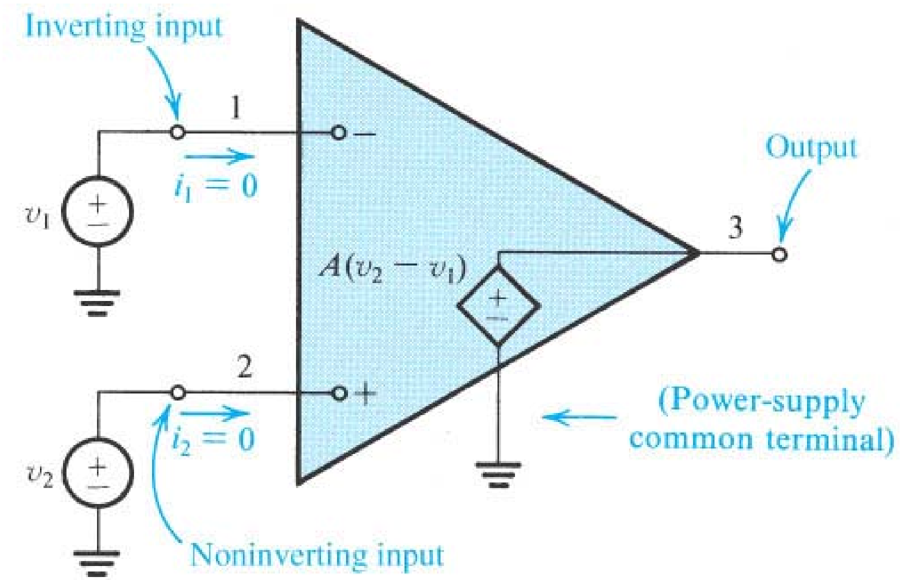
# Lecture 11

## Operational Amplifiers

# Topics

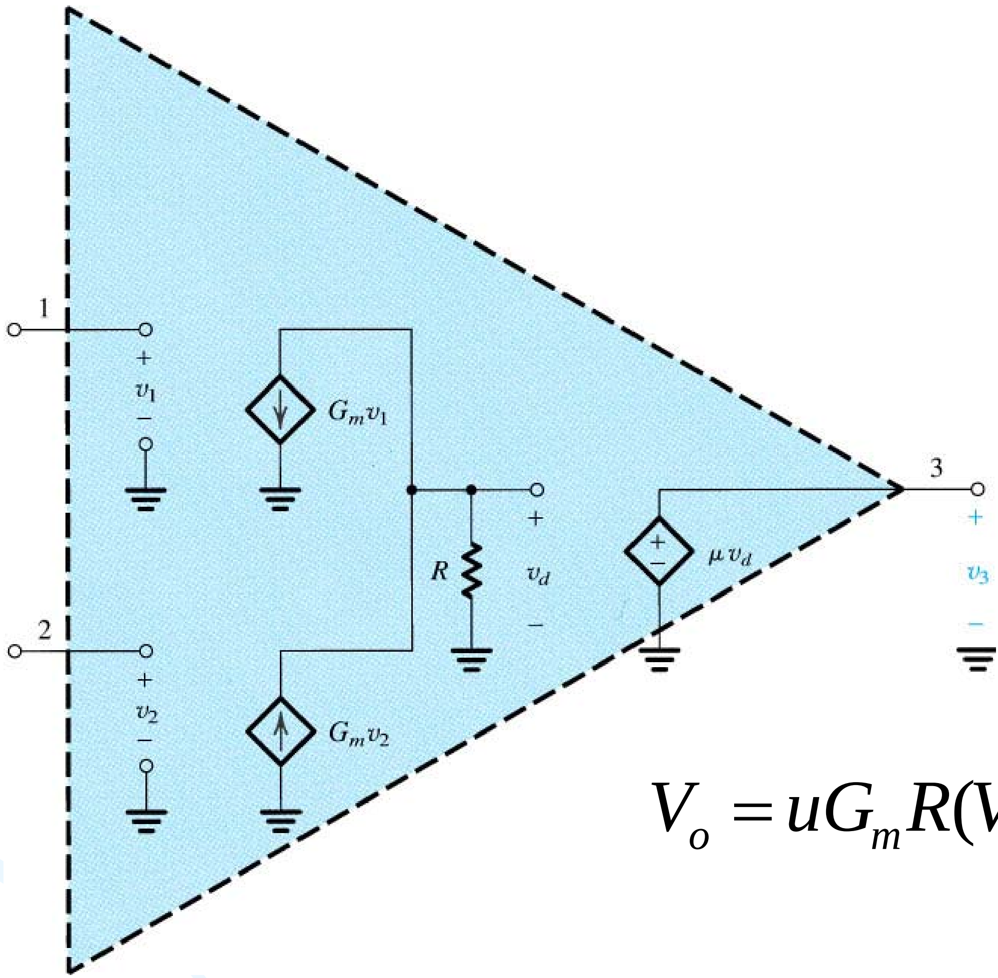
- Ideal op Amplifiers
- Ideal OPA circuits analysis
- Non-ideal op amplifiers
- Non-ideal OPA circuit analysis

# Ideal operational amplifier

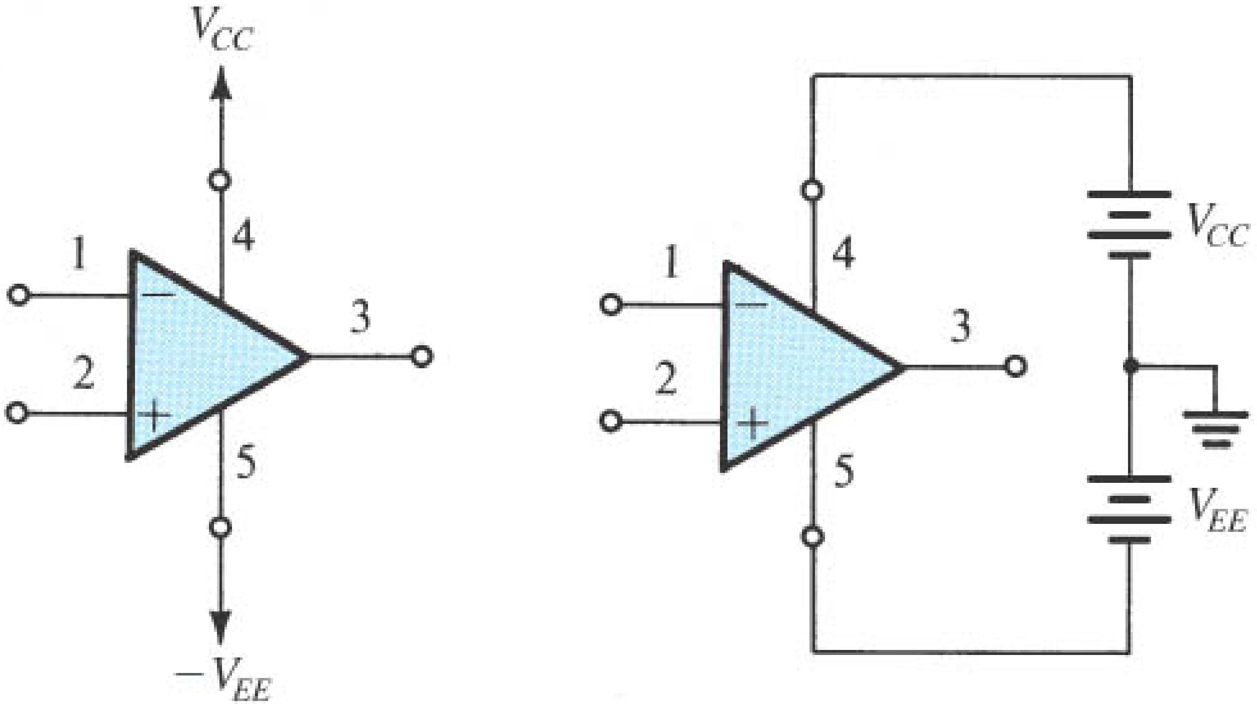



## Ideal OPA characters

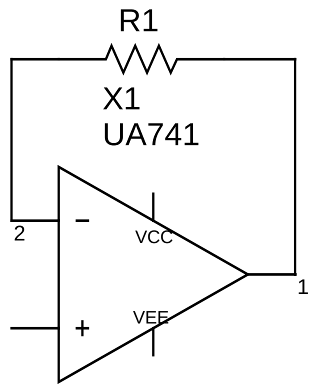
1. Infinite input impedance
2. Zero output impedance
3. Infinite bandwidth
4. Infinite open-loop gain
5. Zero common-mode gain (infinite CMRR)




$$V_o = uG_m R(V_+ - V_-)$$

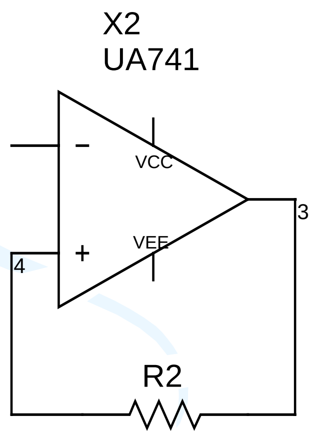



(a) (b)




## Negative feedback

$$V_- = IR + V_o$$
$$V_o = A(V_+ - V_-)$$
$$V_o \uparrow \Rightarrow V_- \uparrow \Rightarrow V_o \downarrow = A(V_+ - V_- \uparrow)$$
$$V_o \downarrow \Rightarrow V_- \downarrow \Rightarrow V_o \uparrow \Rightarrow \ stable$$




## Positive feedback

$$V_+ = IR + V_o$$
$$V_o = A(V_+ - V_-)$$
$$V_o \uparrow \Rightarrow V_+ \uparrow \Rightarrow V_o \uparrow \Rightarrow +V_{sat}$$
$$V_o \downarrow \Rightarrow V_+ \downarrow \Rightarrow V_o \downarrow \Rightarrow -V_{sat} \Rightarrow unstable$$

## Ideal OPA characters

$$R_{in} \to \infty \Rightarrow (i_+ = i_- = 0)$$

$$R_o \to 0$$

$$A \to \infty \Rightarrow (V_+ = V_-)$$

$$BW \to \infty$$

$$CMRR \to \infty$$

## Non-ideal cases

$$R_{in} \neq \infty$$

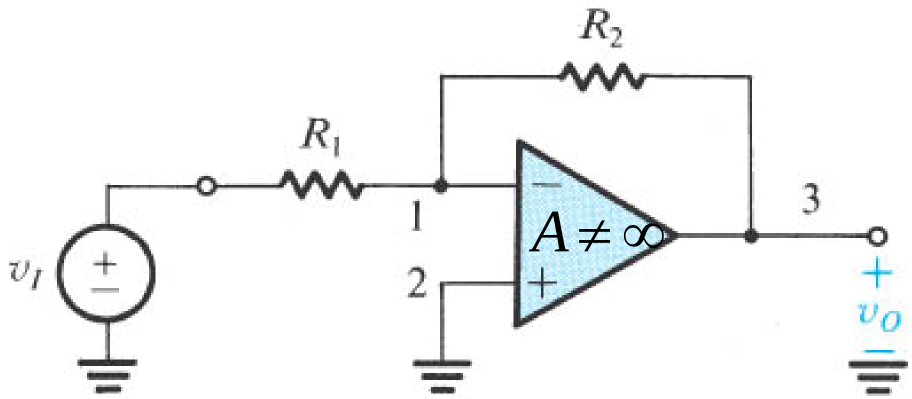
$$R_o \neq 0$$

$$A \neq \infty$$

$$BW \neq \infty$$

$$CMRR \neq \infty$$

# Consider finite open-loop gain $A \neq \infty$




$$\because A \neq \infty$$

$$\therefore v_1 \neq v_2$$

$$\Rightarrow \frac{v_- - v_i}{R_1} + \frac{v_- - v_o}{R_2} = 0 \cdots (1)$$

$$v_0 = A(v_+ - v_-) = -Av_- \cdots (2)$$

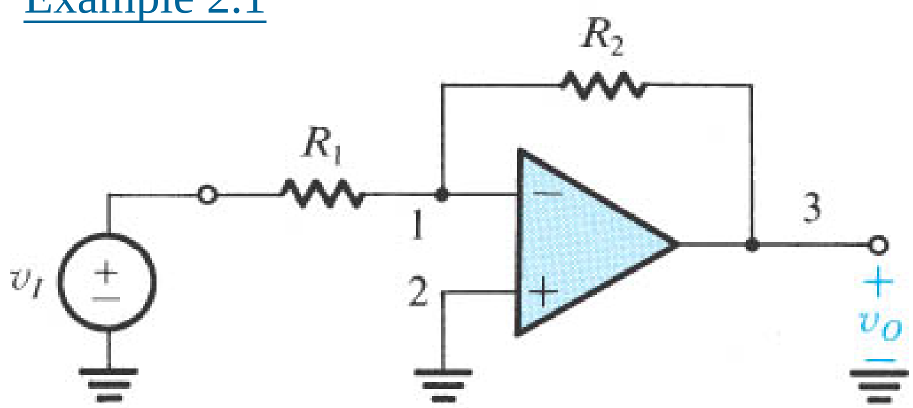
$$\frac{v_o}{v_i} = \frac{-R_2 / R_1}{1 + (1 + R_2 / R_1) / A}$$

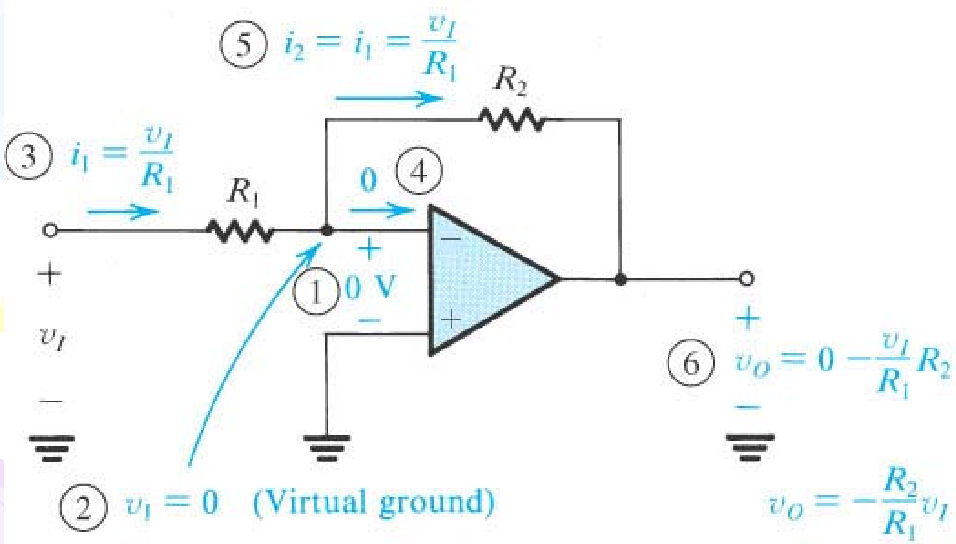
$$if \quad A \to \infty$$

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}$$

# Input and output resistance

Example 2.1




(b)

$$R_{in} = \frac{v_i}{i_i} = R_1$$

$$G_{ain} \equiv \frac{v_O}{v_I}$$

$$\frac{0 - v_I}{R_1} + \frac{0 - v_O}{R_2} = 0$$

$$\Rightarrow G_{ain} = \frac{v_O}{v_I} = -\frac{R_2}{R_1}$$

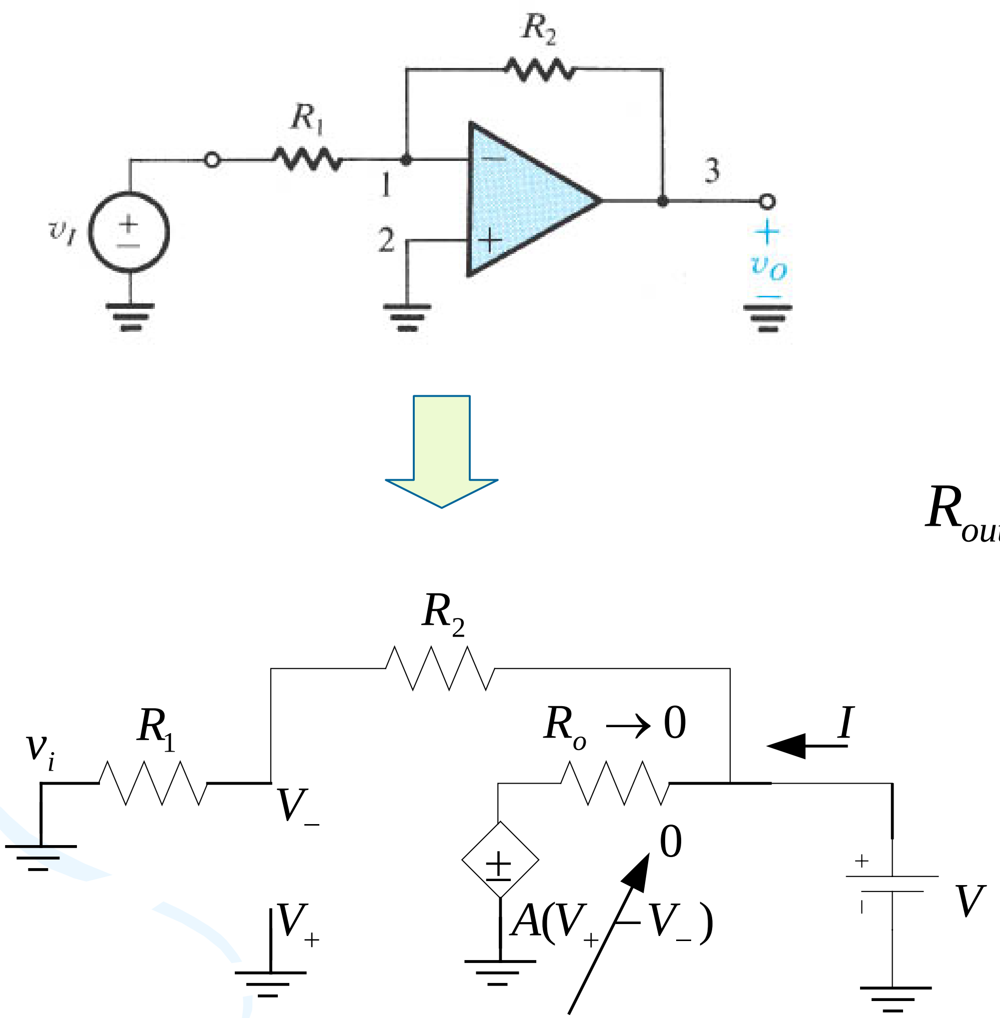
$$if \quad R_1 = 1M$$

$$R_2 = 100M$$

**An impractically large value**

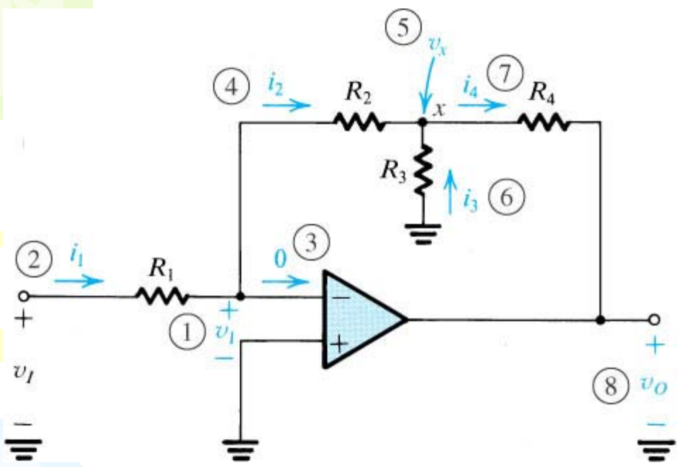
So we may have the problem of input resistance.

# Output resistance




$$R_{out} = \frac{V}{I} = 0$$

## Example 2.2




$$\frac{v_X - 0}{R_2} + \frac{v_X}{R_3} + \frac{v_X - v_o}{R_4} = 0 \cdots (1)$$

$$\frac{0 - v_I}{R_1} + \frac{0 - v_X}{R_2} = 0 \cdots (2)$$

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3})$$

Comparing with Example 2.1
Design a amplifier with a gain –100 and an input resistance of 1M.

Example 2.1:

$$R_1 = 1M$$

$$R_2 = 100M$$

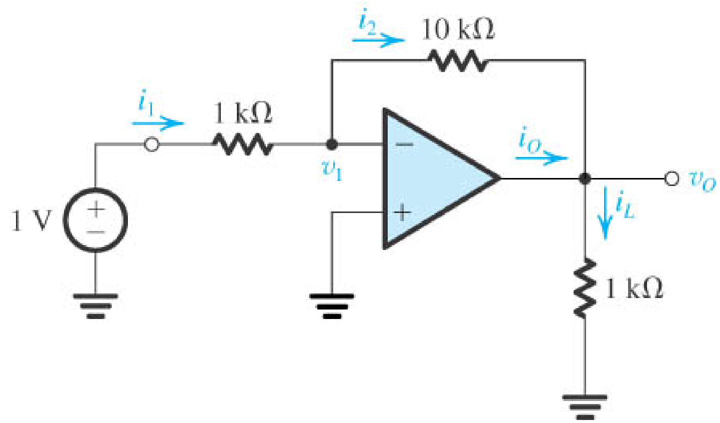
Example 2.2:

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3})$$

$$R_1 = 1M, R_2 = 1M$$

$$R_3 = 10.2k, R_4 = 1M$$

Exercise 2.6




$$\frac{v_- - 1}{1k} + \frac{v_- - v_o}{10k} = 0 \cdots (1)$$

$$v_- = 0 \cdots (2)$$

$$\frac{-1}{1k} + \frac{-v_o}{10k} = 0 \Rightarrow v_o = \frac{-10k}{1k} = -10V$$

*Find*

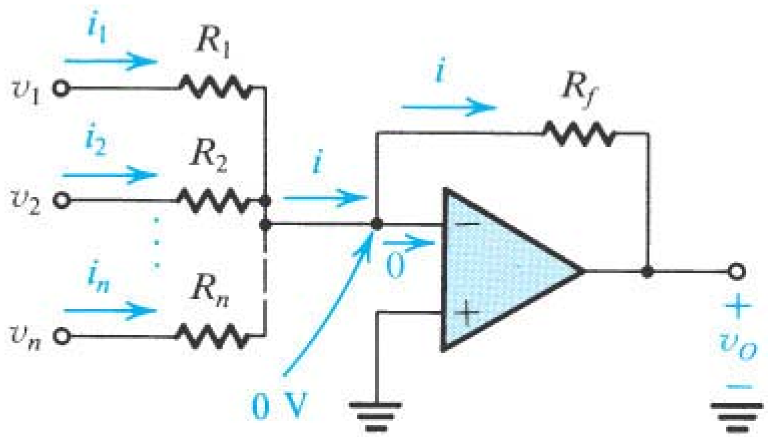
$$i_L = \frac{-10}{1k} = -10mA$$

$$i_2 = \frac{0-(-10)}{10k} = 1mA$$

$$i_0 = -11mA$$

$$i_1 = i_2 = 1mA$$

# Weighted summer



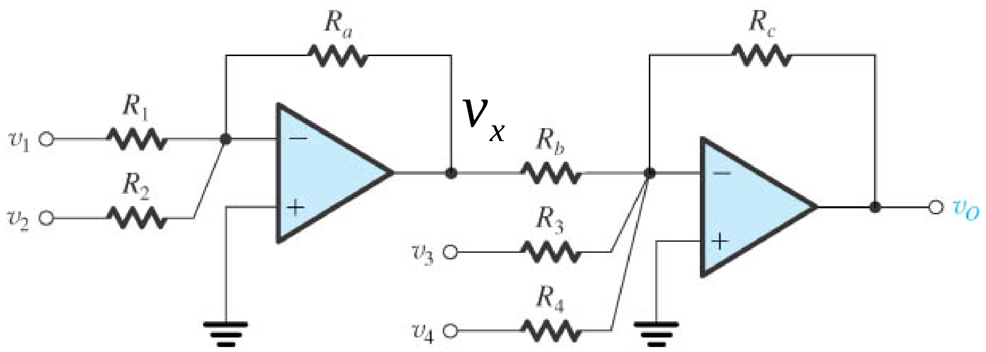

$$v_O = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \cdots + \frac{R_f}{R_n}v_n\right)$$

$$\frac{0-v_1}{R_1}+\frac{0-v_2}{R_2}+\cdots+\frac{0-v_n}{R_n}+\frac{0-v_o}{R_f}=0\cdots(1)$$

$$\Rightarrow \frac{-v_1}{R_1}+\frac{-v_2}{R_2}+\cdots+\frac{-v_n}{R_n}=\frac{v_o}{R_f}$$

$$\Rightarrow v_o=\frac{-R_f}{R_1}v_1+\frac{-R_f}{R_2}v_2+\cdots+\frac{-R_f}{R_n}v_n$$

Exercise D2.8

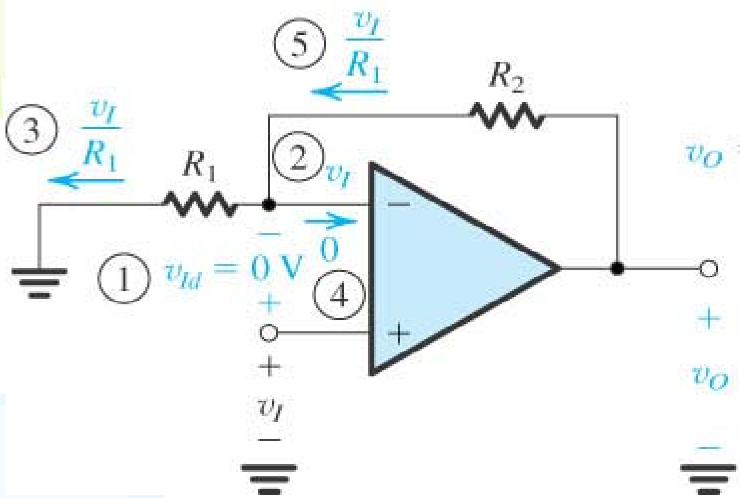



$$v_x = -\left(\frac{R_a}{R_1} v_1 + \frac{R_a}{R_2} v_2\right)$$

$$v_o = -\left(\frac{R_c}{R_3} v_3 + \frac{R_c}{R_4} v_4 + \frac{R_c}{R_b} v_x\right)$$

$$v_o = -\left(\frac{R_c}{R_3} v_3 + \frac{R_c}{R_4} v_4\right) + \frac{R_c}{R_b}\left(\frac{R_a}{R_1} v_1 + \frac{R_a}{R_2} v_2\right)$$

# Non-inverting amplifier




$$v_O = v_I + \frac{v_I}{R_1}R_2 = v_I\left(1 + \frac{R_2}{R_1}\right) \quad ⑥$$

Ideal case: $\dfrac{v_I - 0}{R_1} + \dfrac{v_I - v_o}{R_2} = 0$

$$\Rightarrow v_o = (1 + \frac{R_2}{R_1})v_I$$

Non-ideal case: $A \neq \infty$

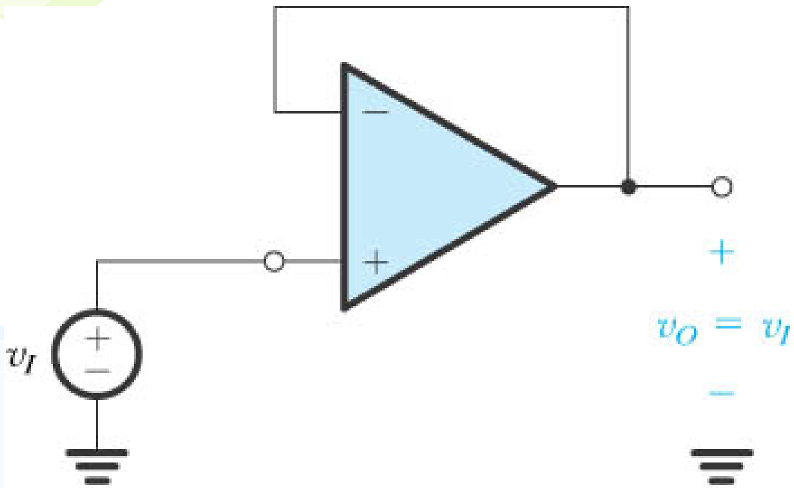
$$\frac{v_i - 0}{R_1} + \frac{v_i - v_o}{R_2} = 0....(1)$$

$$v_o = A(v_i - v_-).........(2)$$

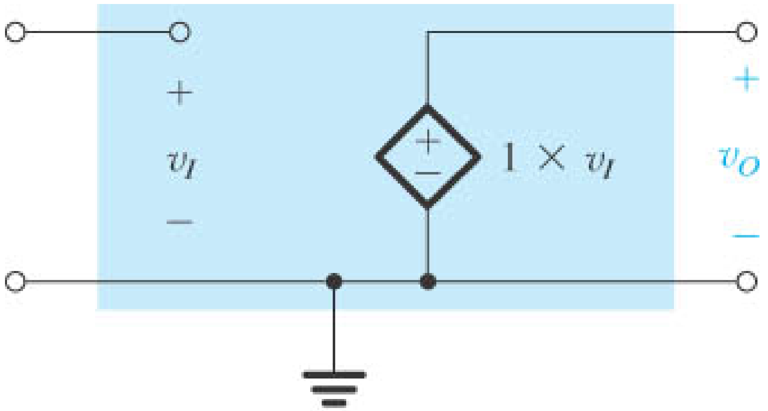
$$(1) \rightarrow v_- = \frac{v_o}{R_2}(\frac{R_1 R_2}{R_1 + R_2})$$

$$\frac{v_o}{v_i} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1 + \frac{R_2}{R_1}}{A}}$$

# Voltage follower

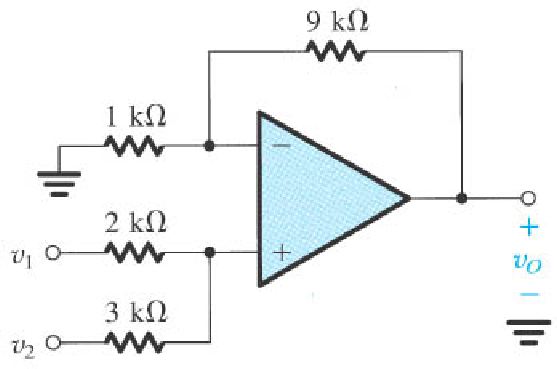



(a)




(b)

## Exercise D2.9




$$v_- = v_+$$

$$\frac{v_- - 0}{1k} + \frac{v_- - v_o}{9k} = 0 \cdots (1)$$

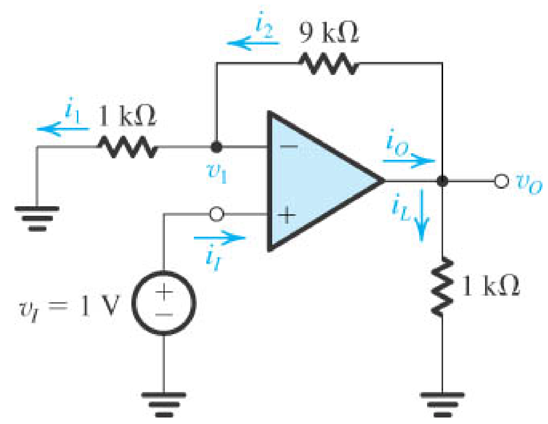
$$\frac{v_- - v_1}{2k} + \frac{v_- - v_2}{3k} = 0 \cdots (2)$$

$$(2) \Rightarrow v_-(\frac{1}{2k} + \frac{1}{3k}) = (\frac{v_1}{2k} + \frac{v_2}{3k})$$

$$(1) \Rightarrow v_o = 9k(\frac{1}{1k} + \frac{1}{9k})v_-$$

$$= 9k(\frac{1}{1k} + \frac{1}{9k})(\frac{2k+3k}{2k \times 3k})(\frac{v_1}{2k} + \frac{v_2}{3k})$$

## Exercise D2.13




$$v_- = v_+ = 1V$$

$$\frac{1-0}{1k} + \frac{1-v_o}{9k} = 0 \cdots (1)$$

$$\Rightarrow v_o = (\frac{1}{1k} + \frac{1}{9k})9k = 10V$$

## Why use difference amplifier ?

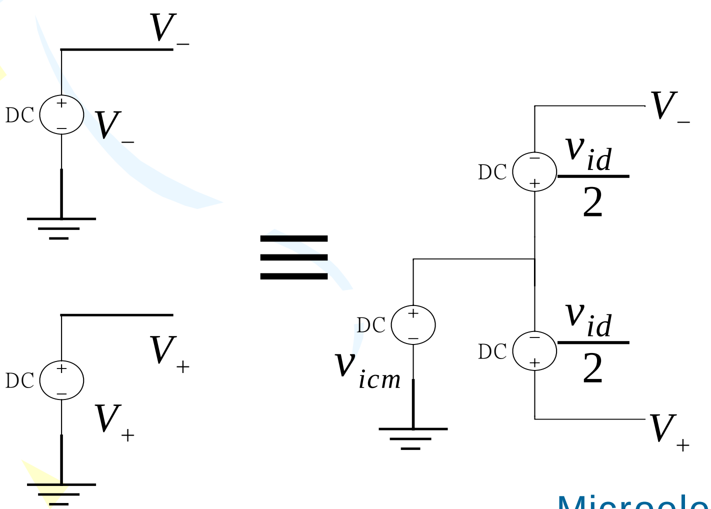
### CMRR ( Common-Mode Rejection ratio )

$$CMRR \equiv 20\log\frac{|A_d|}{|A_{cm}|}$$

$$v_o = A_d v_{id} + A_{cm} v_{icm}$$

Different-mode input

Common-mode input

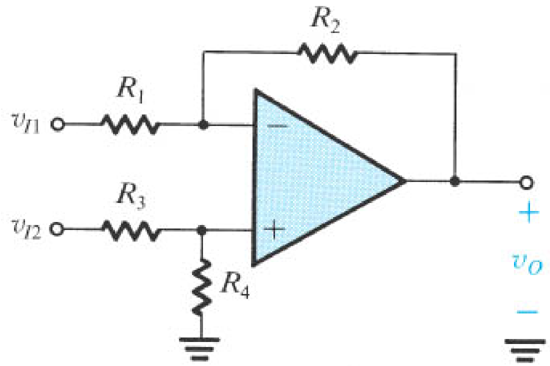



$$v_{id} \equiv v_+ - v_-$$

$$v_{icm} \equiv \tfrac{1}{2}(v_+ + v_-)$$

# Difference Amplifier

Method I:




$$v_- = v_+$$

$$\frac{v_- - v_{i1}}{R_1} + \frac{v_- - v_o}{R_2} = 0 \cdots (1)$$

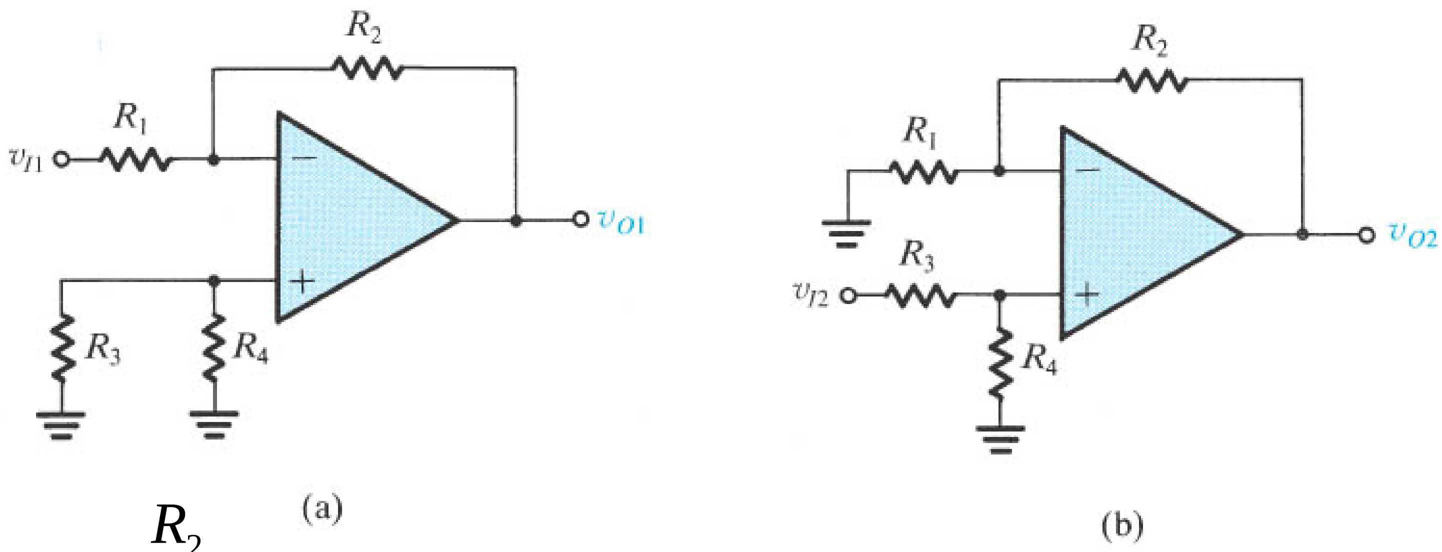
$$\frac{v_+ - v_{i2}}{R_3} + \frac{v_+}{R_4} = 0 \cdots (2)$$

$$(1) \Rightarrow v_o = R_2\left(\frac{v_+ - v_{i1}}{R_1} + \frac{v_+}{R_2}\right) = R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_+ - \frac{R_2}{R_1}v_{i1}$$

$$(2) \Rightarrow v_+\left(\frac{1}{R_3} + \frac{1}{R_4}\right) = \frac{1}{R_3}v_{i2}$$

# Different-mode input

Method I: superposition

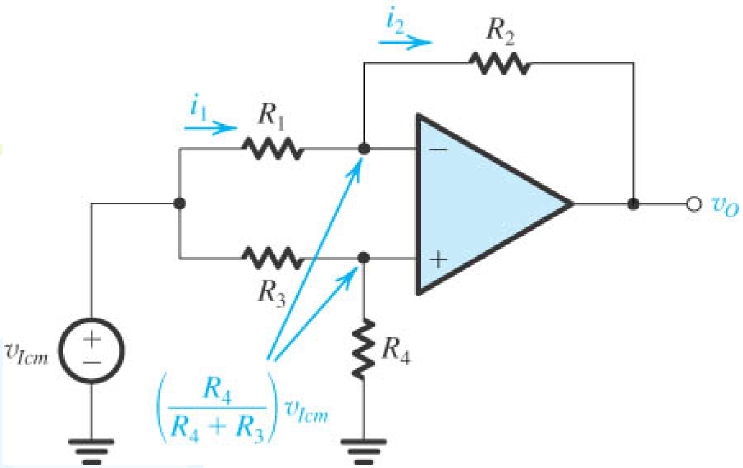



$$v_{o1} = -\frac{R_2}{R_1} v_{i1}$$

$$v_{o2} = \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right) v_{i2} = \frac{R_2}{R_1} v_{i2} (let \qquad R_3 = R_1, R_4 = R_2)$$

$$v_o = v_{o1} + v_{o2} = \frac{R_2}{R_1}(v_{i2} - v_{i1}) = A_d (v_{i2} - v_{i1})$$

# Common-mode input




$$i_1 = \frac{1}{R_1}[v_{icm} - \frac{R_4}{R_3+R_4}v_{icm}] = \frac{R_3}{R_3+R_4}\frac{1}{R_1}v_{icm}$$

$$v_o = \frac{R_4}{R_3+R_4}v_{icm} - i_2R_2$$

$$\because i_1 = i_2 \Rightarrow v_o = [\frac{R_4}{R_3+R_4} - \frac{R_3}{R_3+R_4}\frac{R_2}{R_1}]v_{icm}$$

$$A_{cm} = \frac{R_4}{R_3+R_4}[1 - \frac{R_3}{R_4}\frac{R_2}{R_1}]$$

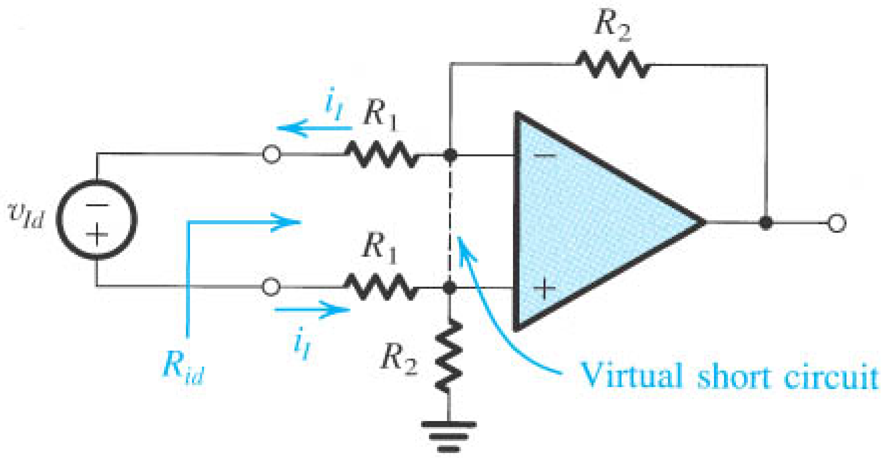
$$\because R_3 = R_1, R_4 = R_2 \Rightarrow \therefore A_{cm} = 0$$

CMRR=infinite

$$if \qquad R_3 \neq R_1, R_4 \neq R_2 \Rightarrow A_{cm} \neq 0$$

CMRR≠infinite

Consider the problem of input resistance




$$v_{id} = i_1 R_1 + i_1 R_1 = 2R_1 i_1$$

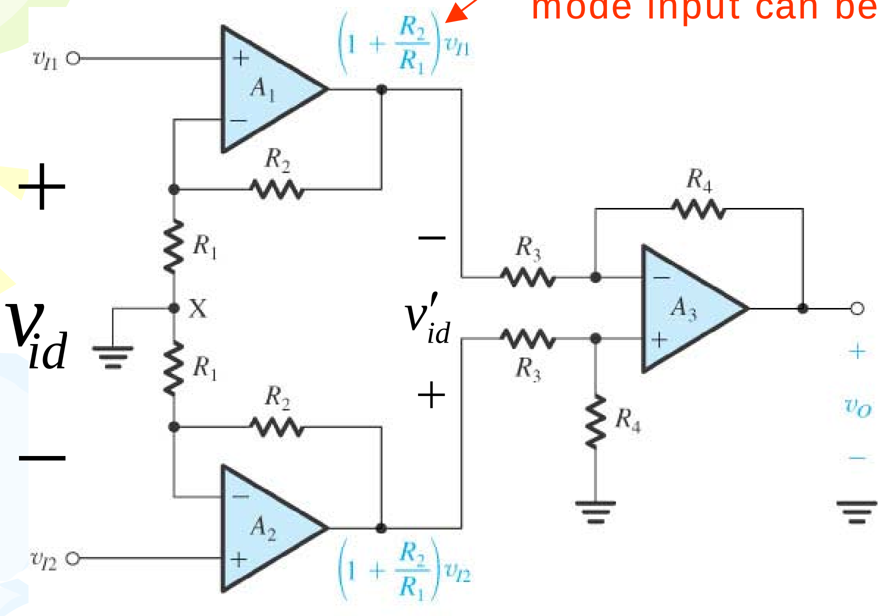
$$\Rightarrow R_{in(diff)} = 2R_1$$

Differential mode input resistance

$$v_o = \frac{R_2}{R_1} v_{id} \qquad if \qquad R_{in} \uparrow \rightarrow R_1 \uparrow \rightarrow v_o \downarrow$$

# Instrumentation Amplifier

Not is differential mode, common mode input can be pass.




(a)

$$v_o = \frac{R_4}{R_3} v'_{id}$$

$$v'_{id} = (1+\frac{R_2}{R_1})(v_{i2} - v_{i1})$$

$$v_o = \frac{R_4}{R_3}(1+\frac{R_2}{R_1})(v_{i2} - v_{i1})$$

Advantages:

1. $R_{in} \to \infty$
2. $A_d = \dfrac{R_4}{R_3}(1+\dfrac{R_2}{R_1})$

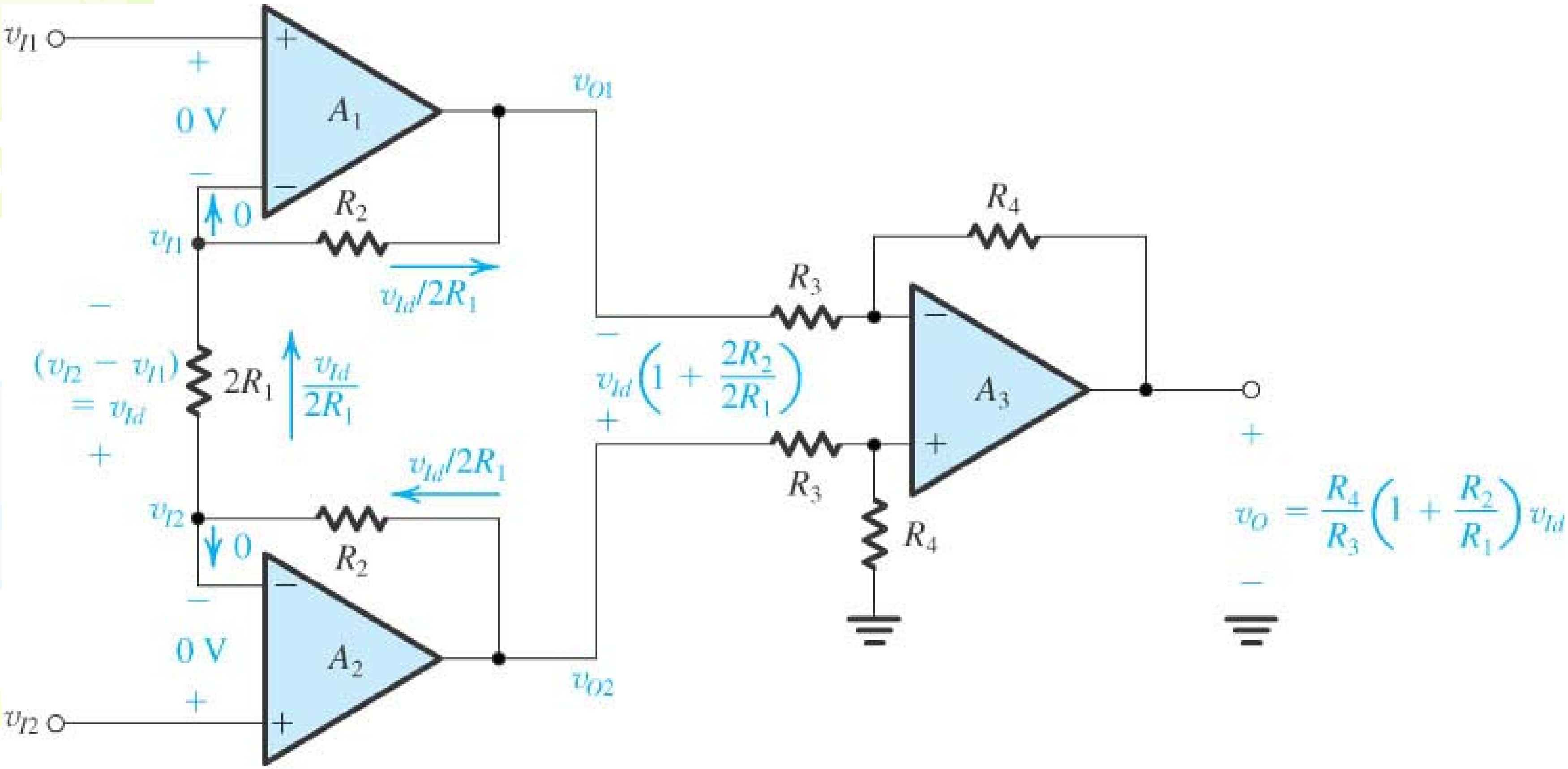
Big differential gain

Defects:

1. Common mode gain=differential mode gain. $v_o \to v_{sat}$
2. Resistance $R_1$ and $R_2$ have to match.

$$v'_{id} = (1+\frac{R_2}{R_1})(v_{i2} - v_{i1})$$

## Remove the point *x*

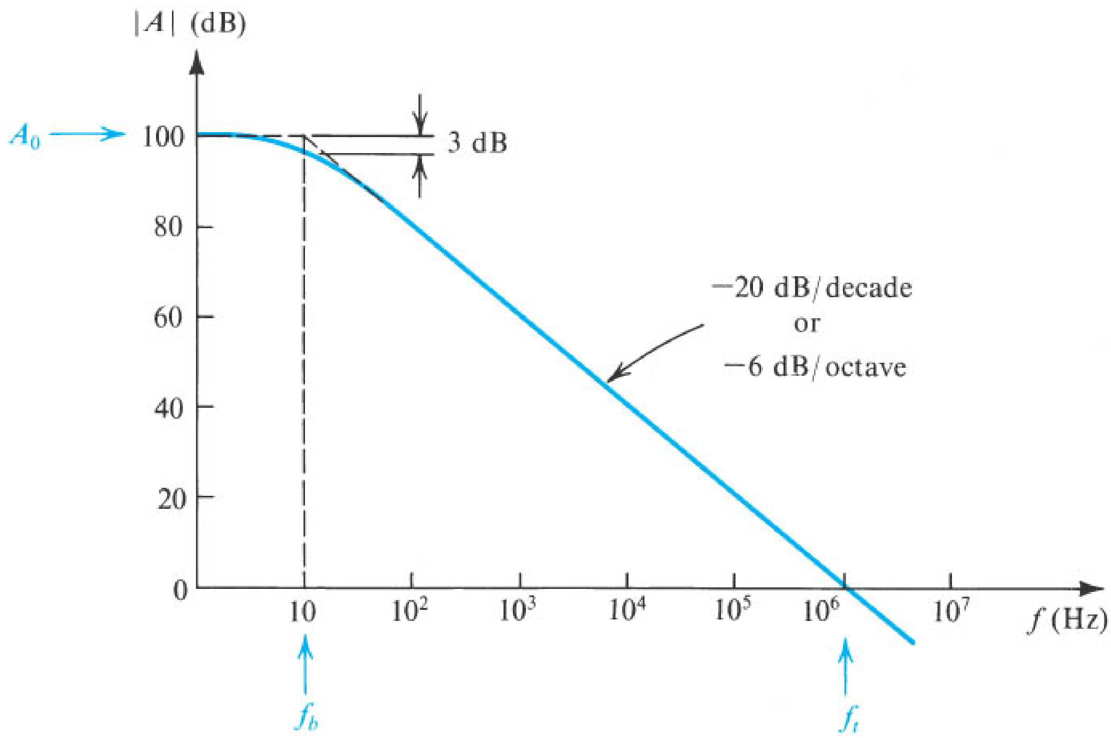



(c)

# DC non-ideal characters

1. Finite open loop gain ( finite CMRR)
2. Finite BW
3. Offset voltage
4. Input bias and offset current

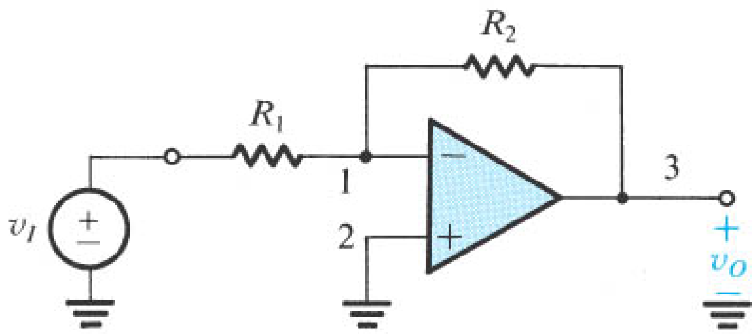
# Frequency response (open-loop)

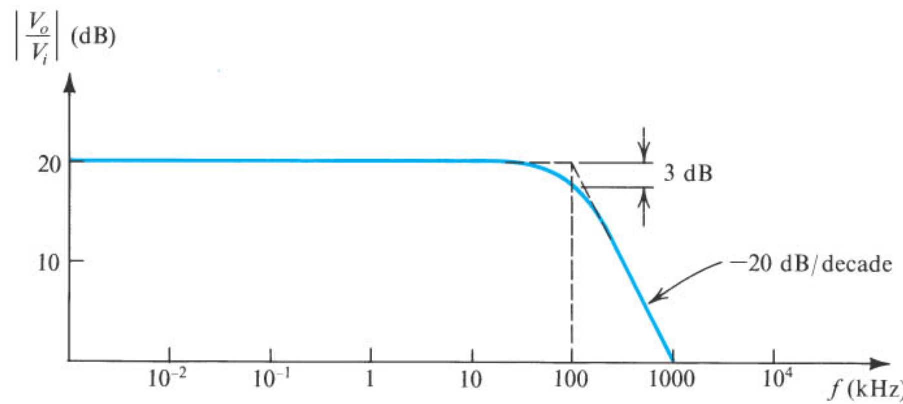



$$A(s) = \frac{A_o}{1 + s/\omega_b} \Rightarrow 0dB : \omega_t/\omega_b = A_o$$

$$\omega_t = A_o \omega_b$$

# Frequency response (closed loop)

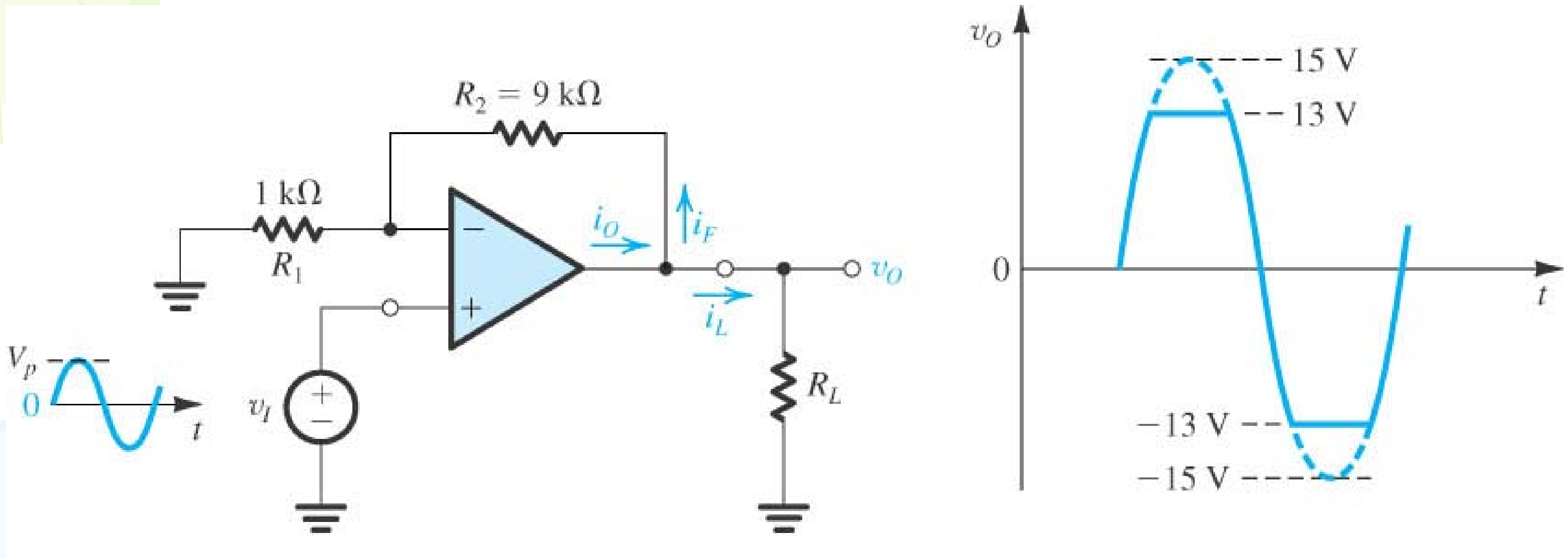



$$\frac{v_o}{v_i} = \frac{-{R_2}/{R_1}}{1+(1+{R_2}/{R_1})/A} \qquad A(s) = \frac{A_o}{1+{s}/{\omega_b}}$$

$$\frac{v_o}{v_i} = \frac{-{R_2}/{R_1}}{1+\frac{1}{A_0}(1+{R_2}/{R_1})+\frac{s}{\omega_t/(1+{R_2}/{R_1})}} \approx \frac{-{R_2}/{R_1}}{1+\frac{s}{\omega_t/(1+{R_2}/{R_1})}} \qquad \omega_{3dB} = \frac{\omega_t}{1+{R_2}/{R_1}}$$

# Output voltage saturation



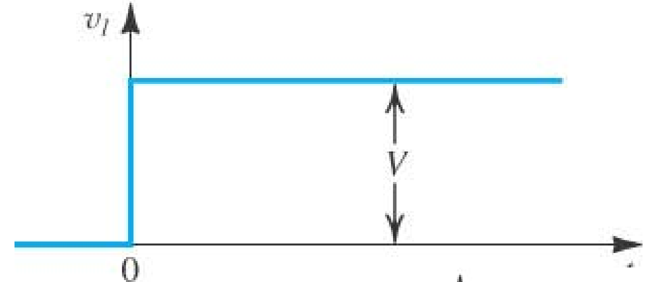

(a) (b)

$$v_{o\max} = \pm 13V$$

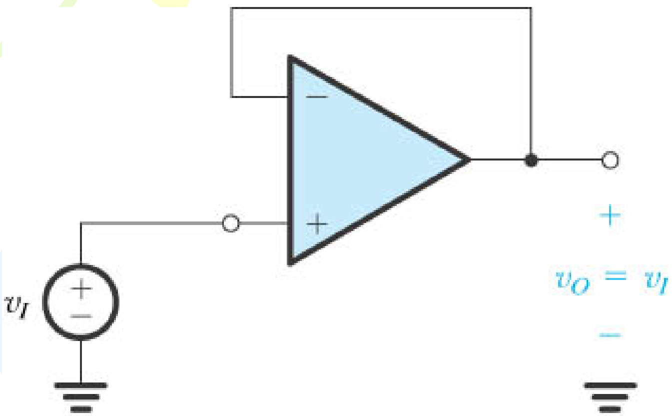
$$i_{o\max} = \pm 20mA$$

# Slew rate

$$\boxed{SR \equiv \left.\frac{dv_o}{dt}\right|_{\max} (V/\mu s)}$$




(b)




(a)

$$\frac{v_o}{v_i} = \frac{1+R_2/R_1}{1+\dfrac{1+R_2/R_1}{A}}$$

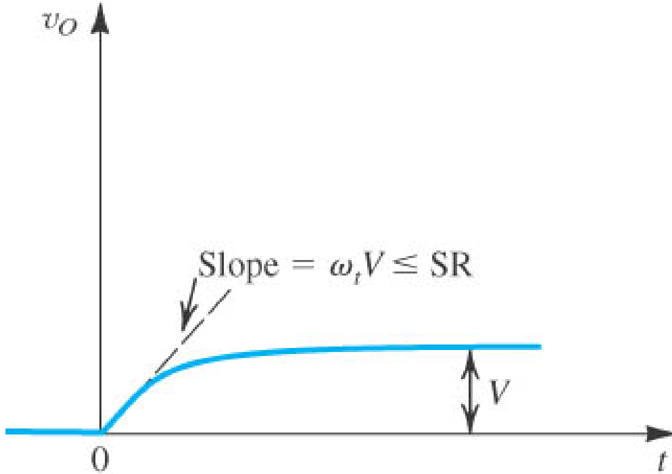
$$A = \frac{A_o}{1+s/\omega_b}$$

$$\frac{v_o}{v_i} \approx \frac{1}{1+\dfrac{s}{\omega_t}} (\because R_2 = 0, R_1 = \infty)$$

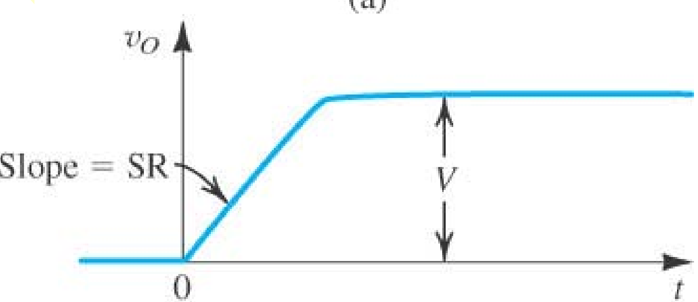
$$\boxed{\omega_t = A_o\omega_b}$$

$$let \quad v_i = V(unit \quad step)$$

$$V_o(s) = \frac{\omega_t}{\omega_t + s}\frac{V}{s} = \frac{-V}{s+\omega_t} + \frac{V}{s} \Rightarrow v_o(t) = V(1-e^{-\omega_t t})$$



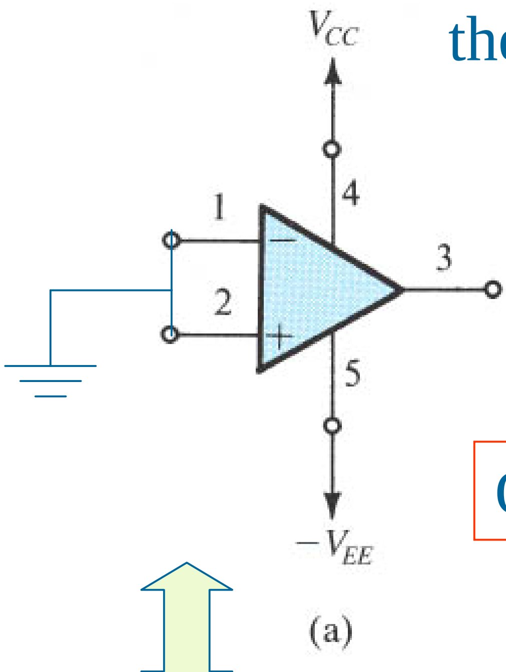

(d)



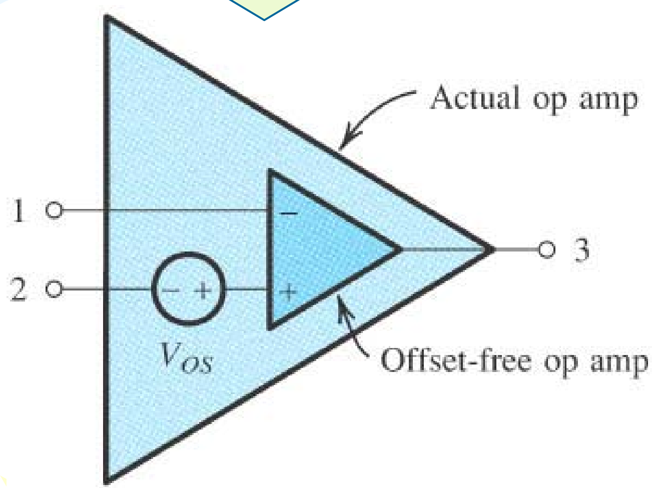

(c)

# Offset voltage

From the component mismatches in the input differential stage

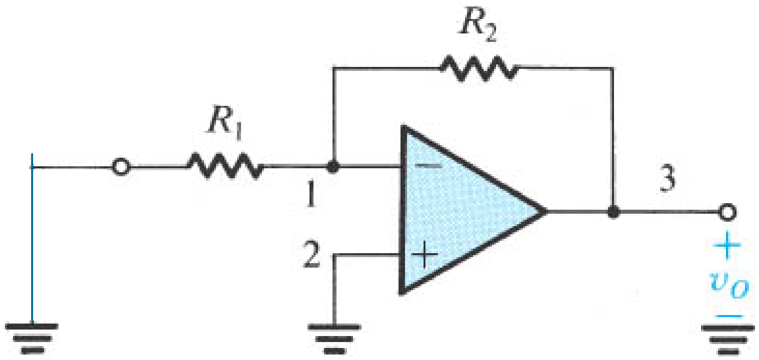



Output = + saturation or - saturation

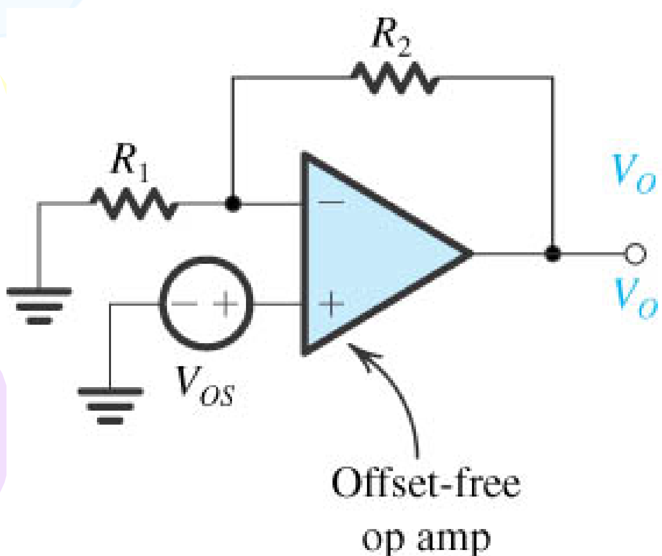



$$v_{os} \approx 1 \sim 5mV$$

May be positive or negative

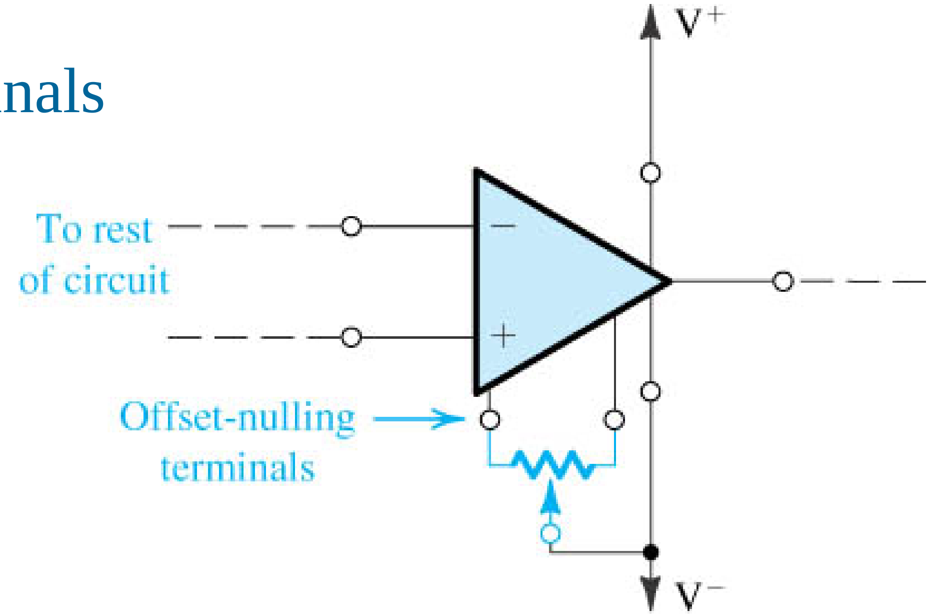



1. Reduce the allowable signal swing
2. When input is dc we would not know the output is due to $v_{os}$ or signals

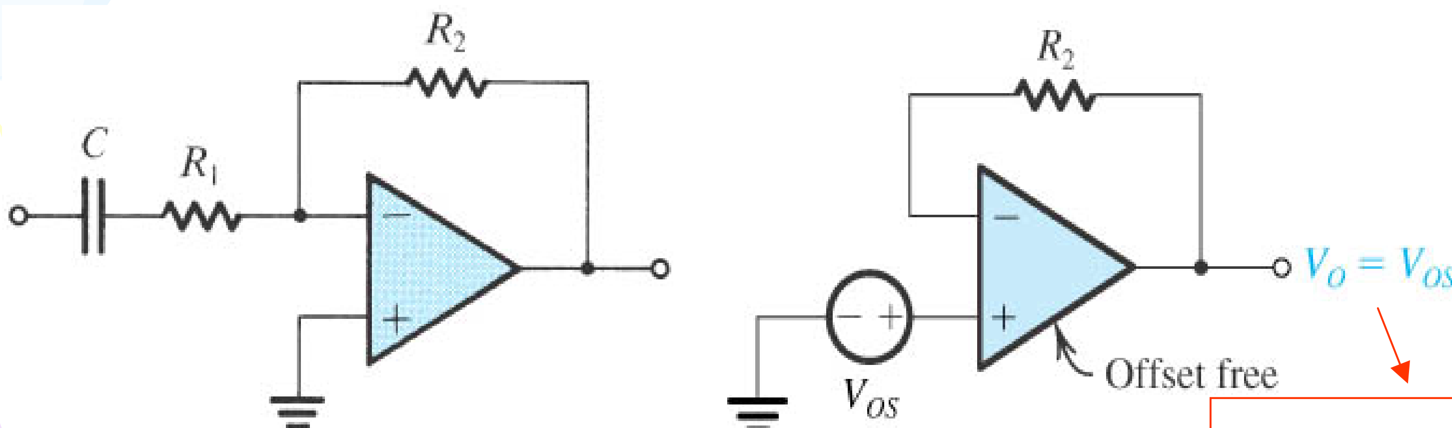



$$V_O = V_{OS}\left(1 + \frac{R_2}{R_1}\right)$$

## Solution 1: Offset-nulling terminals




## Solution 2: Capacitive coupling (only ac signal be amplified)




(a) (b)

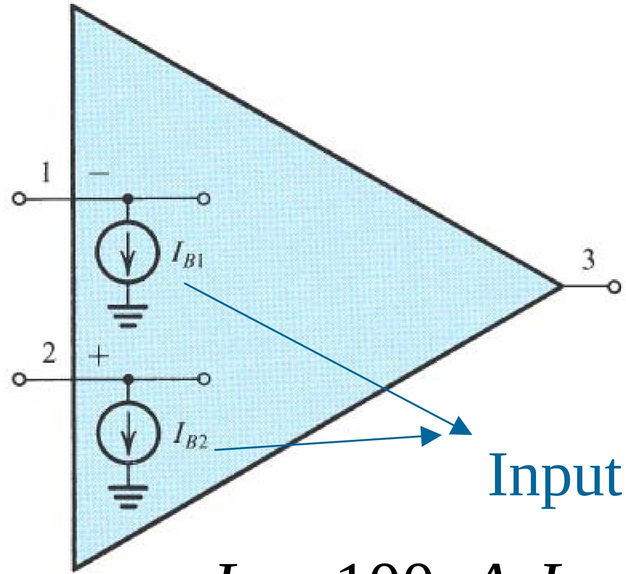
$$\boxed{v_o \neq v_{os}\left(1+\frac{R_2}{R_1}\right)}$$

# Offset current

$$I_B = \frac{I_{B1} + I_{B2}}{2}$$

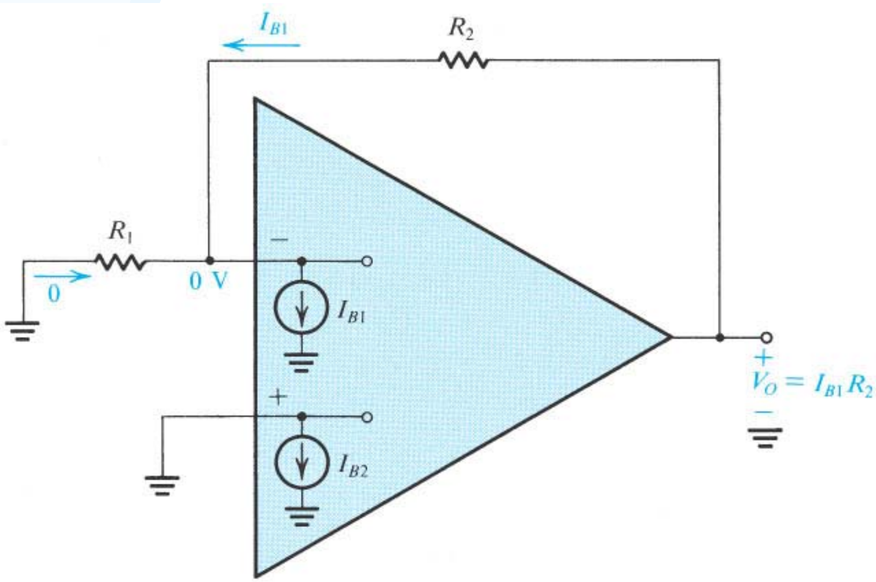
$$I_{os} \equiv \left| I_{B1} - I_{B2} \right|$$

Input offset current




Input bias currents

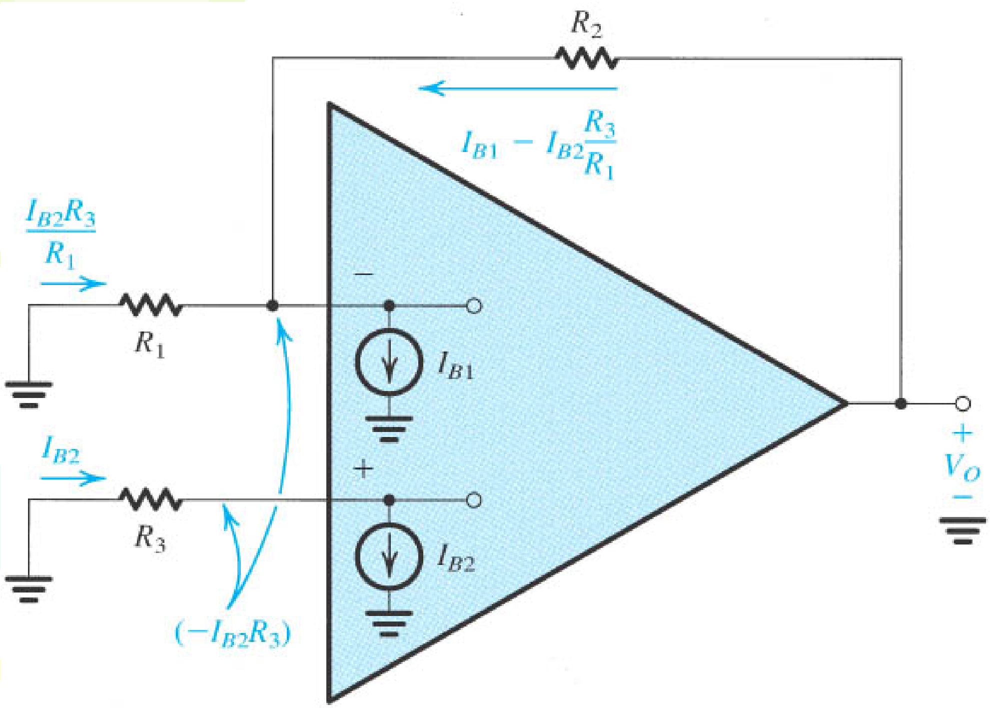
$$I_B \approx 100nA, I_{os} \approx 10nA$$




$$V_o = I_{B1} R_2 \Rightarrow R_2 \downarrow$$

Upper limit $R_2$

# Solution : introducing $R_3$




$$V_o = -I_{B2}R_3 + R_2(I_{B1} - I_{B2}\frac{R_3}{R_1})$$

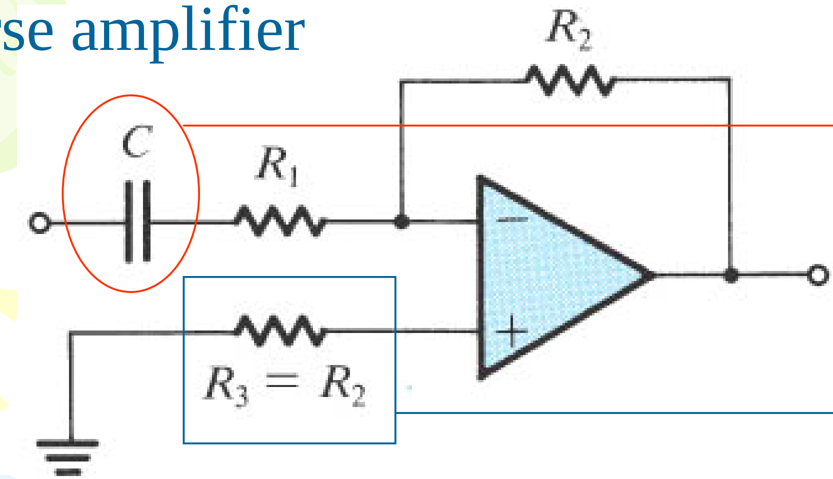
$$if \qquad I_{B1} = I_{B2} = I_B$$

$$V_o = I_B[R_2 - R_3(1+\frac{R_2}{R_1})]$$

$$choose \quad R_3 = \frac{R_2}{(1+\frac{R_2}{R_1})} = R_1 // R_2$$

$$\Rightarrow V_o = 0$$

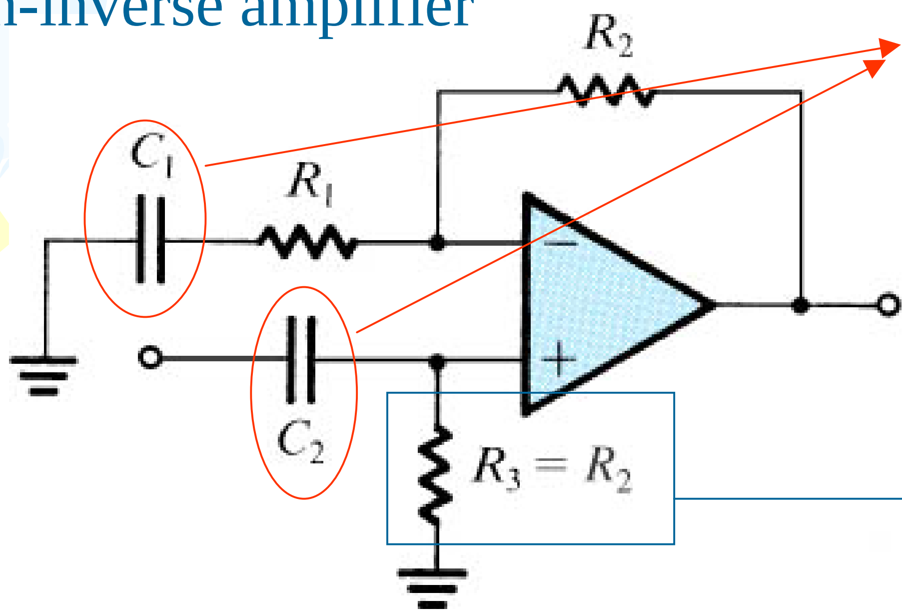
## Inverse amplifier




Reduce the effect of $V_{os}$

Reduce the effect of $I_{os}$

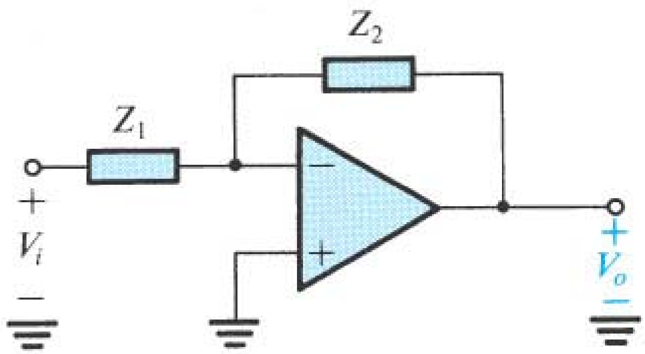
## Non-inverse amplifier



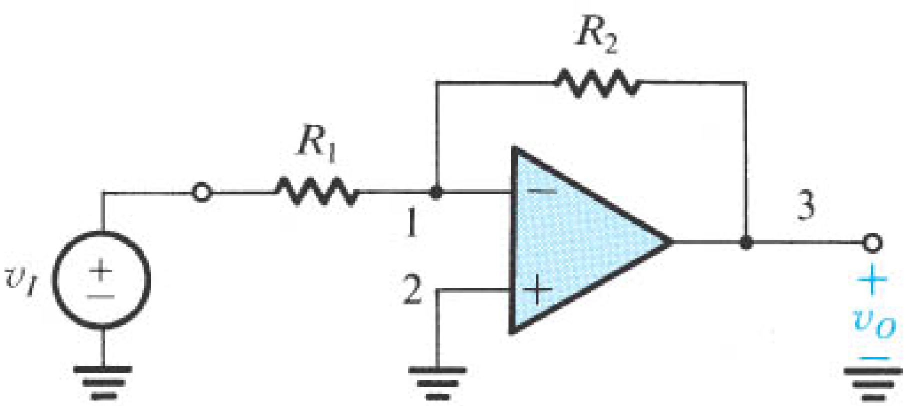

Reduce the effect of $V_{os}$

Reduce the effect of $I_{os}$

# Integrator & Differentiator

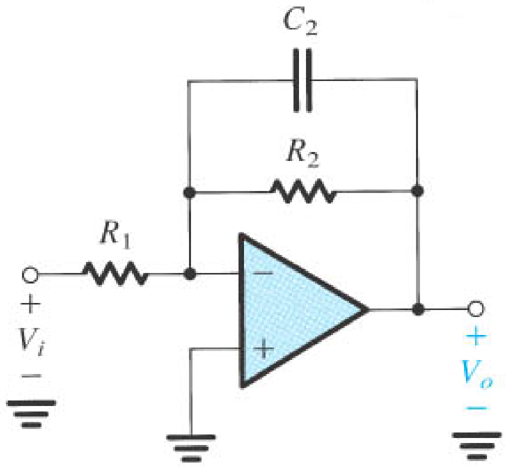



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$




$$\frac{0-v_I}{Z_1} + \frac{0-v_O}{Z_2} = 0$$

$$\Rightarrow G = \frac{v_O}{v_I} = -\frac{Z_2}{Z_1}$$




$$\frac{0-v_i(t)}{R_1}+\frac{0-v_o(t)}{R_2}+C_2\frac{d[0-v_o(t)]}{dt}=0$$

$$\Rightarrow C_2\frac{dv_o(t)}{dt}+\frac{v_o(t)}{R_2}=\frac{-v_i(t)}{R_1}$$

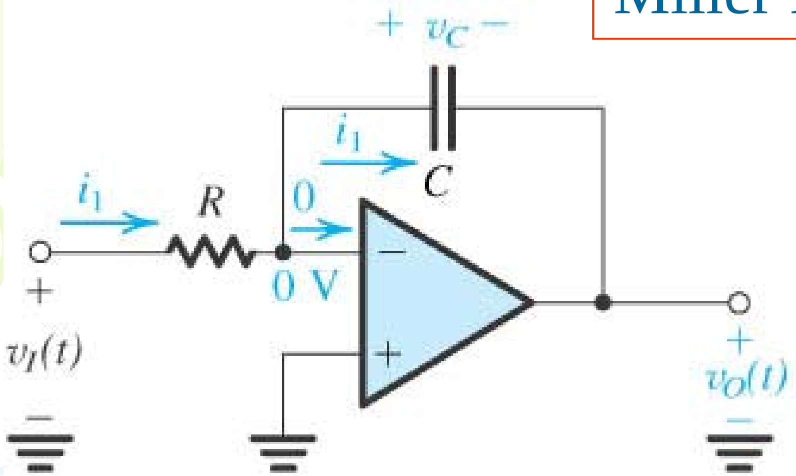
$$\Rightarrow sC_2v_0+\frac{v_o}{R_2}=\frac{-v_i}{R_1}$$

$$\frac{v_O}{v_I}=-\frac{Z_2}{Z_1}=-\frac{1}{R_1/R_2+sC_2R_1}$$

$$\Rightarrow \frac{v_O}{v_I}=-\frac{R_2/R_1}{1+sC_2R_2}$$

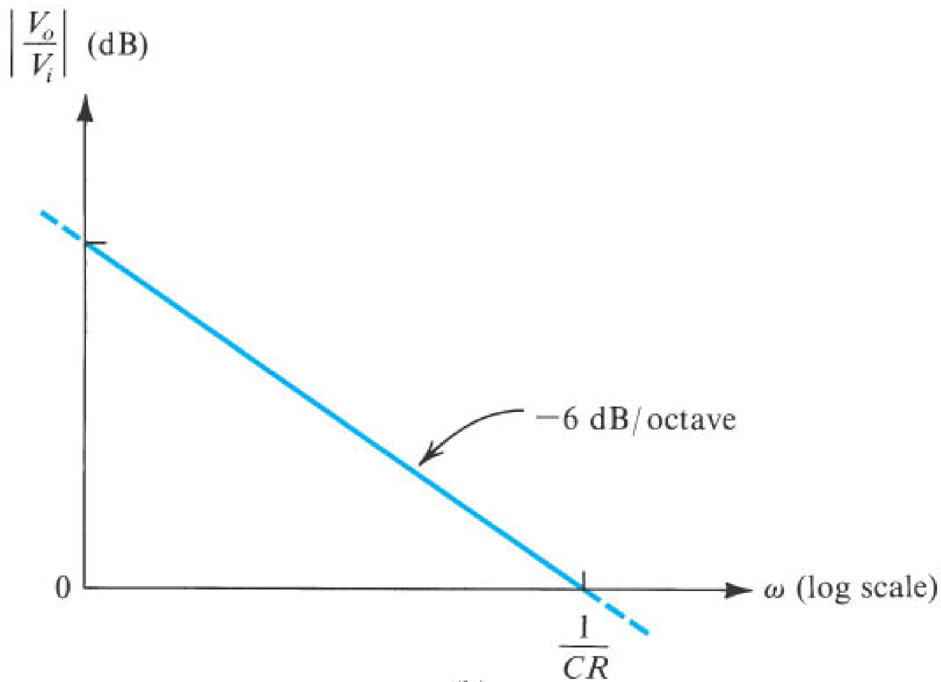
$$\omega_0=\frac{1}{C_2R_2}$$

# Miller Integrator



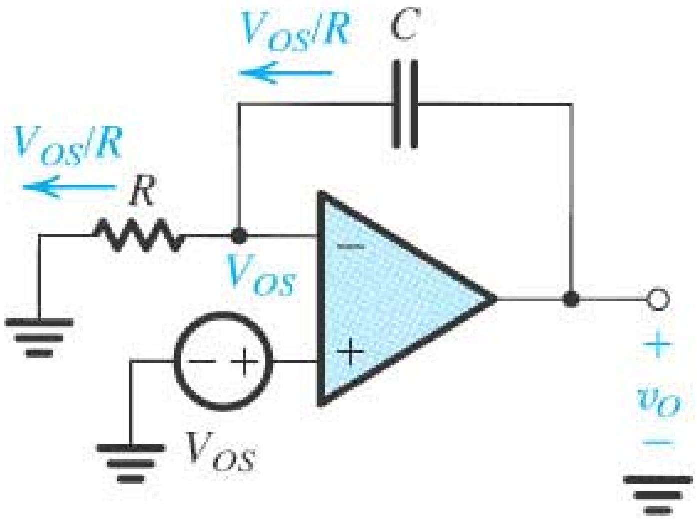

$$v_O(t) = -\frac{1}{CR}\int_0^t v_I(t)\,dt$$

$$\frac{V_o}{V_i} = -\frac{1}{sCR}$$




(b)

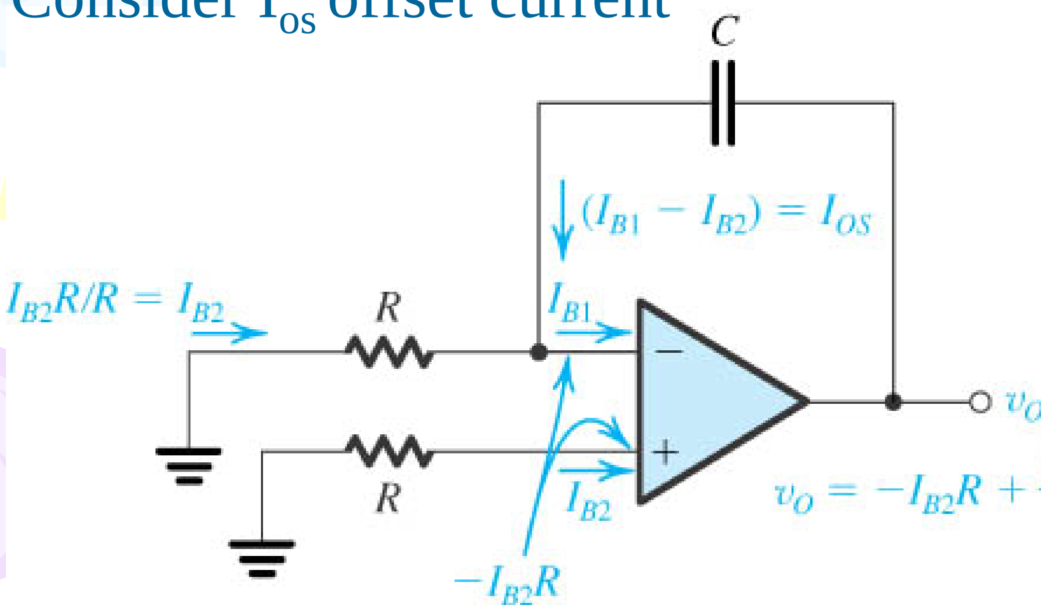
# Consider $V_{os}$ offset voltage




$$v_O = V_{OS} + \frac{1}{C}\int_0^t \frac{V_{OS}}{R}dt$$

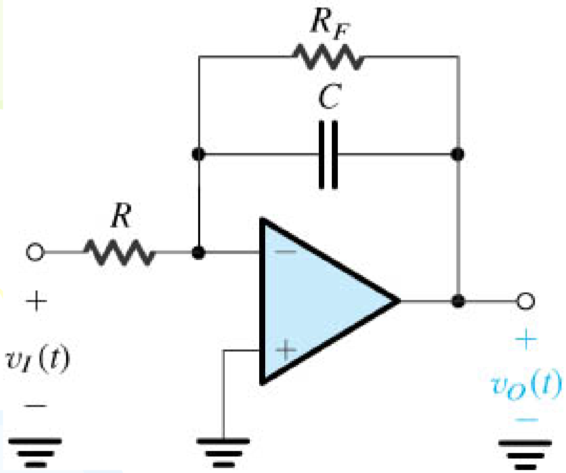
$$= V_{OS} + \cancel{\frac{V_{OS}}{CR}t} \quad \boxed{V_{sat}}$$

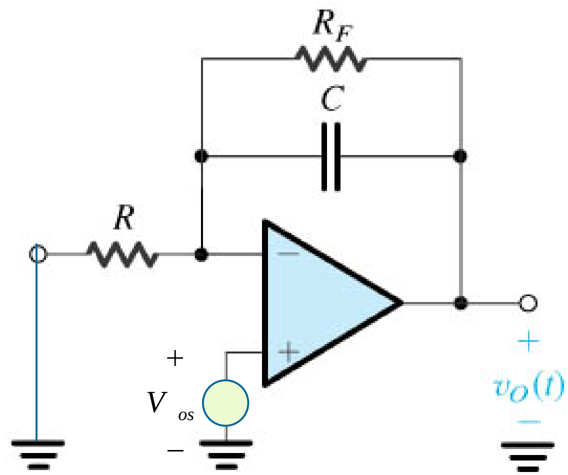
# Consider $I_{os}$ offset current




$$v_O = -I_{B2}R + \cancel{\frac{I_{OS}}{C}t} \quad \boxed{V_{sat}}$$

## Solution :




$$\frac{V_{os}}{R}+\frac{V_{os}-v_o}{1/sC}+\frac{V_{os}-v_o}{R_f}=0$$

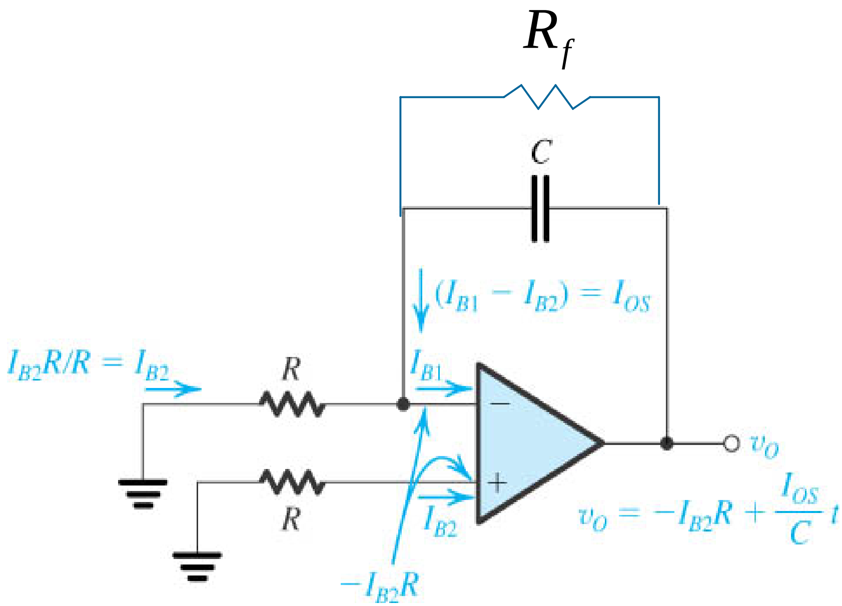
$$\frac{v_o}{V_{os}}=\frac{\frac{1}{R}+\frac{1}{R_f}+sC}{\frac{1}{R_f}+sC}=[1+\frac{\frac{1}{R}}{\frac{1}{R_f}+sC}]$$

$$v_o=[1+\frac{\frac{1}{R}}{\frac{1}{R_f}+sC}]V_{os}=V_{os}+\frac{\frac{1}{RC}}{\frac{1}{R_fC}+s}V_{os}$$

$$\boxed{v_o(t)=V_{os}+\frac{V_{os}}{RC}e^{-\frac{1}{R_fC}t}}$$

$$\boxed{v_o(t)=V_{os}+\frac{V_{os}}{RC}t}$$

$$\frac{-I_{B2}R}{R} + \frac{-I_{B2}R - v_o}{1/sC} + \frac{-I_{B2}R - v_o}{R_f} = 0$$



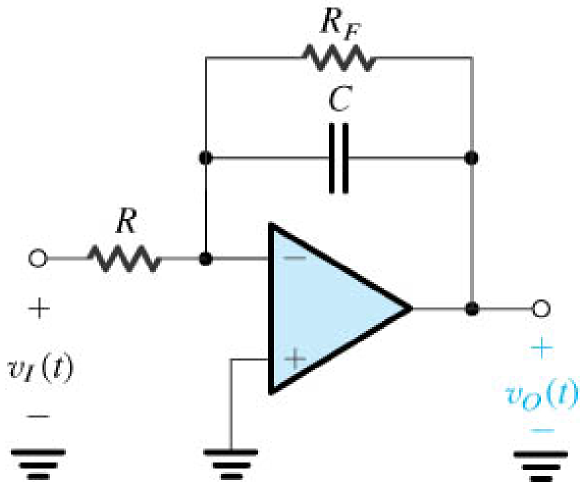

$$(sC + \frac{1}{R_f})v_o = -I_{B2}R[\frac{1}{R} + sC + \frac{1}{R_f}]$$

$$v_o = \frac{-I_{B2}R[\frac{1}{R} + sC + \frac{1}{R_f}]}{(sC + \frac{1}{R_f})} = -I_{B2}R + \frac{-I_{B2}}{(sC + \frac{1}{R_f})} = -I_{B2}R + \frac{-I_{B2}C}{(s + \frac{1}{R_f C})}$$

$$\boxed{v_o(t) = -I_{B2}R + (-I_{B2}C)e^{-\frac{1}{R_f C}t}}$$

$$\boxed{v_o = -I_{B2}R + \frac{I_{os}}{C}t}$$

# Example 2.7 sketch output response




$$R = 10k$$

$$C = 10nF$$

$$R_F = \infty, \qquad R_F = 1M$$

$$V_{sat} = \pm 13V$$

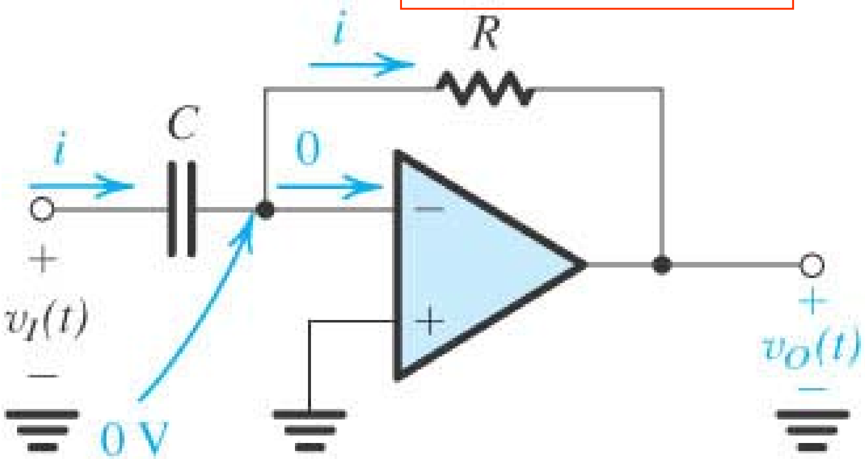
Case I: $R_F = \infty$

$$v_o(t) = -\frac{1}{RC}\int_0^t v_i(t)dt$$

Case II: $R_F = 1M$

$$\frac{v_O}{v_I} = -\frac{R_F / R_1}{1 + sC_2R_F}$$

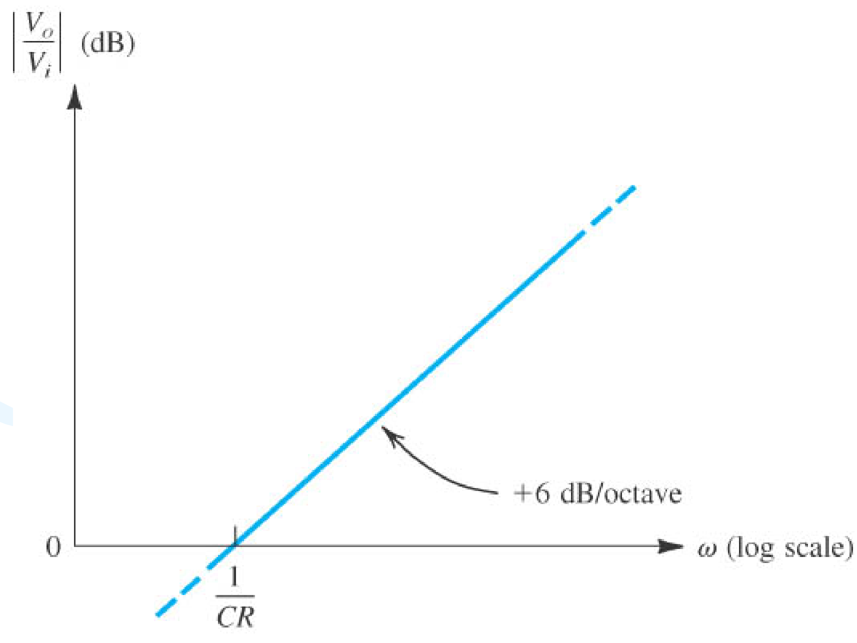
# Differentiator




$$i(t) = C\frac{dv_I(t)}{dt}$$

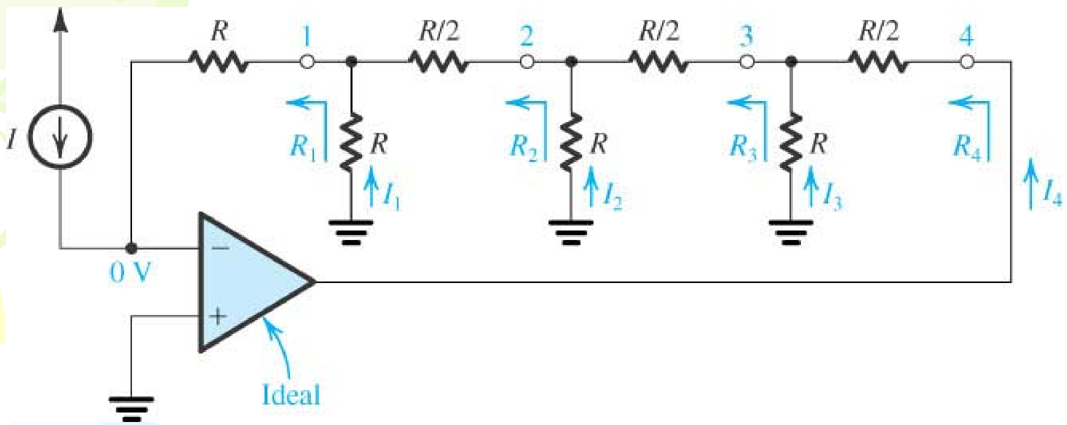
$$v_O(t) = -CR\frac{dv_I(t)}{dt}$$

$$\frac{V_o}{V_i} = -sCR$$

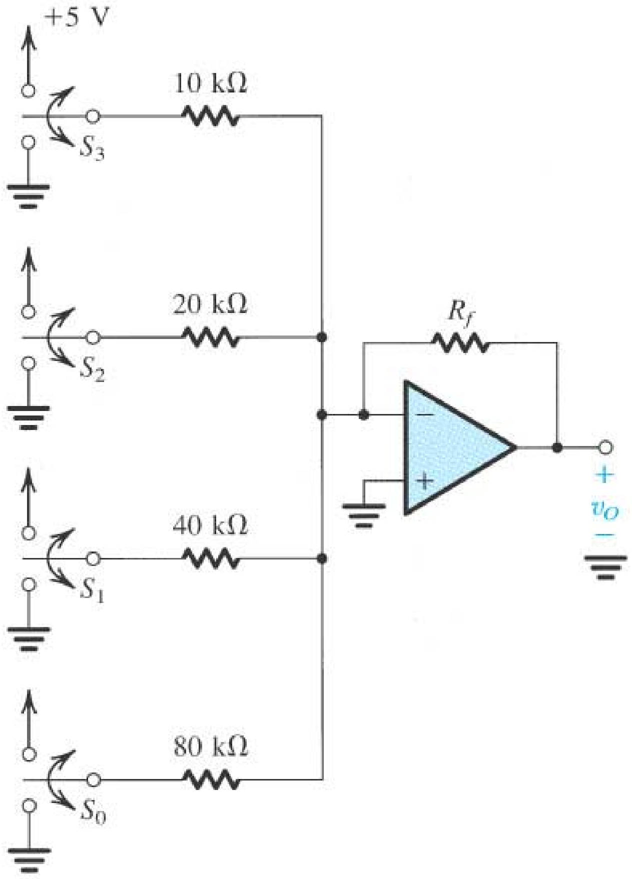



(b)

# Problem *2.31



# Problem *2.43




$$\frac{V_- - S_3}{10k} + \frac{V_- - S_2}{20k} + \frac{V_- - S_1}{40k} + \frac{V_- - S_0}{80k} + \frac{V_- - V_0}{R_f} = 0$$

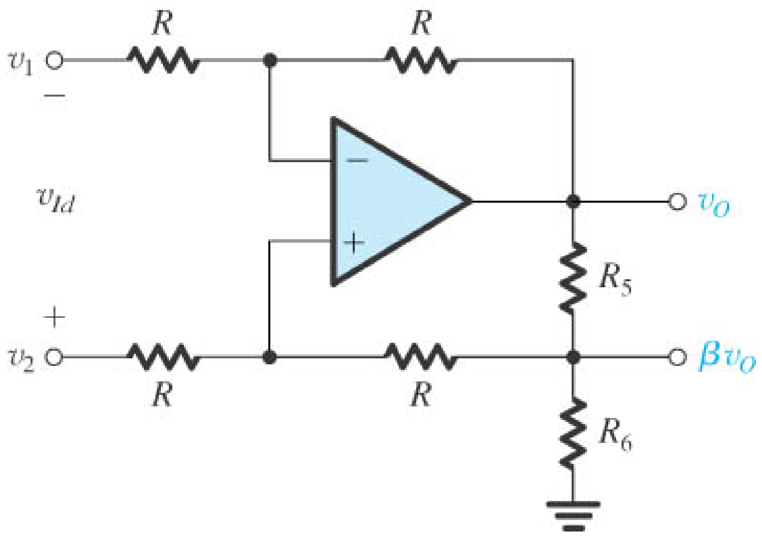
$$\frac{0 - 5S_3}{10k} + \frac{0 - 5S_2}{20k} + \frac{0 - 5S_1}{40k} + \frac{0 - 5S_0}{80k} + \frac{0 - V_0}{R_f} = 0$$

$$V_0 = -5R_f [\frac{S_3}{10k} + \frac{S_2}{20k} + \frac{S_1}{40k} + \frac{S_0}{80k}]$$

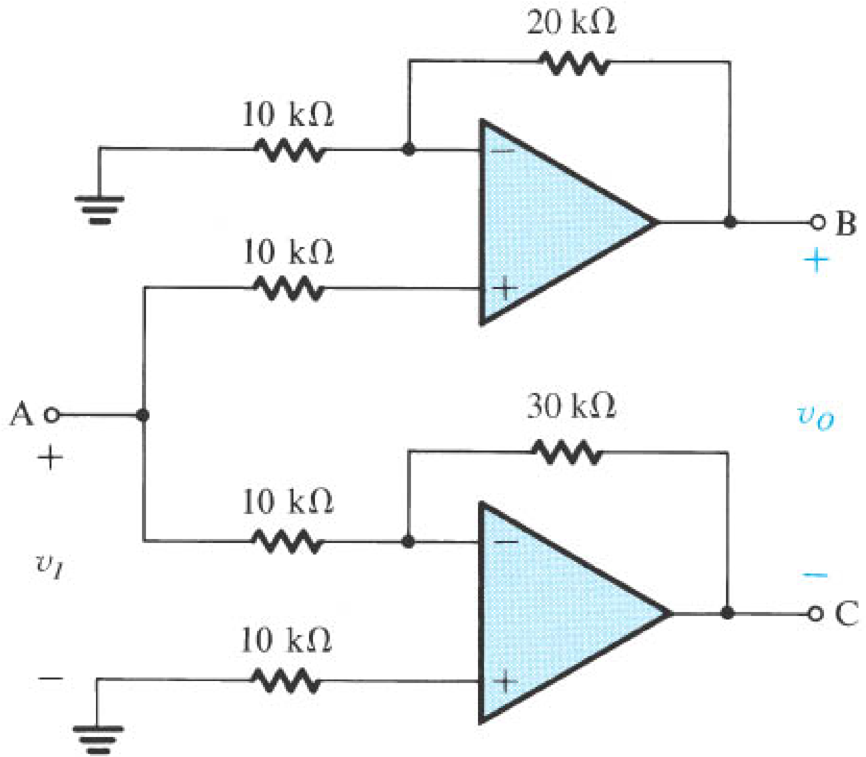
$$V_0 = -\frac{5R_f}{80k}[8S_3 + 4S_2 + 2S_1 + 1S_0]$$

$$= -\frac{R_f}{16k}[8S_3 + 4S_2 + 2S_1 + 1S_0]$$

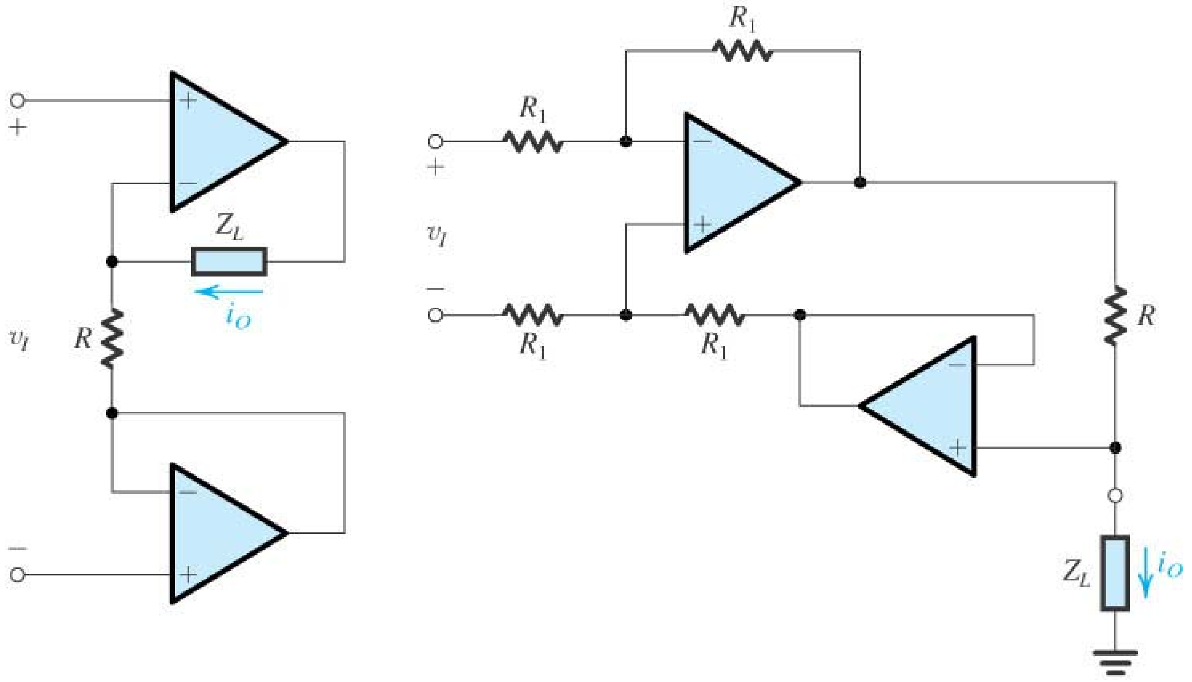
# Problem **2.69



# Problem *2.77



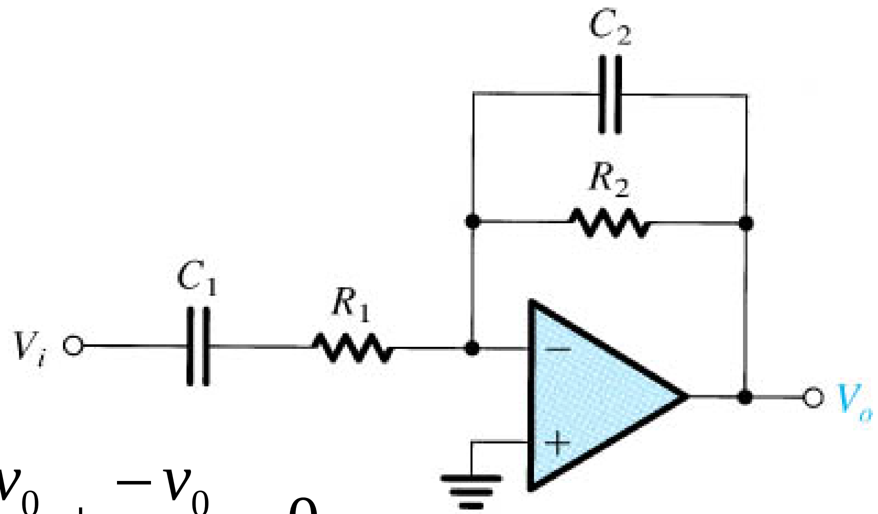
# Problem *2.78




(a)

(b)

# Problem C*2.126




$$\frac{-v_i}{R_1 + \frac{1}{sC_1}} + \frac{-v_0}{R_2} + \frac{-v_0}{\frac{1}{sC_2}} = 0$$

$$\frac{-v_0}{R_2} + \frac{-v_0}{\frac{1}{sC_2}} = \frac{v_i}{R_1 + \frac{1}{sC_1}}$$

$$\frac{-R_2(R_2 + \frac{1}{sC_2})}{R_2} v_o = \frac{v_i}{R_1 + \frac{1}{sC_1}}$$

$$\frac{v_o}{v_i} = -\frac{R_2}{(R_1 + \frac{1}{sC_1})(R_2 + \frac{1}{sC_2})}$$

Non-ideal OP amplifiers

1. Type A: Finite open-loop gain (unknown)
2. Type B: Finite open-loop gain = K
3. Type C: $A_{vo} \neq \infty, R_o \neq 0, R_i \neq \infty$

$$\left.\begin{array}{l}\text{1. Infinite input impedance}\\ \text{2. Zero output impedance}\\ \text{3. Zero common-mode gain}\\ \text{4. Infinite open-loop gain}\\ \text{5. Infinite bandwidth}\end{array}\right\}\ \text{Ideal OPA characters}$$

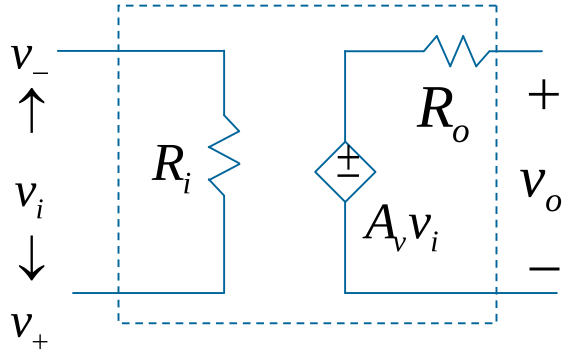
# Type A: Finite open-loop gain (unknown)

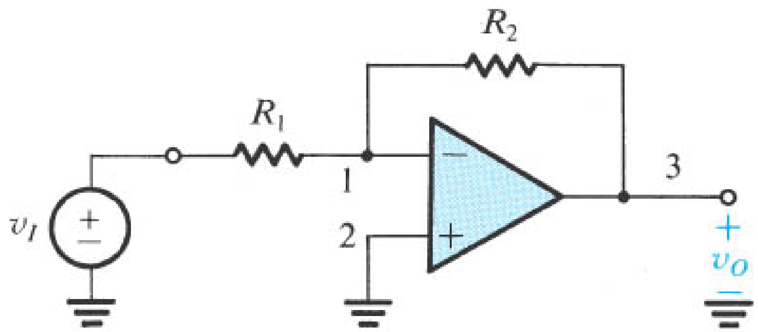
$$\boxed{v_- \neq v_+}$$

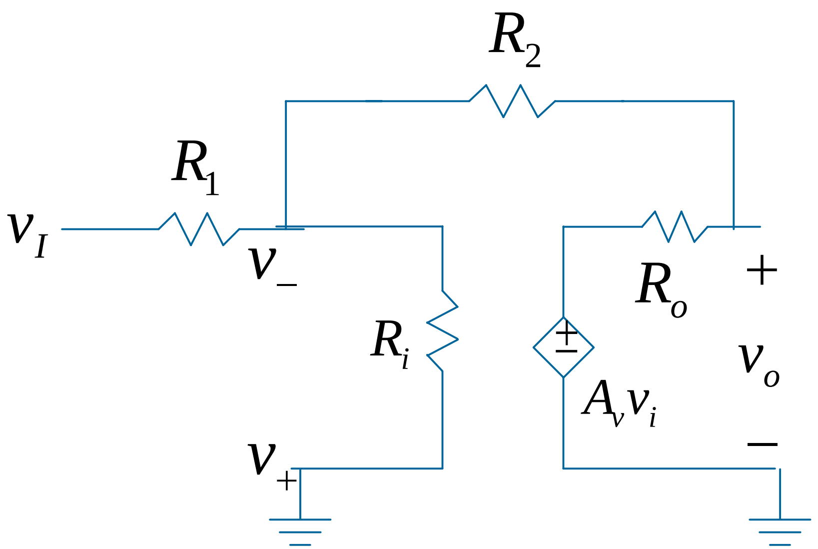
## Type B: Finite open-loop gain =K

$$\boxed{v_o = K(v_- - v_+)}$$

## Type C: $\boxed{A_v \neq \infty, R_o \neq 0, R_i \neq \infty}$




$$\frac{v_- - v_I}{R_1} + \frac{v_- - v_+}{R_i} + \frac{v_- - v_o}{R_2} = 0$$

$$\frac{v_o - v_-}{R_2} + \frac{v_o - A_v(v_- - v_+)}{R_o} = 0$$