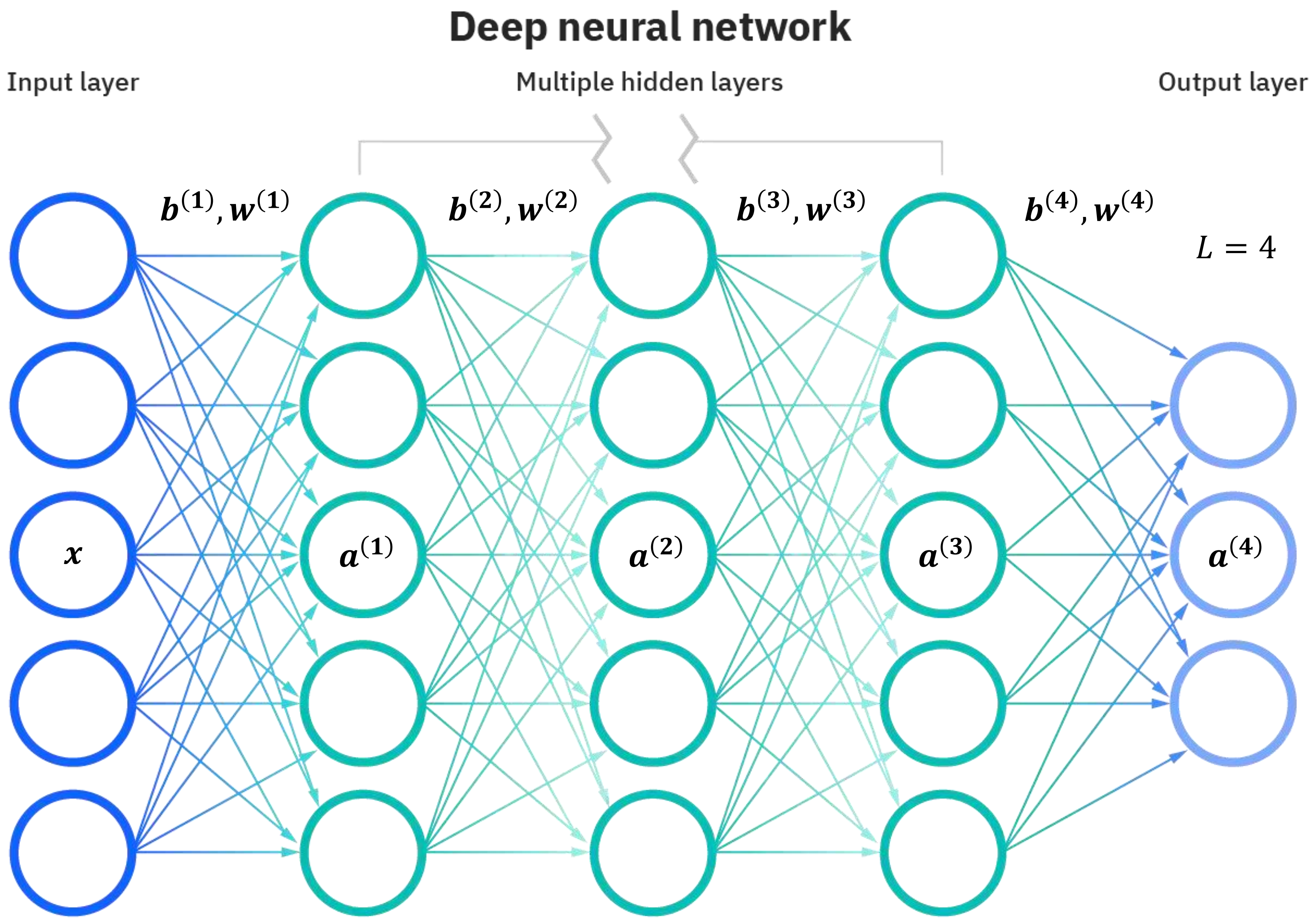
$$sig\, x = \frac{1}{1+e^{-x}} = \frac{e^x}{1+e^x}$$

$$(sig\, x)' = \left(\frac{1}{1+e^{-x}}\right)' = -\frac{-e^{-x}}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}}\frac{e^{-x}}{1+e^{-x}} = sig\, x\, sig(-x)$$

$$1 - sig\, x = 1 - \frac{e^x}{1+e^x} = \frac{1+e^x-e^x}{1+e^x} = \frac{1}{1+e^x} = sig(-x)$$

$$\therefore (sig\, x)' = sig\, x\, sig(-x) = sig\, x\,(1 - sig\, x)$$



$$C = \frac{1}{2} \sum_t \left(a_t^{(L)} - y_t \right)^2$$

$$a_k^{(l)} = sig\, z_k^{(l)}$$

$$z_k^{(l)} = b_k^{(l)} + \sum_t w_{t,k}^{(l)} a_t^{(l-1)}$$

$$C = \frac{1}{2}\sum_t \left(a_t^{(L)} - y_t\right)^2 \qquad a_j^{(L)} = sig\, z_j^{(L)} \qquad z_j^{(L)} = b_j^{(L)} + \sum_t w_{t,j}^{(L)} a_t^{(L-1)}$$

$$\frac{\partial C}{\partial a_j^{(L)}} = a_j^{(L)} - y_j \qquad \frac{\partial a_j^{(L)}}{\partial z_j^{(L)}} = sig\, z_j^{(L)} \left(1 - sig\, z_j^{(L)}\right) \qquad \frac{\partial z_j^{(L)}}{\partial b_j^{(L)}} = 1 \qquad \frac{\partial z_j^{(L)}}{\partial w_{i,j}^{(L)}} = a_i^{(L-1)} \qquad \frac{\partial z_j^{(L)}}{\partial a_i^{(L-1)}} = w_{i,j}^{(L)}$$

$$= a_j^{(L)}\left(1 - a_j^{(L)}\right)$$

$$\left[\Delta\, b_j^{(L)}\right] = \frac{\partial C}{\partial b_j^{(L)}} = \frac{\partial C}{\partial a_j^{(L)}}\frac{\partial a_j^{(L)}}{\partial z_j^{(L)}}\frac{\partial z_j^{(L)}}{\partial b_j^{(L)}} = \frac{\partial C}{\partial a_j^{(L)}}\frac{\partial a_j^{(L)}}{\partial z_j^{(L)}} = \left(a_j^{(L)} - y_j\right) a_j^{(L)}\left(1 - a_j^{(L)}\right)$$

$$\left[\Delta\, w_{i,j}^{(L)}\right] = \frac{\partial C}{\partial w_{i,j}^{(L)}} = \frac{\partial C}{\partial a_j^{(L)}}\frac{\partial a_j^{(L)}}{\partial z_j^{(L)}}\frac{\partial z_j^{(L)}}{\partial w_{i,j}^{(L)}} = \frac{\partial C}{\partial a_j^{(L)}}\frac{\partial a_j^{(L)}}{\partial z_j^{(L)}} a_i^{(L-1)} = \left[\Delta\, b_j^{(L)}\right] a_i^{(L-1)}$$

$$\left[\Delta\, a_i^{(L-1)}\right] = \frac{\partial C}{\partial a_i^{(L-1)}} = \sum_j \frac{\partial C}{\partial a_j^{(L)}}\frac{\partial a_j^{(L)}}{\partial z_j^{(L)}}\frac{\partial z_j^{(L)}}{\partial a_i^{(L-1)}} = \sum_j \frac{\partial C}{\partial a_j^{(L)}}\frac{\partial a_j^{(L)}}{\partial z_j^{(L)}} w_{i,j}^{(L)} = \sum_j \left[\Delta\, b_j^{(L)}\right] w_{i,j}^{(L)}$$

$$a_j^{(L-1)} = sig\, z_j^{(L-1)} \qquad z_j^{(L-1)} = b_j^{(L-1)} + \sum_t w_{t,j}^{(L-1)} a_t^{(L-2)}$$

$$\frac{\partial a_j^{(L-1)}}{\partial z_j^{(L-1)}} = a_j^{(L-1)}\left(1 - a_j^{(L-1)}\right) \quad \frac{\partial z_j^{(L-1)}}{\partial b_j^{(L-1)}} = 1 \quad \frac{\partial z_j^{(L-1)}}{\partial w_{i,j}^{(L-1)}} = a_i^{(L-2)} \quad \frac{\partial z_j^{(L-1)}}{\partial a_i^{(L-2)}} = w_{i,j}^{(L-1)}$$

$$\left[\Delta\, b_j^{(L-1)}\right] = \frac{\partial C}{\partial b_j^{(L-1)}} = \frac{\partial C}{\partial a_j^{(L-1)}} \frac{\partial a_j^{(L-1)}}{\partial z_j^{(L-1)}} \frac{\partial z_j^{(L-1)}}{\partial b_j^{(L-1)}} = \frac{\partial C}{\partial a_j^{(L-1)}} \frac{\partial a_j^{(L-1)}}{\partial z_j^{(L-1)}} = \left[\Delta\, a_j^{(L-1)}\right] a_j^{(L-1)}\left(1 - a_j^{(L-1)}\right)$$

$$\left[\Delta\, w_{i,j}^{(L-1)}\right] = \frac{\partial C}{\partial w_{i,j}^{(L-1)}} = \frac{\partial C}{\partial a_j^{(L-1)}} \frac{\partial a_j^{(L-1)}}{\partial z_j^{(L-1)}} \frac{\partial z_j^{(L-1)}}{\partial w_{i,j}^{(L-1)}} = \frac{\partial C}{\partial a_j^{(L-1)}} \frac{\partial a_j^{(L-1)}}{\partial z_j^{(L-1)}} a_i^{(L-2)} = \left[\Delta\, b_j^{(L-1)}\right] a_i^{(L-2)}$$

$$\left[\Delta\, a_i^{(L-2)}\right] = \frac{\partial C}{\partial a_i^{(L-2)}} = \sum_j \frac{\partial C}{\partial a_j^{(L-1)}} \frac{\partial a_j^{(L-1)}}{\partial z_j^{(L-1)}} \frac{\partial z_j^{(L-1)}}{\partial a_i^{(L-2)}} = \sum_j \frac{\partial C}{\partial a_j^{(L-1)}} \frac{\partial a_j^{(L-1)}}{\partial z_j^{(L-1)}} w_{i,j}^{(L-1)} = \sum_j \left[\Delta\, b_j^{(L-1)}\right] w_{i,j}^{(L-1)}$$

$$a_j^{(l)} = sig\left(b_j^{(l)} + \sum_i w_{i,j}^{(l)} a_i^{(l-1)}\right)$$

$$\left[\Delta\, b_j^{(l)}\right] = \begin{cases} \left(a_j^{(l)} - y_j\right) a_j^{(l)} \left(1 - a_j^{(l)}\right) & if\ l = L \\ \left[\Delta\, a_j^{(l)}\right] a_j^{(l)} \left(1 - a_j^{(l)}\right) & if\ l < L \end{cases}$$

$$\left[\Delta\, w_{i,j}^{(l)}\right] = \left[\Delta\, b_j^{(l)}\right] a_i^{(l-1)}$$

$$\left[\Delta\, a_i^{(l-1)}\right] = \sum_j \left[\Delta\, b_j^{(l)}\right] w_{i,j}^{(l)}$$

$$a_j^{(1)} = sig\left(b_j^{(1)} + \sum_i w_{i,j}^{(1)} x_i\right)$$

$$\boldsymbol{for}\ l = 2\ \boldsymbol{up\ to}\ L:$$

$$\quad a_j^{(l)} = sig\left(b_j^{(l)} + \sum_i w_{i,j}^{(l)} a_i^{(l-1)}\right)$$

$$\left[\Delta\, b_j^{(L)}\right] = \left(a_j^{(L)} - y_j\right) a_j^{(L)} \left(1 - a_j^{(L)}\right)$$

$$\left[\Delta\, w_{i,j}^{(L)}\right] = \left[\Delta\, b_j^{(L)}\right] a_i^{(L-1)}$$

$$\boldsymbol{if}\ L > 1:$$

$$\quad \left[\Delta\, a_i^{(L-1)}\right] = \sum_j \left[\Delta\, b_j^{(L)}\right] w_{i,j}^{(L)}$$

$$\boldsymbol{for}\ l = L - 1\ \boldsymbol{down\ to}\ 2:$$

$$\quad \left[\Delta\, b_j^{(l)}\right] = \left[\Delta\, a_j^{(l)}\right] a_j^{(l)} \left(1 - a_j^{(l)}\right)$$

$$\quad \left[\Delta\, w_{i,j}^{(l)}\right] = \left[\Delta\, b_j^{(l)}\right] a_i^{(l-1)}$$

$$\quad \left[\Delta\, a_i^{(l-1)}\right] = \sum_j \left[\Delta\, b_j^{(l)}\right] w_{i,j}^{(l)}$$

$$\boldsymbol{if}\ L > 1:$$

$$\quad \left[\Delta\, b_j^{(1)}\right] = \left[\Delta\, a_j^{(1)}\right] a_j^{(1)} \left(1 - a_j^{(1)}\right)$$

$$\quad \left[\Delta\, w_{i,j}^{(1)}\right] = \left[\Delta\, b_j^{(1)}\right] x_i$$