(1)

$$success : 1 \to p \qquad failure : 0 \to q \qquad p + q = 1$$

$$\mu_1 = 1 \cdot p = p$$

$${\sigma_1}^2 = (1-\mu_1)^2 p + (0-\mu_1)^2 q = (1-p)^2 p + (0-p)^2(1-p)$$

$$= (1-p)(p - p^2 + p^2) = p(1-p) = pq$$

$$\therefore \mu = np$$

$$\therefore \sigma^2 = npq$$

(2)

$$f(x) = \binom{n}{x} p^x q^{n-x} \qquad @\, x = m : \ f(x) \to maxima$$

$$@\, x = m : \ ln\, f(x) \to maxima$$

$$f = f(x) \qquad f' = f'(x) \qquad f'(m) = [f'(x)]_{x=m}$$

$$f'(m) = 0 \qquad (ln\, f)'(m) = 0$$

$$ln\, f = ln\, f(m) + (ln\, f)'(m)\,(x-m) + \frac{1}{2}(ln\, f)''(m)\,(x-m)^2 + \sum_{k=3}^{\infty} \frac{1}{k!}(ln\, f)^{(k)}(m)\,(x-m)^k$$

$$f = f(m)\, e^{\frac{1}{2}(ln\, f)''(m)(x-m)^2} e^{\sum_{k=3}^{\infty}\frac{1}{k!}(ln\, f)^{(k)}(m)(x-m)^k}$$

(3)

$$ln\, x! = \sum_{k=2}^{x} ln\, k \approx \int_1^x ln\, t\, dt \approx x\, ln\, x - x$$

$$ln\, f = ln\left(\frac{n!}{(n-x)!x!} p^x q^{n-x}\right) = ln\, n! - ln(n-x)! - ln\, x! + ln\, p^x + ln\, q^{n-x}$$

$$\approx n\, ln\, n - n - (n-x)\, ln(n-x) + (n-x) - x\, ln\, x + x + x\, ln\, p + (n-x)\, ln\, q$$

$$= n\, ln\, n - n - n\, ln(n-x) + x\, ln(n-x) + n - x - x\, ln\, x + x + x\, ln\, p + n\, ln\, q - x\, ln\, q$$

$$(ln\, f)' = -n\frac{-1}{n-x} + ln(n-x) + x\frac{-1}{n-x} - 1 - (ln\, x + 1) + 1 + ln\, p - ln\, q$$

$$= \frac{n}{n-x} + ln(n-x) - \frac{x}{n-x} - 1 - ln\, x - 1 + 1 + ln\, p - ln\, q$$

$$= 1 + ln(n-x) - 1 - ln\, x - 1 + 1 + ln\, p - ln\, q = ln(n-x) - ln\, x + ln\, p - ln\, q = ln\left(\frac{(n-x)p}{xq}\right)$$

$$(ln\, f)'(m) = 0 \qquad ln\left(\frac{(n-m)p}{mq}\right) = 0 \qquad \frac{(n-m)p}{mq} = 1$$

$$(n-m)p = mq \qquad np - mp = mq \qquad np = m(p+q) \qquad \therefore m = np = \mu$$

$$(ln\, f)'' = (ln(n-x) - ln\, x + ln\, p - ln\, q)' = -\frac{1}{n-x} - \frac{1}{x}$$

$$(ln\, f)''(m) = -\frac{1}{n-m} - \frac{1}{m} = -\frac{1}{n-np} - \frac{1}{np} = -\frac{1}{nq} - \frac{1}{np} = \frac{-p-q}{npq} = -\frac{1}{npq} = -\frac{1}{\sigma^2}$$

$$(ln\, f)^{(k)}(m) = -(k-2)!\left((n-np)^{-(k-1)} + (-1)^k (np)^{-(k-1)}\right)$$

$$\lim_{n\to\infty} \frac{1}{k!}(ln\, f)^{(k)}(m) = \lim_{n\to\infty}\left(-\frac{1}{k(k-1)} \cdot \frac{p^{k-1} + (-1)^k (1-p)^{k-1}}{n^{k-1}\big(p(1-p)\big)^{k-1}}\right) = 0$$

(4)

$$f \approx f(m)\, e^{\frac{1}{2}(ln\, f)''(m)(x-m)^2} = f(\mu)\, e^{\frac{1}{2}\left(-\frac{1}{\sigma^2}\right)(x-\mu)^2} = f(\mu)\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\therefore \binom{n}{x} p^x (1-p)^{n-x} \approx \binom{n}{\mu} p^{\mu} (1-p)^{n-\mu} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$