Maclaurin Series Proof

(1)

$$\frac{dx^n}{dx} = nx^{n-1}$$

$$\frac{d^2x^n}{dx^2} = n(n-1)x^{n-2}$$

$$\frac{d^3x^n}{dx^3} = n(n-1)(n-2)x^{n-3}$$

$$\vdots$$

$$\frac{d^kx^n}{dx^k} = n(n-1)(n-2)\cdots(n-k+1)x^{n-k}$$

$$\vdots$$

$$\frac{d^nx^n}{dx^n} = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1\cdot x^0 = n!$$

(2)

$$f(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots + c_nx^n + \cdots$$

$$f(0) = c_0$$

$$f'(0) = 1!\,c_1$$

$$f''(0) = 2!\,c_2$$

$$\vdots$$

$$f^{(n)}(0) = n!\,c_n$$

$$\therefore c_n = \frac{f^{(n)}(0)}{n!}$$

(3)

$$\therefore f(x) = \sum_{n=0}^{\infty} c_nx^n = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$