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Department of Electrical and Computer Engineering
ECE417/ECE613: Image Processing and Visual Communications

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Homework 1

- The use of calculators is allowed.
- Work on the Homework independently. Group work not allowed.
- Submit your Homework individually in one single file that includes your answers to all questions.

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1. A 4×4, 4bits/pixel original image passes through a distortion process, resulting in a distorted image, as shown below

0	10	7	5
0	2	9	12
4	2	2	6
10	3	9	15

original image

→ Distortion Process →

2	9	10	5
0	1	6	1
3	6	2	6
11	3	14	14

distorted image

Q1:

Compute the mean absolute error (MAE), the mean squared error (MSE) and the peak signal-to-noise ratio (PSNR) between the original and the distorted images.

① $MAE = \left[|0-2| + |10-9| + |7-10| + |5-5| + |0-0| + |2-1| + |9-6| + |12-1| + |4-3| + |2-6| + |2-2| + |6-6| + |10-11| + |3-3| + |9-14| + |15-14|\right] \times \frac{1}{16}$

$= [2 + 1 + 3 + 1 + 3 + 11 + 1 + 4 + 1 + 5 + 1] \times \frac{1}{16}$

$= 33 \times \frac{1}{16} = 2.0625$

② $$MSE = \frac{\sum_{M,N} [I_1(m,n) - I_2(m,n)]^2}{M \times N}$$

$$= \frac{2^2 + 1^2 + 3^2 + 1^2 + 3^2 + 11^2 + 1^2 + 4^2 + 1^2 + 5^2 + 1^2}{4 \times 4}$$

$$= 11.8125$$

③ $$PSNR = 10 \log_{10} \left(\frac{[2^b - 1]^2}{MSE} \right)$$

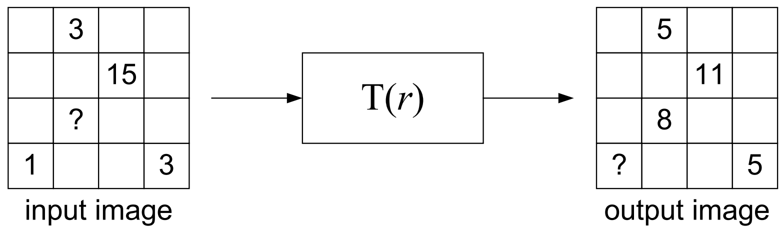
$$= 10 \log_{10} \left(\frac{[2^4 - 1]^2}{11.8125} \right)$$

$$= 12.798$$

2. A 4×4, 4bits/pixel image passes through a point-wise intensity transformation given by

$$s = T(r) = \alpha \log_2(1+r) + \beta$$

where α and β are unknown parameters. Only a few pixels are available in the input and the output images, as shown below.



(a) Find α and β.

(b) What's the value of the pixel with the "?" mark in the output image.

(c) What's the value of the pixel with the "?" mark in the input image.

Q2:

(a) Based on the point-wise intensity transformation, we can get equations below:

$$\begin{cases} \alpha \log_2(1+3) + \beta = 5 \\ \alpha \log_2(1+15) + \beta = 11 \end{cases} \Rightarrow \begin{cases} 2\alpha + \beta = 5 \\ 4\alpha + \beta = 11 \end{cases} \Rightarrow \begin{cases} \alpha = 3 \\ \beta = -1 \end{cases}$$

(b) $\because 3 \times \log_2(1+1) - 1 = 2$

$\therefore$ The "?" mark in the output image values at 2.

(c) $\because 3 \times \log_2(1+r) - 1 = 8$

$$r = 2^3 - 1 = 7$$

$\therefore$ The "?" mark in the input image values at 7.

3. An original image (the intensity values of the pixels are normalized to be between 0 and 1) passes through several point-wise intensity transformations listed below

(1) $s = r^2$

(2) $s = 1 - r$

(3) $s = 0.5r + 0.25$

(4) $s = \begin{cases} 0 & r \le 0.7 \\ 1 & else \end{cases}$

(5) $s = \begin{cases} 0 & r < 0.25 \\ 2r - 0.5 & 0.25 \le r \le 0.75 \\ 1 & 0.75 < r \end{cases}$

The original and transformed images are shown below:

Original (A) (B)

(C) (D) (E)

For each resulting image (A)-(E), determine the corresponding intensity transformation that creates it. Briefly explain your answers.

Q3:

(1) $s = r^2$. The graph could be: (E).

because it allows most of light parts to become darker.

(2) $s = 1 - r$, corresponds to graph (C).

because it exchanges the light parts dark parts.

(3) $s = 0.5r + 0.25$. The graph could be (D).

For example, if $r = 0$ (black), $s = 0.25$

if $r = 1$ (white) $s = 0.75$

The process reduces the contrast.

(4) $s = \begin{cases} 0 & r \le 0.5 \\ 1 & \text{else} \end{cases}$, it corresponds the graph (B).

because it set up a threshold, which only remains two colors: black and white.

(e) $s = \begin{cases} 0 & r < 0.25 \\ 2r - 0.5 & 0.25 \le r \le 0.75 \\ 1 & 0.75 < r \end{cases}$, The graph would be (A).

because for $s = 2r - 0.5$, it increases the contrast.

4. A 4×4, 4bits/pixel original image is given by

6	13	12	13
12	6	7	12
13	7	7	12
14	11	11	14

(a) Apply full-scale contrast stretch to the image. Show your work and sketch the resulting image.

(b) Apply histogram equalization to the image. Show your work and sketch the resulting image.

(c) Sketch the histograms of the original image, the full-scale contrast stretched image, and the histogram-equalized image.

Q4

(a) $r_{min} = 6$, $r_{max} = 14$. $B = 4$. $2^B - 1 = 15$

$$S = \text{round}\left(15 \cdot \frac{r - r_{min}}{r_{max} - r_{min}}\right) = \text{round}\left(\frac{15}{8}(r-6)\right)$$

$6 \rightarrow \text{round}(0) = 0;$

$7 \rightarrow \text{round}\left(\frac{15}{8}\right) = 2;$

$11 \rightarrow \text{round}\left(\frac{15}{8} \times 5\right) = 9;$

$12 \rightarrow \text{round}\left(\frac{15}{8} \times 6\right) = 11;$

$13 \rightarrow \text{round}\left(\frac{15}{8} \times 7\right) = 13;$

$14 \rightarrow \text{round}\left(\frac{15}{8} \times 8\right) = 15$

$\Rightarrow$ The resulting image is:

0	13	11	13
11	0	2	11
13	2	2	11
15	9	9	15

(b)

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H(k)$	0	0	0	0	0	0	2	3	0	0	0	2	4	3	2	0
$Q(k)$	0	0	0	0	0	0	2	5	5	5	5	7	11	14	16	16

original $\longrightarrow$ Intermediate image

$$\begin{array}{|c|c|c|c|}\hline 6 & 13 & 12 & 13 \\ \hline 12 & 6 & 7 & 12 \\ \hline 13 & 7 & 7 & 12 \\ \hline 14 & 11 & 11 & 14 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|c|c|}\hline 2 & 14 & 11 & 14 \\ \hline 11 & 2 & 5 & 11 \\ \hline 14 & 5 & 5 & 11 \\ \hline 16 & 7 & 7 & 16 \\ \hline \end{array}$$

Full-scale Contrast sketch of Intermediate Image

$r_{min} = 2 \qquad r_{max} = 16 \qquad B = 4, \quad 2^B - 1 = 15$

$$S = \text{round}\left[15 \cdot \frac{r-2}{16-2}\right] = \text{round}\left[\frac{15}{14}(r-2)\right]$$

$$\begin{cases} 2 \rightarrow \text{round}(0) = 0; \\ 5 \rightarrow \text{round}(3.214) = 3; \\ 7 \rightarrow \text{round}(5.357) = 5; \\ 11 \rightarrow \text{round}(9.642) = 10; \\ 14 \rightarrow \text{round}(12.86) = 13; \\ 16 \rightarrow \text{round}(15) = 15; \end{cases} \Longrightarrow$$

The final result: histogram equalized image:

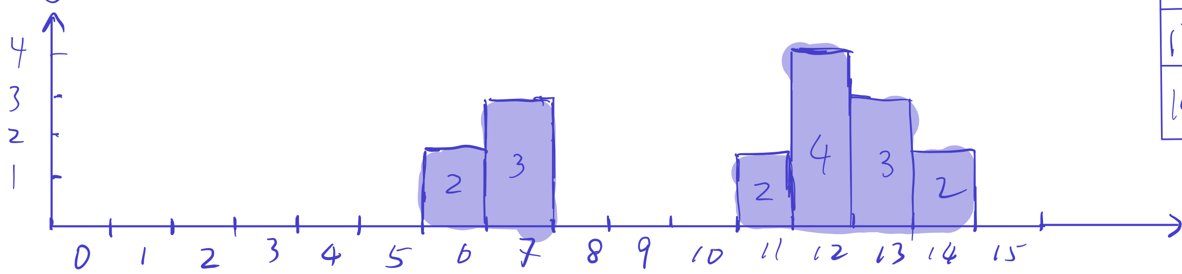
$$\begin{array}{|c|c|c|c|}\hline 0 & 13 & 10 & 13 \\ \hline 10 & 0 & 3 & 10 \\ \hline 13 & 3 & 3 & 10 \\ \hline 15 & 5 & 5 & 15 \\ \hline \end{array}$$

(3)

Histogram:

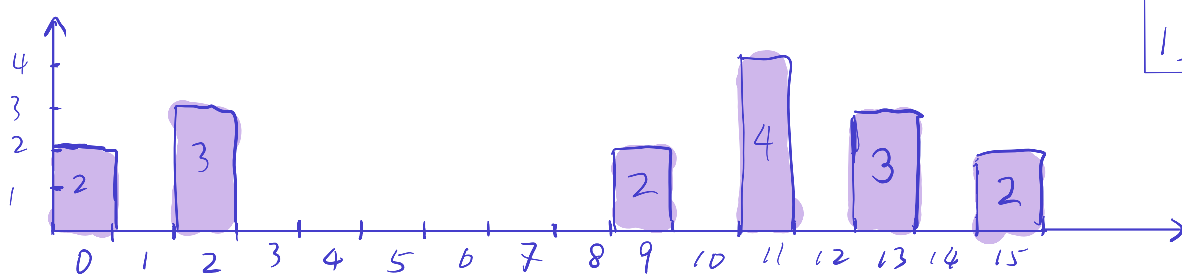
(1) original

6	13	12	13
12	6	7	12
13	7	7	12
14	11	11	14



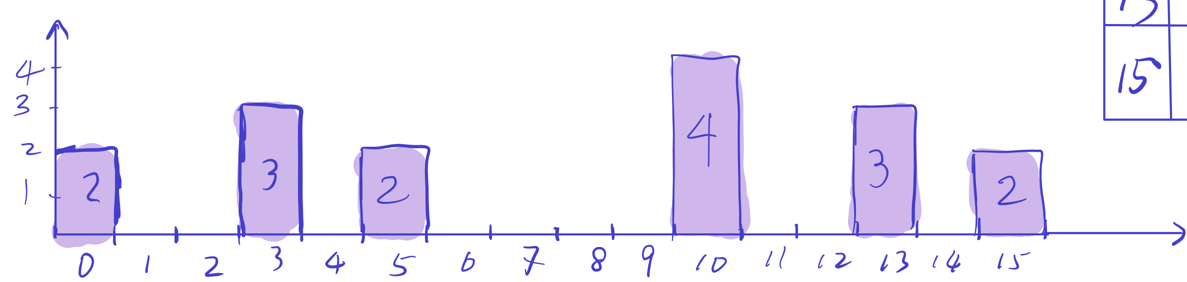
Full-scale sketched

0	13	11	13
11	0	2	11
13	2	2	11
15	9	9	15

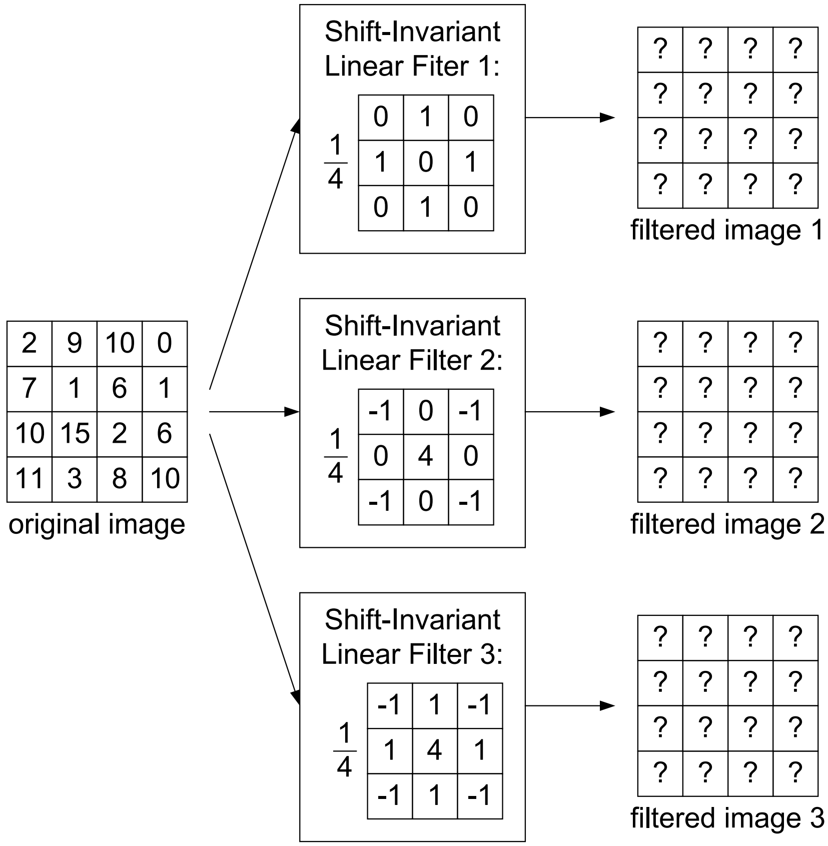


histogram-equalized

0	13	10	13
10	0	3	10
13	3	3	10
15	5	5	15



5. A 4×4 gray-scale original image passes through three spatial linear shift-invariant filters, resulting in three filtered images.



(a) Find the first filtered image (Use zero-padding of the original image and do not round the resulting image pixels into integers).

(b) Find the second filtered image (Use zero-padding of the original image and do not round the resulting image pixels into integers).

(c) Find the third filtered image (Use zero-padding of the original image and do not round the resulting image pixels into integers). Hint: Can you obtain the image from the results of (a) and (b)?

Q5

(a) After zero-padding of the original image, it is like :

0	0	0	0	0	0
0	2	9	10	0	0
0	7	1	6	1	0
0	10	15	2	6	0
0	11	3	8	10	0
0	0	0	0	0	0

for the first linear filter

use left corner element as the sample:

$$\frac{1}{4}\times[0\times0+0\times1+0\times0+0\times1+2\times0+9\times1$$
$$+0\times0+7\times1+1\times0]=\frac{1}{4}\times16=4$$

Therefore the filtered image 1 is:

4	3.25	3.75	2.75
3.25	9.25	3.5	3
8.25	4	8.75	3.25
3.25	8.5	3.75	3.5

(b) Similar as (a).

take left-corner element as a sample example:

$$\frac{1}{4}\Big[0\times(-1) + 0\times 0 + (-1)\times 0 + 0\times 0 + 2\times 4 + 9\times 0 + 0\times(-1)$$
$$+ 7\times 0 + 1\times(-1)\Big] = \frac{7}{4} = 1.75$$

1.75	5.75	9.5	-1.5
1	-5	-1.5	-2
9	7	-1.75	2.5
7.25	0	2.75	9.5

c) Similar as b) and a).

Still, take left-corner element as an example.

$$\frac{1}{4}\Big[0\times(-1) + 0\times 1 + 0\times(-1) + 0\times 1 + 2\times 4 + 9\times 1$$
$$+ 0\times(-1) + 7\times 1 + 1\times(-1)\Big] = 5.75$$

5.75	9	13.25	1.25
4.25	4.25	2	1
17.25	11	7	5.75
10.5	8.5	6.5	13

It can be seen that each element in filered image 1 adds to the corresponding element in filtered image 2 can get the same value in filtered image 3.

6. Two dimensional discrete Fourier transform (2D-DFT).

(a) Compute the 2D-DFT of the 4×4 gray-scale original image given below

7	1	7	1
5	3	5	3
7	1	7	1
5	3	5	3

$\longrightarrow$ 2D-DFT $\longrightarrow$

?	?	?	?
?	?	?	?
?	?	?	?
?	?	?	?

(b) Compute the inverse 2D-DFT of the following three 4×4 patterns, respectively

64	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

0	0	32	0
0	0	0	0
0	0	0	0
0	0	0	0

0	0	0	0
0	0	0	0
0	0	16	0
0	0	0	0

$\longrightarrow$ Inverse 2D-DFT $\longrightarrow$

?	?	?	?
?	?	?	?
?	?	?	?
?	?	?	?

?	?	?	?
?	?	?	?
?	?	?	?
?	?	?	?

?	?	?	?
?	?	?	?
?	?	?	?
?	?	?	?

(c) Add the three resulting images in (b) and compare it with the original image. Comment on it.

Q6:

(a) Because $N=4$,

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}, \quad F_4^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$\tilde{X} = F_4 X F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \\ 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 8 & 24 & 8 \\ 0 & 0 & 0 & 0 \\ 4 & -4 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 64 & 0 & 32 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b)

For $\tilde{X} = \begin{bmatrix} 64 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$X = \frac{1}{N^2} F_N^* \tilde{X} F_N^* = \frac{1}{16} F_4^* \tilde{X} F_4^*$$

$$= \frac{1}{16} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 64 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$

For $\tilde{X} = \begin{bmatrix} 0 & 0 & 32 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$X = \frac{1}{16} F_4^* \tilde{X} F_4^* = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \end{bmatrix}$$

For $\tilde{X} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$X = \frac{1}{16} F_4^* \tilde{X} F_4^* = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

c) $\begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \\ 2 & -2 & 2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$

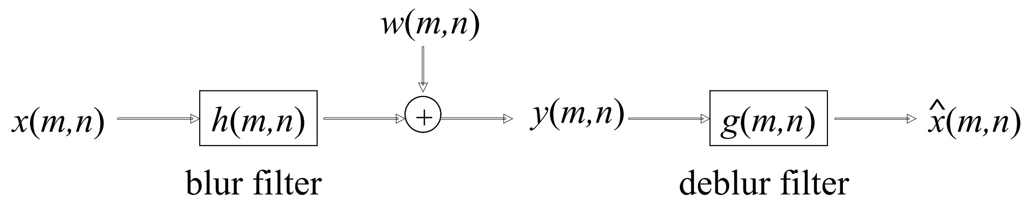
$= \begin{bmatrix} 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \\ 7 & 1 & 7 & 1 \\ 5 & 3 & 5 & 3 \end{bmatrix} =$ original image

The added result is the same as the origin image.
It shows Inverse 2D-DFT can do linear operation.
By distribution law of 2D-DFT:

$$\mathcal{F}\{x\} = \mathcal{F}\{x_1 + x_2 + x_3\} = \mathcal{F}\{x_1\} + \mathcal{F}\{x_2\} + \mathcal{F}\{x_3\} = \tilde{X}_1 + \tilde{X}_2 + \tilde{X}_3$$

and for linearity: $\tilde{X} = \tilde{X}_1 + \tilde{X}_2 + \tilde{X}_3 \xrightarrow[\text{INV.}]{\text{2D-DFT}} X_1 + X_2 + X_3 = X$.

7. An image distortion (blurring followed by additive white Gaussian noise contamination) and restoration model is shown as below. The signal variance and the noise variance are given by $\sigma_X^2 = 100$ and $\sigma_W^2 = 25$, respectively.



The 2D-DFT of the blur filter $h(m,n)$ is given by

$$H(u,v) = \begin{bmatrix} 1 & -0.2-0.2j & 0 & -0.2+0.2j \\ -0.2-0.2j & 0.05j & 0 & 0.05 \\ 0 & 0 & 0 & 0 \\ -0.2+0.2j & 0.05 & 0 & -0.05j \end{bmatrix}$$

(a) Design a deblur filter $G(u,v)$, i.e., the 2D-DFT of $g(m,n)$, using the inverse filtering approach;

(b) Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 0.03$;

(c) Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 0.1$;

(d) Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 0.3$;

(e) Design a deblur filter $G(u,v)$ using the Wiener filtering approach;

(f) Find $h(m,n)$;

(g) Find $g(m,n)$ for pseudo-inverse filtering with $\delta = 0.1$, i.e., the corresponding spatial domain deblur filter designed for Question (c).

Q7

(a)

$$\frac{1}{-0.2-0.2j} = -2.5+2.5j$$

$$G(u,v) = \frac{1}{H(u,v)} = \begin{bmatrix} 1 & -2.5+2.5j & \text{INF} & -2.5-2.5j \\ -2.5+2.5j & -20j & \text{INF} & 20 \\ \text{INF} & \text{INF} & \text{INF} & \text{INF} \\ -2.5-2.5j & 20 & \text{INF} & +20j \end{bmatrix}$$

(b) pseudo-inverse filter, with $\delta = 0.03$

$$G(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & |H(u,v)| > 0.03 \\ 0 & |H(u,v)| \le 0.03 \end{cases} = \begin{bmatrix} 1 & -2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & -20j & 0 & 20 \\ 0 & 0 & 0 & 0 \\ -2.5-2.5j & 20 & 0 & +20j \end{bmatrix}$$

take $-0.2-0.2j$ as an example.

$$|-0.2-0.2j| = \sqrt{0.2^2+0.2^2} = 0.283 > 0.03$$

(c) Similar as part (b), now $\delta = 0.1$

$$G(u,v) = \begin{bmatrix} 1 & -2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2.5-2.5j & 0 & 0 & 0 \end{bmatrix}$$

d) Similar as part (a), but now $\sigma = 0.3$.

$$G(u,v) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

e) For Wiener filter, with $\sigma_x^2 = 100$,

and $\sigma_w^2 = 25$.

$$\therefore K = \frac{\sigma_w^2}{\sigma_x^2} = \frac{25}{100} = 0.25$$

$$G(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K}$$

$$H^*(u,v) = \begin{bmatrix} 1 & -0.2+0.2j & 0 & -0.2-0.2j \\ -0.2+0.2j & -0.05j & 0 & 0.05 \\ 0 & 0 & 0 & 0 \\ -0.2-0.2j & 0.05 & 0 & 0.05j \end{bmatrix}$$

$$|H(u,v)|^2 = \begin{bmatrix} 1 & 0.08 & 0 & 0.08 \\ 0.08 & 2.5\times10^{-3} & 0 & 2.5\times10^{-3} \\ 0 & 0 & 0 & 0 \\ 0.08 & 2.5\times10^{-3} & 0 & 2.5\times10^{-3} \end{bmatrix}$$

$$G(u,v) = \begin{bmatrix} 0.8 & -0.61+0.61j & 0 & -0.61-0.61j \\ -0.61+0.61j & -0.2j & 0 & 0.2 \\ 0 & 0 & 0 & 0 \\ -0.61-0.61j & 0.2 & 0 & 0.2j \end{bmatrix}$$

f) Because $N=4$

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \qquad F_4^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$h(m,n) = \frac{1}{16} F_4^* H(u,v) F_4^*$$

$$= \frac{1}{16}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 1 & -0.2-0.2j & 0 & -0.2+0.2j \\ -0.2-0.2j & 0.05j & 0 & 0.05 \\ 0 & 0 & 0 & 0 \\ -0.2+0.2j & 0.05 & 0 & -0.05j \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \frac{1}{16}\begin{bmatrix} 0.6 & -0.15-0.15j & 0 & -0.15+0.15j \\ 1.4 & -0.25-0.25j & 0 & -0.25+0.25j \\ 1.4 & -0.25-0.25j & 0 & -0.25+0.25j \\ 0.6 & -0.15-0.15j & 0 & -0.15+0.15j \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \begin{bmatrix} 0.01875 & 0.05625 & 0.05625 & 0.01875 \\ 0.05625 & 0.11875 & 0.11875 & 0.05625 \\ 0.05625 & 0.11875 & 0.11875 & 0.05625 \\ 0.01875 & 0.05625 & 0.05625 & 0.01875 \end{bmatrix}$$

(g) $g(m,n)$ for question (c), in which

$$G(u,v) = \begin{bmatrix} 1 & 2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2.5-2.5j & 0 & 0 & 0 \end{bmatrix}$$

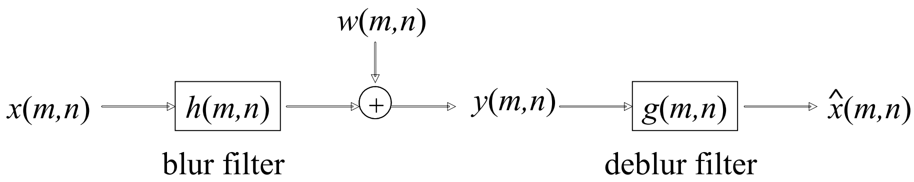
Because $N=4$, $\quad F_4^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$

$$g(m,n) = \frac{1}{16} F_4^* G(u,v) F_4^*$$

$$= \frac{1}{16} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 1 & -2.5+2.5j & 0 & -2.5-2.5j \\ -2.5+2.5j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2.5-2.5j & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= \begin{bmatrix} -0.5625 & -0.5625 & 0.0625 & 0.0625 \\ -0.5625 & -0.5625 & 0.0625 & 0.0625 \\ 0.0625 & 0.0625 & 0.6875 & 0.6875 \\ 0.0625 & 0.0625 & 0.6875 & 0.6875 \end{bmatrix}$$

8. An image distortion (blurring followed by additive white Gaussian noise contamination) and restoration model is shown in the figure below. The signal variance and the noise variance are given by $\sigma_X^2 = 400$ and $\sigma_W^2 = 100$, respectively.



The blur filter $h(m,n)$ is known as

$$h(m,n) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(a) Find $H(u,v)$, i.e., the blur filter in the frequency (2D-DFT) domain.

(b) Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 1$.

(c) Design a deblur filter $G(u,v)$ using the pseudo-inverse filtering approach with $\delta = 3$.

(d) Design a deblur filter $G(u,v)$ using the Wiener filtering approach.

Q8:

(a) Because $N=4$. $\Rightarrow$ $F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

$$H(u,v) = F_4 \, h(m,n) \, F_4$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -2-2j & 0 & -2+2j \\ -2-2j & 2j & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -2+2j & 2 & 0 & -2j \end{bmatrix}$$

(b) pseudo-inverse filter, with $\delta = 1$

$$G(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & |H(u,v)| > 1 \\ 0 & |H(u,v)| \le 1 \end{cases}$$

$$= \begin{bmatrix} 0.05 & -0.25+0.25j & 0 & -0.25-0.25j \\ -0.25+0.25j & -0.5j & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ -0.25-0.25j & 0.5 & 0 & 0.5j \end{bmatrix}$$

c) pseudo-inverse filter, with $\delta = 3$

$$G(u,v) = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

d) Wiener filter, with $\sigma_x^2 = 400$, $\sigma_w^2 = 100$

$$K = \frac{\sigma_w^2}{\sigma_x} = \frac{100}{400} = 0.25$$

$$G(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K}$$

$$H^*(u,v) = \begin{bmatrix} 20 & -2+2j & 0 & -2-2j \\ -2+2j & -2j & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -2-2j & 2 & 0 & +2j \end{bmatrix}$$

$$|H(u,v)|^2 = \begin{bmatrix} 400 & 8 & 0 & 8 \\ 8 & 4 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 8 & 4 & 0 & 4 \end{bmatrix}$$

$$\therefore G(u,v) = \begin{bmatrix} 0.05 & -0.24+0.24j & 0 & -0.24-0.24j \\ -0.24+0.24j & -0.47j & 0 & 0.47 \\ 0 & 0 & 0 & 0 \\ -0.24-0.24j & 0.47 & 0 & 0.47j \end{bmatrix}$$

9. A 4×4, 4bits/pixel original image is shown as below

7	2	7	2
2	7	2	7
7	2	7	2
2	7	2	7

(a) The original image passes through a linear shift-invariant filter given by

$$\frac{1}{6}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Find the filtered image (use zero-padding of the original image), and round the resulting image pixels to integers. Compute the mean absolute error (MAE), the mean squared error (MSE), and the peak signal to noise ratio (PSNR) between the original image and the filtered image (after rounding).

(b) The original image passes through a 3×3 median filter. Find the filtered image (use replicate-padding of the original image). Compute the MAE, the MSE, and the PSNR between the original and the filtered images.

Q9:

After zero-padding, the original image change to

(a)

0	0	0	0	0	0
0	7	2	7	2	0
0	2	7	2	7	0
0	7	2	7	2	0
0	2	7	2	7	0
0	0	0	0	0	0

The filtered image is:

3	4	3	3
4	4	5	3
3	5	4	4
3	3	4	3

Take the left-top element as an example:

$(0\times1 + 0\times1 + 2\times1 + 2\times1 + 7\times2)\times\frac{1}{6} = 3$

- $MAE = [4+2+4+1+2+3+3+4$
$+4+3+3+2+1+4+2+4] \times \frac{1}{16}$

$= 46 \times \frac{1}{4 \times 4}$

$= 2.875$

- $MSE = [4^2+2^2+4^2+1^2+2^2+3^2+3^2+4^2$
$+4^2+3^2+3^2+2^2+1^2+4^2+2^2+4^2] \times \frac{1}{16}$

$= 9.375$

- $PSNR = 10 \times \log_{10}\left(\frac{[2^4-1]^2}{9.375}\right)$ because of 4 bits/pixel

$= 13.802$

(b) After replicating-padding, the original image change to

7	7	2	7	2	2
7	7	2	7	2	2
2	2	7	2	7	7
7	7	2	7	2	2
2	2	7	2	7	7
2	2	7	2	7	7

The filtered image (by 3×3 median filter) will be:

7	7	2	2
7	7	2	2
2	2	7	7
2	2	7	7

$$MAE = \frac{1}{16} \times \left[(0+5+5+0) \times 4 \right] = 2.5$$

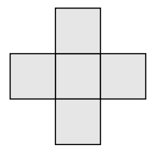
$$MSE = \frac{1}{16} \times \left[5^2 \times 8 \right] = 12.5$$

$$PSNR = 10 \log_{10} \left(\frac{[2^4-1]^2}{12.5} \right) = 12.553$$

10. A 4×4 image is given by

9	8	7	6
8	7	13	5
7	6	5	4
6	1	4	3

(a) Filter the image using a Median filter (after replicate-padding), where the filter mask is given by



(b) Filter the image using a Min filter (after replicate-padding) using the same filter mask as in (a).

(c) Filter the image using a Max filter (after replicate-padding) using the same filter mask as in (a).

(d) Filter the image using an Order Statistics filter (after replicate padding) using the same filter mask as in (a). The weighting vector of the order statistics filter is defined as

$$\{w_i\} = \left\{0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right\}.$$

Round the resulting pixels to integers.

Ans:

After replicate-padding, the image changes to:

9	9	8	7	6	6
9	9	8	7	6	6
8	8	7	13	5	5
7	7	6	5	4	4
6	6	1	4	3	3
6	6	1	4	3	3

(a) Take the median filter, the filtered image would be as:

9	8	7	6
8	8	7	5
7	6	5	4
6	4	4	3

Take the top-left-corner element as an example.

$\{8, 8, 9, 9, 9\}$, which the median is 9

(b) Take the min. filter, the filtered image would be like:

8	7	6	5
7	6	5	4
6	1	4	3
1	1	1	3

Take the left-bottom element as an example.

$\{1, 6, 6, 6, 7\}$

the minimum number is 1.

(c) Take the max filter, the filtered image would be as:

9	9	13	7
9	13	13	13
8	7	13	5
7	6	5	4

Still, take the left-bottom element as an example.

$\{1, 6, 6, 6, 7\}$.

the maximum number is 7.

(d) Take the order statistic filter

with $\{w_i\} = \{0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\}$.

the filtered image would be:

9	8	7	6
8	8	6	5
7	6	5	4
6	4	4	3

Take the left-corner element as an example:

$\{8, 8, 9, 9, 9\}$

the result would be:

$8 \times \frac{1}{3} + 9 \times \frac{1}{3} + 9 \times \frac{1}{3} = 8.67 \approx 9$

11. A 7×7 image is given by

3	3	1	3	3	3	4
0	3	3	3	3	3	3
3	3	3	2	3	3	12
12	3	3	3	3	12	12
10	12	2	3	3	12	12
12	14	12	12	12	12	11
11	12	12	12	10	12	12

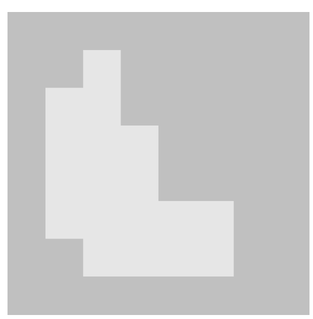
Use Prewitt gradient operator to find its edges. Use

$$|g(m,n)| = |g_1(m,n)| + |g_2(m,n)|$$

to estimate the gradient magnitude, and use $T = 22$ as the threshold for edge detection. Note: no padding of the boundaries, and the result will be a 5x5 binary edge image.

Q11: See below

12. An image is given below



Use iterative quadtree split-and-merge algorithm to segment the image. Give the result after each step (split or merge) in each iteration.

Question C11).

Use Prewitt gradient operator,

$$g_1 = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad g_2 = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Therefore. $g_1(m,n)$ will be:

1	-1	2	1	10
-6	-1	0	10	18
-17	-10	1	19	27
-17	-11	1	18	17
-7	-11	-1	9	10

$g_2(m,n)$ will be:

2	1	1	-1	8
12	0	0	9	18
15	9	0	10	9
20	29	27	18	8
11	19	26	16	7

$$|g(m,n)| = |g_1(m,n)| + |g_2(m,n)|$$

=

3	2	3	2	18
18	1	0	19	36
32	19	1	29	36
37	40	28	36	25
18	30	27	25	17

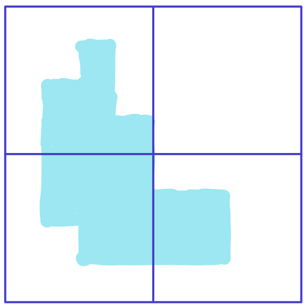
$T = 22$, we can get the edge map be:

0	0	0	0	0
0	0	0	0	1
1	0	0	1	1
1	1	1	1	1
0	1	1	1	0

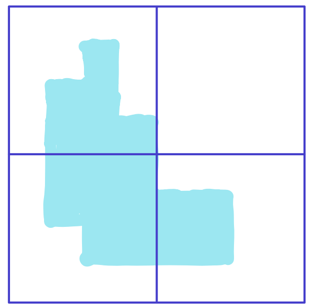
Question 12)

First iteration:

split →

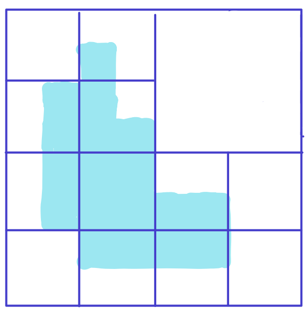


merge →

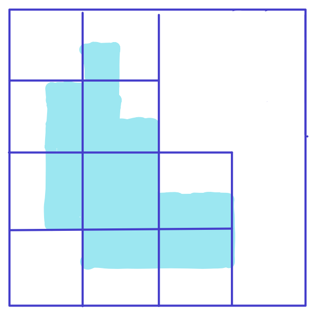


Second iteration:

split →

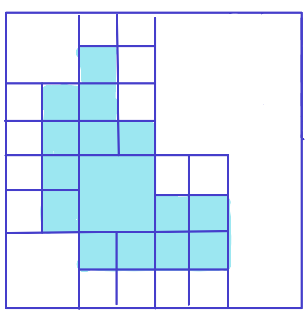


merge →



Third iteration:

split →



merge →