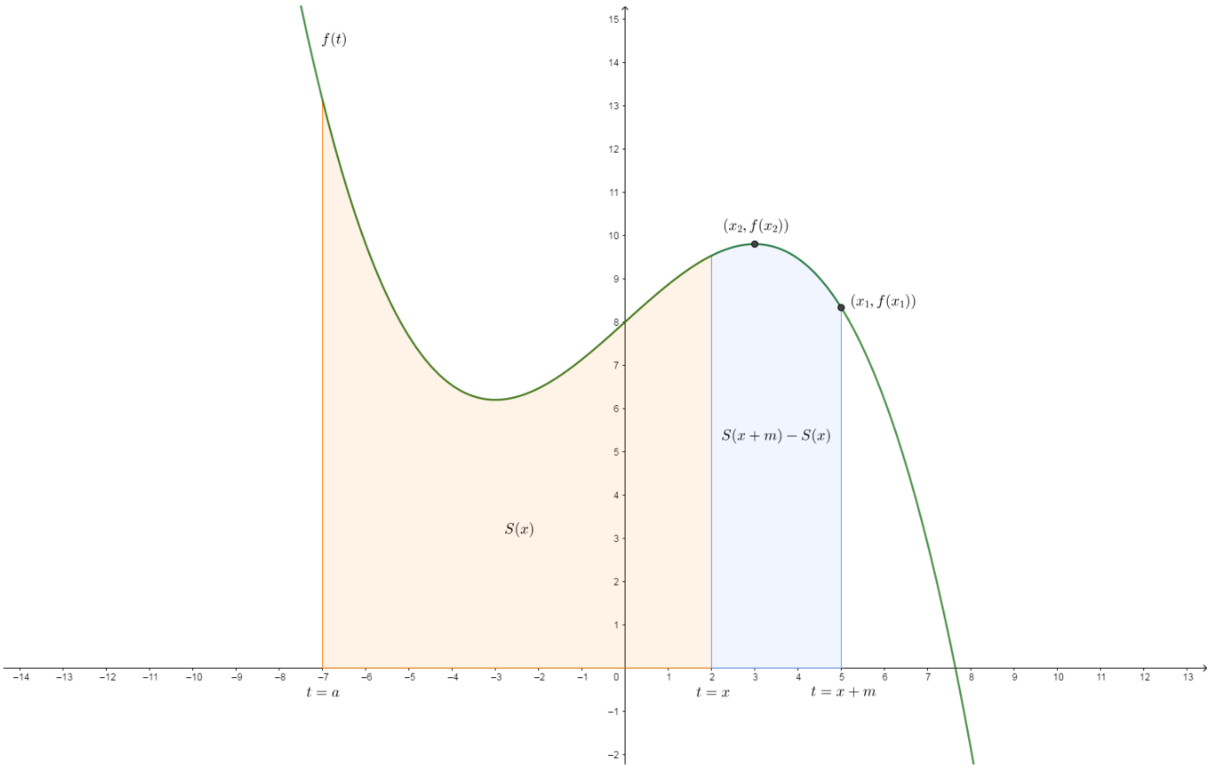
$$Lowest\ Point: (x_1, f(x_1)) \qquad Highest\ Point: (x_2, f(x_2))$$

$$x \le x_1 \le x + m \qquad x \le x_2 \le x + m$$

$$S(x) = \int_a^x f(t)\, dt$$



$$m\, f(x_1) \le S(x+m) - S(x) \le m\, f(x_2)$$

$$f(x_1) \le \frac{S(x+m) - S(x)}{m} \le f(x_2)$$

$$m \to 0$$

$$x \le x_1 \le x + 0 \qquad x \le x_2 \le x + 0$$

$$x_1 \to x \qquad x_2 \to x$$

$$f(x) \le \lim_{m \to 0} \frac{S(x+m) - S(x)}{m} \le f(x)$$

$$\lim_{m \to 0} \frac{S(x+m) - S(x)}{m} = \frac{d\, S(x)}{dx} = S'(x) = f(x)$$

$$F'(x) = f(x)$$

$$S(x) = F(x) + C$$

$$S(a) = \int_a^a f(t)\,dt = 0$$

$$F(a) + C = 0$$

$$C = -F(a)$$

$$S(x) = F(x) - F(a)$$

$$S(b) = F(b) - F(a)$$

$$\int_a^b f(t)\,dt = \int_a^b f(x)\,dx = \int_a^b F'(x)\,dx = F(b) - F(a)$$

$$\therefore \int_a^b F'(x)\,dx = F(b) - F(a)$$

Resources - resources.zip