1. $u[n]+u[-n]$ is equal to

(A) 2 (B) $1+\delta[n]$

(C) $2+\delta[n]$ (D) 1

2. The even part of a function $x[n]=u[n]-u[n-4]$ is

(A) $\frac{1}{2}\{1+\delta[n]-u[n-4]-u[-n-4]\}$

(B) $\frac{1}{2}\{u[n+3]-u[n-4]+\delta[n]\}$

(C) $\frac{1}{2}\{u[n]+u[-n]-u[n-4]-u[-n-4]\}$

(D) Above all

3. The energy of signal $A\delta[n]$ is

(A) A^2 (B) $\frac{A^2}{2}$

(C) $\frac{A^2}{4}$ (D) 0

4. The energy of signal $nu[n]$ is

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n+1)(2n+1)}{6}$

(C) $\left(\frac{n(n+1)}{2}\right)^2$ (D) ∞

5. The power of signal $u[n]$ is

(A) n (B) 1

(C) $\frac{1}{2}$ (D) ∞

Statement for Q.6–11:

$x[n]$ and $y[n]$ are given in fig. P5.2.6–11 respectively. Choose the sketch for the signal given in question.

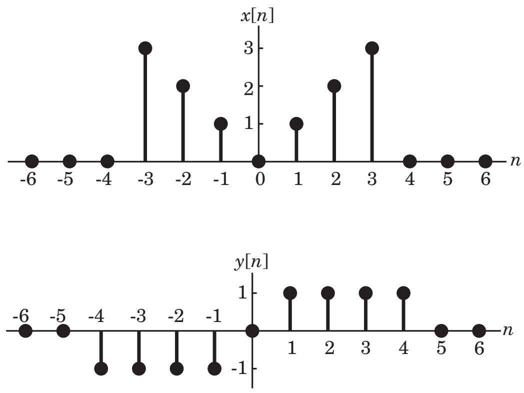
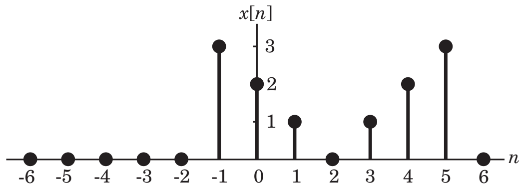


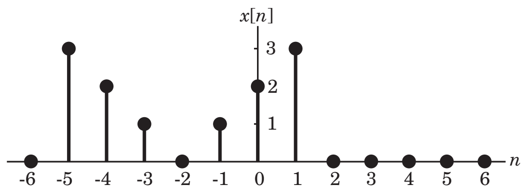
Fig. P5.2.6–11

6. $x[2n]$

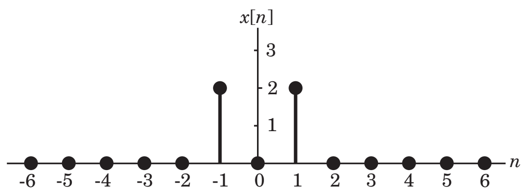
(A)



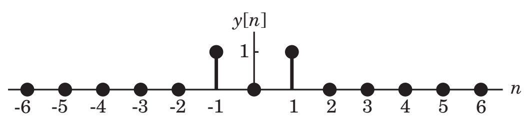
(B)



(C)

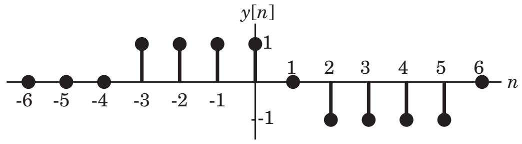


(D)

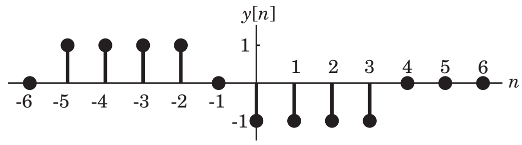


7. $y[1-n]$

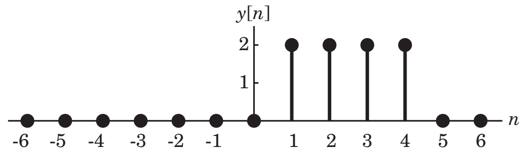
(A)



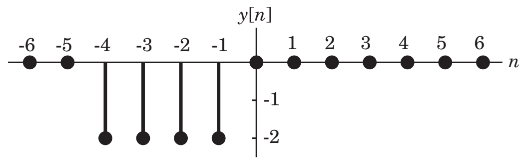
(B)



(C)

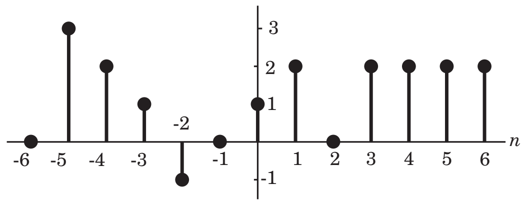


(D)

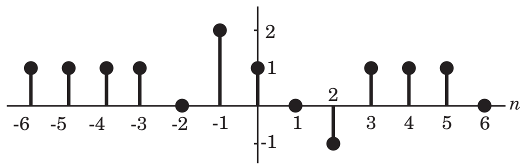


8. $x[n-2]+y[n+2]$

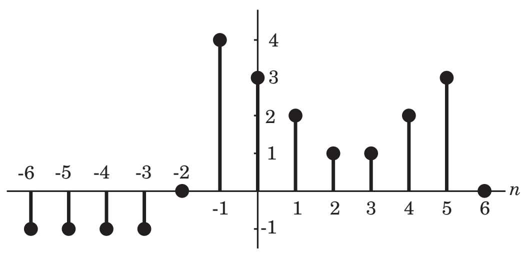
(A)



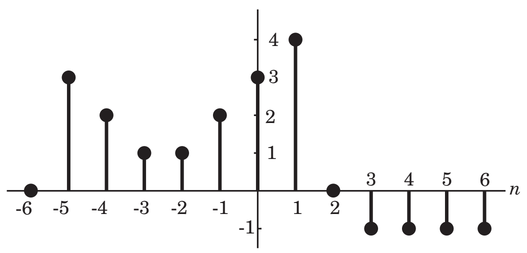
(B)



(C)

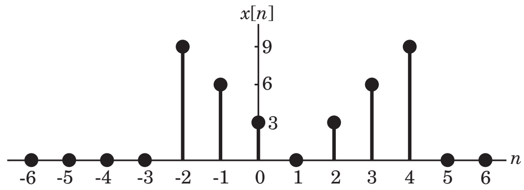


(D)

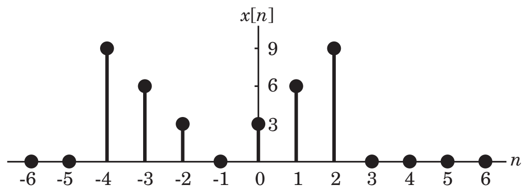


9. $x[3n-1]$

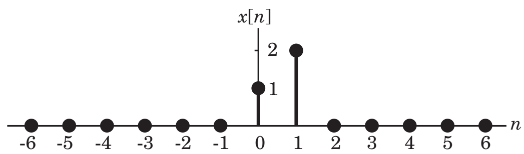
(A)



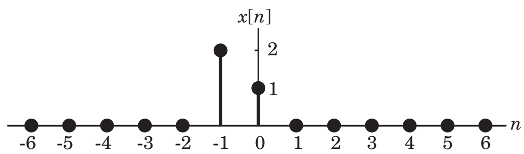
(B)



(C)

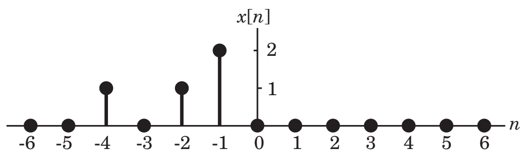


(D)

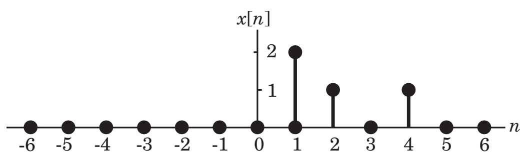


10. $x[3-n]y[-n]$

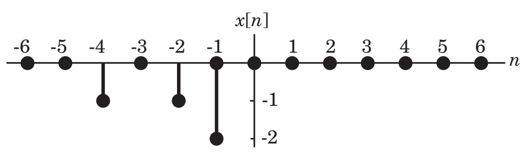
(A)



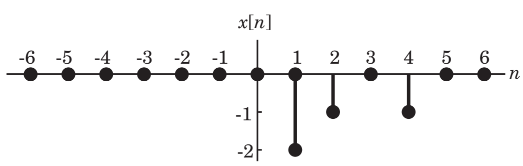
(B)



(C)

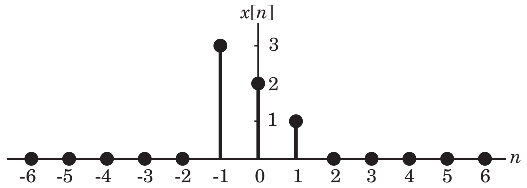


(D)

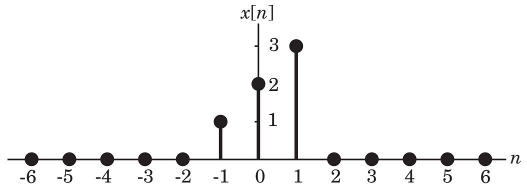


11. $x[n+2]y[n-2]$

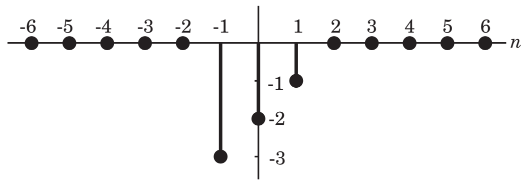
(A)



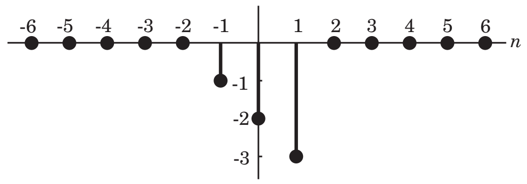
(B)



(C)



(D)



Statement for Q.12–15:

A discrete-time signal is given. Determine the period of signal and choose correct option.

12. $x[n] = \cos\frac{\pi n}{9} + \sin\left(\frac{\pi n}{7} + \frac{1}{2}\right)$

(A) periodic with period $N = 126$

(B) periodic with period $N = 32$

(C) periodic with period $N = 252$

(D) Not periodic

13. $x[n] = \cos\left(\frac{n}{8}\right)\cos\left(\frac{\pi n}{8}\right)$

(A) Periodic with period 16π

(B) periodic with period $16(\pi + 1)$

(C) periodic with period 8

(D) Not periodic

14. $x[n] = \cos\left(\frac{\pi n}{2}\right) - \sin\left(\frac{\pi n}{8}\right) + 3\cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right)$

(A) periodic with period 16

(B) periodic with period 4

(C) periodic with period 2

(D) Not periodic

15. $x[n] = 2e^{j\left(\frac{n}{6} - \pi\right)}$

(A) periodic with 12π

(B) periodic with 12

(C) periodic with 11π

(D) Not periodic

16. The sinusoidal signal has fundamental period $N = 10$ samples. The smallest angular frequency, for which $x[n]$ is periodic, is

(A) $\frac{1}{10}$ rad/cycle

(B) 10 rad/cycle

(C) 5 rad/cycle

(D) $\frac{\pi}{5}$ rad/cycle

17. Let $x[n], -5 \le n \le 3$ and $h[n], 2 \le n \le 6$ be two finite duration signals. The range of their convolution is

(A) $-7 \le n \le 9$

(B) $-3 \le n \le 9$

(C) $2 \le n \le 3$

(D) $-5 \le n \le 6$

Statement for Q.18–26:

$x[n]$ and $h[n]$ are given in the question. Compute the convolution $y[n] = x[n] * h[n]$ and choose correct option.

18. $x[n] = \{1, 2, 4\}$, $h[n] = \{1, 1, 1, 1, 1\}$

(A) $\{1, 3, 7, 7, 7, 6, 4\}$

(B) $\{1, 3, 3, 7, 7, 6, 4\}$

(C) $\{1, 2, 4\}$

(D) $\{1, 3, 7\}$

19. $x[n] = \{1, 2, 3, 4, 5\}$, $h[n] = \{1\}$

(A) $\{1, 3, 6, 10, 15\}$

(B) $\{1, 2, 3, 4, 5\}$

(C) $\{1, 4, 9, 16, 20\}$

(D) $\{1, 4, 6, 8, 10\}$

20. $x[n] = \{1, 2, -1\}$, $h[n] = x[n]$

(A) $\{1, 4, 1\}$

(B) $\{1, 4, 2, -4, 1\}$

(C) $\{1, 2, -1\}$

(D) $\{2, 4, -2\}$

21. $x[n] = \{\underset{\uparrow}{1}, -2, 3\}, \quad h[n] = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

(A) $\{\underset{\uparrow}{1}, -2, 4, 1, 1, 1\}$

(B) $\{\underset{\uparrow}{0}, 0, 3\}$

(C) $\{\underset{\uparrow}{0}, 0, 3, 1, 1, 1, 1\}$

(D) $\{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

22. $x[n] = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}, \quad h[n] = \{1, \underset{\uparrow}{-2}, 3\}$

(A) $\{0, \underset{\uparrow}{0}, 1, -1, 2, 2, 1, 3\}$

(B) $\{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

(C) $\{1, \underset{\uparrow}{-2}, 3, 1, 1, 2, 1, 1\}$

(D) $\{\underset{\uparrow}{1}, -2, 3, 1, 1, 1, 1\}$

23. $x[n] = \{1, 1, \underset{\uparrow}{0}, 1, 1\}, \quad h[n] = \{1, -2, -3, \underset{\uparrow}{4}\}$

(A) $\{1, -1, -2, \underset{\uparrow}{4}, 1, 1\}$

(B) $\{\underset{\uparrow}{1}, -1, -2, 4, 1, 1\}$

(C) $\{1, -1, \underset{\uparrow}{-5}, 2, 3, -5, 1, 4\}$

(D) $\{1, -1, -5, 2, 3, \underset{\uparrow}{-5}, 1, 4\}$

24. $x[n] = \{1, 2, \underset{\uparrow}{0}, 2, 1\}, \quad h[n] = x[n]$

(A) $\{\underset{\uparrow}{1}, 4, 4, 4, 10, 4, 4, 4, 1\}$

(B) $\{1, 4, 4, 4, \underset{\uparrow}{10}, 4, 4, 4, 1\}$

(C) $\{1, 4, \underset{\uparrow}{4}, 10, 4, 4, 4, 1\}$

(D) $\{1, 4, 4, 10, 4, 4, 4, \underset{\uparrow}{1}\}$

25. $x[n] = \{1, 4, \underset{\uparrow}{-3}, 6, 4\}, \quad h[n] = \{2, \underset{\uparrow}{-4}, 3\}$

(A) $\{2, 4, -19, \underset{\uparrow}{36}, -25, 2, 12\}$

(B) $\{4, -19, \underset{\uparrow}{36}, -25\}$

(C) $\{1, 4, \underset{\uparrow}{-3}, 6, 4\}$

(D) $\{\underset{\uparrow}{1}, 4, -3, 6, 4\}$

26. $x[n] = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0 & \text{elsewhere} \end{cases}$

$h(n) = \delta[n] - \delta[n-1] + \delta[n-4]$

(A) $\delta[n] - 2\delta[n-1] + 4\delta[n-4] + \delta[n-5]$

(B) $\delta[n+2] + \delta[n+1] - \delta[n] + 2\delta[n-3] + \delta[n-4] + \delta[n-5]$

(C) $\delta[n+2] - \delta[n+1] + \delta[n] + 2\delta[n-3] - \delta[n-4] + 2\delta[n-5]$

(D) $\delta[n] + 2\delta[n-1] + 4\delta[n-5] + \delta[n-5]$

Statement for Q.27–30:

In question $y[n]$ is the convolution of two signal. Choose correct option for $y[n]$.

27. $y[n] = (-1)^n * 2^n u[2n+2]$

(A) $\frac{4}{6}$

(B) $\frac{4}{6} u[-n+2]$

(C) $\frac{8}{3}(-1)^n u[-n+2]$

(D) $\frac{8}{3}(-1)^n$

28. $y[n] = \frac{1}{4^n} u[n] * u[n+2]$

(A) $\left(\frac{1}{3} - \frac{1}{4^n}\right) u[n]$

(B) $\left(\frac{1}{3} - \frac{12}{4^n}\right) u[n+2]$

(C) $\left(\frac{4}{3} - \frac{1}{12}\left(\frac{1}{4}\right)^n\right) u[n+2]$

(D) $\left(\frac{16}{3} - \frac{1}{4^n}\right) u[n+2]$

29. $y[n] = 3^n u[-n+3] * u[n-2]$

(A) $\begin{cases} \frac{3^n}{2}, & n \le 5 \\ \frac{83}{2}, & n \ge 6 \end{cases}$

(B) $\begin{cases} 3^n, & n \le 5 \\ \frac{83}{2}, & n \ge 6 \end{cases}$

(C) $\begin{cases} \frac{3^n}{2}, & n \le 5 \\ \frac{81}{2}, & n \ge 6 \end{cases}$

(D) $\begin{cases} \frac{3^n}{6}, & n \le 5 \\ \frac{81}{2}, & n \ge 6 \end{cases}$

30. $y[n] = u[n+3] * u[n-3]$

(A) $(n+1)u[n]$

(B) $nu[n]$

(C) $(n-1)u[n]$

(D) $u[n]$

31. The convolution of $x[n] = \cos\left(\frac{\pi}{2}n\right)u[n]$ and $h[n] = u[n-1]$ is $f[n]u[n-1]$. The function $f[n]$ is

(A) $\begin{cases} 1, & n = 4m+1, \quad 4m+2 \\ 0, & n = 4m, \quad 4m+3 \end{cases}$

(B) $\begin{cases} 0, & n = 4m+1, \quad 4m+2 \\ 1, & n = 4m, \quad 4m+3 \end{cases}$

(C) $\begin{cases} 1, & n = 4m+1, \quad 4m+3 \\ 0, & n = 4m, \quad 4m+2 \end{cases}$

(D) $\begin{cases} 0, & n = 4m+1, \quad 4m+3 \\ 1, & n = 4m, \quad 4m+2 \end{cases}$

Statement for Q.32–38:

Let P be linearity, Q be time invariance, R be causality and S be stability. In question discrete time input $x[n]$ and output $y[n]$ relationship has been given. In the option properties of system has been given. Choose the option which match the properties for system.

32. $y[n] = rect\,(x[n])$

(A) P, Q, R (B) Q, R, S

(C) R, S, P (D) S , P, Q

33. $y[n] = nx[n]$

(A) P, Q, R, S (B) Q, R, S

(C) P, R (D) Q, S

34. $y[n] = \sum_{m=-\infty}^{n+1} u[m]$

(A) P, Q, R, S (B) R, S

(C) P, Q (D) Q, R

35. $y[n] = \sqrt{x[n]}$

(A) Q, R, S (B) R, S, P

(C) S, P, Q (D) P, Q, R

36. $x[n]$ as shown in fig. P5.2.36

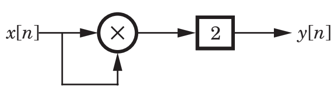


Fig. P5.2.36

(A) P, Q, R, S (B) Q, R, S

(C) P, Q (D) R, S

37. $x[n]$ as shown in fig. P5.2.37

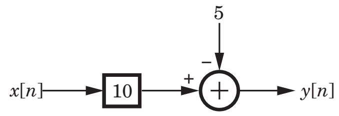


Fig. P5.2.37

(A) P, Q, R, S (B) Q, R, S

(C) P, R, S (D) P, Q, S

38. $x[n]$ as shown in fig. P5.2.38

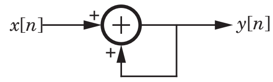


Fig. P5.2.38

(A) P, Q, R, S (B) P, Q, R

(C) P, Q (D) Q, R, S

Statement for Q.39–41:

Two discrete time systems S_1 and S_2 are connected in cascade to form a new system as shown in fig. P5.2.39–41.

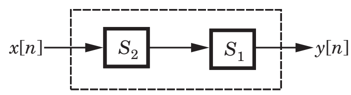


Fig. P5.2.39–41.

39. Consider the following statements

(a) If S_1 and S_2 are linear, the S is linear

(b) If S_1 and S_2 are nonlinear, then S is nonlinear

(c) If S_1 and S_2 are causal, then S is causal

(d) If S_1 and S_2 are time invariant, then S is time invariant

True statements are :

(A) a, b, c (B) b, c, d

(C) a, c, d (D) All

40. Consider the following statements

(a) If S_1 and S_2 are linear and time invariant, then interchanging their order does not change the system.

(b) If S_1 and S_2 are linear and time varying, then interchanging their order does not change the system.

True statement are

(A) Both a and b (B) Only a

(C) Only b (D) None

41. Consider the statement

(a) If S_1 and S_2 are noncausal, the S is non causal

(b) If S_1 and/or S_2 are unstable, the S is unstable.

True statement are :

(A) Both a and b (B) Only a

(C) Only b (D) None

42. The following input output pairs have been observed during the operation of a time invariant system :

$$x_1[n] = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{S} y_1[n] = \{\underset{\uparrow}{0}, 1, 2\}$$

$$x_2[n] = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{S} y_2[n] = \{\underset{\uparrow}{0}, 1, 0, 2\}$$

$$x_3[n] = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{S} y_3[n] = \{1, \underset{\uparrow}{2}, 1\}$$

The conclusion regarding the linearity of the system is

(A) System is linear

(B) System is not linear

(C) One more observation is required.

(D) Conclusion cannot be drawn from observation.

43. The following input output pair have been observed during the operation of a linear system:

$$x_1[n] = \{-1, \underset{\uparrow}{2}, 1\} \xleftrightarrow{S} y_1[n] = \{\underset{\uparrow}{1}, 2, -1, 0, 1\}$$

$$x_2[n] = \{1, \underset{\uparrow}{-1}, -1\} \xleftrightarrow{S} y_2[n] = \{-1, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3[n] = \{\underset{\uparrow}{0}, 1, 1\} \xleftrightarrow{S} y_3[n] = \{\underset{\uparrow}{1}, 2, 1\}$$

The conclusion regarding the time invariance of the system is

(A) System is time-invariant

(B) System is time variant

(C) One more observation is required

(D) Conclusion cannot be drawn from observation

44. The stable system is

(A) $y[n] = x[n] + 1.1y[n-1]$

(B) $y[n] = x[n] - \frac{1}{2}(y[n-1] + y[n-2])$

(C) $y[n] = x[n] - (1.5y[n-1] + 0.4y[n-2])$

(D) Above all

45. The system shown in fig. P5.2.45 is

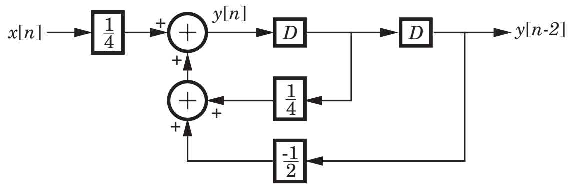


Fig. P5.2.45

(A) Stable and causal

(B) Stable but not causal

(C) Causal but unstable

(D) unstable and not causal

46. The impulse response of a LTI system is given as

$$h[n] = \left(-\frac{1}{2}\right)^n u[n].$$

The step response is

(A) $\frac{1}{3}\left(2 - \left(-\frac{1}{2}\right)^{n+1}\right)u[n]$ (B) $\frac{1}{3}\left(2 - \left(-\frac{1}{2}\right)^{n}\right)u[n]$

(C) $\frac{1}{3}\left(2 + \left(-\frac{1}{2}\right)^{n+1}\right)u[n]$ (D) $\frac{1}{3}\left(2 + \left(-\frac{1}{2}\right)^{n}\right)u[n]$

47. The difference equation representation for a system is

$$y[n] - \frac{1}{2}y[n-1] = 2x[n], \quad y[-1] = 3$$

The natural response of system is

(A) $\frac{3}{2}\left(-\frac{1}{2}\right)^n u[n]$ (B) $\frac{2}{3}\left(-\frac{1}{2}\right)^n u[n]$

(C) $\frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$ (D) $\frac{2}{3}\left(\frac{1}{2}\right)^n u[n]$

48. The difference equation representation for a system is

$$y[n] - 2y[n-1] + y[n-2] = x[n] - x[n-1]$$

If $y[n] = 0$ for $n < 0$ and $x[n] = \delta[n]$, then $y[2]$ will be

(A) 2 (B) –2

(C) –1 (D) 0

49. Consider a discrete-time system S whose response to a complex exponential input $e^{j\pi n/2}$ is specified as

$$S : e^{j\pi n/2} \quad \Rightarrow \quad e^{j\pi 3n/2}$$

The system is

(A) definitely LTI

(B) definitely not LTI

(C) may be LTI

(D) information is not sufficient.

50. Consider the two system

$$S_1 : y[n] = \begin{cases} x[n+1], & n \geq 0 \\ x[n], & n \leq -1 \end{cases}$$

$$S_2 : y[n] = \begin{cases} x\left[\dfrac{n}{2}\right], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

The invertible system is

(A) S_1 (B) S_2

(C) Both S_1 and S_2 (D) None of the above

Statement for Q.51–52:

Consider the cascade of the following two system S_1 and S_2, as shown in fig. P5.2.51–52

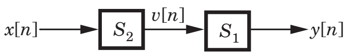


Fig. P5.2.51–52

S_1 : Causal LTI $\quad v[n] = \dfrac{1}{2} v[n-1] + x[n]$

S_2 : Causal LTI $\quad y[n] = a y[n-1] + b v[n]$

The difference equation for cascaded system is

$$y[n] = -\frac{1}{8} y[n-2] + \frac{3}{4} y[n-1] + x[n]$$

51. The value of a is

(A) $\dfrac{1}{4}$ (B) 1

(C) 4 (D) 2

52. The value of b is

(A) $\dfrac{1}{4}$ (B) 1

(C) 4 (D) 2

****

Solutions

1. (B)

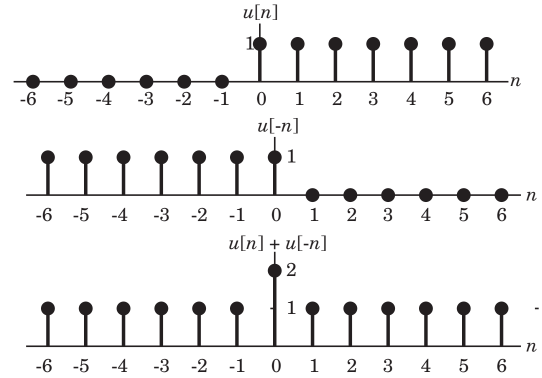


Fig. S5.2.1

2. (D) All describe the even part of x

3. (A) $E = \sum_{-\infty}^{\infty} |A\delta[n]|^2 = A^2 \sum_{0}^{\infty} (1) = A^2$

4. (D) $E = \sum_{-\infty}^{\infty} |nu[n]|^2 = \sum_{0}^{\infty} n^2 = \infty$

5. (C) $P = \lim_{N \to \infty} \dfrac{1}{2N+1} \sum_{n=-N}^{n=N} (u[n])^2 = \dfrac{N+1}{2N+1} = \dfrac{1}{2}$

6. (B) Let $v[n] = x[2n]$, $\quad v[0] = x[0] = 0$

$v[1] = x[2] = 1, \quad v[2] = x[4] = 0$

$v[-1] = x[-2] = 1, \quad v[-2] = x[-4] = 0$

So $x[n] = \{3, \ 2, \ 1, \ \underset{\uparrow}{0}, \ 1, \ 2, \ 3\}$

$x[2n] = \{2, \ \underset{\uparrow}{0}, \ 2\}$

7. (A) $y[n] = \{-1, \ -1, \ -1, \ -1, \ \underset{\uparrow}{0}, \ 1, \ 1, \ 1, \ 1\}$

$y[n+1] = \{-1, \ -1, \ -1, \ -1, \ 0, \ \underset{\uparrow}{1}, \ 1, \ 1, \ 1\}$

$y[-n+1] = \{1, \ 1, \ 1, \ \underset{\uparrow}{1}, \ 0, \ -1, \ -1, \ -1, \ -1\}$

8. (C) $x[n-2] = \{3, \ \underset{\uparrow}{2}, \ 1, \ 0, \ 1, \ 2, \ 3\}$

$y[n+2] = \{-1, \ -1, \ -1, \ -1, \ 0, \ 1, \ \underset{\uparrow}{1}, \ 1, \ 1\}$

$x[n-2] + y[n+2]$

$= \{-1, \ -1, \ -1, \ -1, \ 0, \ 4, \ \underset{\uparrow}{3}, \ 2, \ 1, \ 1, \ 2, \ 3\}$

9. (C) $x[n-1] = \{3, \ 2, \ \underset{\uparrow}{1}, \ 0, \ 1, \ 2, \ 3\}$

$x[3n-1] = \{\underset{\uparrow}{1}, \ 2\}$

10. (D) $x[3-n] = \{\underset{\uparrow}{3}, \ 2, \ 1, \ 0, \ 1, \ 2, \ 3\}$

$y[-n] = \{1, \ 1, \ 1, \ 1, \ \underset{\uparrow}{0}, \ -1, \ -1, \ -1, \ -1\}$

$x[3-n]y[-n] = \{\underset{\uparrow}{0}, \ -2, \ -1, \ 0, \ -1\}$

11. (D) $x[n+2] = \{3, \ 2, \ 1, \ 0, \ 7, \ \underset{\uparrow}{2}, \ 3\}$

$y[n-2] = \{-1, \ -1, \ -\underset{\uparrow}{1}, \ -1, \ 0, \ 1, \ 1, \ 1, \ 1\}$

$x[n+2]y[n-2] = \{0, \ -1, \ -\underset{\uparrow}{2}, \ -3, \ 0 \ .....\}$

12. (A) Both signal are periodic,
$N_1 = 18$, $N_2 = 14$,
N = LCM(18, 14) =126

13. (D) $\cos\left(\dfrac{N}{8}\right)$ is not periodic. So $x[n]$ is not periodic.

14. (A) $N_1 = 4$, $N_2 = 16$, $N_3 = 8$,
N = LCM(4, 16, 8) =16

15. (D) $\dfrac{2\pi}{N} = \dfrac{1}{6} \Rightarrow N = \dfrac{\pi}{3}$, Not periodic.

16. (D) $\Omega = \dfrac{2\pi m}{N}$ The smallest value of Ω is attained with $m = 1$, $\Omega = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$ rad/cycle.

17. (B) $L_1 = N_1 + M_1$, $L_2 = N_2 + M_2$
$N_1 = -5$, $N_2 = 3$, $M_1 = 2$, $M_2 = 6$

18. (A)

	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

Fig. S5.2.18

$y[n] = \{1, \ 3, \ 7, \ 7, \ 7, \ 6, \ 4\}$

19. (C) $y[n] = \{1, 2, 3, 4, 5\}$

	1	2	3	4	5
1	1	2	3	4	5

Fig. S5.2.19

20. (B) $y[n] = \{1, 4, \ 2, -4, 1\}$

	1	2	-1
1	1	1	-1
2	2	4	-2
3	-1	-2	-1

Fig. S5.2.20

21. (D)

	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

Fig. S5.2.21

$y[n] = \{0, \ 0, \ \underset{\uparrow}{1}, \ -1, \ 2, \ 2, \ 1, \ 3\}$

22. (A)

	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

Fig S5.2.22

$y[n] = \{0, \ 0, \ 1, \ -\underset{\uparrow}{1}, \ 2, \ 2, \ 1, \ 3\}$

23. (D)

	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	4	4	0	4	4

Fig. S5.2.23

$y[n] = \{1, \ -1, \ -5, \ 2, \ 3, \ -\underset{\uparrow}{5}, \ 1, \ 4\}$

24. (B) $y[n] = \{1,\ 4,\ 4,\ \underset{\uparrow}{10},\ 4,\ 4,\ 4,\ 1\}$

	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

Fig. S5.2.24

25. (A) $y[n] = \{2,\ 4,\ -19,\ \underset{\uparrow}{36},\ -25,\ 2,\ 12\}$

	1	4	-3	6	4
2	2	8	-6	12	8
-4	-4	-16	12	-24	-16
3	3	12	-9	18	12

Fig. S5.2.25

26. (B) $x[n] = \{1,\ 2,\ \underset{\uparrow}{1},\ 1\}$, $h[n] = \{\underset{\uparrow}{1},\ -1,\ 0,\ 0,\ 1\}$

	1	2	1	1
1	1	2	1	-1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1

Fig. S5.2.26

$y[n] = \{1,\ 1,\ \underset{\uparrow}{-1},\ 0,\ 0,\ 2,\ 1,\ 1\}$

$y[n] = \delta[n+2] + \delta[n+1] - \delta[n] + 2\delta[n-3] + \delta[n-4] + \delta[n-5]$

27. (D) $y[n] = \sum_{k=n-2}^{\infty} (-1)^k 2^{n-k} = 2^n \sum_{k=n-2}^{\infty} \left(-\frac{1}{2}\right)^k$

$$= \frac{2^n \left(-\frac{1}{2}\right)^{n-2}}{1 + \frac{1}{2}} = \frac{8}{3}(-1)^n$$

28. (C) For $n + 2 < 0$ or $n < -2$, $y[n] = 0$

for $n + 2 \geq 0$ or $n \geq -2$, $y[n] = \sum_{k=0}^{n+2} \frac{1}{4}k = \frac{4}{3} - \frac{1}{12}\frac{1}{4^n}$,

$$\Rightarrow \quad y[n] = \left(\frac{4}{3} - \frac{1}{12}\left(\frac{1}{4}\right)^n\right) u[n+2]$$

29. (D) For $n - 2 \leq 3$ or $n \leq 5$, $y[n] = \sum_{k=\infty}^{n-2} 3^k = \frac{3^n}{6}$

for $n - 2 \geq 4$ or $n \geq 6$, $y[n] = \sum_{k=-\infty}^{3} 3^k = \frac{81}{2}$,

$$\Rightarrow \quad y[n] = \begin{cases} \frac{3^n}{6}, & n \leq 5 \\ \frac{81}{2}, & n \geq 6 \end{cases}$$

30. (A) For $n - 3 < -3$ or $n < 0$, $y[n] = 0$

for $n - 3 \geq -3$ or $n > 0$, $y[n] = \sum_{k=-3}^{n-3} 1 = n + 1$,

$y[n] = (n+1)u[n]$

31. (A) For $n - 1 < 0$ or $n < 1$, $y[n] = 0$

For $n - 1 \geq 0$ or $n \geq 1$, $y[n] = \sum_{k=0}^{n-1} \cos\left(\frac{\pi}{2}k\right)$

$$\Rightarrow \quad y[n] = \begin{cases} 1, & n = 4m+1,\ 4m+2 \\ 0, & n = 4m,\ 4m+3 \end{cases}$$

32. (B) $y_1[n] = rect\,(v[n])$, $y_2[n] = rect\,(kv[n])$

$y_2[n] \neq k\,y_1[n]$ (Not Homogeneous not linear)

$y_1[n] = rect\,(v[n])$, $y_2[n] = rect\,(v[n - n_o])$

$y_1[n - n_o] = rect\,(v[n - n_o]) = y_2[n]$ (Time Invariant)

At any discrete time $n = n_o$, the response depends only on the excitation at that discrete time. (Causal)

No matter what values the excitation may have the response can only have the values zero or one. (Stable)

33. (C) $y_1[n] = nv[n]$, $y_2[n] = nkv[n]$

$ky_1[n] = y_2[n]$ (Homogeneous)

Let $x_1[n] = v[n]$ then $y_1[n] = nv[n]$

Let $x_2[n] = w[n]$ then $y_2[n] = nw[n]$

Let $x_3[n] = v[n] + w[n]$

Then $y_3[n] = n(v[n] + w[n]) = nv[n] + nw[n]$

$= y_1[n] + y_2[n]$ (Additive)

Since the system is homogeneous and additive, it is also linear.

$y_1[n - n_o] = (n - n_o)v[n - n_o] \neq y_n[n] = nv[n - n_o]$

(Time variant)

At any discrete time, $n = n_o$ the response depends only on the excitation at that same time. (Causal)

If the excitation is a constant, the response is unbounded as n approaches infinity. (Unstable)

34. (C) $y_1[n] = \sum_{m=-\infty}^{n+1} v[m]$, $y_2[n] = \sum_{m=-\infty}^{n+1} kv[m]$

$y_2[n] = ky_1[n]$ (Homogeneous)

$y_1[n] = \sum_{n=-\infty}^{n+1} v[m]$, $y_2[n] = \sum_{n=-\infty}^{n+1} w[m]$

$y_3[n] = \sum_{m=-\infty}^{n+1}(v[n] + w[m])$

$= \sum_{m=-\infty}^{n+1} v[m] + \sum_{m=-\infty}^{n+1} w[n] = y_1[n] + y_2[n]$ (Additive)

Since the system is homogeneous and additive it is also linear

$y_1[n] = \sum_{m=-\infty}^{n+1} v[n]$, $y_2[n] = \sum_{m=-\infty}^{n+1} v[m - n_o]$

$y_1[n - n_o] = \sum_{m=-\infty}^{n-n_o+1} v[m] = \sum_{q=-\infty}^{n+1} v[q - n_o] = y_2[n]$

(Time Invariant)

At any discrete time, $n = n_o$, the response depends on the excitation at the next discrete time in future.

(Anti causal)

If the excitation is a constant, the response increases without bound. (Unstable)

35. (A) $y_1[n] = \sqrt{v[n]}$, $y_2 = \sqrt{kv[n]} = \sqrt{k}\sqrt{v[n]}$

$ky_1[n] = k\sqrt{v[n]} \neq y_2[n]$ (Not Homogeneous Not linear)

$y_1[n] = \sqrt{v[n]}$, $y_2[n] = \sqrt{v[n - n_o]}$

$y_1[n - n_o] = \sqrt{v[n - n_o]} = y_2[n]$ (Time Invariant)

At any discrete time $n = n_o$, the response depends only on the excitation at that time (Causal)

If the excitation is bounded, the response is bounded.

(Stable).

36. (B) $y[n] = 2x^2[n]$

Let $x_1[n] = v[n]$ then $y_1[n] = 2v^2[n]$

Let $x_2[n] = kv[n]$ then $y_2[n] = 2k^2v^2[n]$

$ky[n] \neq y_2[n]$ (Not homogeneous Not linear)

Let $x_1[n] = v[n]$ then $y_1[n] = 2v^2[n]$

Let $x_2[n] = v[n - n_o]$ then $y_2[n] = 2v^2[n - n_o]$

$y_1[n - n_o] = 2v[n - n_o] = y_2[n]$ (Time invariant)

At any discrete time, $n = n_o$, the response depends only on the excitation at that time. (Causal)

If the excitation is bounded, the response is bounded.

(Stable).

37. (B) $y_1[n] = 10v[n] - 5$, $y_2[n] = 10kv[n] - 5$

$y_2[n] \neq ky_1[n]$ (Not Homogeneous so not linear)

$y_1[n] = 10v[n] - 5$, $y_2[n] = 10v[n - n_o] - 5$

$y_1[n - n_o] = 10v[n - n_o] - 5$, $= y_2[n]$ (Time Invariant)

At any discrete time, $n = n_o$ the response depends only on the excitation at that discrete time and not on any future excitation. (Causal)

If the excitation is bounded, the response is bounded.

(Stable).

38. (B) $y[n] = x[n] + y[n-1]$,

$y[n-1] = x[n-1] + y[n-2]$

$y[n] = x[n] + x[n-1] + y[n-2]$, Then by induction

$y[n] = x[n-1] + x[n-2] + \dots x[n-k] + \dots = \sum_{k=0}^{\infty} x[n-k]$

Let $m = n - k$ then $y[n] = \sum_{m=n}^{-\infty} x[m] = \sum_{m=-\infty}^{n} x[m]$

$y_1[n] = \sum_{m=-\infty}^{n} v[m]$, $y_2[n] = \sum_{m=-\infty}^{n} kv[m] = ky_1[n]$

(Homogeneous)

$y_3[n] = \sum_{m=-\infty}^{n}\left(v[m] + w[m]\right) = \sum_{m=-\infty}^{\infty} v[m] + \sum_{m=-\infty}^{n} w[m]$

$= y_1[n] + y_2[n]$ (Additive)

System is Linear.

$y_1[n] = \sum_{m=-\infty}^{\infty} v[m]$, $y_2 = \sum_{m=-\infty}^{n} v[n - n_o]$

$y_1[n]$ can be written as

$y_1[n - n_o] = \sum_{m=-\infty}^{n-n_o} v[m] = \sum_{q=-\infty}^{n} v[q - n_o] = y_2[n]$

(Time Invariant)

At any discrete time $n = n_o$ the response depends only on the excitation at that discrete time and previous discrete time. (Causal)

If the excitation is constant, the response increase without bound. (Unstable)

39. (C) Only statement (b) is false. For example

$S_1 : y[n] = x[n] + b$,

$S_2 : y[n] = x[n] - b$, where $b \neq 0$

$S\{x[n]\} = S_2\{S_1\{x[n]\}\} = S_2\{x[n] + b\} = x[n]$

Hence S is linear.

40. (B) For example

$S_1 \; : \; y[n] = nx[n]$

$S_2 \; : \; y[n] = nx[n+1]$

If $x[n] = \delta[n]$ then $S_2\{S_1\{\delta[n]\}\} = S_2[0] = 0$,

$S_1\{S_2\{\delta[n]\}\} = S_1\{\delta[n+1]\} = -\delta[n+1] \neq 0$

41. (D) $S_1 : y[n] = x[n+1]$ non causal

$S_2 : y[n] = x[n-2]$ causal

$S : y[n] = x[n-1]$ which is causal (false)

$S_1 : y[n] = e^{x[n]}$ stable,

$S_2 : y[n] = \ln(x[n])$ unstable

But $S : y[n] = x[n]$ stable (false)

42. (B) System is not linear. This is evident from observation of the pairs

$x_3[n] \;\Leftrightarrow\; y_3[n]$ and $x_2[n] \;\Leftrightarrow\; y_2[n]$

If the system were linear $y_2[n]$ would be of the form

$y_2[n] = \{3, \;\; 6, \;\; \underset{\uparrow}{3}\}$

43. (B) Since system is linear $x_1[n] + x_2[n] = \delta[n]$.

The impulse response of the system is

$y_1[n] + y_2[n] = \{0, \; 3, \; -1, \; 2, \; 1\}$

If the system were time invariant, the response to $x_3[n]$ would be $x_3[n] * (y_1[n] + y_2[n]) = \{\underset{\uparrow}{3}, \; 2, \; 1, \; 3, \; 1\}$

But this is not the case.

44. (B) For (A) $\alpha = -1.1$ greater than 1 so unstable.

For (B) $\alpha_{12} = -0.25 \pm j0.66$. Its magnitude are less than one so stable.

For (C) $\alpha_{12} = -1.153, \;\; -0.3469$

One magnitude is greater than one so unstable.

45. (A) The difference equation is

$$y[n] = \frac{1}{4}x[n] + \frac{1}{4}y[n-1] - \frac{1}{2}y[n-2]$$

$$4y[n] - y[n-1] + 2y[n-2] = x[n]$$

Eigen values are

$4\alpha^2 - \alpha + 2 = 0, \quad \alpha_{12} = 0.125 \pm j0.696$

Its magnitude are less than 1. Thus system is stable.

$$y[n] = \frac{1}{4}\left(x[n] + y[n-1] + 2y[n-2]\right)$$

Response at time, n, depends on the excitation at time n and the responses at previous time. It does not depend on any future values of the excitation.

46. (A) For $n < 0 = s[n] = 0$

For $n \geq 0$, $\quad s[n] = \sum_{k=0}^{n}\left(-\frac{1}{2}\right)^k = \frac{1}{3}\left(2 + \left(-\frac{1}{2}\right)^n\right)$

47. (C) Characteristic equation $r - \frac{1}{2} = 0$

$$y^{(n)}[n] = C\left(\frac{1}{2}\right)^n, \quad y[-1] = 3 = C\left(\frac{1}{2}\right)^{-1}, \quad C = \frac{3}{2},$$

$$\Rightarrow \quad y^{(n)}[n] = \frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$$

48. (D) For $n = 0$, $\quad y[0] = x[0] - x[-1]$

For $n = 1$, $y[1] = 2y[0] + x[1] - x[0]$

$\Rightarrow \quad y[1] = x[0] + x[1] - 2x[-1]$

For $n = 2$, $y[2] = 2y[1] - y[0] + x[2] - x[1]$

$y[2] = x[0] + x[1] + x[2] - 3x[-1]$

$y[2] = \delta[0] + \delta[1] + \delta[2] - 3\delta[-1] = 0$

49. (B) The input $e^{j\pi n/2}$ must produce the output in the form $Ae^{j\pi n/2}$. The output in this case is $e^{j3\pi n/2}$. This violates the Eigen function property of LTI system. Therefore, S is definitely not LTI system.

50. (B) S_2 is Invertible System $y[n] = x[2n]$, S_1 is not invertible because $\delta[n]$, $2\delta[n]$ etc. result in $y[n] = 0$.

51. (A) $y[n] = ay[n+1] + bv[n]$,

$$v[n] = \frac{1}{b}y[n] - \frac{a}{b}y[n-1],$$

$$v[n-1] = \frac{1}{b}y[n-1] - \frac{a}{b}y[n-2]$$

Weighting the previous equation by $\frac{1}{2}$ and subtracting from the one before

$$v[n] - \frac{1}{2}v[n-1] = \frac{1}{b}y[n] - \frac{a}{b}y[n-1] - \frac{1}{2b}y[n-1] + \frac{a}{2b}y[n-2]$$

$$x[n] = \frac{1}{b}y[n] - \left(\frac{a}{b} + \frac{1}{2b}\right)y[n-1] = \frac{a}{2b}y[n-2]$$

$$y[n] = \left(a + \frac{1}{2}\right)y[n-1] - \frac{a}{2}y[n-2] + bx[n]$$

Comparing $\quad a = \frac{1}{4}, \quad b = 1$

52. 52. (B) $b = 1$.

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