

(1)

$$\text{success} : 1 \rightarrow p \quad \text{failure} : 0 \rightarrow q \quad p + q = 1$$

$$\mu_1 = 1 \cdot p = p$$

$$\begin{aligned} \sigma_1^2 &= (1 - \mu_1)^2 p + (0 - \mu_1)^2 q = (1 - p)^2 p + (0 - p)^2 (1 - p) \\ &= (1 - p)(p - p^2 + p^2) = p(1 - p) = pq \end{aligned}$$

$$\therefore \mu = np$$

$$\therefore \sigma^2 = npq$$

(2)

$$f(x) = \binom{n}{x} p^x q^{n-x} \quad @ x = m : f(x) \rightarrow \text{maxima}$$

$$@ x = m : \ln f(x) \rightarrow \text{maxima}$$

$$f = f(x) \quad f' = f'(x) \quad f'(m) = [f'(x)]_{x=m}$$

$$f'(m) = 0 \quad (\ln f)'(m) = 0$$

$$\ln f = \ln f(m) + (\ln f)'(m)(x - m) + \frac{1}{2}(\ln f)''(m)(x - m)^2 + \sum_{k=3}^{\infty} \frac{1}{k!}(\ln f)^{(k)}(m)(x - m)^k$$

$$f = f(m) e^{\frac{1}{2}(\ln f)''(m)(x-m)^2} e^{\sum_{k=3}^{\infty} \frac{1}{k!}(\ln f)^{(k)}(m)(x-m)^k}$$

(3)

$$\ln x! = \sum_{k=2}^x \ln k \approx \int_1^x \ln t \, dt \approx x \ln x - x$$

$$\ln f = \ln \left(\frac{n!}{(n-x)!x!} p^x q^{n-x} \right) = \ln n! - \ln(n-x)! - \ln x! + \ln p^x + \ln q^{n-x}$$

$$\approx n \ln n - n - (n-x) \ln(n-x) + (n-x) - x \ln x + x + x \ln p + (n-x) \ln q$$

$$= n \ln n - n - n \ln(n-x) + x \ln(n-x) + n - x - x \ln x + x + x \ln p + n \ln q - x \ln q$$

$$(\ln f)' = -n \frac{-1}{n-x} + \ln(n-x) + x \frac{-1}{n-x} - 1 - (\ln x + 1) + 1 + \ln p - \ln q$$

$$= \frac{n}{n-x} + \ln(n-x) - \frac{x}{n-x} - 1 - \ln x - 1 + 1 + \ln p - \ln q$$

$$= 1 + \ln(n-x) - 1 - \ln x - 1 + 1 + \ln p - \ln q = \ln(n-x) - \ln x + \ln p - \ln q = \ln \left(\frac{(n-x)p}{xq} \right)$$

$$(\ln f)'(m) = 0 \quad \ln \left(\frac{(n-m)p}{mq} \right) = 0 \quad \frac{(n-m)p}{mq} = 1$$

$$(n-m)p = mq \quad np - mp = mq \quad np = m(p+q) \quad \therefore m = np = \mu$$

$$(\ln f)'' = (\ln(n-x) - \ln x + \ln p - \ln q)' = -\frac{1}{n-x} - \frac{1}{x}$$

$$(\ln f)''(m) = -\frac{1}{n-m} - \frac{1}{m} = -\frac{1}{n-np} - \frac{1}{np} = -\frac{1}{nq} - \frac{1}{np} = \frac{-p-q}{npq} = -\frac{1}{npq} = -\frac{1}{\sigma^2}$$

$$(\ln f)^{(k)} = (k-2)! \left(-(n-x)^{-(k-1)} + (-1)^{k-1} x^{-(k-1)} \right)$$

$$(\ln f)^{(k)}(m) = (k-2)! \left(-(n-np)^{-(k-1)} + (-1)^{k-1} (np)^{-(k-1)} \right) = (k-2)! \cdot \frac{-p^{k-1} + (-q)^{k-1}}{(npq)^{k-1}}$$

$$\lim_{n \rightarrow \infty} \frac{1}{k!} (\ln f)^{(k)}(m) = \lim_{n \rightarrow \infty} \left(\frac{1}{k(k-1)} \cdot \frac{-p^{k-1} + (-q)^{k-1}}{n^{k-1}(pq)^{k-1}} \right) = 0$$

(4)

$$f \approx f(m) e^{\frac{1}{2}(\ln f)''(m)(x-m)^2} = f(\mu) e^{\frac{1}{2}\left(-\frac{1}{\sigma^2}\right)(x-\mu)^2} = f(\mu) e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\therefore \binom{n}{x} p^x (1-p)^{n-x} \approx \binom{n}{\mu} p^{\mu} (1-p)^{n-\mu} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$