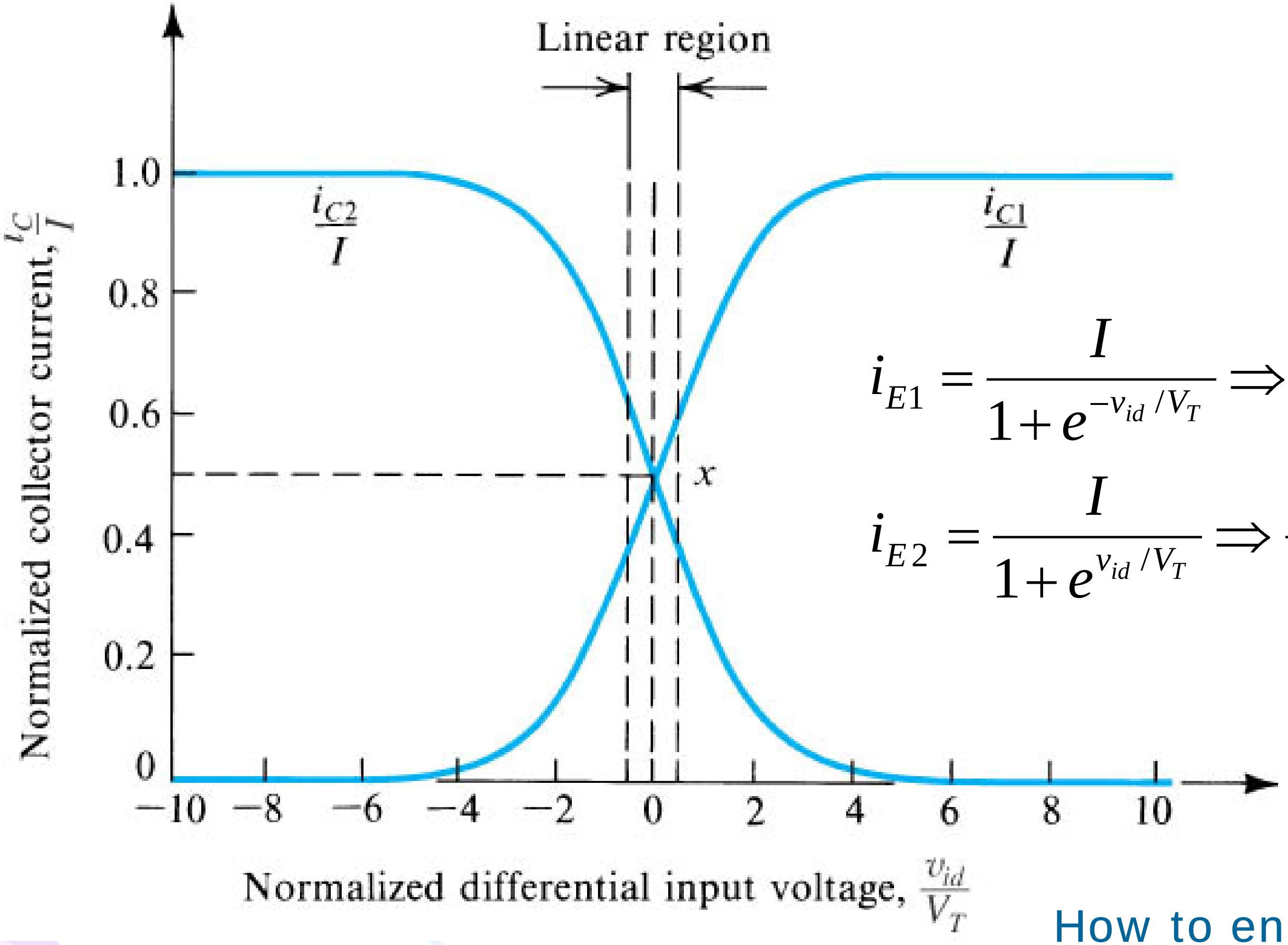
$$\frac{i_{E1}}{i_{E1}+i_{E2}} = \frac{1}{1+e^{(v_{B2}-v_{B1})/V_T}}$$

$$\frac{i_{E2}}{i_{E1}+i_{E2}} = \frac{1}{1+e^{(v_{B1}-v_{B2})/V_T}}$$

$$i_{E1}+i_{E2} = I$$

$$i_{E1} = \frac{I}{1+e^{-v_{id}/V_T}}$$

$$i_{E2} = \frac{I}{1+e^{v_{id}/V_T}}$$



$$i_{E1} = \frac{I}{1+e^{-v_{id}/V_T}} \Rightarrow \frac{i_{C1}}{I} \approx \frac{1}{1+e^{-v_{id}/V_T}}$$

$$i_{E2} = \frac{I}{1+e^{v_{id}/V_T}} \Rightarrow \frac{i_{C2}}{I} \approx \frac{1}{1+e^{v_{id}/V_T}}$$

How to enhance linear region？

Figure 7.14 Transfer characteristics of the BJT differential pair of Fig. 7.12 assuming α . 1.

$$R_e \uparrow \rightarrow v_E \uparrow \rightarrow v_{BE} \downarrow \rightarrow i_C \downarrow$$

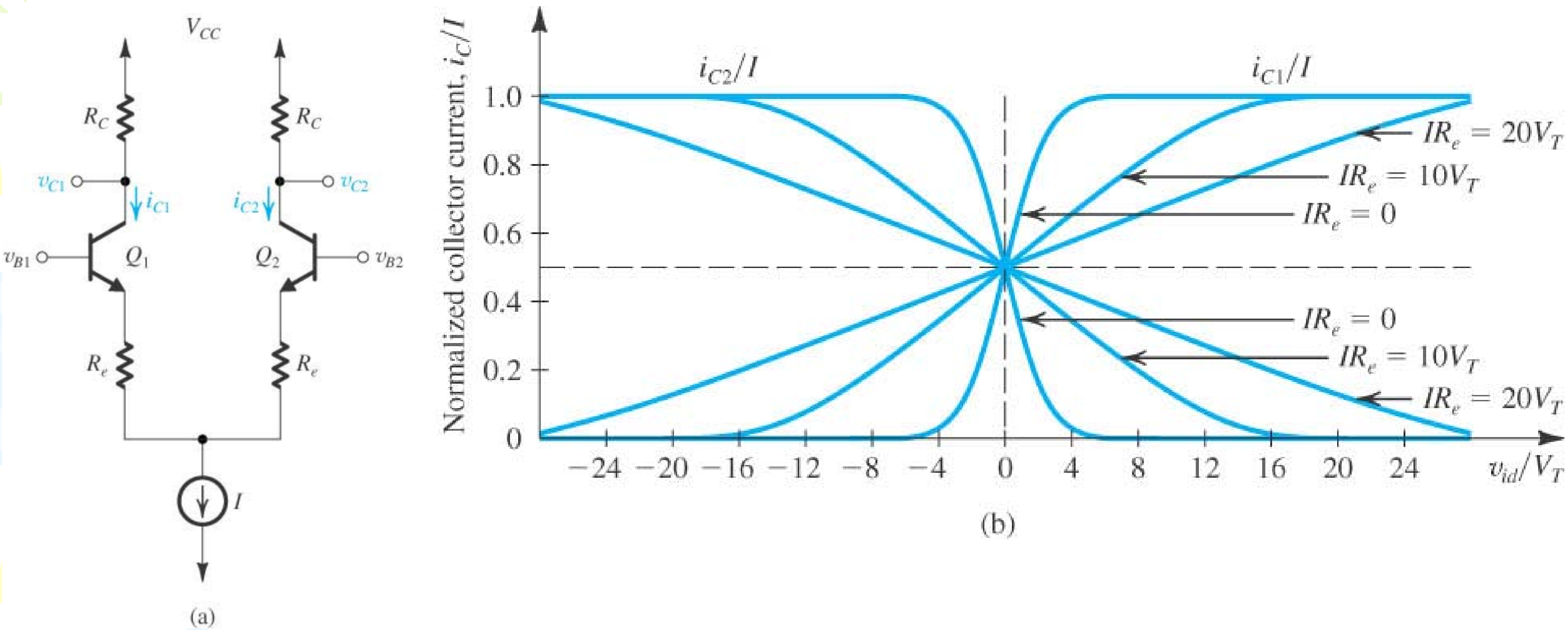


Figure 7.15 The transfer characteristics of the BJT differential pair **(a)** can be linearized **(b)** (i.e., the linear range of operation can be extended) by including resistances in the emitters.

large signal analysis (AC+DC)

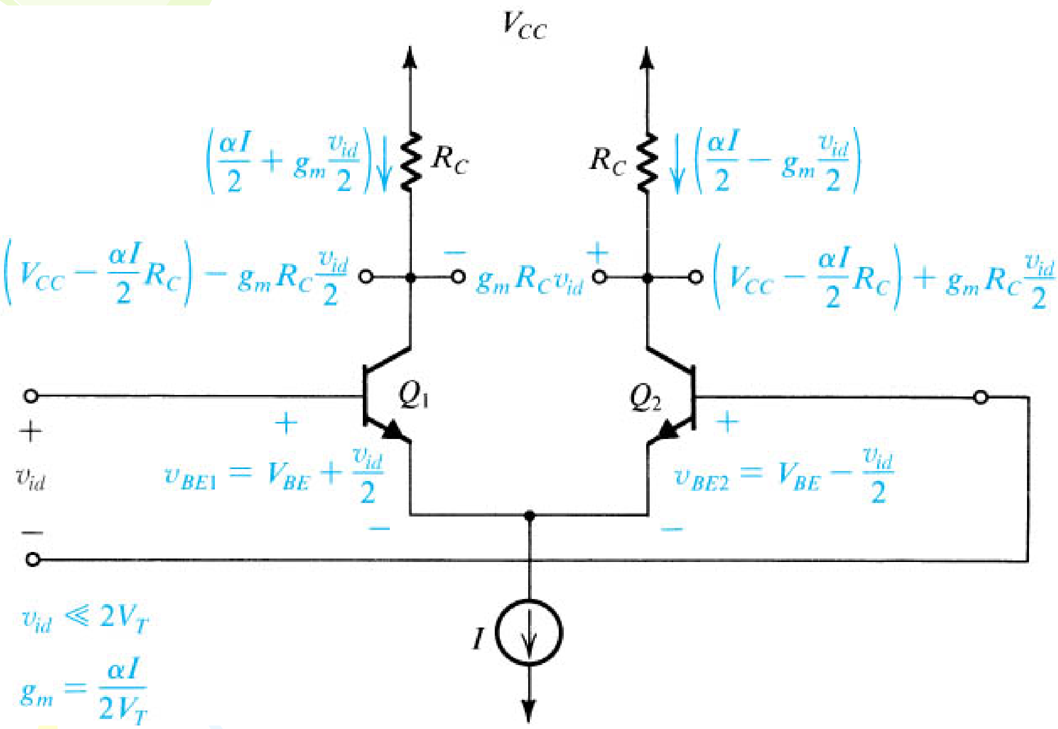


Figure 7.16 The currents and voltages in the differential amplifier when a small differential input signal v_{id} is applied.

$$i_{C1} = \frac{\alpha I}{1 + e^{-v_{id}/V_T}} \cdots (1)$$

$$i_{C2} = \frac{\alpha I}{1 + e^{v_{id}/V_T}}$$

$$(1) \times \frac{e^{v_{id}/2V_T}}{e^{v_{id}/2V_T}} \Rightarrow i_{C1} = \frac{\alpha I e^{v_{id}/2V_T}}{e^{v_{id}/2V_T} + e^{-v_{id}/2V_T}}$$

$$let \quad v_{id} << 2V_T$$

$$\Rightarrow i_{C1} \approx \frac{\alpha I \left(1 + \frac{v_{id}}{2V_T}\right)}{1 + \frac{v_{id}}{2V_T} + 1 - \frac{v_{id}}{2V_T}}$$

Taylor series

$$\Rightarrow i_{C1} = \frac{\alpha I}{2} + \frac{\alpha I}{2V_T}\frac{v_{id}}{2} = I_C + \frac{I_C}{V_T}\frac{v_{id}}{2}$$

$$\Rightarrow i_{C2} = \frac{\alpha I}{2} - \frac{\alpha I}{2V_T}\frac{v_{id}}{2} = I_C - \frac{I_C}{V_T}\frac{v_{id}}{2}$$

$$i_c = \frac{\alpha I}{2V_T}\frac{v_{id}}{2} = \frac{g_m}{2} V_{id} \quad \text{AC}$$

$$v_{BE}\Big|_{Q_1} = V_{BE} + \frac{v_{id}}{2}$$

$$v_{BE}\Big|_{Q_2} = V_{BE} - \frac{v_{id}}{2}$$

Small signal analysis (AC)

$$g_m = I_C / V_T = \frac{\alpha I / 2}{V_T}$$

$$r_e = V_T / I_E = \frac{V_T}{I / 2}$$

$$i_e = \frac{v_{id}}{2r_e}$$

$$i_c = \alpha i_e = \frac{\alpha v_{id}}{2r_e} = g_m \frac{v_{id}}{2}$$

$$i_e = \frac{v_{id}}{2r_e}$$

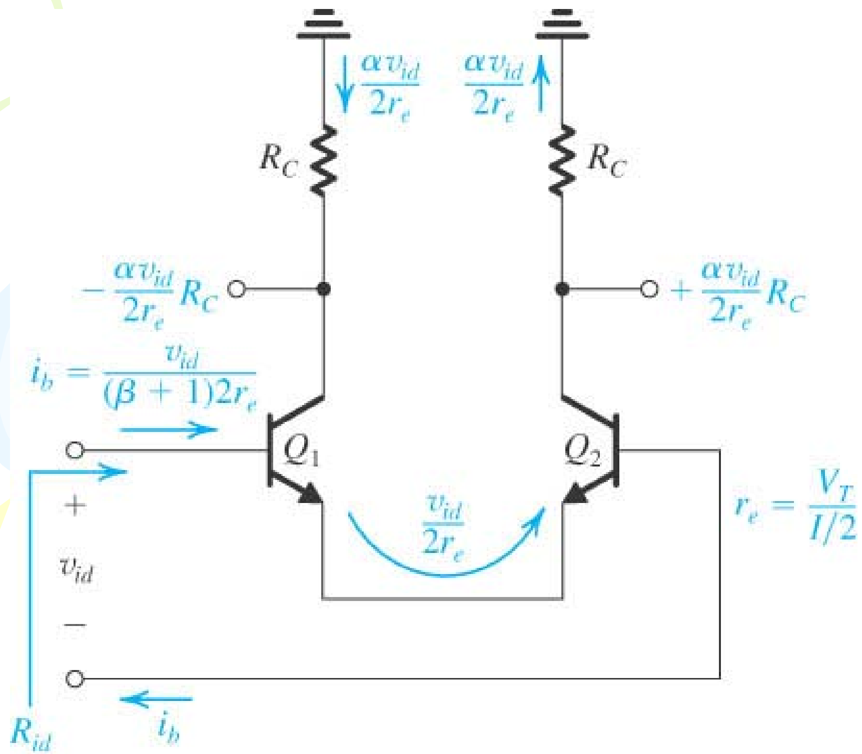
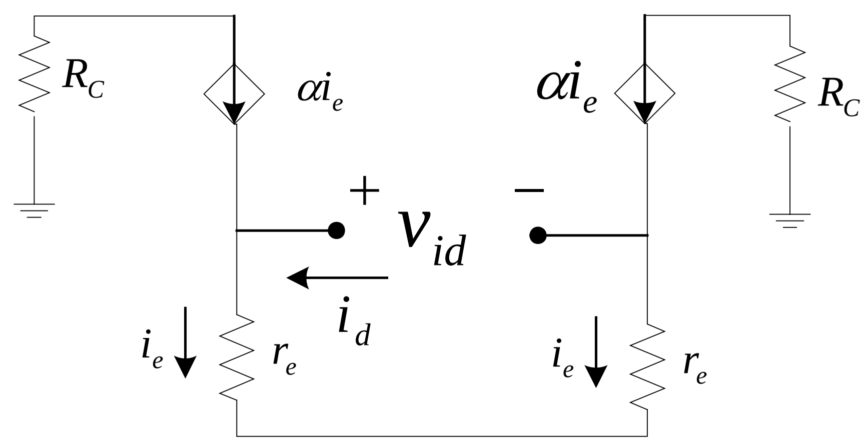
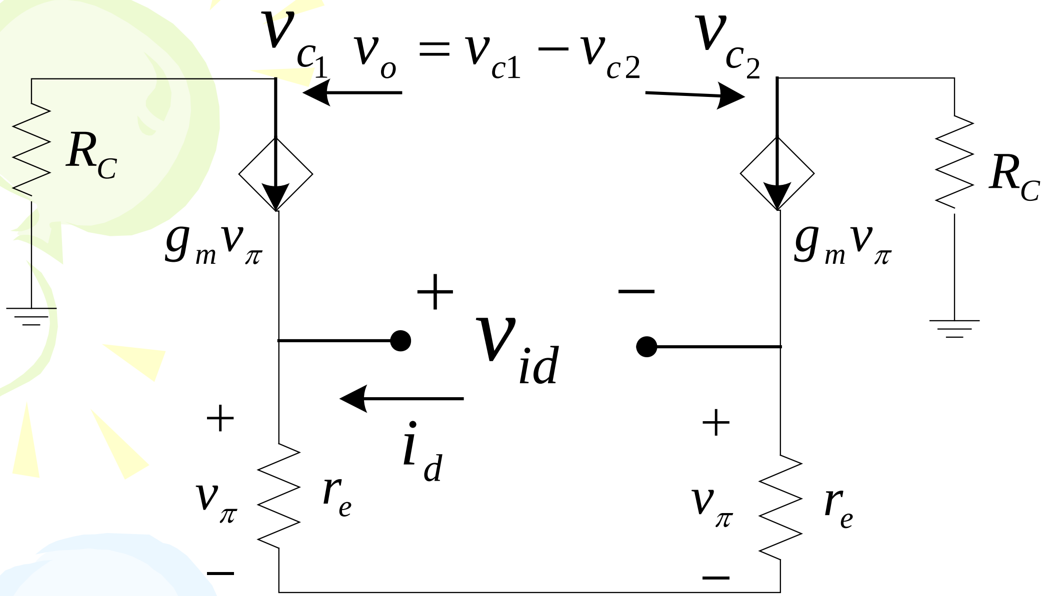


Figure 7.17 A simple technique for determining the signal currents in a differential amplifier excited by a differential voltage signal v_{id}; dc quantities are not shown.





$$R_{id} = \frac{v_{id}}{i_d} = \frac{2r_e i_e}{i_e / (1+\beta)} = 2(1+\beta)r_e$$

$\boxed{\text{Input differential resistance}}$

$$v_{C1} = -g_m R_C \frac{v_{id}}{2}$$

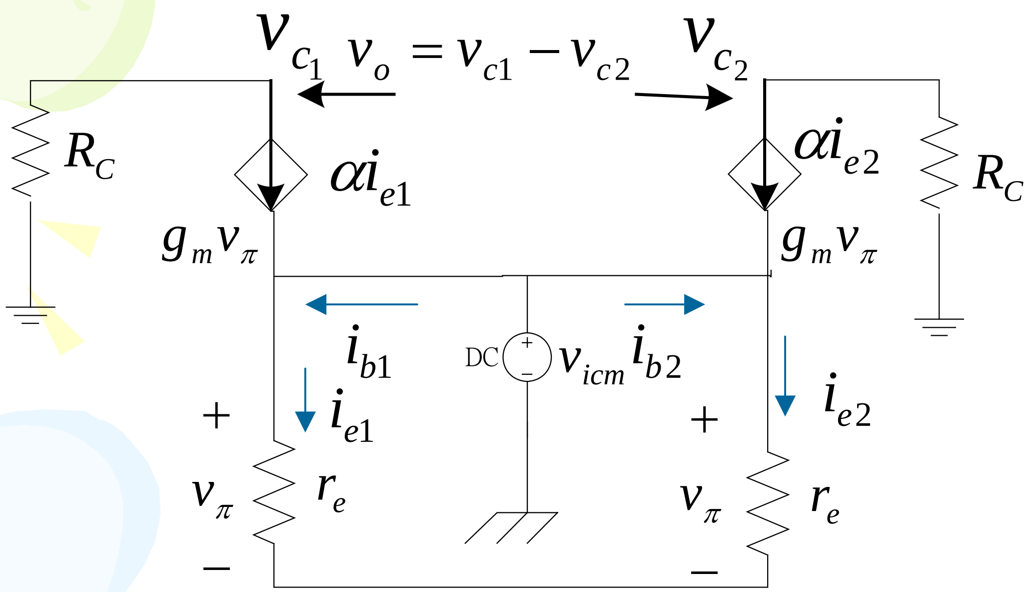
$$v_{C2} = +g_m R_C \frac{v_{id}}{2}$$

$$g_m = \frac{I_C}{V_T} = \frac{\alpha I_E}{V_T} = \frac{\alpha}{r_e}$$

$$A_d = \frac{v_{c1} - v_{c2}}{v_{id}} = -g_m R_C$$

$\boxed{\text{Differential voltage gain}}$

Common mode



$$v_{c1} = -\alpha i_{e_1} R_C$$

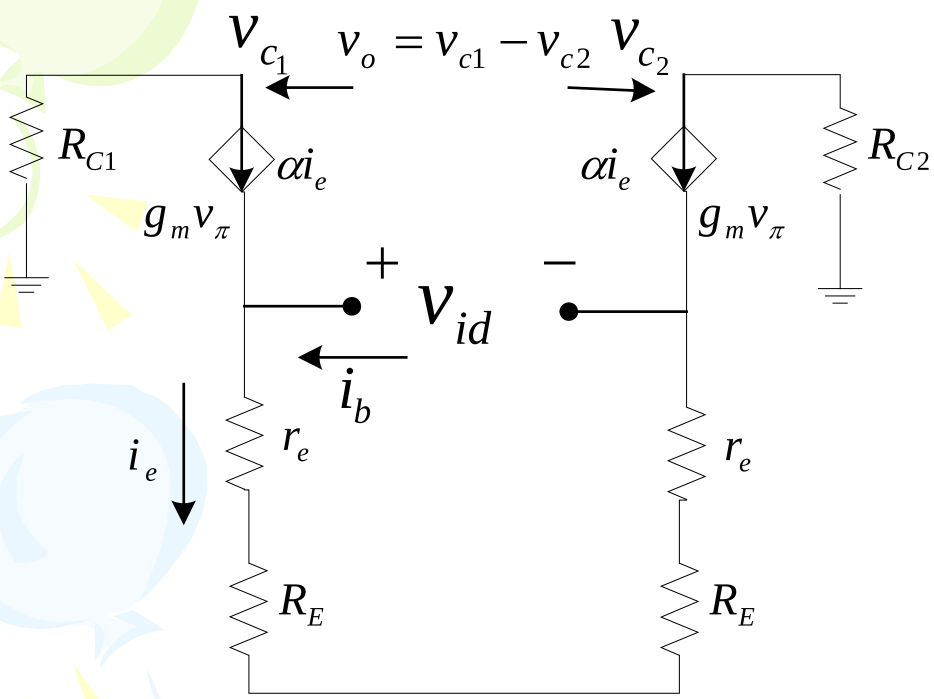
$$v_{c2} = -\alpha i_{e_2} R_C$$

$$v_o = v_{c1} - v_{c2} = -\alpha R_c (i_{e_1} - i_{e_2})$$

$$\therefore i_{e_1} = i_{e_2} = 0 \Rightarrow v_o = -\alpha R_c (i_{e_1} - i_{e_2}) = o$$

$$\boxed{if \quad R_{C1} \neq R_{C2} \Rightarrow v_o \neq 0}$$

External emitter resistance R_E



$$i_e = \frac{v_{id}}{2r_e + 2R_E}$$

$$i_b = \frac{i_e}{\beta+1} = \frac{v_{id}/(2r_e + 2R_E)}{\beta+1}$$

$$R_{id} = \frac{v_{id}}{i_b} = (\beta+1)(2r_e + 2R_E)$$

Input differential resistance

$$v_{C1} = -g_m R_C \frac{v_{id}}{2}$$

$$v_{C2} = +g_m R_C \frac{v_{id}}{2}$$

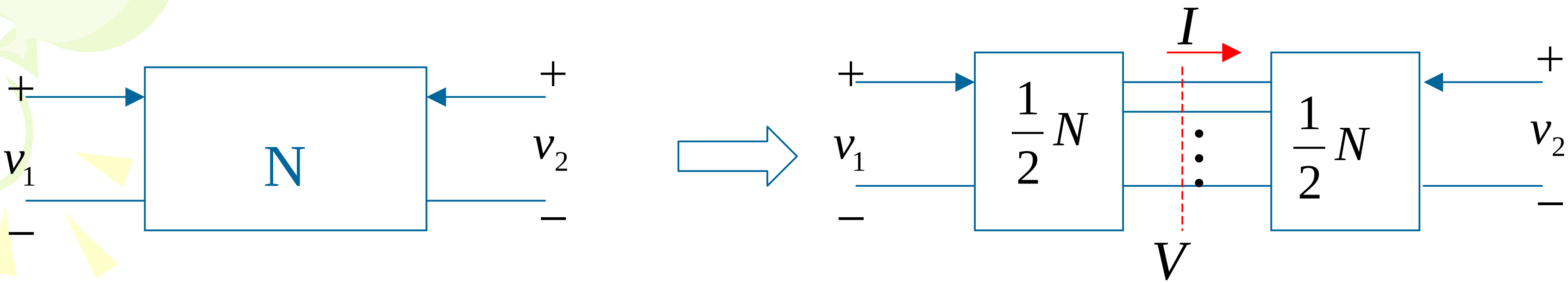
$$g_m = \frac{I_c}{V_T} = \frac{\alpha}{r_e + R_E}$$

Differential voltage gain

$$A_d = \frac{v_{c1} - v_{c2}}{v_{id}} = -g_m R_C$$

$$\boxed{\begin{array}{l} R_E \uparrow \Rightarrow R_{id} \uparrow \\ R_E \uparrow \Rightarrow A_d \downarrow \end{array}}$$

Bartlett Bisection theorem

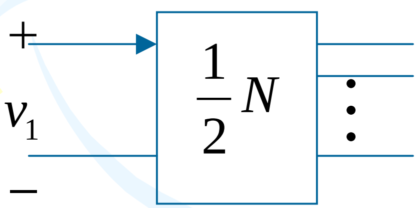


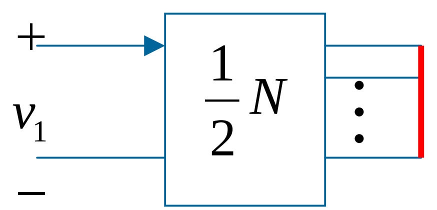
1. $v_1 = v_2 \rightarrow I = 0$

2. $v_1 = -v_2 \rightarrow V = 0$

Common Mode

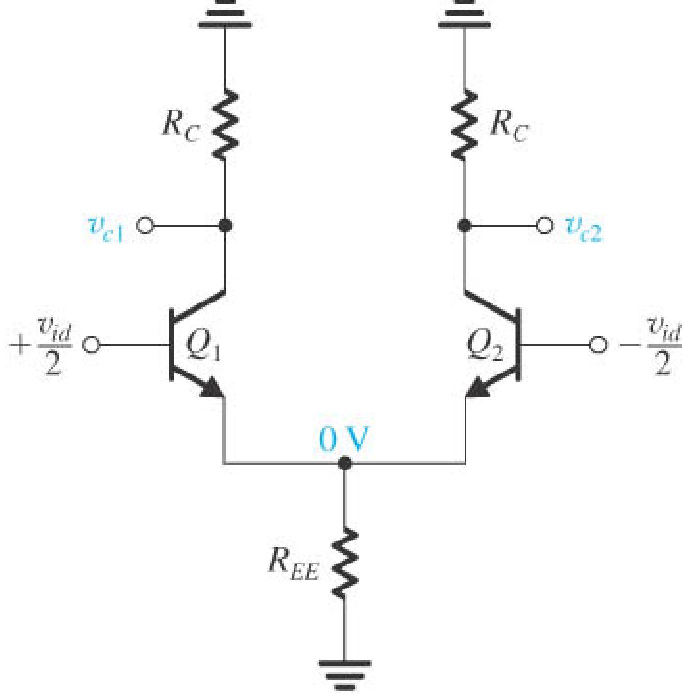
Differential Mode





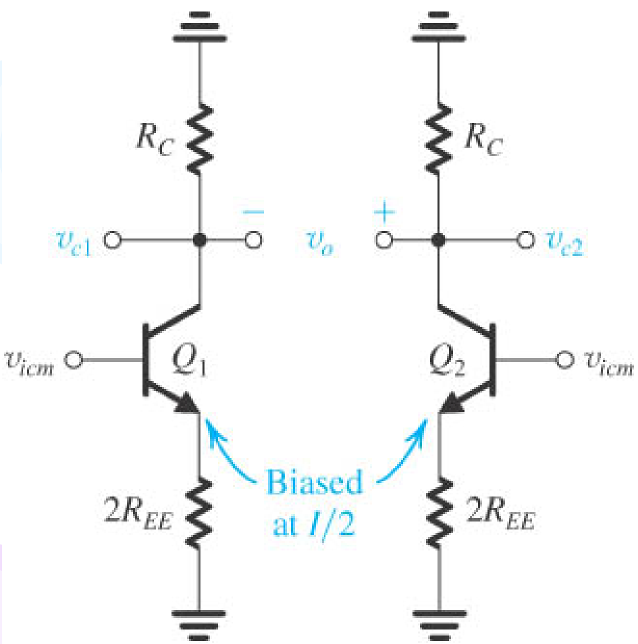
I=0 open circuit
Common-mode

V=0 short circuit
Differential-mode



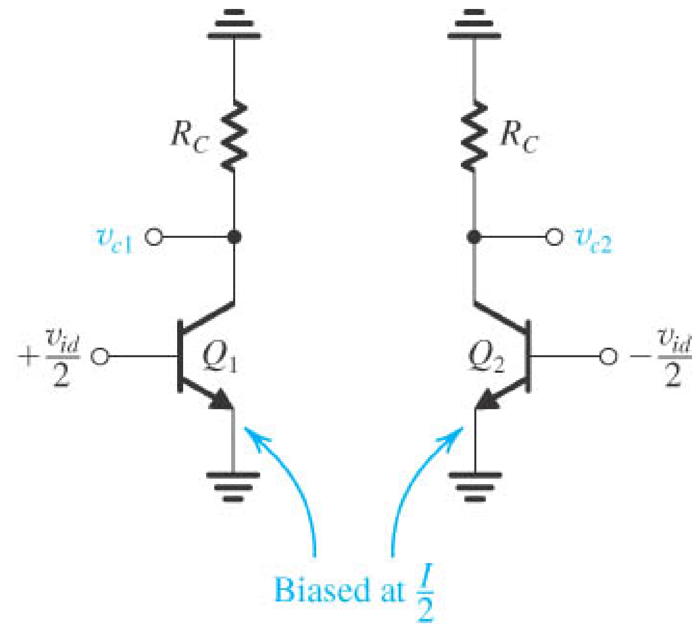
(a)

Common Mode



(b)

Differential Mode



(b)

Equivalence of the differential amplifier to a CE amplifier

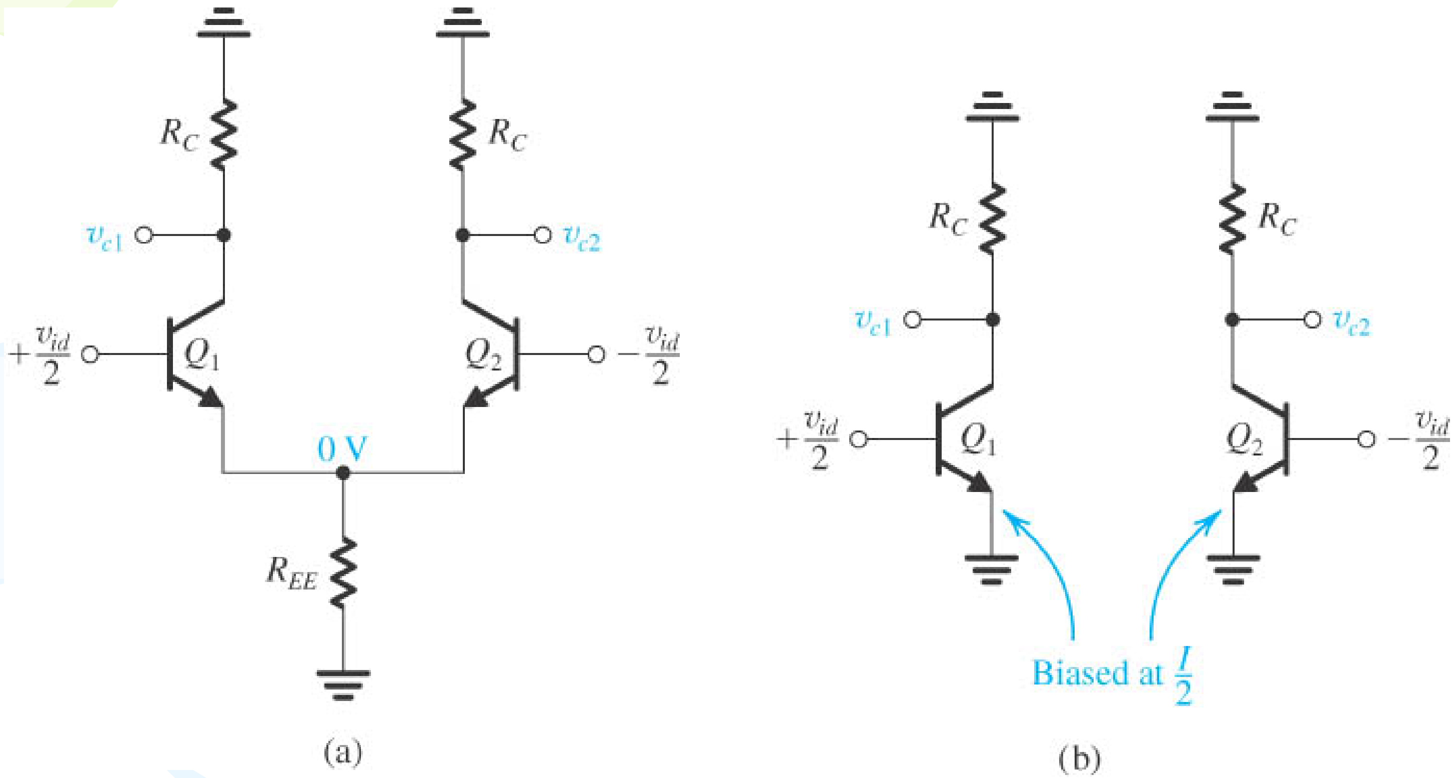
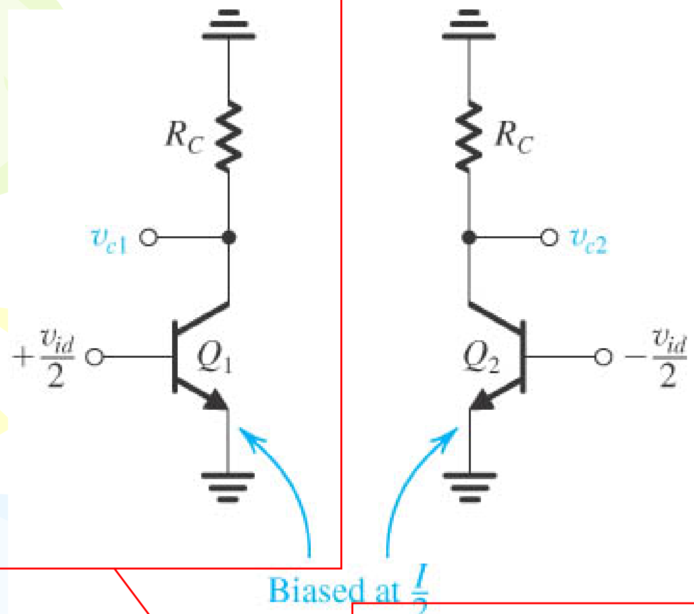


Figure 7.19 Equivalence of the BJT differential amplifier in **(a)** to the two common-emitter amplifiers in (b). This equivalence applies only for differential input signals. Either of the two common-emitter amplifiers in **(b)** can be used to find the differential gain, differential input resistance, frequency response, and so on, of the differential amplifier.



(b)

Differential half-circuit

$$\frac{V_{c1}}{V_{id}} = -\frac{g_m}{2}(r_o // R_c)$$

$$\frac{V_{c2}}{V_{id}} = \frac{g_m}{2}(r_o // R_c)$$

$$A_d = \frac{v_{c1}}{v_{id}/2} = -g_m(R_C // r_o)$$

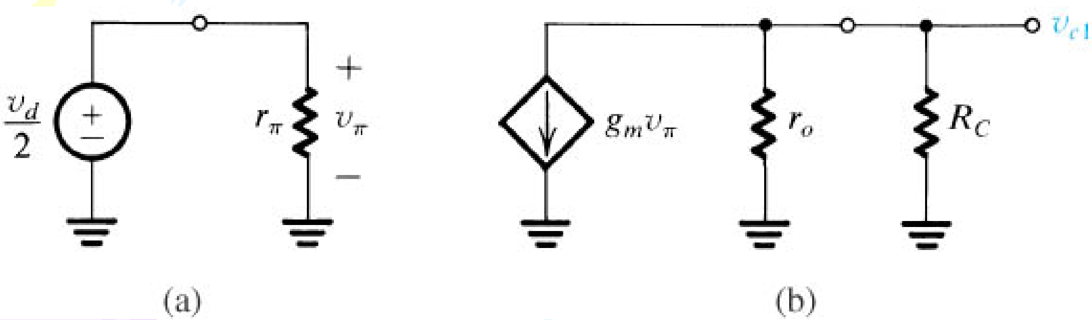


Figure 7.21 **(a)** The differential half-circuit and **(b)** its equivalent circuit model.

$$i = \frac{V_\pi}{r_\pi}$$

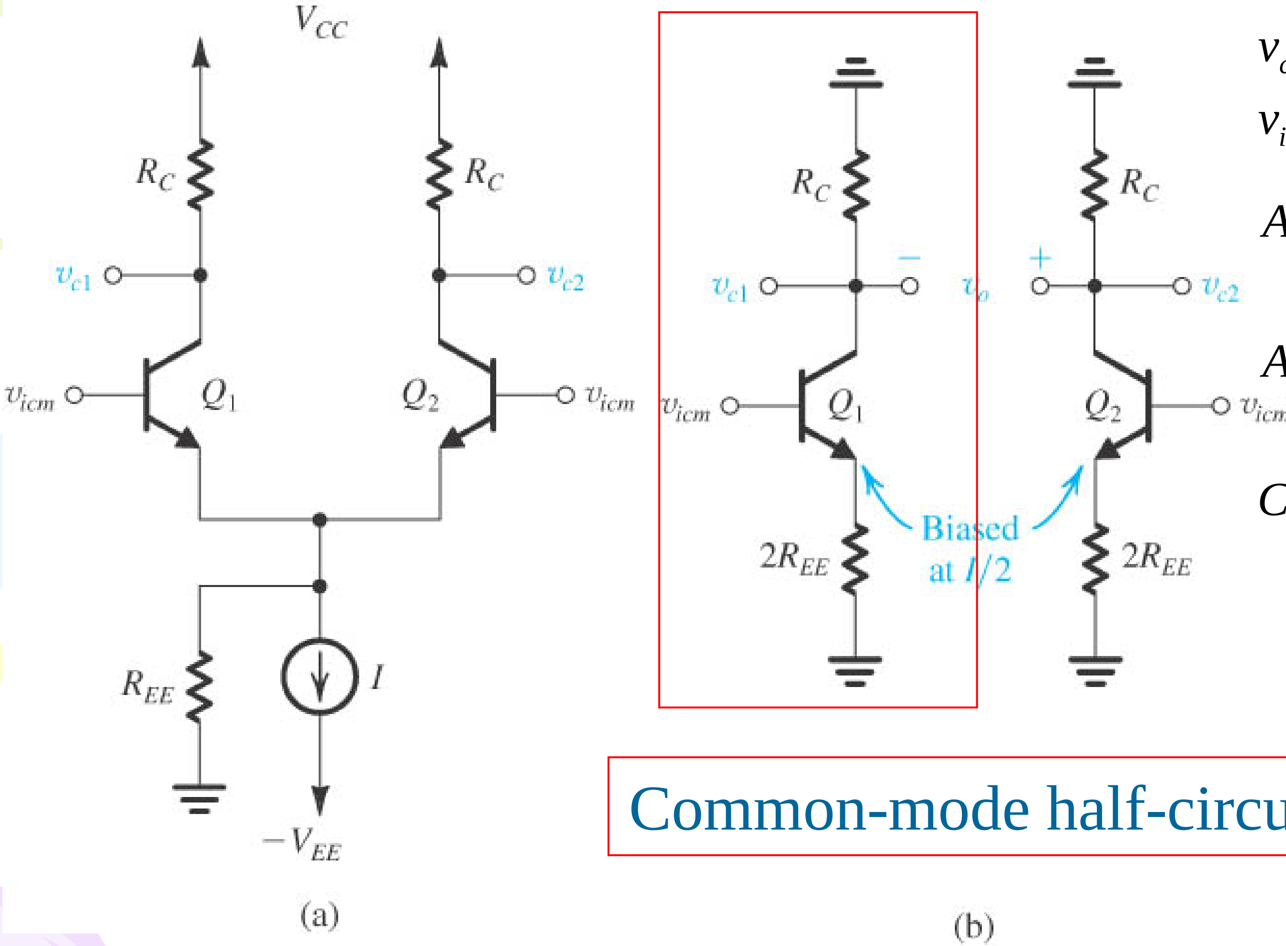
$$\frac{V_{id}}{2} = V_\pi$$

$$R_{id} = \frac{V_{id}}{i} = \frac{2V_\pi}{V_\pi / r_\pi} = 2r_\pi$$

$\boxed{2(1+\beta)r_e}$

Input differential resistance

Common mode gain et CMRR (I=0→ open circuit)



$$v_{c1} = -\alpha i_e R_c$$

$$v_{icm} = i_e (r_e + 2R_{EE})$$

$$A_{cm}\Big|_{\frac{1}{2}} = -\frac{\alpha R_C}{(r_e + 2R_{EE})} \approx -\frac{\alpha R_C}{(2R_{EE})}$$

$$A_d = \frac{1}{2} g_m R_C$$

$$CMRR\Big|_{\frac{1}{2}} = \left|\frac{A_d}{A_{cm}}\right| \approx g_m R_{EE}$$

Common-mode half-circuit

Figure 7.22 **(a)** The differential amplifier fed by a common-mode voltage signal v_{icm}. **(b)** Equivalent "half-circuits" for common-mode calculations.

Common mode gain at CMRR (Asymmetric case)

$$v_{c1} = -\alpha i_e R_C$$

$$v_{c2} = -\alpha i_e (R_C + \Delta R_C)$$

$$v_o = v_{c1} - v_{c2} = -\alpha i_e \Delta R_C$$

$$v_{icm} = i_e (2R_{EE} + r_e)$$

$$A_{cm} = \frac{-\alpha \Delta R_C}{2R_{EE} + r_e} \approx \frac{\Delta R_C}{2R_{EE}} = \boxed{-\frac{\alpha R_C}{2R_{EE}} \frac{\Delta R_C}{R_C}}$$

$$v_{icm} \equiv \frac{v_1 + v_2}{2}$$

$$v_{id} \equiv v_1 - v_2$$

$$v_o = A_d (v_1 - v_2) + A_{cm} (\frac{v_1 + v_2}{2})$$

Last page

$$v_{c1} = -v_{icm} \frac{\alpha R_C}{2R_{EE} + r_e} \approx -v_{icm} \frac{\alpha R_C}{2R_{EE}}$$

$$v_{c2} \approx -v_{icm} \frac{\alpha R_C}{2R_{EE}}$$

<<

$$A_{cm} = \boxed{-\frac{\alpha R_C}{2R_{EE}}}$$

$$A_d = \frac{1}{2} g_m R_C$$

$$CMRR = \left| \frac{A_d}{A_{cm}} \right| \approx g_m R_{EE}$$

Common-mode input resistance

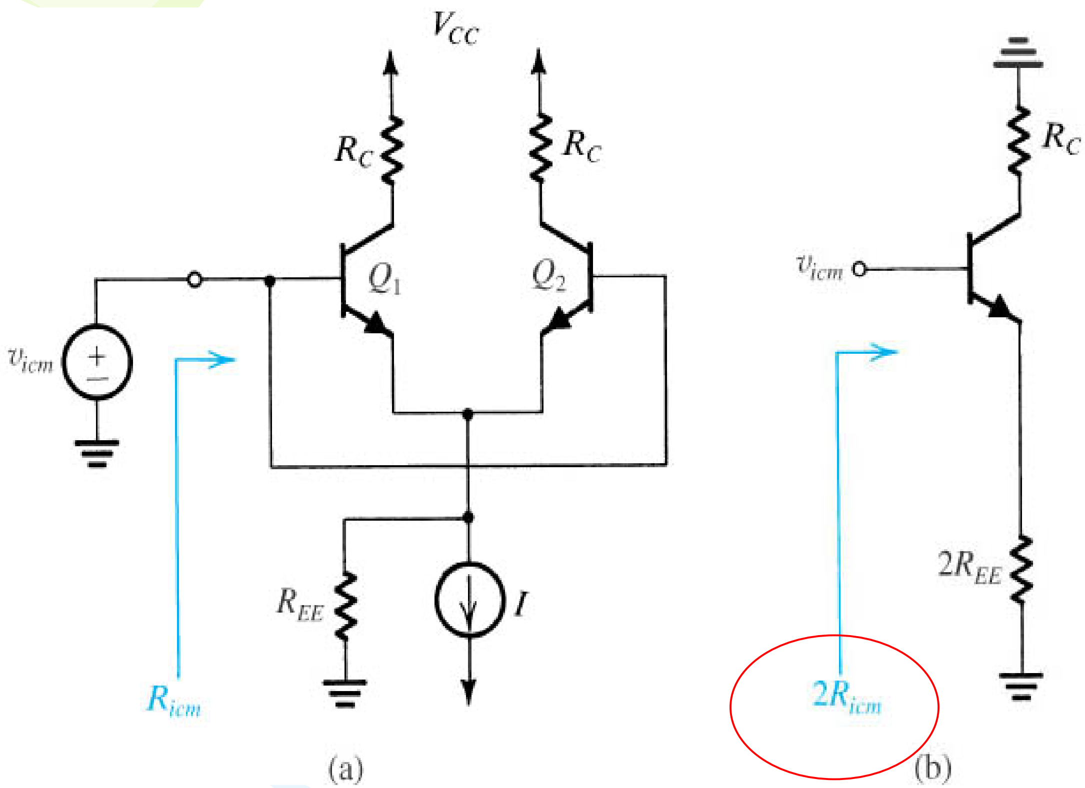


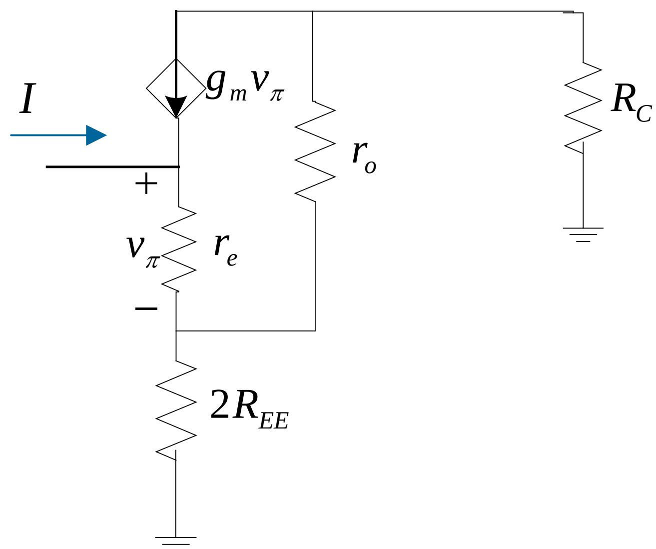
Figure 7.23 **(a)** Definition of the input common-mode resistance R_{icm}. (b) The equivalent common-mode half-circuit.

$$V = i_e(r_e + 2R_{EE})$$

$$I = i_b = \frac{i_e}{1+\beta}$$

$$2R_{icm} \approx (\beta+1)(2R_{EE} // r_o)$$

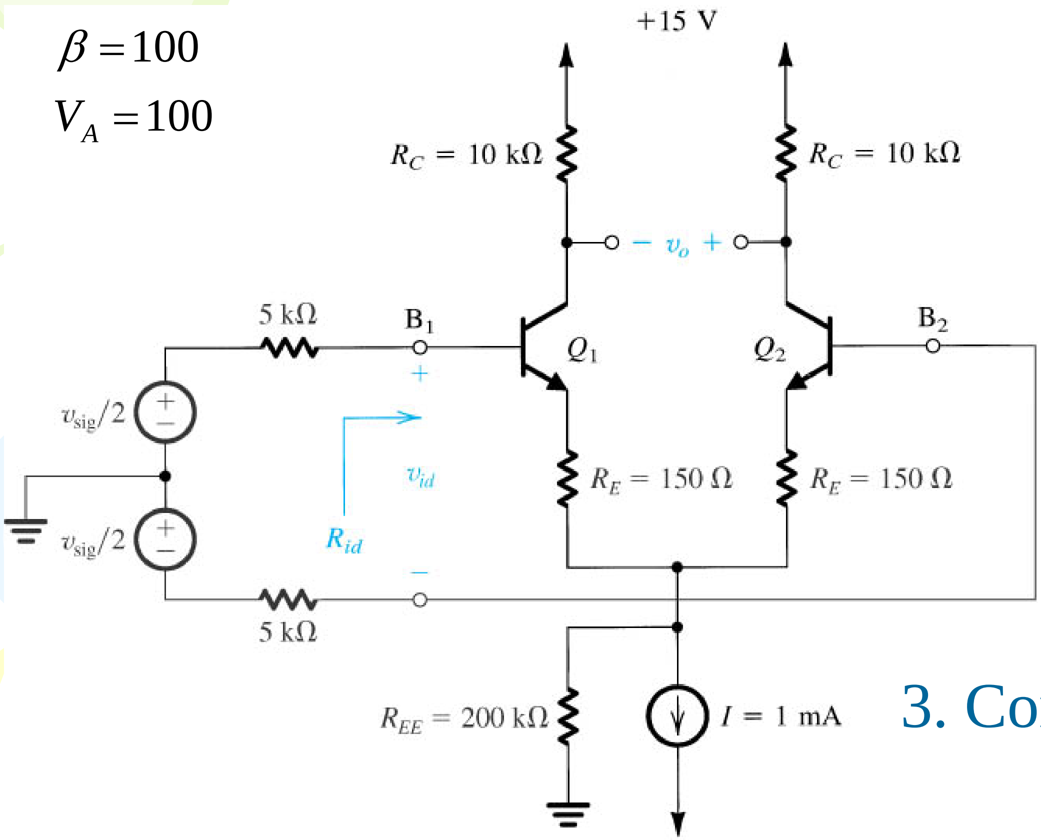
$$R_{icm} \approx (\beta+1)(R_{EE} // \frac{r_o}{2})$$



Example 7.1

$\beta = 100$

$V_A = 100$



1. Input differential resistance

$$r_{e1} = r_{e2} = \frac{V_T}{I_E} = \frac{25mV}{0.5mA} = 50$$

$$R_{id} = 2(\beta + 1)(r_e + R_E) = 40k$$

2. Differential voltage gain

$$A_d = \frac{v_o}{v_{id}} \frac{v_{id}}{v_s} = \frac{2R_C}{2(r_e + R_E)} \frac{R_{id}}{R_s + R_{id}} = 40$$

3. Common-mode gain in worst case

$$A_{cm} = \frac{R_C}{2R_{EE} + (r_e + R_E)} \frac{\Delta R_C}{R_C}$$

$$\Delta R_C = 0.02 R_C$$

$$A_{cm} = 5 \times 10^{-4}$$

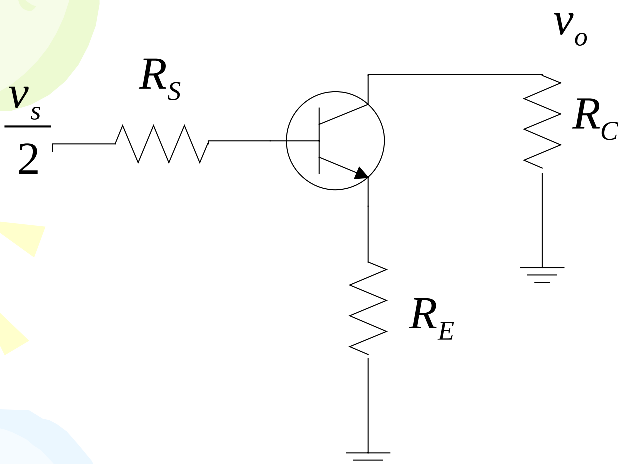
$$CMRR = 20 \log \left| \frac{A_d}{A_{cm}} \right| = 98dB$$

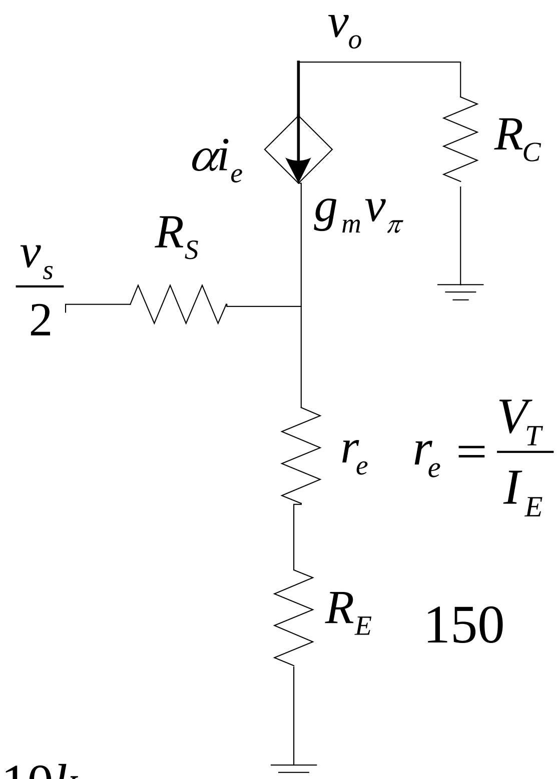
4. Input common-mode resistance

$$r_o = \frac{V_A}{I/2} = 200k$$

$$R_{icm} \approx (\beta + 1)(R_{EE} // \frac{r_o}{2}) = 6.7M$$

Differential mode





$$r_e = \frac{V_T}{I_E} = \frac{25m}{0.5m} = 50$$

$$R_{id\frac{1}{2}} = (1+\beta)(50+150)$$

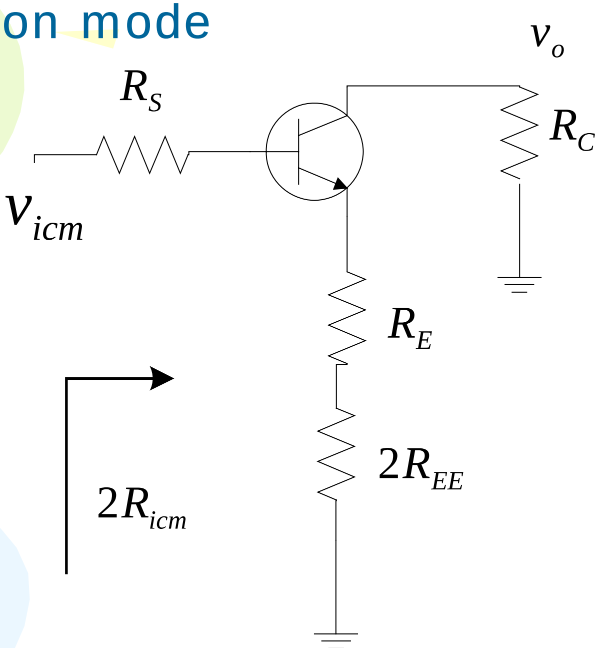
$$R_{id} = 2(1+\beta)(50+150) = 40k$$

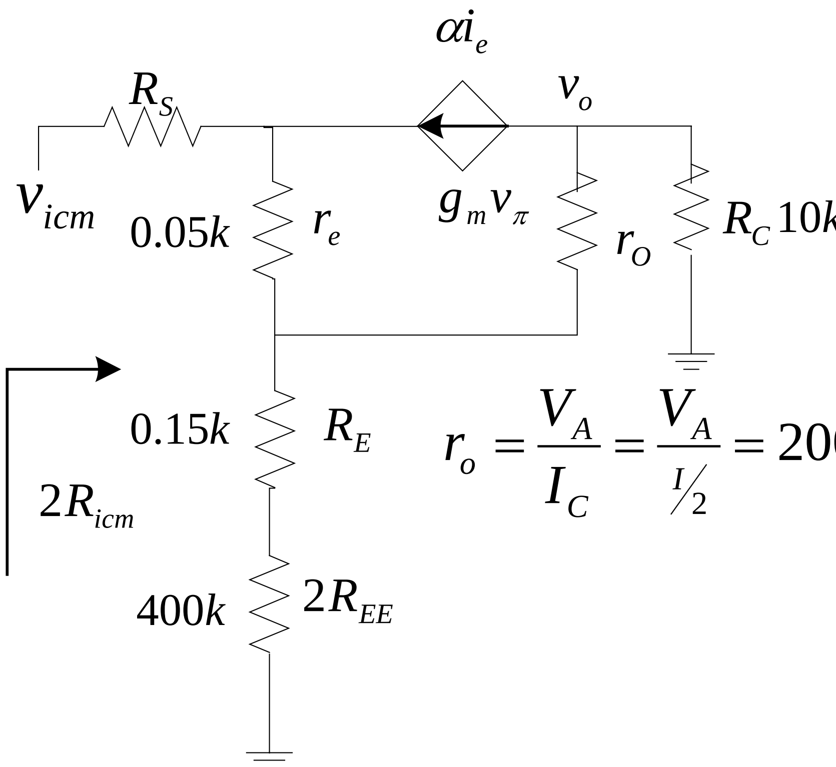
$$A_d = \frac{v_{c1}}{v_s/2} = \frac{-\alpha i_e R_c}{R_s i_b + 0.2 i_e} = \frac{-\alpha(1+\beta)i_b 10k}{5k i_b + 0.2k(1+\beta)i_b}$$

$$\frac{v_{c1}}{v_s} = \frac{1}{2}\frac{-\alpha(1+\beta)10k}{5k+0.2k(1+\beta)}$$

$$\frac{v_{c1}-v_{c2}}{v_s} = 2\frac{v_{c1}}{v_s} = \frac{-\alpha(1+\beta)10k}{5k+0.2k(1+\beta)} = 40$$

Common mode





$$r_o = \frac{V_A}{I_C} = \frac{V_A}{I/2} = 200k$$

$$if \quad R_{C1} = R_c + 1\% \neq R_{C2} = R_c - 1\%$$

$$v_{c1} = -\beta i_b (R_C)$$

$$v_{c2} = -\beta i_b (R_C + \Delta R_C)$$

$$\Delta R_C = R_C \times 0.02$$

$$v_{icm} = 5k i_b + (1+\beta) i_b (400.2k)$$

$$A_{cm} = \frac{v_{c1} - v_{c2}}{v_{icm}} = \frac{\beta(\Delta R_C)}{5k + (1+\beta)(400.2k)}$$

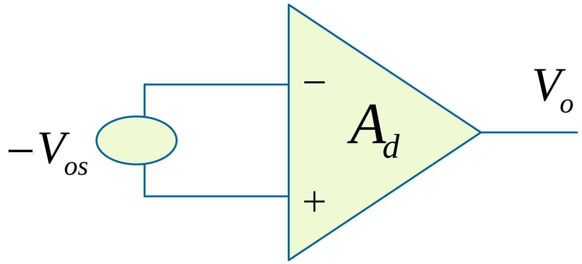
$$2R_{icm} \approx (1+\beta)(400k \,//\, 200k)$$

$$R_{icm} \approx \frac{1}{2}(1+\beta)(400k \,//\, 200k) = 6.7M$$

7-4.2 Input offset voltage

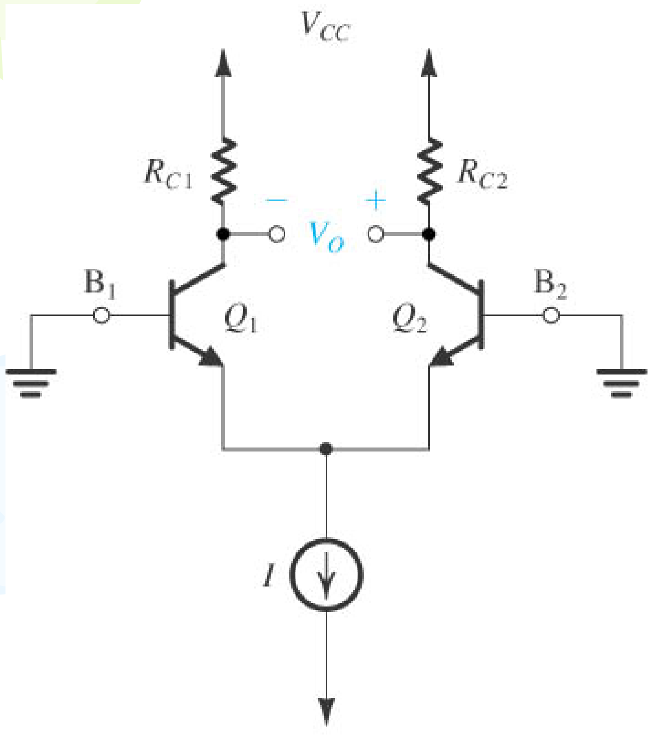
$$\boxed{V_{os} \equiv \frac{V_o\big|_{v_i=0}}{A_d}}$$

Solution : Add a $-V_{os}$



$$V_o = V_{os}A_d + (-V_{os})A_d = 0$$

7-4.2 Input offset voltage



(a)

Case 1 : different R_C

Case 2 : different Q

$$\text{let}\quad R_{C1} \neq R_{C2}, Q_1 = Q_2$$

$$R_{C1} = R_C + \frac{\Delta R_C}{2}$$

$$R_{C2} = R_C - \frac{\Delta R_C}{2}$$

$$V_{C1} = V_{CC} - \left(\frac{\alpha I}{2}\right)\left(R_C + \frac{\Delta R_C}{2}\right)$$

$$V_{C2} = V_{CC} - \left(\frac{\alpha I}{2}\right)\left(R_C - \frac{\Delta R_C}{2}\right)$$

$$V_o = V_{C1} - V_{C2} = \frac{\alpha I}{2}\Delta R_C$$

$$V_{os} \equiv \frac{V_o}{A_d} = \frac{\frac{\alpha I}{2}\Delta R_C}{g_m R_C} = \frac{\frac{\alpha I}{2}\Delta R_C}{\frac{I_E}{V_T} R_C}$$

$$\boxed{V_{os} = V_T \frac{\Delta R_C}{R_C}}$$

$$consider \quad Q_1 \neq Q_2 \Rightarrow I_{S1} \neq I_{S2}$$

$$I_C = I_S e^{V_{BE}/V_T}$$

internal

$$I_{S1} = I_S + \frac{\Delta I_S}{2}$$

$$I_{S2} = I_S - \frac{\Delta I_S}{2}$$

$$\because V_{BE1} = V_{BE2}$$

$$\therefore I_{E1} = \frac{I}{2}(1 + \frac{\Delta I_S}{2I_S})$$

$$\therefore I_{E2} = \frac{I}{2}(1 - \frac{\Delta I_S}{2I_S})$$

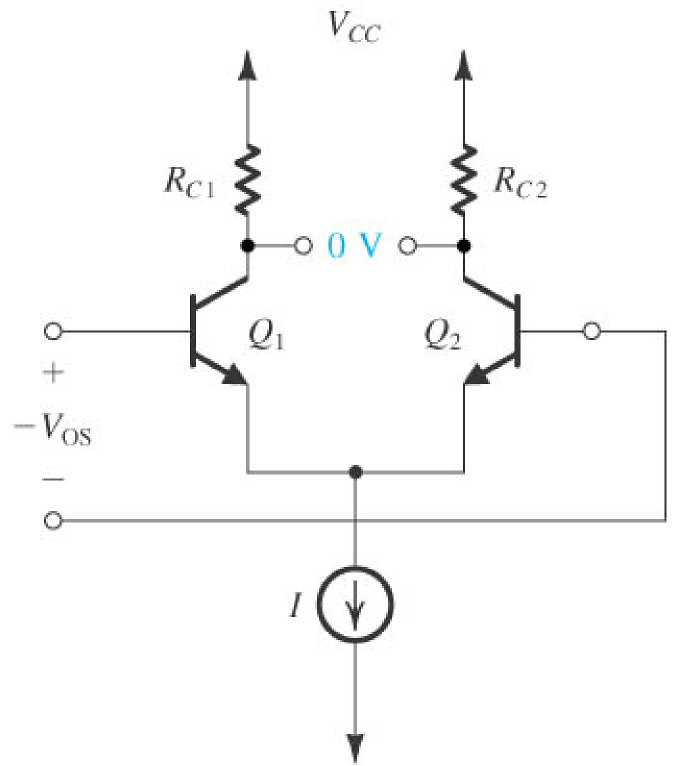
$$\Rightarrow V_O = \alpha \frac{I}{2}\frac{\Delta I_S}{I_S}R_C$$

$$|V_{os}| = V_T(\frac{\Delta I_S}{I_S})$$

Consider Q and R_C

$$\boxed{V_{os} = \sqrt{(V_T \frac{\Delta R_C}{R_C})^2 + (V_T \frac{\Delta I_S}{I_S})^2}}$$

Solution : Add a $-V_{os}$



(b)

Input offset current

$$I_{B1} = I_{B2} = \frac{I/2}{\beta+1} \cdots symmetric \quad case$$

$$let \quad \beta_1 \neq \beta_2 \Rightarrow I_{B1} \neq I_{B2}$$

$$I_{os} = \left| I_{B1} - I_{B2} \right|$$

$$let \quad \beta_1 = \beta + \frac{\Delta\beta}{2}, \beta_2 = \beta - \frac{\Delta\beta}{2}$$

$$\Rightarrow I_{B1} = \frac{I_{E1}}{(1+\beta_1)} = \frac{I}{2}\frac{1}{\beta+1+\Delta\beta/2} \approx \frac{I}{2}\frac{1}{\beta+1}(1-\frac{\Delta\beta}{2\beta})$$

$$\Rightarrow I_{B2} = \frac{I_{E2}}{(1+\beta_2)} = \frac{I}{2}\frac{1}{\beta+1-\Delta\beta/2} \approx \frac{I}{2}\frac{1}{\beta+1}(1+\frac{\Delta\beta}{2\beta})$$

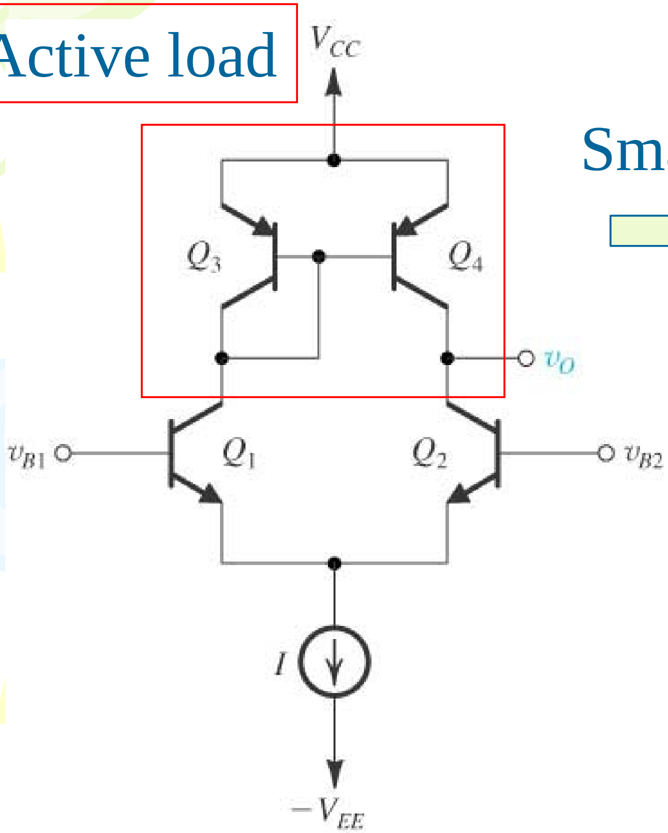
$$I_{os} = \frac{I}{2(\beta+1)}(\frac{\Delta\beta}{\beta})$$

$$\because I_B \equiv \frac{I_{B1}+I_{B2}}{2} = \frac{I}{2(\beta+1)}$$

$$\Rightarrow I_{os} = I_B(\frac{\Delta\beta}{\beta})$$

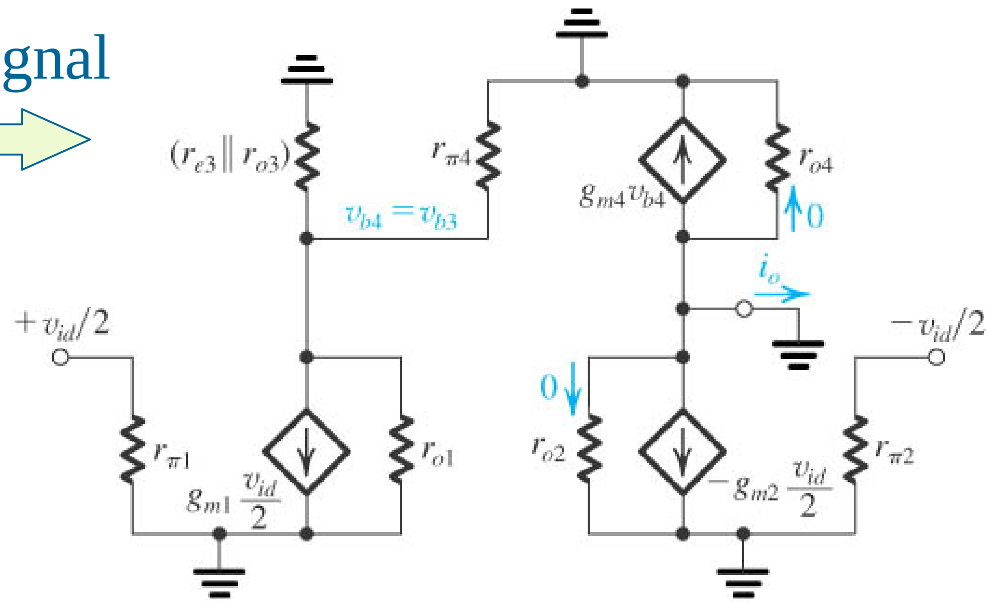
7-5.5 Differential amplifier with active load

Active load



(a)

Small-signal



(b)

Passive load R_C

$$A_d = -g_m R_C$$

$$A_{cm\frac{1}{2}} \approx -\frac{\alpha R_C}{2R_{EE}}$$

$$CMRR \approx g_m R_{EE}$$

$$R_{id} = (1+\beta)(2r_e + 2R_E) = 2r_\pi + 2(1+\beta)R_E$$

$$R_{icm} \approx (1+\beta)(R_{EE} // \frac{r_0}{2})$$

$$R_o = R_C // r_0$$

Active load Q_3 Q_4

$$A_d = -g_m (r_{o4} // r_{o2}) \uparrow$$

$$A_{cm\frac{1}{2}} \approx \frac{r_{o4}}{\beta_3 R_{EE}} \downarrow$$

$$CMRR \approx g_m (r_o // r_{o4}) \frac{\beta_3 R_{EE}}{r_{o4}} \uparrow$$

$$R_{id} = 2r_\pi \downarrow$$

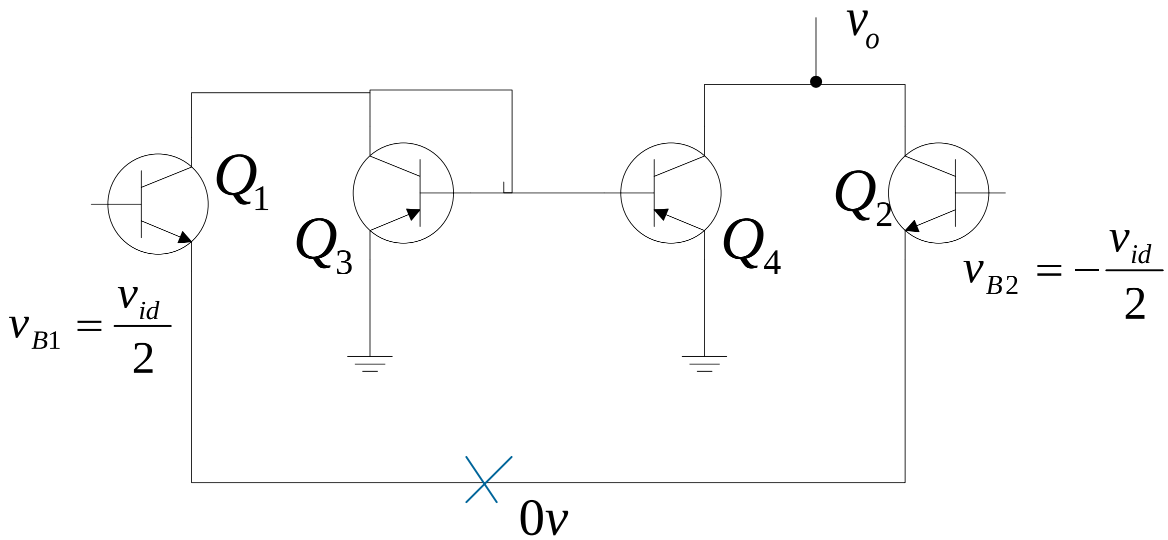
$$R_{icm} \approx$$

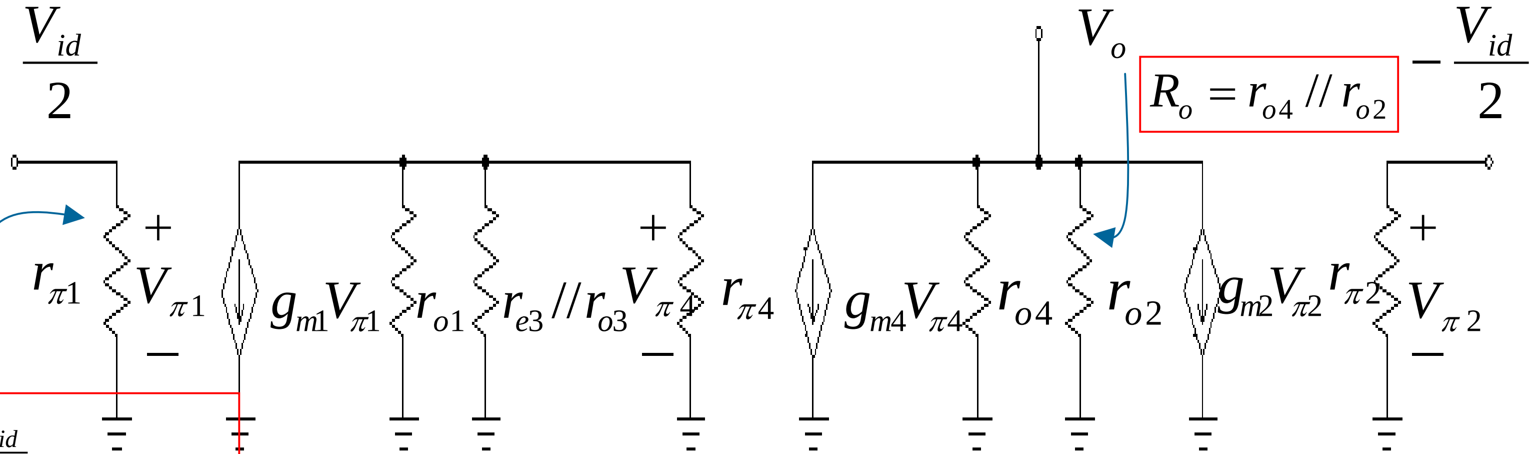
$$R_o = r_{o4} // r_{o2} \uparrow$$

Improving:

1. Differential gain
2. Common-mode gain and CMRR
3. Input offset voltage

Differential amplifier with active load equivalent-circuit Differential-mode





$$R_{id} = \frac{v_{id}}{I}$$

$$\frac{v_{id}}{2} = I r_\pi \Rightarrow R_{id} = 2 r_\pi$$

$$v_{b3} = -g_{m1}\left(\frac{v_{id}}{2}\right)(r_{e3} // r_{o3} // r_{o1} // r_{\pi 4}) \approx -g_{m1} r_{e3}\left(\frac{v_{id}}{2}\right)$$

$$\because v_{b4} = v_{b3} \Rightarrow g_{m4} v_{b4} = -g_{m4} g_{m1} r_{e3}\left(\frac{v_{id}}{2}\right)$$

$$i_o = g_{m2}\left(\frac{v_{id}}{2}\right) - g_{m4} v_{b4}$$

$$\therefore i_o = g_{m2}\left(\frac{v_{id}}{2}\right) + g_{m4} g_{m1} r_{e3} \frac{v_{id}}{2}$$

$$\because g_{m1} = g_{m2} = g_{m4} = g_m$$

$$g_m \approx \frac{I/2}{V_T}$$

$$r_{e3} = \alpha_3 / g_{m3} \approx 1 / g_m$$

$$\Rightarrow G_M = \frac{i_o}{v_{id}} = g_m$$

$$v_o = (-g_m v_{\pi 4} - g_m v_{\pi 2})(r_{04} // r_{o2})$$

$$v_o = [-g_m v_{\pi 4} - g_m(-\frac{v_{id}}{2})](r_{04} // r_{o2})$$

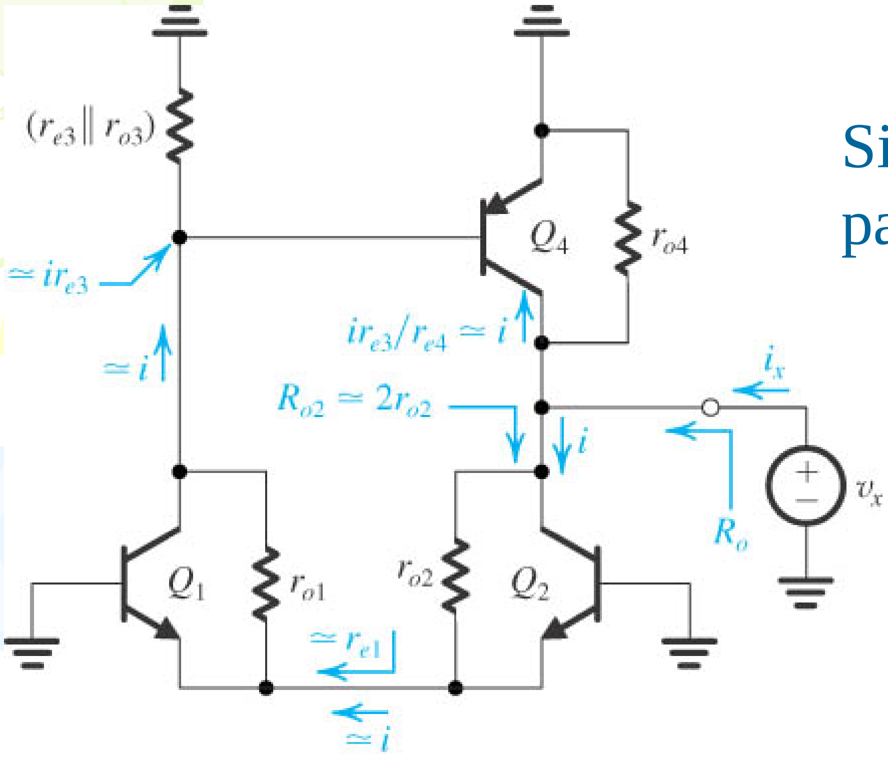
$$v_{\pi 4} = -g_m v_{\pi 1}(r_{01} // r_{e3} // r_{o3} // r_{\pi 4}) \approx -g_m \frac{v_{id}}{2} r_{e3}$$

$$v_o = g_m\left(\frac{v_{id}}{2}\right)(r_{04} // r_{o2})[g_m(r_{01} // r_{e3} // r_{o3} // r_{\pi 4}) + 1]$$

$$v_o \approx g_m\left(\frac{v_{id}}{2}\right)(r_{04} // r_{o2})[g_m r_{e3} + 1]$$

$$A_d = \frac{v_o}{v_{id}} \approx g_m\left(\frac{1}{2}\right)(r_{04} // r_{o2})[\overbrace{g_m r_{e3}}^{1} + 1]$$

$$\boxed{A_d = \frac{v_o}{v_{id}} = g_m(r_{04} // r_{o2})}$$



(c)

$$R_{o2} = r_{o2}[1 + g_{m2}(r_{e1} // r_{\pi 2}]$$
$$\cong r_{o2}(1 + g_{m2} r_{e1}) \approx 2r_{o2}$$

Since four transistors have the same parameters

$$i = \frac{v_x}{R_{o2}} = \frac{v_x}{2r_{o2}}$$

$$i_x = 2i + \frac{v_x}{r_{o4}} = \frac{v_x}{r_{o2}} + \frac{v_x}{r_{o4}}$$

$$\Rightarrow R_o \equiv \frac{v_x}{i_x} = r_{o2} // r_{o4}$$

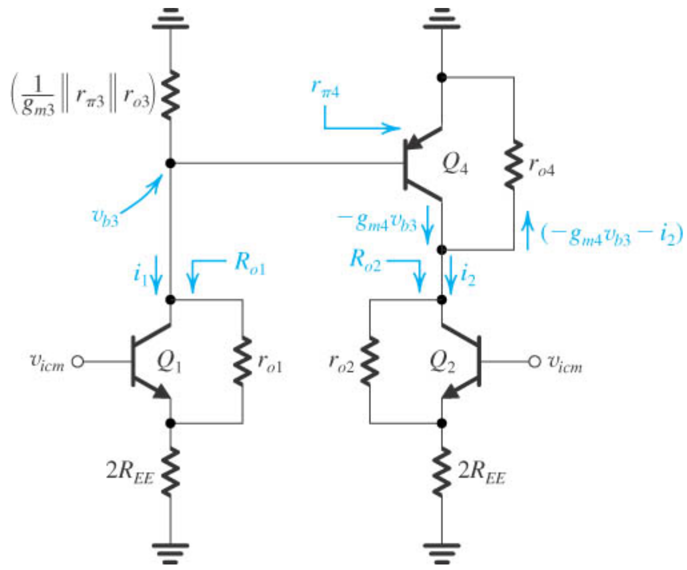
$$A_d \equiv \frac{v_o}{v_{id}} = G_M R_o = g_m (r_{o2} // r_{o4})$$

$$\because r_{o2} = r_{o4} = r_o$$

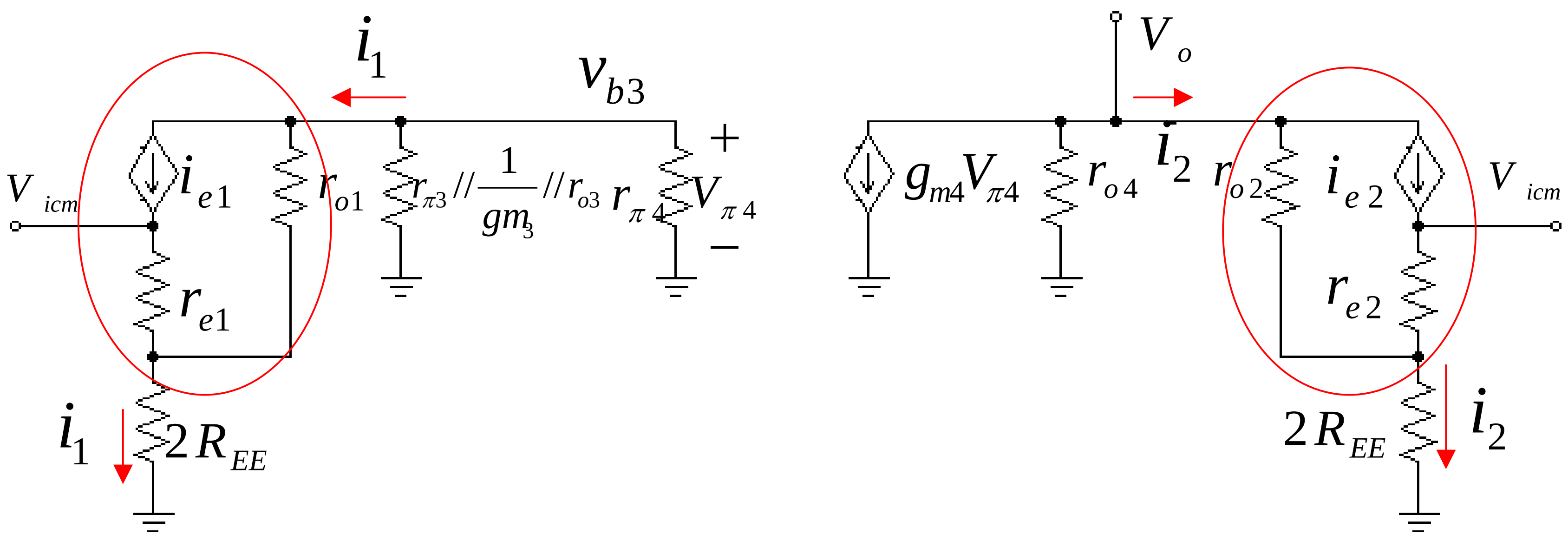
$$A_d \equiv g_m \frac{r_o}{2}$$

$$R_{id} = 2r_\pi$$

Common-mode gain at CMRR



Differential amplifier with active load equivalent-circuit Common-mode



$$i_1 \approx i_2 \approx \frac{v_{icm}}{2R_{EE}}$$

$$v_{b3} = -i_1\left(\frac{1}{g_{m3}} // r_{\pi 3} // r_{o3} // r_{\pi 4}\right)$$

$$i_{c4} = g_{m4} v_{b3}$$

$$v_o = (-g_{m4} v_{b3} - i_2) r_{o4}$$

$$A_{cm} \equiv \frac{v_o}{v_{icm}} = \frac{r_{o4}}{2R_{EE}}\left[g_{m4}\left(\frac{1}{g_{m3}} // r_{\pi 3} // r_{o3} // r_{\pi 4}\right) - 1\right]$$

$$= -\frac{r_{o4}}{2R_{EE}} \frac{\frac{1}{r_{\pi 3}} + \frac{1}{r_{\pi 4}} + \frac{1}{r_{o3}}}{g_{m3} + \frac{1}{r_{\pi 3}} + \frac{1}{r_{\pi 4}} + \frac{1}{r_{o3}}}$$

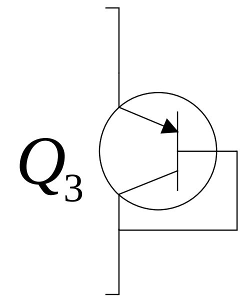
$$let \quad g_{m3} = g_{m4}, r_{\pi 3} = r_{\pi 4}, r_{o3} >> r_{\pi 3}, r_{o3} >> r_{\pi 4}$$

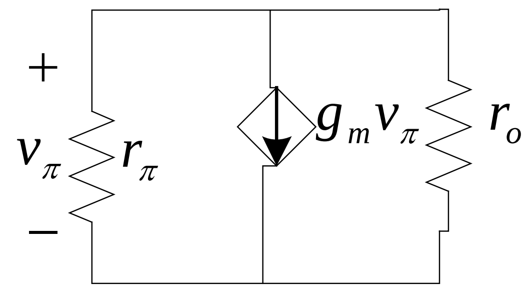
$$A_{cm} \equiv \frac{v_o}{v_{icm}} = -\frac{r_{o4}}{2R_{EE}} \frac{\frac{1}{r_{\pi 3}}}{g_{m3} + \frac{1}{r_{\pi 3}}} \approx -\frac{r_{o4}}{2R_{EE}} \frac{2}{\beta_3} = - - \frac{r_{o4}}{\beta_3 R_{EE}}$$

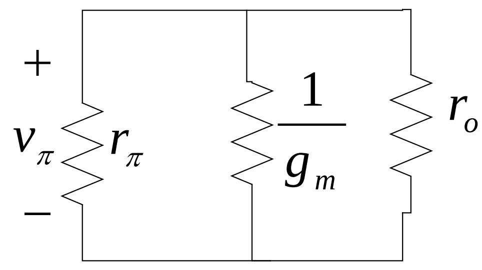
$$CMRR \equiv \frac{|A_d|}{|A_{cm}|} = g_m (r_{o2} // r_{o4})\left(\frac{\beta_3 R_{EE}}{r_{o4}}\right)$$

$$when \quad r_{o2} = r_{o4} = r_o$$

$$CMRR = \frac{1}{2} \beta_3 g_m R_{EE}$$







Input offset voltage

$$\frac{I_4}{I_3} = \frac{1}{1+2/\beta_P} \cdots \beta_P \equiv \beta_3 = \beta_4$$

$$I_4 = \frac{\alpha I/2}{1+2/\beta_P}$$

$$\Delta i = \frac{\alpha I}{2} - \frac{\alpha I/2}{1+2/\beta_P} = \frac{\alpha I}{2}\frac{2/\beta_P}{1+2/\beta_P} \approx \frac{\alpha I}{\beta_P}$$

$$V_{os} = -\frac{\Delta i}{G_M} = -\frac{\alpha I/\beta_P}{\alpha I/2V_T} = -\frac{2V_T}{\beta_P}$$

Exercise 7-13