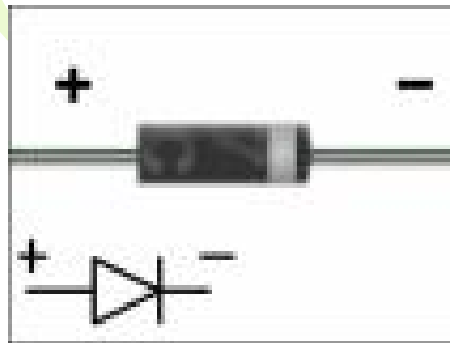


# Lecture 02

## Diodes



圖片來自;[www.personeel.glr.nl/koster/elektro/diodes.JP](http://www.personeel.glr.nl/koster/elektro/diodes.JP)

A decorative graphic on the left side of the slide featuring three balloons: a green one at the top, a light blue one in the middle, and a purple one at the bottom. Each balloon has a string and several small yellow triangular flags attached to it.

# topics

- Semiconductor physics
- Diode forward characteristic
- Diode reverse characteristic
- Special diodes

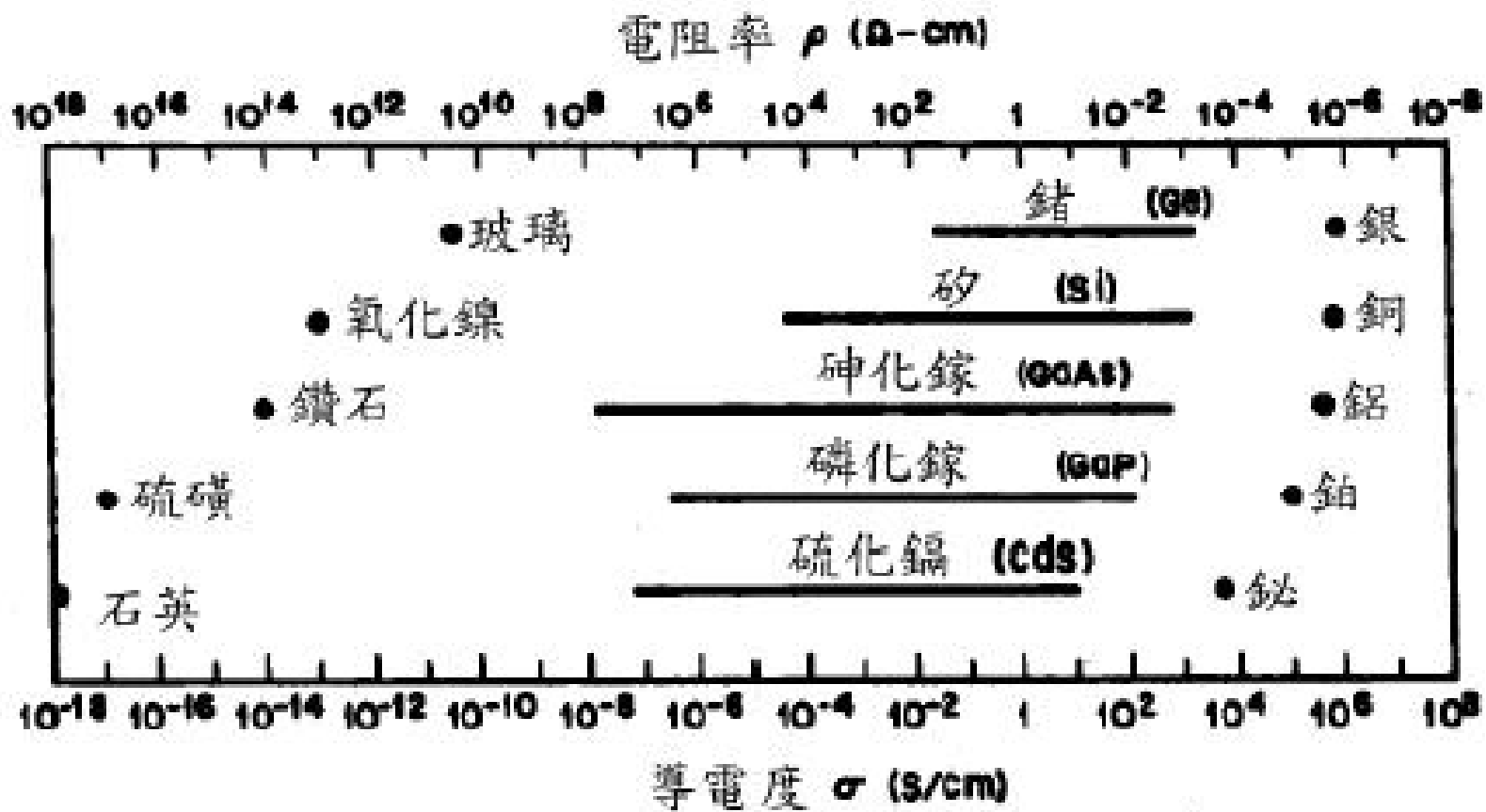


# Solid-state material

- Insulator ( $\text{SiO}_2$ )
  - Strong covalent bonds, no free electron
  - Conductivity  $\sigma < 10^{-8} (\Omega\text{-cm})^{-1}$
  - Energy gap  $E_g \geq 3\text{eV}$
- Semi-conductor (Si, Ge)
  - Conductivity  $10^{-8} < \sigma < 10^3 (\Omega\text{-cm})^{-1}$
  - Energy gap  $0 < E_g < 3\text{eV}$
- Conductor (Cu, Ag)
  - Weak covalent bonds, many free electrons
  - Conductivity  $\sigma > 10^3 (\Omega\text{-cm})^{-1}$
  - Energy gap  $E_g = 0\text{eV}$

$$1\text{eV} \equiv 1.6 \times 10^{-19} \text{ Joule}$$

*eV: eletron voltage*



# Energy gap

*Energy level*



Infinite many atoms

*high energy*

*Low energy*

conduction band

Energy-level gap

valance band

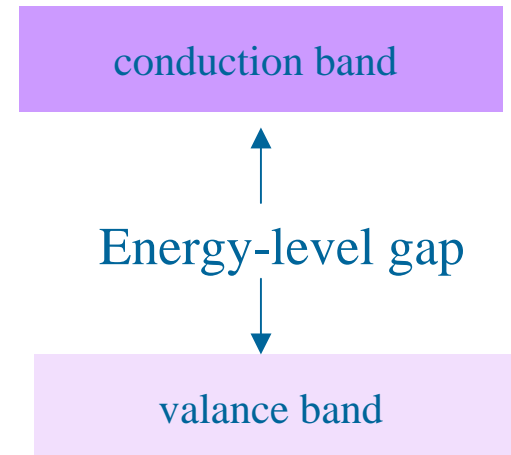
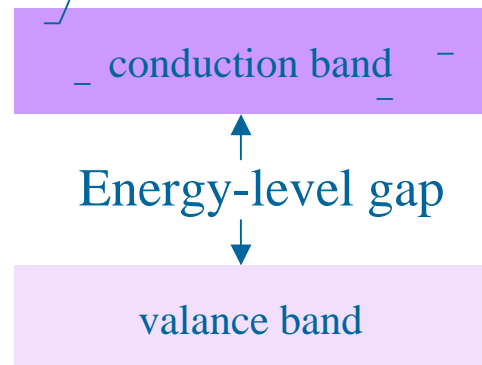
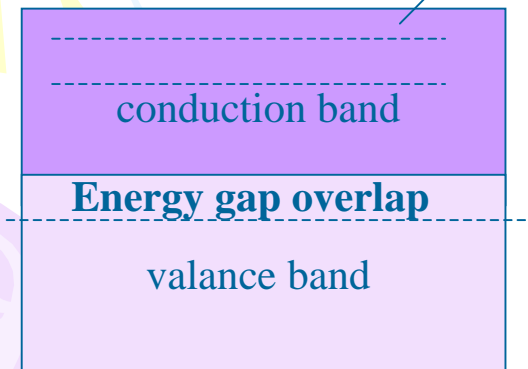
<http://oldsite.vislab.usyd.edu.au/photonics/devices/semicdev/doping2.html>

## One atom

Level 3 : high energy

Level 1 : low energy

electrons



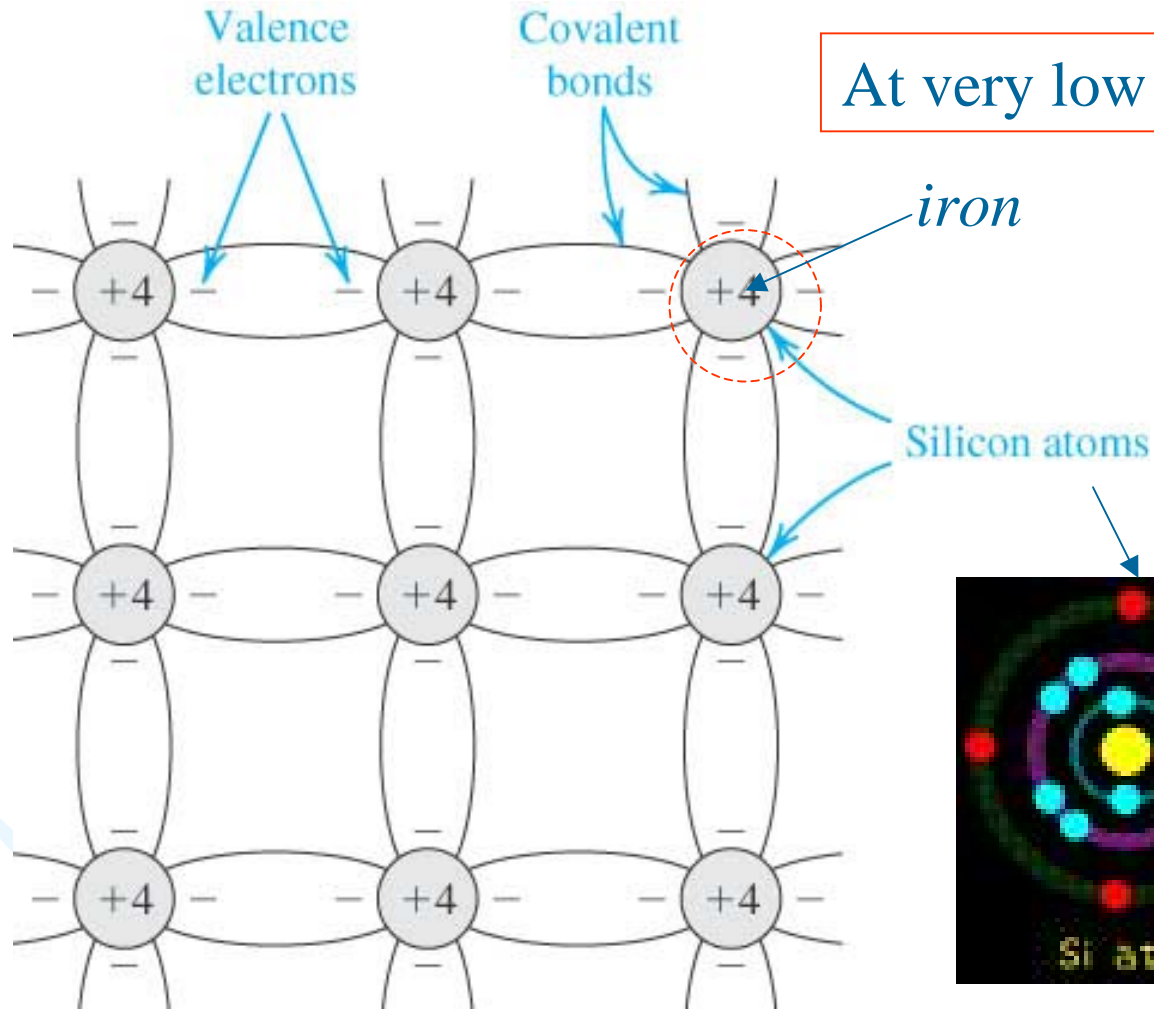
# Famous Semi-conductors

- Element semi-conductor
  - Si
    - Cheap
    - Energy gap > Ge → small leakage current
    - Stable oxide
  - Ge
    - First transistor
    - Small energy gap
    - Unstable oxide
- Compound semi-conductor

– GaAs

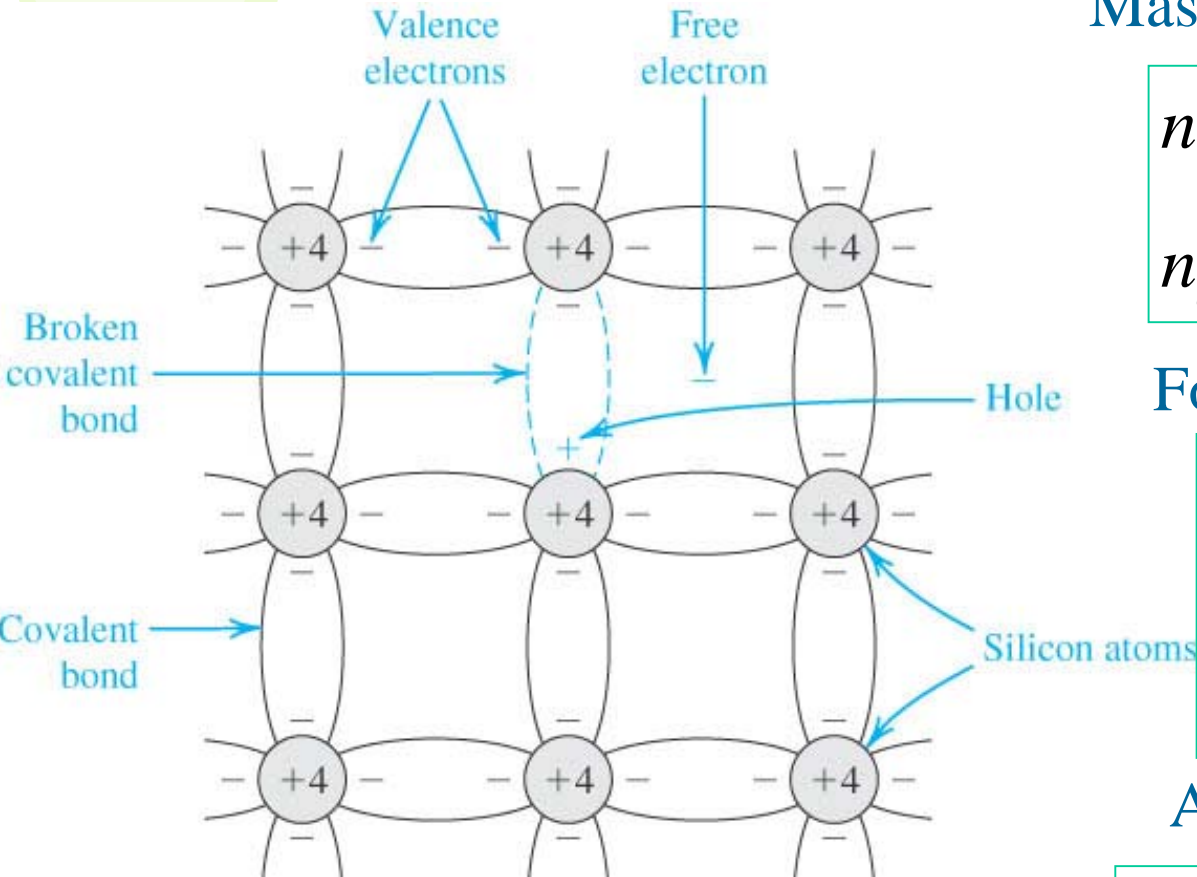
半導體種類	固有電子濃度 $n_i$ (cm <sup>-3</sup> )	帶溝 $E_g$ (eV)
鍺(Ge)	$2.4 \times 10^{13}$	0.67
矽(Si)	$1.45 \times 10^{10}$	1.12
砷化鎵(GaAs)	$1.79 \times 10^6$	1.42

# Intrinsic semiconductor (silicon)



<http://oldsite.vislab.usyd.edu.au/photonics/devices/semicdev/doping2.html>

# Intrinsic semiconductor at room temperature



Free electrons and holes generated by thermal ionization, so the concentration is same  $n = p = n_i$

Mass-action law:

$$n = p = n_i \Rightarrow np = n_i^2$$

$$n_i^2 = BT^3 e^{-\frac{E_G}{KT}}$$

For silicon semi-conductor

Material parameter

$$B = 5.4 \times 10^{31}$$

$$E_G = 1.12 \text{ eV}$$

Boltzmann's constant

$$k = 8.62 \times 10^{-5} \text{ eV/K}$$

At room temperature

$$T \approx 300 \text{ K}$$

$$n_i \approx 1.45 \times 10^{10} \text{ carriers/cm}^3$$



# Hole and electrons moving

## 1. drift

Drift velocity  $v = \mu E$

Resistivity  $R = \rho l / A$

Charge density  $\rho \equiv nq (\Omega - cm)$

conductivity  $\sigma \equiv nq\mu$

$$J = \sigma E$$

$$J_p = qp\mu_p E$$

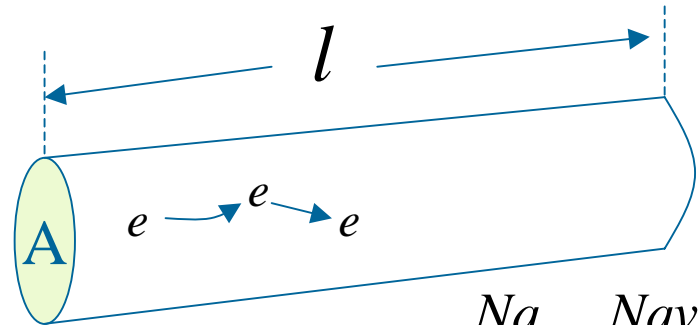
$$J_n = qn\mu_n E$$

$$J_{drift} = q(p\mu_p + n\mu_n)E$$

$E$  : electric field strength (V/cm)

$\mu$  : mobility of hole/electron ( $cm^2/V\text{-sec}$ )

$$E = \frac{f}{q}$$



$$I \equiv \frac{Nq}{T} = \frac{Nqv}{L}$$

$$J = \frac{I}{A} \Rightarrow J = \frac{Nqv}{AL}$$

$$n \equiv \frac{N}{LA} \Rightarrow J = nqv$$

Electron density

For intrinsic silicon :

$$\mu_p = 480 cm^2 / V \cdot s$$

$$\mu_n = 1350 cm^2 / V \cdot s$$

## 2. diffusion

$$J_p = -qD_p \frac{dp}{dx}$$

$$J_n = qD_n \frac{dn}{dx}$$

$$q = 1.6 \times 10^{-19} \text{ C}$$

$J$  : current density

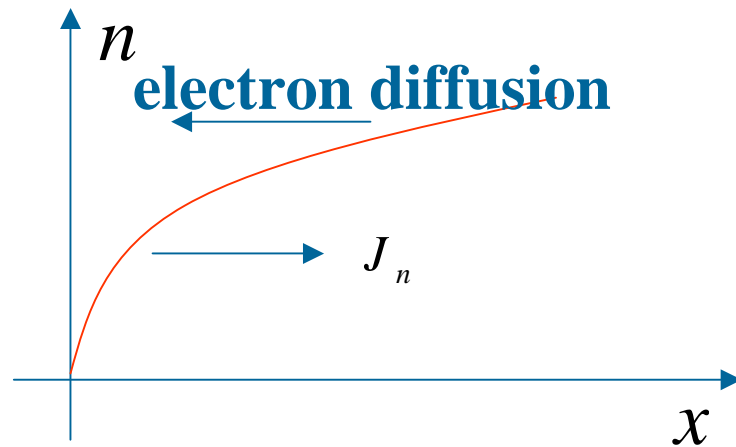
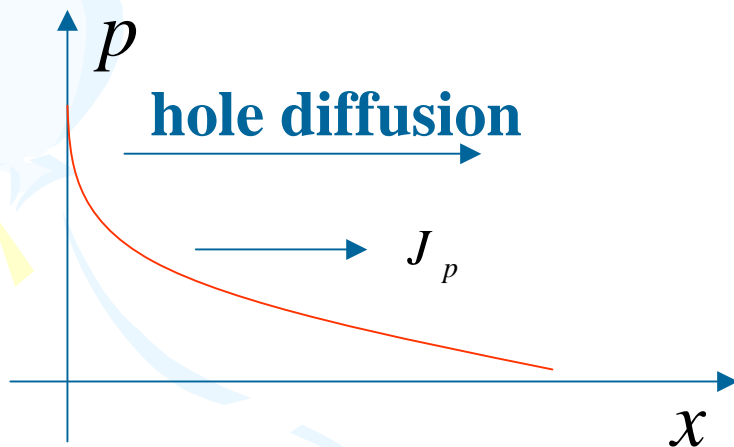
$q$  : electron charge

$D$  : diffusivity of hole/electron

For intrinsic silicon :

$$D_p = 12 \text{ cm}^2/\text{s}$$

$$D_n = 34 \text{ cm}^2/\text{s}$$

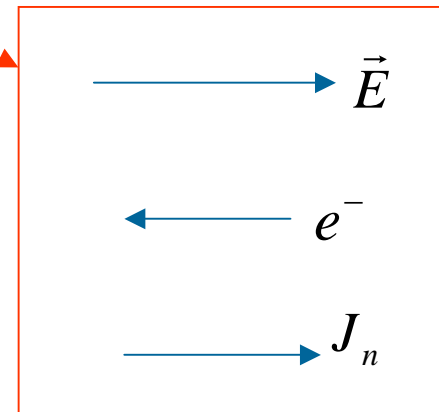
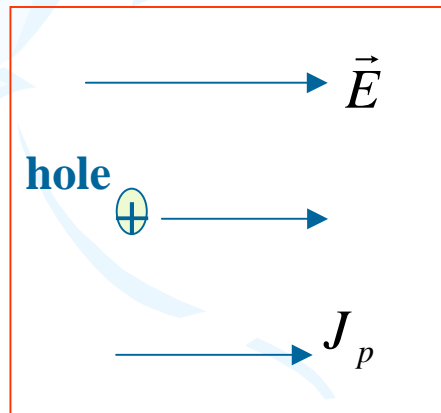


**The conventional current direction is the positive charge flow direction**

Currents in semi-conductor = drift current + diffusion current

$$J_p = pq\mu_p E - qD_p \frac{dp}{dx}$$

$$J_n = nq\mu_n E + qD_n \frac{dn}{dx}$$



## Einstein relationship : relationship between drift current and diffusion current

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$$

$V_T$  thermal voltage  
溫度伏特當量

At room temperature ( $20^\circ\text{C}$ )  $V_T \approx 25\text{mV}$

$$V_T \equiv \frac{kT}{q}$$

Boltzmann's constant

$$k = 1.38 \times 10^{-23} \text{ joules/kelvin}$$

$$T = 273 + ^\circ\text{C}$$

$$q = 1.6 \times 10^{-19} \text{ coulomb}$$

$$T = 20^\circ\text{C} \rightarrow 293^\circ\text{K} \Rightarrow V_T \approx 25\text{mV}$$

$$T = 27^\circ\text{C} \rightarrow 300^\circ\text{K} \Rightarrow V_T \approx 26\text{mV}$$

# Current in solid-state material

- Insulator

$$J_T = 0$$

- Semi-conductor

$$J_T = J_p + J_n$$
$$J_p = pq\mu_p E - qD_p \frac{dp}{dx}$$
$$J_n = nq\mu_n E + qD_n \frac{dn}{dx}$$

- Conductor

$$J_T = J_n = nq\mu_n E$$

# N-type impure semiconductor (donor)

Majority carriers are negative charges : electrons

$N_D$ : Donor atoms concentration

$n_{no}$ : free electrons concentration

In thermal equilibrium

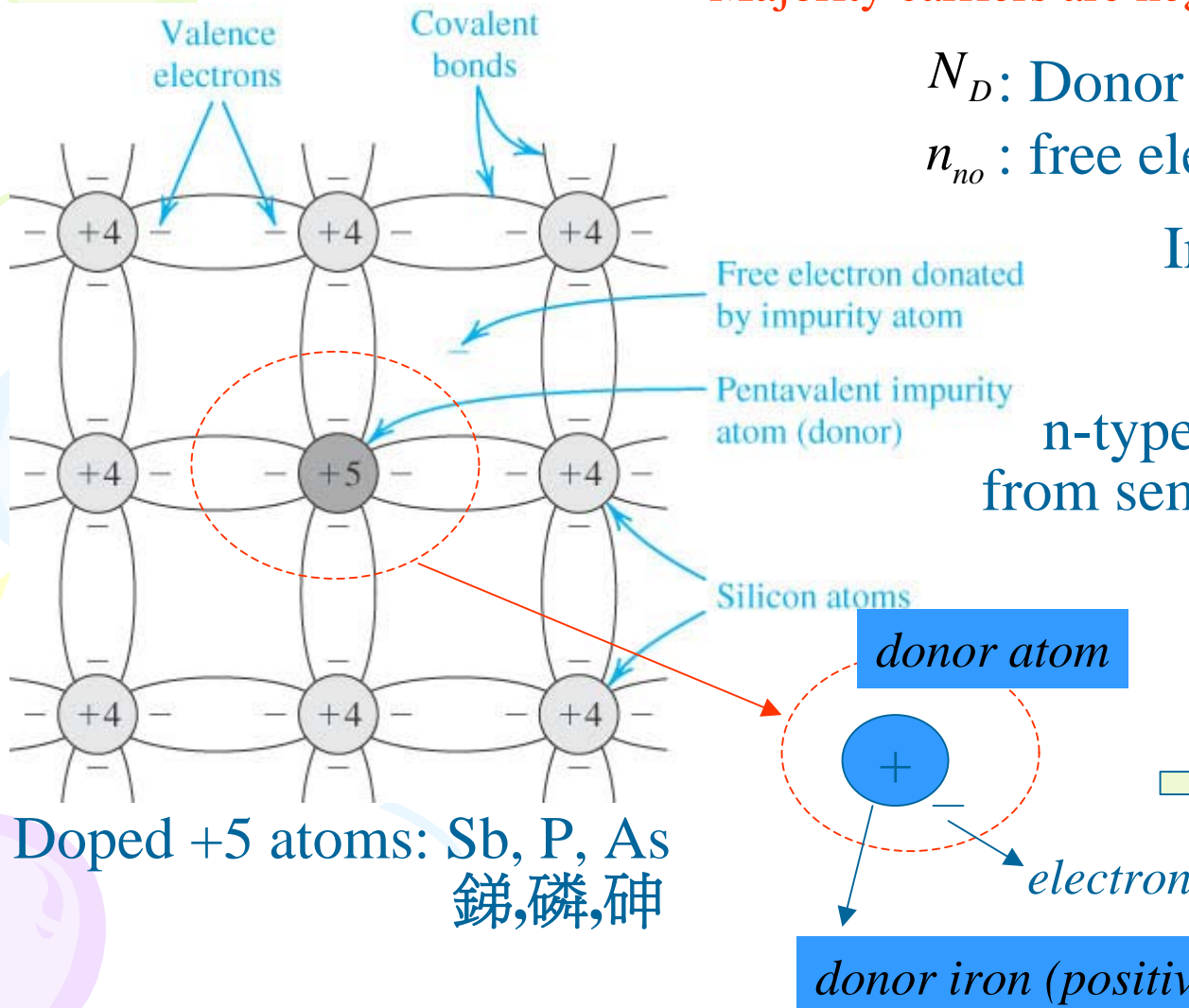
$$n_{no} \approx N_D$$

n-type  
from semiconductor physic

**Mass-action law**

$$n_{no} p_{no} = n_i^2$$

$$p_{no} \approx \frac{n_i^2}{N_D}$$



# P-type impure semiconductor (acceptor)

Majority carriers are positive charges : holes

$N_A$  : Acceptor atoms concentration

$p_{po}$  : free holes concentration

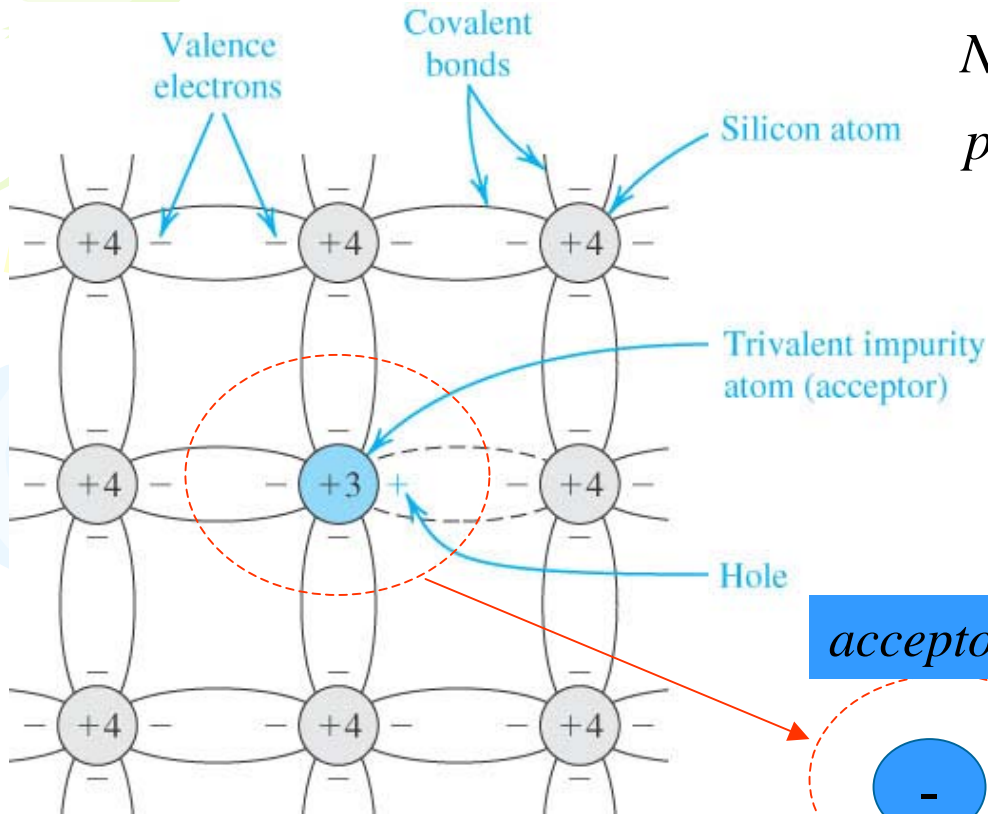
In thermal equilibrium

$$p_{po} \approx N_A$$

from semiconductor physic

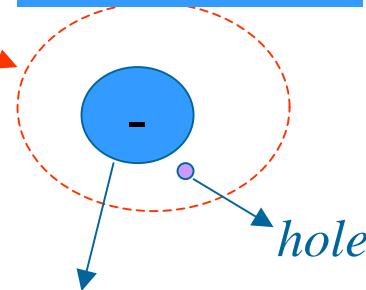
$$p_{po} n_{po} = n_i^2$$

$$n_{po} \approx \frac{n_i^2}{N_A}$$



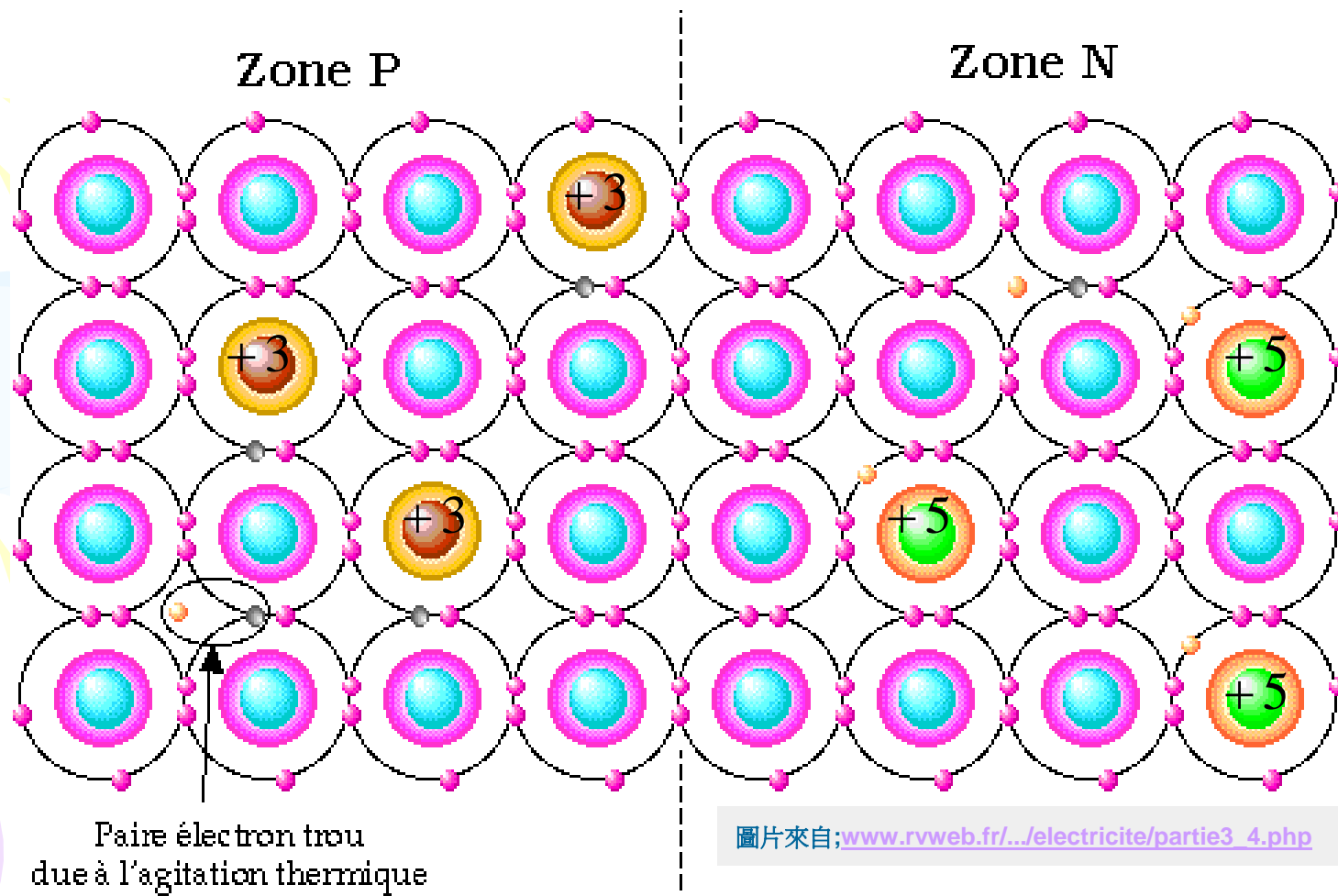
Doped +3 atoms: B, Ga, In,  
硼, 鎵, 銦

acceptor atom



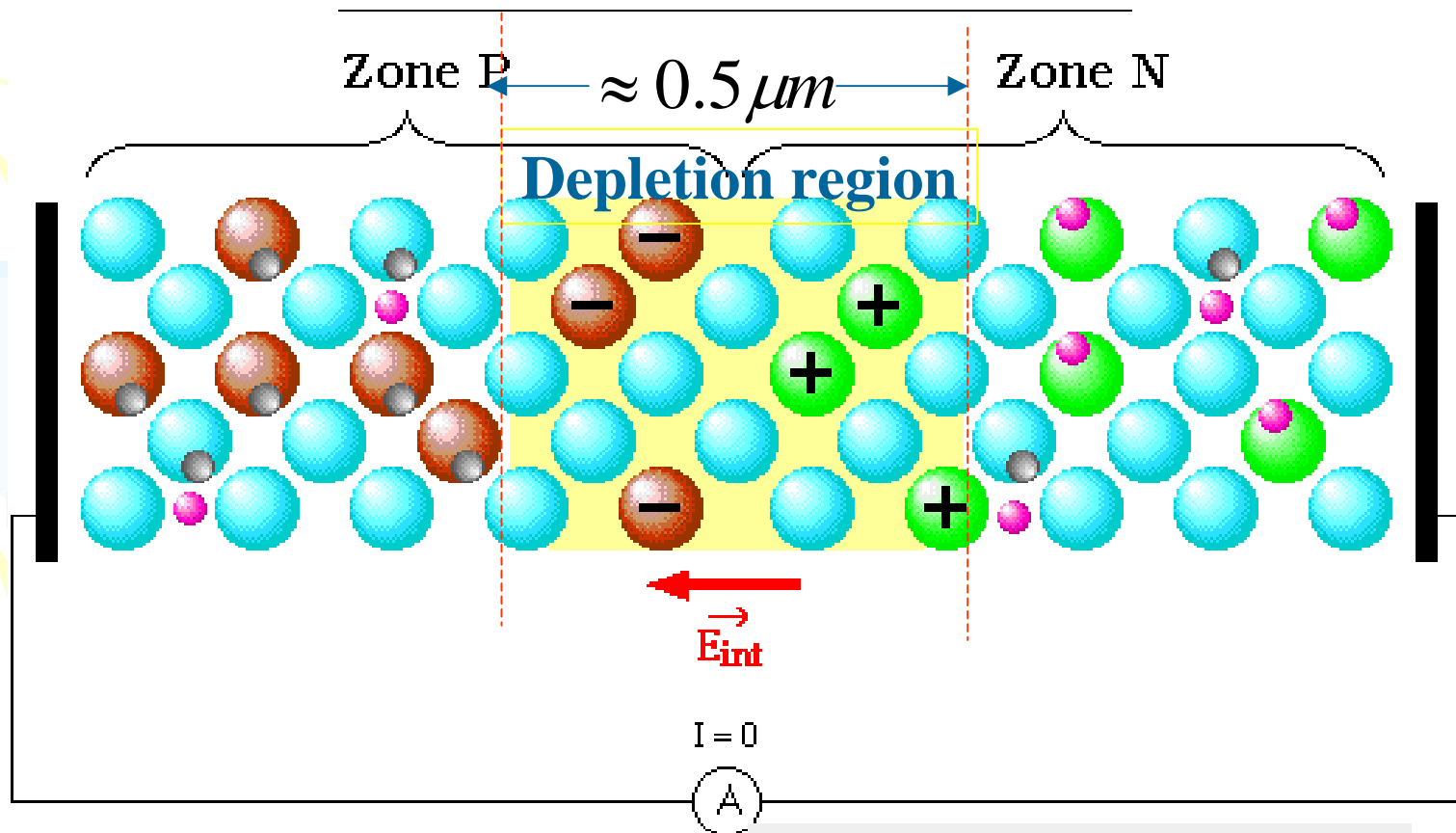
acceptor ion (negative ion)

# Fonction PN



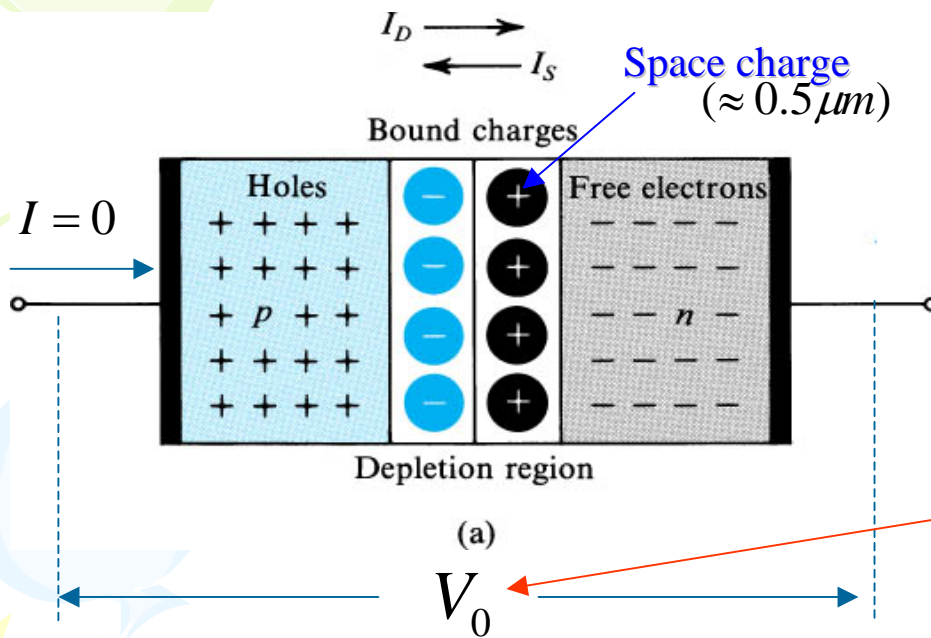


# *Fonction PN sans champ extérieur*



圖片來自: [www.rvweb.fr/.../electricite/partie3\\_4.php](http://www.rvweb.fr/.../electricite/partie3_4.php)

# PN Junction under open-circuit



$I_D$  : Diffusion current

$I_S$  : Drift current

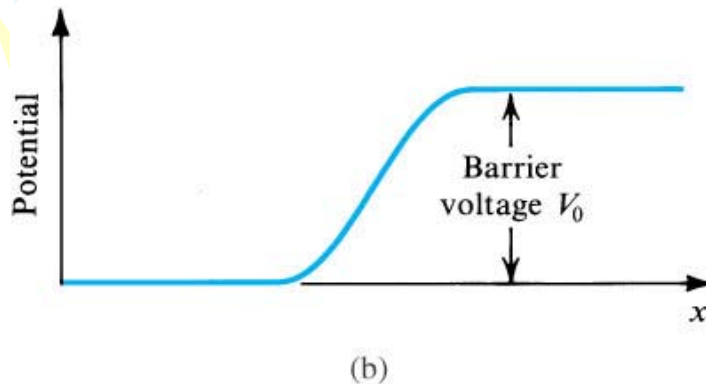
$$\therefore I = 0 \rightarrow I_D = I_S$$

**Contact difference of potential**

$$V_o = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

For silicon at room temperature

$$V_o = 0.6 \sim 0.8V$$



$$J_p = pq\mu_p E - qD_p \frac{dp}{dx}$$

$$J_n = nq\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = 0 \Rightarrow pq\mu_p E = qD_p \frac{dp}{dx}$$

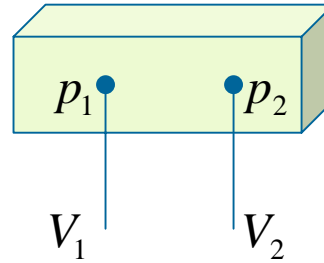
$$E = \frac{D_p}{p\mu_p} \frac{dp}{dx}$$

$$\therefore \frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = V_T$$

$$\Rightarrow E = \frac{V_T}{p} \frac{dp}{dx}$$

$$\therefore E = -\frac{dV}{dx}$$

$$\Rightarrow dV = -V_T \frac{dp}{p}$$



$$\int dV = \int -V_T \frac{1}{p} dp$$

$$\rightarrow V_2 - V_1 = -V_T (\ln p_2 - \ln p_1) = V_T \ln \frac{p_1}{p_2}$$

**Boltzmann equation**  $\frac{V_{21}}{V_T}$   
 $\Rightarrow p_1 = p_2 e^{\frac{V_{21}}{V_T}}$

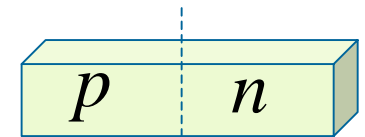
$$J_n = 0 \Rightarrow n_1 = n_2 e^{\frac{-V_{21}}{V_T}}$$

$$V_o = V_{21} = V_T \ln \frac{p_1}{p_2}$$

$$V_o = V_T \ln \frac{N_A N_D}{n_i^2}$$

**Mass-action law**

$$n_1 p_1 = n_2 p_2 = n_i^2$$



$$p_1 = p_{po} = N_A$$

$$p_2 = p_{n0} = \frac{n_i^2}{N_D}$$

# Depletion region

## Poisson equation

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} = -\frac{\rho(x)}{\epsilon}$$

$$E(x) = \int_{-W_p}^{W_n} \frac{\rho(x)}{\epsilon} dx$$

$$V(x) = -\int_{-W_p}^{W_n} E(x) dx$$

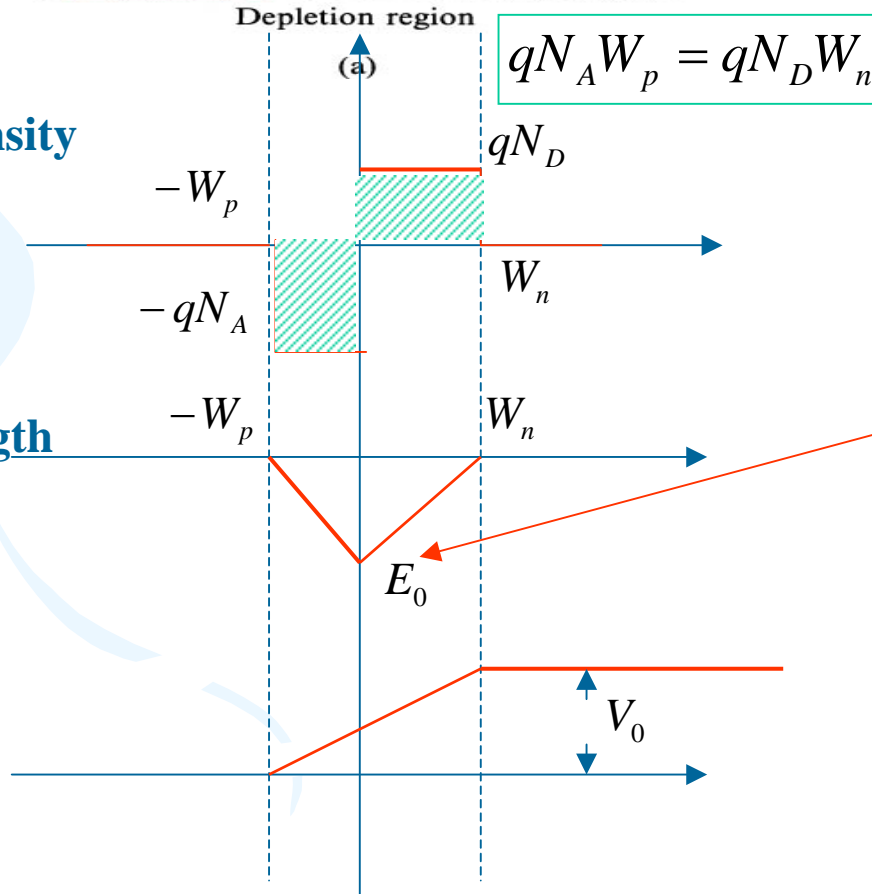
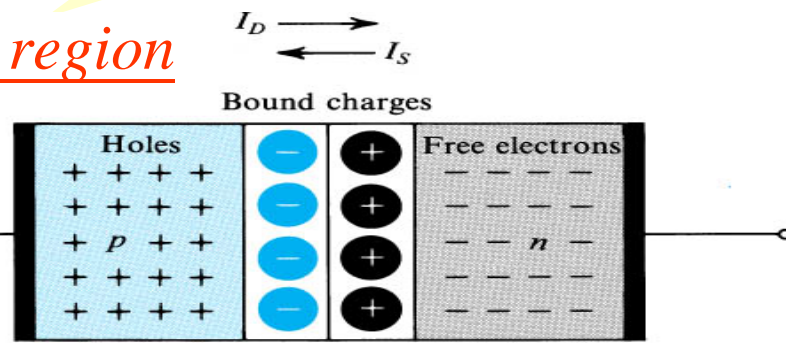
$$E_0 = -\frac{qN_A W_p}{\epsilon} = -\frac{qN_D W_n}{\epsilon}$$

$$\begin{aligned} V_0 &= -\int_{-W_p}^0 E(x) dx - \int_0^{W_n} E(x) dx \\ &= \frac{qN_A W_p^2}{2\epsilon} + \frac{qN_D W_n^2}{2\epsilon} \end{aligned}$$

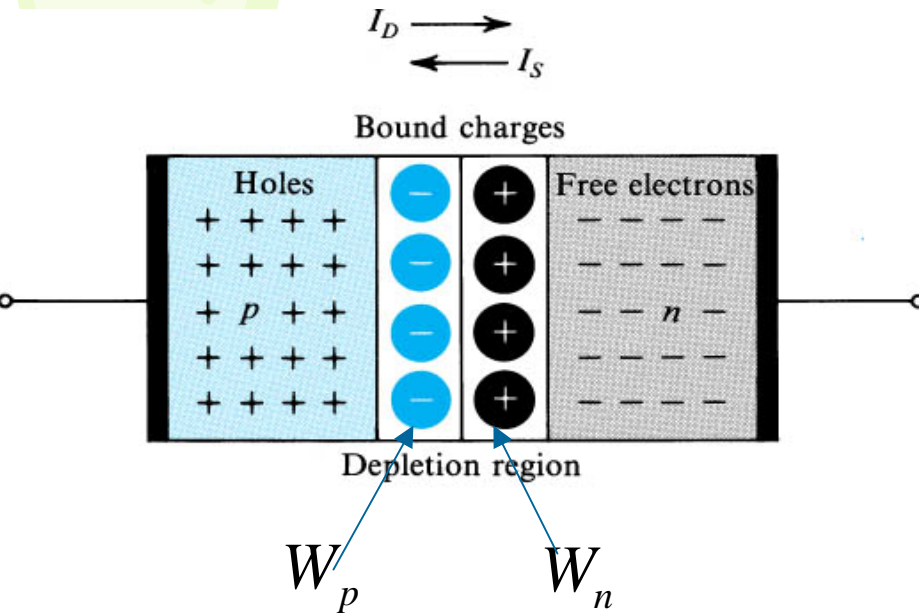
$\rho(x)$   
Charge density

$E(x)$   
Field strength

$V(x)$



## Depletion width



## Charge equality

$$qW_pAN_A = qW_nAN_D$$

$$\Rightarrow \frac{W_n}{W_p} = \frac{N_A}{N_D}$$

$$\Rightarrow W_nN_D = W_pN_A$$

$$W = W_p + W_n \Rightarrow W = \frac{N_D}{N_A}W_n + W_n$$

$$\Rightarrow W_n = \frac{N_A}{N_A + N_D}W \Rightarrow W_p = \frac{N_D}{N_A + N_D}W$$

$$V_0 = \frac{qN_AW_p^2}{2\epsilon} + \frac{qN_DW_n^2}{2\epsilon}$$

$$V_0 = \frac{q}{2\epsilon} \left( \frac{N_A N_D}{N_A + N_D} \right) W^2$$

$$W_{dep} = W_n + W_p = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0}$$

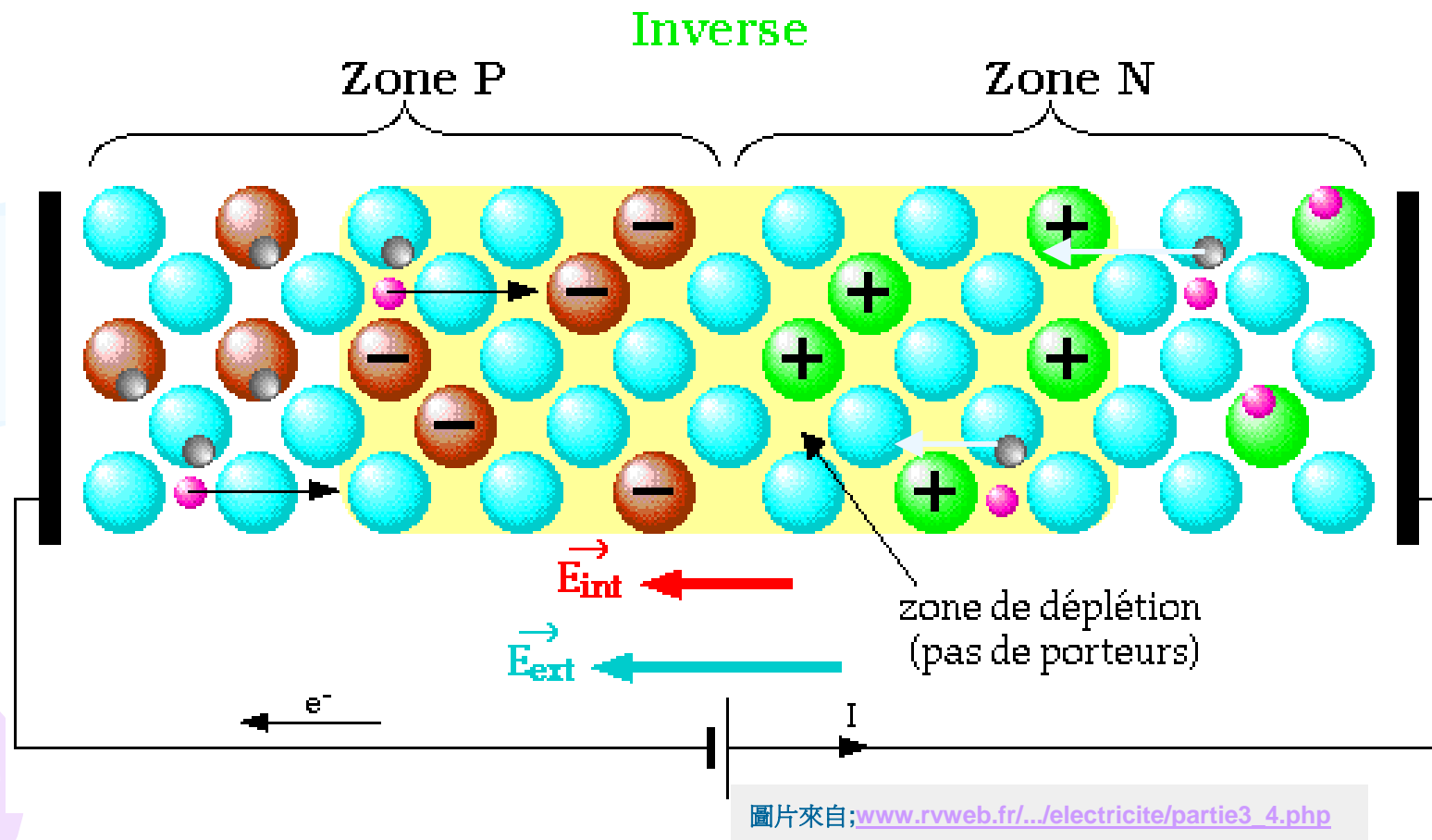
$\epsilon_s$  : Silicon Electrical permirrivivity  
容電係數

For silicon

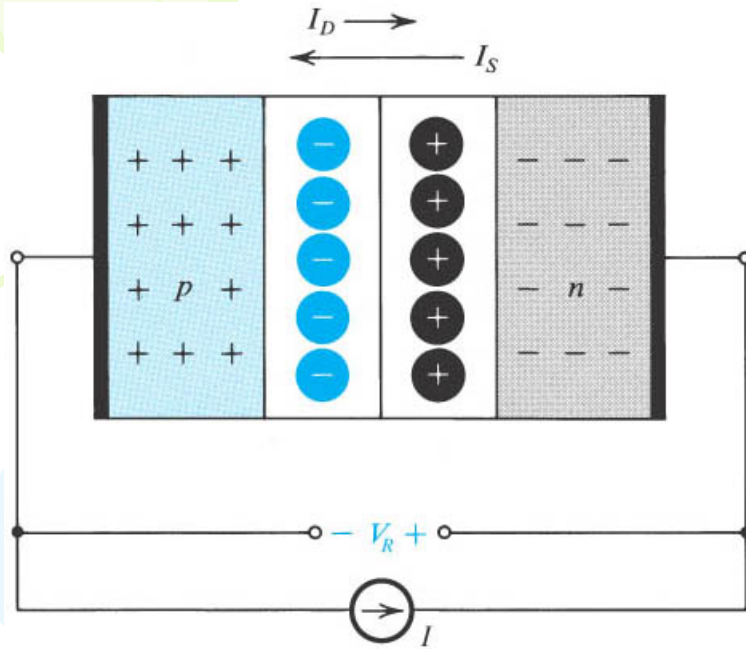
$$\epsilon_s = 1.04 \times 10^{-12} \text{ F/cm}$$

$$W_{dep} = 0.1\mu m \sim 1\mu m$$

# Fonction PN avec champ extérieur



## PN Junction reverse bias



$$I_S > I_D \quad I_S - I_D = I$$

$$q_J = q_N = qN_D W_n A$$

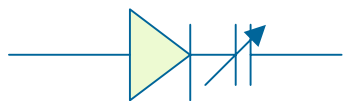
$$q_J = q \frac{N_A N_D}{N_A + N_D} A W_{dep}$$

$$W_{dep} = W_n + W_p = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)}$$

## Junction capacitance

$$C_j = \left. \frac{dq_J}{dV_R} \right|_{V_R=V_0}$$

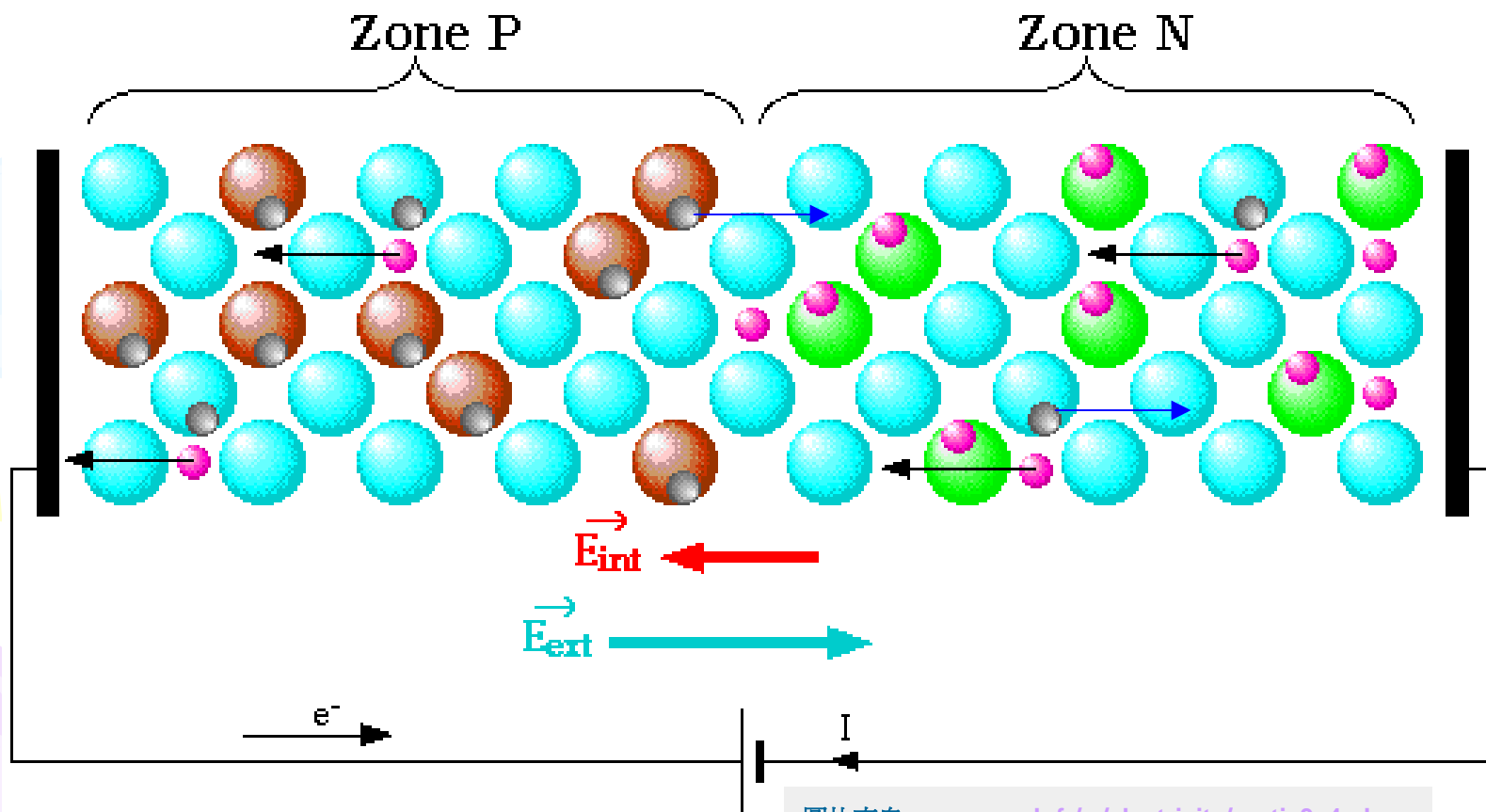
$$\Rightarrow C_j = \frac{\epsilon_s A}{W_{dep}}$$

$$C_j = \frac{C_{jo}}{\sqrt{1 + \frac{V_R}{V_0}}} \quad \text{varactor}$$


$$V_R = 0 \rightarrow C_{jo} = A \sqrt{\frac{q\epsilon_s}{2} \left( \frac{N_A N_D}{N_A + N_D} \right) \frac{1}{V_0}}$$

# *Fonction PN avec champ extérieur*

Direct



圖片來自;[www.rvweb.fr/.../electricite/partie3\\_4.php](http://www.rvweb.fr/.../electricite/partie3_4.php)



# Minority carriers distribution in forward bias (the story about reverse saturation current)

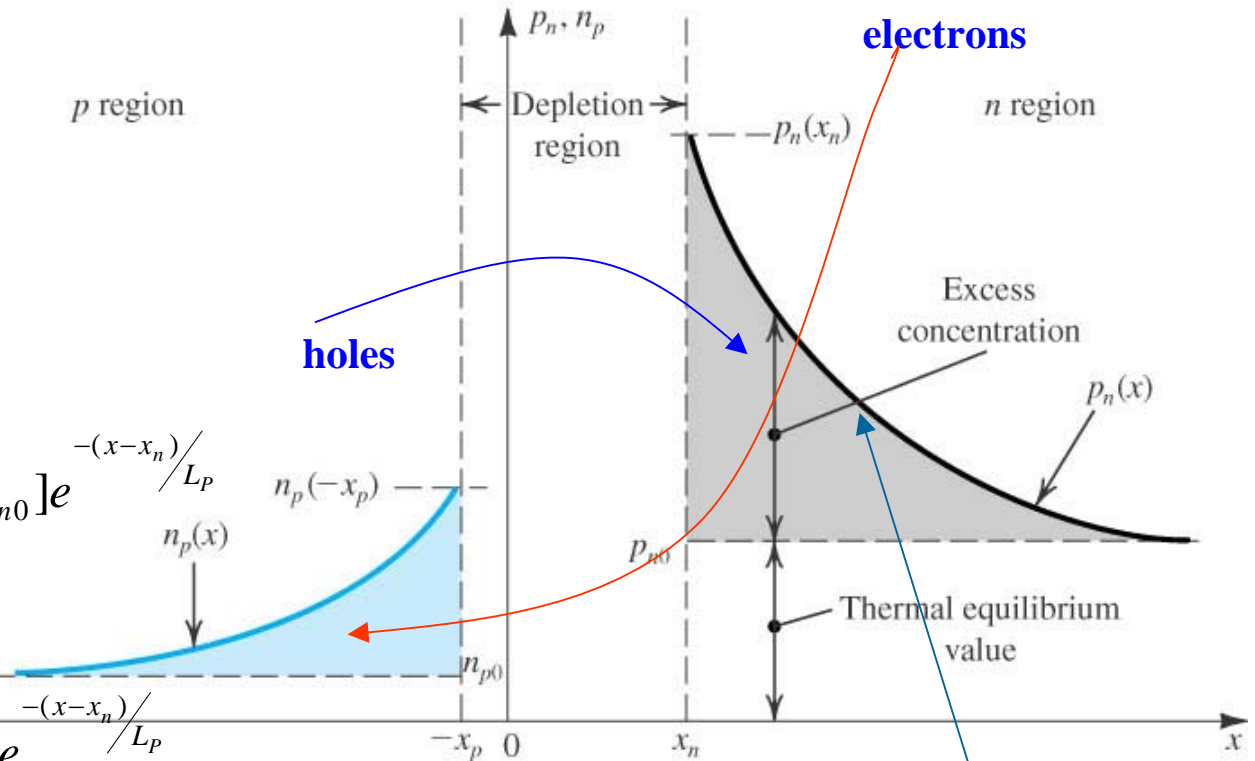
$$p_1 = p_2 e^{\frac{V_{21}}{V_T}} \quad \text{page 18}$$

$$\rightarrow p_n(x_n) = p_{n0} e^{\frac{V}{V_T}}$$

$$p_n(x) = p_{n0} + [p_n(x_n) - p_{n0}] e^{-\frac{(x-x_n)}{L_p}}$$

$$\therefore J_p = -qD_p \frac{dp}{dx}$$

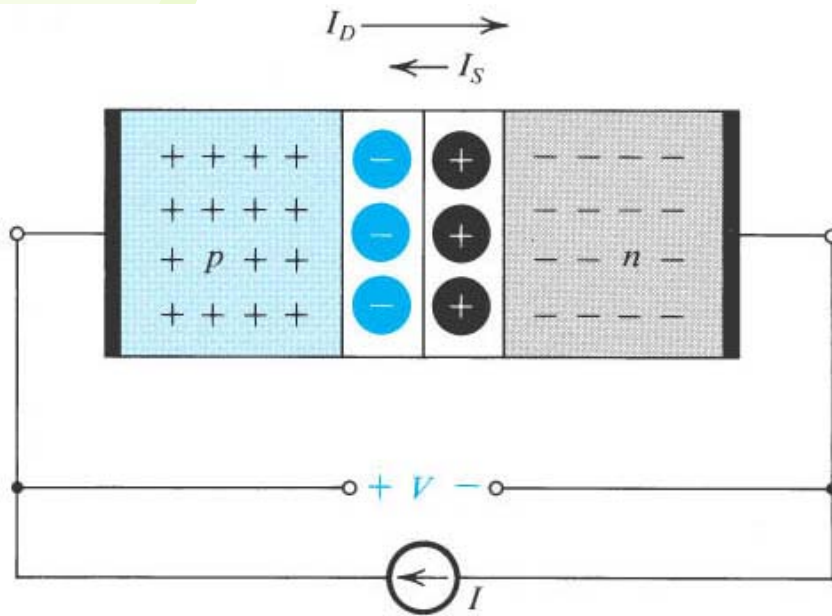
$$\Rightarrow J_p = q \frac{D_p}{L_p} p_{n0} (e^{\frac{V}{V_T}} - 1) e^{-\frac{(x-x_n)}{L_p}}$$



**Total loss by recombination  
= External electric field inject electrons**

**Inject holes from P region to diffuse away from the junction into the N region and disappear by recombination**

# PN Junction under forward bias



$$J_p = q \frac{D_p}{L_p} p_{no} (e^{v/v_T} - 1)$$

$$J_n = q \frac{D_n}{L_n} n_{po} (e^{v/v_T} - 1)$$

$$I = A(J_p + J_n)$$

$$I = Aqn_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_p}{L_p N_D} \right) (e^{v/v_T} - 1)$$

$$I_s = Aqn_i^2 \left( \frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$$

$$I = I_s (e^{v/v_T} - 1)$$

$$I_S < I_D$$

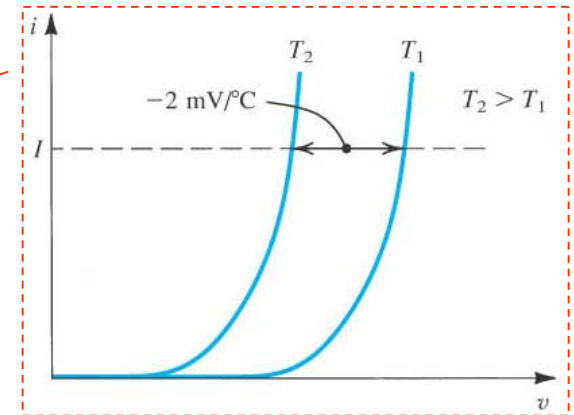
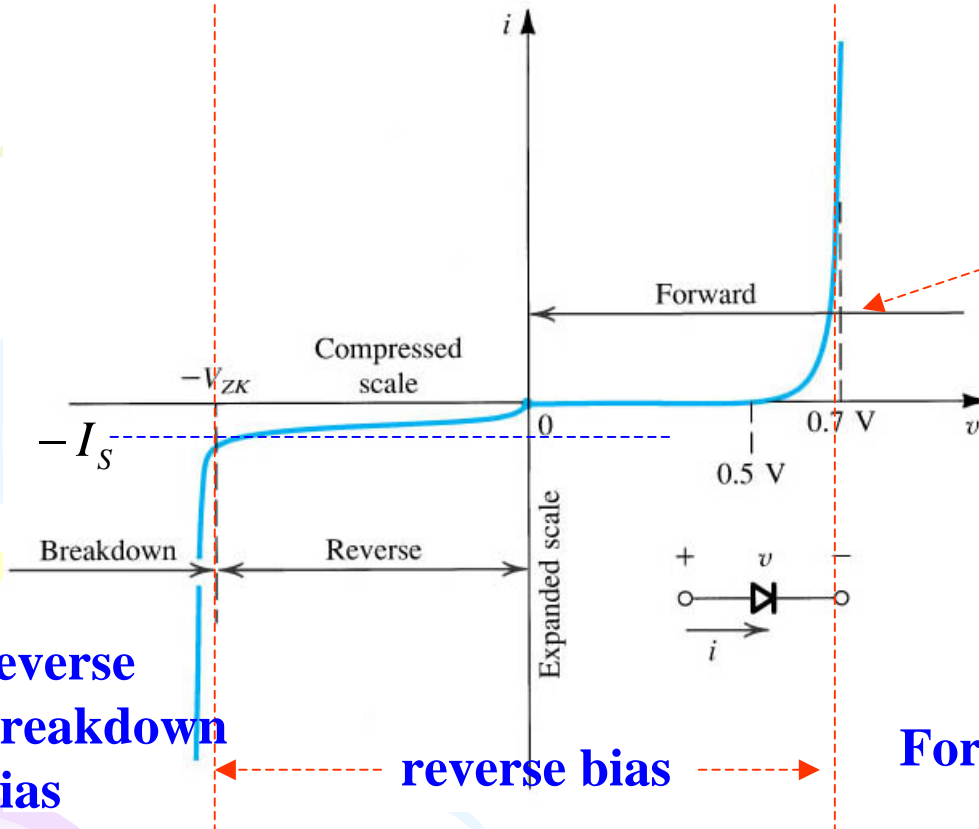
Reverse saturation current

$$I = I_s (e^{v/v_T} - 1)$$

if  $V \ll 0$

# Diode's $i$ - $v$ relationship

$$i = I_s (e^{v/nV_T} - 1)$$



$n = 1, 2$  Ideality factor which depending on diode's material and physical structure

## Forward bias

$$i = I_s (e^{v/nV_T} - 1)$$

$$i \gg I_s \longrightarrow \text{Forward}$$

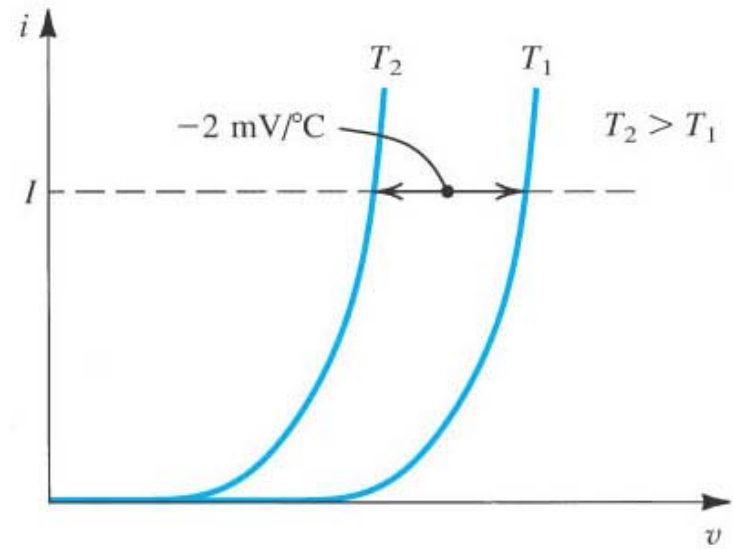
$$\Rightarrow i \approx I_s e^{v/nV_T}$$

$$\Rightarrow v = nV_T \ln \frac{i}{I_s}$$

$$I_1 \approx I_s e^{V_1/nV_T}$$

$$I_2 \approx I_s e^{V_2/nV_T}$$

$$\Rightarrow V_2 - V_1 = nV_T \ln \frac{I_2}{I_1} = 2.3nV_T \log \frac{I_2}{I_1}$$



$n$  : Ideality factor which depending on diode's material and physical structure

$$n = \begin{cases} 1 & Ge \\ 1 & Si(I_D \geq 25mA) \\ 2 & Si(I_D \leq 25mA) \end{cases}$$

Open current  $\longrightarrow V_D = 0 \Rightarrow I_D = 0$

Forward bias  $\longrightarrow V_D > 0 \Rightarrow I_D \approx I_s e^{\frac{V_D}{nV_T}}$

reverse bias  $\longrightarrow V_D < 0 \Rightarrow I_D = -I_s$

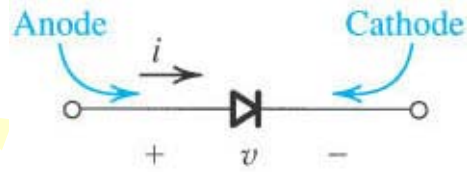
Cut-in voltage  $V_r$

$V_r = 0.7 \rightarrow Si$

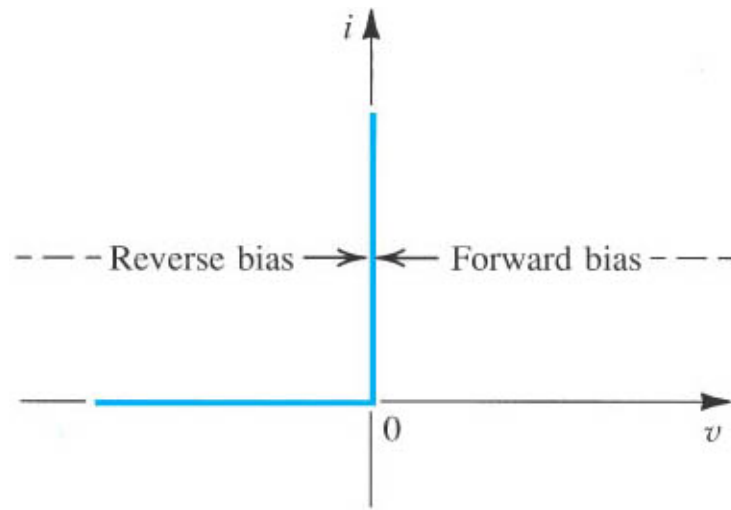
$V_r = 0.25 \rightarrow Ge$

$V_r = 1.2 \rightarrow GaAs$

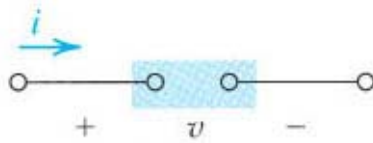
# Diode Model I (ideal model)



(a)

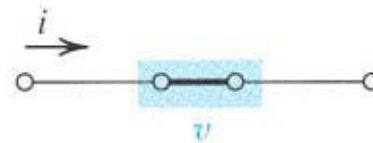


(b)



$$v < 0 \Rightarrow i = 0$$

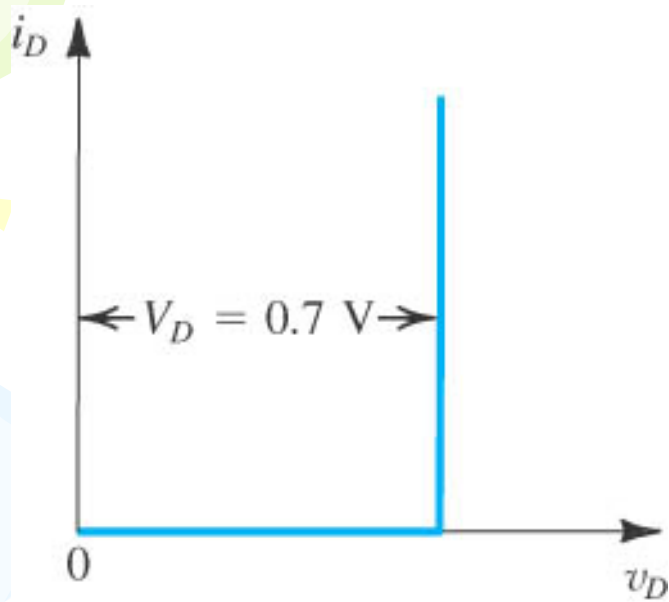
(c)



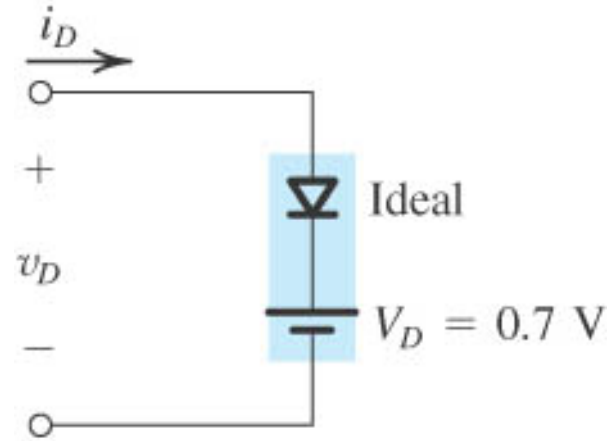
$$i > 0 \Rightarrow v = 0$$

(d)

## Diode Model II (constant-Voltage Drop model )

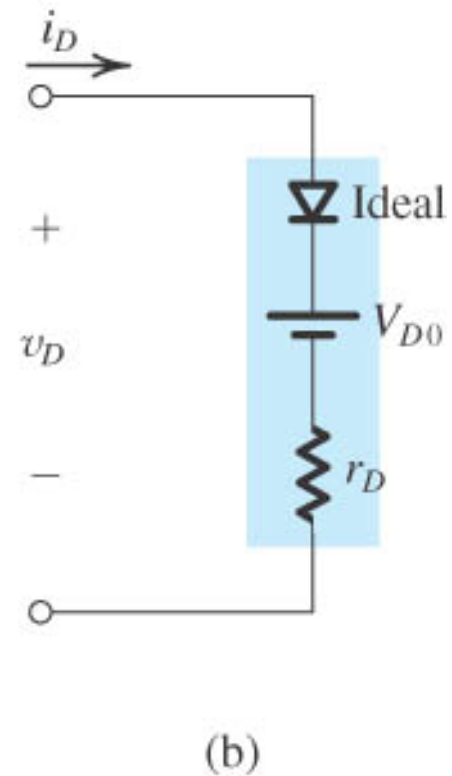
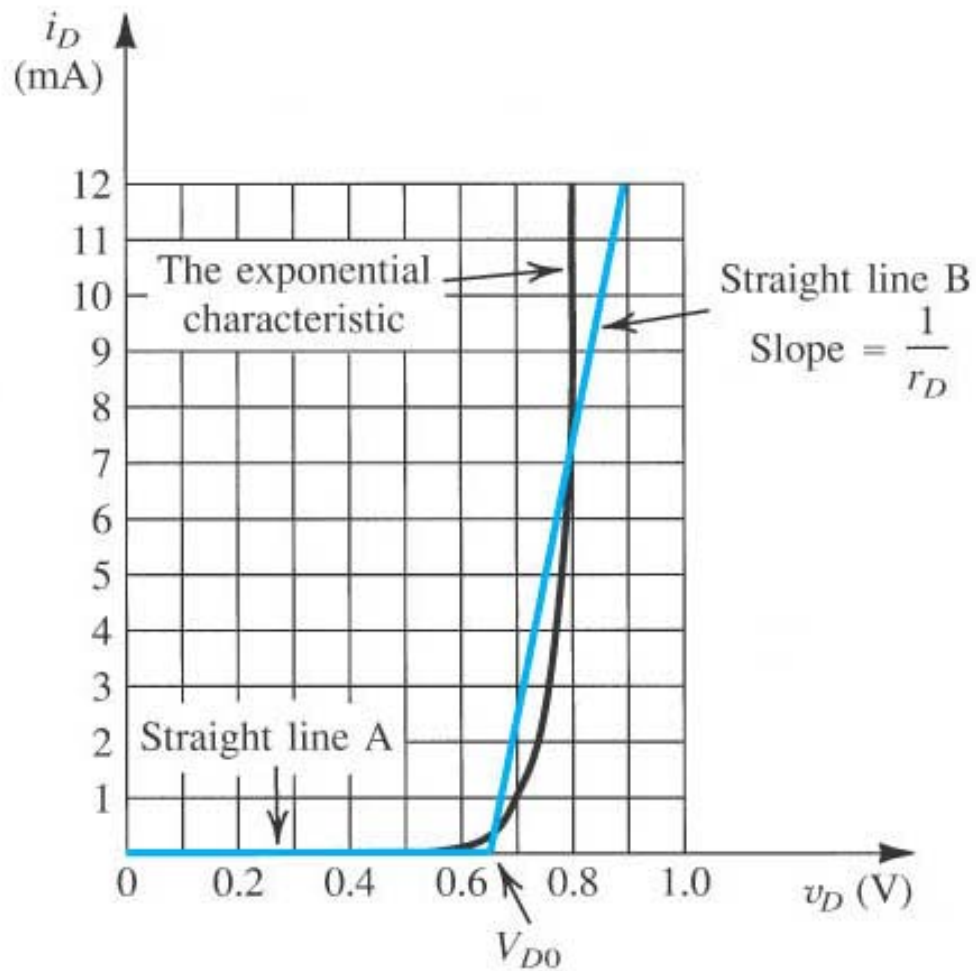


(a)

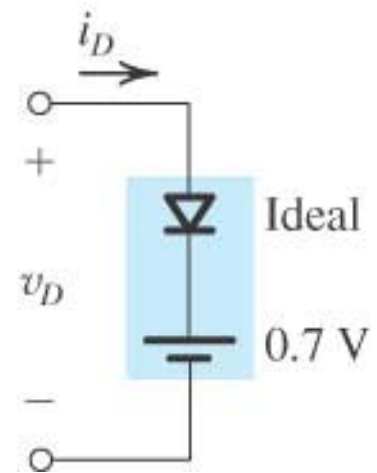
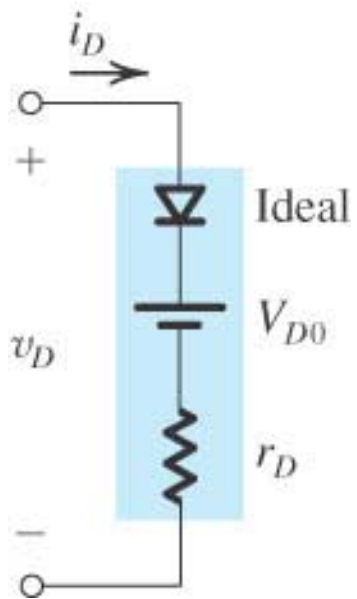
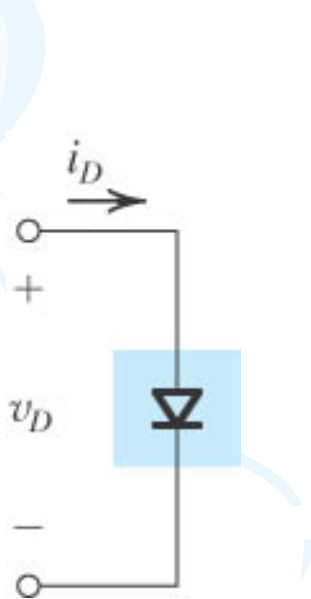
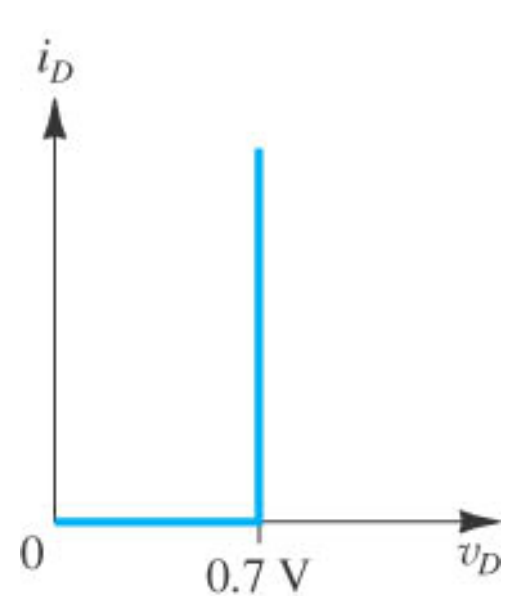
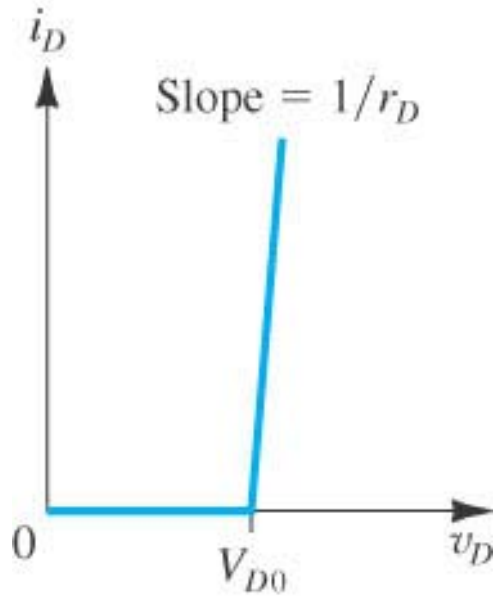
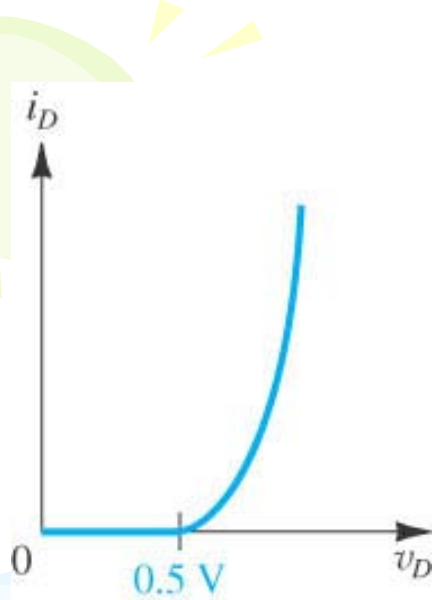


(b)

## Diode Model III (Piecewise-linear model )

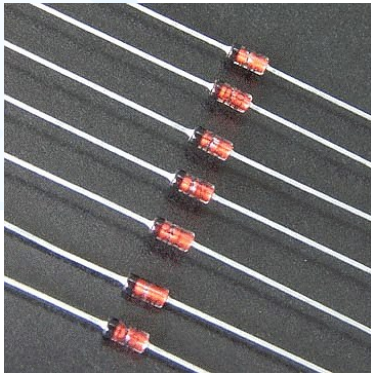
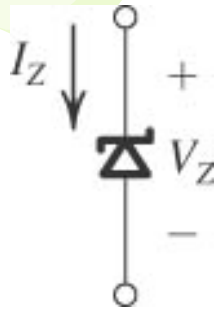




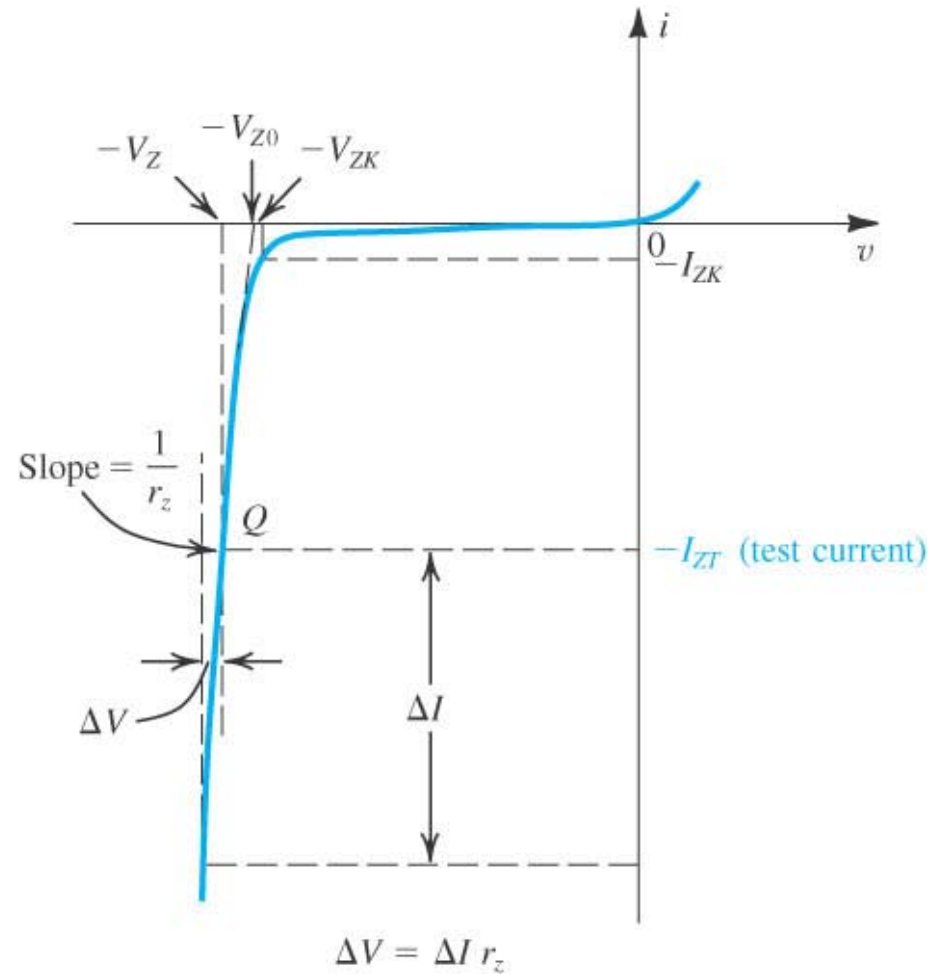
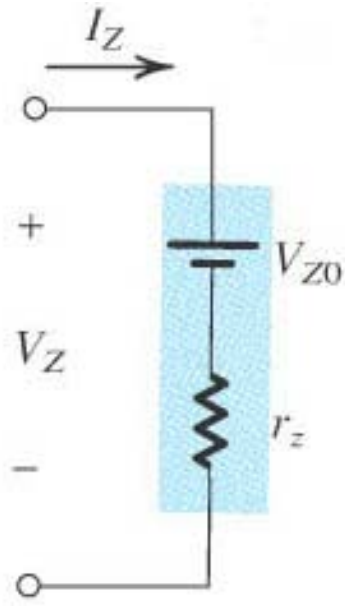


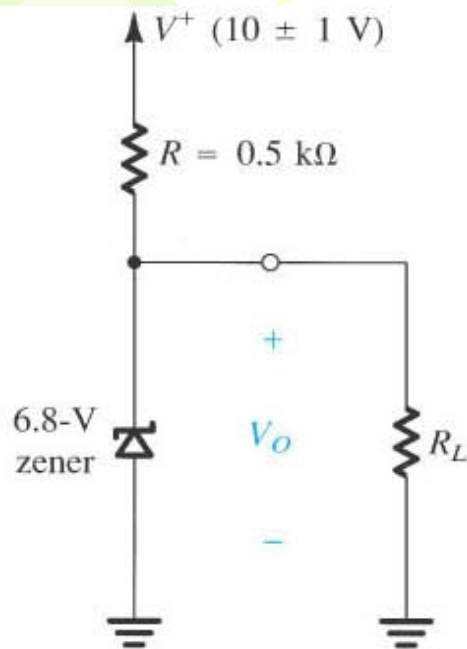
**Table 3.1** Modeling the Diode Forward Characteristic

# Special Diodes (Zener Diode)

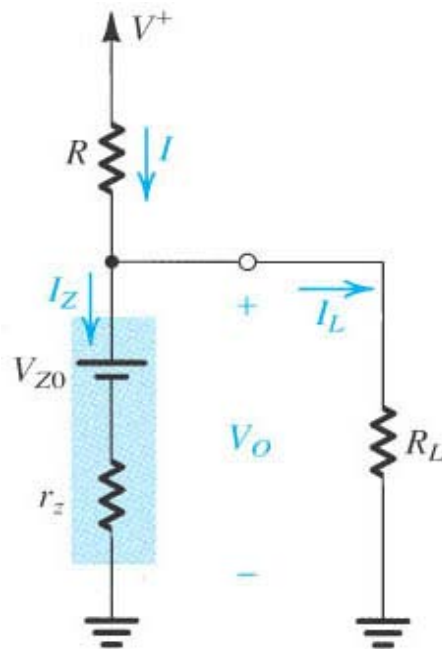


圖片來自: [www.alibaba.com/catalog/11418809/0\\_5w\\_Series](http://www.alibaba.com/catalog/11418809/0_5w_Series)





(a)



(b)

$$I = I_z + I_L$$

$$\Rightarrow \frac{V - V_z}{R} = \frac{P_z}{V_z} + \frac{V_z}{R_L}$$



$$I = I_{z(\max)} + I_{L(\min)}$$

$$\Rightarrow \frac{V - V_z}{R} = \frac{P_{z(\max)}}{V_z} + \frac{V_z}{R_{L(\max)}}$$

$$I = I_{z(\min)} + I_{L(\max)}$$

$$\Rightarrow \frac{V - V_z}{R} = \frac{P_{z(\min)}}{V_z} + \frac{V_z}{R_{L(\min)}}$$

**Ideal case:**  $r_z = 0$

$R_L \uparrow (\max) \rightarrow I_L \downarrow \because R = K \therefore I = \text{fixed} \Rightarrow I_z \uparrow (\max)$

$R_L \downarrow (\min) \rightarrow I_L \uparrow \because R = K \therefore I = \text{fixed} \Rightarrow I_z \downarrow (\min)$

## The others Special Diodes

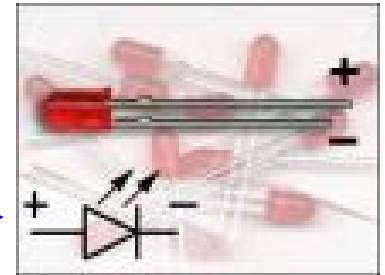
### 1. Schottky Diode

- Metal + semiconductor (unipolar)



### 2. Light emitting diode (LED)

- GaAs diode
- GaN diode (B, G)



[www.personeel.glr.nl/koster/elektro/ledjes.JPG](http://www.personeel.glr.nl/koster/elektro/ledjes.JPG)

### 3. Tunnel diode



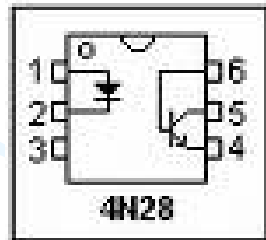
[hyperphysics.phy-astr.gsu.edu/.../tundio.html](http://hyperphysics.phy-astr.gsu.edu/.../tundio.html)

### 4. Photo diode (PD)

### 5. Opto coupler (LED + PD)



[home.swipnet.se/.../hifi\\_100pr/photodiode.jpg](http://home.swipnet.se/.../hifi_100pr/photodiode.jpg)



[www.du.edu/~etuttle/electron/circ43.gif](http://www.du.edu/~etuttle/electron/circ43.gif)



[www.geda.seul.org/.../analog/photodiode-1\\_tn.png](http://www.geda.seul.org/.../analog/photodiode-1_tn.png)

# LED

- 1963 Red
- 1993 Blue
- 1996 white

2006