# KENYATTA UNIVERSITY

## UNIVERSITY EXAMINATIONS 2008/2009

## SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR SCIENCE

## SMA 201: CALCULUS III

**DATE:** MONDAY 6$^{TH}$ APRIL 2009 **TIME:** 11.00 A.M. – 1.00 P.M.

**INSTRUCTIONS**

Answer Question One and any other two questions

**QUESTION ONE (30 MARKS)**

a) Find $\lim_{x \to 1} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right)$ (3 marks)

b) Obtain the first three nonzero terms in the Maclaurin series for

$$f(x) = \frac{x}{1-x}$$ (4 marks)

c) Find the value of $c$ guaranteed by Cauchy Mean Value Theorem if

$f(x) = \frac{x^3}{x} - 4x$ and $g(x) = x^2$ defined on $[0,3]$.

(4 marks)

d) Find $\frac{\partial w}{\partial x}$ at the point (2, -1, 1) if

$W = x^2 + y^2 + z^2$, $z^3 - xy + yz + y^3 = 0$ and where x and y are the independent variables. (4 marks)

e) Obtain the equation of the tangent plane of the curve $x^2 + y^2 - 2xy - x + 3y - z = -4$ at a point (2, -3, 18). (4 marks)

f) Find the Fourier transform of the function

$$f(x) = \begin{cases} 1; & 0 < x < \pi \\ 0; & \pi < x < 2\pi \\ f; & f(x + 2\pi) \end{cases}$$

(3 marks)

g) Evaluate $\iint_R f(x,y)dxdy$ where $f(x,y) = x + 2y$ and $R$ is the region defined by $1 \le x \le 4$ and $1 \le y \le 2$. (4 marks)

i) Use Green's theorem in the plane to evaluate $\oint_c (x^2 + xy^3)\,dx + (y^2 - 2xy)\,dy$ where c is a square with vertices (0,0), (2,0), (2,2) and (0,2). (4 marks)

**QUESTION TWO (20 MARKS)**

a) i) State the Mean Value Theorem. (3 marks)

ii) The function $y = |1 - x^2|$ defined on $-2 \le x \le 2$, has a horizontal tangent at $x = 0$ even though the function is not differentiable at $x = -1$ and $x = 1$. Does this contradict the Mean Value Theorem? Explain. (4 marks)

b) Suppose you know that f(x) is differentiable and the $f'(x)$ always has a value between -1 and +1. Show that $|f(x) - f(a)| \le |x - a|$. (3 marks)

c) Two runners start a race at the same time and finish in a tie. Prove that at some time during the race they have the same velocity. [Hint; consider *(f(t)* = *g(t)* – *h(t)* – *h(t)* where g and h are the position functions of the two runners.] (6 marks)

d) Verify the validity of Rolle's Theorem in $f(x) = x^3 - 6x^2 + 11x - 6$

(4 marks)

## QUESTION THREE (20 MARKS)

a) Find the third order Taylor polynomial of *lnx* about the point 1. Use this in order to approximate *ln(1.03)*. Also give an estimation of the error of approximation.

(6 marks)

b) Find $\lim_{x \to \infty} \left(1 + \frac{3}{x} + \frac{5}{x^2}\right)^x$

(5 marks)

c) A car is moving with speed 20m/sec and acceleration $2\text{m/s}^2$ at a given instant. Using a second degree Taylor polynomial, estimate how far the car moves in the next second. Would it be reasonable to use this polynomial to estimate the distance traveled during the next minute? Explain (5 marks)

d) Approximate the function $f(x) = x^{\frac{1}{3}}$ by a Taylor polynomial of degree 2 at a = 8.

(4 marks)

## QUESTION FOUR (20 MARKS)

a) A mirror in the shape of a rectangle capped by a semi-circle is to have a perimeter P. Choose the radius of the semi-circular part so that the mirror has maximum area. (8 marks)

b) Postal corporation must make a rectangular steel box whose three dimensions must have a sum of 120 cm. What is the maximum volume the box can have and what are its dimensions? (8 marks)

c) Find the Fourier series for the function defined by

$$f(x) = \begin{cases} -x & -\pi < x < 0 \\ 0 & 0 < x < \pi \end{cases}$$

and so that $f(x) = f(x + 2\pi)$. (4 marks)

## QUESTION FIVE (20 MARKS)

a) State the Divergence theorem and use it to evaluate

$\iint_R F.ds$ where

$$\underset{\sim}{F}\ (x,y,z) = xy\,\underset{\sim}{i} + (y^2 + e^{xz^2})\,\underset{\sim}{j} + \sin(xy)\,\underset{\sim}{k}$$

and S is the surface of the region bounded by the parabolic cylinder $z = 1\text{-}x^2$ and the planes $z = 0,\ y = 0$ and $y + z = 2.$ (7 marks)

b) Verify Stokes Theorem for

$\underset{\sim}{A} = (2x - y)\,\underset{\sim}{i} - yz^2\,\underset{\sim}{j} - y^2 z\,\underset{\sim}{k}$ , where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary . (8 marks)

c) Show that the volume of the region in space bounded above by the surface $z = 1 - x^2 - y^2$, on the sides by the planes $x = 0,\ y = 0,\ x + y = 1$ and below by the plane. $z=0$ is given by the triple integral below and hence evaluate it

$$\int_0^1 \int_0^{1-y} \int_0^{1-x^2-y^2} dz\ dx\ dy.$$ (5 marks)

=====================