1. Rotation Matrices

1) 2D Rotation

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

2) 3D Rotation

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. 2D Orbital Trajectory

$\overrightarrow{r_0}$: Initial Position Vector

$\overrightarrow{v_0}$: Initial Velocity Vector

a : Semi-major Axis

ε : Eccentricity

r_p : Periapsis

r_a : Apoapsis

θ_p : Argument of Periapsis

$$r_0 = |\overrightarrow{r_0}| \qquad v_0 = |\overrightarrow{v_0}| \qquad H = |\overrightarrow{r_0} \times \overrightarrow{v_0}|$$

$$a = \frac{\mu r_0}{2\mu - r_0 {v_0}^2} \qquad \varepsilon = \sqrt{1 - \frac{H^2}{\mu a}}$$

$$r = \frac{H^2}{\mu(1 + \varepsilon \cos\theta)}$$

$$\begin{cases} \theta = 0 & r_p = a(1 - \varepsilon) \\ \theta = \pi & r_a = a(1 + \varepsilon) \end{cases}$$

$$\theta_0 = cos^{-1}\left(\frac{1}{\varepsilon}\left(\frac{H^2}{\mu r_0} - 1\right)\right)$$

$$\theta_p = \begin{cases} {tan_2}^{-1}\overrightarrow{r_0} - \theta_0 & (\overrightarrow{r_0} \cdot \overrightarrow{v_0} \geq 0) \\ {tan_2}^{-1}\overrightarrow{r_0} + \theta_0 & (\overrightarrow{r_0} \cdot \overrightarrow{v_0} < 0) \end{cases}$$

$$Orbit2D(\mu, \overrightarrow{r_0}, \overrightarrow{v_0}) = R_z(\theta_p)\,\vec{r}$$

3. 3D Orbital Trajectory

$\vec{H}$: Specific Angular Momentum Vector

ϕ : Inclination

θ : Longitude of the Ascending Node

$$\vec{H} = \overrightarrow{r_0} \times \overrightarrow{v_0}$$

$$\vec{H} = H_x\hat{\imath} + H_y\hat{J} + H_z\hat{k}$$

$$\phi = cos^{-1}\frac{H_z}{|\vec{H}|}$$

$$\theta = tan_2{}^{-1}\left(H_x\hat{\imath} + H_y\hat{\jmath}\right)$$

$$\overrightarrow{r_1} = R_y(-\phi)\, R_z(-\theta)\, \overrightarrow{r_0}$$

$$\overrightarrow{v_1} = R_y(-\phi)\, R_z(-\theta)\, \overrightarrow{v_0}$$

$$Orbit3D(\mu, \overrightarrow{r_0}, \overrightarrow{v_0}) = R_z(\theta)\, R_y(\phi)\, Orbit2D(\mu, \overrightarrow{r_1}, \overrightarrow{v_1})$$

4. Position and Velocity

E : Eccentricity Anomaly

t : Time since Periapsis

P : Orbital Period

θ : True Anomaly

$$E_0 = \begin{cases} \cos^{-1}\left(\frac{1}{\varepsilon}\left(1 - \frac{r_0}{a}\right)\right) & (\overrightarrow{r_0} \cdot \overrightarrow{v_0} \geq 0) \\ 2\pi - \cos^{-1}\left(\frac{1}{\varepsilon}\left(1 - \frac{r_0}{a}\right)\right) & (\overrightarrow{r_0} \cdot \overrightarrow{v_0} < 0) \end{cases}$$

$$t_0 = \sqrt{\frac{a^3}{\mu}}\,(E_0 - \varepsilon \sin E_0)$$

$$E - \varepsilon \sin E = t\sqrt{\frac{\mu}{a^3}} \qquad P = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$r = a(1 - \varepsilon \cos E)$$

$$\theta = \begin{cases} \cos^{-1}\left(\frac{1}{\varepsilon}\left(\frac{H^2}{\mu r} - 1\right)\right) & \left(t \,\%\, P \leq \frac{P}{2}\right) \\ 2\pi - \cos^{-1}\left(\frac{1}{\varepsilon}\left(\frac{H^2}{\mu r} - 1\right)\right) & \left(t \,\%\, P > \frac{P}{2}\right) \end{cases}$$

$$\vec{v} = \frac{\sqrt{\mu a}}{r}\left(-\sin E\,\hat{\imath} + \sqrt{1 - \varepsilon^2}\cos E\,\hat{\jmath}\right)$$