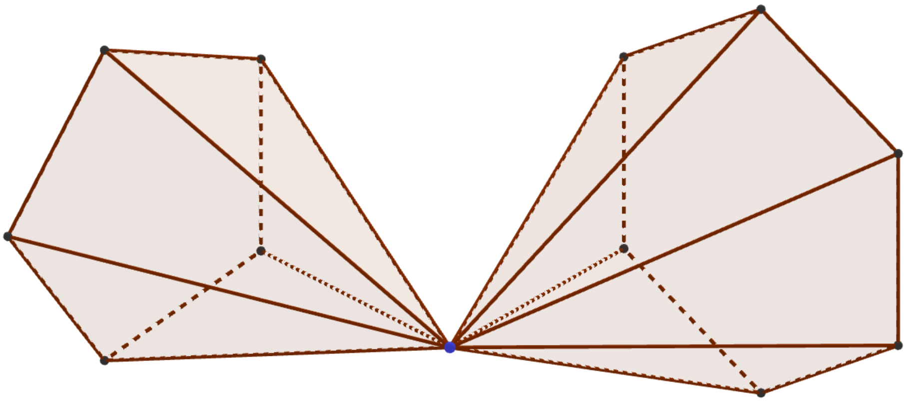
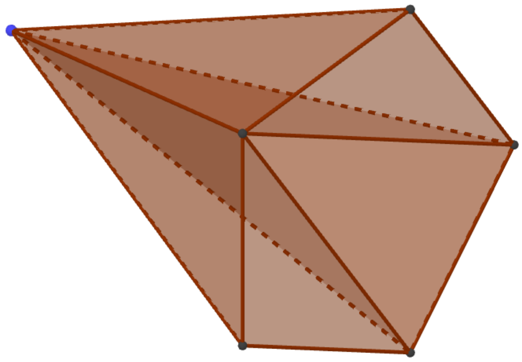
(1)



$$\phi^2 - \phi - 1 = 0$$
$$\phi^2 = \phi + 1$$
$$\phi = \frac{1+\sqrt{5}}{2}$$

Pentagon	*Hexagon*
$\frac{1}{2}\hat{\imath} + \frac{3}{2}\phi\hat{\jmath} + 0\hat{k}$	$\left(1+\frac{1}{2}\phi\right)\hat{\imath} - \phi\hat{\jmath} - \frac{1}{2}\hat{k}$
$1\hat{\imath} + \left(\frac{1}{2}+\phi\right)\hat{\jmath} + \frac{1}{2}\phi\hat{k}$	$\left(\frac{1}{2}+\phi\right)\hat{\imath} - \frac{1}{2}\phi\hat{\jmath} - \hat{k}$
$\left(1+\frac{1}{2}\phi\right)\hat{\imath} + \phi\hat{\jmath} + \frac{1}{2}\hat{k}$	$\frac{3}{2}\phi\hat{\imath} + 0\hat{\jmath} - \frac{1}{2}\hat{k}$
$\left(1+\frac{1}{2}\phi\right)\hat{\imath} + \phi\hat{\jmath} - \frac{1}{2}\hat{k}$	$\frac{3}{2}\phi\hat{\imath} + 0\hat{\jmath} + \frac{1}{2}\hat{k}$
$1\hat{\imath} + \left(\frac{1}{2}+\phi\right)\hat{\jmath} - \frac{1}{2}\phi\hat{k}$	$\left(\frac{1}{2}+\phi\right)\hat{\imath} - \frac{1}{2}\phi\hat{\jmath} + \hat{k}$
	$\left(1+\frac{1}{2}\phi\right)\hat{\imath} - \phi\hat{\jmath} + \frac{1}{2}\hat{k}$

(2) Volume of **P**entagonal Pyramid



$$\frac{1}{6}\left(2\left\|\begin{matrix}\frac{1}{2} & \frac{3}{2}\phi & 0\\ 1 & \frac{1}{2}+\phi & \frac{1}{2}\phi\\ 1+\frac{1}{2}\phi & \phi & \frac{1}{2}\end{matrix}\right\|+\left\|\begin{matrix}\frac{1}{2} & \frac{3}{2}\phi & 0\\ 1+\frac{1}{2}\phi & \phi & \frac{1}{2}\\ 1+\frac{1}{2}\phi & \phi & -\frac{1}{2}\end{matrix}\right\|\right)$$

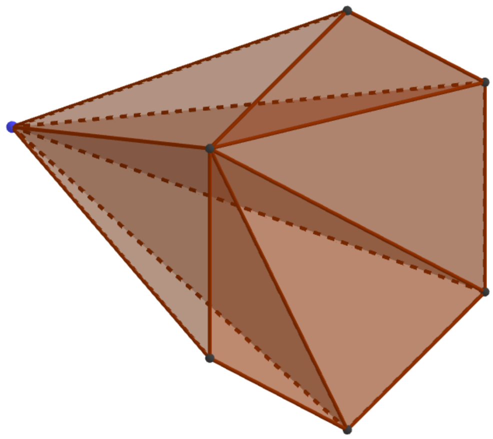
$$=\frac{1}{6}\left(2\left\|\begin{matrix}\frac{1}{2} & 0 & 0\\ 1 & \frac{1}{2}-2\phi & \frac{1}{2}\phi\\ 1+\frac{1}{2}\phi & -\frac{3}{2}-\frac{7}{2}\phi & \frac{1}{2}\end{matrix}\right\|+\left\|\begin{matrix}\frac{1}{2} & \frac{3}{2}\phi & 0\\ 1+\frac{1}{2}\phi & \phi & \frac{1}{2}\\ 0 & 0 & -1\end{matrix}\right\|\right)$$

$$=\frac{1}{6}\left(2\left|\frac{1}{2}\begin{vmatrix}\frac{1}{2}-2\phi & \frac{1}{2}\phi\\ -\frac{3}{2}-\frac{7}{2}\phi & \frac{1}{2}\end{vmatrix}\right|+\left|-\begin{vmatrix}\frac{1}{2} & \frac{3}{2}\phi\\ 1+\frac{1}{2}\phi & \phi\end{vmatrix}\right|\right)$$

$$=\frac{1}{6}\left(2\left|\frac{1}{4}\begin{vmatrix}\frac{1}{2}-2\phi & \phi\\ -\frac{3}{2}-\frac{7}{2}\phi & 1\end{vmatrix}\right|+\left|-\phi\begin{vmatrix}\frac{1}{2} & \frac{3}{2}\\ 1+\frac{1}{2}\phi & 1\end{vmatrix}\right|\right)$$

$$=\frac{1}{6}\left(2\left|1+\frac{3}{4}\phi\right|+\left|\frac{3}{4}+\frac{7}{4}\phi\right|\right)=\frac{11}{24}+\frac{13}{24}\phi$$

(3) Volume of **H**exagonal Pyramid



$$\frac{1}{6}\left(2\left|\begin{vmatrix} 1+\frac{1}{2}\phi & -\phi & -\frac{1}{2} \\ \frac{1}{2}+\phi & -\frac{1}{2}\phi & -1 \\ \frac{3}{2}\phi & 0 & -\frac{1}{2} \end{vmatrix}\right| + 2\left|\begin{vmatrix} 1+\frac{1}{2}\phi & -\phi & -\frac{1}{2} \\ \frac{3}{2}\phi & 0 & -\frac{1}{2} \\ \frac{3}{2}\phi & 0 & \frac{1}{2} \end{vmatrix}\right|\right)$$

$$= \frac{1}{6}\left(2\left|\begin{vmatrix} 1-\phi & -\phi & -\frac{1}{2} \\ \frac{1}{2}-2\phi & -\frac{1}{2}\phi & -1 \\ 0 & 0 & -\frac{1}{2} \end{vmatrix}\right| + 2\left|\phi\begin{vmatrix} \frac{3}{2}\phi & -\frac{1}{2} \\ \frac{3}{2}\phi & \frac{1}{2} \end{vmatrix}\right|\right)$$

$$= \frac{1}{6}\left(2\left|-\frac{1}{2}\begin{vmatrix} 1-\phi & -\phi \\ \frac{1}{2}-2\phi & -\frac{1}{2}\phi \end{vmatrix}\right| + 2\left|\frac{3}{4}\phi^2\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}\right|\right)$$

$$= \frac{1}{6}\left(2\left|\frac{1}{2}\phi\begin{vmatrix} 1-\phi & 1 \\ \frac{1}{2}-2\phi & \frac{1}{2} \end{vmatrix}\right| + 2\left|\frac{3}{2}\phi^2\right|\right)$$

$$= \frac{1}{6}\left(2\left|\frac{3}{4}+\frac{3}{4}\phi\right| + 2\left|\frac{3}{2}+\frac{3}{2}\phi\right|\right) = \frac{3}{4}+\frac{3}{4}\phi$$

(4) **V**olume of Truncated Icosahedron

$$12P + 20H = 12\left(\frac{11}{24} + \frac{13}{24}\phi\right) + 20\left(\frac{3}{4} + \frac{3}{4}\phi\right)$$

$$= \frac{11}{2} + \frac{13}{2}\phi + 15 + 15\phi = \frac{41}{2} + \frac{43}{2}\phi$$

$$= \frac{41}{2} + \frac{43}{2} \cdot \frac{1+\sqrt{5}}{2} = \frac{82}{4} + \frac{43+43\sqrt{5}}{4} = \frac{125+43\sqrt{5}}{4}$$

$$\therefore V = \frac{125+43\sqrt{5}}{4}$$

Resources - resources.zip