(1)

$$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k} \quad \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$
$$a = |\vec{a}| \quad b = |\vec{b}|$$

$$\vec{a}\cdot\vec{b} = ab\cos C \quad \cos C = \frac{\vec{a}\cdot\vec{b}}{ab} \quad \sin C = \sqrt{1-\cos^2 C}$$
$$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

(2)

$$|\vec{a}\times\vec{b}| = ab\sin C = ab\sqrt{1-\cos^2 C} = ab\sqrt{1-\frac{(\vec{a}\cdot\vec{b})^2}{(ab)^2}} = ab\sqrt{\frac{(ab)^2-(\vec{a}\cdot\vec{b})^2}{(ab)^2}} = \sqrt{(ab)^2-(\vec{a}\cdot\vec{b})^2}$$
$$= \sqrt{(a_1{}^2+a_2{}^2+a_3{}^2)(b_1{}^2+b_2{}^2+b_3{}^2)-(a_1b_1+a_2b_2+a_3b_3)^2}$$
$$= \sqrt{a_1{}^2b_1{}^2 + a_1{}^2b_2{}^2 + a_1{}^2b_3{}^2 + a_2{}^2b_1{}^2 + a_2{}^2b_2{}^2 + a_2{}^2b_3{}^2 + a_3{}^2b_1{}^2 + a_3{}^2b_2{}^2 + a_3{}^2b_3{}^2 - (a_1{}^2b_1{}^2 + a_2{}^2b_2{}^2 + a_3{}^2b_3{}^2 + 2a_1b_1a_2b_2 + 2a_2b_2a_3b_3 + 2a_3b_3a_1b_1)}$$
$$= \sqrt{a_1{}^2b_2{}^2 + a_1{}^2b_3{}^2 + a_2{}^2b_1{}^2 + a_2{}^2b_3{}^2 + a_3{}^2b_1{}^2 + a_3{}^2b_2{}^2 - 2a_1b_1a_2b_2 - 2a_2b_2a_3b_3 - 2a_3b_3a_1b_1}$$
$$= \sqrt{a_1{}^2b_2{}^2 - 2a_1b_1a_2b_2 + a_2{}^2b_1{}^2 + a_1{}^2b_3{}^2 - 2a_3b_3a_1b_1 + a_3{}^2b_1{}^2 + a_2{}^2b_3{}^2 - 2a_2b_2a_3b_3 + a_3{}^2b_2{}^2}$$
$$= \sqrt{(a_1b_2-a_2b_1)^2 + (a_1b_3-a_3b_1)^2 + (a_2b_3-a_3b_2)^2}$$

(3)

$$\begin{cases}\hat{\imath}\times\hat{\jmath} = \hat{k}\\ \hat{\jmath}\times\hat{k} = \hat{\imath}\\ \hat{k}\times\hat{\imath} = \hat{\jmath}\end{cases} \quad \begin{cases}(1\hat{\imath}+0\hat{\jmath}+0\hat{k})\times(0\hat{\imath}+1\hat{\jmath}+0\hat{k}) = (0\hat{\imath}+0\hat{\jmath}+1\hat{k})\\ (0\hat{\imath}+1\hat{\jmath}+0\hat{k})\times(0\hat{\imath}+0\hat{\jmath}+1\hat{k}) = (1\hat{\imath}+0\hat{\jmath}+0\hat{k})\\ (0\hat{\imath}+0\hat{\jmath}+1\hat{k})\times(1\hat{\imath}+0\hat{\jmath}+0\hat{k}) = (0\hat{\imath}+1\hat{\jmath}+0\hat{k})\end{cases}$$

$$a_1b_2-a_2b_1 = (\vec{a}\times\vec{b})_3 \quad a_1b_3-a_3b_1 = -(\vec{a}\times\vec{b})_2 \quad a_2b_3-a_3b_2 = (\vec{a}\times\vec{b})_1$$
$$\vec{a}\times\vec{b} = (\vec{a}\times\vec{b})_1\hat{\imath} + (\vec{a}\times\vec{b})_2\hat{\jmath} + (\vec{a}\times\vec{b})_3\hat{k} = (a_2b_3-a_3b_2)\hat{\imath} - (a_1b_3-a_3b_1)\hat{\jmath} + (a_1b_2-a_2b_1)\hat{k}$$

(4)

$$\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix} = \begin{vmatrix}a_2 & a_3\\ b_2 & b_3\end{vmatrix}\hat{\imath} - \begin{vmatrix}a_1 & a_3\\ b_1 & b_3\end{vmatrix}\hat{\jmath} + \begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}\hat{k} = (a_2b_3-a_3b_2)\hat{\imath} - (a_1b_3-a_3b_1)\hat{\jmath} + (a_1b_2-a_2b_1)\hat{k}$$

$$\therefore \vec{a}\times\vec{b} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix} = (a_2b_3-a_3b_2)\hat{\imath} - (a_1b_3-a_3b_1)\hat{\jmath} + (a_1b_2-a_2b_1)\hat{k}$$