

(1)

$$\lim_{x \rightarrow a} f(x) = 0 \qquad \lim_{x \rightarrow a} g(x) = 0$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{g(x) - g(a)} = \lim_{x \rightarrow a} \frac{\frac{f(x) - f(a)}{x - a}}{\frac{g(x) - g(a)}{x - a}} = \frac{f'(a)}{g'(a)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

(2)

$$\lim_{x \rightarrow a} f(x) = \infty \qquad \lim_{x \rightarrow a} g(x) = \infty$$

$$\lim_{m \rightarrow 0} \frac{f(x+m)}{g(x+m)} = \frac{f(x)}{g(x)}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{\substack{x \rightarrow a \\ m \rightarrow 0}} \frac{f(x+m) - f(x)}{g(x+m) - g(x)} = \lim_{\substack{x \rightarrow a \\ m \rightarrow 0}} \frac{\frac{f(x+m) - f(x)}{m}}{\frac{g(x+m) - g(x)}{m}} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$