Polynomial Derivatives

$$\frac{d}{dx}f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

(1)

$$\frac{d}{dx}x^n = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}(x+\Delta x - x)\sum_{k=1}^{n}\{(x+\Delta x)^{n-k}x^{k-1}\}$$

$$= \sum_{k=1}^{n}\left(x^{n-k}x^{k-1}\right) = \sum_{k=1}^{n} x^{n-1} = nx^{n-1}$$

(2)

$$\frac{d}{dx}cf(x) = \lim_{\Delta x \to 0} \frac{cf(x+\Delta x) - cf(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} c\left\{\frac{f(x+\Delta x) - f(x)}{\Delta x}\right\} = c\frac{d}{dx}f(x)$$

(3)

$$\frac{d}{dx}\{f(x) \pm g(x)\}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) \pm g(x+\Delta x) - \{f(x) \pm g(x)\}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x) \pm g(x+\Delta x) \mp g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x) \pm \{g(x+\Delta x) - g(x)\}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \pm \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$= \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$