

Signals & Systems

Signals & Systems

UNIT I

2 Mark Questions and Answers

1. Define Signal.

Signal is a physical quantity that varies with respect to time, space or any other independent variable.

Or

It is a mathematical representation of the system

Eg $y(t) = t$. and $x(t) = \sin t$.

2. Define system.

A set of components that are connected together to perform the particular task.

3. What are the major classifications of the signal?

- (i) Discrete time signal
- (ii) Continuous time signal

4. Define discrete time signals and classify them.

Discrete time signals are defined only at discrete times, and for these signals, the independent variable takes on only a discrete set of values.

Classification of discrete time signal:

- 1. Periodic and Aperiodic signal
- 2. Even and Odd signal

5. Define continuous time signals and classify them.

Continuous time signals are defined for a continuous set of values of the independent variable. In the case of continuous time signals the independent variable is continuous.

For example:

- (i) A speech signal as a function of time
- (ii) Atmospheric pressure as a function of altitude

Classification of continuous time signal:

- (i) Periodic and Aperiodic signal
- (ii) Even and Odd signal

6. Define discrete time unit step & unit impulse.

Discrete time Unit impulse is defined as

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

Unit impulse is also known as unit sample.

Discrete time unit step signal is defined by

$$U[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

7. Define continuous time unit step and unit impulse.

Continuous time unit impulse is defined as

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

Continuous time Unit step signal is defined as

$$U(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

8. Define unit ramp signal.

Continuous time unit ramp function is defined by

$$r(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$

A ramp signal starts at $t=0$ and increases linearly with time 't'.

9. Define periodic signal. and nonperiodic signal.

A signal is said to be periodic, if it exhibits periodicity. i.e.,

$$X(t+T) = X(t), \text{ for all values of } t.$$

Periodic signal has the property that it is unchanged by a time shift of T .

A signal that does not satisfy the above periodicity property is called an aperiodic signal.

10. Define even and odd signal ?

A discrete time signal is said to be even when,

$$x[-n] = x[n].$$

The continuous time signal is said to be even when,

$$x(-t) = x(t)$$

For example, $\cos \omega n$ is an even signal.

The discrete time signal is said to be odd when

$$x[-n] = -x[n]$$

The continuous time signal is said to be odd when

$$x(-t) = -x(t)$$

Odd signals are also known as nonsymmetrical signal.
Sine wave signal is an odd signal.

11. Define Energy and power signal.

A signal is said to be energy signal if it have finite energy and zero power.

A signal is said to be power signal if it have infinite energy and finite power.

If the above two conditions are not satisfied then the signal is said to be neither energy nor power signal

12. Define unit pulse function.

Unit pulse function $\Pi(t)$ is obtained from unit step signals

$$\Pi(t) = u(t+1/2) - u(t-1/2)$$

The signals $u(t+1/2)$ and $u(t-1/2)$ are the unit step signals shifted by $1/2$ units in the time axis towards the left and right, respectively.

13. Define continuous time complex exponential signal.

The continuous time complex exponential signal is of the form

$$x(t) = Ce^{at}$$

where c and a are complex numbers.

14. What is continuous time real exponential signal.

Continuous time real exponential signal is defined by

$$x(t) = Ce^{at}$$

where c and a are complex numbers. If c and a are real, then it is called as real exponential.

15. What is continuous time growing exponential signal?

Continuous time growing exponential signal is defined as

$$x(t) = Ce^{at}$$

where c and a are complex numbers.

If a is positive, as t increases, then $x(t)$ is a growing exponential.

16. What is continuous time decaying exponential?

Continuous time growing exponential signal is defined as

$$x(t) = Ce^{at}$$

where c and a are complex numbers.

If a is negative, as t increases, then $x(t)$ is a decaying exponential.

17. What are the types of Fourier series?

1. Exponential Fourier series
2. Trigonometric Fourier series

18. Write down the exponential form of the fourier series representation of a periodic signal?

$$x(t) = \sum a_k e^{jk\omega_0 t}$$

Here the summation is taken from $-\infty$ to ∞ .

$$a_k = 1/T \int x(t) e^{-jk\omega_0 t} dt$$

Here the integration is taken from 0 to T.

The set of coefficients $\{a_k\}$ are often called the fourier series coefficients or spectral coefficients.

The coefficient a_0 is the dc or constant component of $x(t)$.

19. Write down the trigonometric form of the fourier series representation of a periodic signal?

$$x(t) = a_0 + \sum [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

where

$$a_0 = 1/T \int x(t) dt$$

$$a_n = 1/T \int x(t) \cos n\omega_0 t dt$$

$$b_n = 1/T \int x(t) \sin n\omega_0 t dt$$

20. Write short notes on dirichlets conditions for fourier series.

- $x(t)$ must be absolutely integrable
- The function $x(t)$ should be single valued within the interval T.
- The function $x(t)$ should have finite number of discontinuities in any finite interval of time T.
- The function $x(t)$ should have finite number of maxima & minima in the interval T.

21. State Time Shifting property in relation to fourier series.

$$x(t-t_0) \xrightarrow{\text{FS}} a_k e^{-jk\omega_0 t}$$

Time shifting property states that; when a periodic signal is shifted in time, the magnitudes of its fourier series coefficients, remain unaltered.

22. State parseval's theorem for continuous time periodic signals.

Parseval's relation for continuous time periodic signals is

$$1/T \int |x(t)|^2 dt = \sum |a_k|^2$$

Parseval's relation states that the total average power in a periodic signal equals the sum of the average power in all of its harmonic components.

UNIT II

23. Define continuous time system.

A continuous time system is a system in which continuous time input signals are applied and result in continuous time output signals.

24. Define fourier transform pair.

Consider the aperiodic signal $x(t)$ & Fourier transform of $x(t)$ is defined as

$$X(j\omega) = \int x(t) e^{-j\omega t} dt \text{ -----(1)}$$

Inverse fourier transform of $x(t)$ is given by

$$x(t) = 1/2\pi \int X(j\omega) e^{j\omega t} d\omega \text{ -----(2)}$$

Equations (1)& (2) are referred to as the fourier transform pair.

25. Write short notes on dirichlets conditions for fourier transform.

- $x(t)$ be absolutely integrable
- $x(t)$ have a finite number of maxima and minima within any finite interval.
- $x(t)$ have a finite number of discontinuities within any finite interval.
Furthermore each of these discontinuities must be finite.

26. Explain how aperiodic signals can be represented by fourier transform.

Consider the aperiodic signal $x(t)$ & Fourier transform of $x(t)$ is defined as

$$X(j\omega) = \int x(t) e^{-j\omega t} dt \text{ -----(1)}$$

Inverse fourier transform of $x(t)$ is given by

$$x(t) = 1/2\pi \int X(j\omega) e^{j\omega t} d\omega \text{ -----(2)}$$

27. State convolution property in relation to fourier transform.

$$Y(t) = x(t) * h(t)$$

$$Y(j\omega) = H(j\omega)X(j\omega)$$

Convolution property states that convolution in time domain corresponds to multiplication in the frequency domain.

28 . State parseval's relation for continuous time fourier transform.

If $x(t)$ and $X(j\omega)$ are a fourier transform pair then

$$\int |x(t)|^2 dt = 1/2 \pi \int |X(j\omega)|^2 d\omega$$

29. what is the use of laplace transform?

Laplace transform is an another mathematical tool used for analysis of signals and systems. Laplace transform is used for analysis of unstable systems.

30. What are the types of laplace transform?

1. Bilateral or two sided laplace transform.
2. Unilateral or single sided laplace transform.

31. Define Bilateral and unilateral laplace transform.

The bilateral laplace transform is defined as

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

Here the integration is taken from $-\infty$ to ∞ . Hence it is called bilateral laplace transform

The unilateral laplace transform is defined as

$$X(s) = \int_0^{\infty} x(t) e^{-st} dt$$

Here the integration is taken from 0 to ∞ . Hence it is called unilateral laplace transform.

32. Define inverse laplace transform.

The inverse laplace transform is given as

$$x(t) = 1/2\pi j \int_{\sigma-j\infty}^{\sigma+j\infty} X(s) e^{st} ds$$

Here the integration is taken from $\sigma-j\infty$ to $\sigma+j\infty$.

33. State the linearity property for laplace transform.

Let $x_1(t) \longleftrightarrow X_1(s)$ and $x_2(t) \longleftrightarrow X_2(s)$ be the two laplace transform pairs. Then linearity property states that

$$L[a_1 x_1(t) + a_2 x_2(t)] = a_1 X_1(s) + a_2 X_2(s)$$

Here a_1 and a_2 are constants.

34. State the time shifting property for laplace transform.

Let $x(t) \longleftrightarrow X(s)$ be a laplace transform pair. If $x(t)$ is delayed by time t_0 , then its laplace transform is multiplied by e^{-st_0} .

$$L[x(t-t_0)] = e^{-st_0} X(s)$$

35. Region of convergence of the laplace transform.

The range of values of s for which the integral i.e., $\int x(t)e^{-st} dt$ converges is referred to as the region of convergence of the laplace transform.

36. What is pole zero plot.

The representation of $X(s)$ through its poles and zeros in the s-plane is referred to as pole zero plot.

37. State initial value theorem and final value theorem for laplace transform.

If $L[x(t)] = X(s)$, then initial value theorem states that
$$x(0) = \lim_{s \rightarrow \infty} sX(s)$$

If $L[x(t)] = X(s)$, then final value theorem states that
$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

38. State Convolution property.

The laplace transform of convolution of two functions is equivalent to multiplication of their laplace transforms.

$$L[x_1(t) * x_2(t)] = X_1(s)X_2(s)$$

39. Define a causal system.

The causal system generates the output depending upon present & past inputs only. A causal system is non anticipatory.

40. What is meant by linear system?

A linear system should satisfy superposition principle. A linear system should satisfy $F[ax_1(t) + bx_2(t)] \rightarrow ay_1(t) + by_2(t)$

$$y_1(t) = F[x_1(t)]$$

$$y_2(t) = F[x_2(t)]$$

41. Define time invariant system.

A system is time invariant if the behavior and characteristics of the system are fixed over time.

A system is time invariant if a time shift in the input signal results in an identical time shift in the output signal.

For example, a time invariant system should produce $y(t-t_0)$ as the output when $x(t-t_0)$ is the input.

42. Define stable system?

When the system produces bounded output for bounded input, then the system is called bounded input & bounded output stable.

If the signal is bounded, then its magnitude will always be finite.

43. Define memory and memoryless system.

The output of a memory system at any specified time depends on the inputs at that specified time and at other times. Such systems have memory or energy storage elements.

The system is said to be static or memoryless if its output depends upon the present input only.

44 .Define invertible system.

A system is said to be invertible if the input is get from the output input. Otherwise the system is noninvertible system.

45 .What is superposition property?

If an input consists of the weighted sum of several signals, then the output is the superposition that is, the weighted sum of the responses of the system to each of those signals

UNIT III

46. Why CT signals are represented by samples.

- A CT signal can not be processed in the digital processor or computer.
- To enable the digital transmission of CT signals.

47. What is meant by sampling.

A sampling is a process by which a CT signal is converted into a sequence of discrete samples with each sample representing the amplitude of the signal at the particular instant of time.

48. State Sampling theorem.

A band limited signal of finite energy, which has no frequency components higher than the W hertz, is completely described by specifying the values of the signal at the instant of time separated by $1/2W$ seconds and

A band limited signal of finite energy, which has no frequency components higher than the W hertz, is completely recovered from the knowledge of its samples taken at the rate of $2W$ samples per second.

49. What is meant by aliasing.

When the high frequency interferes with low frequency and appears as low then the phenomenon is called aliasing.

50. What are the effects aliasing.

Since the high frequency interferes with low frequency then the distortion is generated.

The data is lost and it can not be recovered.

51. How the aliasing process is eliminated.

- i). Sampling rate $f_s \geq 2W$.
- ii). Strictly band limit the signal to 'W'.

This can be obtained by using the Low pass filter before

the sampling process. It is also called as antialiasing filter.

52. Define Nyquist rate and Nyquist interval.

When the sampling rate becomes exactly equal to '2W' samples/sec, for a given bandwidth of W hertz, then it is called Nyquist rate.
Nyquist interval is the time interval between any two adjacent samples.
Nyquist rate = 2W Hz
Nyquist interval = 1/2W seconds.

53. Define sampling of band pass signals.

A bandpass signal $x(t)$ whose maximum bandwidth is '2W' can be completely represented into and recovered from its samples, if it is sampled at the minimum rate of twice the band width.

54. Define Z transform.

The Z transform of a discrete time signal $x[n]$ is denoted by $X(z)$ and it is given as $X(z) = \sum x[n] z^{-n}$. and the value n range from $-\infty$ to $+\infty$. Here 'z' is the complex variable. This Z transform is also called as bilateral or two sided Z transform.

55. What are the two types of Z transform?

- (i) Unilateral Z transform
- (ii) Bilateral Z transform

56. Define unilateral Z transform.

The unilateral Z transform of signal $x[n]$ is given as

$$X(z) = \sum x[n] z^{-n}$$

The unilateral and bilateral Z transforms are same for causal signals.

57. What is region of Convergence.

The region of convergence or ROC is specified for Z transform, where it converges.

58. What are the Properties of ROC.

- i. The ROC of a finite duration sequence includes the entire z- plane, except $z=0$ and $|z|=\infty$.
- ii. ROC does not contain any poles.
- iii. ROC is the ring in the z-plane centered about origin.
- iv. ROC of causal sequence (right handed sequence) is of the form $|z| > r$.
- v. ROC of left handed sequence is of the form $|z| < r$.
- vi. ROC of two sided sequence is the concentric ring in the z plane.

59. What is the time shifting property of Z transform.

$$x[n] \longleftrightarrow X(Z) \text{ then}$$

$$x[n-k] \longleftrightarrow Z^{-k} X[Z].$$

60. What is the differentiation property in Z domain.

$$x[n] \longleftrightarrow X(Z) \text{ then}$$

$$nx[n] \longleftrightarrow -z \frac{d}{dz} \{X(Z)\}.$$

61. State convolution property of Z transform.

The convolution property states that if

$$x_1[n] \longleftrightarrow X_1(Z) \text{ and}$$

$$x_2[n] \longleftrightarrow X_2(Z) \text{ then}$$

$$x_1[n] * x_2[n] \longleftrightarrow X_1(Z) X_2(Z)$$

That is convolution of two sequences in time domain is equivalent to multiplication of their Z transforms.

62. State the methods to find inverse Z transform.

- Partial fraction expansion
- Contour integration
- Power series expansion
- Convolution method.

63. State multiplication property in relation to Z transform.

This property states that if ,

$$x_1[n] \longleftrightarrow X_1(Z) \text{ and}$$

$$x_2[n] \longleftrightarrow X_2(Z) \text{ then}$$

$$x_1[n] x_2[n] \longleftrightarrow \frac{1}{2\pi j} \oint X_1(v) \cdot X_2(Z/v) v^{-1} dv$$

Here c is a closed contour .It encloses the origin and lies in the Roc which is common to both $X_1(v)$. $X_2(1/v)$

64. State parseval's relation for Z transform.

If $x_1[n]$ and $x_2[n]$ are complex valued sequences, then the parseval's relation states that

$$\sum x_1[n] x_2^*[n] = \frac{1}{2\pi j} \oint X_1(v) \cdot X_2^*(1/v^*) v^{-1} dv.$$

65. What is the relationship between Z transform and fourier transform.

$$X(z) = \sum x[n] z^{-n} \text{-----1.}$$

$$X(w) = \sum x[n] e^{-j\omega n} \text{-----2}$$

$$X(z) \text{ at } z = e^{j\omega} \text{ is } = X(w).$$

When z- transform is evaluated on unit circle (ie. $|z| = 1$) then it becomes fourier transform.

Unit IV

66. What is meant by step response of the DT system.

The output of the system $y(n)$ is obtained for the unit step input $u(n)$ then it is said to be step response of the system.

67. Define Transfer function of the DT system.

The Transfer function of DT system is defined as the ratio of Z transform of the system output to the input.

That is, $H(z) = Y(z)/X(z)$,

68. Define impulse response of a DT system.

The impulse response is the output produced by DT system when unit impulse is applied at the input. The impulse response is denoted by $h(n)$.

The impulse response $h(n)$ is obtained by taking inverse Z transform from the transfer function $H(z)$.

69. State the significance of difference equations.

The input and output behaviour of the DT system can be characterized with the help of linear constant coefficient difference equations.

70. Write the difference equation for Discrete time system.

The general form of constant coefficient difference equation is

$$Y(n) = -\sum a_k y(n-k) + \sum b_k x(n-k)$$

Here n is the order of difference equation. $x(n)$ is the input and $y(n)$ is the output.

71. Define frequency response of the DT system.

The frequency response of the system is obtained from the Transfer function by replacing $z = e^{j\omega}$

I.e., $H(z) = Y(z)/X(z)$, Where $z = e^{j\omega}$

72. What is the condition for stable system.

A LTI system is stable if

$$\sum |h(n)| < \infty.$$

Here the summation is absolutely summable

73. What are the blocks used for block diagram representation.

The block diagrams are implemented with the help of scalar multipliers, adders and multipliers

74. State the significance of block diagram representation.

The LTI systems are represented with the help of block diagrams. The block diagrams are more effective way of system description. Block Diagrams indicate how individual calculations are performed. Various blocks are used for block diagram representation.

75. What are the properties of convolution?

i. Commutative

- ii. Associative.
- iii. Distributive

76. State the Commutative properties of convolution?

Commutative property of Convolution is

$$x(t)*h(t)=h(t)*x(t)$$

77. State the Associative properties of convolution

Associative Property of convolution is

$$[x(t)*h_1(t)]*h_2(t)=x(t)*[h_1(t)*h_2(t)]$$

78. State Distributive properties of convolution

The Distributive Property of convolution is

$$\{x(t)*[h_1(t)+h_2(t)]\}=x(t)*h_1(t)+x(t)*h_2(t)$$

79. Define causal system.

For a LTI system to be causal if $h(n)=0$, for $n<0$.

80. What is the impulse response of the system $y(t)=x(t-t_0)$.

Answer:

$$h(t)=\delta(t-t_0)$$

81. What is the condition for causality if $H(z)$ is given.

A discrete LTI system with rational system function $H(z)$ is causal if and only if

- i. The ROC is the exterior of the circle outside the outermost pole.
- ii. When $H(z)$ is expressed as a ratio of polynomials in z , the order of the numerator can not be greater than the order of the denominator.

82. What is the condition for stability if $H(z)$ is given.

A discrete LTI system with rational system function $H(z)$ is stable if and only if all of the poles $H(z)$ lies inside the unit circle. That is they must all have magnitude smaller than 1.

83. Check whether the system is causal or not, the $H(z)$ is given by $(z^3 + z)/(z+1)$.

The system is not causal because the order of the numerator is greater than denominator.

84. Check whether the system is stable or not, the $H(z)$ is given by $(z/z-a)$, $|a|<1$.

The system is stable because the poles at $z = a$ lies inside the unit circle.

85. Determine the transfer function for the system described by the difference equation $y(n) - y(n-1) = x(n) - x(n-2)$.

By taking z transform on both sides the transfer function

$$H(z) = (z^2 - 1)/(z^2 - z).$$

UNIT V

86. How the discrete time system is represented.

The DT system is represented either Block diagram representation or difference equation representation.

87. What are the classification of the system based on unit sample response.

- a. FIR (Finite impulse Response) system.
- b. IIR (Infinite Impulse Response) system.

88. What is meant by FIR system.

If the system has finite duration impulse response then the system is said to be FIR system.

89. What is meant by IIR system.

If the system has infinite duration impulse response then the system is said to be IIR system.

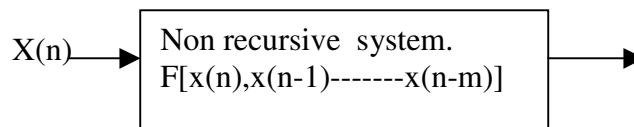
90. What is recursive system.

If the present output is dependent upon the present and past value of input then the system is said to be recursive system.

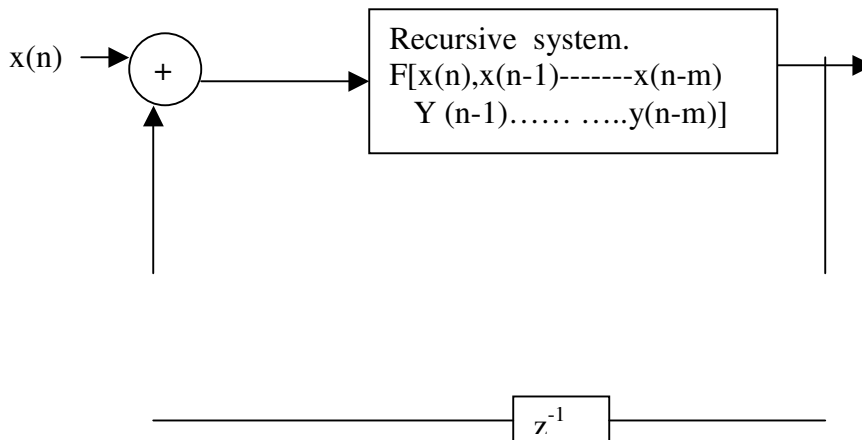
91. What is Non recursive system.

If the present output is dependent upon the present and past value of input and past value of output then the system is said to be non recursive system.

92. What is the block diagram representation of recursive system.



93. What is the block diagram representation of non recursive system.



94. What is the difference between recursive and non recursive system

A recursive system have the feed back and the non recursive system have no feed back .And also the need of memory requirement for the recursive system is less than non recursive system.

95. Define realization structure.

The block diagram representation of a difference equation is called realization structure. These diagram indicate the manner in which the computations are performed.

96. What are the different types of structure realization.

- i. Direct form I
- ii. Direct form II
- iii. Cascade form
- iv. Parallel Form.

97. What is natural response?

This is output produced by the system only due to initial conditions .Input is zero for natural response. Hence it is also called zero input Response.

98. What is zero input Response?

This is output produced by the system only due to initial conditions .Input is zero for zero input response.

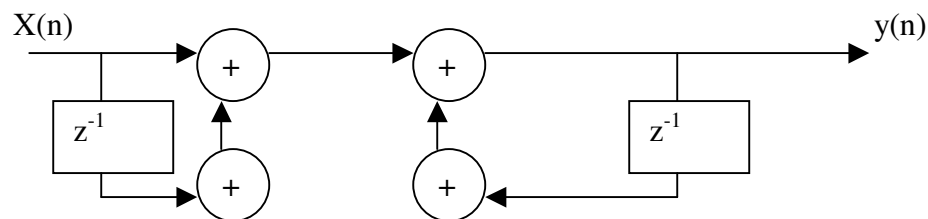
99. What is forced response.

This is the output produced by the system only due to input .Initial conditions are considered zero for forced response. It is denoted by $y^{(f)}(t)$.

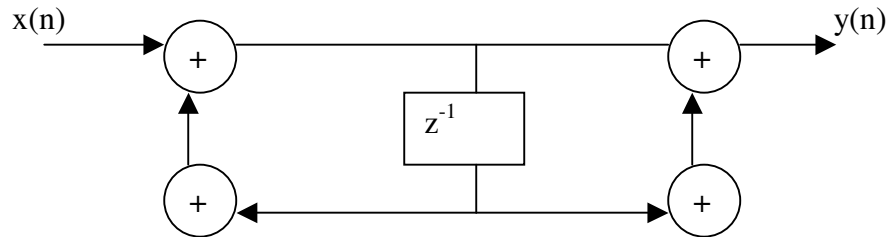
100. What is complete response?

The complete response of the system is equal to the sum of natural response and forced response .Thus initial conditions as well as input both are considered for complete response.

101. Give the direct form I structure.



102. Give the direct form II structure..

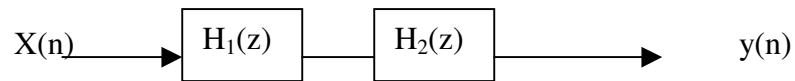


103. How the Cascade realization structure obtained..

The given transfer function $H(z)$ is split into two or more sub systems.

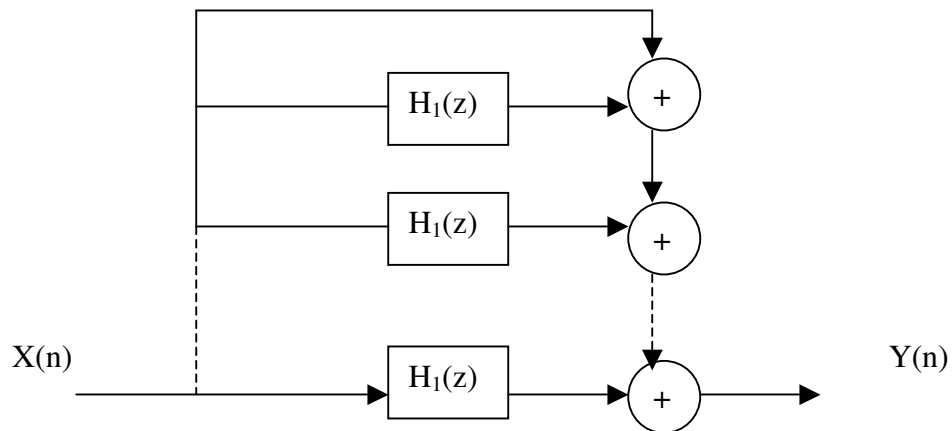
That is for eg.

$$H(z) = H_1(z) H_2(z)$$



104. Give the parallel for Realization structure.

$$H(z) = c + H_1(z) + H_2(z) + \dots + H_k(z)$$



105. What is transformed structure representation.

The flow graph reversal theorem states that if the directions of all branches are reversed and positions of input and output is interchanged, the system function remain unchanged. Such structure is called transposed structure

16 MARK QUESTIONS**1. Find the trigonometric fourier series representation of a periodic signal $x(t)=t$, for the interval of $t = -1$ to $t = 1$?**

The trigonometric fourier series representation of a periodic signal is

$$x(t) = a_0 + \sum [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

where

$$a_0 = 1/T \int x(t) dt = 0$$

$$a_n = 1/T \int x(t) \cos n\omega_0 t dt = 0$$

$$b_n = 1/T \int x(t) \sin n\omega_0 t dt = 2/\pi [- (-1)^n / n]$$

$$x(t) = \sum [2/\pi [- (-1)^n / n]] \sin n\pi t.$$

Summation varies $n=1$ to ∞ .

2. Find the exponential fourier series for half wave rectified sine wave.

Answer:

$$a_k = A/\pi (n^2 - 1)$$

3. Find the energy and power of the signal.

i. $X(t) = r(t) - r(t-2).$

$$\text{Answer : } E = \lim_{T \rightarrow \infty} \int |x(t)|^2 dt = \infty..$$

$$P = \lim_{T \rightarrow \infty} 1/2T \int |x(t)|^2 dt = 2W..$$

ii. $x(n) = (1/3)^n u(n).$

$E = \text{Infinite}$ and power is finite. So it is power signal

$$\text{Answer : } E = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x(n)|^2 = 9/8..$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \int |x(t)|^2 dt = 0..$$

E= Finite and power is zero. So it is energy signal.

4. Find the DTFS of $x(n) = 5 + \sin(n\pi/2) + \cos(n\pi/4)$.

$$x(n) = \sum a_k e^{jk\omega_0 n}$$

Here the summation is taken from $-\infty$ to ∞ .

$$a_k = 1/T \int x(t) e^{-jk\omega_0 t} dt$$

5. Find the fourier series representation for Full wave rectified sine wave.

Answer:

$$X(t) = \frac{2}{\pi} + \frac{4}{\pi} \sum \frac{1}{1-4n^2} \cos n \omega_0 t$$

6. Consider a continuous time system with impulse response $h(t) = e^{-at} u(t)$ to the input signal $x(t) = e^{-bt} u(t)$. Find the system response.

Answer:

$$y(t) = 1/(b-a) [e^{-at} u(t) - e^{-bt} u(t)]$$

7. Find the laplace transform of $x(t) = \delta(t) - 4/3 e^{-t} u(t) + 1/3 e^{2t} u(t)$.

Answer:

$$X(s) = (s-1)^2 / ((s+1)(s-2))$$

8. Find the invers laplace transform of $X(s) = 1/((s+1)(s+2))$.

Answer:

$$x(t) = e^{-t} u(t) - e^{-2t} u(t)$$

9. State and prove parseval's theorem for Fourier transform.

Answer:

If $x(t)$ and $X(j\omega)$ are a fourier transform pair then

$$\int |x(t)|^2 dt = 1/2 \int |X(j\omega)|^2 d\omega$$

10 . Determine the Fourier transform of the signal $x(t) = e^{-at} u(t)$, $a > 0$, plot the Magnitude and Phase Spectrum.

11 . Determine the System reponse of the given differential equation $y''(t) + 3y'(t) = x(t)$, Where $x(t) = e^{-2t} u(t)$.

12. State and prove sampling theorem.

Sampling.

A sampling is a process by which a CT signal is converted into a sequence of discrete samples with each sample representing the amplitude of the signal at the particular instant of time.

Sampling theorem.

A band limited signal of finite energy, which has no frequency components higher than the W hertz, is completely described by specifying the values of the signal at the instant of time separated by $1/2W$ seconds and A band limited signal of finite energy, which has no frequency components higher than the W hertz, is completely recovered from the knowledge of its samples taken at the rate of $2W$ samples per second.

Aliasing.

When the high frequency interferes with low frequency and appears as low frequency then the phenomenon is called aliasing.

Effects aliasing.

Since the high frequency interferes with low frequency then the distortion is generated.

The data is lost and it can not be recovered.

Overcome of aliasing .

- i). Sampling rate $f_s \geq 2W$.
- ii). Strictly band limit the signal to ' W '.

This can be obtained by using the Low pass filter before the sampling process. It is also called as antialiasing filter.

Nyquist rate and Nyquist interval.

When the sampling rate becomes exactly equal to ' $2W$ ' samples/sec, for a given bandwidth of W hertz, then it is called Nyquist rate.

Nyquist interval is the time interval between any two adjacent samples.

Nyquist rate = $2W$ Hz

Nyquist interval = $1/2W$ seconds.

Then Sampling theorem Explanation.

13. Explain the Discrete time processing of CT Signals.

A bandpass signal $x(t)$ whose maximum bandwidth is ' $2W$ ' can be completely represented into and recovered from its samples, if it is sampled at the minimum rate of twice the band width.



The above diagram explains the Discrete time Processing of CT signals..

14. i. Determine the Z transform of following functions

$$x[n] = (-1)^n 2^{-n} u(n)$$

Answer:

$$X(z) = 1/(1 + 1/2 Z^{-1})$$

ii. Find the Z transform of the following and determine ROC

$$x[n] = \{8, 3, -2, 0, 4, 6\}$$

15. i. Determine the Z transform of following functions

$$x[n] = (-1)^n 2^{-n} u(n)$$

Answer:

$$X(z) = 1/(1 + 1/2 Z^{-1})$$

ii. State and prove the Time shifting Convolution properties of Z transform.

$$x[n] \longleftrightarrow X(Z) \text{ then}$$

Proof:

$$x[n-k] \longleftrightarrow Z^{-k} X[Z].$$

convolution property of Z transform.

The convolution property states that if

$$x_1[n] \longleftrightarrow X_1(Z) \text{ and}$$

$$x_2[n] \longleftrightarrow X_2(Z) \text{ then}$$

Proof:

$$x_1[n] * x_2[n] \longleftrightarrow X_1(Z) X_2(Z)$$

That is convolution of two sequences in time domain is equivalent to multiplication of their Z transforms.

16. (i). Determine the Z Transform and Plot the ROC for the sequence

$$x[n] = a^n u[n] - b^n u[n], \text{ } b > a$$

$$\text{Answers: } X(Z) = Z/(Z-a) - Z/(Z-b), |z| > b.$$

(ii) Compute the inverse Z transform of $X(Z) = (z + 0.5)/(z + 0.6)(z + 0.8)$.

$|z| > 0.8$, using residue method.

$$\text{Answers: } -0.5(-0.6)^{n-1}u(n-1) + 1.5(-0.8)^{n-1}u(n-1).$$

17. Find the inverse z Transform of the function $X(Z) = 1/(1 - 1.5Z^{-1} + 0.5Z^{-2})$.

Using power series method for $|Z| > 1$ and $|Z| < 1$.

$$\text{Answers: 1. For } |Z| > 1, x(n) = \{1, 1.5, 1.75, 1.875, \dots\}$$

2 For $|Z| < 1$, $x(n) = \{ \dots, 62, 30, 14, 6, 2, 0, 0 \}$

18. Find the inverse z Transform of the function $X(Z) = Z/(Z-1)(Z-2)(Z-3)$.

Using partial fraction method for ROC $|Z| > 3$, $3 > |Z| > 2$ and $|Z| < 1$.

- Answers: 1. For $|Z| > 3$, $x(n) = 0.5u(n) - 2^n u(n) + 0.5(3)^n u(n)$.
 2. For $3 > |Z| > 2$, $x(n) = 0.5u(n) - 2^n u(n) - 0.5(3)^n u(-n-1)$.
 3. For $|Z| < 1$, $x(n) = -0.5u(-n-1) + 2^n u(-n-1) - 0.5(3)^n u(-n-1)$.

19. Determine the discrete time fourier transform of

$x[n] = a^n u(n)$ for $-1 < a < 1$

Answer:

$$X(e^{j\omega}) = 1/(1 - ae^{-j\omega})$$

20. Determine the output of the discrete time linear time invariant system whose input and unit sample response are given as follows.

$$x[n] = (1/2)^n u[n]$$

$$h[n] = (1/4)^n u[n]$$

Solution:

The impulse response of the discrete time linear time invariant system is

$$Y[n] = \sum x[k] h[n-k];$$

21. Determine the output of the discrete time linear time invariant system whose input and unit sample response are given as follows.

$$x[n] = b^n u[n]$$

$$h[n] = a^n u[n]$$

Solution:

The impulse response of the discrete time linear time invariant system is

$$Y[n] = \sum x[k] h[n-k];$$

22. The sequences $x[n]$ and $h[n]$ are given as follows.

$$x[n] = \{1, 1, 0, 1, 1\}$$

$$h[n] = \{1, -2, -3, 4\}$$

Compute the convolution of these two sequences.

Answer:

$$Y[n] = \{1, -1, -5, 2, 3, -5, 1, 4\}$$

23. Write short notes on properties of convolution.

Explain each property with mathematical definition.

1. Commutative property
2. Distributive property
3. Associative property

24. Determine system transfer function and impulse response of discrete time system described by the difference equation

$$y(n) - 5/6 y(n-1) + 1/6 y(n-2) = x(n) - 1/2 x(n-1).$$

Answers.

$$H(z) = 1/(1 - 1/3 z^{-1})$$

$$\text{And } h(n) = (1/3)^n u(n), |z| > 1/3.$$

25. Obtain the Direct form I and Direct form II realization for the system described by the difference equation $y(n) - \frac{5}{6}y(n-1) + y(n-2) = x(n) + x(n-1)$.

Answers. $Y(Z) - \frac{5}{6}Z^{-1}Y(Z) + \frac{1}{6}Z^{-2}Y(Z) = W(Z)$

$$Y(Z) = W(Z) + \frac{5}{6}Z^{-1}Y(Z) - \frac{1}{6}Z^{-2}Y(Z)$$

Draw the block diagram representation for direct I and Direct II form.

26. Obtain the Cascade and parallel form.

realization for the system described by the difference equation $y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + 3x(n-1) + 2x(n-2)$

Answers.

$$H(Z) = Y(Z)/X(Z) = (1 + 3Z^{-1} + 2Z^{-2}) / (1 - \frac{1}{4}Z^{-1} - \frac{1}{8}Z^{-2})$$

$$H(Z) = H_1(z) H_2(z)$$

$$\text{Where } H_1(z) = (1 + z^{-1}) / (1 - \frac{1}{2}z^{-1}) \text{ and } H_2(z) = (1 + 2z^{-1}) / (1 - \frac{1}{4}z^{-1})$$

Draw the block diagram representation for Cascade form.

For parallel form obtain the transfer function in the form of

$$H(Z) = Y(Z)/X(Z) = 16 + \frac{22}{(1 + Z^{-1})} - \frac{37}{(1 + 2Z^{-1})}$$

Draw the block diagram representation for parallel form.

27. Obtain the Direct form I and Direct form II realization for the system described by the differential equation $d^2y(t)/dt^2 + 5dy(t)/dt + 4y(t) = dx(t)/dt$.

Solution : The procedure is same as to DT system . Here take the laplace transform and find the transfer function $H(S) = Y(S)/X(S)$.

Draw the block diagram representation for direct I and Direct II form. Here replace Z^{-1} by $1/S$.

28. Obtain the Cascade and parallel form realization for the system described by the differential equation $d^2y(t)/dt^2 + 5dy(t)/dt + 4y(t) = dx(t)/dt$

Answers.

$$H(S) = Y(S)/X(S)$$

$$H(S) = H_1(s) H_2(s).$$

Draw the block diagram representation for Cascade form. And Parallel form.

29. Obtain the direct form I, Direct form II, Cascade and parallel realization of the system described by the difference equation

$$y(n) = 0.75y(n-1) - 0.125y(n-2) + x(n) + 7x(n-1) + x(n-2).$$

Solution : Draw the Four Form realization structure using the above procedure.