The background features several large, stylized, overlapping swirls in light green, light blue, and light purple. Scattered throughout are numerous small, yellow, starburst-like shapes of varying sizes.

Lecture 13

MOSFET Differential Amplifiers



topics

- Ideal characteristics of differential amplifier
 - Input differential resistance
 - Input common-mode resistance
 - Differential voltage gain
 - CMRR
- Non-ideal characteristics of differential amplifier
 - Input offset voltage
 - Input biasing and offset current
- Differential Amplifier with active load
- Frequency response

MOS differential pair

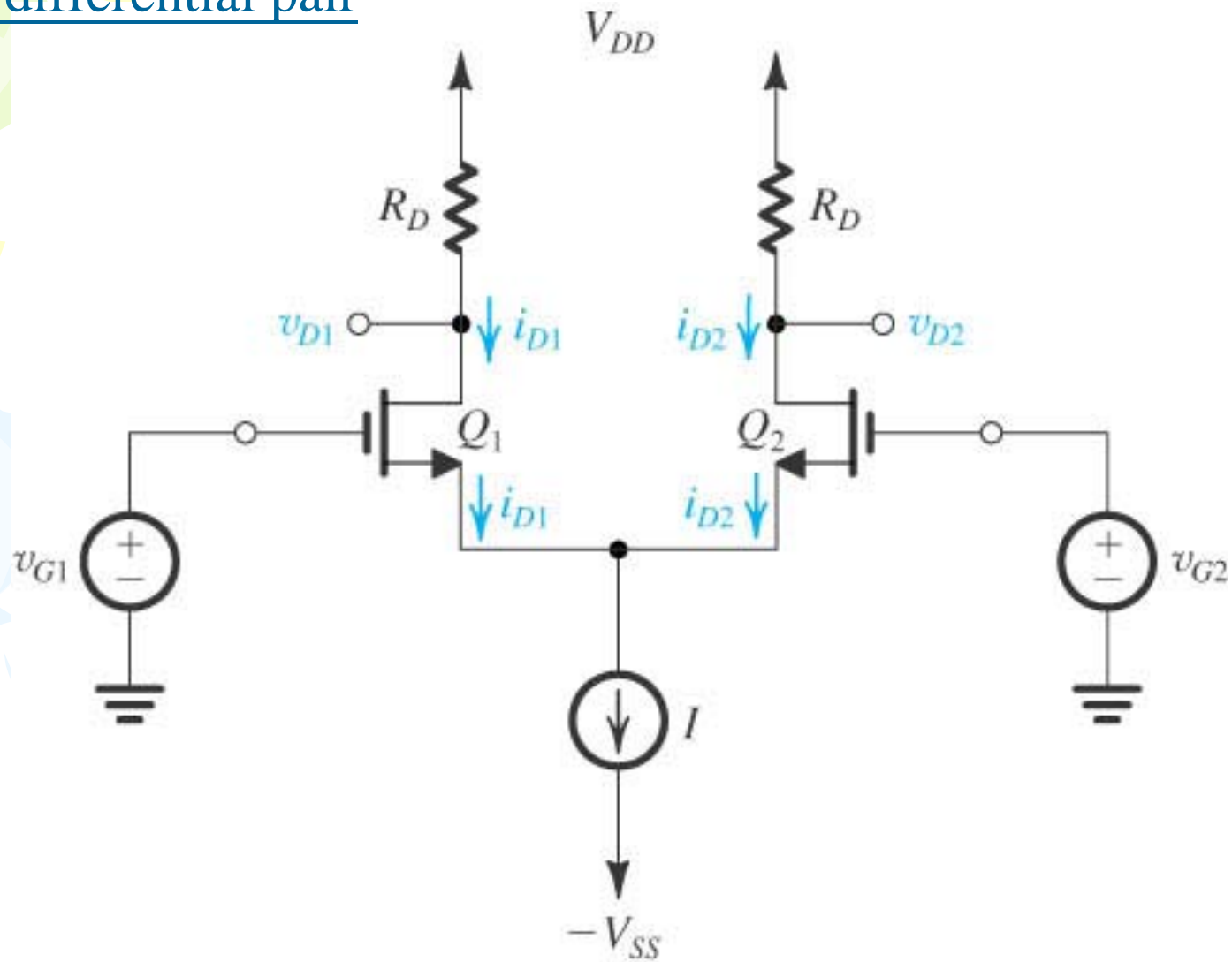


Figure 7.1 The basic MOS differential-pair configuration.

Common mode operation

BJT's differential pair V_{CM} no bound

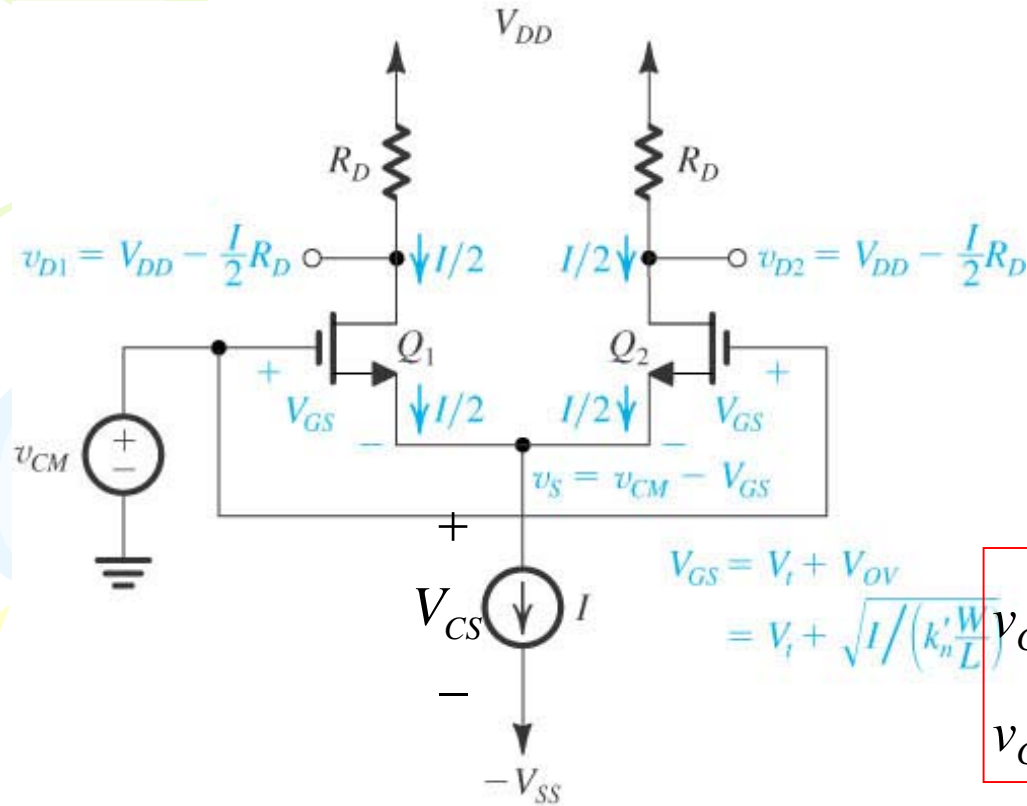


Figure 7.2 The MOS differential pair with a common-mode input voltage v_{CM} .

$$\therefore Q_1 = Q_2$$

$$\therefore I_{D1} = I_{D2} = \frac{I}{2}$$

$$v_s = v_{CM} - V_{GS}$$

$$I_D = \frac{I}{2} = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

$$v_{D1} = v_{D2} = V_{DD} - I_D R_D$$

Q_1 and Q_2 in saturation mode

$$v_{DS} > v_{GS} - V_t$$

$$(V_{DD} - I_D R_D) - v_s > v_{CM} - v_s - V_t$$

$$v_{CM(\max)} = V_t + V_{DD} - \frac{I}{2} R_D$$

$$v_{CM(\min)} = -V_{ss} + V_{CS} + V_t + (V_{GS} - V_t)$$

$$v_{GS} - V_t = v_{CM} - v_s - V_t$$

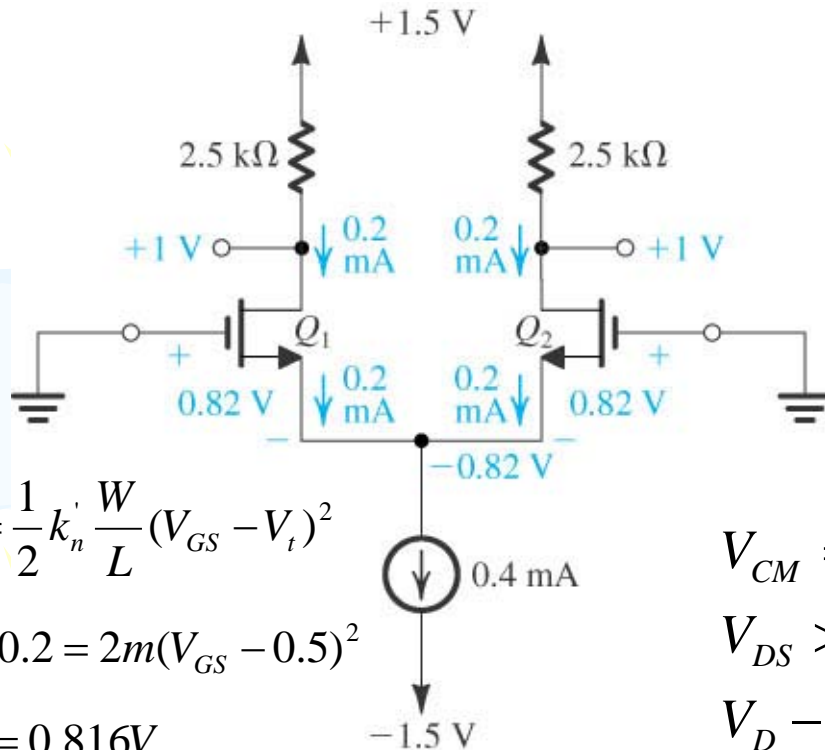
$$= v_{CM} - (V_{CS} - V_{SS}) - V_t$$

$$v_{CM} = V_{CS} - V_{SS} + v_{GS}$$

Make sure current source is working

Exercise 7.1

$$V_{DD} = V_{SS} = 1.5V, k'_n \frac{W}{L} = 4mA/V^2, V_t = 0.5V, I = 0.4mA, R_D = 2.5k$$



$$I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$

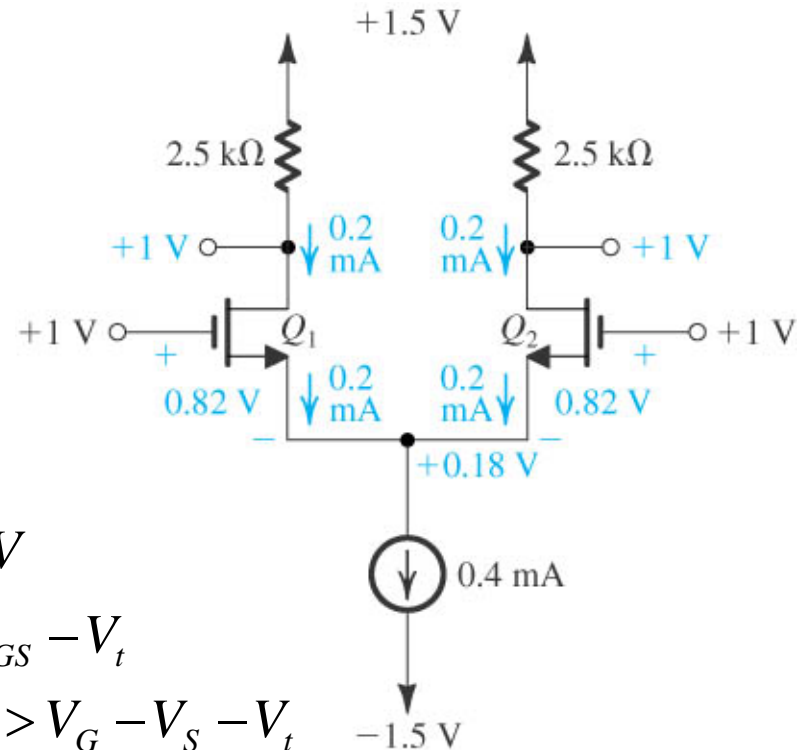
$$\frac{I}{2} = 0.2 = 2m(V_{GS} - 0.5)^2$$

$$V_{GS} = 0.816V$$

$$V_{DS} = 1.5 - 0.2m \times 2.5k = 1V^{(a)}$$

$$V_{DS} > V_{GS} - V_t = 0.816 - 0.5$$

Saturation mode



$$V_{CM} = 1V$$

$$V_{DS} > V_{GS} - V_t$$

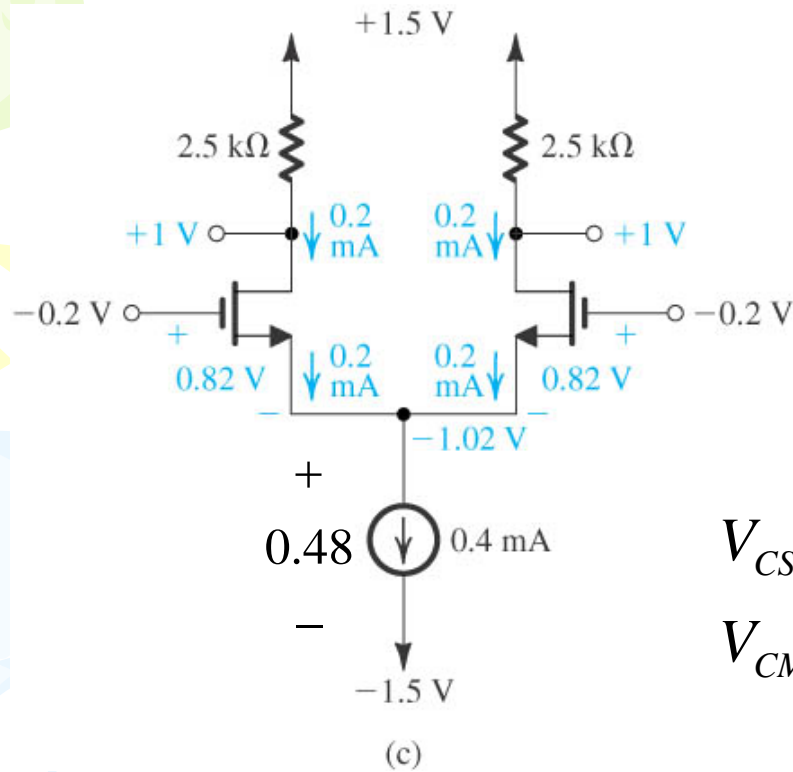
$$V_D - V_S > V_G - V_S - V_t$$

$$V_D > V_G - V_t \Rightarrow V_D > V_{CM} \oplus V_t$$

$$\Rightarrow V_D = 1 > 1 - 0.5$$

Saturation mode

$$V_{CM(max)} = 1.5V$$



$$V_{CS(\min)} = 0.48$$

$$V_{CM(\min)} = 0.82 + 0.48 - 1.5 = -0.2 \text{ V}$$

Figure 7.3 (Continued)

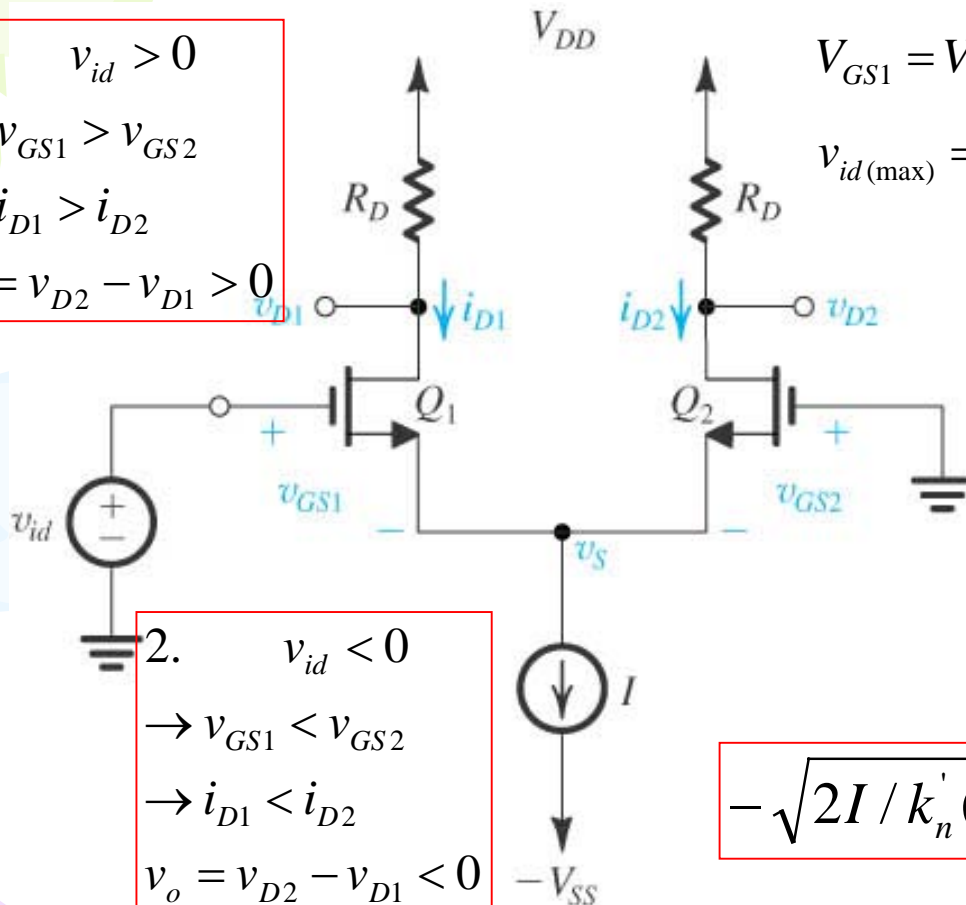
Differential mode operation

$$1. \quad v_{id} > 0$$

$$\rightarrow v_{GS1} > v_{GS2}$$

$$\rightarrow i_{D1} > i_{D2}$$

$$v_o = v_{D2} - v_{D1} > 0$$



$$2. \quad v_{id} < 0$$

$$\rightarrow v_{GS1} < v_{GS2}$$

$$\rightarrow i_{D1} < i_{D2}$$

$$v_o = v_{D2} - v_{D1} < 0$$

$$I_{D1} = I = \frac{1}{2} k'_n \frac{W}{L} (V_{GS1} - V_t)^2$$

$$V_{GS1} = V_t + \sqrt{2I / k'_n (W / L)}$$

$$v_{id(max)} = v_{GS1} + v_s = V_t + \sqrt{2I / k'_n (W / L)} - v_{GS2}$$

$$\because Q_2 \text{ off} \Rightarrow v_{GS2} = V_t$$

$$\text{if } v_{id} > v_{id(max)} \Rightarrow i_{D1} = I$$

$$\Rightarrow v_{GS1} = V_t + \sqrt{2I / k'_n (W / L)}$$

$$v_{id} \uparrow \rightarrow v_s \uparrow \rightarrow Q_2 \Rightarrow \text{off}$$

$$-\sqrt{2I / k'_n (W / L)} \leq v_{id} \leq \sqrt{2I / k'_n (W / L)}$$

Figure 7.4 The MOS differential pair with a differential input signal v_{id} applied. With v_{id} positive: $v_{GS1} > v_{GS2}$, $i_{D1} > i_{D2}$, and $v_{D1} < v_{D2}$; thus $(v_{D2} - v_{D1})$ will be positive. With v_{id} negative: $v_{GS1} < v_{GS2}$, $i_{D1} < i_{D2}$, and $v_{D1} > v_{D2}$; thus $(v_{D2} - v_{D1})$ will be negative.

Large signal operation

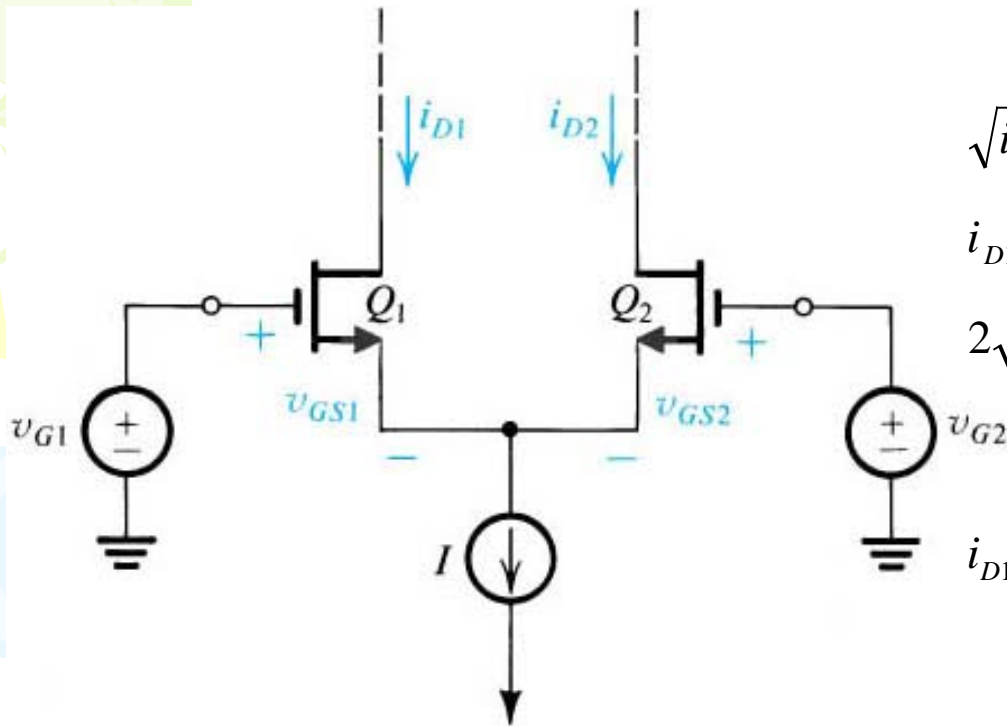


Figure 7.5 The MOSFET differential pair for the purpose of deriving the transfer characteristics, i_{D1} and i_{D2} versus $v_{id} = v_{G1} - v_{G2}$.

$$i_{D1} = \frac{1}{2} k_n' \frac{W}{L} (v_{GS1} - V_t)^2$$

$$i_{D2} = \frac{1}{2} k_n' \frac{W}{L} (v_{GS2} - V_t)^2$$

$$v_{id} = v_{GS1} - v_{GS2} = v_{G1} - v_{G2}$$

$$\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{\frac{1}{2} k_n' \frac{W}{L}} v_{id}$$

$$i_{D1} + i_{D2} = I$$

$$2\sqrt{i_{D1} i_{D2}} = I - \frac{1}{2} k_n' \frac{W}{L} v_{id}^2$$

$$i_{D1} = \frac{I}{2} + \sqrt{\frac{1}{2} k_n' \frac{W}{L}} I \left(\frac{v_{id}}{2} \right) \sqrt{1 - \left(\frac{v_{id} / 2}{\sqrt{\frac{1}{2} k_n' \frac{W}{L}}} \right)^2}$$

$$i_{D1} = \frac{I}{2} - \sqrt{\frac{1}{2} k_n' \frac{W}{L}} I \left(\frac{v_{id}}{2} \right) \sqrt{1 - \left(\frac{v_{id} / 2}{\sqrt{\frac{1}{2} k_n' \frac{W}{L}}} \right)^2}$$

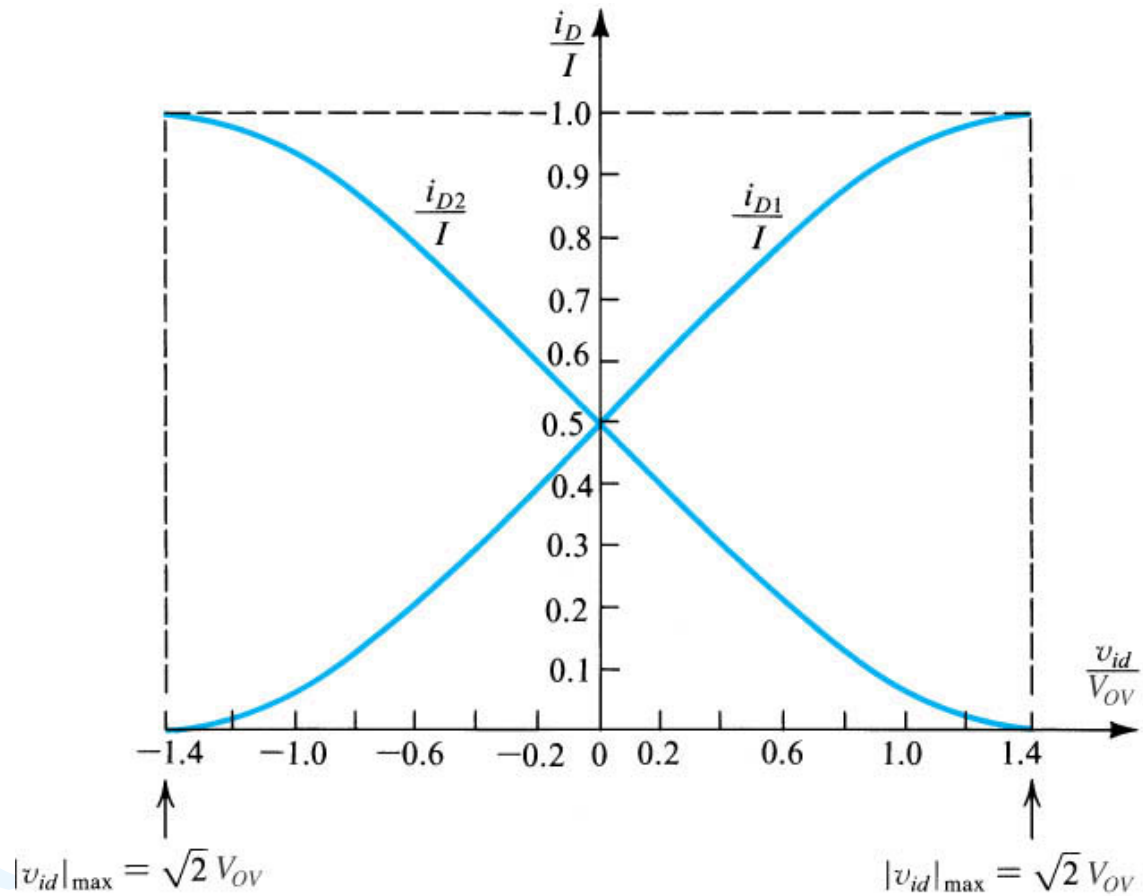


Figure 7.6 Normalized plots of the currents in a MOSFET differential pair. Note that V_{OV} is the overdrive voltage at which Q_1 and Q_2 operate when conducting drain currents equal to $I/2$.

$$i_{D1} = \frac{I}{2} + \sqrt{\frac{1}{2} k_n' \frac{W}{L} I \left(\frac{v_{id}}{2} \right)} \sqrt{1 - \left(\frac{v_{id} / 2}{\sqrt{\frac{1}{2} k_n' \frac{W}{L}}} \right)^2}$$

More k is bigger more linear range of v_{id}

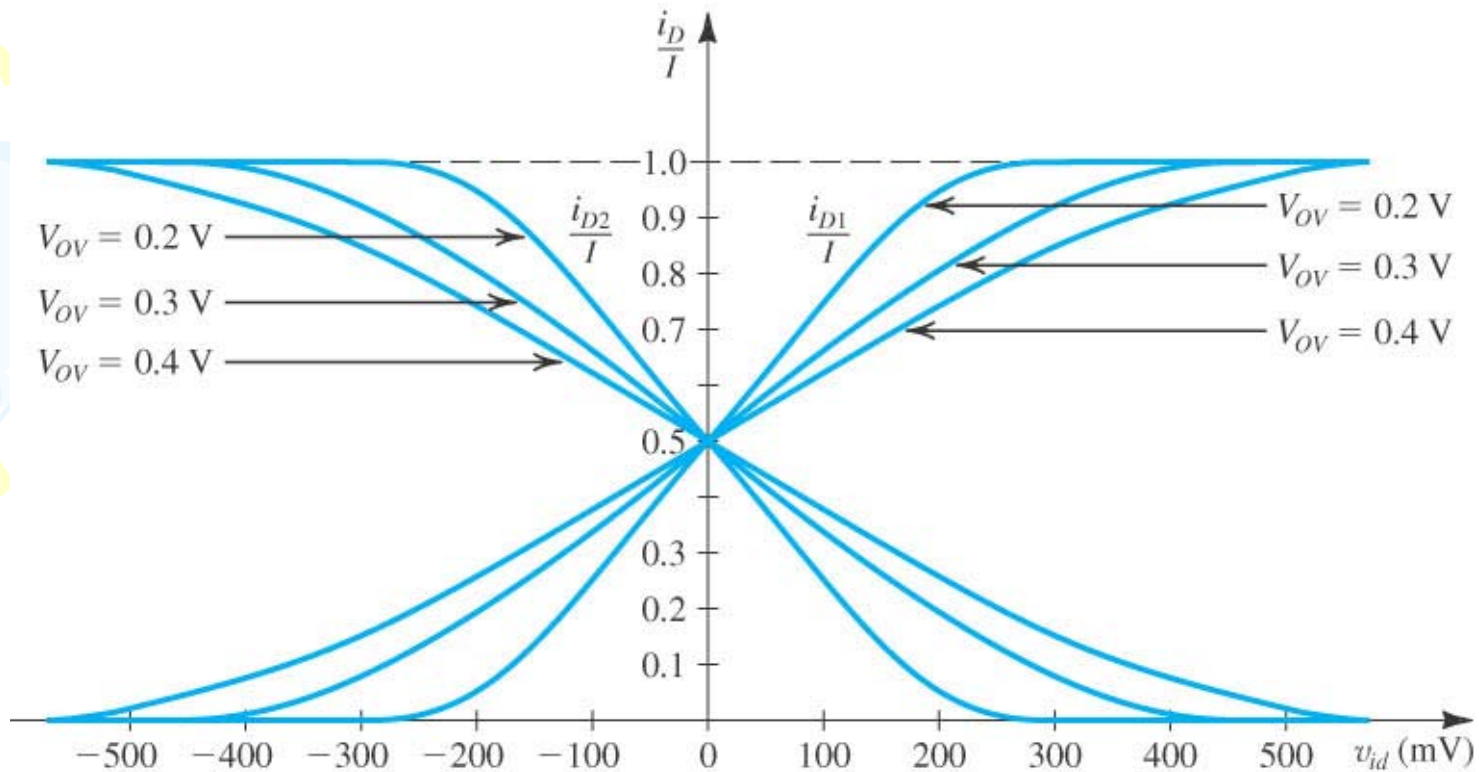
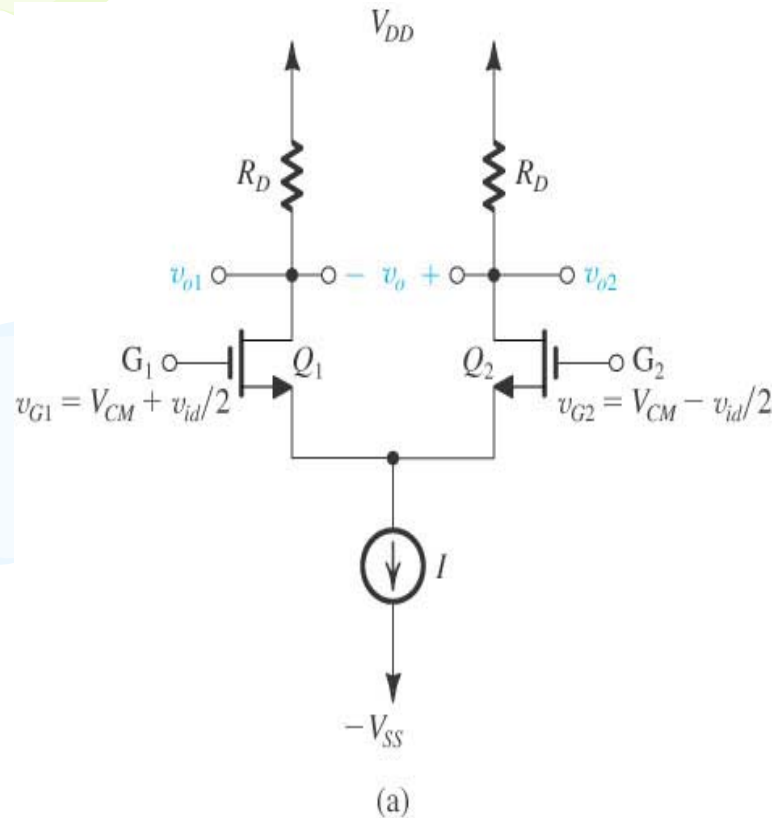


Figure 7.7 The linear range of operation of the MOS differential pair can be extended by operating the transistor at a higher value of V_{OV} .

7-2.1 Small signal operation (differential gain)



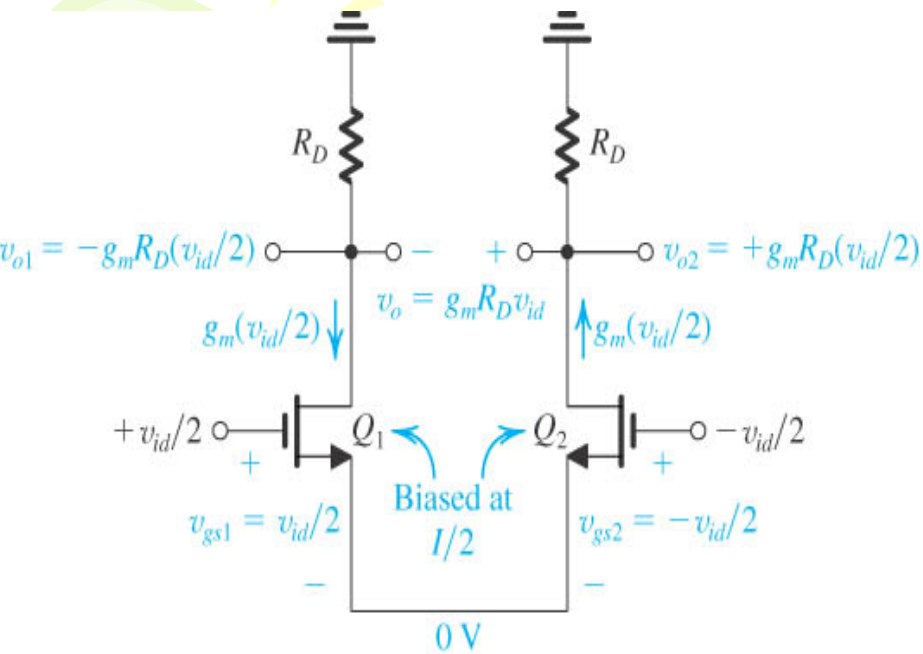
$$v_{G1} = v_{CM} + \frac{1}{2}v_{id}$$

$$v_{G2} = v_{CM} - \frac{1}{2}v_{id}$$

$$v_{CM} = \frac{v_1 + v_2}{2}$$

$$v_{id} = \frac{v_1 - v_2}{2}$$

Figure 7.8 Small-signal analysis of the MOS differential amplifier: (a) The circuit with a common-mode voltage applied to set the dc bias voltage at the gates and with v_{id} applied in a complementary (or balanced) manner. (b) The circuit prepared for small-signal analysis. (c) An alternative way of looking at the small-signal operation of the circuit.



(b)

$$v_{o1} = -g_m \frac{v_{id}}{2} (R_D // r_{o1})$$

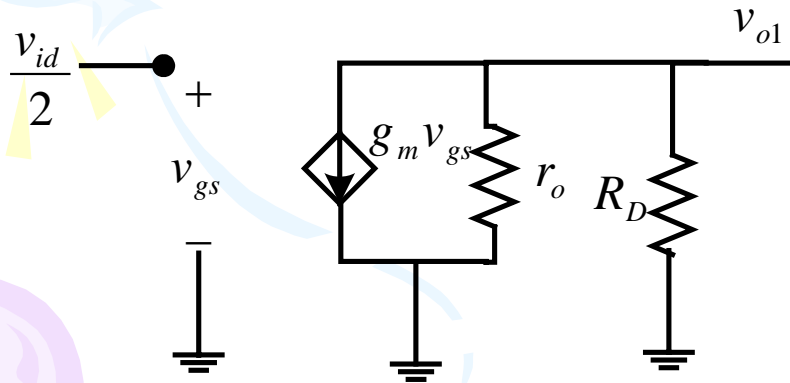
$$v_{o2} = g_m \frac{v_{id}}{2} (R_D // r_{o2})$$

$$\frac{v_{o1}}{v_{id}} = -g_m \frac{1}{2} (R_D // r_{o1}), \frac{v_{o2}}{v_{id}} = g_m \frac{1}{2} (R_D // r_{o2})$$

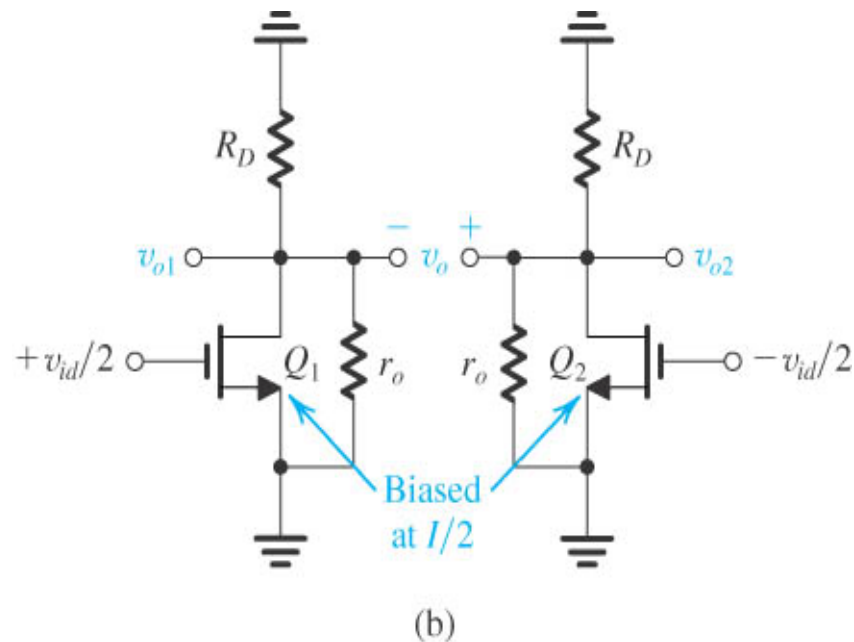
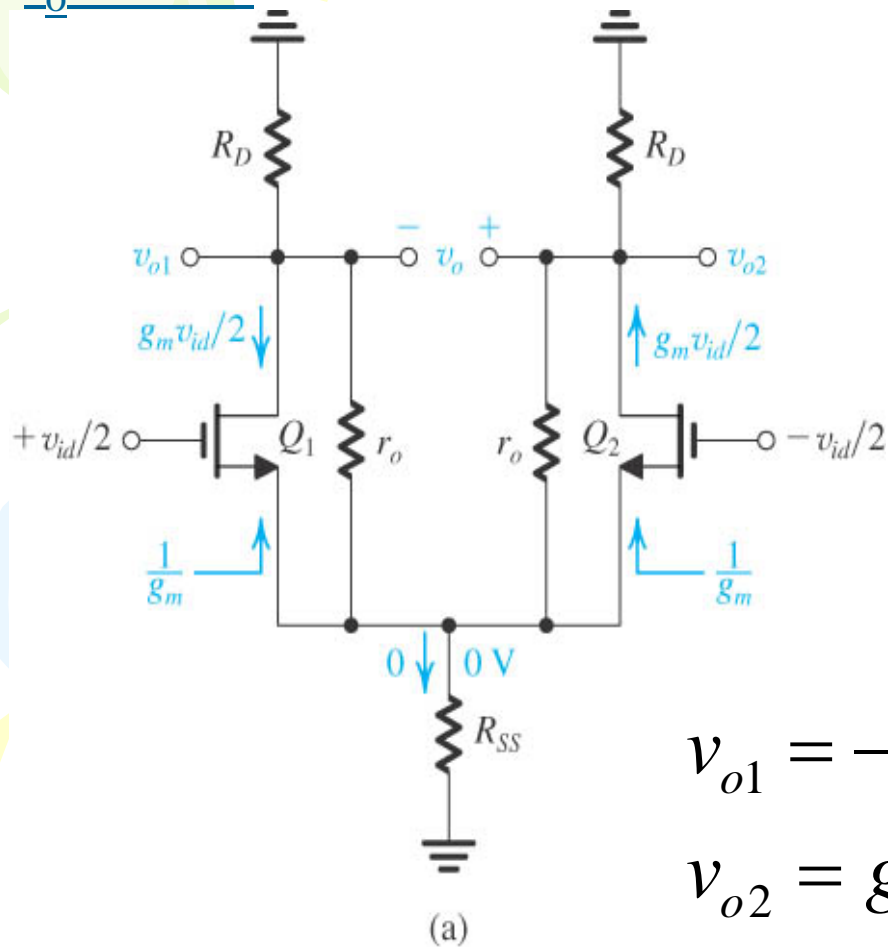
$$A_d \equiv \frac{v_{o2} - v_{o1}}{v_{id}} = g_m (R_D // r_o)$$

$$R_{id} = \infty$$

$$R_{o\frac{1}{2}} = r_o // R_D$$



r_o effects



$$v_{o1} = -g_m (R_D // r_o) (v_{id} / 2)$$

$$v_{o2} = g_m (R_D // r_o) (v_{id} / 2)$$

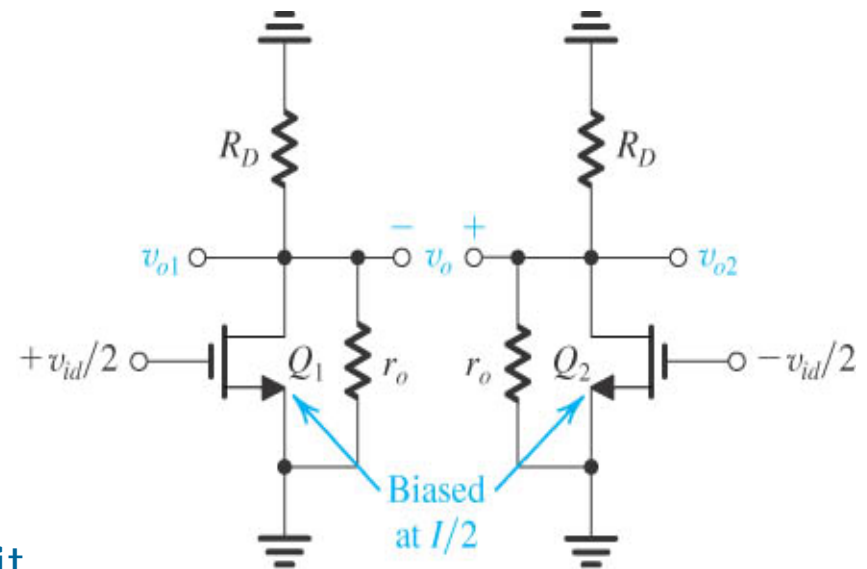
$$v_o = v_{o2} - v_{o1} = g_m (R_D // r_o) v_{id}$$

$$v_{o1} = -g_m(R_D // r_o)(v_{id} / 2)$$

$$v_{o2} = g_m(R_D // r_o)(v_{id} / 2)$$

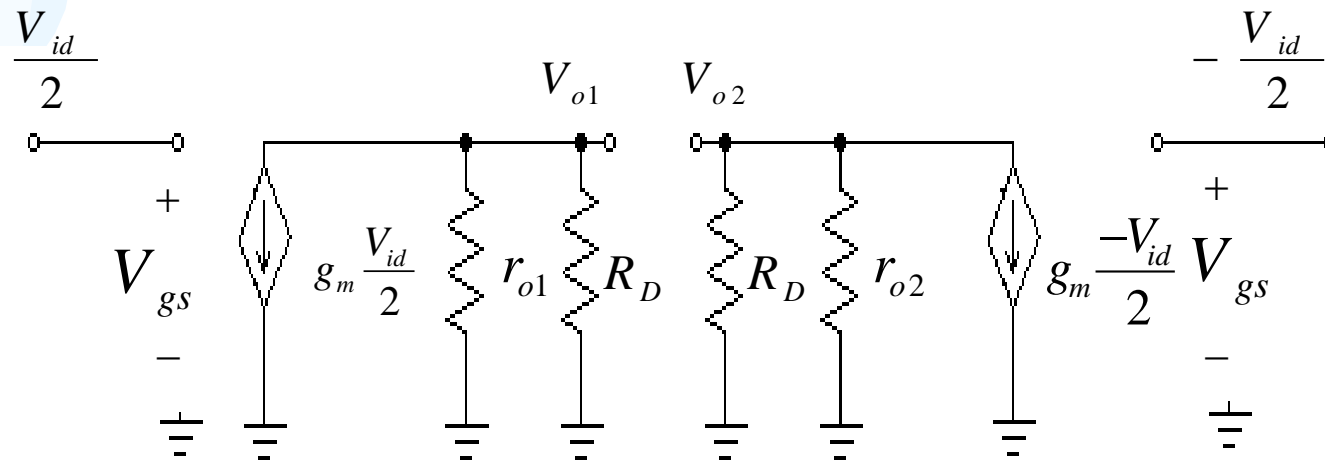
$$v_o = v_{o2} - v_{o1} = g_m(R_D // r_o)v_{id}$$

$$A_d = \frac{v_{o2} - v_{o1}}{v_{id}} = g_m(R_D // r_o)$$

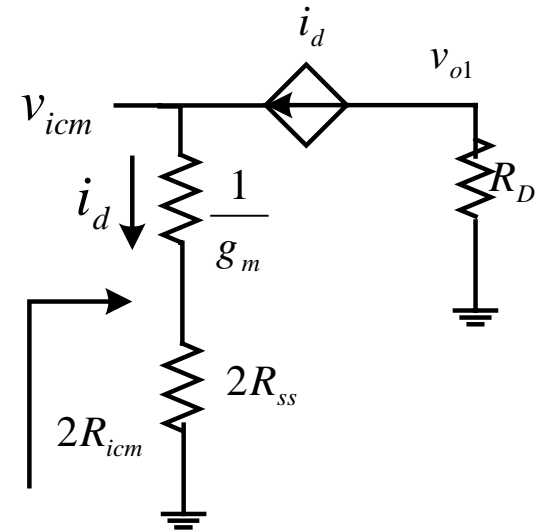
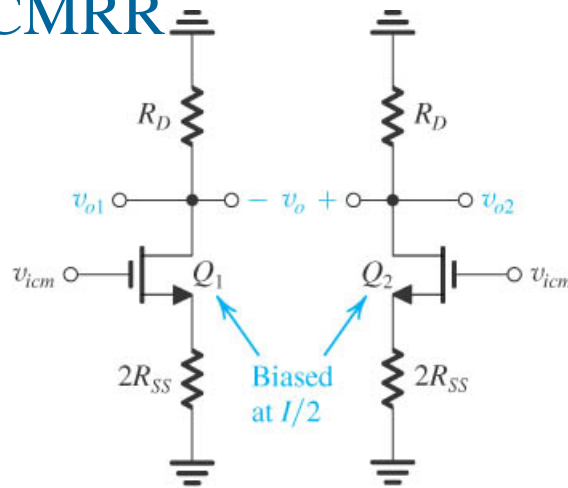
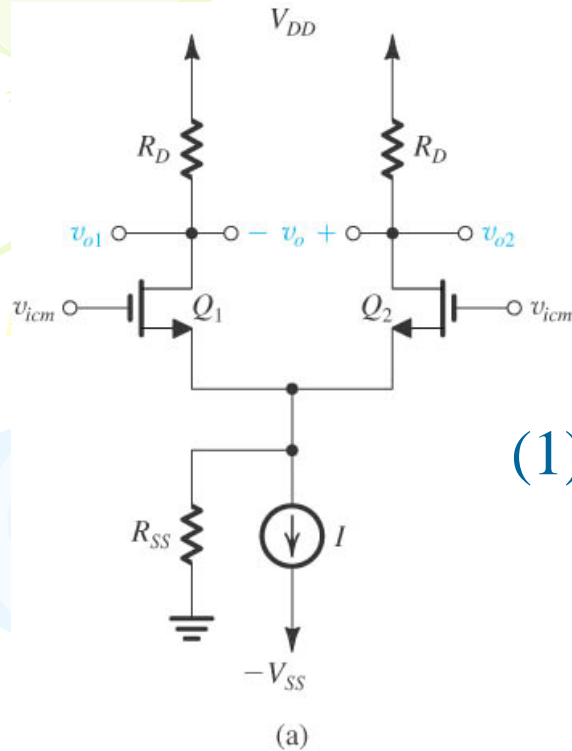


(b)

Differential-mode equivalent circuit



Common-mode gain et CMRR



(1) Half circuit of differential pair

$$|A_{cm}|_{\frac{1}{2}} = R_D / 2R_{ss}, |A_d|_{\frac{1}{2}} = \frac{1}{2} g_m R_D$$

$$CMRR \equiv \left| \frac{A_d}{A_{cm}} \right| = g_m R_{ss}$$

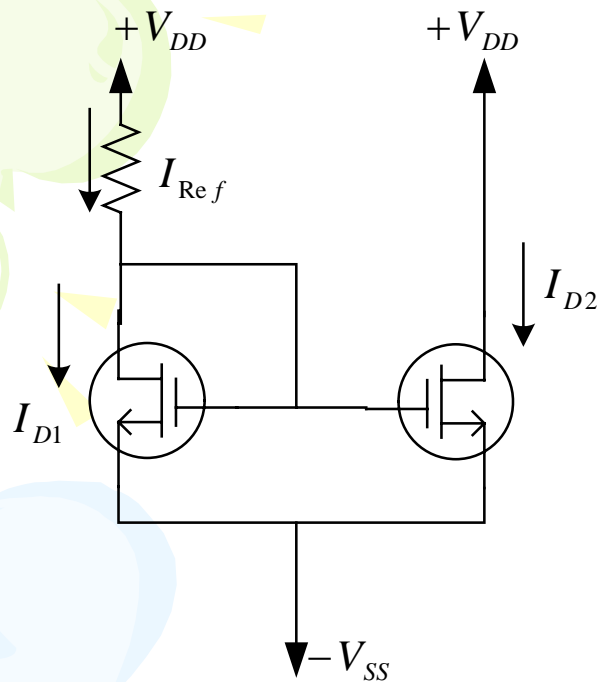
$$\frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} = - \frac{R_D}{1/g_m + 2R_{ss}}$$

$$\because R_{ss} \gg 1/g_m \Rightarrow \frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} = - \frac{R_D}{2R_{ss}}$$

(2) Full circuit

$$|A_{cm}| = (v_{o2} - v_{o1}) / v_{icm} = 0, |A_d| = (v_{o2} - v_{o1}) / v_{id} = g_m R_D$$

$$CMRR \equiv \left| \frac{A_d}{A_{cm}} \right| = \infty$$

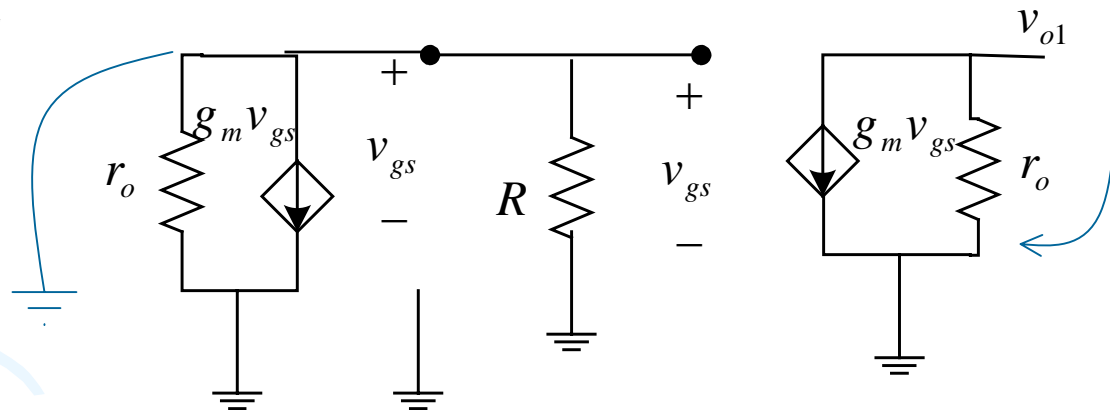


$$I_D = \frac{1}{2} k'_n \frac{W}{L} (v_{GS} - V_t)^2$$

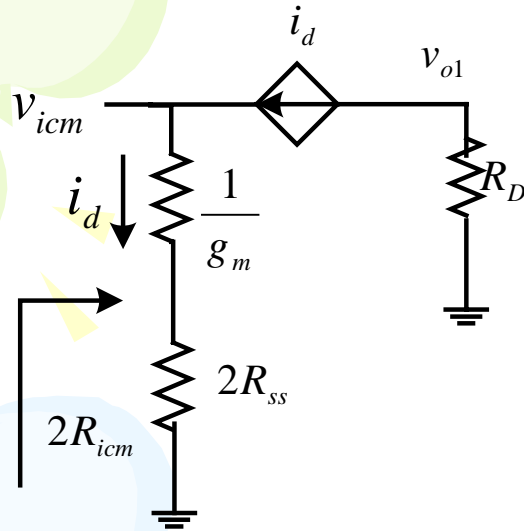
$$v_{GS1} = v_{GS2} \Rightarrow I_{D1} = I_{D2} = I_{ref}$$

$$I_{D1} = \frac{V_{DD} + V_{SS} - V_{GS}}{R}$$

$$R_o = r_o = R_{SS}$$



Non zero common gain due to R_D mismatch



$$\frac{v_{o1}}{v_{icm}} = \frac{v_{o1}}{v_{icm}} = -\frac{R_D}{1/g_m + 2R_{ss}}$$

$$\because R_{ss} \gg 1/g_m \Rightarrow \frac{v_{o1}}{v_{icm}} = \frac{v_{o1}}{v_{icm}} = -\frac{R_D}{2R_{ss}}$$

consider $R_{D1} \neq R_{D2}$

$$v_{o1} \cong -\frac{R_D}{2R_{ss}} v_{icm}$$

$$v_{o2} \cong -\frac{R_D + \Delta R_D}{2R_{ss}} v_{icm}$$

$$v_{o2} - v_{o1} = -\frac{\Delta R_D}{2R_{ss}} v_{icm}$$

$$A_{cm} = -\frac{\Delta R_D}{2R_{ss}} = -\frac{R_D}{2R_{ss}} \frac{\Delta R_D}{R_D}$$

$$A_d = -g_m R_D$$

$$CMRR \equiv \left| \frac{A_d}{A_{cm}} \right| = 2g_m R_{ss} / \frac{\Delta R_D}{R_D}$$

Non zero common gain due to g_m mismatch

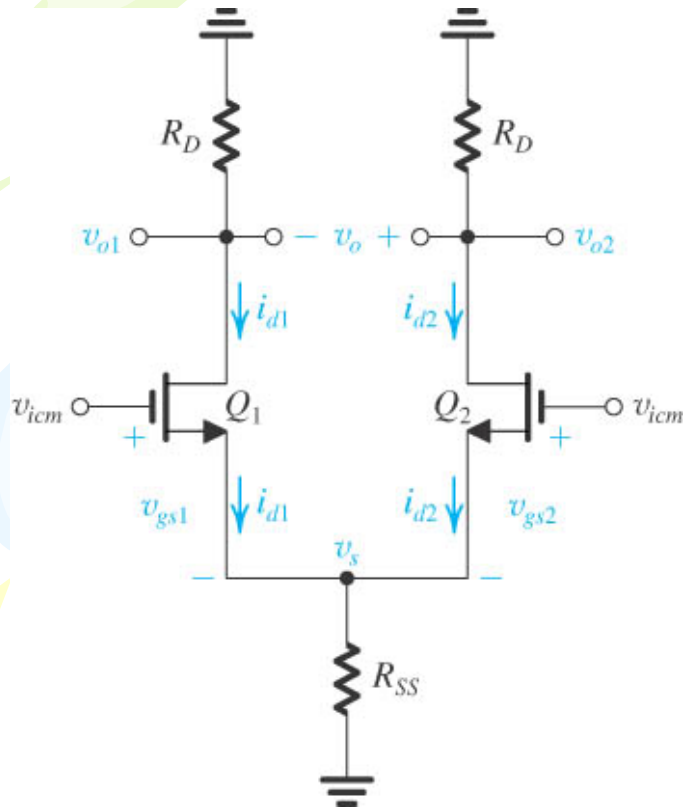


Figure 7.11 Analysis of the MOS differential amplifier to determine the common-mode gain resulting from a mismatch in the g_m values of Q_1 and Q_2 .

consider $g_{m1} \neq g_{m2}$

$$\because v_{gs1} = v_{gs2} \Rightarrow \frac{i_{d1}}{i_{d2}} = \frac{g_{m1}}{g_{m2}}$$

$$i_d = \frac{\mu C_{ox} W}{2L} (v_{gs} - V_t)^2$$

$$g_m = \frac{\mu C_{ox} W}{L} (v_{gs} - V_t)$$

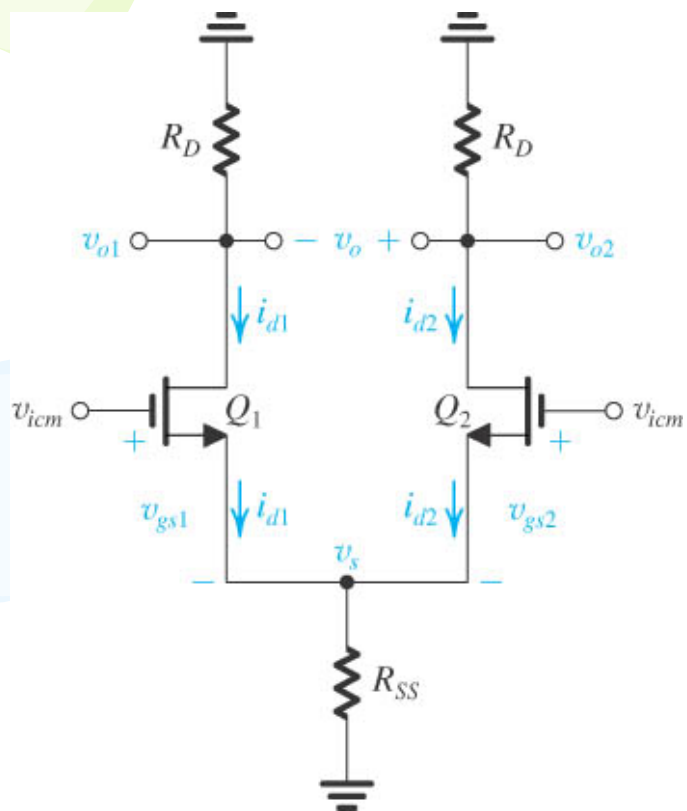
$$v_s = (i_{d1} + i_{d2}) R_{ss} \Rightarrow (i_{d1} + i_{d2}) = \frac{v_s}{R_{ss}}$$

$$v_s \approx v_{icm} \Rightarrow (i_{d1} + i_{d2}) = \frac{v_{icm}}{R_{ss}}$$

$$i_{d1} = \frac{g_{m1} v_{icm}}{(g_{m1} + g_{m2}) R_{ss}}$$

$$i_{d2} = \frac{g_{m2} v_{icm}}{(g_{m1} + g_{m2}) R_{ss}}$$

$$\text{let } \Delta g_m = g_{m1} - g_{m2}$$



$$\Rightarrow i_{d1} = \frac{g_{m1} v_{icm}}{2g_m R_{ss}}$$

$$\Rightarrow i_{d2} = \frac{g_{m2} v_{icm}}{2g_m R_{ss}}$$

$$v_{o2} - v_{o1} = -i_{d2} R_D + i_{d1} R_D = \frac{\Delta g_m R_D v_{icm}}{2g_m R_{ss}}$$

$$A_{cm} = \frac{R_D}{R_{ss}} \frac{\Delta g_m}{2g_m}$$

$$A_d = -g_m R_D$$

$$CMRR \equiv \left| \frac{A_d}{A_{cm}} \right| = 2g_m R_{ss} / \frac{\Delta g_m}{g_m}$$

Input offset voltage

1. $R_{D1} \neq R_{D2}$
2. $Q_1 \neq Q_2$ $\frac{W_1}{L_1} \neq \frac{W_2}{L_2}$
3. $Q_1 \neq Q_2$ $V_{t1} \neq V_{t2}$

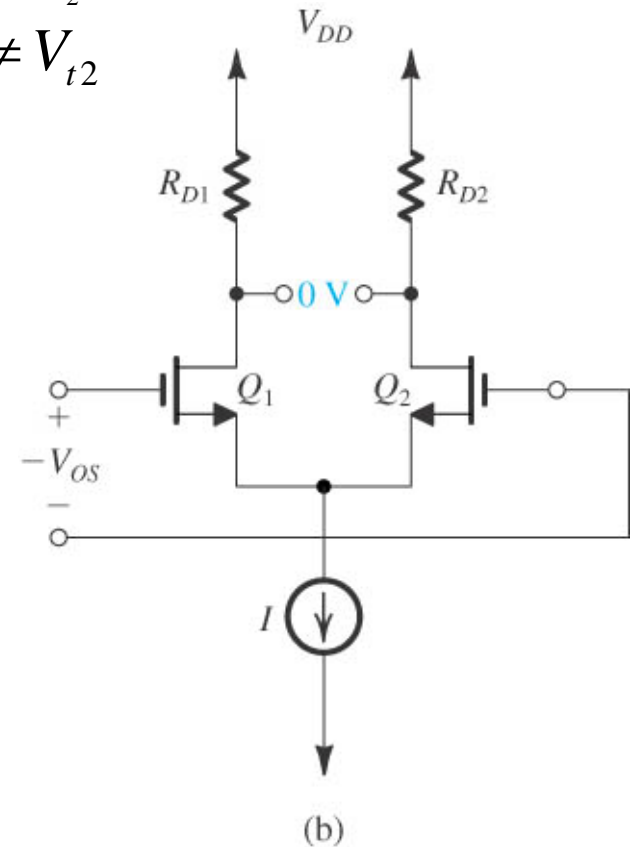
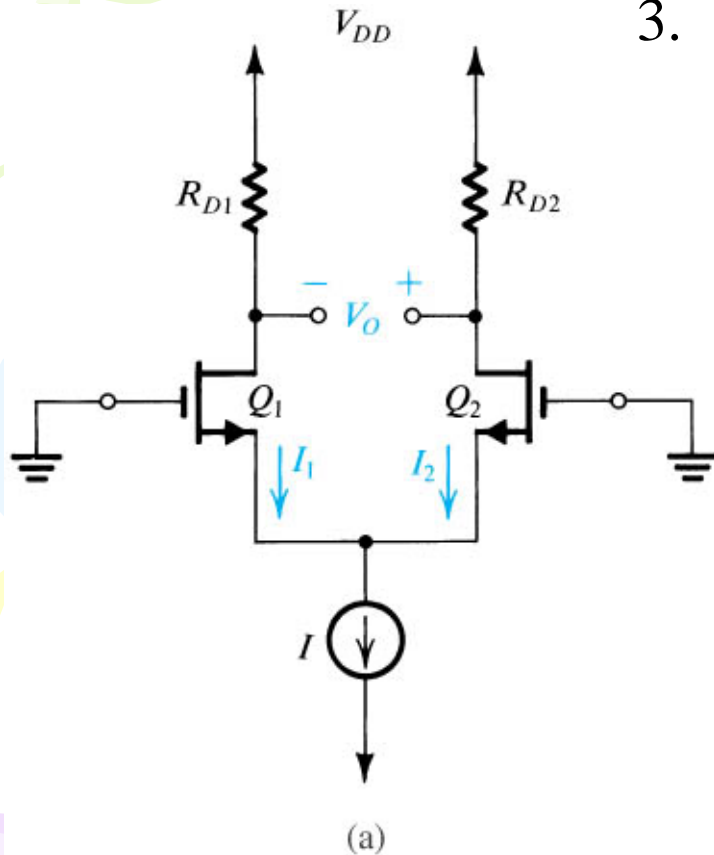


Figure 7.25 (a) The MOS differential pair with both inputs grounded. Owing to device and resistor mismatches, a finite dc output voltage V_o results. (b) Application of a voltage equal to the input offset voltage V_{OS} to the terminals with opposite polarity reduces V_o to zero.

consider $R_{D1} \neq R_{D2}$

$$R_{D1} = R_D + \frac{\Delta R_D}{2}$$

$$R_{D2} = R_D - \frac{\Delta R_D}{2}$$

$$V_{D1} = V_{DD} - \frac{I}{2} \left(R_D + \frac{\Delta R_D}{2} \right)$$

$$V_{D2} = V_{DD} - \frac{I}{2} \left(R_D - \frac{\Delta R_D}{2} \right)$$

$$V_O = V_{D2} - V_{D1} = \frac{I}{2} \Delta R_D$$

$$V_{os} = \frac{V_O}{A_d} = \frac{V_O}{g_m R_D} = \frac{\frac{I}{2} \Delta R_D}{g_m R_D}$$

$$= \frac{\frac{\mu C_{ox} W}{2L} (V_{GS} - V_t)^2 \Delta R_D}{\frac{\mu C_{ox} W}{L} (V_{GS} - V_t) R_D} = \left(\frac{V_{GS} - V_t}{2} \right) \left(\frac{\Delta R_D}{R_D} \right)$$

$$V_{os} = \left(\frac{V_{OV}}{2} \right) \left(\frac{\Delta R_D}{R_D} \right)$$

consider $Q_1 \neq Q_2$

$$\left(\frac{W}{L} \right)_1 = \frac{W}{L} + \frac{1}{2} \Delta \left(\frac{W}{L} \right)$$

$$\left(\frac{W}{L} \right)_2 = \frac{W}{L} - \frac{1}{2} \Delta \left(\frac{W}{L} \right)$$

$$I_1 = \frac{I}{2} + \frac{I}{2} \frac{\Delta (W / L)}{(W / L)}$$

$$I_2 = \frac{I}{2} - \frac{I}{2} \frac{\Delta (W / L)}{(W / L)}$$

$$\Delta I = \frac{I}{2} \frac{\Delta (W / L)}{2 (W / L)}$$

$$V_{os} = \left(\frac{V_{OV}}{2} \right) \left(\frac{\Delta (W / L)}{(W / L)} \right)$$

consider $V_{t1} \neq V_{t2}$

$$V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$V_{t2} = V_t - \frac{\Delta V_t}{2}$$

$$I_1 = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t - \frac{\Delta V_t}{2})^2 = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 [1 - \frac{\Delta V_t}{2(V_{GS} - V_t)}]^2$$

$$I_1 \approx \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 (1 - \frac{\Delta V_t}{V_{GS} - V_t})$$

$$I_{12} \approx \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 (1 + \frac{\Delta V_t}{V_{GS} - V_t})$$

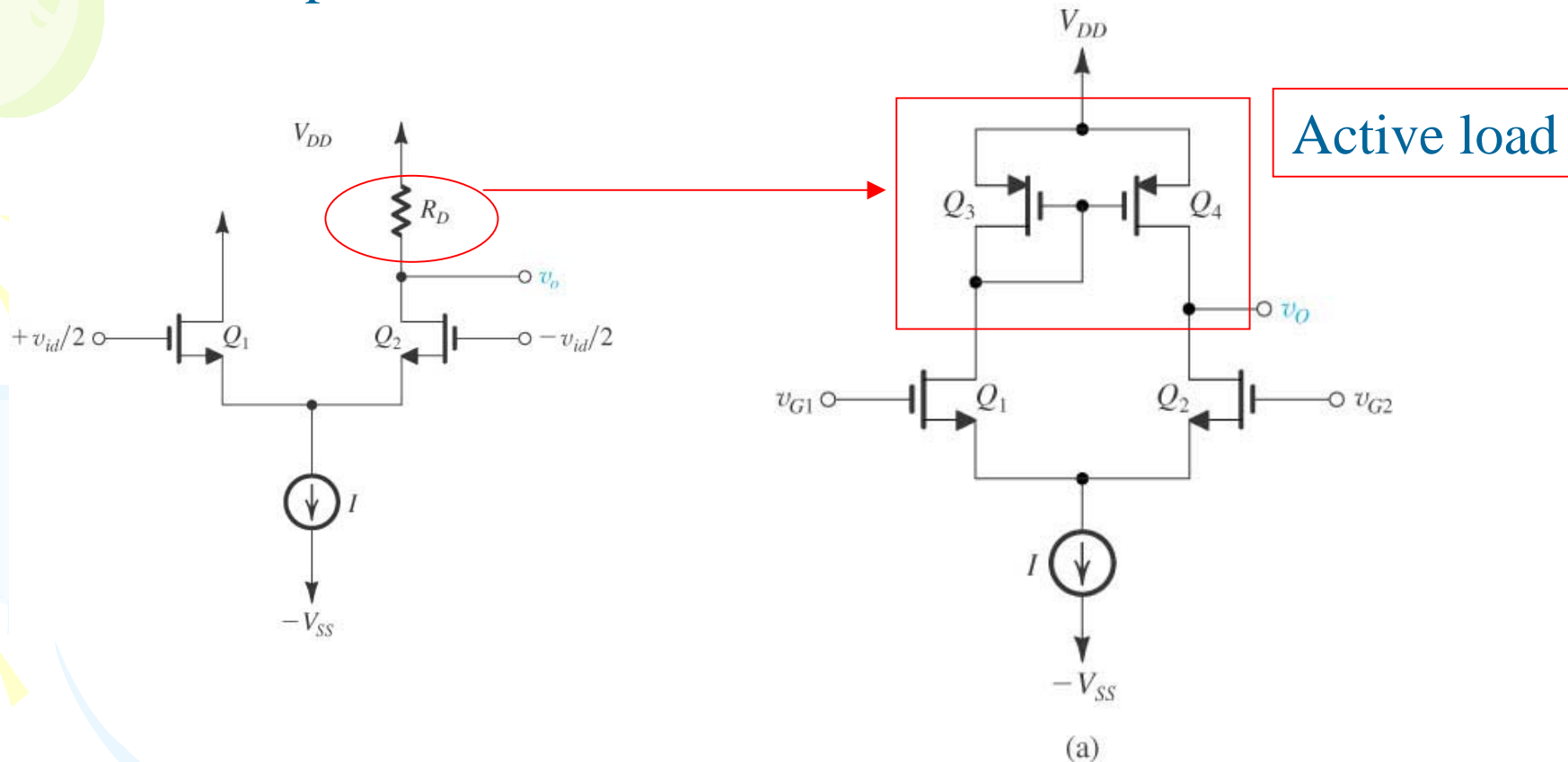
$$\frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 = \frac{I}{2}$$

$$\Delta I = \frac{I}{2} \frac{\Delta V_t}{V_{GS} - V_t} = \frac{I}{2} \frac{\Delta V_t}{V_{OV}}$$

$$V_{os} = \Delta V_t$$

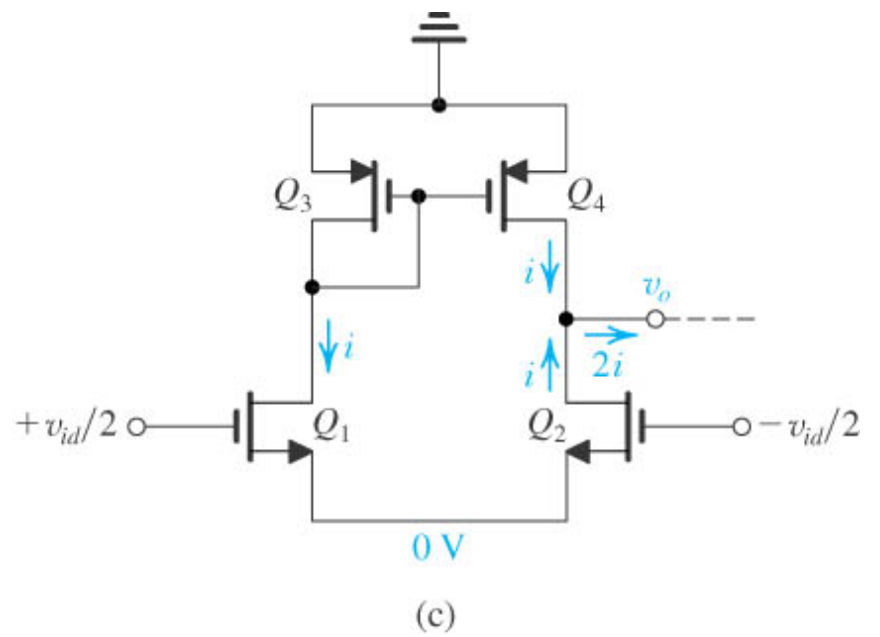
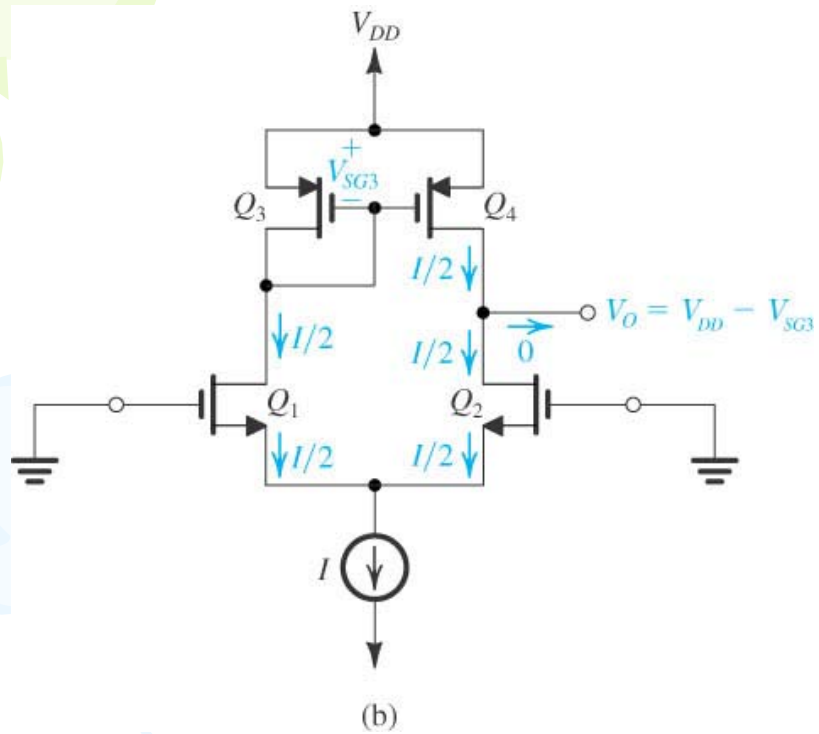
$$V_{os} = \sqrt{(\frac{V_{OV}}{2} \frac{\Delta R_D}{R_D})^2 + (\frac{V_{OV}}{2} \frac{\Delta(W/L)}{(W/L)})^2 + (\Delta V_t)^2}$$

Differential amplifier with active load

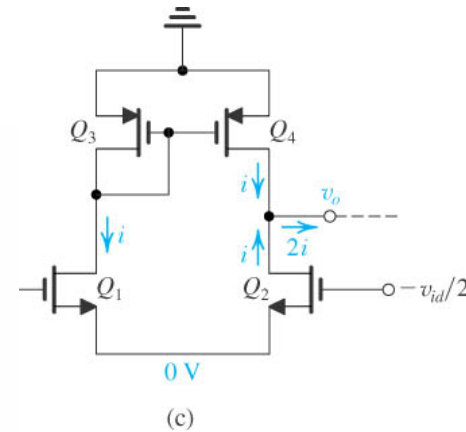
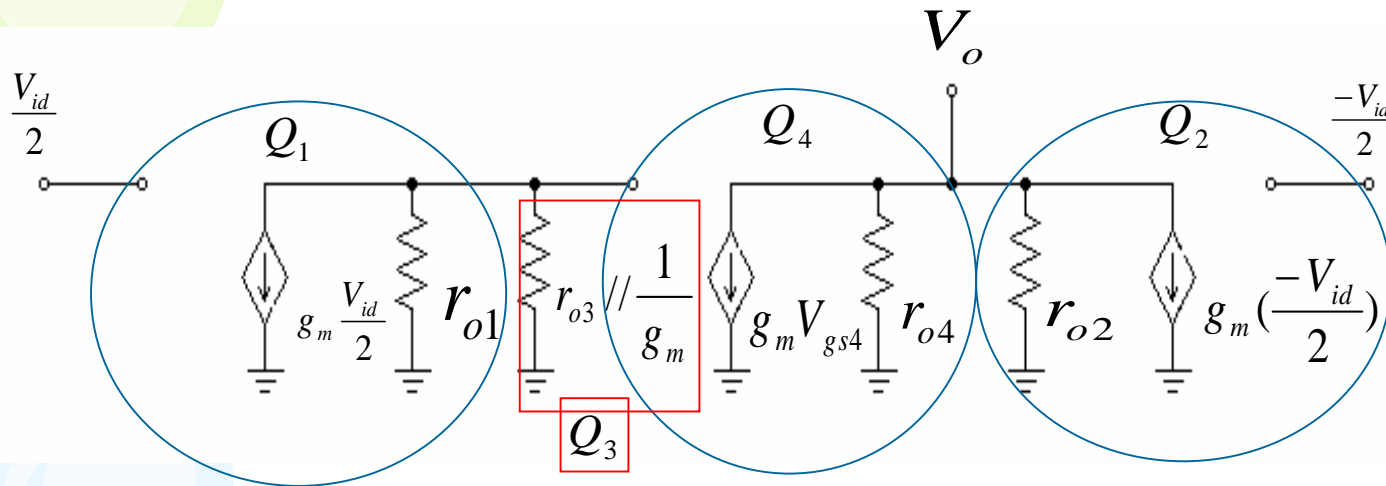


Improving:

1. Differential gain
2. Common-mode gain et CMRR
3. Input offset voltage



Differential-mode equivalent circuit with active load



$$R_o = r_{o2} // r_{o4}$$

active load

$$v_o = -g_m \left(\frac{-v_{id}}{2} + v_{gs4} \right) (r_{o2} // r_{o4})$$

$$v_{gs4} = -g_m \frac{v_{id}}{2} (r_{o1} // r_{o3} // \frac{1}{g_m}) \approx -g_m \frac{v_{id}}{2} \frac{1}{g_m} = -\frac{v_{id}}{2}$$

$$A_d \equiv \frac{v_o}{v_{id}} = \frac{-g_m \left(\frac{-v_{id}}{2} + \frac{-v_{id}}{2} \right) (r_{o2} // r_{o4})}{v_{id}} = g_m (r_{o2} // r_{o4})$$

$$\text{when } r_{o2} = r_{o4} = r_o$$

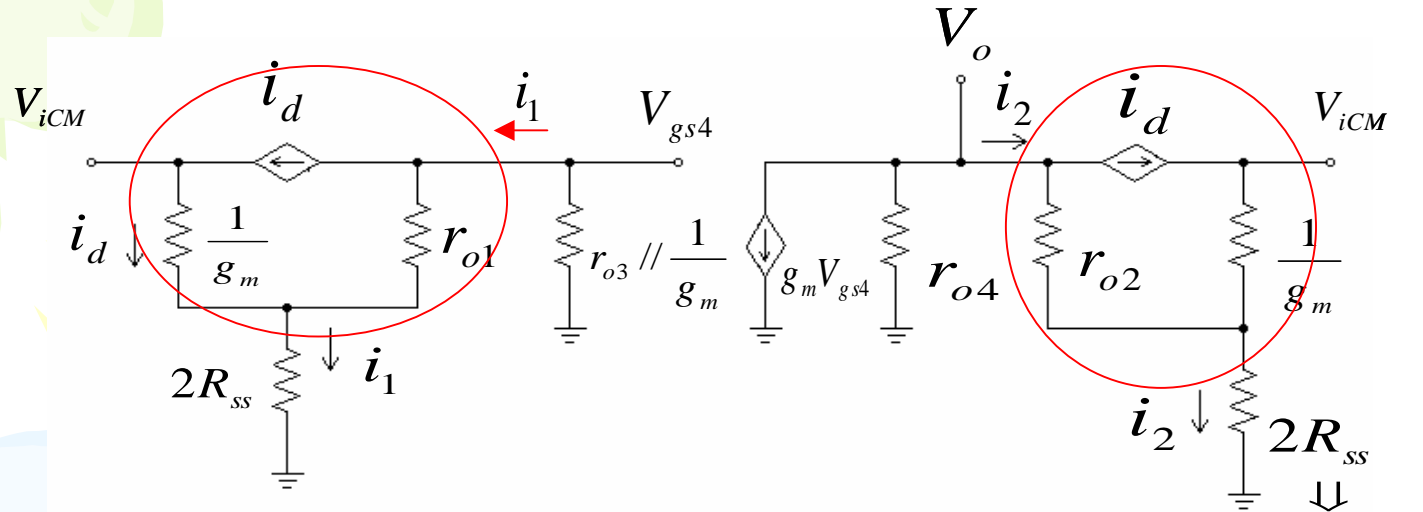
$$A_d = \frac{g_m}{2} r_o$$

active load

Passive load

$$A_d = g_m (R_D // r_o)$$

Common-mode equivalent circuit with active load



$$v_o = -r_{o4}(g_m v_{gs4} + i_2)$$

$$v_{gs4} = -i_1(r_{o4} \parallel \frac{1}{g_m})$$

$$v_{icm} \approx i_1 2R_{SS}$$

$$v_{icm} \approx i_2 2R_{SS}$$

$$v_o = -r_{o4}(g_m v_{gs4} + \frac{v_{icm}}{2R_{SS}}) \quad (R_{ss} \approx r_o)$$

$$v_{gs4} = -\frac{v_{icm}}{2R_{SS}}(r_{o3} \parallel \frac{1}{g_m})$$

$$v_o = -r_{o4}[-\frac{g_m v_{icm}}{2R_{SS}}(r_{o3} \parallel \frac{1}{g_m}) + \frac{v_{icm}}{2R_{SS}}]$$

$$A_{cm} = \frac{v_o}{v_{icm}} = -\frac{r_{o4}}{2R_{SS}} \frac{1}{1 + r_{o3}g_m}$$

1. Find the transconductance G_m

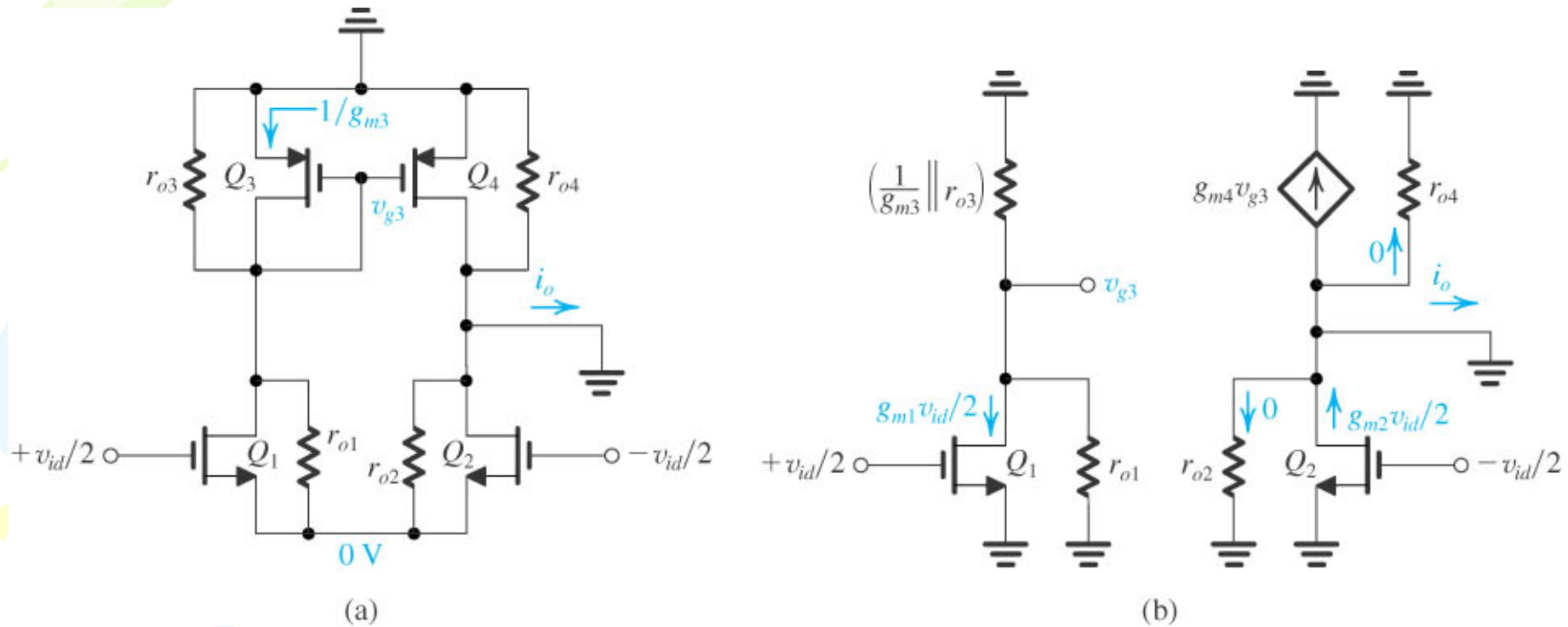
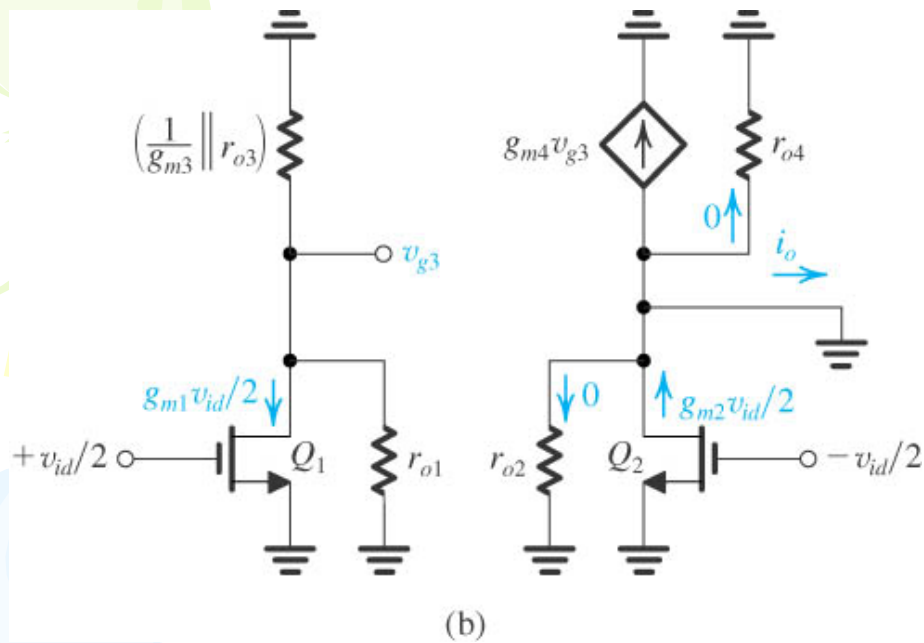


Figure 7.29 Determining the short-circuit transconductance G_m ; i_o/v_{id} of the active-loaded MOS differential pair.



$$v_{g3} = -g_{m1} \left(\frac{v_{id}}{2} \right) \left(\frac{1}{g_{m3}} \parallel r_{o3} \parallel r_{o1} \right)$$

$$r_{o1}, r_{o3} \gg (1/g_{m3})$$

$$v_{g3} \approx -\frac{g_{m1}}{g_{m3}} \left(\frac{v_{id}}{2} \right)$$

$$i_o = -g_{m4} v_{g3} + g_{m2} \left(\frac{v_{id}}{2} \right)$$

$$i_o = -g_{m1} \left(\frac{g_{m4}}{g_{m3}} \right) \left(\frac{v_{id}}{2} \right) + g_{m2} \left(\frac{v_{id}}{2} \right)$$

$$\because g_{m3} = g_{m4}, g_{m1} = g_{m2} = g_m$$

$$i_o = g_m v_{id}$$

$$G_m = g_m$$

$$R_{o2} = r_{o2} + (1 + g_{m2}r_{o2})(1/g_{m1})$$

$$\Rightarrow R_{o2} = 2r_{o2}$$

$$R_o \equiv \frac{v_x}{\dot{l}_x} = r_{o2} // r_{o4}$$

$$A_d \equiv \frac{v_o}{v_{id}} = G_m R_o = g_m (r_{o2} // r_{o4})$$
$$A_d = \frac{g_m}{2} r_o$$

Common-mode gain et CMRR

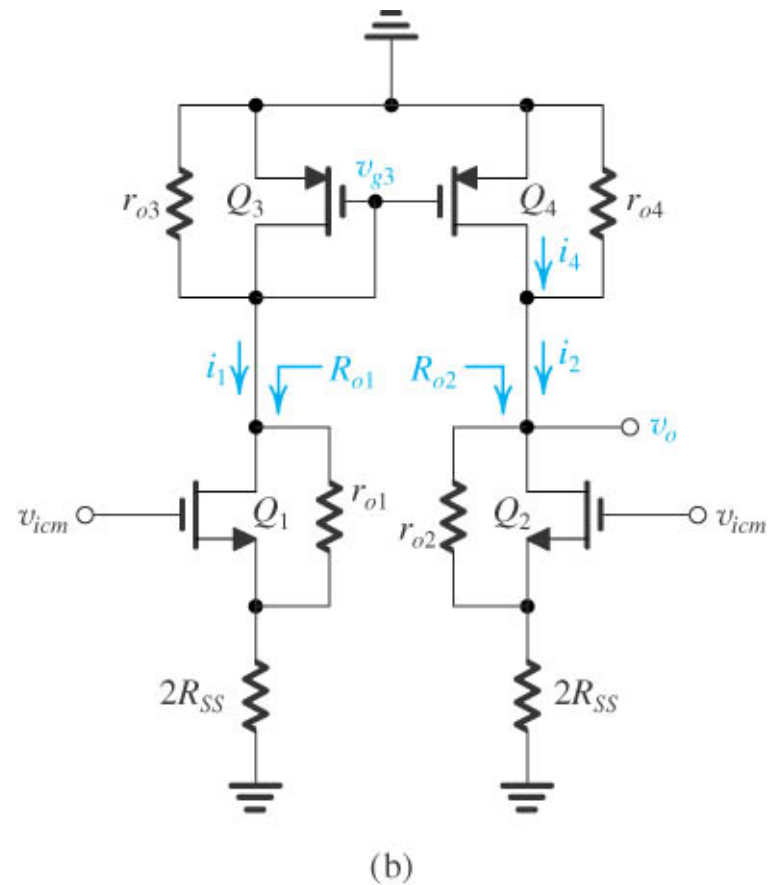
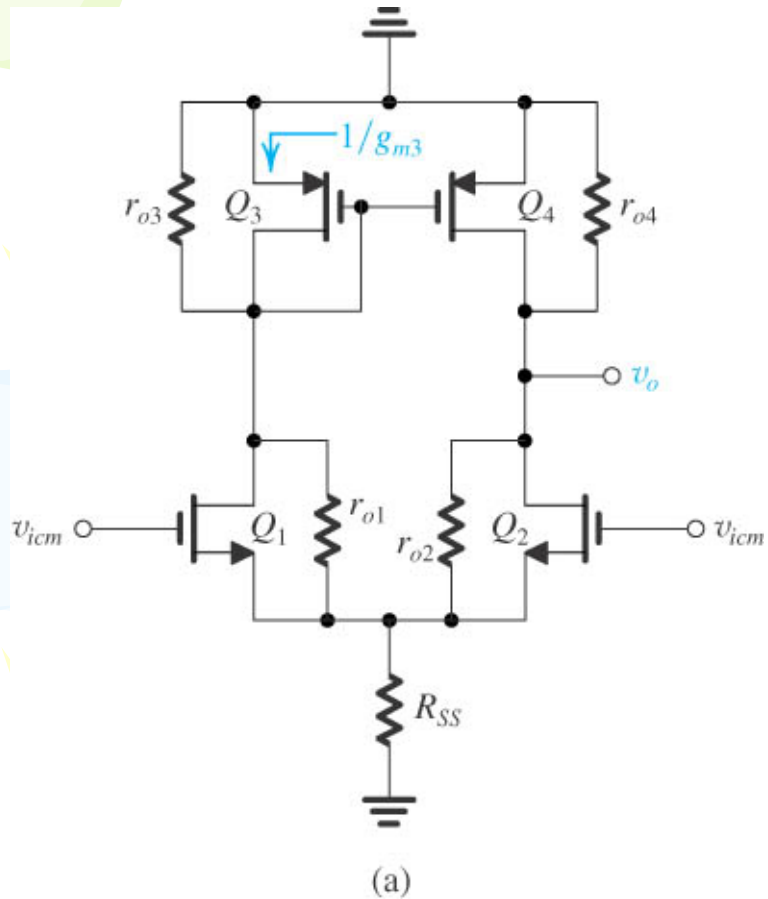
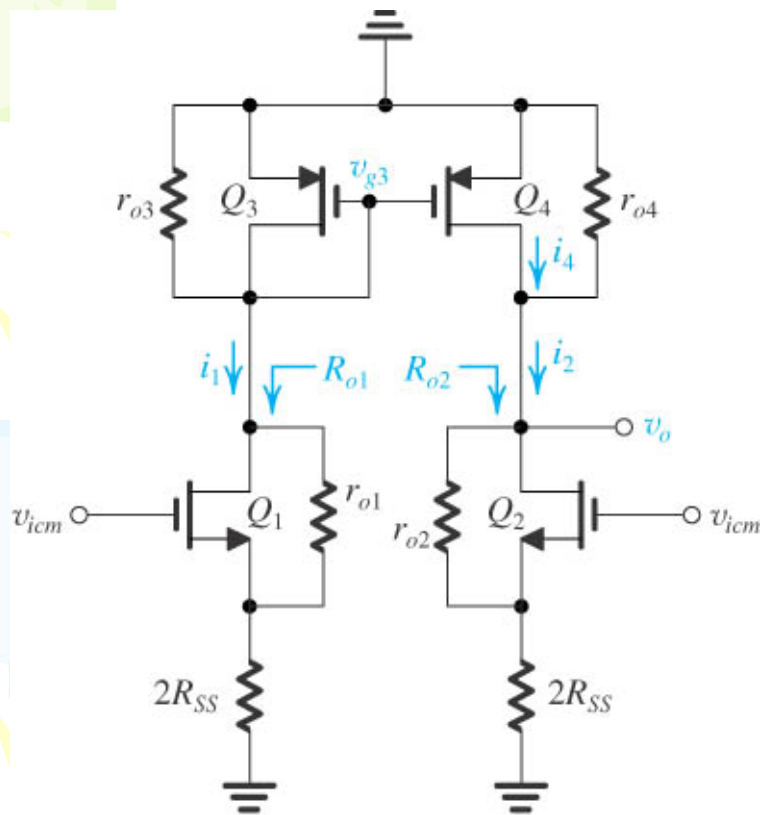


Figure 7.31 Analysis of the active-loaded MOS differential amplifier to determine its common-mode gain.



(b)

$$i_1 = i_2 \approx \frac{v_{icm}}{2R_{ss}}$$

$$R_{o1} = R_{o2} = r_o + 2R_{ss} + 2g_m r_o R_{ss}$$

$$v_{g3} = -i_1 \left(\frac{1}{g_{m3}} // r_{o3} \right)$$

$$i_4 = -g_{m4} v_{g3} = g_{m4} i_1 \left(\frac{1}{g_{m3}} // r_{o3} \right)$$

$$v_o = (i_4 - i_2) r_{o4} = \left[g_{m4} i_1 \left(\frac{1}{g_{m3}} // r_{o3} \right) - i_2 \right] r_{o4}$$

$$A_{cm} \equiv \frac{v_o}{v_{icm}} = -\frac{1}{2R_{ss}} \frac{r_{o4}}{1 + g_{m3} r_{o3}}$$

$$\because g_{m3} r_{o3} \gg 1, r_{o3} = r_{o4}$$

$$A_{cm} \approx -\frac{1}{2g_{m3} R_{ss}}$$

$$CMRR \equiv \left| \frac{A_d}{A_{cm}} \right| = [g_m (r_{o2} // r_{o4})] [2g_{m3} R_{ss}]$$

$$\text{let } r_{o2} = r_{o4} = r_o, g_{m3} = g_m$$

$$CMRR = g_m r_o g_m R_{ss}$$