Gaussian Integral

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$\left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)^2 = \int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy$$

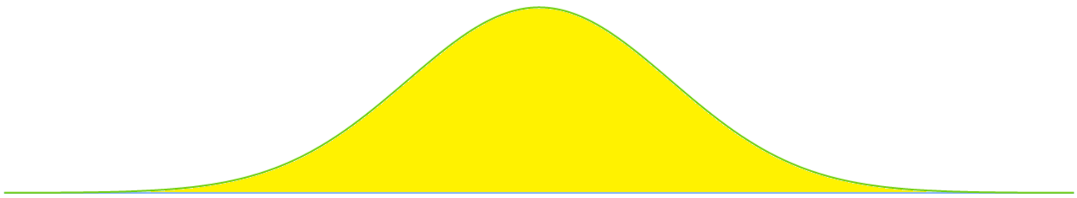
$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2} dx\, dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx\, dy$$

Changing the domain to polar coordinates,

$$x^2 + y^2 = r^2 \qquad\qquad dxdy = rd\theta dr$$

$$= \int_0^{2\pi} \int_0^{\infty} e^{-r^2} rdr\, d\theta = \frac{1}{2}\int_0^{2\pi} \int_0^{\infty} e^{-t} dt\, d\theta = -\frac{1}{2}\int_0^{2\pi} \left[e^{-t}\right]_0^{\infty} d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} d\theta = \frac{1}{2}\left[\theta\right]_0^{2\pi} = \frac{1}{2} \cdot 2\pi = \pi$$



$$\therefore \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$