Lecture 10

Frequency response

topics

- Bode diagram
- BJT's Frequency response
- MOSFET Frequency response

$$v_i(t) = V_i \sin \omega t$$

Amplifier

$$v_o(t) = V_o \sin(\omega t + \phi)$$

Magnitude:
$$\frac{V_o}{V_i}$$
 Phase: ϕ

$$T(s) = \frac{V_o(s)}{V_i(s)}$$

 $T(s) = \frac{V_o(s)}{V_i(s)} \qquad s = \sigma + j\omega \implies s = j\omega$ Steady-state response

$$T(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

$$rac{\left|V_{o}(j\omega)
ight|}{\left|V_{i}(j\omega)
ight|}$$

Magnitude:
$$\frac{|V_o(j\omega)|}{|V_i(j\omega)|}$$
 Phase: $\frac{\angle V_o(j\omega)}{\angle V_i(j\omega)}$

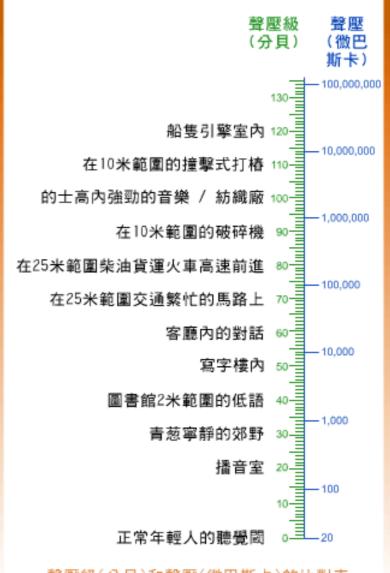
dB (decibel)= Decimal + Bell

$$dB \equiv 10\log_{10}\frac{p_1}{p_2}$$

$$\therefore p = i^2 z = \frac{v^2}{z}$$

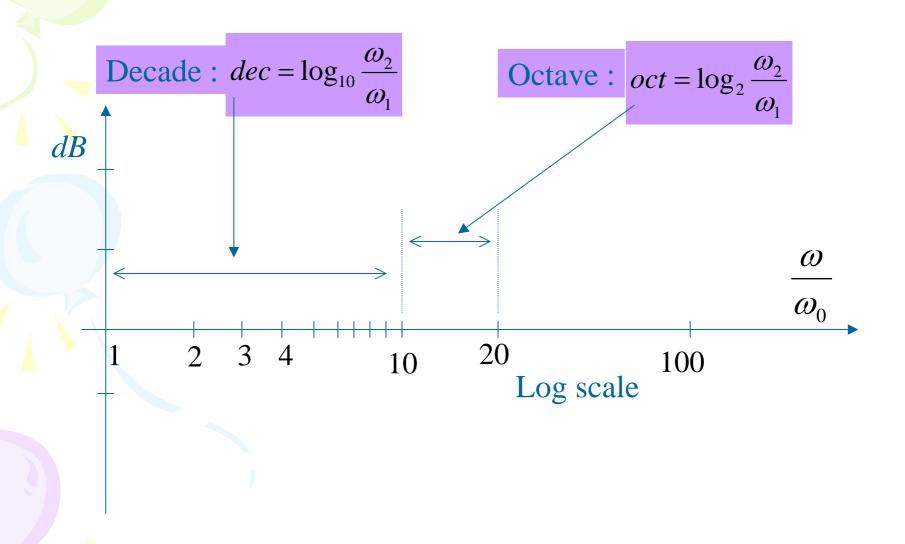
$$\Rightarrow dB = 20\log_{10}\frac{v_1}{v_2} = 20\log_{10}\frac{i_1}{i_2}$$

Human hearing frequency zone : 10Hz~24kHz Most Sensitive frequency zone : 2kHz~5kHz



聲壓級(分貝)和聲壓(微巴斯卡)的比對表

Logarithmic coordinate



Example(low pass)

$$V_o = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_i \Rightarrow T(\omega) = \frac{1}{1 + (j\omega)RC}$$

let
$$\omega_0 = \frac{1}{RC} \Rightarrow T(\omega) = \frac{1}{1 + (j\omega)RC} = \frac{1}{1 + (\frac{j\omega}{\omega_0})}$$

if
$$\omega = \omega_0 \Rightarrow |T(\omega)| = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \angle T(\omega) = 0 - \tan^{-1} 1 = -45^{\circ}$$

$$T(\omega) = \frac{1}{1 + (\frac{s}{\omega_0})} = \frac{1}{1 + (\frac{j\omega}{\omega_0})}$$

Magnitude:

$$\left| (1+j\frac{\omega}{\omega_0})^{-1} \right|_{dB} = -20\log\sqrt{1+(\frac{\omega}{\omega_0})^2} \qquad \omega >> \omega_0 \Rightarrow 1+j\frac{\omega}{\omega_0} \approx \frac{\omega}{\omega_0}$$

$$=-10\log[1+(\frac{\omega}{\omega_0})^2] \qquad \Rightarrow dB \approx -20\log\frac{\omega}{\omega_0}$$

$$dB = -[20\log\omega - 20\log\omega_0]$$

$$\omega << \omega_0 \Rightarrow \frac{\omega}{\omega_0} \approx 0$$

$$\Rightarrow dB = -10\log 1 = 0$$

$$dB = -[20\log \omega - 20\log \omega_0]$$

$$\omega = \omega_0 \Rightarrow 1 + j1 \Rightarrow dB = -10\log 2 = -3.01$$

$$\Rightarrow aB = -10\log 1 = 0$$

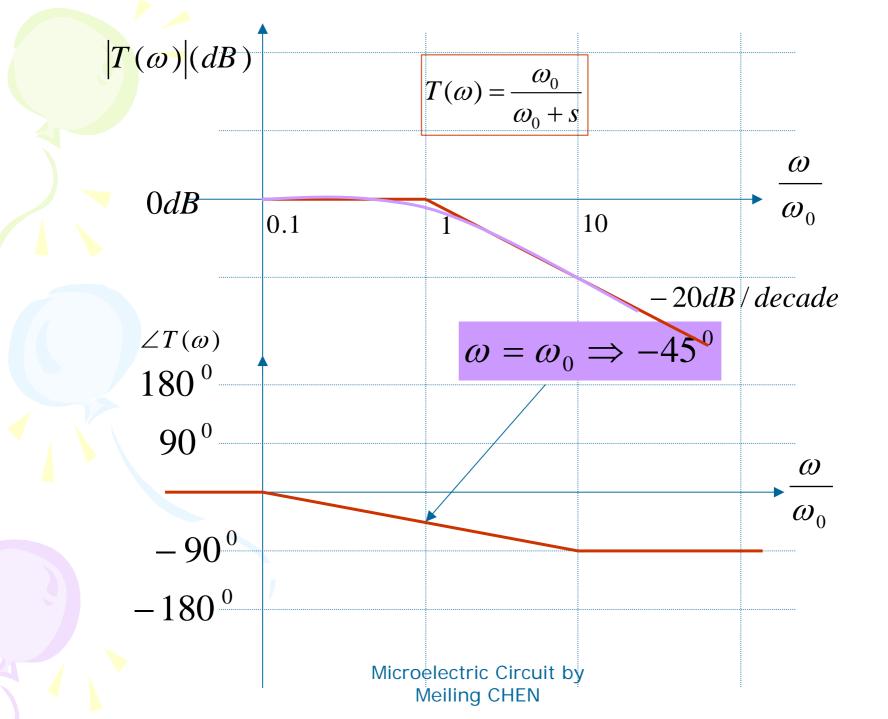
Phase:
$$\angle (1+j\frac{\omega}{\omega_0}) = 0^0 - \tan^{-1}\frac{\omega}{\omega_0}$$

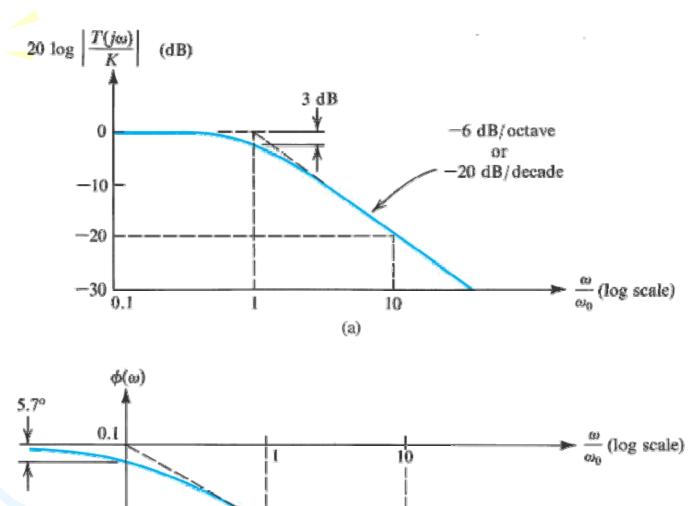
$$\omega \ll \omega_0 \Rightarrow \frac{\omega}{\omega_0} \approx 0 \Rightarrow \angle T(\omega) \approx \tan^{-1} 0 = 0^{\circ}$$

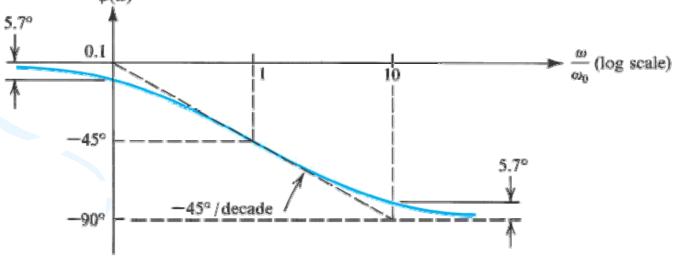
$$\omega >> \omega_0 \Rightarrow \frac{\omega}{\omega_0} \approx \infty \Rightarrow \angle T(\omega) \approx -\tan^{-1} \infty = -90^{\circ}$$

Microelectric Circuit by

Meiling CHEN







Example(high pass)

$$V_i$$
 C
 V_o
 C

$$V_o = \frac{R}{R + \frac{1}{sC}} V_i \Rightarrow T(\omega) = \frac{j\omega RC}{1 + (j\omega)RC}$$

let
$$\omega_0 = \frac{1}{RC} \Rightarrow T(\omega) = \frac{j\omega RC}{1 + (j\omega)RC} = \frac{(\frac{j\omega}{\omega_0})}{1 + (\frac{j\omega}{\omega_0})}$$

if
$$\omega = \omega_0 \Rightarrow |T(\omega)| = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \angle T(\omega) = 90 - \tan^{-1} 1 = 45^{\circ}$$

$$T(s) = \frac{s}{s + \omega_0}$$
 $T(\omega) = \frac{j\omega}{j\omega + \omega_0}$

Magnitude:

$$\left| \frac{j\omega}{j\omega + \omega_0} \right|_{m} = 20\log \omega - 20\log \sqrt{\omega^2 + \omega_0^2}$$

$$= 20\log\omega - 10\log[\omega^2 + \omega_0^2]$$

$$\omega >> \omega_0 \Rightarrow 20 \log \frac{1}{1 + \frac{\omega_0}{i\omega}}$$

$$\Rightarrow$$
 $-20\log(1+\frac{\omega_0}{i\omega})$

$$\Rightarrow dB = -20 \log 1 = 0$$

Phase:

$$\angle \frac{j\omega}{j\omega + \omega_0} = 90 - \tan^{-1} \frac{\omega}{\omega_o}$$

$$\omega >> \omega_0 \Rightarrow \frac{\omega}{\omega_0} \approx \infty \Rightarrow \angle T(\omega) \approx 90 - \tan^{-1} \infty = 0^{\circ}$$

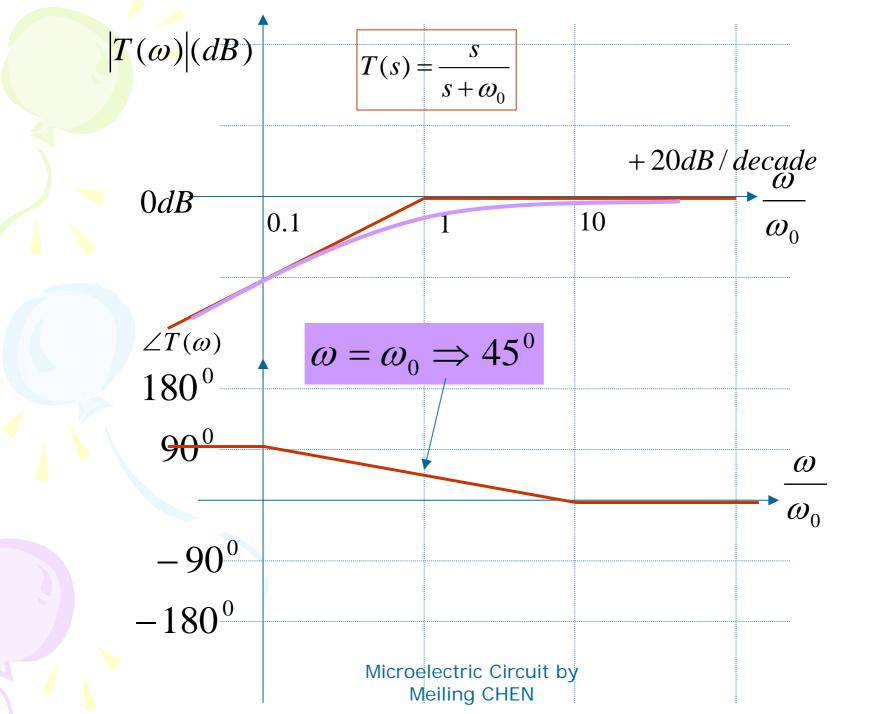
$$\omega \ll \omega_0 \Rightarrow \frac{\omega}{\omega_0} \approx 0 \Rightarrow \angle T(\omega) \approx 90 - \tan^{-1} 0 = 90^\circ$$

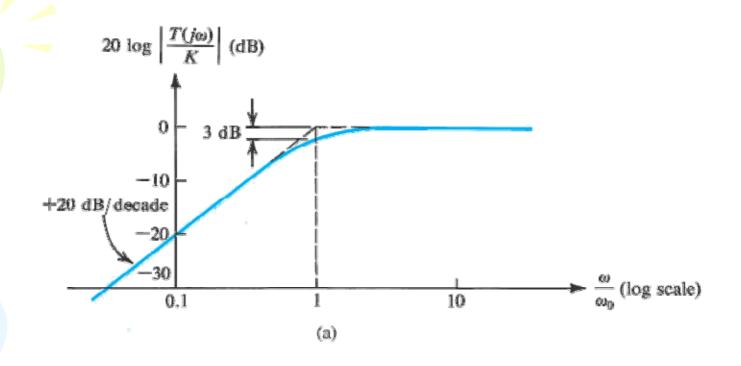
$$\omega \ll \omega_0 \Rightarrow dB \approx 20 \log \frac{\omega}{\omega_0}$$

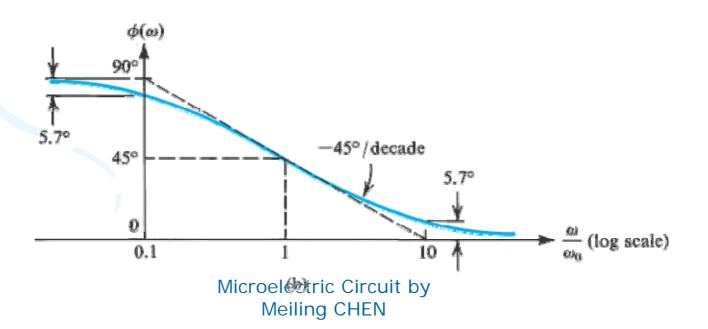
$$dB = 20\log\omega - 20\log\omega_0$$

$$\omega = 0.1\omega_0 \Rightarrow dB = 20\log 0.1 = -20$$

$$\omega = \omega_0 \Rightarrow \frac{1}{1+i1} \Rightarrow dB = -20\log\sqrt{2} = -3.01$$







$$dB \equiv 20\log |T(\omega)|$$

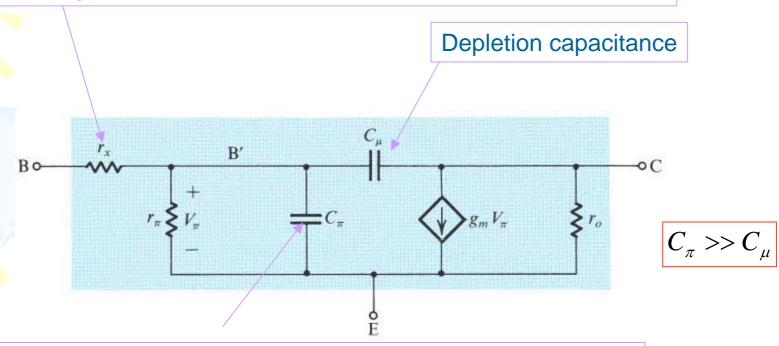
The bandwidth represents the distance between the two points in the frequency domain where the signal is $\frac{1}{\sqrt{2}}$ of the maximum signal strength.

$$20\log|T(\omega)|$$
if $|T(\omega)| = \frac{1}{\sqrt{2}}$

$$\Rightarrow 20\log|T(\omega)| = 10\log 2 = 3dB$$

BJT high frequency model

Splitting resistance (refinement the lumped-component circuit)



Emitter-base capacitance

= diffusion capacitance + Base-Emitter junction capacitance

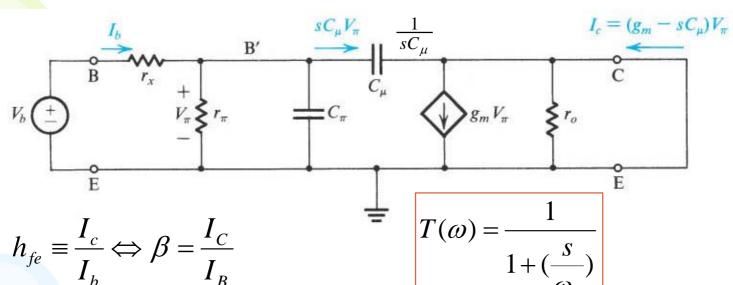
KTC9013 Technical data

ELECTRICAL CHARACTERISTICS (Ta=25°C)

CHARACTERISTIC	SYMBOL	TEST CONDITION	MIN.	TYP.	MAX.	UNIT
Collector Cut-off Current	I _{CBO}	V_{CB} =35V, I_{E} =0	-	_	0.1	μΑ
Emitter Cut-off Current	I_{EBO}	V_{EB} =5V, I_{C} =0	_	_	0.1	μΑ
DC Current Gain	h _{FE} (Note)	V _{CE} =1V, I _C =50mA	64	-	246	
Collector-Emitter Saturation Voltage	V _{CE(sat)}	I _C =100mA, I _B =10mA	-	0.1	0.25	V
Base-Emitter Voltage	V_{BE}	$I_C=100 \text{mA}, V_{CE}=1 \text{V}$	_	0.8	1.0	V
Transition Frequency	f_{T}	V _{CE} =6V, I _C =20mA, f=100MHz	140	_	_	MHz
Collector Output Capacitance	Cob	V_{CB} =6 V , I_{E} =0, f =1 M H z	-	7.0	-	pF

Note: h_{FE} Classification D:64~91, E:78~112, F:96~135, G:118~166, H:144~202, I:176~246

How to find C_{π} by datasheet ?



$$h_{fe} \equiv \frac{I_c}{I_b} \Leftrightarrow \beta = \frac{I_C}{I_B}$$

$$I_c = g_m V_{\pi} - \frac{V_{\pi}}{\frac{1}{sC_{\mu}}} = (g_m - sC_{\mu})V_{\pi}$$

$$I_b = \frac{V_{\pi}}{(r_{\pi} // C_{\pi} // C_{\mu})}$$

$$I_b = \frac{1}{(r_\pi // C_\pi // C_\mu)}$$

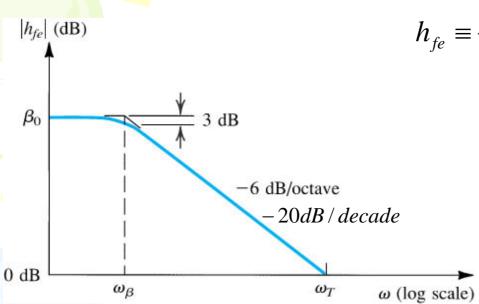
$$h_{fe} \equiv \frac{I_c}{I_b} = \frac{g_m - sC_\mu}{1/r_\pi + s(C_\pi + C_\mu)}$$
 Low frequency ß

3-dB frequency

$$\Rightarrow h_{fe} \approx \frac{g_{m}r_{\pi}}{1 + s(C_{\pi} + C_{\mu})r_{\pi}} = \frac{\beta_{0}}{1 + s(C_{\pi} + C_{\mu})r_{\pi}}$$

$$= \frac{\beta_{0}}{1 + s(C_{\pi} + C_{\mu})r_{\pi}}$$
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 $\omega_{\beta} = \frac{1}{(C_{\pi} + C_{\mu})r_{\pi}}$



 f_T

$$h_{fe} \equiv \frac{I_c}{I_b} = \frac{\beta_0}{1 + s(C_{\pi} + C_{\mu})r_{\pi}} \Leftrightarrow \frac{1}{1 + s/\omega_{\beta}}$$

$$20\log\beta - 20\log\sqrt{1 + (\omega_T/\omega_\beta)^2} = 0$$

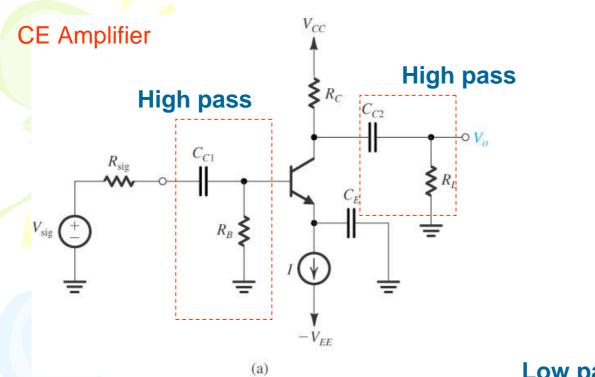
$$\beta = \sqrt{1 + (\omega_T / \omega_\beta)^2}$$

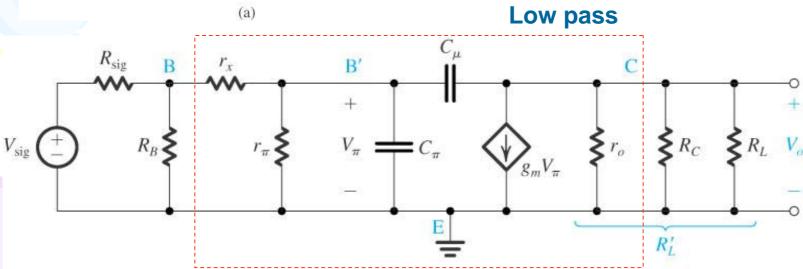
$$\omega_T \approx \omega_\beta \beta :: \beta >> 1$$

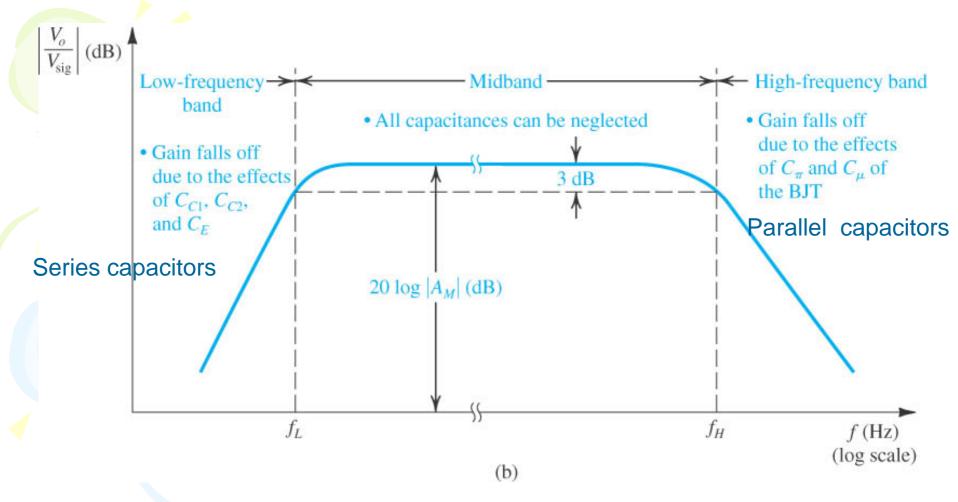


$$\omega_T \approx \omega_\beta \beta = \frac{g_m}{C_\pi + C_\mu}$$

$$f_T = \frac{g_m}{2\pi (C_{\pi} + C_{\mu})}$$



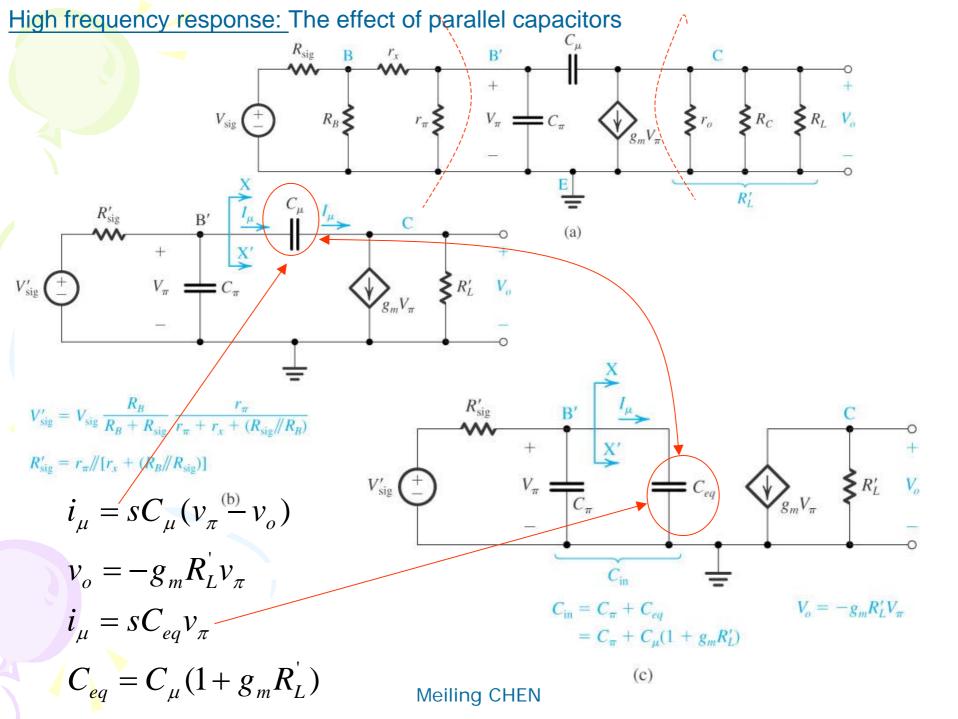




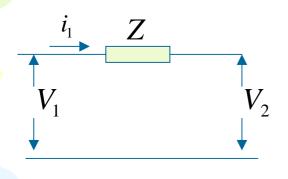
Bandwidth
$$BW \equiv f_H - f_L \approx f_H (:: f_L << f_H)$$

Midband amplitude
$$A_{M} = \frac{v_{o}}{v_{sig}} = -\frac{(R_{B} // r_{\pi})}{(R_{B} // r_{\pi}) + R_{s}} g_{m} (r_{o} // R_{C} // R_{L})$$

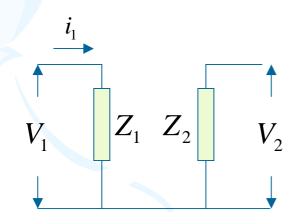
Microelectric Circuit by Meiling CHEN



Miller's theorem



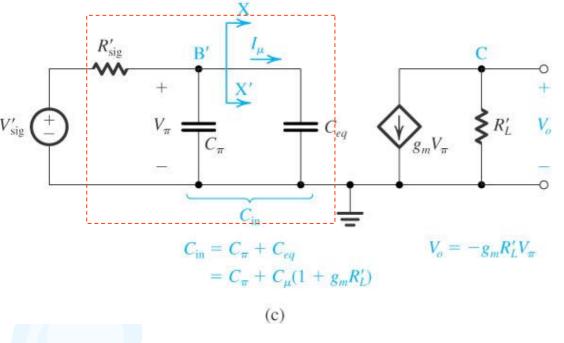
$$\frac{V_2}{V_1} = k$$

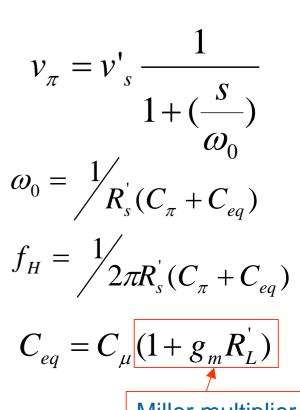


$$i_{1} = \frac{V_{1} - V_{2}}{Z} = \frac{V_{1}}{Z_{1}}$$

$$\frac{V_{2}}{V_{1}} = k \Rightarrow Z_{1} = \frac{Z}{1 - k}$$

$$Z_{2} = \frac{k}{k - 1} Z$$





Miller multiplier

$$\frac{V_o}{V_{\rm sig}} \left| \text{ (dB)} \right|$$

$$\frac{V_o}{V_{\rm sig}} \left| \text{ (dB)} \right|$$

$$\frac{-6 \text{ dB/octave}}{-20 \text{ dB/decade}}$$

$$\frac{-20 \log |A_M|}{f_H} = \frac{1}{2\pi C_{\rm in} R_{\rm sig}'}$$

$$f_H = \frac{1}{2\pi C_{\rm in} R_{\rm sig}'}$$

(d)

Example 5.18 Find the Midband gain and upper 3 dB frequency of CE amplifier.

The parameters of Hybrid π model

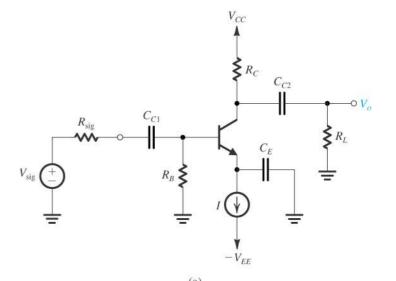
$$g_{m} = \frac{I_{C}}{V_{T}} = 40mA/V$$

$$r_{\pi} = \frac{V_{T}}{I_{B}} = 2.5k\Omega$$

$$r_{o} = \frac{V_{A}}{I_{C}} = 100k\Omega$$

$$C_{\pi} + C_{\mu} = \frac{g_{m}}{\omega_{T}} = 8pF = C_{\pi} + 1$$

$$C_{\pi} = 7pF$$



$$V_{CC} = V_{EE} = 10V$$
 $I = 1mA$
 $R_B = 100k\Omega, R_C = 8k\Omega$
 $R_s = 5k\Omega, R_L = 5k\Omega$
 $\beta = 100, V_A = 100V$
 $C_\mu = 1pF, r_x = 50\Omega$
 $f_T = 800Mhz$

24

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$$A_{M} = \frac{v_{o}}{v_{sig}} = -\frac{R_{B}}{R_{B} + R_{s}} \frac{r_{\pi}}{(R_{B} // R_{s}) + r_{\pi} + r_{x}} g_{m}(r_{o} // R_{C} // R_{L}) = -39$$

$$C_{eq} = C_{\mu}(1 + g_{m}R_{L}^{'})$$

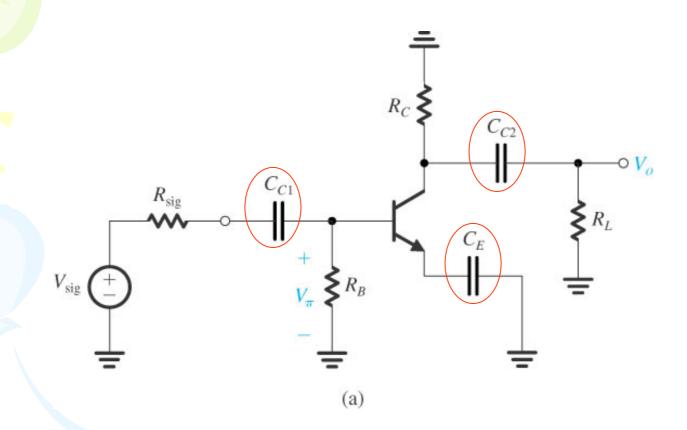
$$C_{in} = C_{\pi} + C_{eq} = 128 \, pF$$

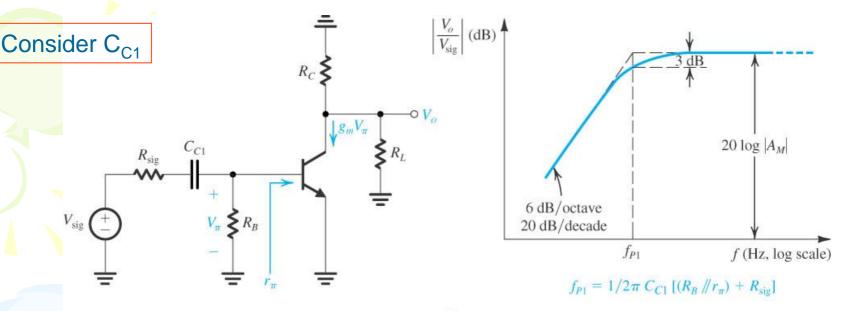
$$R_{s}' = r_{\pi} / [r_{s} + (R_{R} / / R_{s})] = 1.65 k\Omega$$

$$\omega_0 = \frac{1}{R_s'}(C_{\pi} + C_{eq})$$

$$f_H = \frac{1}{2\pi R_s} (C_{\pi} + C_{eq}) = 754kHZ$$

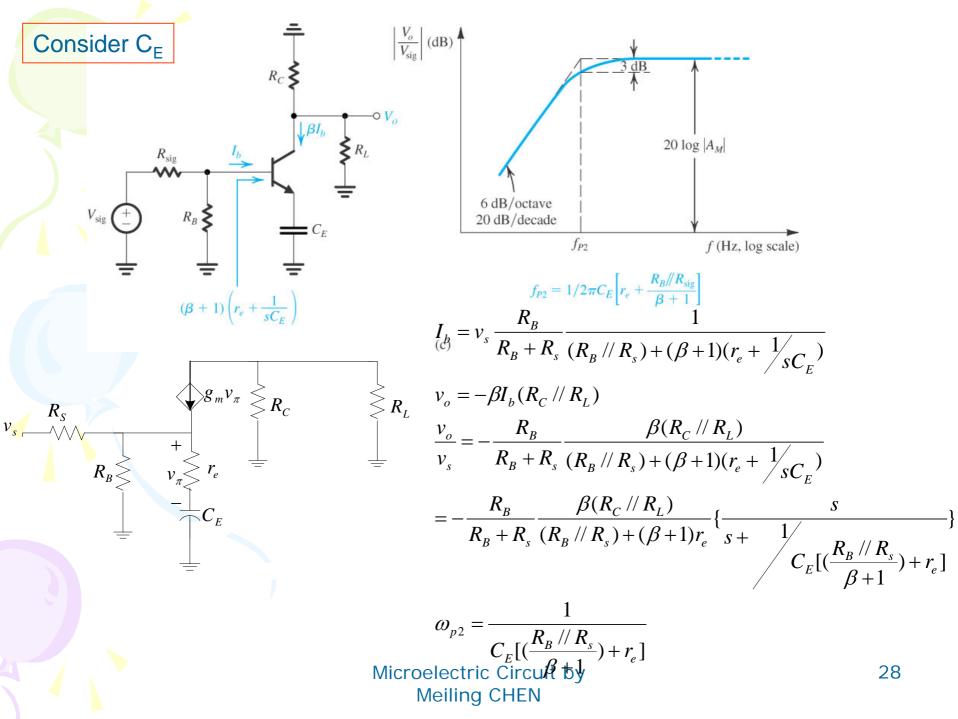
Low frequency response: The effects of the series capacitors

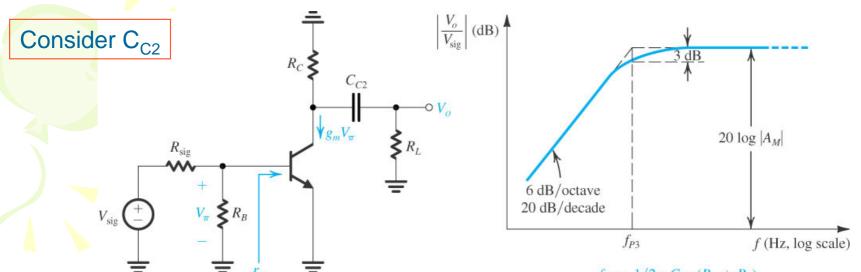




$$\omega_{p1} = \frac{1}{C_{C1}[(R_B // r_{\pi}) + R_s]}$$
 Microelectric Circuit by Meiling CHEN

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$$f_{P3} = 1/2\pi C_{C2} (R_C + R_L)$$

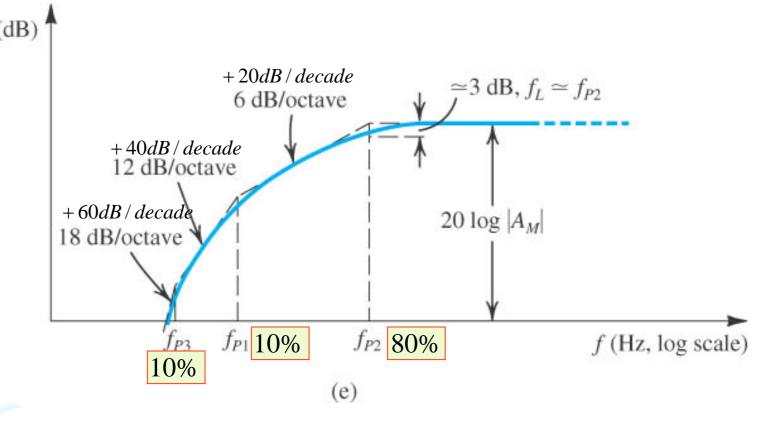
$$v_{\pi} = v_{s} \frac{R_{B} // r_{\pi}}{(R_{B} // r_{\pi}) + R_{s}}$$

$$v_{o} = -g_{m} v_{\pi} \frac{R_{C}}{R_{C} + R_{L} + \frac{1}{sC_{C2}}} R_{L}$$

$$\frac{v_o - g_m v_\pi}{R_C + R_L + \frac{1}{sC_{C2}}} R_L$$

$$\frac{v_o}{v_s} = \frac{-(R_B // r_\pi)}{(R_B // r_\pi) + R_s} g_m (R_C // R_L) \frac{s}{s + \frac{1}{C_{C2}(R_C + R_L)}}$$

$$\omega_{p3} = \frac{1}{C_{C2}(R_C + R_L)}$$



$$\frac{v_o}{v_s} = -A_M \left(\frac{s}{s + \omega_{p1}}\right) \left(\frac{s}{s + \omega_{p2}}\right) \left(\frac{s}{s + \omega_{p3}}\right)$$

$$\omega_{p1} = \frac{1}{C_{C1}[(R_B // r_{\pi}) + R_s]}$$

$$R_{C1}$$

$$\omega_{p2} = \frac{1}{C_E[(\frac{R_B // R_s}{\beta + 1}) + r_e]}$$
dominant R_E

$$\omega_{p3} = \frac{1}{C_{C2}(R_C + R_L)}$$

$$R_{C2}$$

Example 5.19 Find the C_{C1} , C_{C2} and C_{E} the CE amplifier of the example 5.18.

$$\omega_{p1} = \frac{1}{C_{C1}[(R_B // r_{\pi}) + R_s]}$$

$$\omega_{p2} = \frac{1}{C_E[(\frac{R_B // R_s}{\beta + 1}) + r_e]}$$

$$\omega_{p3} = \frac{1}{C_{C2}(R_C + R_L)}$$

$$C_E[(\frac{R_B/R_s}{\beta+1}) + r_e] = 0.8 \times 2\pi f_L \Rightarrow C_E = 27.6 \mu F$$

$$C_{C1}[(R_B // r_{\pi}) + R_s] = 0.1 \times 2\pi f_L \implies C_{C1} = 2.1 \mu F$$

$$C_{C2}(R_C + R_L) = 2\pi\omega f_L \Rightarrow C_{C2} = 1.2\mu F$$

$$V_{CC} = V_{FE} = 10V$$

$$I = 1mA$$

$$R_B = 100k\Omega, R_C = 8k\Omega$$

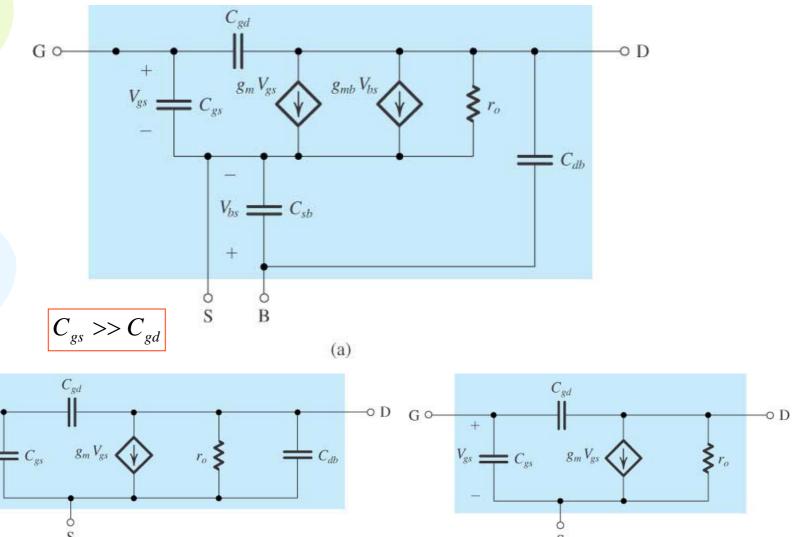
$$R_s = 5k\Omega, R_L = 5k\Omega$$

$$\beta = 100, g_m = 40$$

$$r_{\pi} = 2.5k\Omega$$

$$f_L = 100hz$$

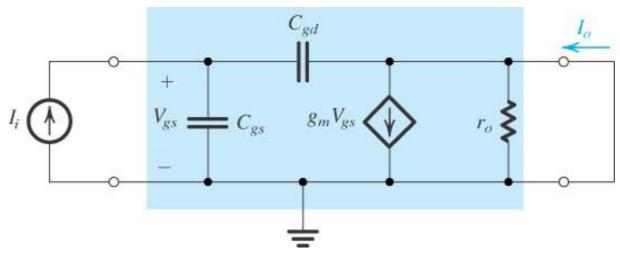
MOSFET high frequency model



(b)

G O

(c)



$$I_o = g_m v_{gs} - sC_{gd} v_{gs} \approx g_m v_{gs}$$

$$v_{gs} = \frac{I_i}{s(C_{gs} + C_{gd})}$$

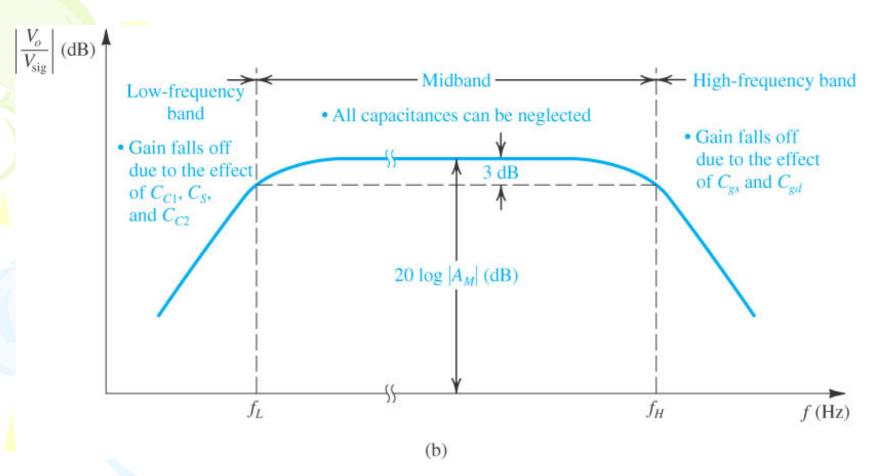
$$\frac{I_o}{I_i} = \frac{g_m}{s(C_{gs} + C_{gd})} = \frac{g_m}{j\omega(C_{gs} + C_{gd})}$$

if
$$\omega = \frac{g_m}{(C_{gs} + C_{gd})} \Rightarrow \frac{I_o}{I_i} = 1 \Rightarrow 0dB$$

$$\omega_T = \frac{g_m}{(C_{gs} + C_{gd})}$$

$$f_T = \frac{g_m}{2\pi (C_{gs} + C_{gd})}$$

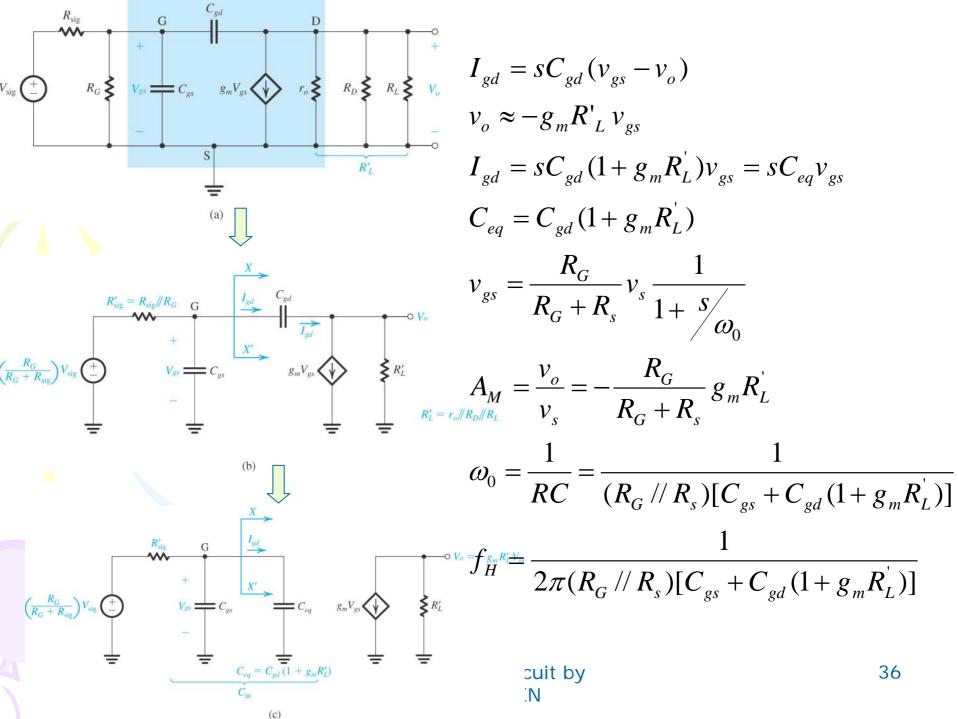
High frequency response: The effect of parallel capacitors R_D C_{C2} C_{C1} $R_{\rm sig}$ V_{sig} $R_{\rm sig}$ G R_G R_D $V_{\rm sig}$ (R_L S R'_L (a)

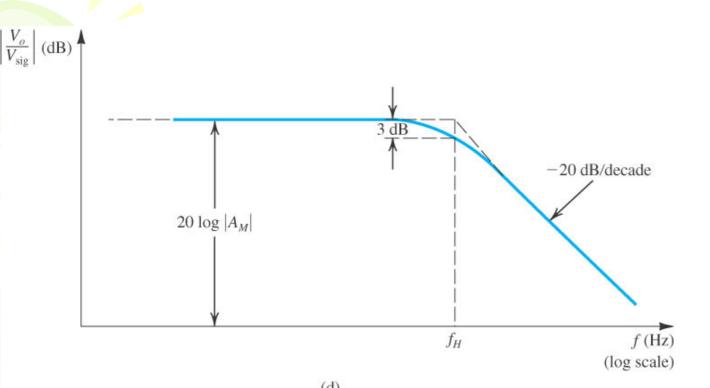


Bandwidth
$$BW \equiv f_H - f_L \approx f_H (:: f_L << f_H)$$

$$A_{M} = \frac{v_{o}}{v_{sig}} = -\frac{R_{G}}{R_{G} + R_{s}} g_{m} (r_{o} // R_{D} // R_{L})$$

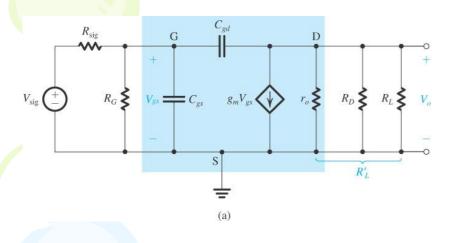
Microelectric Circuit by Meiling CHEN

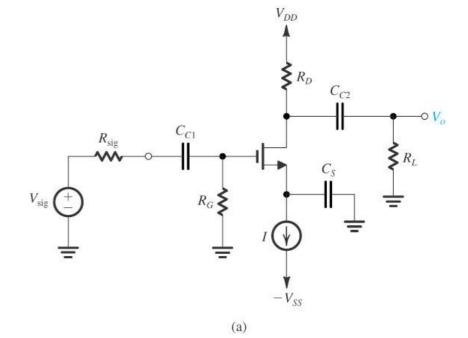




$$\begin{split} A_{M} &= -\frac{R_{G}}{R_{G} + R_{s}} g_{m} R_{L}^{'} \\ \omega_{0} &= \frac{1}{RC} = \frac{1}{(R_{G} /\!/ R_{s})[C_{gs} + C_{gd} (1 + g_{m} R_{L}^{'})]} \\ f_{H} &= \frac{1}{2\pi (R_{G} /\!/ R_{s})[C_{gs} + C_{gd} (1 + g_{m} R_{L}^{'})]} \\ &\stackrel{\text{Microelectric Circuit by}}{\text{Meiling CHEN}} \end{split}$$

Example 4.12 Find the Midband gain and upper 3 dB frequency of CS amplifier.





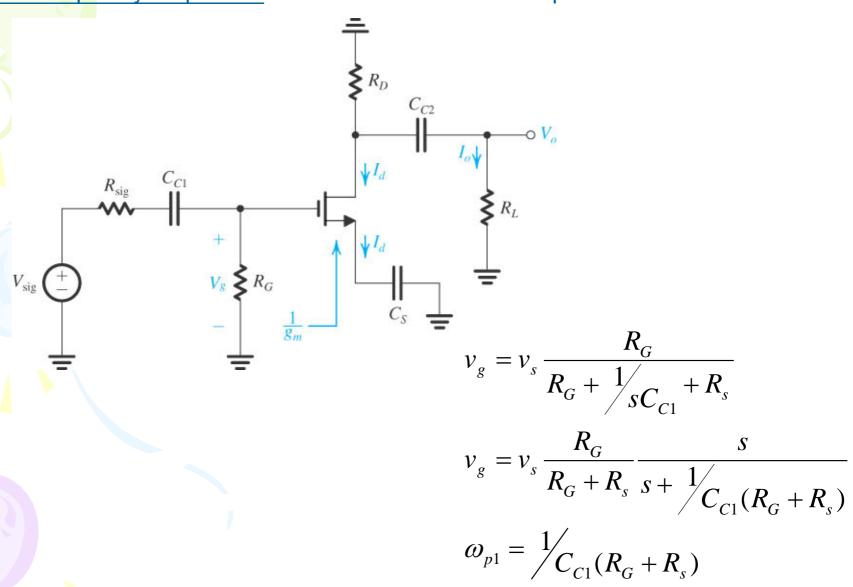
$$A_{M} = -\frac{R_{G}}{R_{G} + R_{s}} g_{m} R_{L}$$

$$\omega_0 = \frac{1}{RC} = \frac{1}{(R_G // R_s)[C_{gs} + C_{gd}(1 + g_m R_L^{'})]}$$

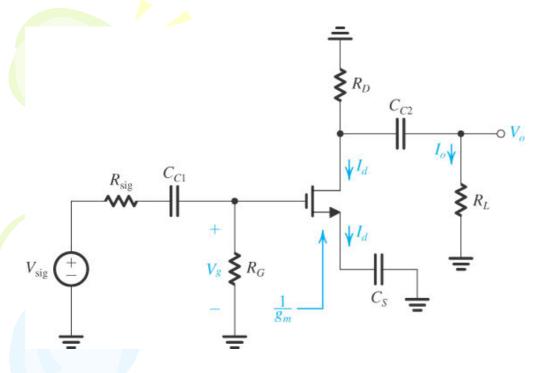
$$f_{H} = \frac{1}{2\pi (R_{G} // R_{s})[C_{gs} + C_{gd}(1 + g_{m}R_{L}^{'})]}$$

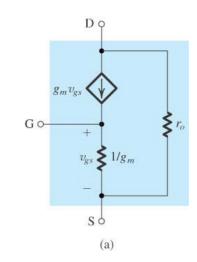
$$\begin{split} I &= mA \\ R_S &= 100k\Omega, R_G = 4.7M\Omega \\ R_D &= 15k\Omega, R_L = 15k\Omega \\ g_m &= 1, r_0 = 150k\Omega \\ C_{gs} &= 1pF, C_{gd} = 0.4pF \end{split}$$

Low frequency response: The effects of the series capacitors



Microelectric Circuit by Meiling CHEN





$$i_d = \frac{v_g}{\frac{1}{g_m} + \frac{1}{sC_s}}$$

$$=g_{m}v_{g}\frac{s}{s+g_{m}/C_{s}}$$

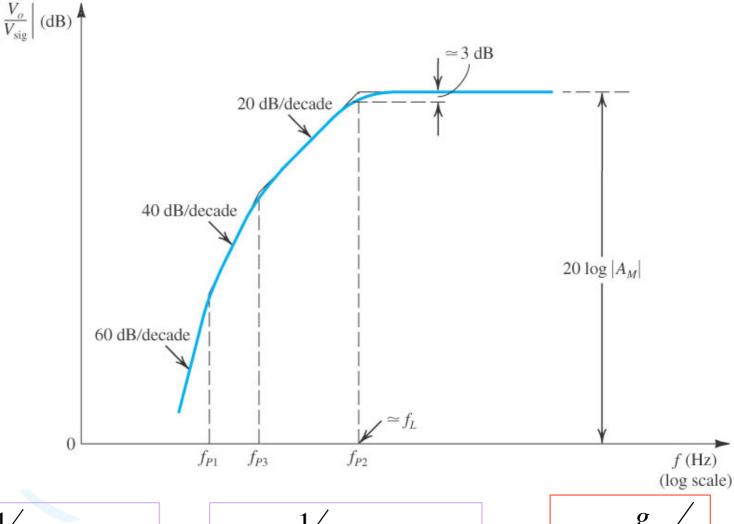
$$\omega_{p3} = \frac{g_m}{C_s}$$

$$i_o = -i_d \frac{R_D}{R_D + 1/sC_{C2} + R_L}$$

$$v_o = i_o R_L = -i_d (R_D // R_L) \frac{s}{s + 1/C_{C2} (R_D + R_L)}$$

$$\omega_{p2} = \frac{1}{C_{C2}(R_D + R_L)}$$

Microelectric Circuit by Meiling CHEN



$$\omega_{p1} = \frac{1}{C_{C1}}(R_G + R_s)$$

$$\omega_{p3} = \frac{1}{C_{C2}}(R_D + R_L)$$

$$\omega_{p2} = \frac{g_m}{C_s}$$

$$\frac{v_o}{v_s} = -\frac{R_G}{R_G + R_s} g_m (R_D /\!/ R_L) (\frac{s}{s + \omega_{p1}}) (\frac{s}{s + \omega_{p2}}) (\frac{s}{s + \omega_{p3}})$$
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Example 4.13 Find the C_{C1} , C_{C2} and C_s the CS amplifier of the example 4.12.

$$f_L = f_{p2} = \frac{1}{2\pi \left(\frac{C_s}{g_m}\right)}$$

$$\Rightarrow C_s = \frac{g_m}{2\pi f_I} = 1.6 \mu F$$

$$f_{p1} = 10Hz = \frac{1}{2\pi C_{C1}}(R_G + R_s)$$

$$\Rightarrow C_{C1} = \frac{1}{2\pi f_{p1}}(R_G + R_s) = 3.3nF$$

$$f_{p3} = 10Hz = \frac{1}{2\pi C_{C2}}(R_D + R_L)$$

$$C_{C2} = \frac{1}{2\pi f_L}(R_D + R_L) = 0.53\mu F$$

$$I = mA$$

$$R_S = 100k\Omega, R_G = 4.7M\Omega$$

$$R_D = 15k\Omega, R_L = 15k\Omega$$

$$g_m = 1$$

$$C_{gs} = 1pF, C_{gd} = 0.4pF$$

$$f_L = 100Hz$$

$$f_{p1} = f_{p3} = \frac{100Hz}{10}$$