

The background features several large, stylized, overlapping swirls in light green, light blue, and light purple. Scattered throughout the background are numerous small, yellow, triangular shapes, some pointing towards the center and others pointing outwards, creating a dynamic and festive feel.

Lecture 03

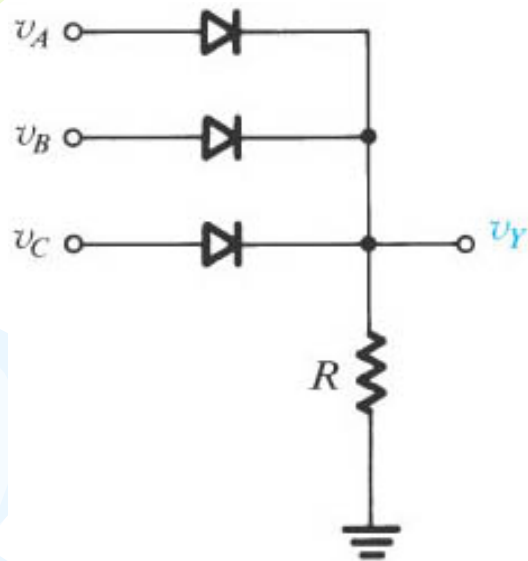
Diode circuits



topics

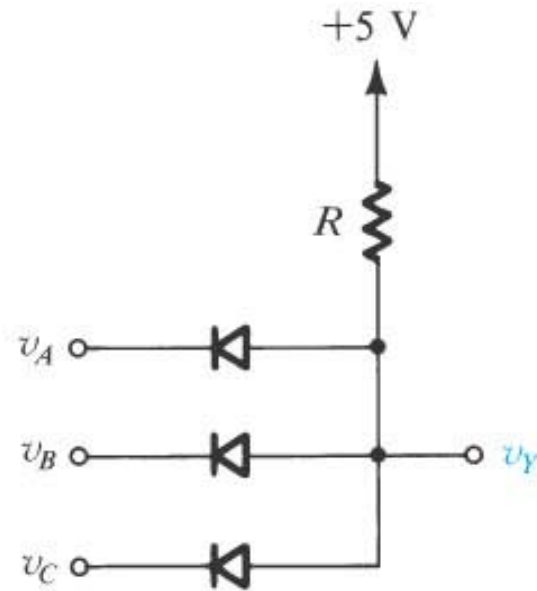
- Rectifier circuit
- Limiting and clamping circuits
- Small signal model

Diode logic gates (ideal diode)



(a)

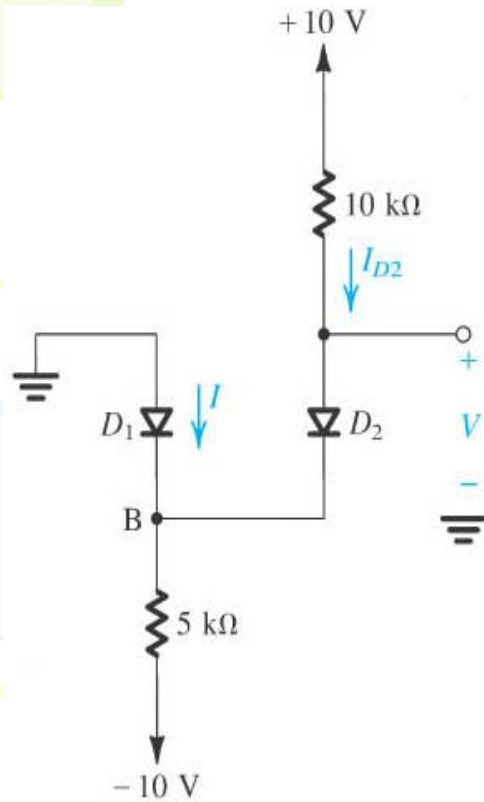
$$Y = A + B + C$$



(b)

$$Y = ABC$$

Example (ideal diode)



(a)

Assume D_1 and D_2 are ideal diodes and conducting

$$I_{D2} = \frac{10 - 0}{10k} = 1mA$$

$$0 = 5k \times (I + I_{D2}) - 10v$$

$$\Rightarrow I = 1mA$$

Example (ideal diode)

Assuming that D1 and D2 are ideal diodes and conducting

$$I_{D2} = \frac{10 - 0}{5k} = 2mA$$

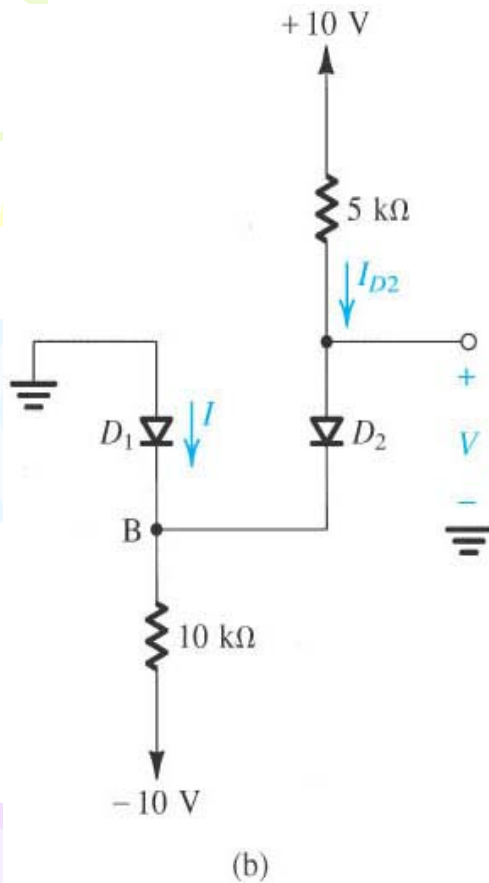
$$0 = 10k \times (I + I_{D2}) - 10v$$

$$\Rightarrow I = -1mA \quad \text{Contradictory result}$$

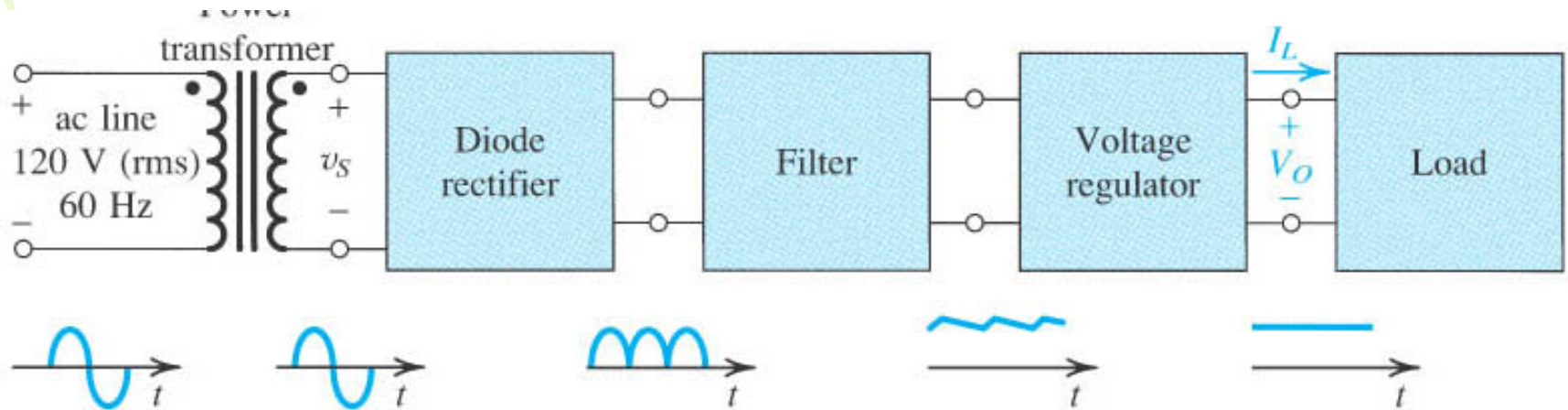
Assuming that D1 is off and D2 is on

$$I_{D2} = \frac{10 - (-10)}{15k} = 1.33mA$$

$$V_B = 10k \times I_{D2} - 10v = 3.3v$$



AC→DC Rectifier



Rectifier parameters:

- Crest factor (C.F.)
- Form factor (F.F.)
- Ripple factor (R.F.)

$$C.F. = \frac{V_{\max}}{V_{rms}}$$

$$F.F. = \frac{V_{rms}}{V_{average}}$$

$$R.F. = \frac{V_{ripple-rms}}{V_{average}}$$

$$V_{average} = \frac{1}{T} \int_0^T V_o(t) dt$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T V_o^2(t) dt}$$

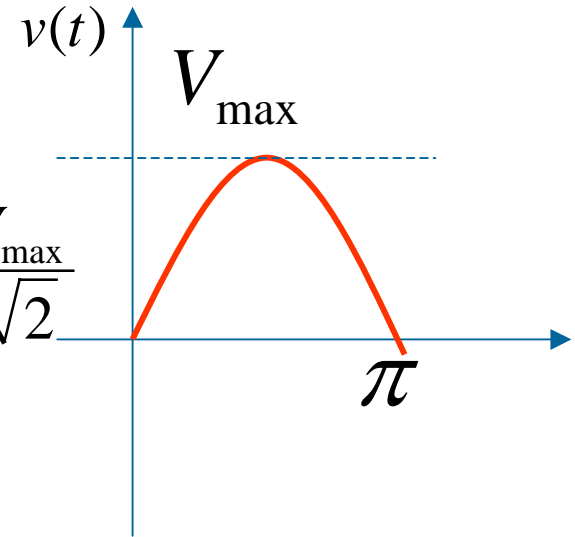
Root-mean-square

$$V_{average} = \frac{1}{T} \int_0^T v(t) dt = \frac{1}{\pi} \int_0^{\pi} V_{\max} \sin \theta d\theta = \frac{2V_{\max}}{\pi}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} = \sqrt{\frac{1}{\pi} \int_0^{\pi} V_{\max}^2 \sin^2 \theta d\theta} = \frac{V_{\max}}{\sqrt{2}}$$

$$\int_0^{\pi} \sin \theta d\theta = -\cos \theta \Big|_0^{\pi} = 2$$

$$\int_0^{\pi} \sin^2 \theta d\theta = \int_0^{\pi} \frac{1 - \cos 2\theta}{2} d\theta = \frac{\pi}{2}$$



Full-wave rectifier

$$V_{average} = \frac{2V_{\max}}{\pi}$$

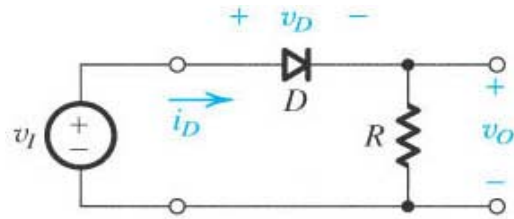
$$V_{rms} = \frac{V_{\max}}{\sqrt{2}}$$

Half-wave rectifier

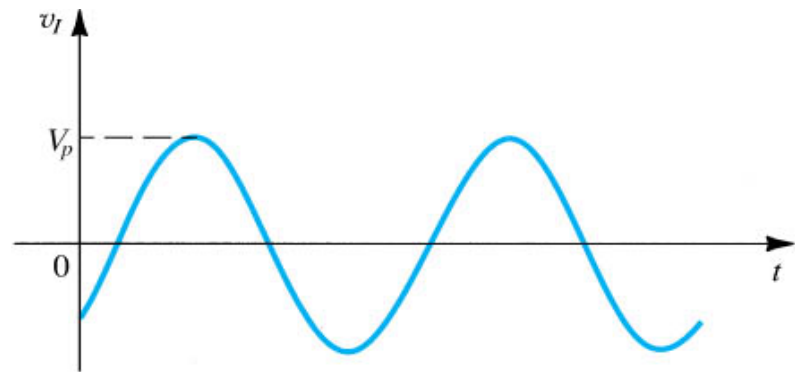
$$V_{average} = \frac{V_{\max}}{\pi}$$

$$V_{rms} = \frac{V_{\max}}{2}$$

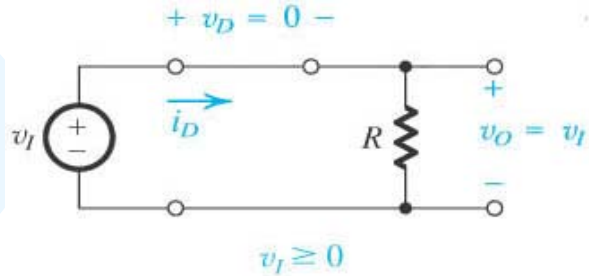
Using ideal model



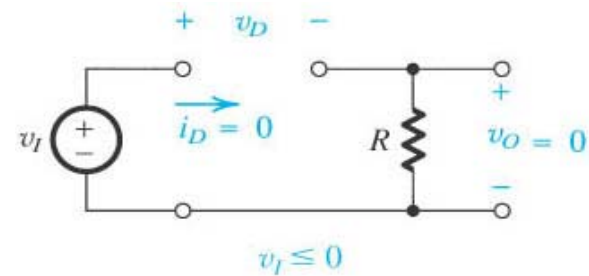
(a)



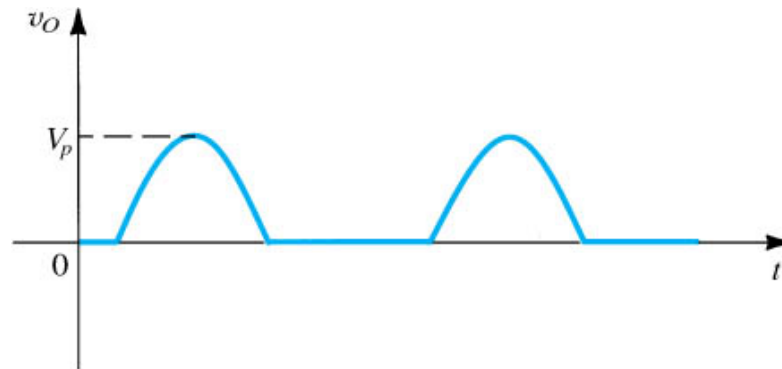
(b)



(c)



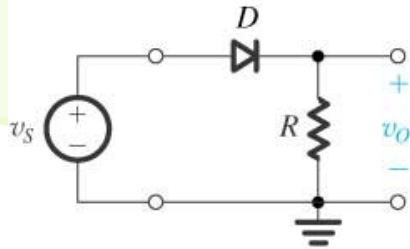
(d)



(e)

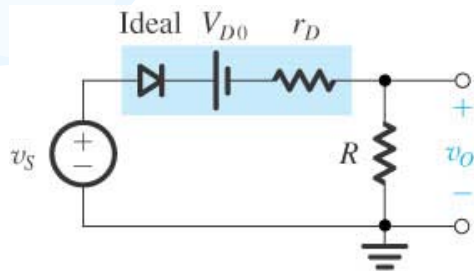
Half-wave rectifier

Using piecewise-linear model

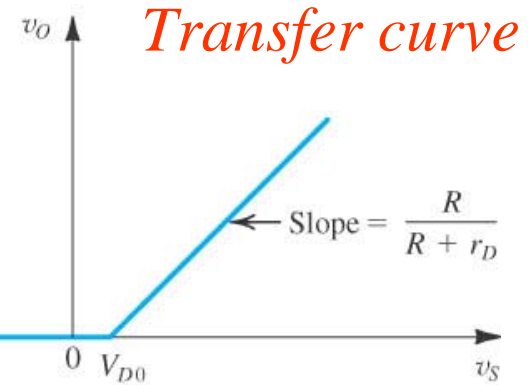


(a)

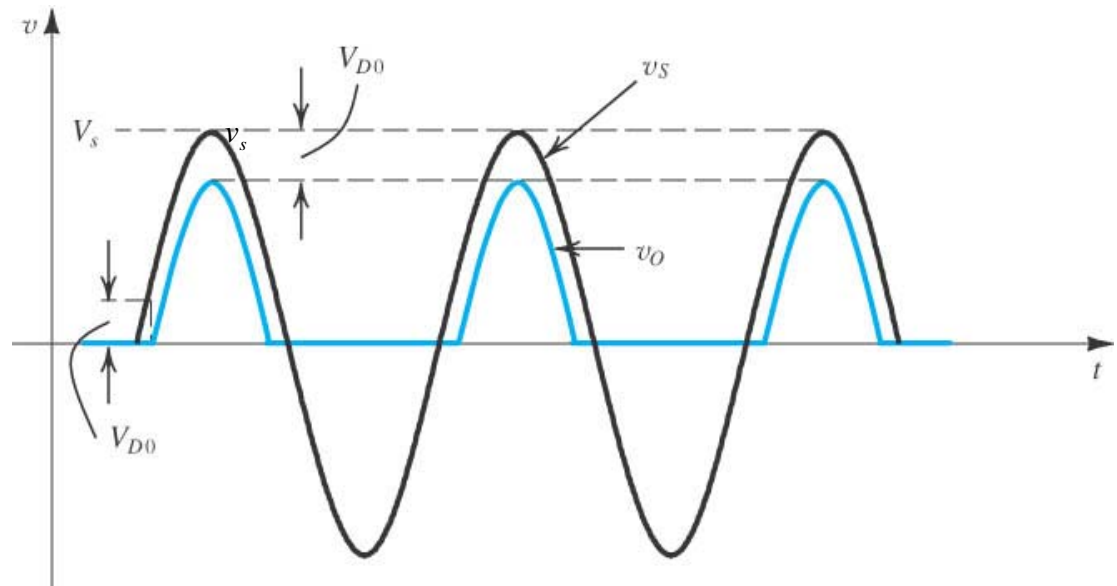
$$v_o = \frac{R}{R + r_D} v_s - \frac{R}{R + r_D} V_{D0}$$



(b)



(c)

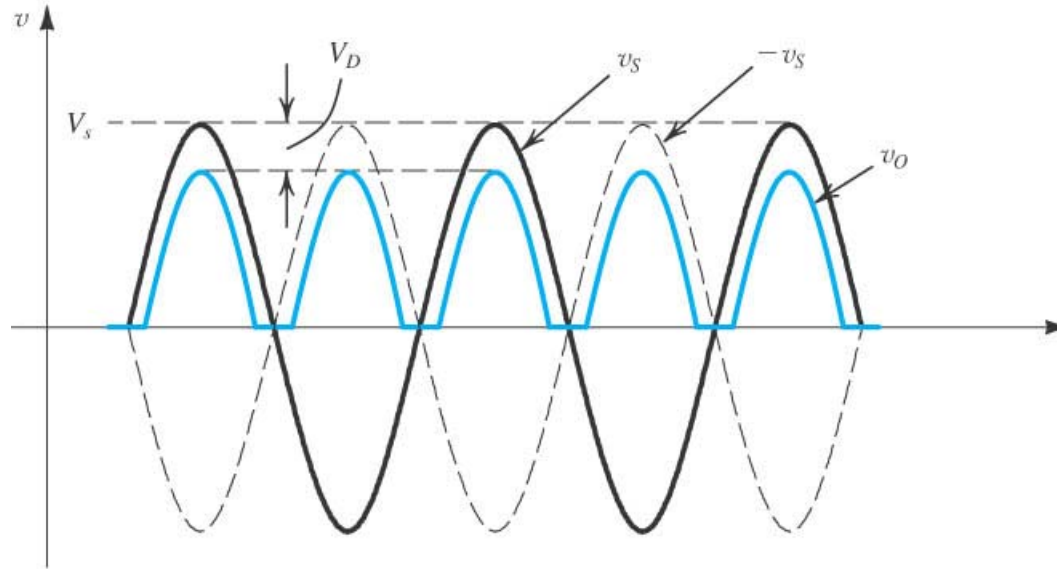
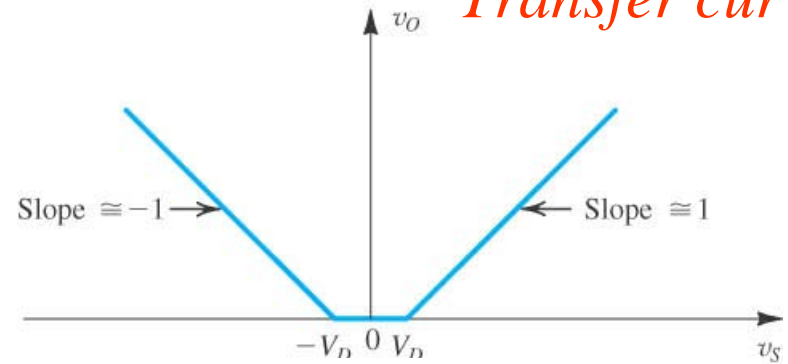
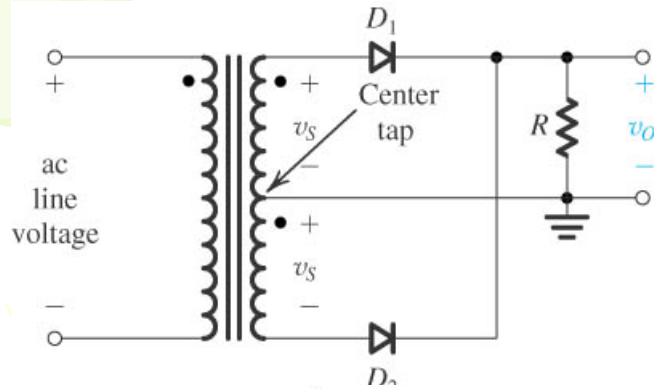


(d)

PIV (peak inverse voltage) = v_s

Using piecewise-linear model (center-tapped Rectifier)

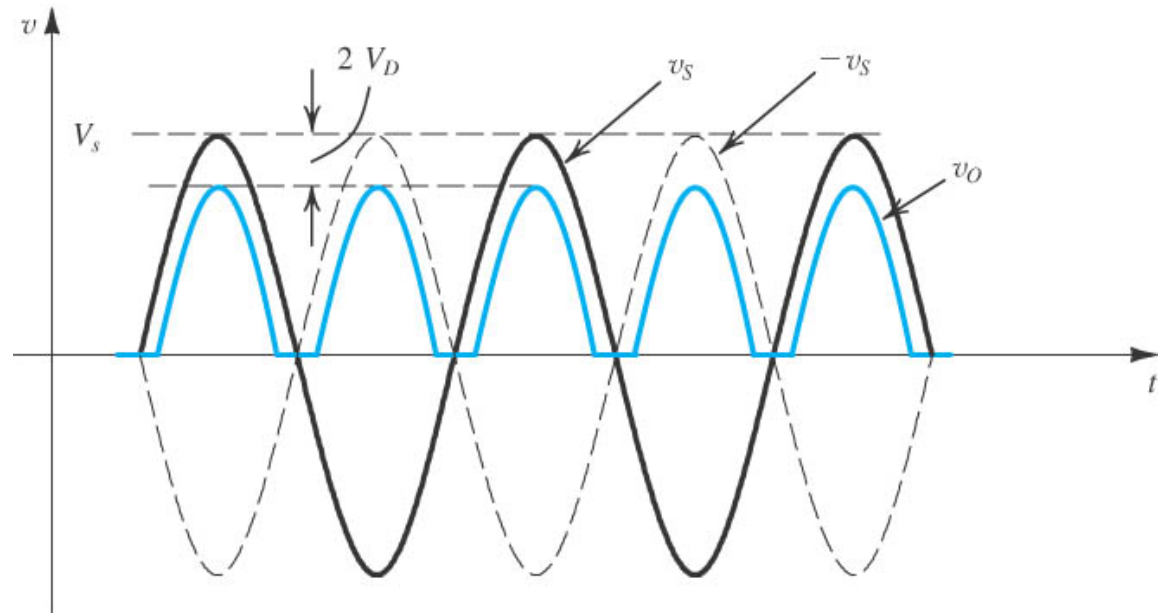
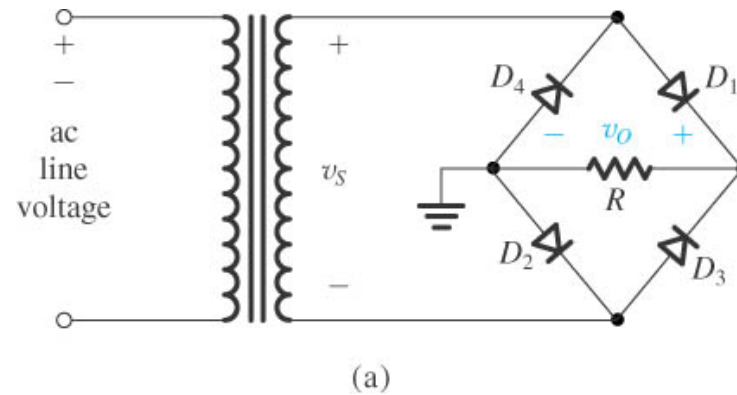
Transfer curve



(c)

$$\text{PIV (peak inverse voltage)} = 2v_s - V_D$$

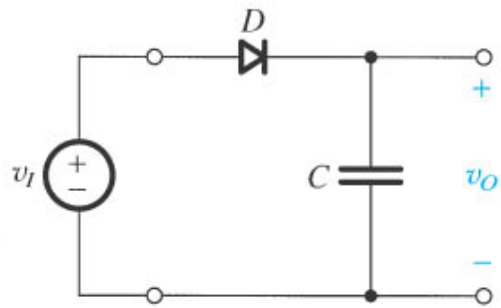
Bridge rectifier



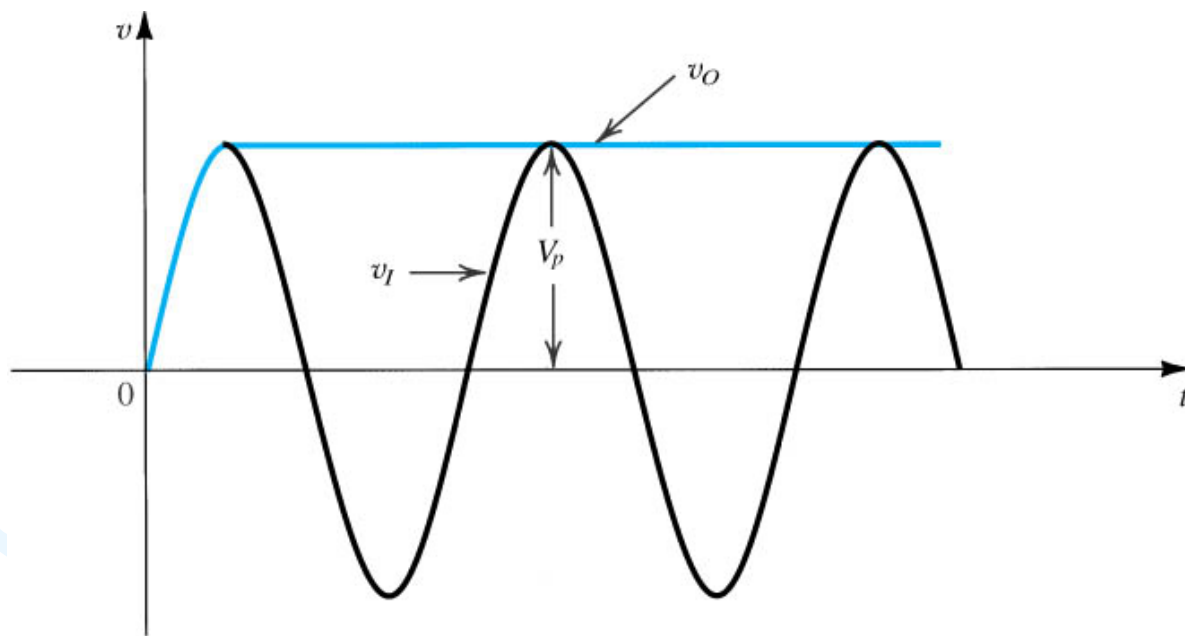
$$\text{PIV (peak inverse voltage)} = v_s - 2V_D + V_D = v_s - V_D$$

	Half-wave	Center-tapped	bridge
v_i	$v_{\max} \sin \omega t$	$v_{\max} \sin \omega t$	$v_{\max} \sin \omega t$
v_0	$V_{\max} - V_D$	$V_{\max} - V_D$	$V_{\max} - 2V_D$
$V_{average}$	$\frac{V_{\max} - V_D}{\pi}$	$\frac{2(V_{\max} - V_D)}{\pi}$	$\frac{2(V_{\max} - V_D)}{\pi}$
V_{rms}	$\frac{V_{\max} - V_D}{2}$	$\frac{V_{\max} - V_D}{\sqrt{2}}$	$\frac{V_{\max} - V_D}{\sqrt{2}}$
PIV	V_{\max}	$2V_{\max} - V_D$	$V_{\max} - V_D$
$F.F.$	$\frac{\pi}{2}$	$\frac{\pi}{2\sqrt{2}}$	$\frac{\pi}{2\sqrt{2}}$
$R.F.$	$\sqrt{(\frac{\pi}{2})^2 - 1}$	$\sqrt{(\frac{\pi}{2\sqrt{2}})^2 - 1}$	$\sqrt{(\frac{\pi}{2\sqrt{2}})^2 - 1}$

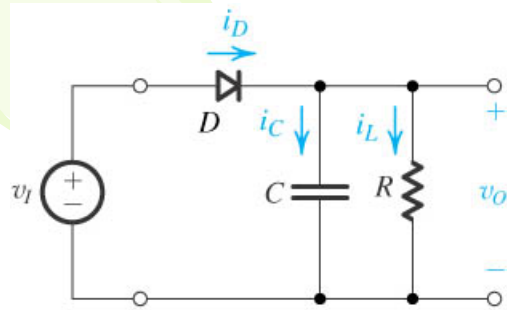
Filter



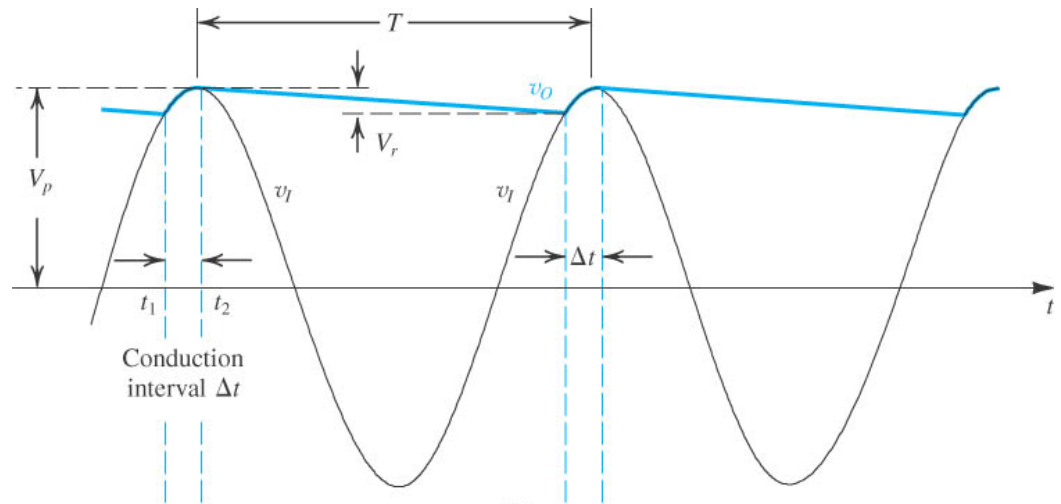
(a)



(b)



(a)



$$i_D = i_C + i_L = C \frac{dv_I}{dt} + \frac{v_o}{R}$$

$$C \frac{dv_o}{dt} + \frac{v_o}{R} = 0$$

$$\frac{dv_o(t)}{dt} = -\frac{v_o(t)}{RC}, \quad v_o(0) = V_p$$

$$v_o(t) = ke^{-\frac{t}{RC}}, \quad k = V_p$$

$$v_o(t) = V_p e^{-\frac{t}{RC}}$$

$$RC \gg T$$

$$V_p - V_r = V_p e^{-\frac{T}{RC}}$$

$$e^{-\frac{T}{RC}} \approx 1 - \frac{T}{RC}$$

$$V_r \approx V_p \frac{T}{RC} = \frac{V_p}{fRC}$$

Average diode current $i_{D\max}$

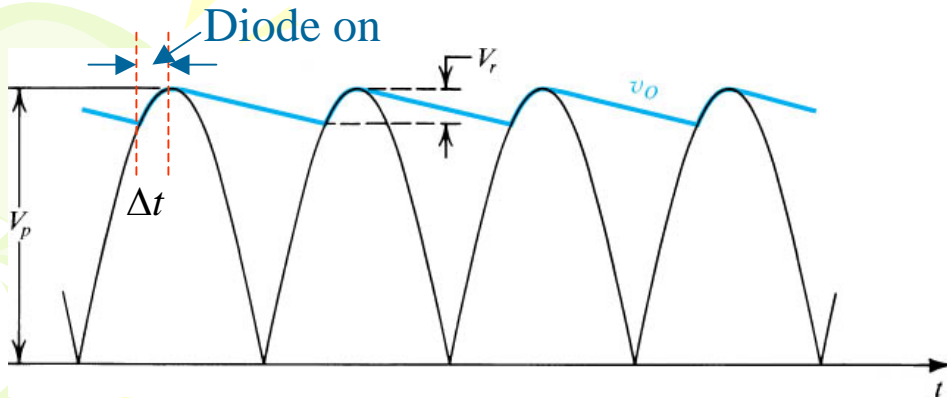
$$i_{D\max} = I_L (1 + 2\pi \sqrt{\frac{2V_p}{V_r}})$$

Half wave

Average diode current i_{Dav}

$$i_{Dav} = I_L (1 + \pi \sqrt{\frac{2V_p}{V_r}})$$

next page



$$V_p - V_r = V_p e^{-\frac{T}{2RC}}$$

$$e^{-\frac{T}{2RC}} \approx 1 - \frac{T}{2RC}$$

full wave

$$V_r \approx V_p \frac{T}{RC} = \frac{V_p}{2fRC}$$

Average diode current i_{Dav}

$$V_p \cos(\omega \Delta t) = V_p - V_r$$

$$\Rightarrow V_p \left[1 - \frac{1}{2} (\omega \Delta t)^2 \right] = V_p - V_r$$

$$\frac{V_p}{2} (\omega \Delta t)^2 = V_r \Rightarrow \omega \Delta t = \sqrt{\frac{2V_r}{V_p}}$$

$$Q_c = i_c \Delta t = (i_{Dav} - I_L) \Delta t$$

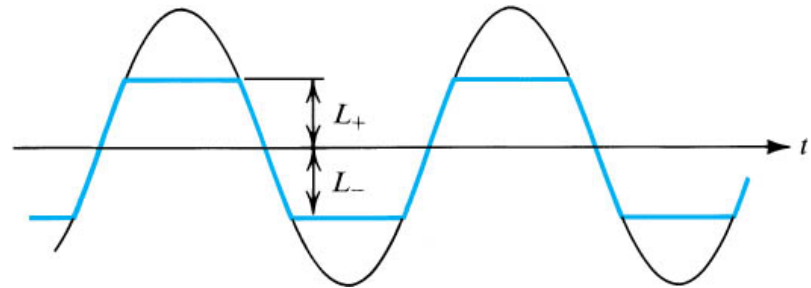
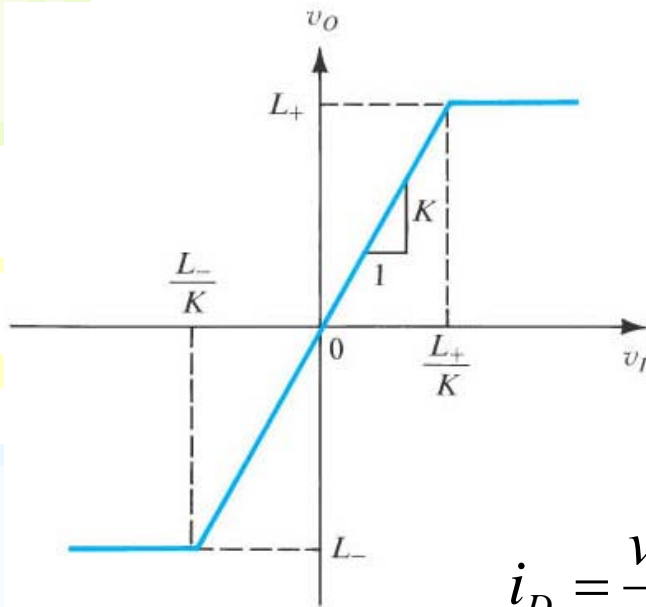
$$\Rightarrow i_{Dav} = I_L + \frac{Q_c}{\Delta t} = I_L + \frac{CV_r}{\Delta t}$$

$$i_{Dav} = \frac{V_p}{R} \left(1 + \pi \sqrt{\frac{V_p}{2V_r}} \right)$$

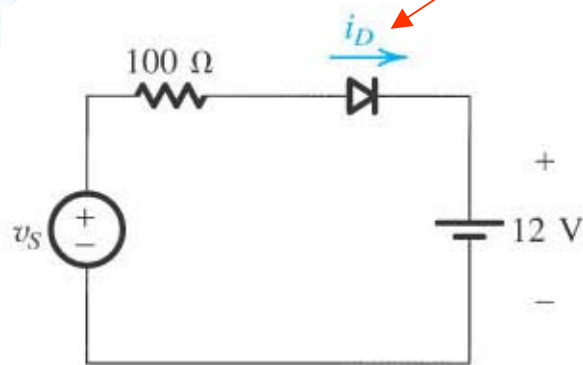
Maximum diode current i_{Dmax}

$$i_{Dmax} = \frac{V_p}{R} \left(1 + 2\pi \sqrt{\frac{V_p}{2V_r}} \right)$$

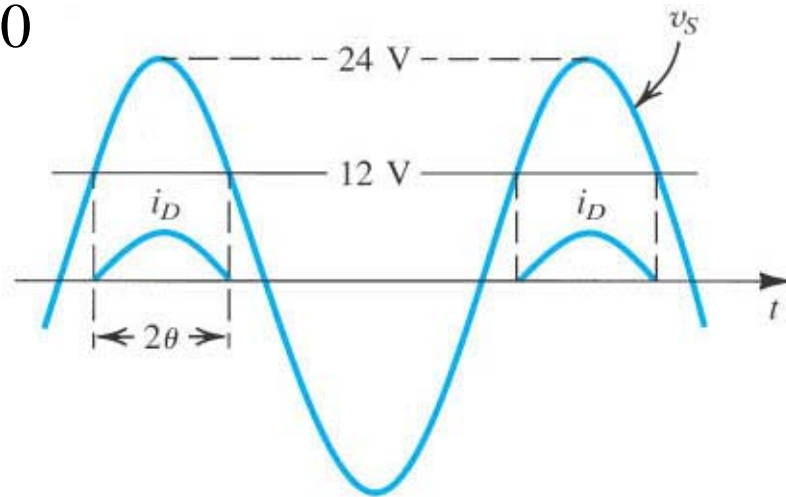
Limiter circuits



$$i_D = \frac{v_s - 12}{100}$$

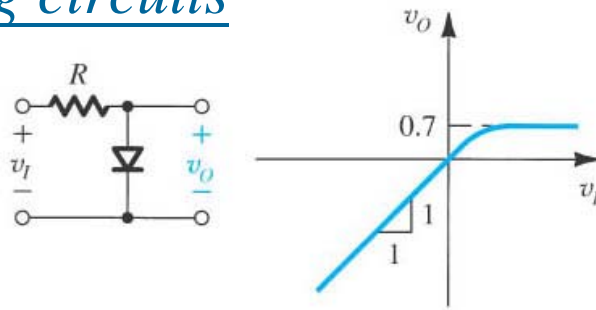


(a)

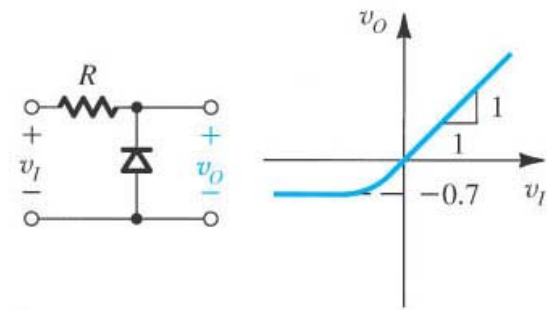


(b)

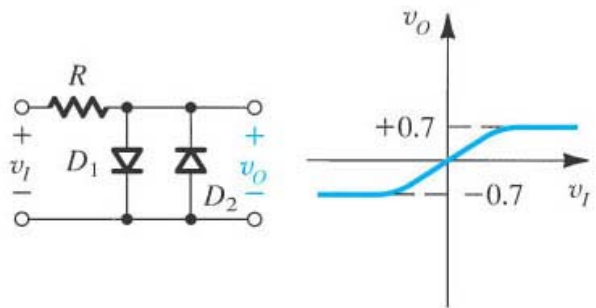
Basic limiting circuits



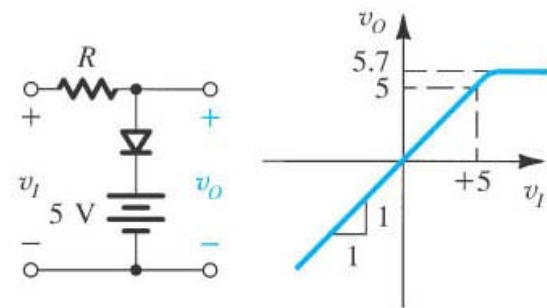
(a)



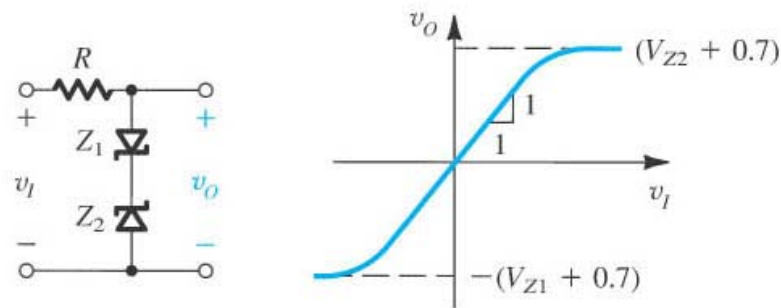
(b)



(c)

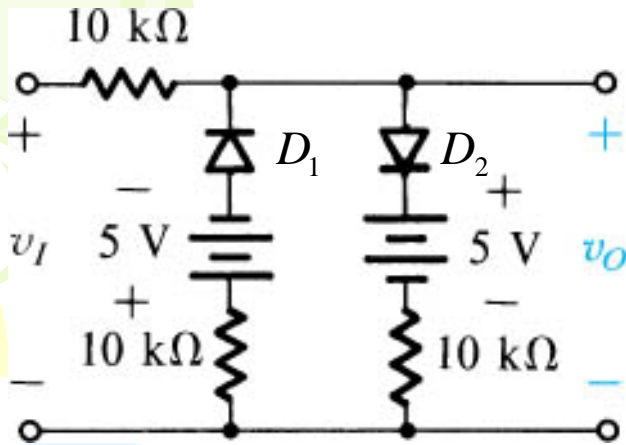


(d)



(e)

example



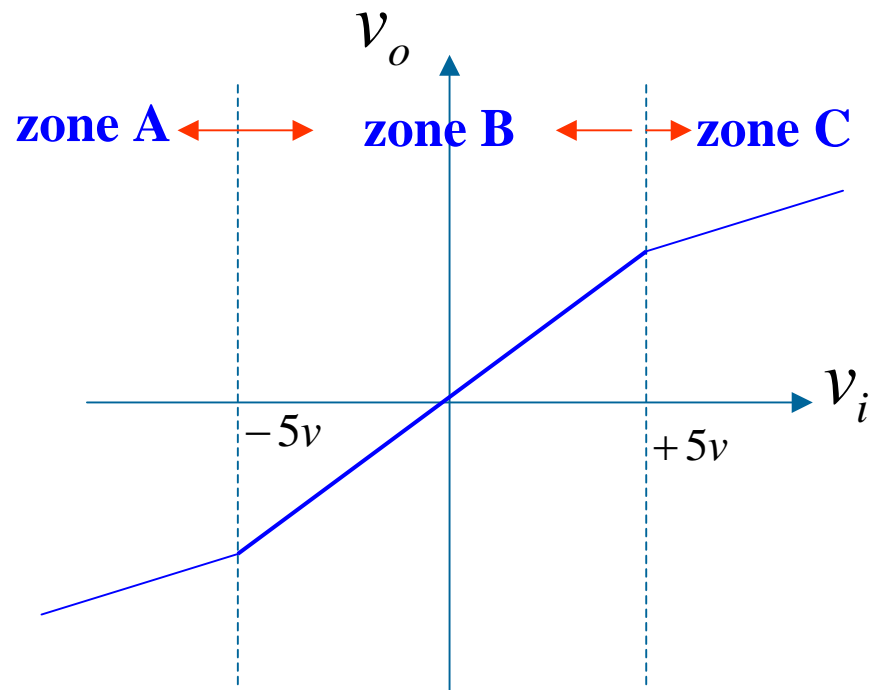
zone A : D_2 off, D_1 on

$$v_o = \frac{10k}{10k + 10k} v_i + \frac{10k}{10k + 10k} (-5v)$$

$$v_o = \frac{1}{2} v_i - 2.5v$$

zone B : D_2 off, D_1 off

$$v_o = v_i$$



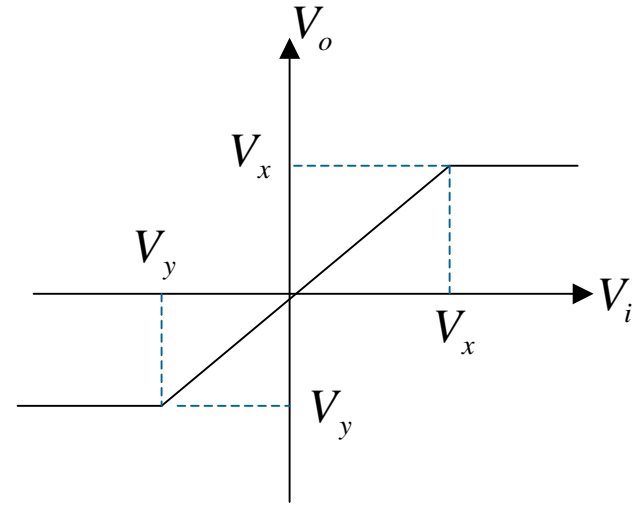
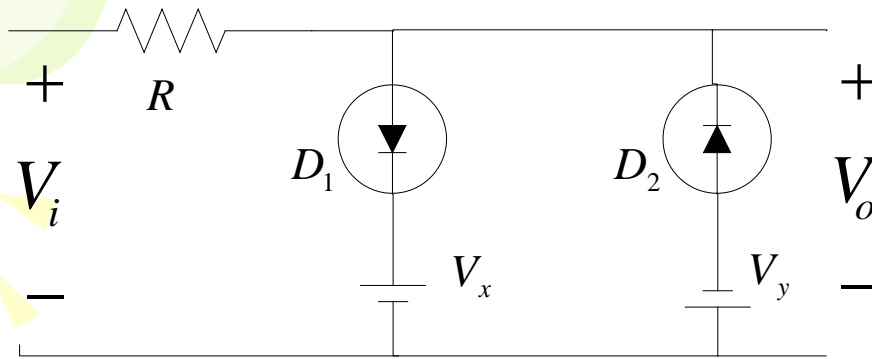
zone C : D_2 on, D_1 off

$$v_o = \frac{10k}{10k + 10k} v_i + \frac{10k}{10k + 10k} (5v)$$

$$v_o = \frac{1}{2} v_i + 2.5v$$

example

Ideal diodes



zone A : D_1 off, D_2 on

$$V_i < V_y \Rightarrow V_o = V_y$$

zone C : D_1 on, D_2 off

$$V_i > V_x \Rightarrow V_o = V_x$$

zone B : D_2 off, D_1 off

$$V_x > V_i > V_y \Rightarrow V_o = V_i$$

example

$$V_i \gg V_A$$

$D_1 \text{ on}, D_2 \text{ off}$

$$\Rightarrow V_o = 10V$$

$$V_i \ll V_B$$

$D_1 \text{ off}, D_2 \text{ on}$

$$\Rightarrow V_o = 7.5V$$

$$V_A > V_i > V_B \Rightarrow D_1 \text{ on}, D_2 \text{ on}$$

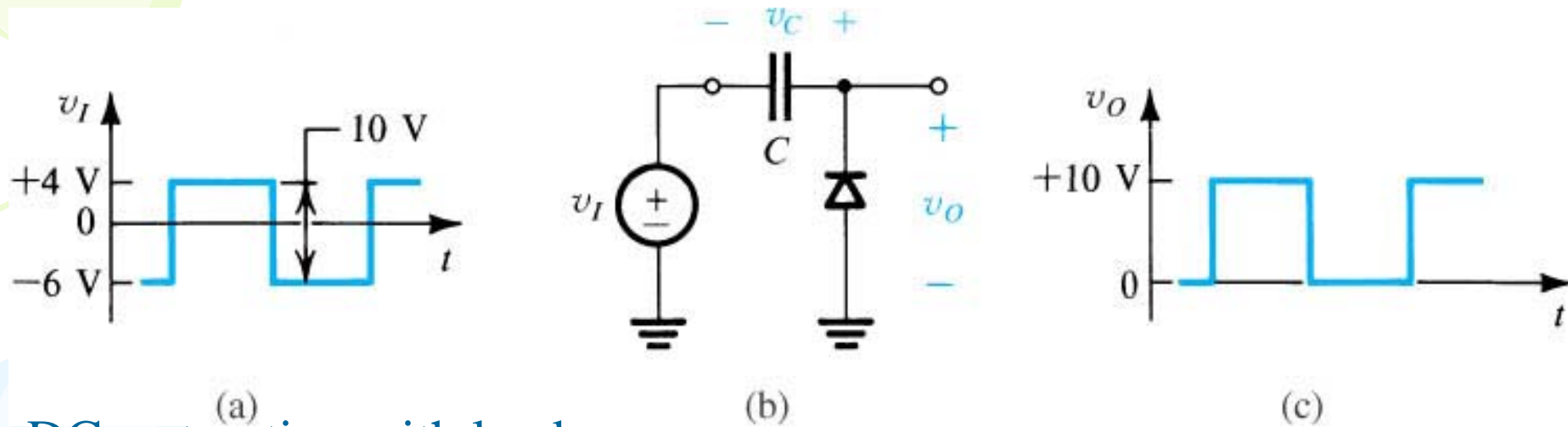
$$V_o = V_i \frac{(10//5)}{15+(10//5)} + 2.5 \frac{(15//5)}{10+(15//5)} + 10 \frac{(10//15)}{5+(10//15)}$$

$$= \frac{1}{11} (2V_i + 67.5)$$

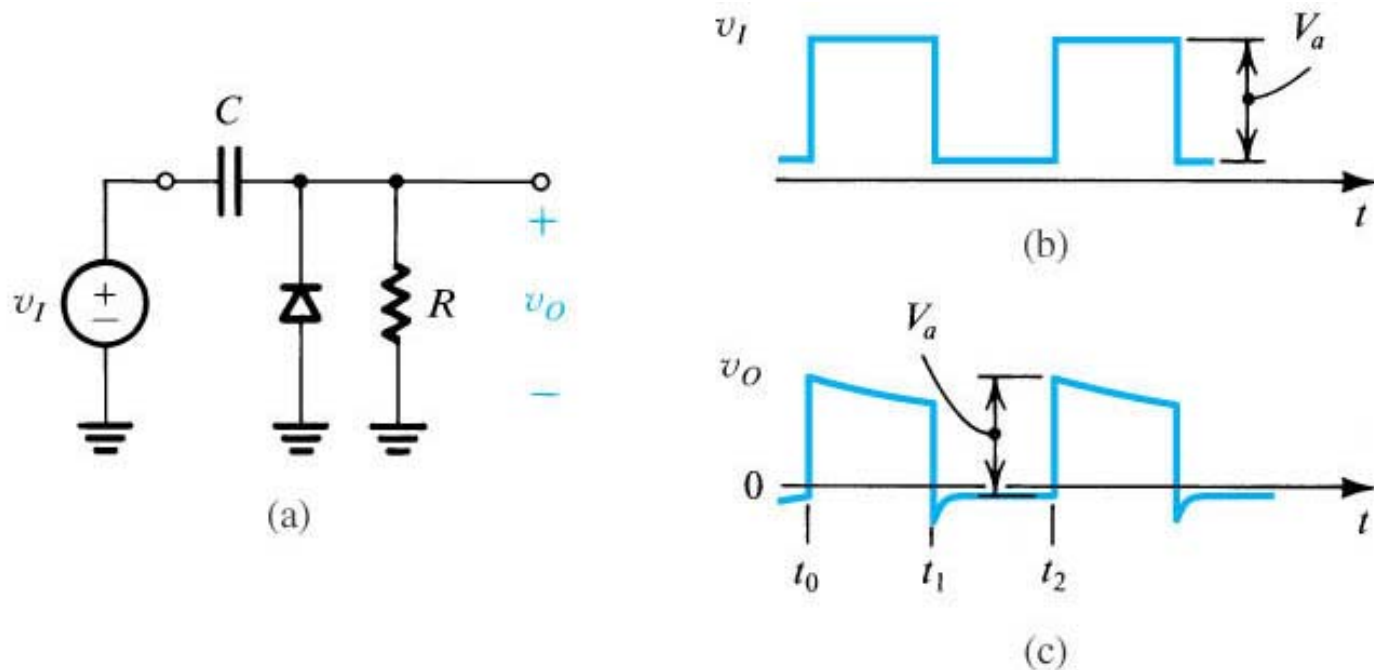
$$V_o = 7.5V = \frac{1}{11} (2V_B + 67.5)$$

$$V_o = 10V = \frac{1}{11} (2V_A + 67.5)$$

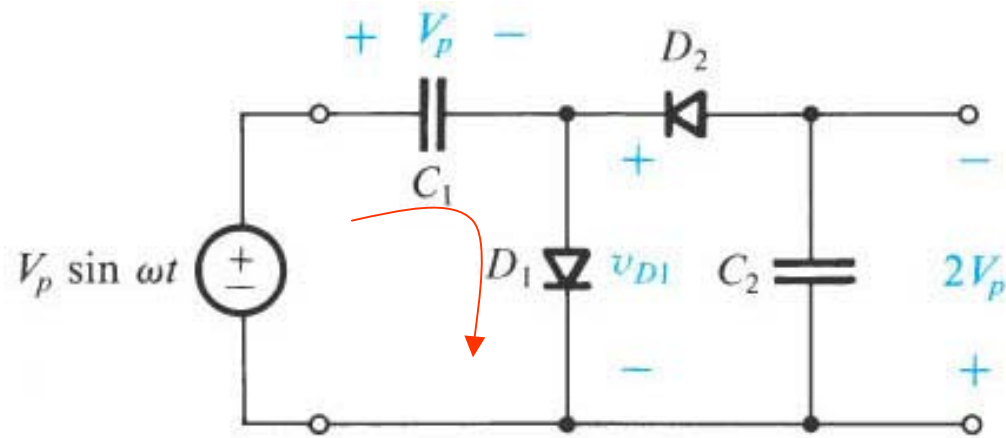
DC restoration (clamping circuit)



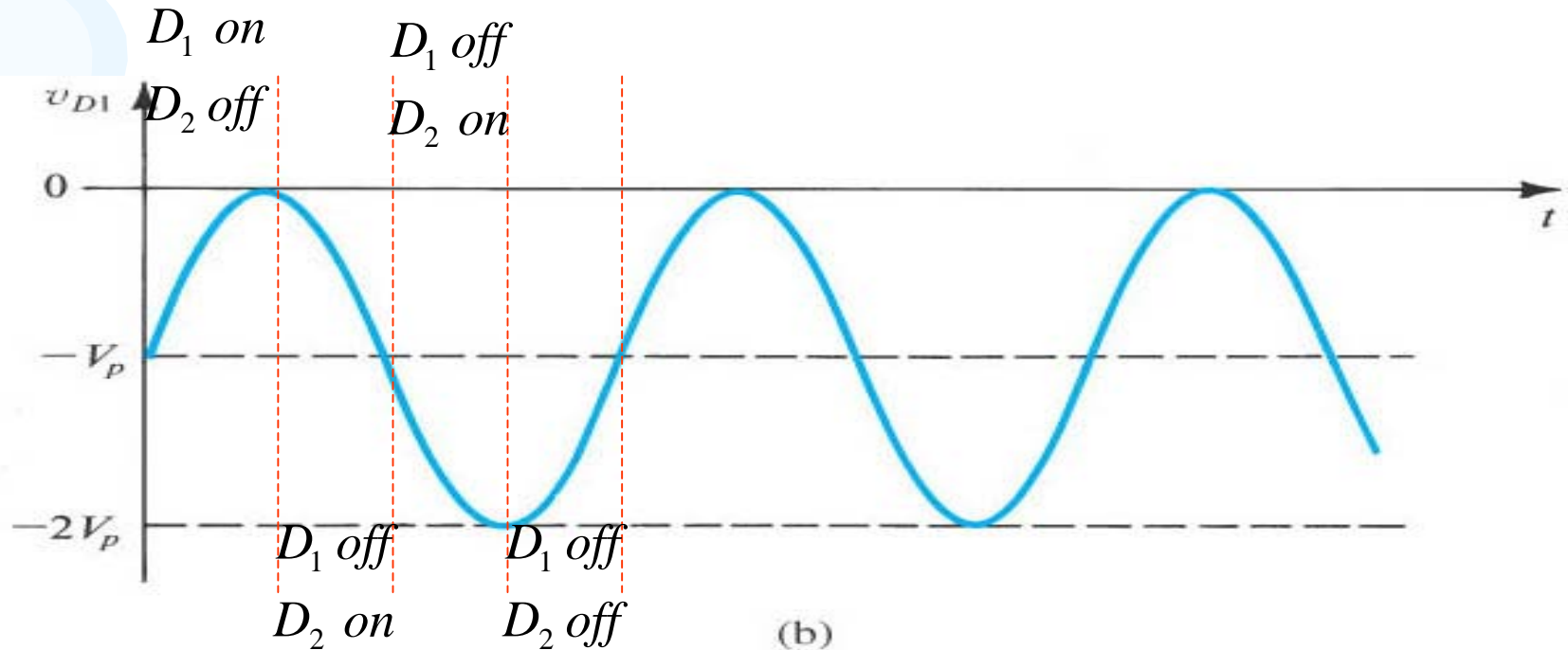
DC restoration with load



Voltage doubler

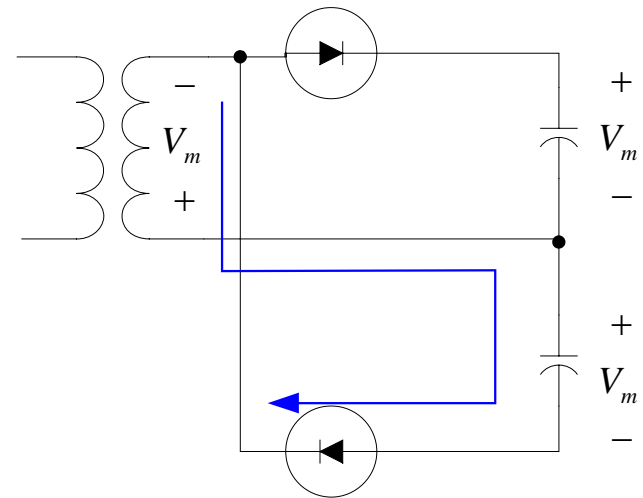
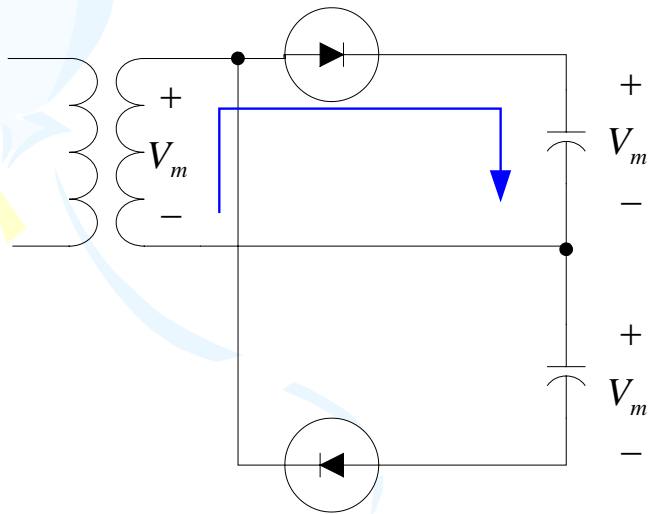
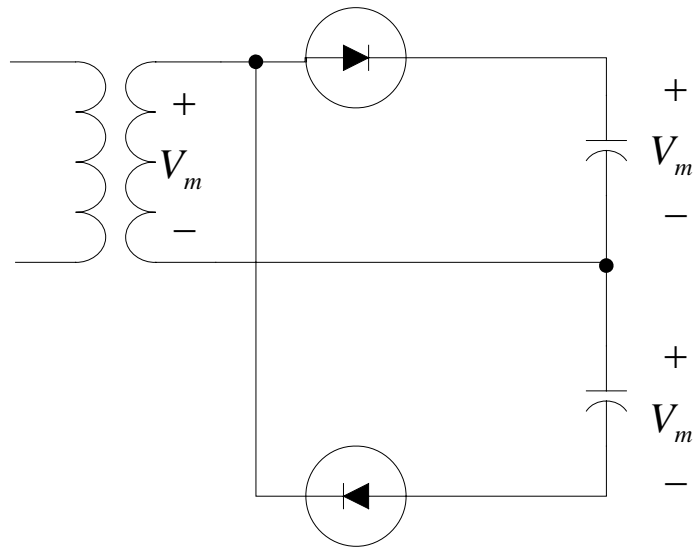


(a)

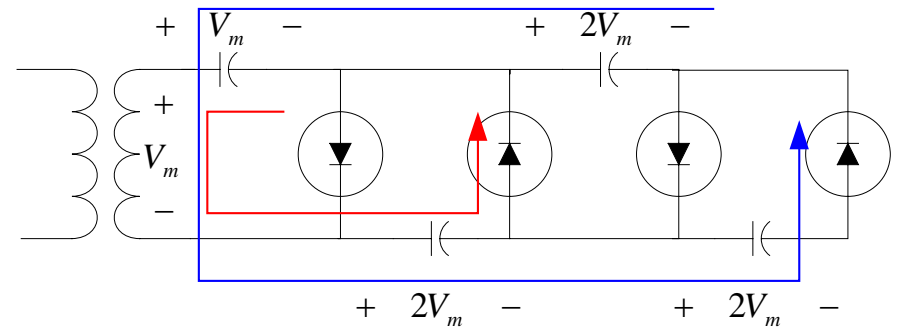
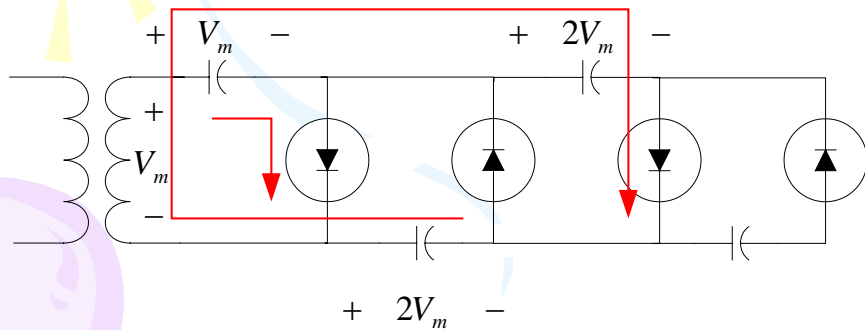
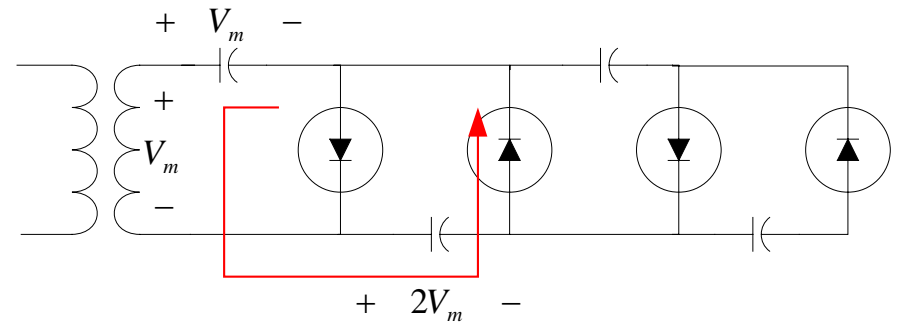
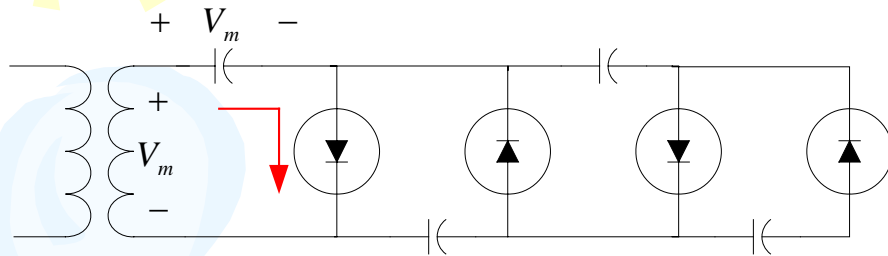
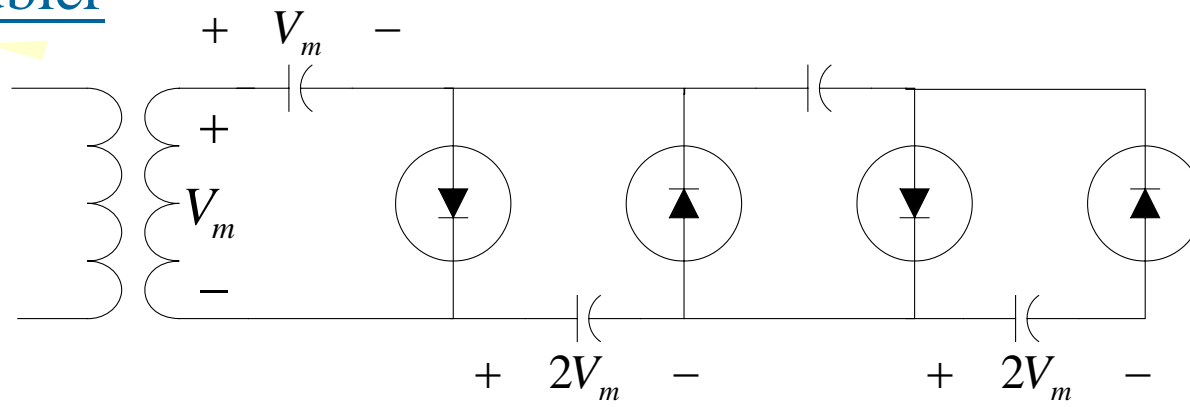


(b)

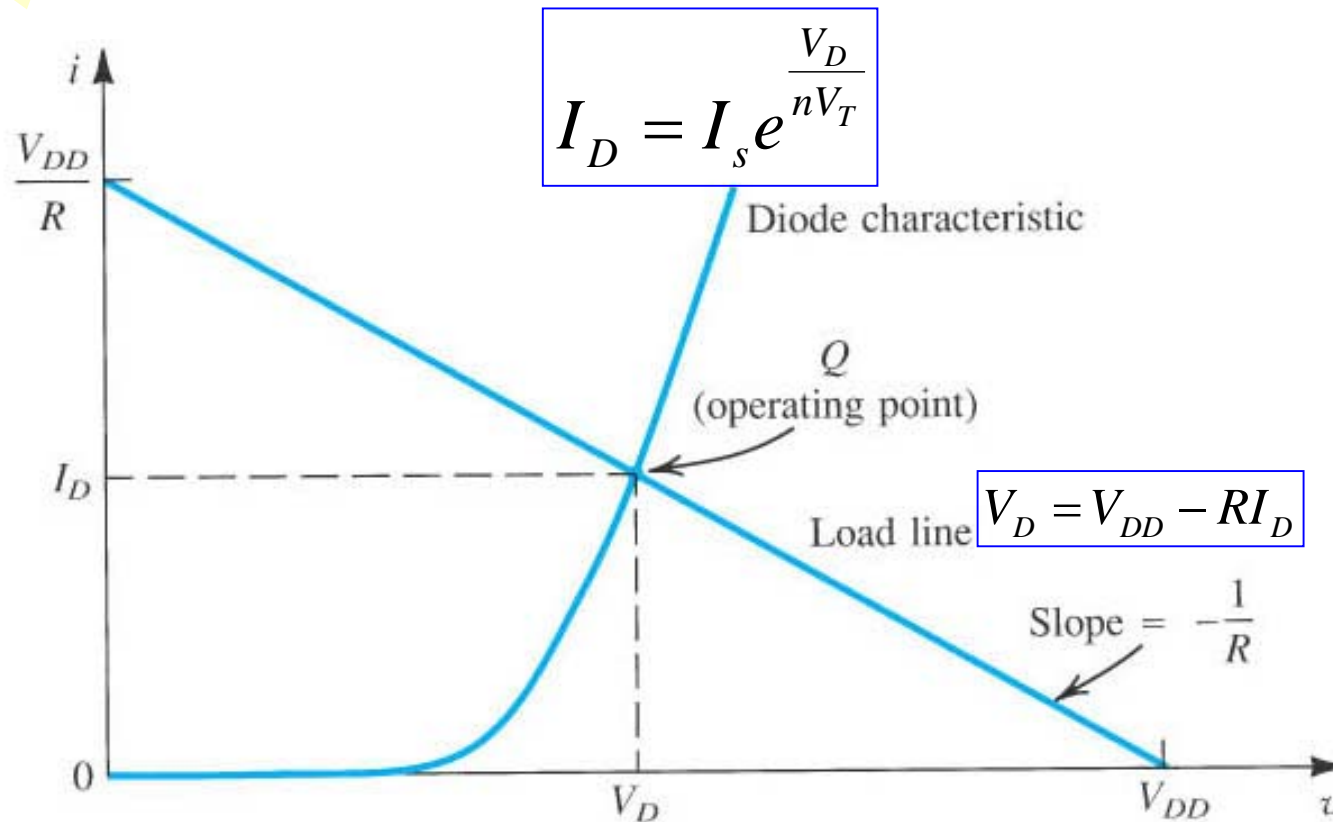
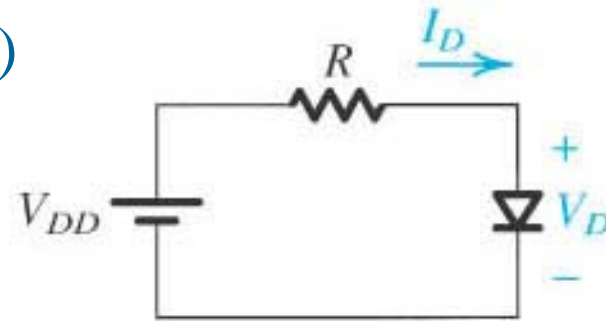
Voltage doubler

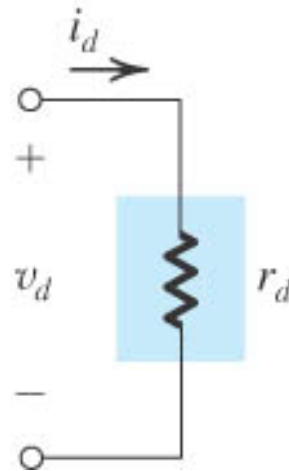
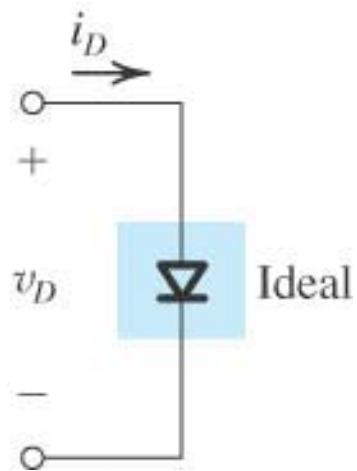
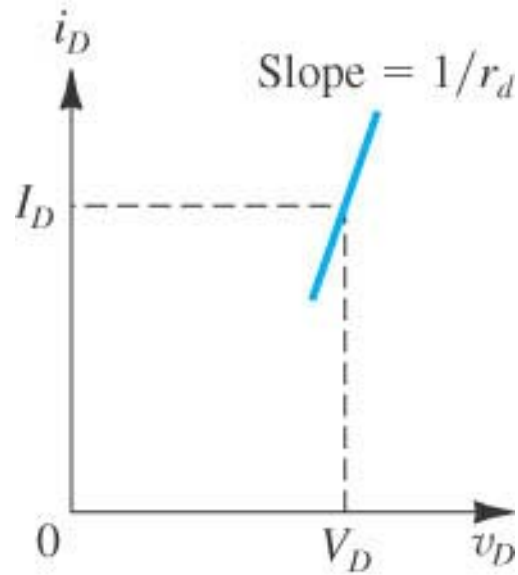
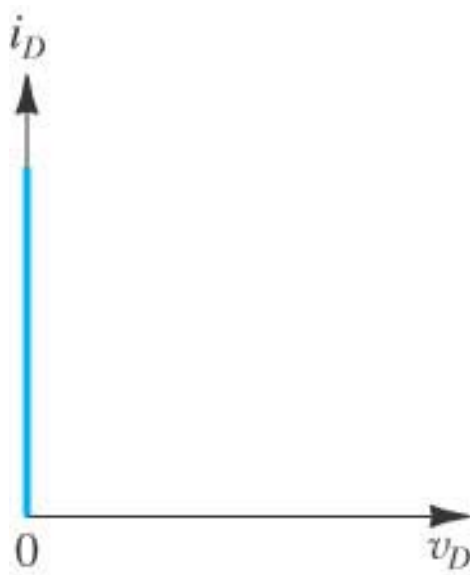


Voltage doubler

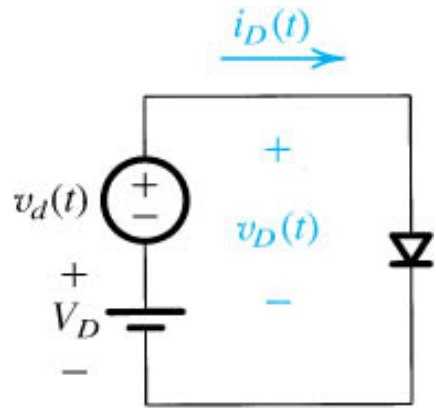


Operating point (DC analysis)





small signal analysis (AC analysis)



$$I_D = I_s e^{\frac{V_D}{nV_T}}$$

$$v_D = V_D + v_d(t)$$

$$i_D = I_s e^{\frac{v_D}{nV_T}} = I_s e^{\frac{(V_D + v_d(t))}{nV_T}} = I_s e^{\frac{V_D}{nV_T}} e^{\frac{v_d(t)}{nV_T}}$$

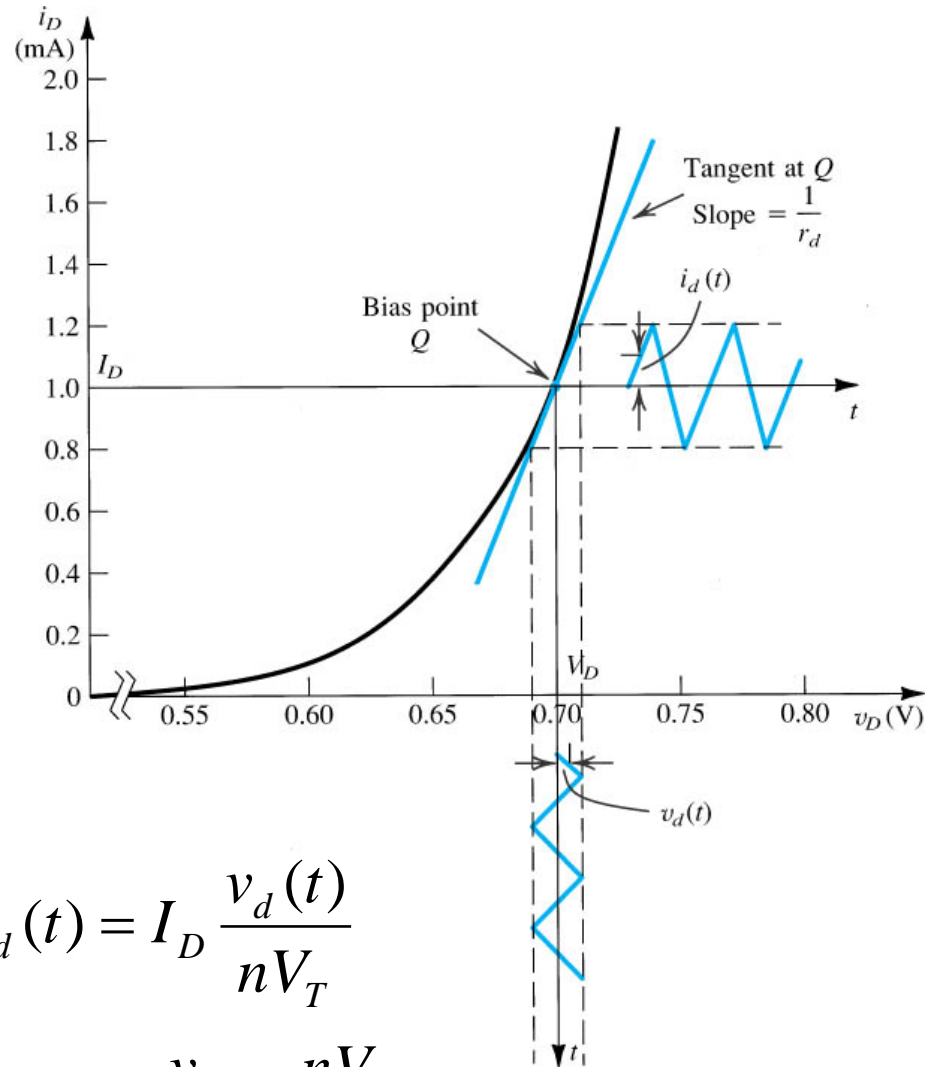
$$i_D = I_D e^{\frac{v_d(t)}{nV_T}}$$

$$\frac{v_d(t)}{nV_T} \ll 1 \quad \text{small signal}$$

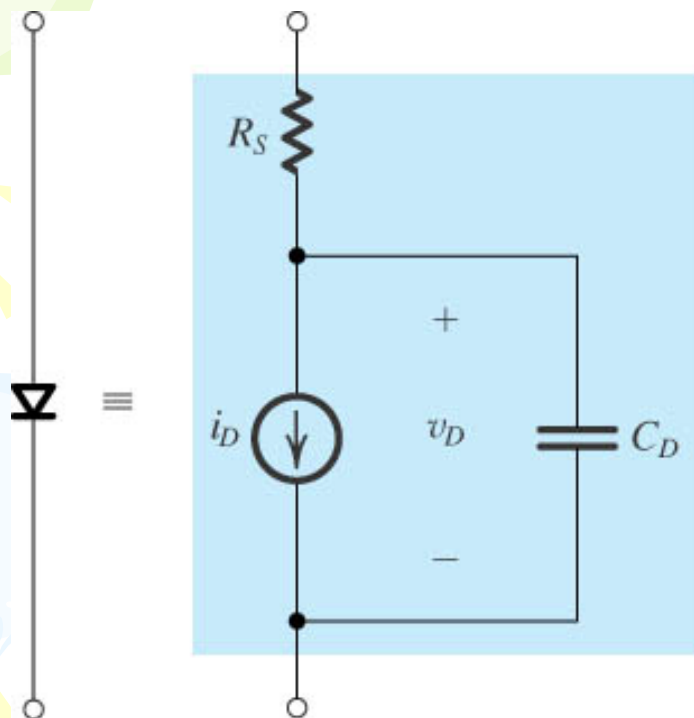
$$i_D \approx I_D \left(1 + \frac{v_d(t)}{nV_T} \right)$$

$$i_d(t) = I_D \frac{v_d(t)}{nV_T}$$

$$\Rightarrow r_d = \frac{v_d}{i_d} = \frac{nV_T}{I_D} \quad (b)$$



Diode Model (small signal model)



$$i_D = I_S (e^{v_D/nV_T} - 1)$$

$$C_D = C_d + C_j = \frac{\tau_T}{V_T} I_S e^{v_D/nV_T} + C_{j0} \left/ \left(1 - \frac{v_D}{V_0} \right)^m \right.$$