

Polynomial Derivatives

$$\frac{d}{dx}f(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

(1)

$$\begin{aligned}\frac{d}{dx}x^n &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^n - x^n}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} (x + \Delta x - x) \sum_{k=1}^n \{(x + \Delta x)^{n-k} x^{k-1}\} \\&= \sum_{k=1}^n (x^{n-k} x^{k-1}) = \sum_{k=1}^n x^{n-1} = nx^{n-1}\end{aligned}$$

(2)

$$\begin{aligned}\frac{d}{dx}cf(x) &= \lim_{\Delta x \rightarrow 0} \frac{cf(x + \Delta x) - cf(x)}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} c \left\{ \frac{f(x + \Delta x) - f(x)}{\Delta x} \right\} = c \frac{d}{dx}f(x)\end{aligned}$$

(3)

$$\begin{aligned}\frac{d}{dx}\{f(x) \pm g(x)\} &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) \pm g(x + \Delta x) - \{f(x) \pm g(x)\}}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x) \pm g(x + \Delta x) \mp g(x)}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x) \pm \{g(x + \Delta x) - g(x)\}}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \pm \lim_{\Delta x \rightarrow 0} \frac{g(x + \Delta x) - g(x)}{\Delta x} \\&= \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)\end{aligned}$$