Lecture 06

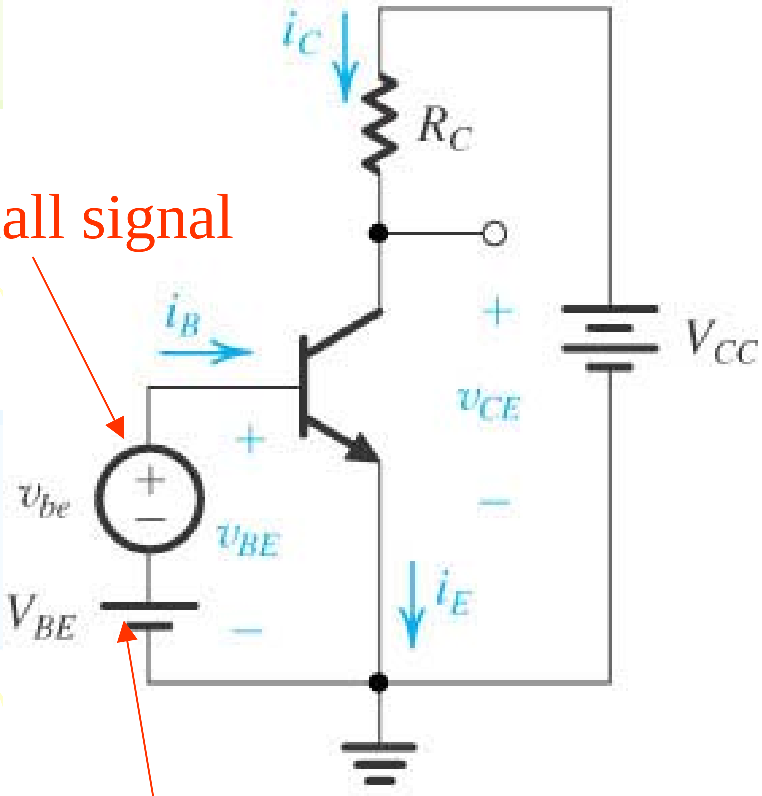
BJT Circuits

topics

- Small-signal operation
  - Hybrid π model
  - T-model
  - Early Effect
- Single-stage BJT amplifiers
  - Amplifier basic structure
  - CE Amplifier
  - CB Amplifier
  - CC Amplifier

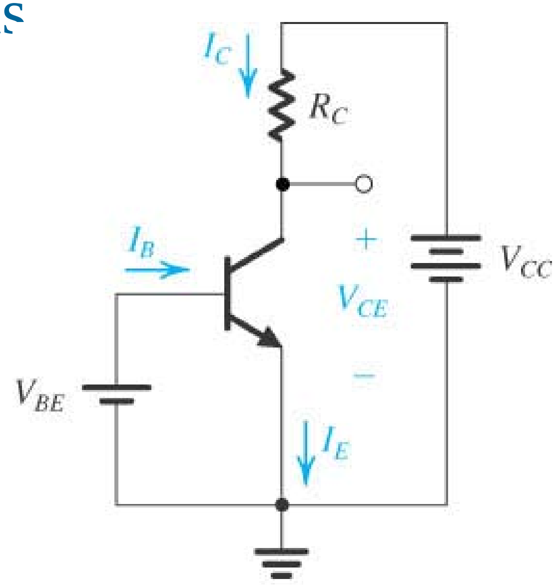
DC analysis

Small signal



(a)

DC analysis



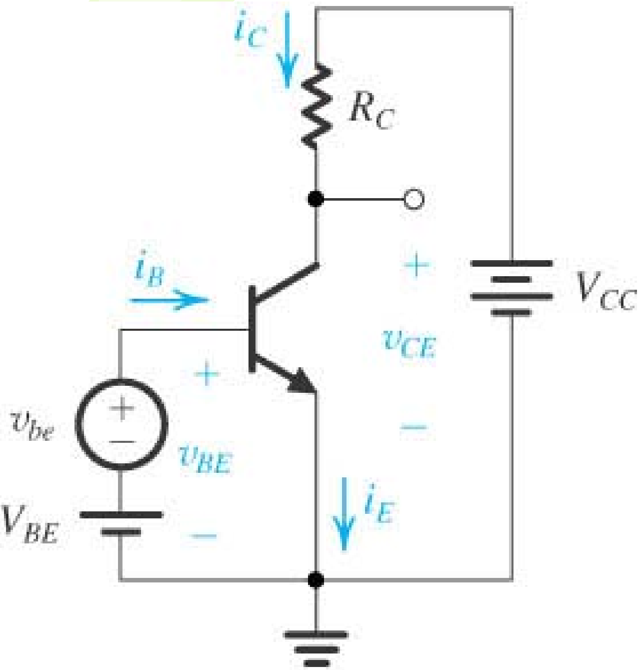
(b)

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}$$

$$I_E = \frac{I_C}{\alpha}$$

$$I_B = \frac{I_C}{\beta}$$

$$V_C = V_{CE} = V_{CC} - I_C R_C$$



(a)

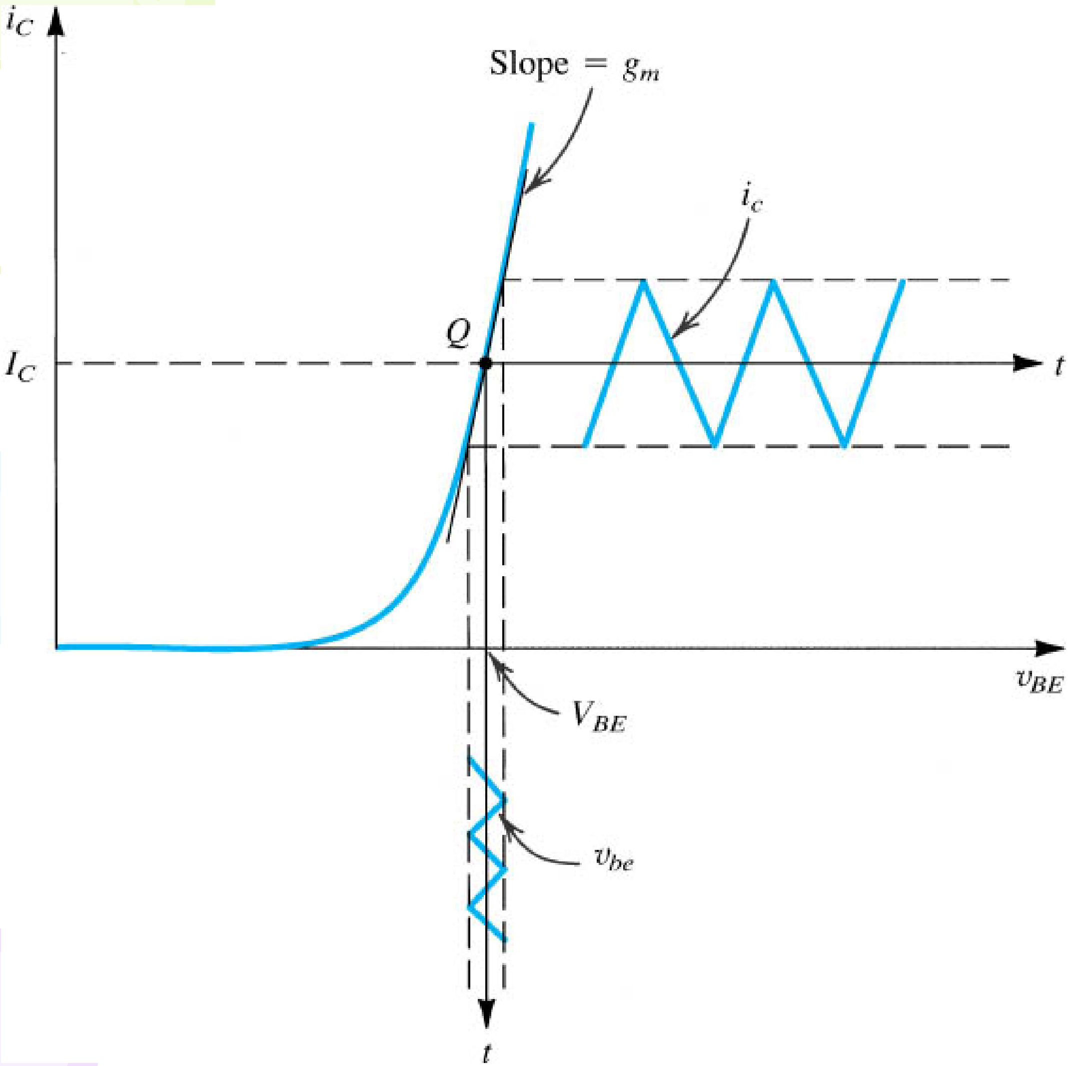
$$i_C = I_S e^{\frac{v_{BE}}{V_T}} = I_S e^{(\frac{V_{BE}+v_{be}}{V_T})} = I_S e^{\frac{V_{BE}}{V_T}} e^{\frac{v_{be}}{V_T}} = I_C e^{\frac{v_{be}}{V_T}}$$

$$i_C \approx I_C(1+\frac{v_{be}}{V_T}) \leftarrow if \quad v_{be} << V_T$$

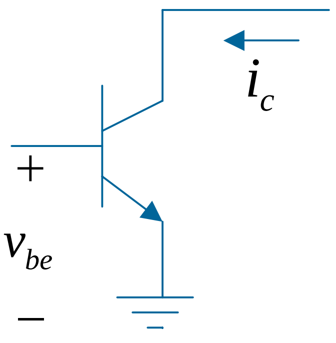
$$i_c = \frac{I_C}{V_T} v_{be} = g_m v_{be}$$

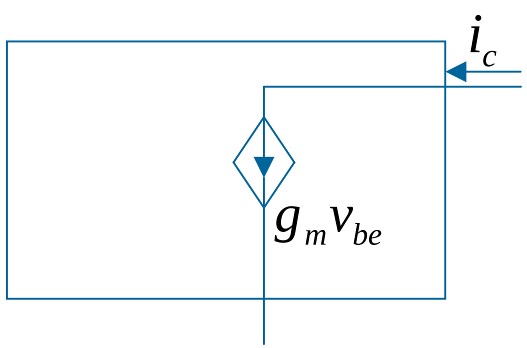
$$g_m = \frac{I_C}{V_T}$$

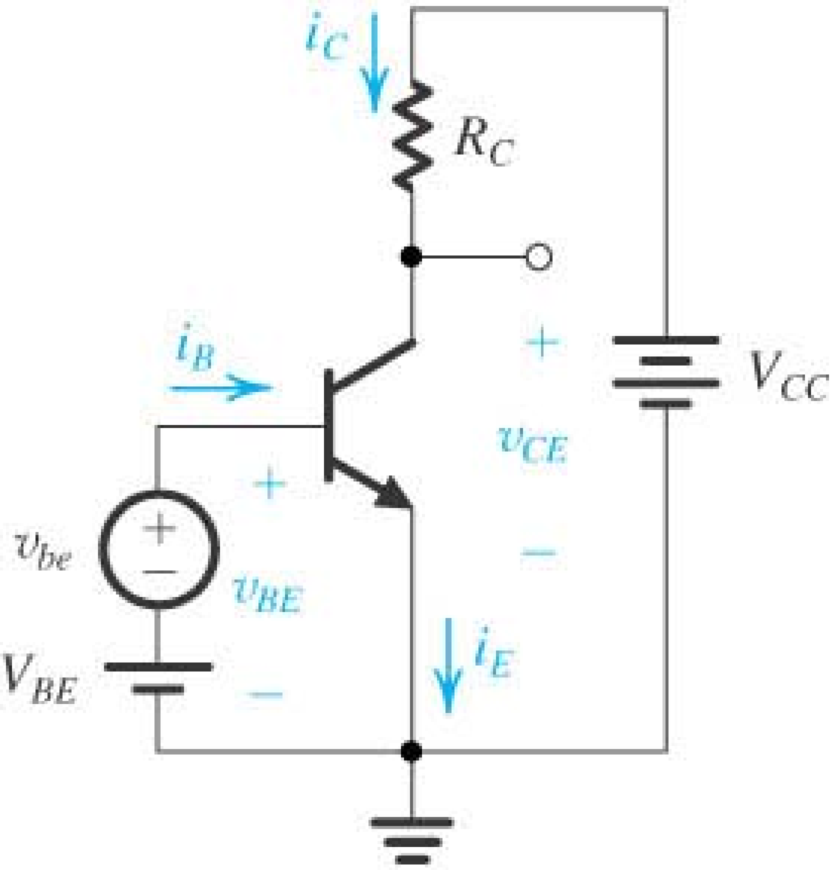
transconductance



$$g_m = \frac{\partial i_C}{\partial v_{BE}}\bigg|_{i_C = I_C}$$







(a)

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta}\frac{I_C}{V_T}v_{be} \quad i_c$$

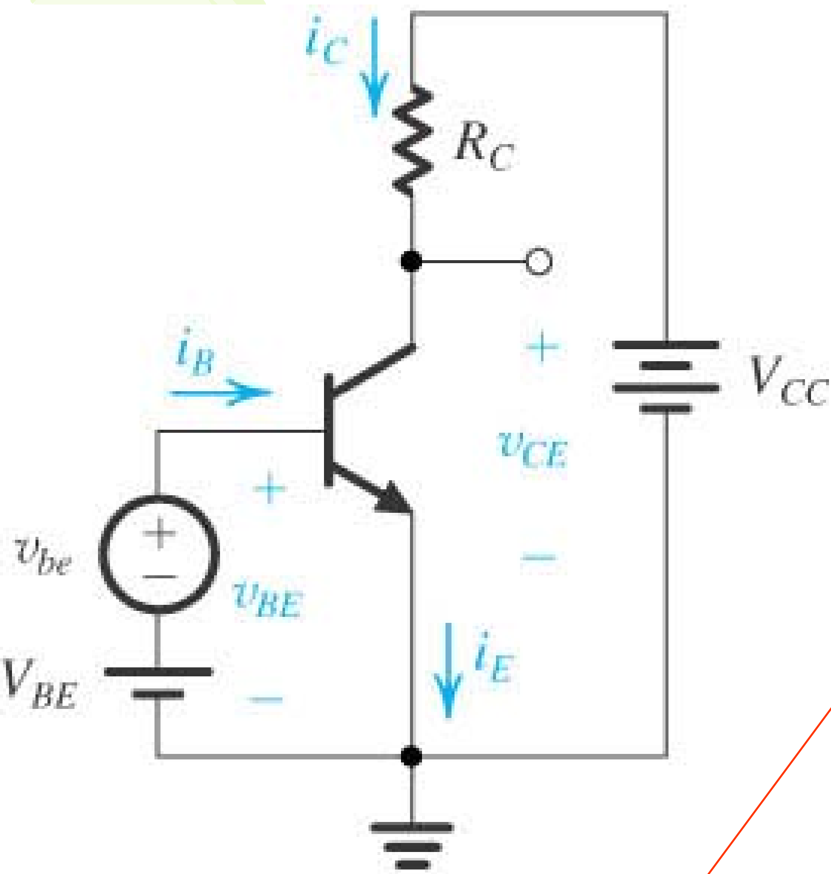
$$i_B = I_B + i_b$$

$$i_b = \frac{1}{\beta}\frac{I_C}{V_T}v_{be} = \frac{g_m}{\beta}v_{be}$$

$$r_\pi \equiv \frac{v_{be}}{i_b} \Rightarrow r_\pi = \frac{\beta}{g_m}$$

$$\therefore g_m = \frac{I_C}{V_T} \Rightarrow r_\pi = \frac{V_T}{I_B}$$

Small-signal input resistance



(a)

$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha}$$

$$i_E = I_E + i_e \qquad \boxed{i_c}$$

$$i_e = \frac{I_C}{\alpha V_T} v_{be} = \frac{I_E}{V_T} v_{be}$$

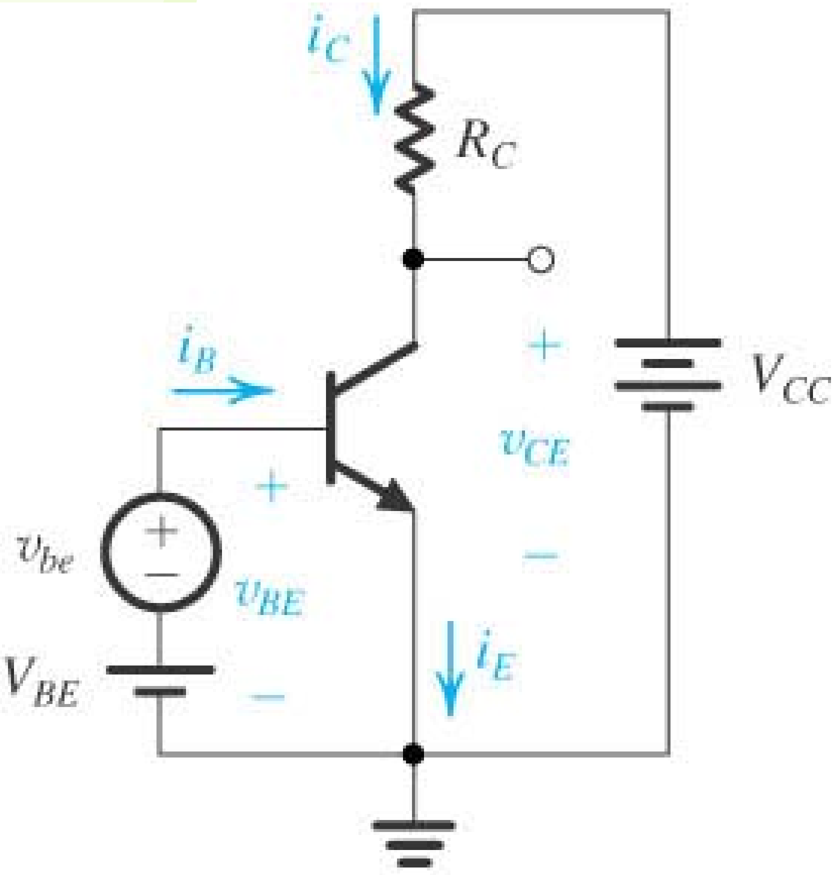
$$r_e \equiv \frac{v_{be}}{i_e} \Rightarrow r_e = \frac{V_T}{I_E}$$

$$\therefore g_m = \frac{I_C}{V_T} \Rightarrow r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$$

$$v_{be} = i_b r_\pi = i_e r_e$$

$$r_\pi = \frac{i_e}{i_b} r_e \Rightarrow r_\pi = (\beta + 1) r_e$$

Emitter resistance



(a)

Voltage gain

$$A_v \equiv \frac{v_c}{v_{be}}$$

$$v_C = V_{CC} - i_C R_C$$

$$= V_{CC} - (I_C + i_c) R_C$$

$$= (V_{CC} - I_C R_C) - i_c R_C$$

$$= V_C - i_c R_C$$

$$v_c = -i_c R_C = -g_m v_{be} R_C$$

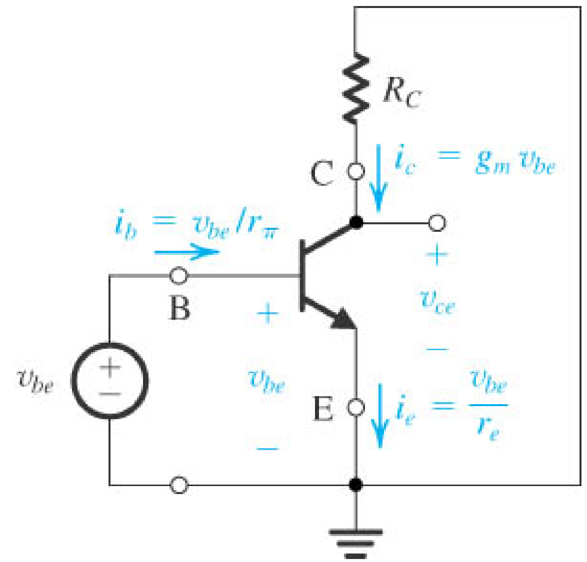
$$\Rightarrow A_v = -g_m R_C = -\frac{I_C R_C}{V_T}$$

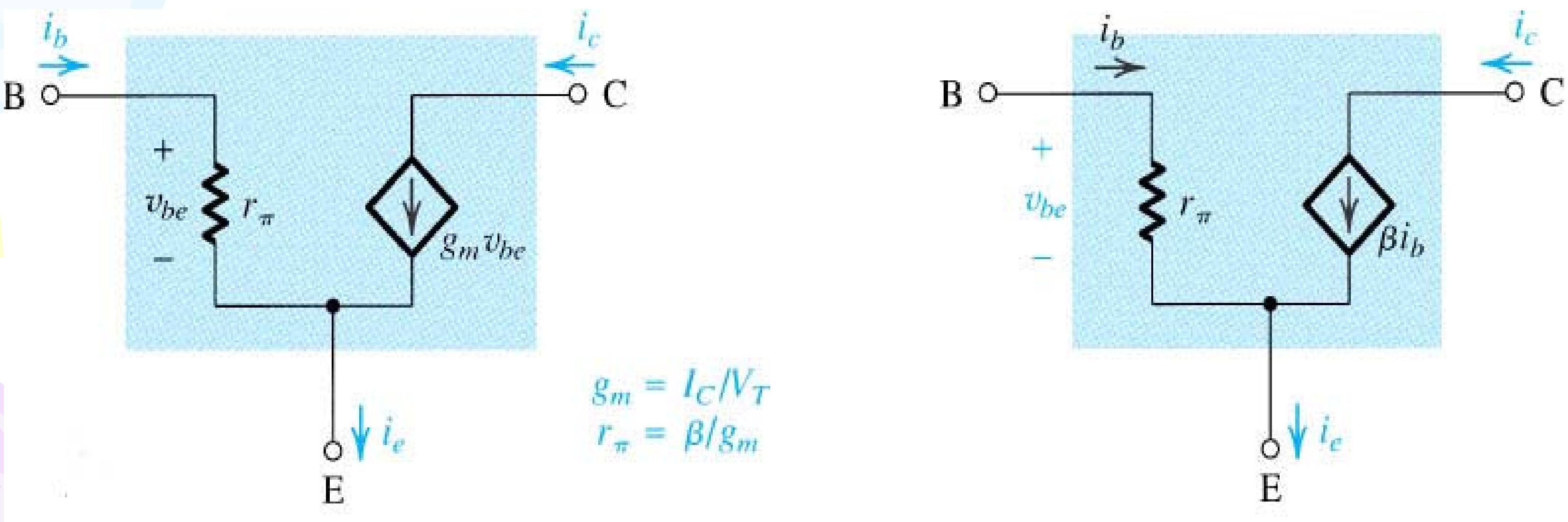
Hybird- π Model

$$i_e = \frac{v_{be}}{r_\pi} + g_m v_{be} = \frac{v_{be}}{r_\pi}(1 + g_m r_\pi)$$

$$= \frac{v_{be}}{r_\pi}(1+\beta) = \frac{v_{be}}{r_\pi/(1+\beta)} = \frac{v_{be}}{r_e}$$

$$g_m v_{be} = g_m (i_b r_\pi) = \beta i_b$$





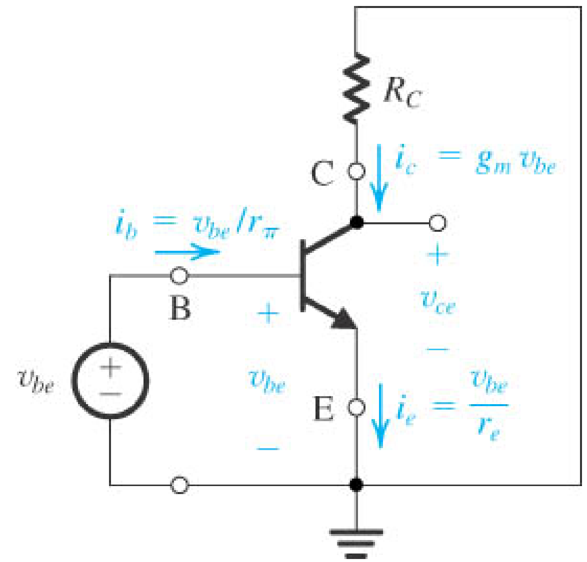
(a) (b)

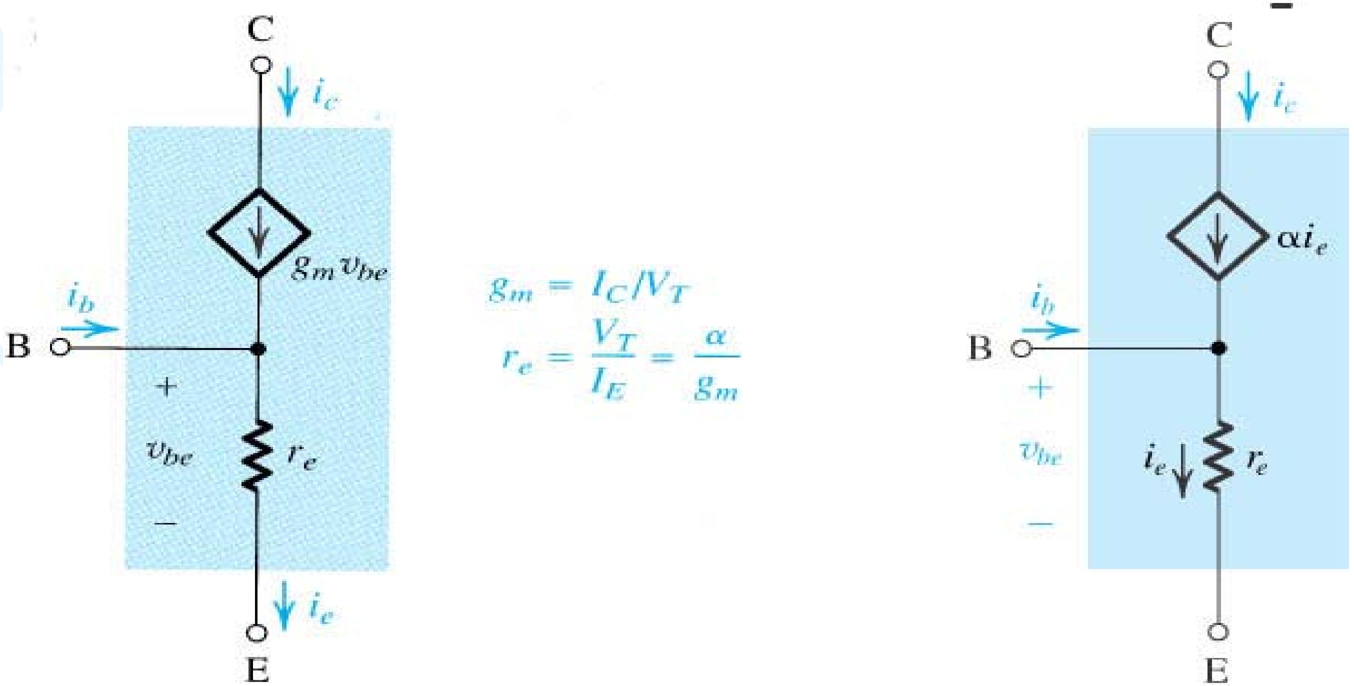
T- Model

$$i_b = \frac{v_{be}}{r_e} + g_m v_{be} = \frac{v_{be}}{r_e}(1 + g_m r_e)$$

$$= \frac{v_{be}}{r_e}(1+\alpha) = \frac{v_{be}}{r_e}\left(1 - \frac{\beta}{\beta+1}\right) = \frac{v_{be}}{r_\pi}$$

$$g_m v_{be} = g_m (i_e r_e) = \alpha i_e$$

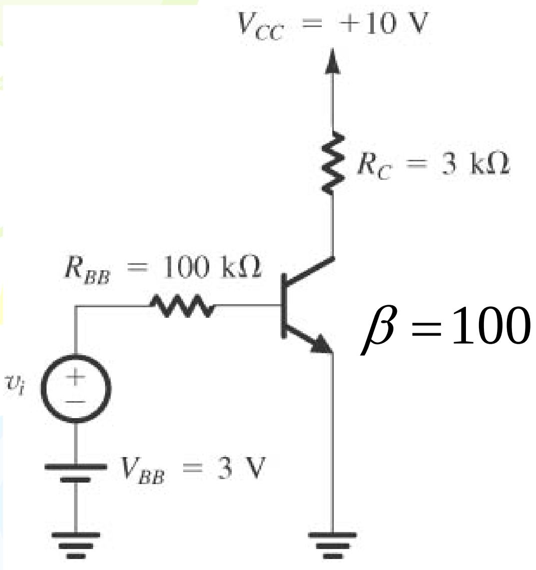




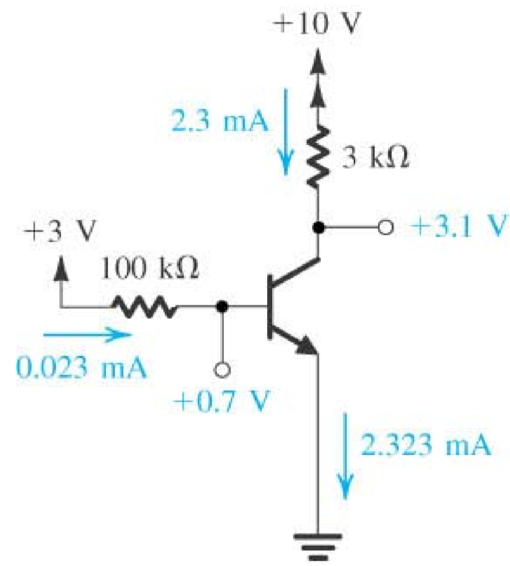
(a) (b)

Example 5.14

DC analysis



(a)



(b)

DC analysis

$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB}} = \frac{3-0.7}{100k} = 0.023mA$$

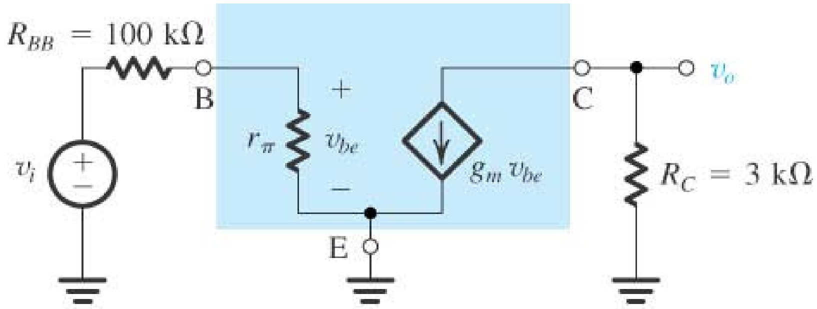
$$I_C = \beta I_B = 100 \times 0.023 = 2.3mA$$

$$I_E = \frac{I_C}{\alpha} = \frac{2.3}{0.99} = 2.32mA$$

$$V_C = V_{CC} - I_C R_C = 10 - 2.3 \times 3 = 3.1V$$

Confirm transistor in active mode

Small-signal analysis



(c)

$$r_e = \frac{V_T}{I_E} = \frac{25mV}{2.32mA} = 10.8\Omega$$

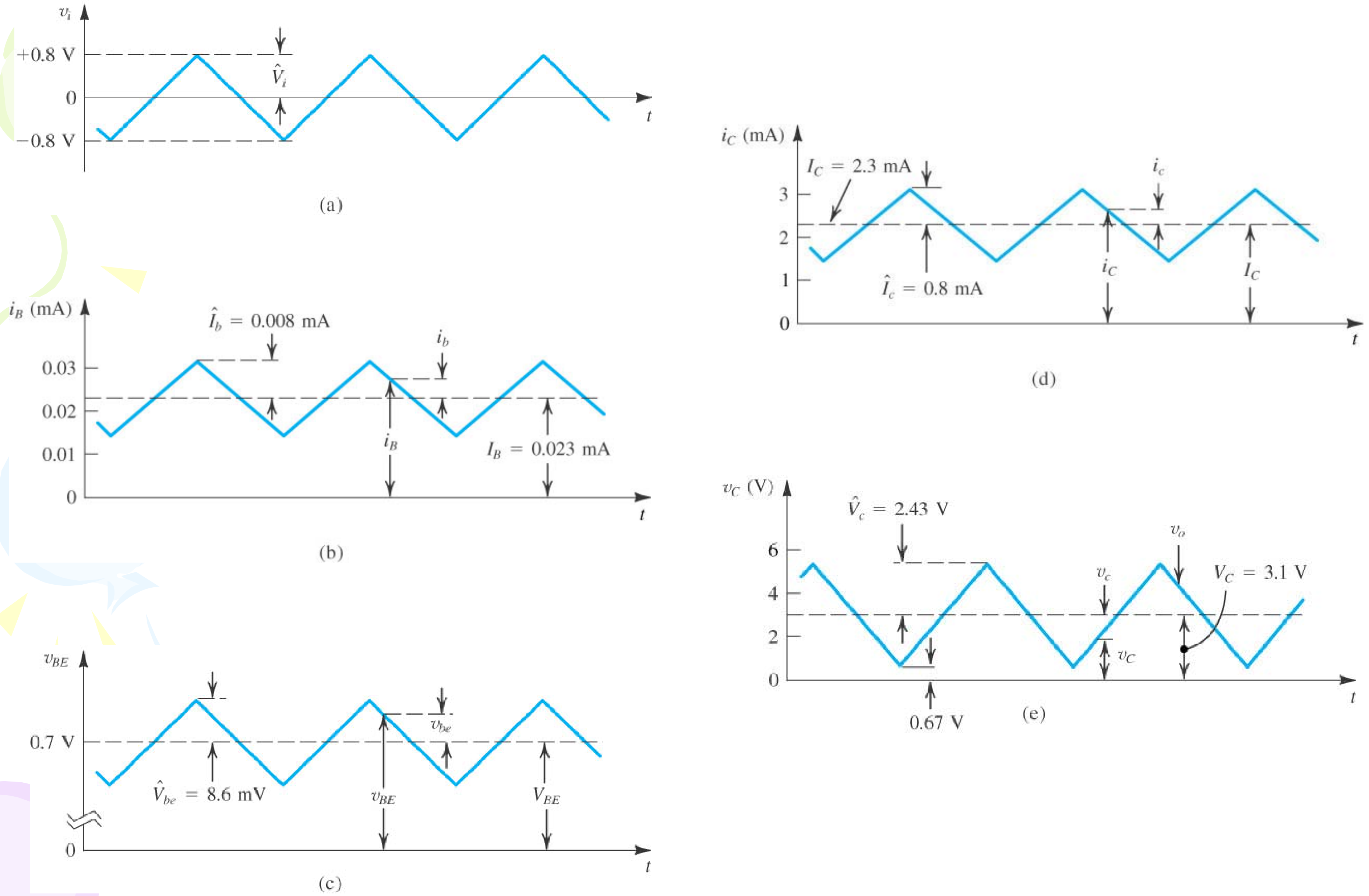
$$g_m = \frac{I_C}{V_T} = \frac{2.3mA}{25mV} = 92\, {}^{mA}\!/_{V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{92} = 1.09k\Omega$$

$$v_{be} = v_i \frac{r_\pi}{r_\pi + R_{BB}} = v_i \frac{1.09}{101.09} = 0.011 v_i$$

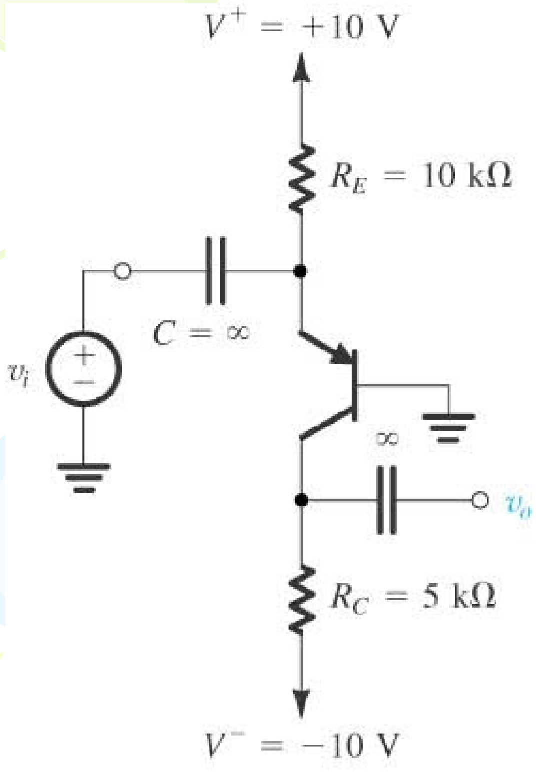
$$v_o = -g_m v_{be} R_C = -92 \times 0.011 v_i \times 3 = -3.04 v_i$$

$$A_v = \frac{v_o}{v_i} = -3.04$$



v_i
+0.8 V
0
−0.8 V
$\hat{V}_i$
t
(a)
i_B (mA)
$\hat{I}_b$ = 0.008 mA
i_b
0.03
0.02
0.01
0
i_B
I_B = 0.023 mA
t
(b)
v_{BE}
0.7 V
v_{be}
$\hat{V}_{be}$ = 8.6 mV
v_{BE}
V_{BE}
0
t
(c)
i_C (mA)
I_C = 2.3 mA
i_c
3
2
1
0
i_C
I_C
$\hat{I}_c$ = 0.8 mA
t
(d)
v_C (V)
$\hat{V}_c$ = 2.43 V
v_o
6
4
2
0
v_c
V_C = 3.1 V
v_C
0.67 V
t
(e)

Example 5.16



(a)

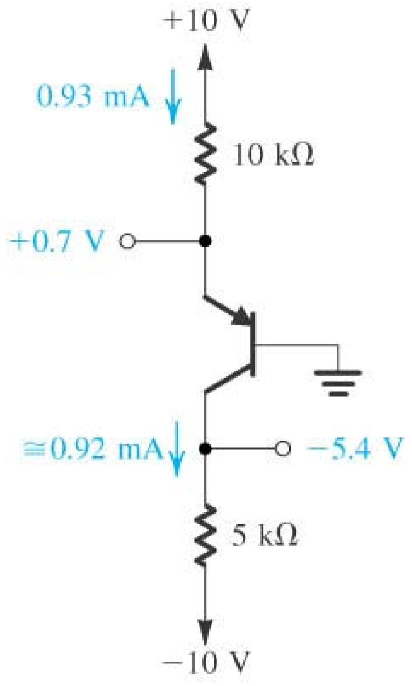
DC analysis

$$I_E = \frac{10-0.7}{10k} = 0.93mA$$

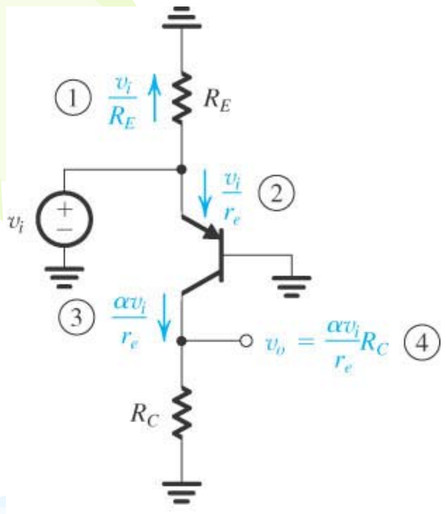
$$I_C = \alpha I_E = 0.99 \times 0.93 = 0.92mA$$

$$V_C = I_C R_C - 10V = -5.4V$$

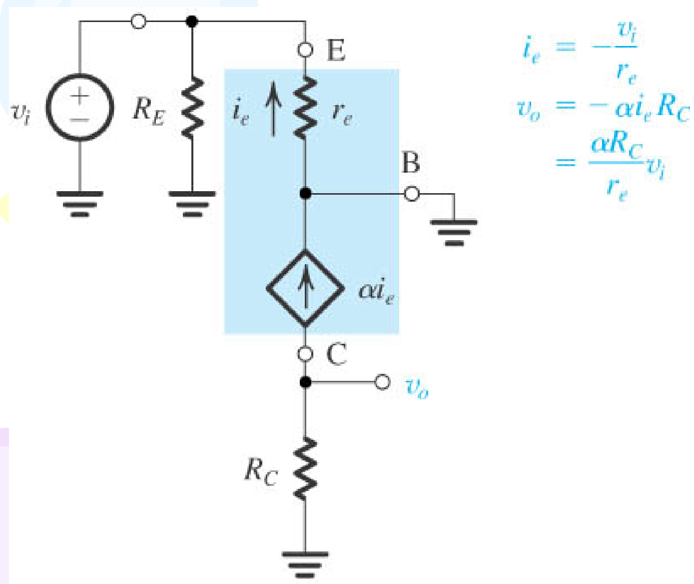
Confirm transistor in active mode



(b)



(d)



(c)

Small-signal analysis

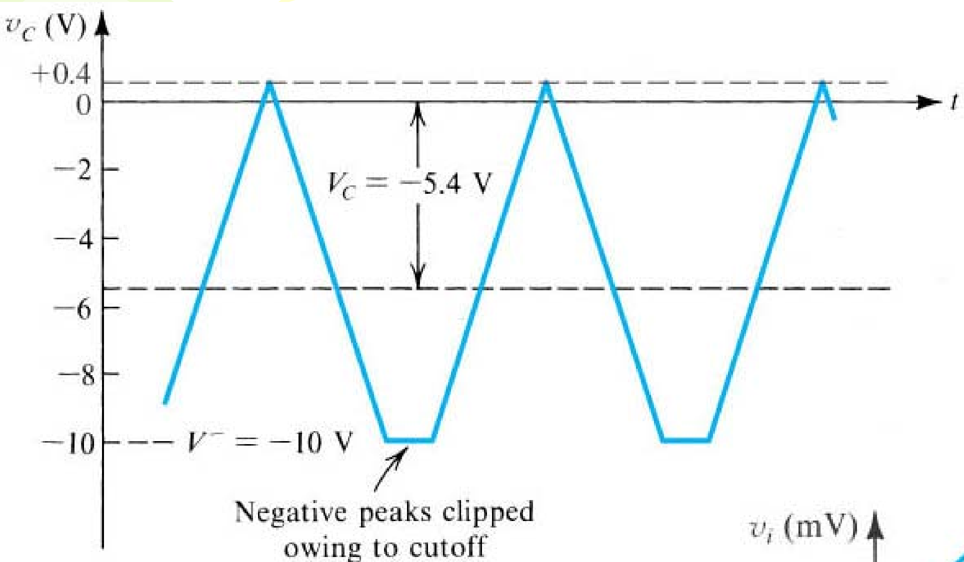
$$r_e = \frac{V_T}{I_E} = \frac{25mV}{0.93mA} = 27\Omega$$

$$v_o = -\alpha i_e R_C$$

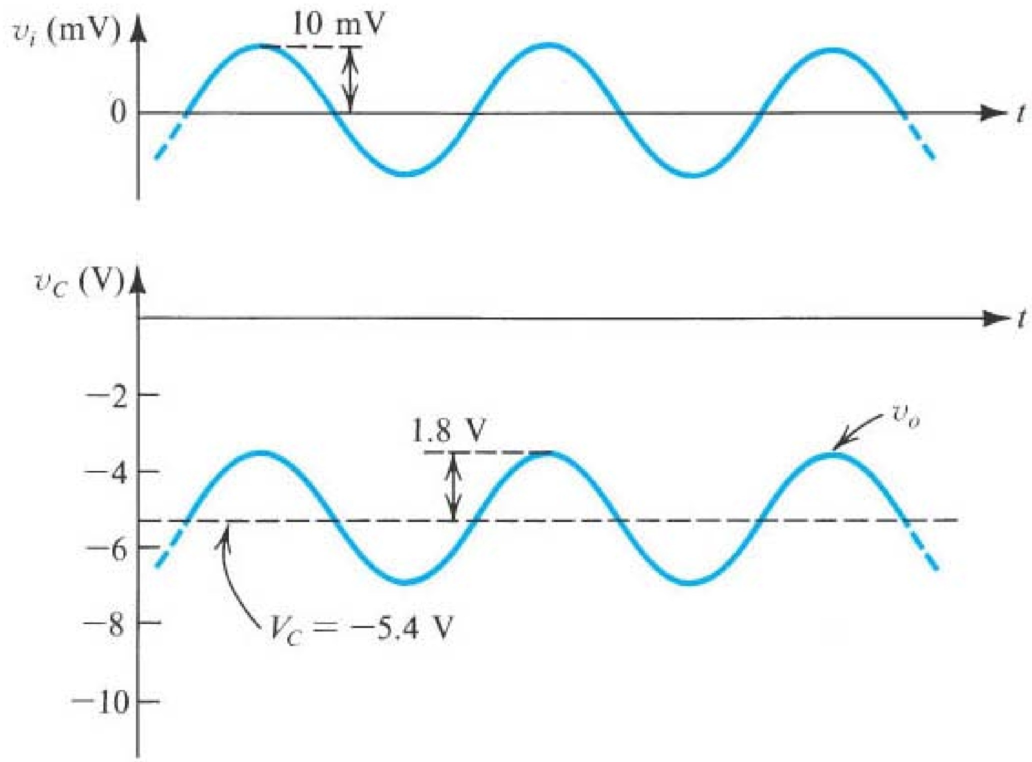
$$i_e = -\frac{v_i}{r_e}$$

$$\Rightarrow v_o = \alpha \frac{R_C}{r_e} v_i$$

$$A_v = \frac{v_o}{v_i} = \alpha \frac{R_C}{r_e} = 0.99 \frac{5k}{27} = 183.3$$

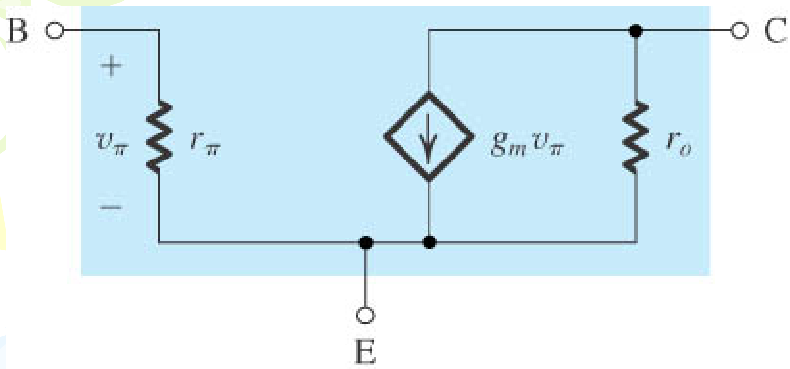


v_C (V)
+0.4
0
−2
−4
−6
−8
−10
t
$V_C = -5.4$ V
$V^- = -10$ V
Negative peaks clipped owing to cutoff

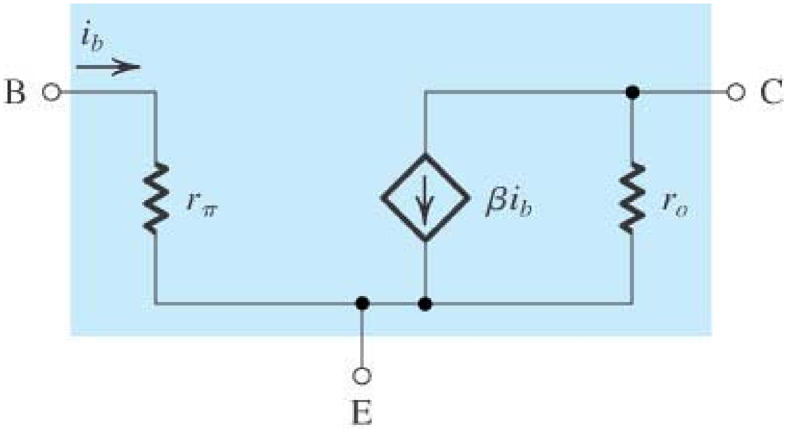


v_i (mV)
10 mV
0
t
v_C (V)
t
−2
−4
−6
−8
−10
1.8 V
v_o
$V_C = -5.4$ V

Small-signal model with Early Effect



(a)

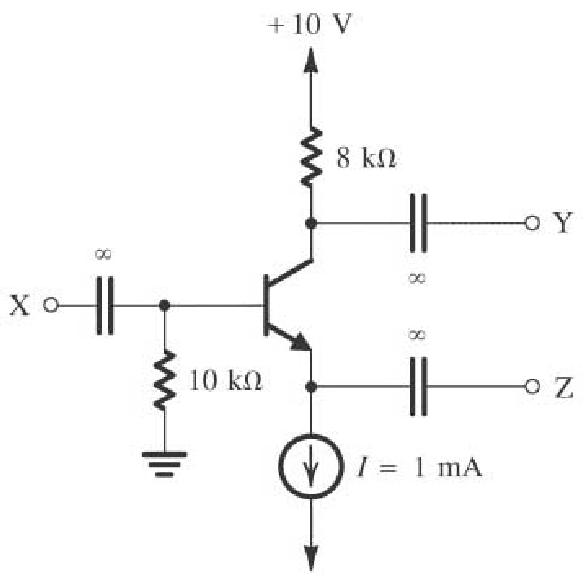


(b)

$$r_o = \frac{V_A + V_{CE}}{I_E} \approx \frac{V_A}{I_E}$$

$$V_A \rightarrow \quad Early \quad voltage$$

Exercise 5.40



$$given$$

$$I_E = 1mA$$

$$\beta = 100$$

$$V_A = 100V$$

$$I_C = \alpha I_E = 0.99 \times 1mA = 0.99mA$$

$$V_B = -I_B \times 10k = -\frac{0.99m}{100} \times 10k = -0.099V$$

$$V_E = V_B - 0.7V = -0.799V$$

Active mode

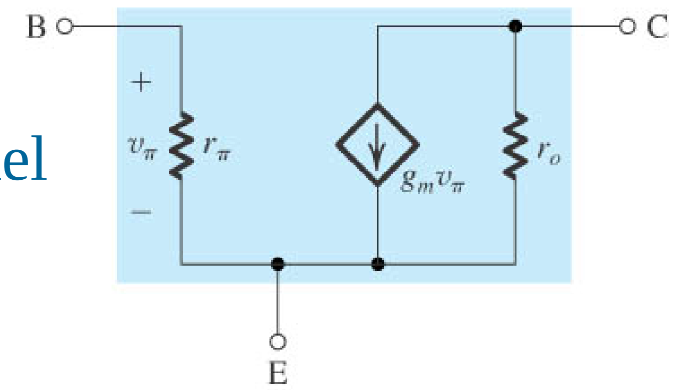
$$V_C = I_C R_C - 10V = -0.99m \times 8k + 10V = 2.08V$$

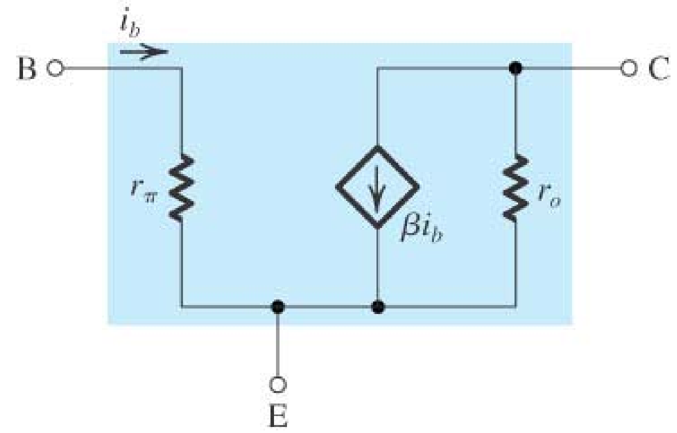
$$r_o = \frac{V_A}{I_E} = \frac{100mV}{1mA} = 100\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{0.99mA}{25mV} = 0.0396\, {}^{mA}\!/\!{}_{V}$$

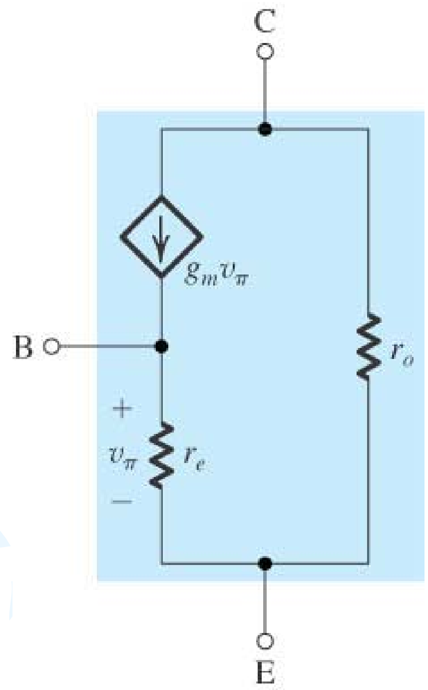
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.0396} = 2.525k\Omega$$

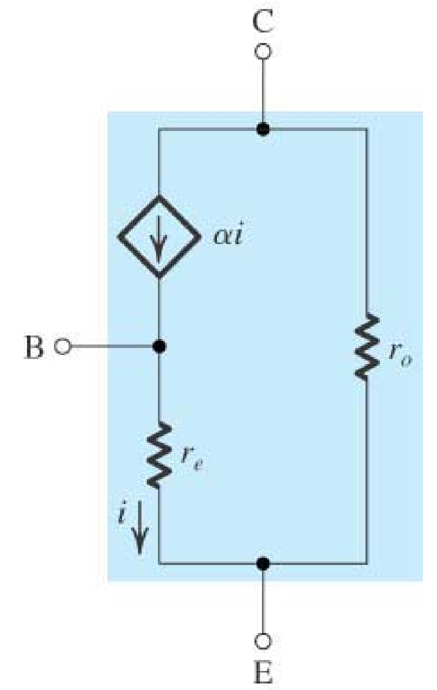
Hybrid- π Model





T Model



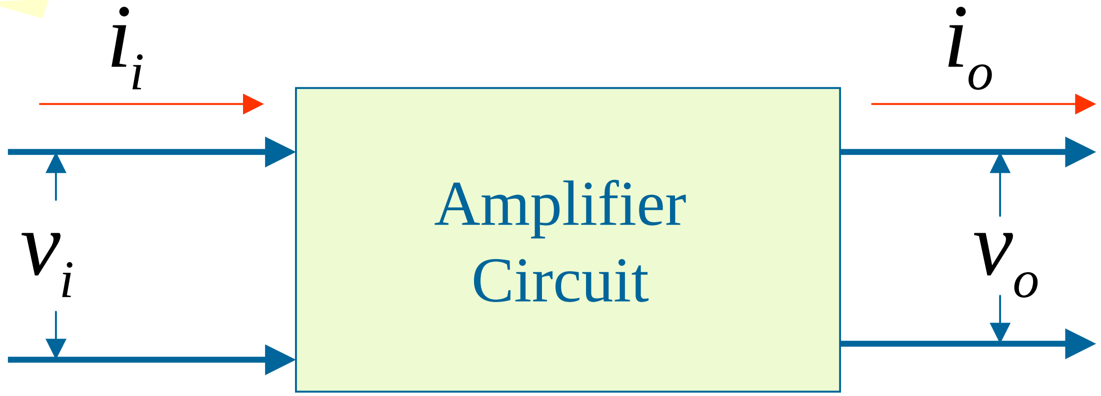


Signal-stage BJT amplifiers

- Common-Emitter (CE) Amplifier
- Common-Base (CB) Amplifier
- Common-Collector (CC) Amplifier

Amplifier Categories

- Voltage Amplifier
- Current Amplifier
- Transconductance Amplifier
- Transresistance Amplifier



$$gain \equiv \frac{output}{input}$$

$$power\,gain \quad A_P \equiv \frac{v_o i_o}{v_i i_i}$$

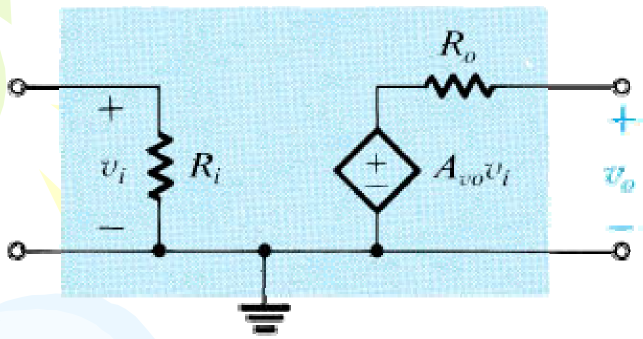
$$voltage\,gain \quad A_v \equiv \frac{v_o}{v_i}$$

$$current\,gain \quad A_i \equiv \frac{i_o}{i_i}$$

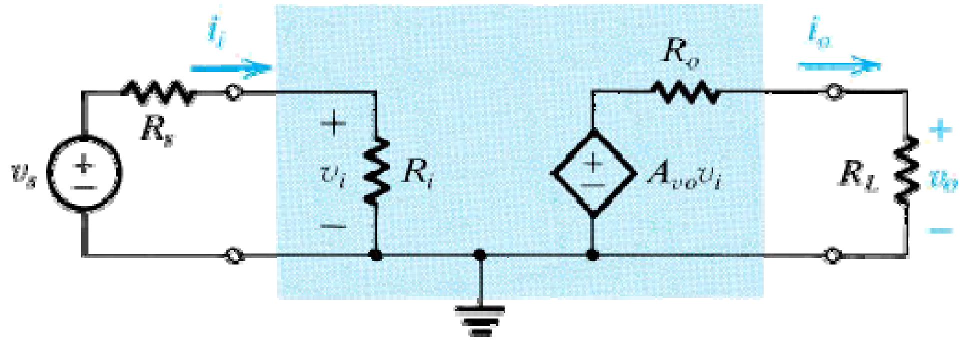
$$transconduc\tan ce \quad G_m \equiv \frac{i_o}{v_i}$$

$$transresis\tan ce \quad R_m \equiv \frac{v_o}{i_i}$$

Voltage Amplifier



(a)



(b)

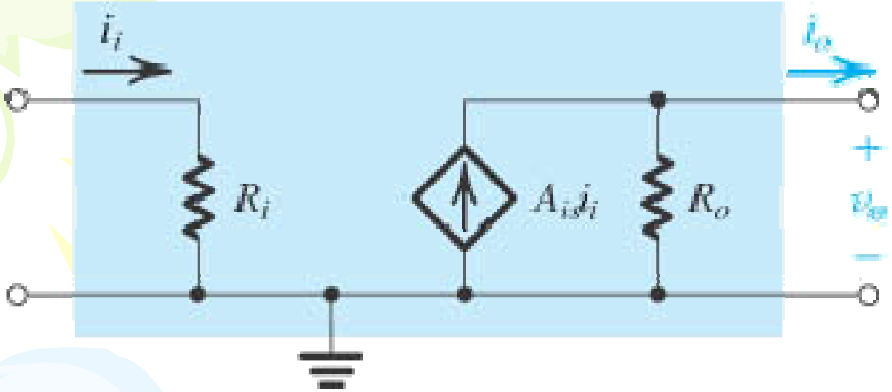
Open-Circuit voltage Gain :

$$\boxed{A_{vo} \equiv \left.\frac{v_o}{v_i}\right|_{i_o=0}}$$

Ideal voltage amplifier :

$$\boxed{\begin{aligned} R_i &= \infty \\ R_o &= 0 \end{aligned}}$$

Current Amplifier



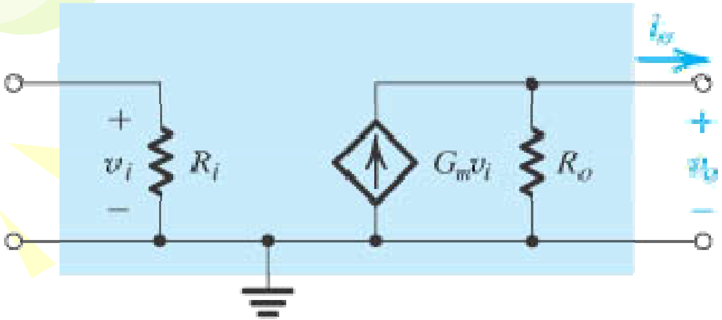
Short-Circuit current Gain :

$$\boxed{A_{is} \equiv \left.\frac{i_o}{i_i}\right|_{v_o=0}}$$

Ideal voltage amplifier :

$$\boxed{\begin{aligned} R_i &= 0 \\ R_o &= \infty \end{aligned}}$$

Transconductance Amplifier



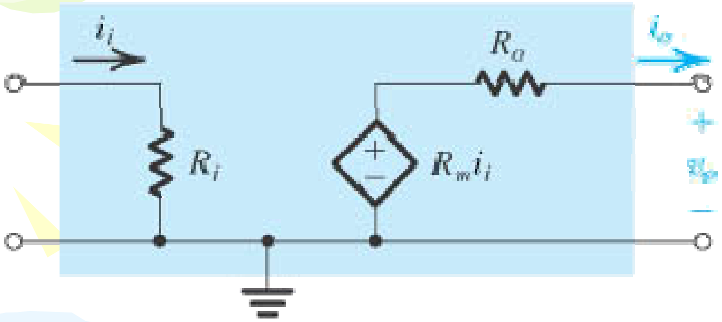
Short-Circuit Transconductance :

$$\boxed{G_m \equiv \left.\frac{i_o}{v_i}\right|_{v_o=0}}$$

Ideal transconductance amplifier :

$$\boxed{\begin{aligned} R_i &= \infty \\ R_o &= \infty \end{aligned}}$$

Transresistance Amplifier



Open-Circuit Transresistance :

$$R_m \equiv \left.\frac{v_o}{i_i}\right|_{i_o=0}$$

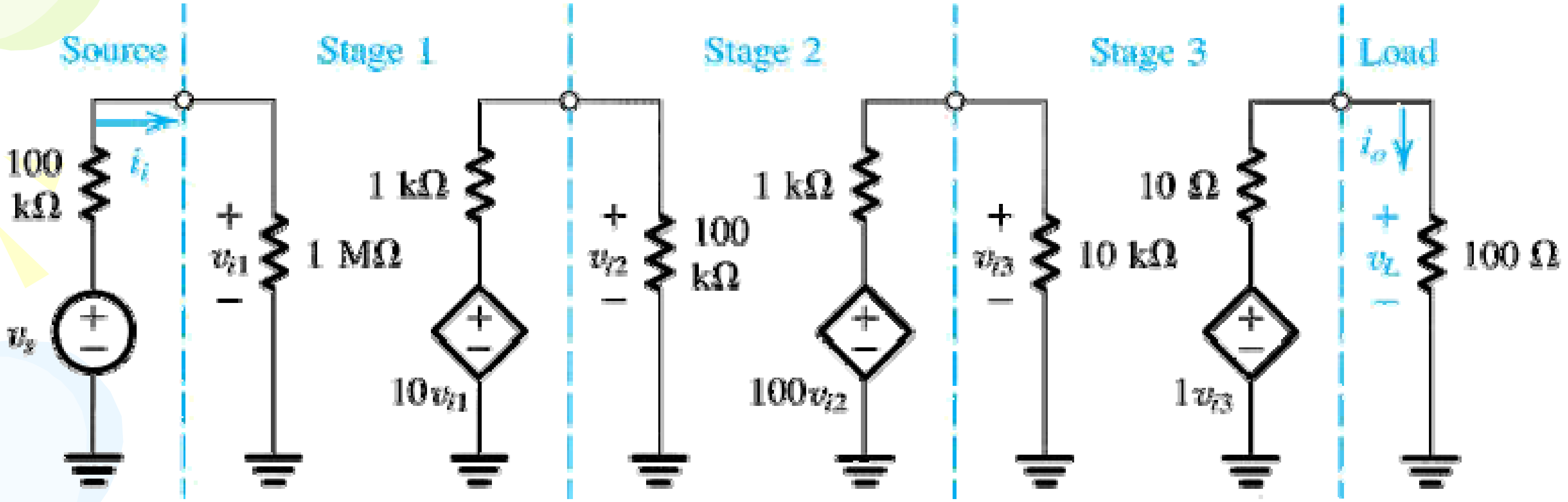
Ideal Transresistance amplifier :

$$R_i = 0$$

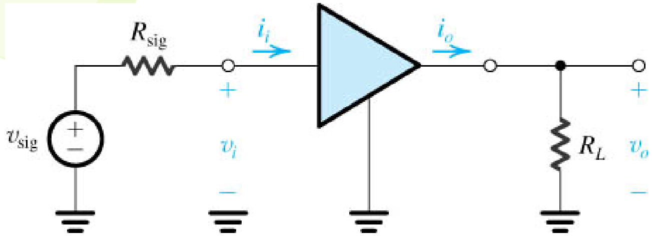
$$R_o = 0$$

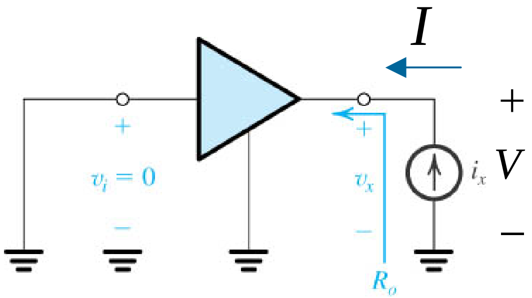
Example 1.3

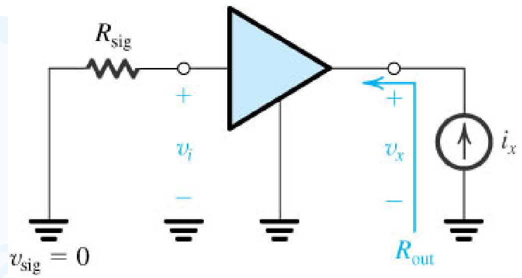
Cascade Amplifier

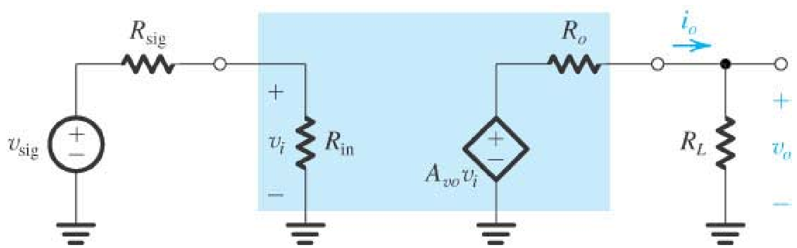


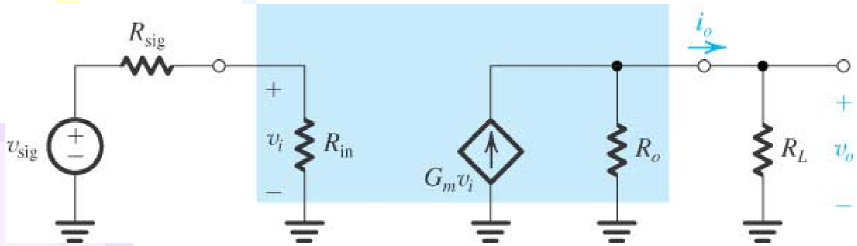
Amplifier basic structure

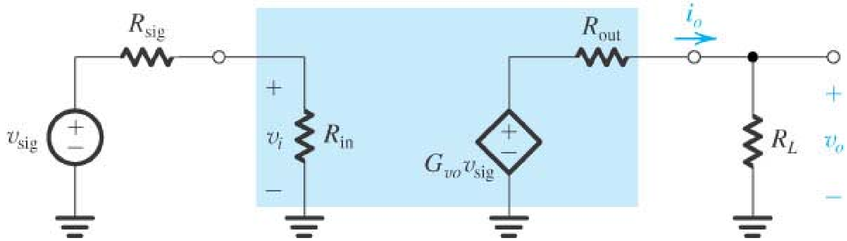




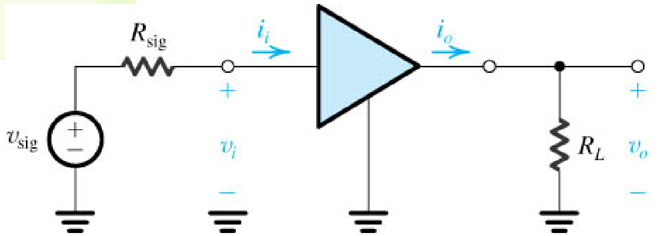


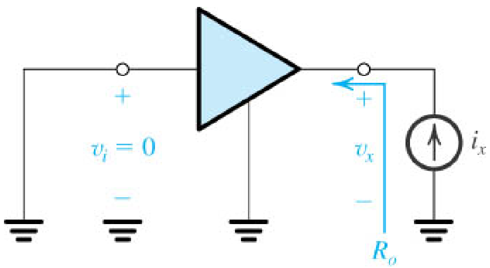






Definitions:





No load input resistance $R_i \equiv \left.\dfrac{v_i}{i_i}\right|_{R_L=\infty}$

Input resistance $R_{in} \equiv \dfrac{v_i}{i_i}$

Open-circuit voltage gain $A_v \equiv \left.\dfrac{v_o}{v_i}\right|_{R_L=\infty}$

Voltage gain $A_v \equiv \dfrac{v_o}{v_i}$

Short-circuit current gain $A_{is} \equiv \left.\dfrac{i_o}{i_i}\right|_{R_L=0}$

Current gain $A_i \equiv \dfrac{i_o}{i_i}$

Short-circuit transconductance $G_m \equiv \left.\dfrac{i_o}{v_i}\right|_{R_L=0}$

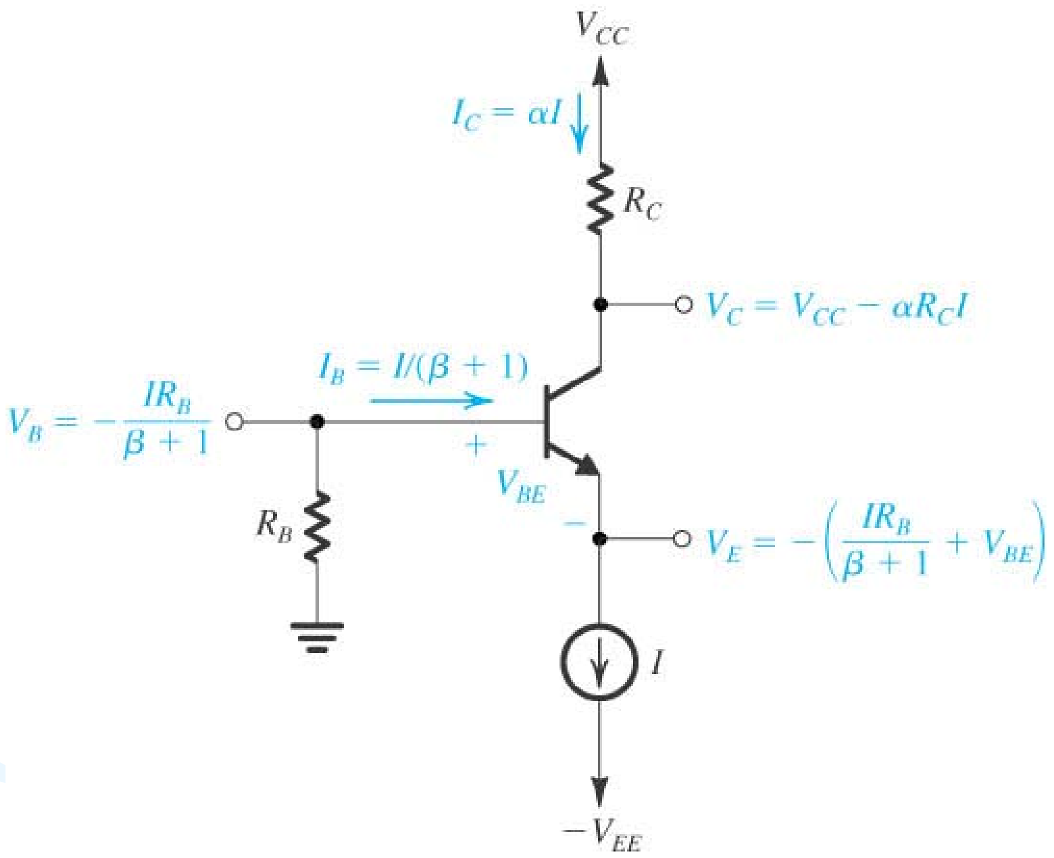
Amplifier propre output resistance $R_o \equiv \left.\dfrac{v_x}{i_x}\right|_{v_i=0}$

Output resistance $R_o \equiv \left.\dfrac{v_x}{i_x}\right|_{v_{signal}=0}$

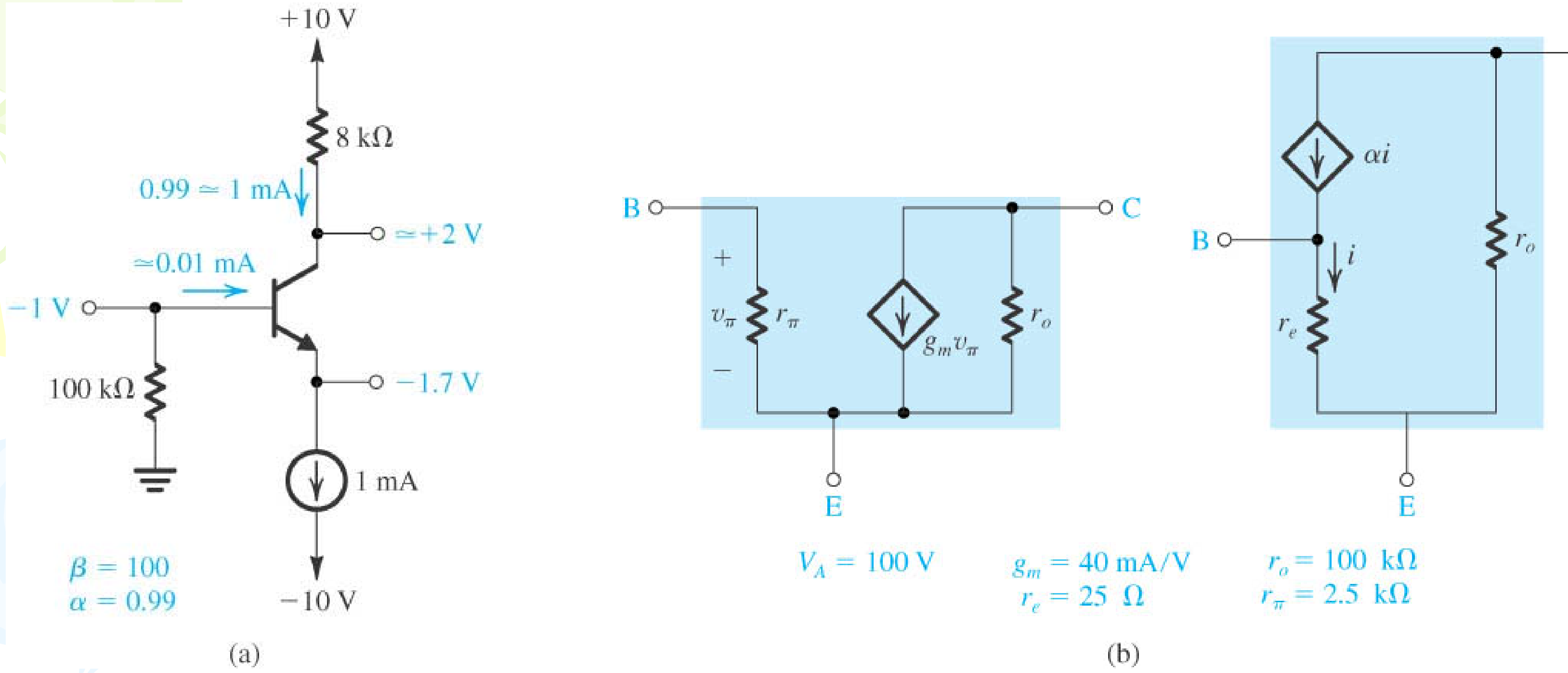
Open-circuit overall voltage gain $G_{vo} \cong \left.\dfrac{v_o}{v_{signal}}\right|_{R_L=\infty}$

Overall voltage gain $G_v \equiv \dfrac{v_o}{v_{signal}}$

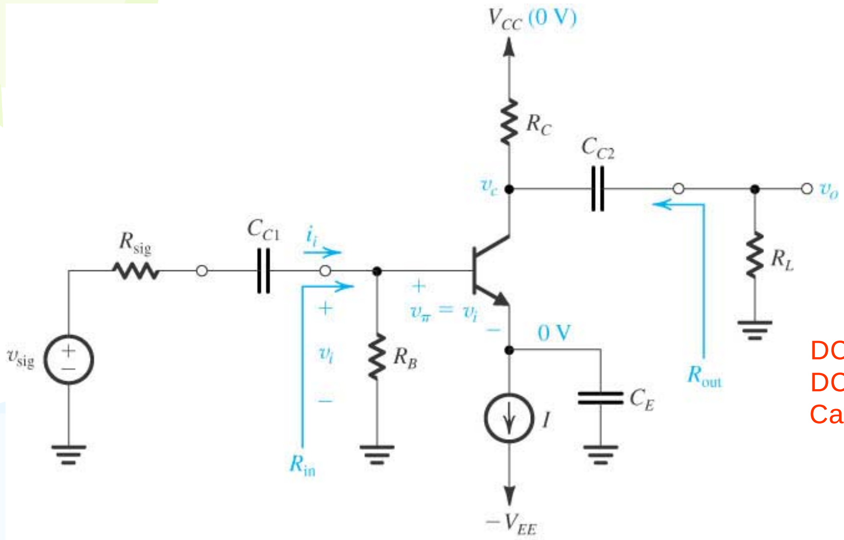
Constant current bias (we can also use the others biasing structures)



Exercise 5.41

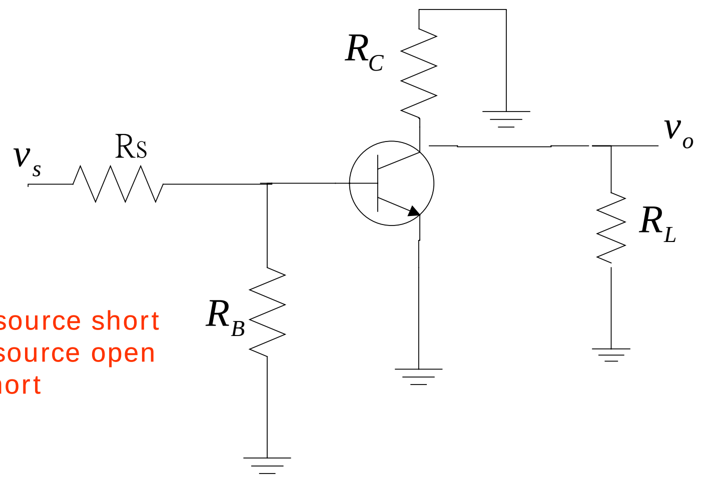


Common-Emitter amplifier



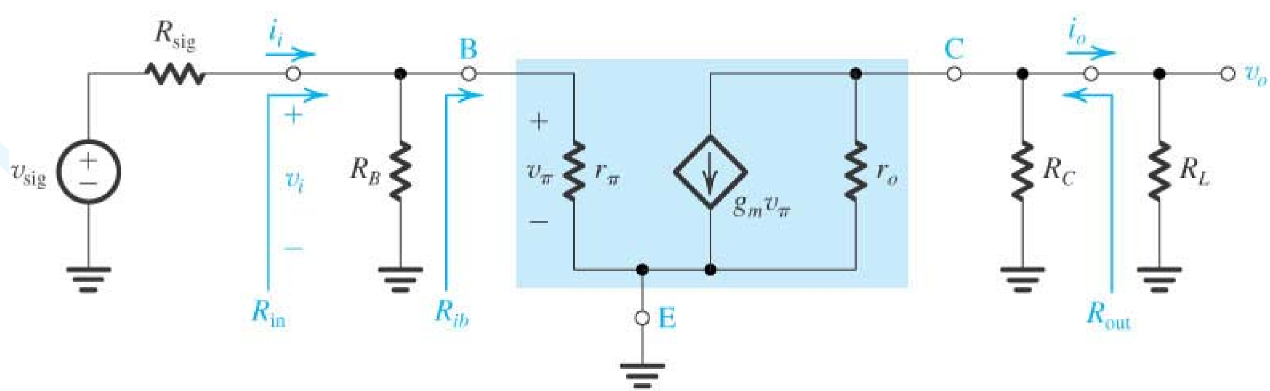
(a)

Small signal CKT

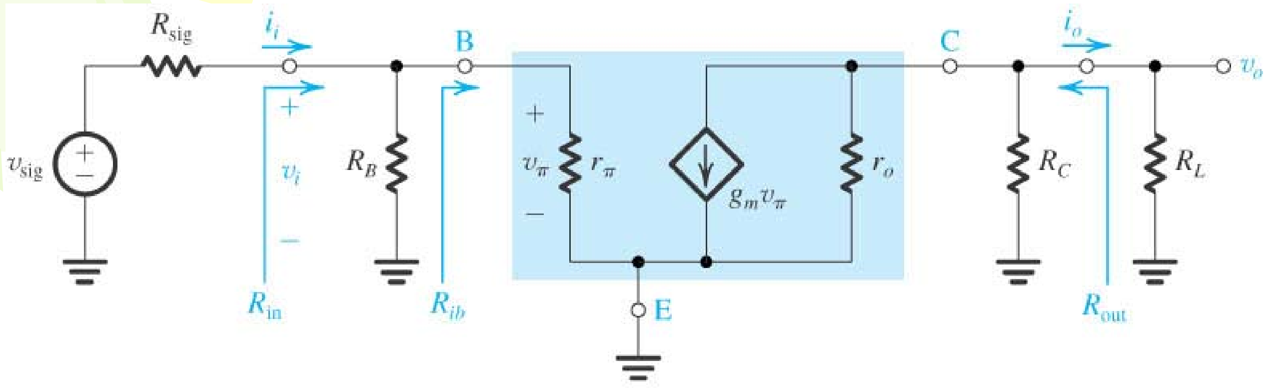


DC voltage source short
DC current source open
Capacitor short

Replace transistor by Pi-model



(b)



(b)

$$R_{in} = \frac{v_i}{i_i} = R_B \,//\, r_\pi$$

$$R_{ib} = r_\pi$$

$$if \quad R_B >> r_\pi \rightarrow R_{in} \approx R_{ib} = r_\pi$$

$$v_i = v_{signal} \frac{R_{in}}{R_{signal} + R_{in}} \approx v_{signal} \frac{r_\pi}{R_{signal} + r_\pi}$$

$$v_i = v_\pi$$

$$v_o = -g_m v_\pi (r_o \,//\, R_C \,//\, R_L)$$

$$A_v = -g_m (r_o \,//\, R_C \,//\, R_L)$$

$$R_{out} = R_C \,//\, r_o$$

$$G_v \equiv \frac{v_o}{v_{signal}}$$

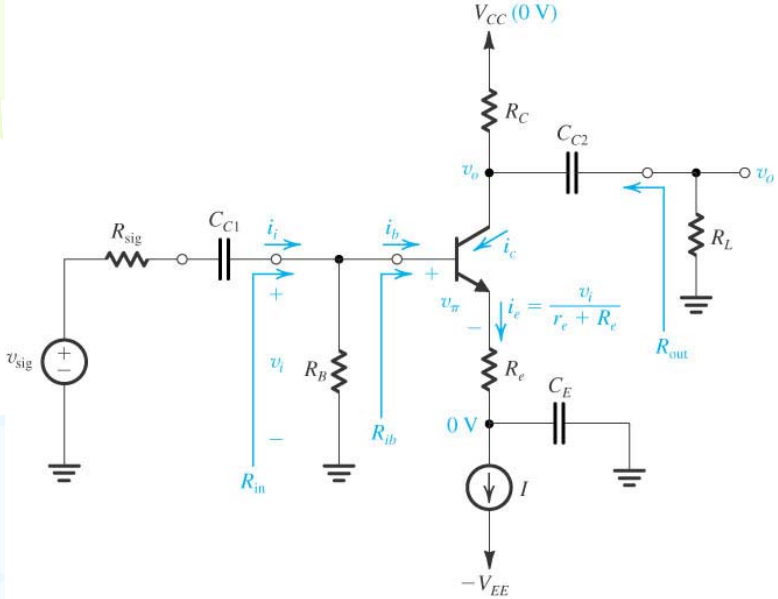
$$= \frac{-g_m (r_o \,//\, R_C \,//\, R_L)}{R_{signal} + R_{in}} R_{in}$$

$$= \frac{-\beta (R_C \,//\, R_L \,//\, r_o)}{r_\pi + R_{signal}}$$

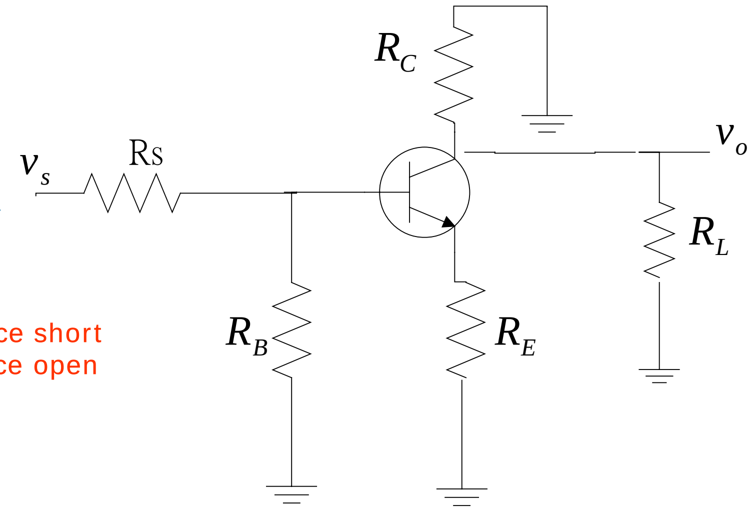
$$i_{os} = -g_m v_\pi$$

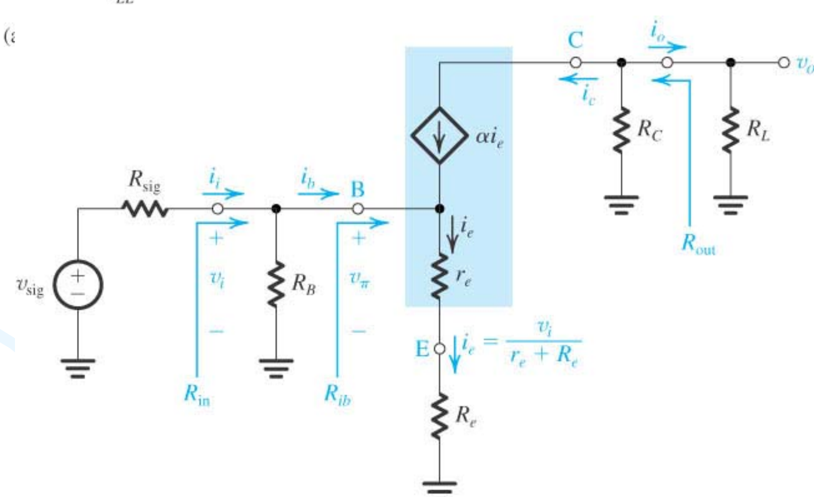
$$A_{is} \equiv \frac{i_{os}}{i_i} = -g_m R_{in}$$

Common-Emitter amplifier



DC voltage source short
DC current source open
Capacitor short

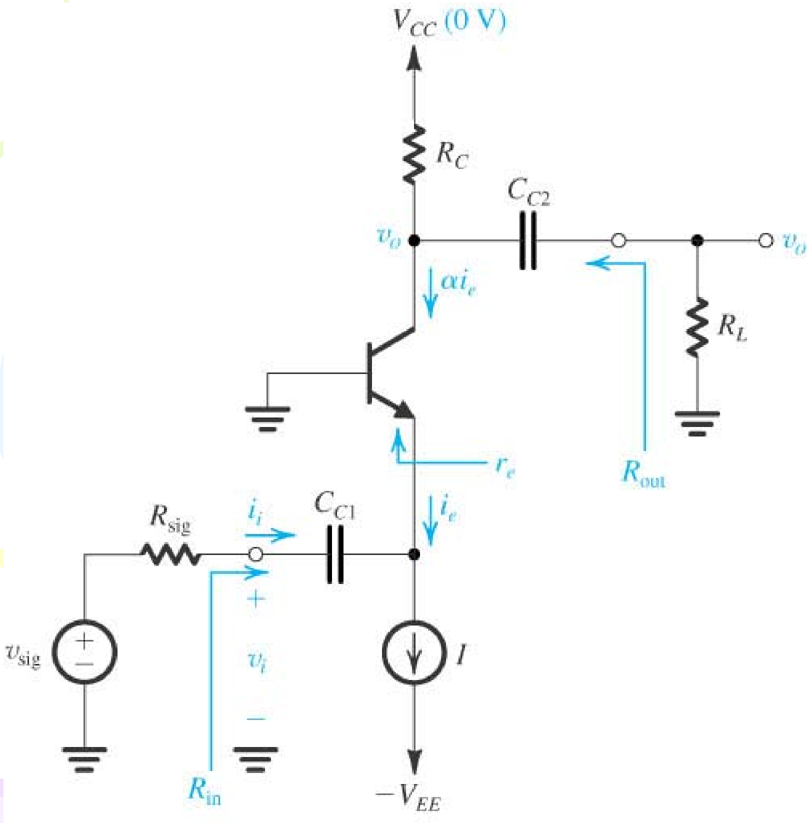




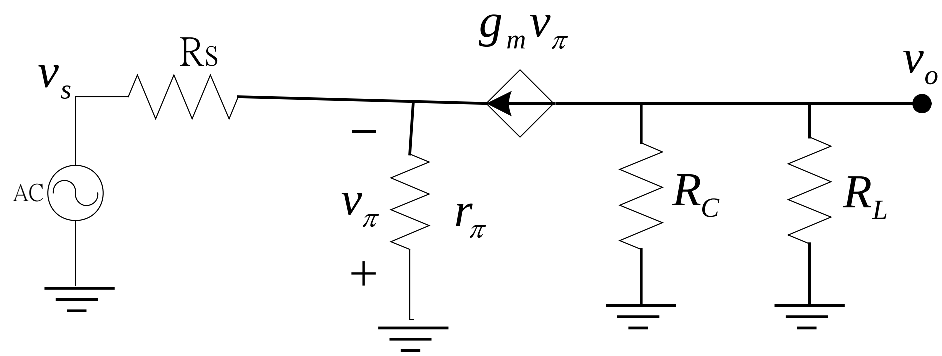
(b)

Replace transistor by T-model

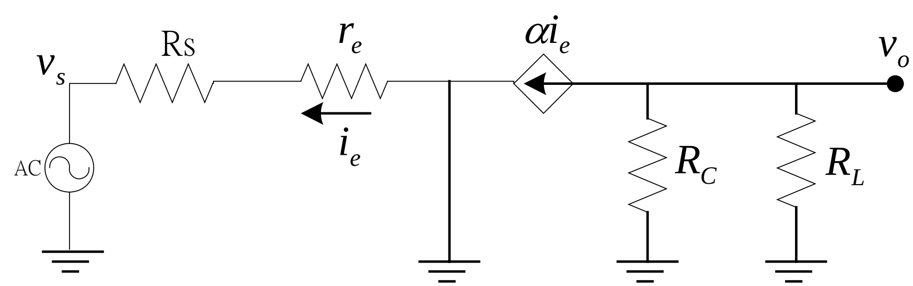
Common-Base amplifier



(a)

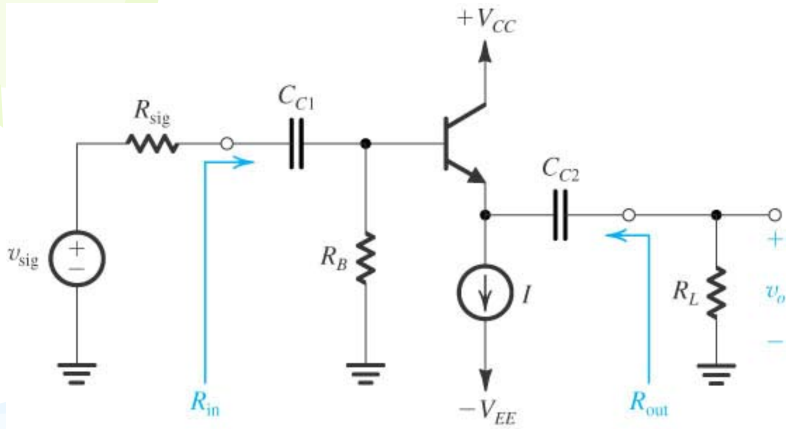


π Model equivalent Circuit

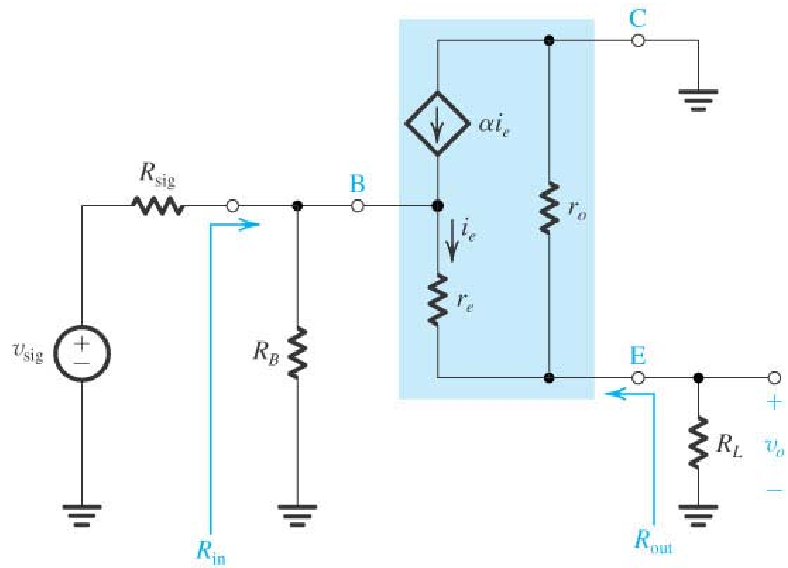


T-Model equivalent Circuit

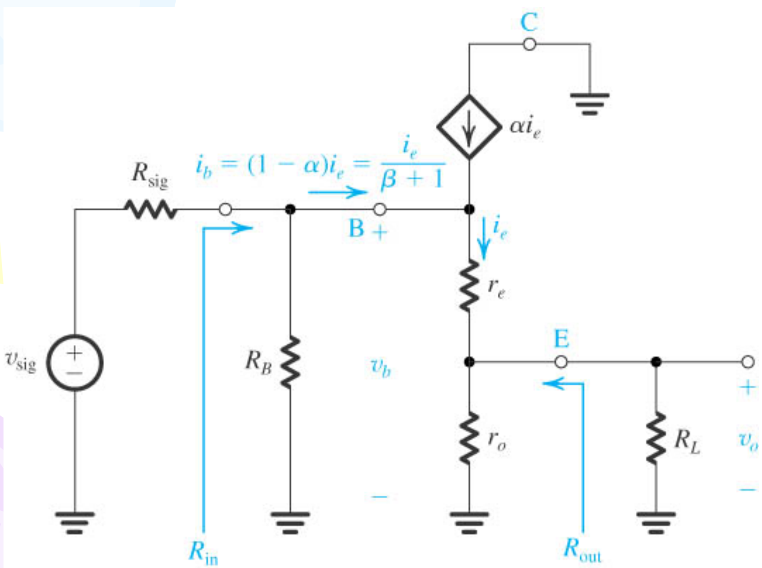
Common-collector amplifier



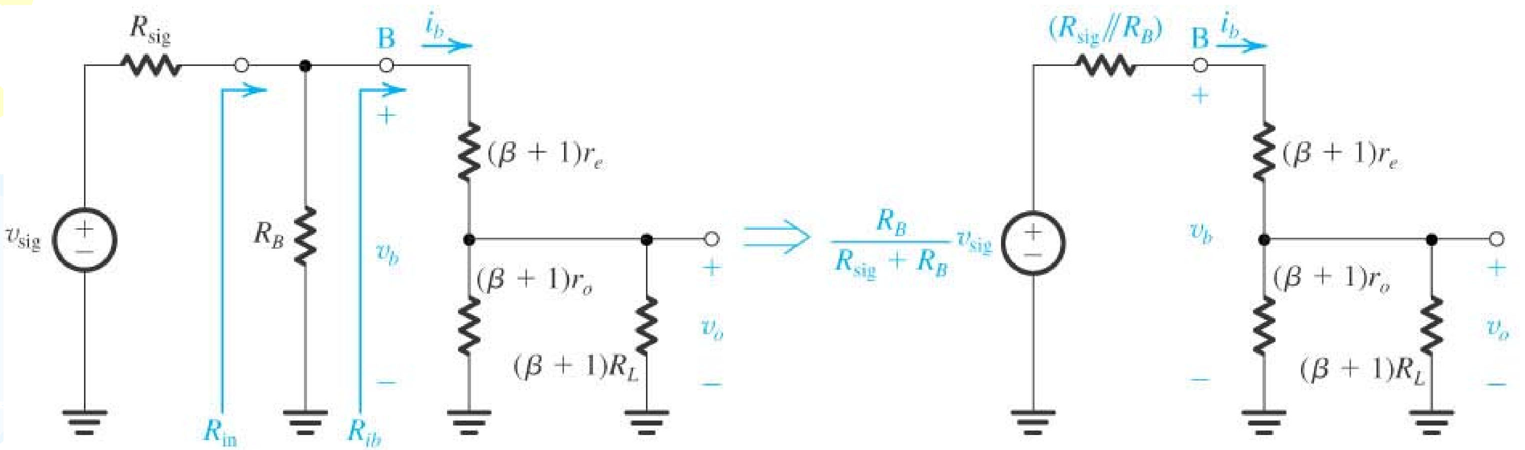
(a)



(b)



(c)

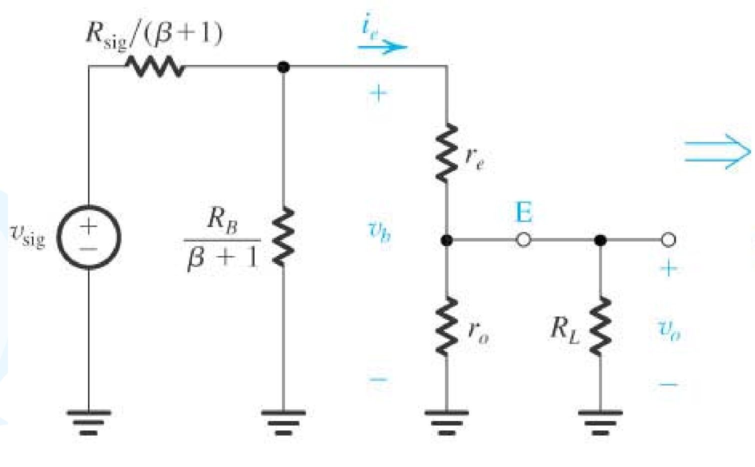


$$R_{in} = R_B /\!/ (\beta + 1)[r_e + (r_o /\!/ R_L)]$$

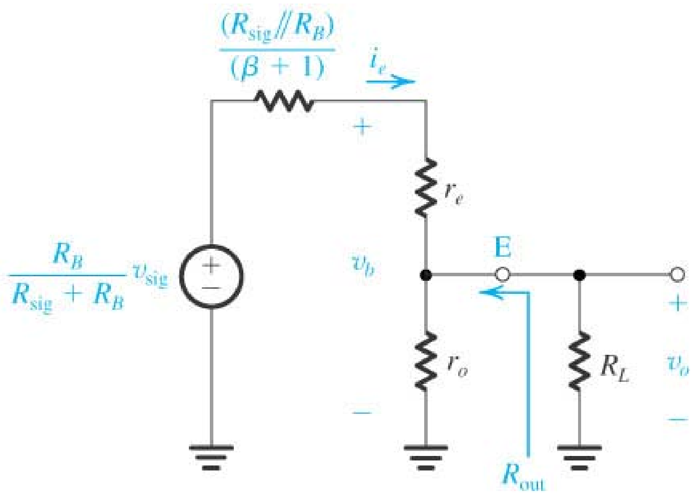
(a)

$$G_v = \frac{v_o}{v_{sig}} = \frac{R_B}{R_{sig} + R_B} \frac{(\beta + 1)(r_o /\!/ R_L)}{(R_{sig} /\!/ R_B) + (\beta + 1)[r_e + (r_o /\!/ R_L)]}$$

(b)

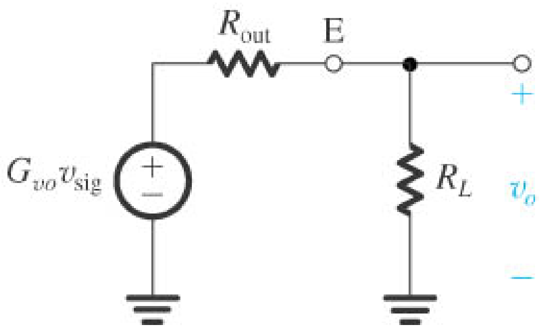


(a)



$$G_v = \frac{v_o}{v_{sig}} = \frac{R_B}{R_{sig}+R_B}\,\frac{(r_o//R_L)}{\frac{(R_{sig}//R_B)}{\beta+1} + r_e + (r_o//R_L)}$$

(b)

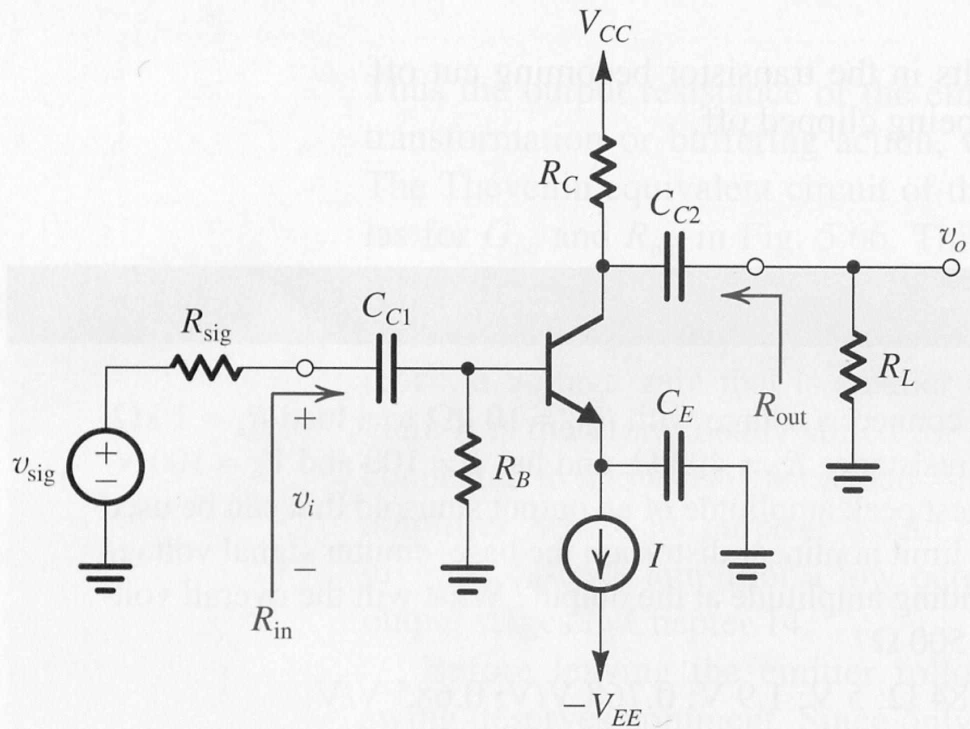


$$G_{vo} = \frac{R_B}{R_{sig} + R_B} \frac{r_o}{\frac{(R_{sig} /\!/ R_B)}{(\beta + 1)} + r_e + r_o}$$

$$R_{out} = r_o /\!/ \left(r_e + \frac{R_{sig} /\!/ R_B}{\beta + 1} \right)$$

Summary I

Common Emitter



$$R_{in} = R_B \parallel r_\pi = R_B \parallel (\beta+1)r_e$$

$$A_v = -g_m(r_o \parallel R_C \parallel R_L)$$

Inverse phase

$$R_{out} = r_o \parallel R_C$$

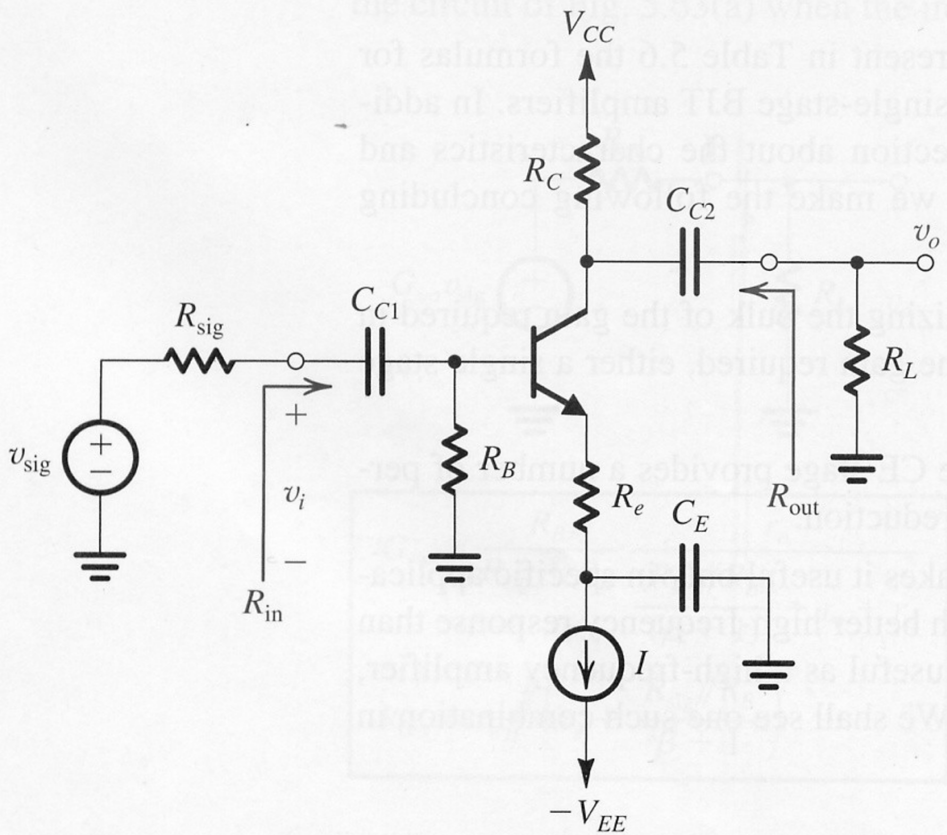
$$G_v = -\frac{(R_B \parallel r_\pi)}{(R_B \parallel r_\pi) + R_{sig}} g_m(r_o \parallel R_C \parallel R_L)$$

$$\cong -\frac{\beta(r_o \parallel R_C \parallel R_L)}{r_\pi + R_{sig}}$$

$$A_{is} = -g_m R_{in} \cong -\beta$$

R_i	middle	A_v	High(>1)
R_o	high	A_i	High(>1)

Common Emitter with Emitter Resistance



Neglecting r_o:

$$R_{in} = R_B \,\|\, (\beta+1)(r_e+R_e)$$

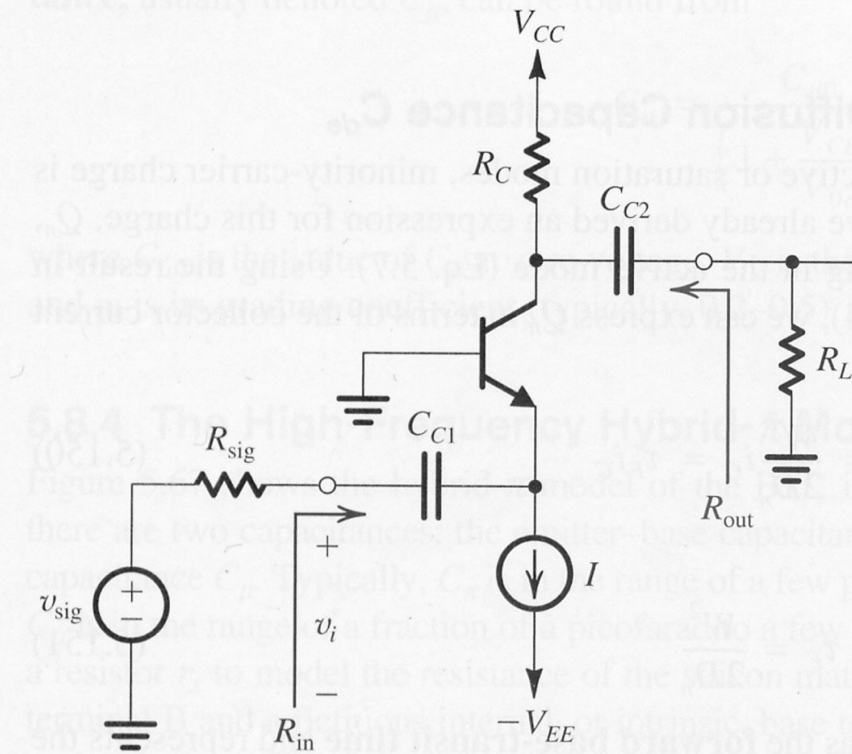
$$A_v = -\frac{\alpha(R_C \,\|\, R_L)}{r_e+R_e} \cong \frac{-g_m(R_C \,\|\, R_L)}{1+g_m R_e}$$

$$R_{out} = R_C$$

$$G_v \cong -\frac{\beta(R_C \,\|\, R_L)}{R_{sig}+(\beta+1)(r_e+R_e)}$$

$$\frac{v_\pi}{v_i} \cong \frac{1}{1+g_m R_e}$$

Common Base



Neglecting r_o:

$$R_{in} = r_e$$

$$A_v = g_m(R_C \parallel R_L)$$

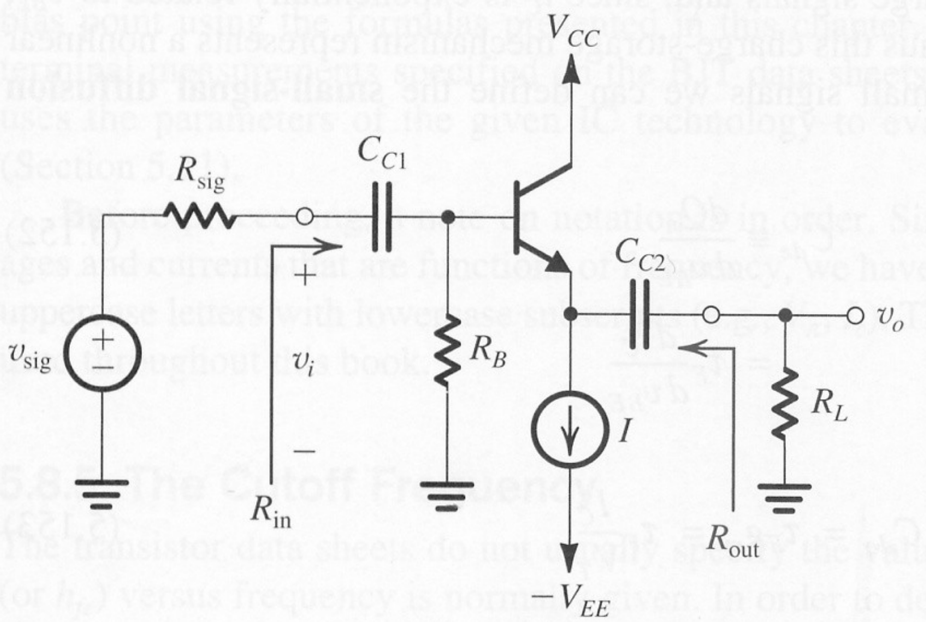
$$R_{out} = R_C$$

$$G_v = \frac{\alpha(R_C \parallel R_L)}{R_{sig} + r_e}$$

$$A_{is} \cong \alpha$$

R_i	low	A_v	Low (<1)
R_o	highest	A_i	Low (<=1)

Common Collector or Emitter Follower



$$R_{in} = R_B \parallel (\beta+1)[r_e + (r_o \parallel R_L)]$$

$$A_v = \frac{(r_o \parallel R_L)}{(r_o \parallel R_L) + r_e}$$

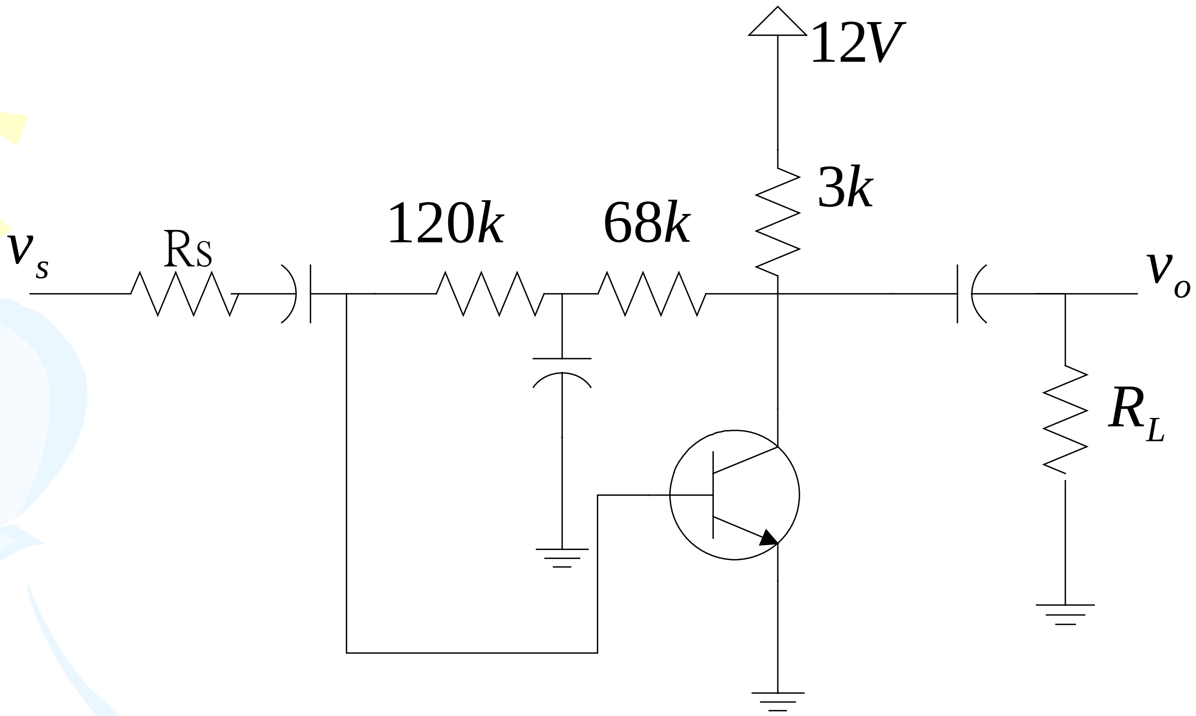
$$R_{out} = r_o \parallel \left[r_e + \frac{R_{sig} \parallel R_B}{\beta+1} \right]$$

$$G_v = \frac{R_B}{R_B + R_{sig}} \frac{(r_o \parallel R_L)}{\frac{R_{sig} \parallel R_B}{\beta+1} + r_e + (r_o \parallel R_L)}$$

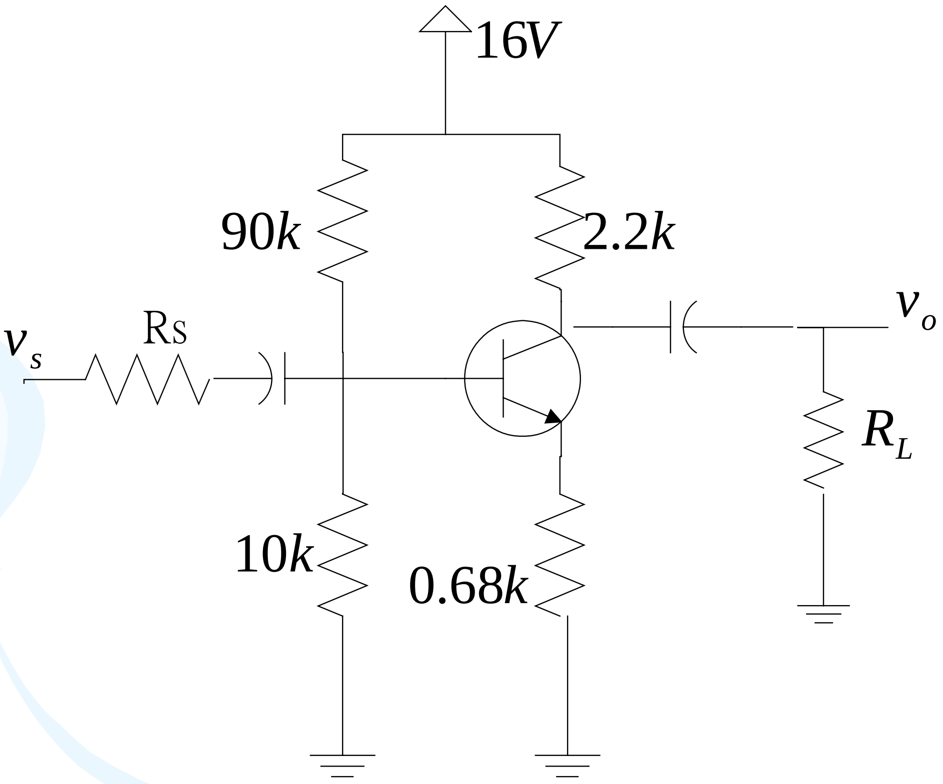
$$A_{is} \cong \beta + 1$$

R_i	high	A_v	Low (<=1)
R_o	low	A_i	Highest (>1)

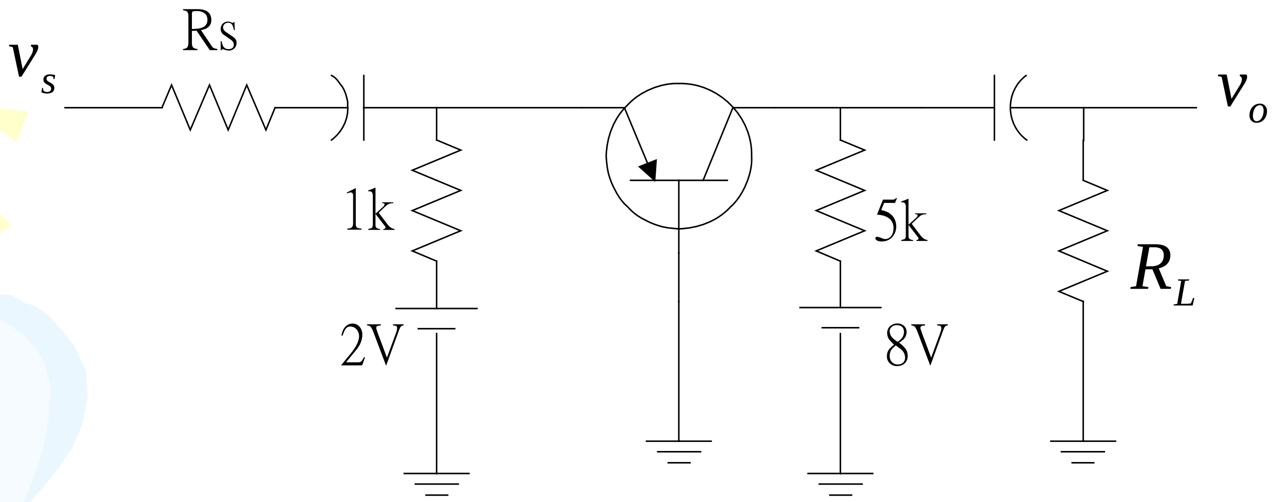
Common-Emitter amplifier (with collector feedback bias)



Common-Emitter amplifier with emitter resistance (by using self-bias)



Common-Base amplifier (by using self-bias)



Common-Collector amplifier (by using self-bias)