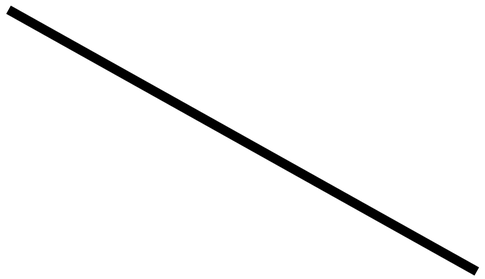
Vectors

A vector is a mathematical quantity
that has a magnitude and a direction.

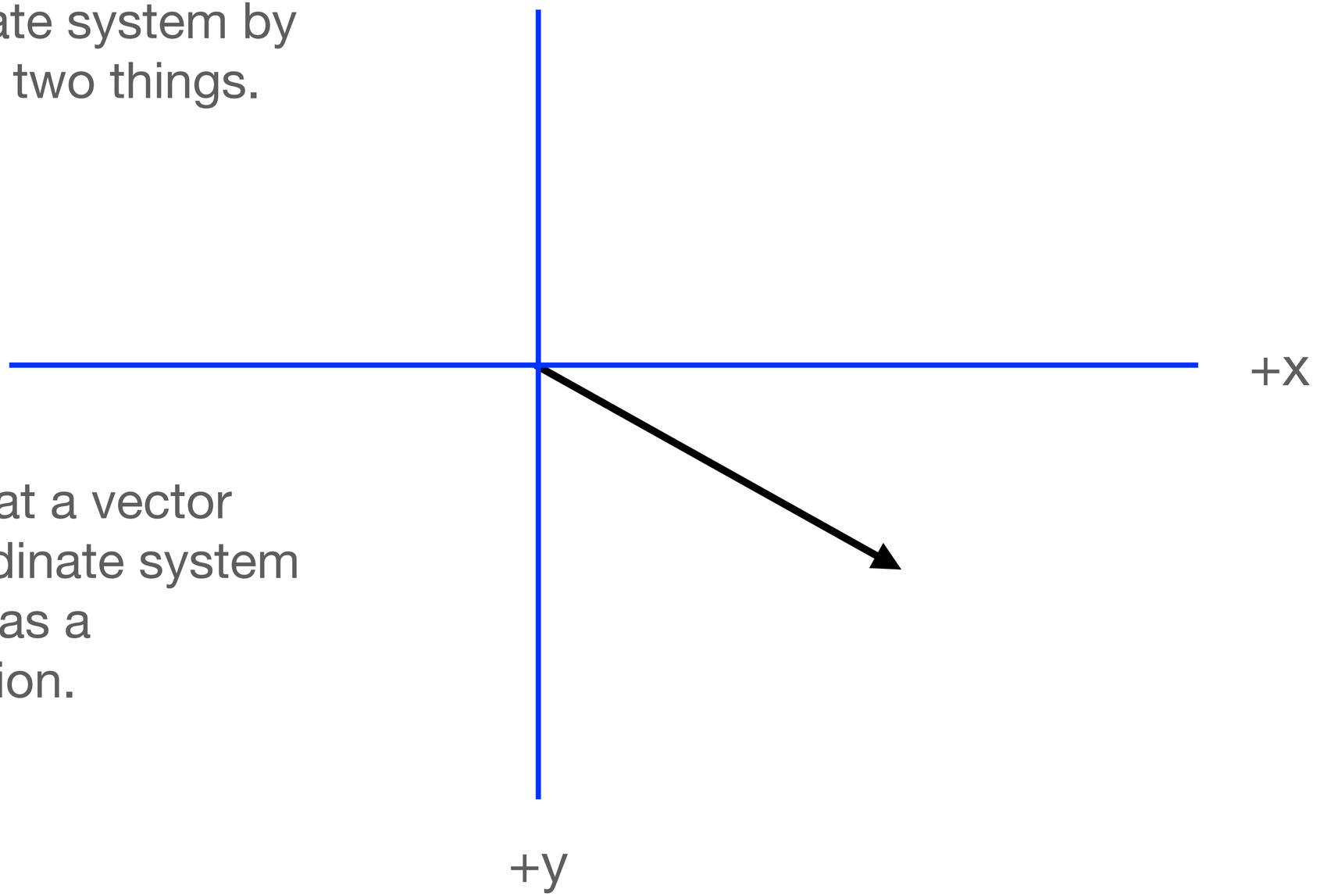
A vector can be represented visually
by a line segment.

Does this line segment represent a vector?

What is its magnitude and what is its direction?

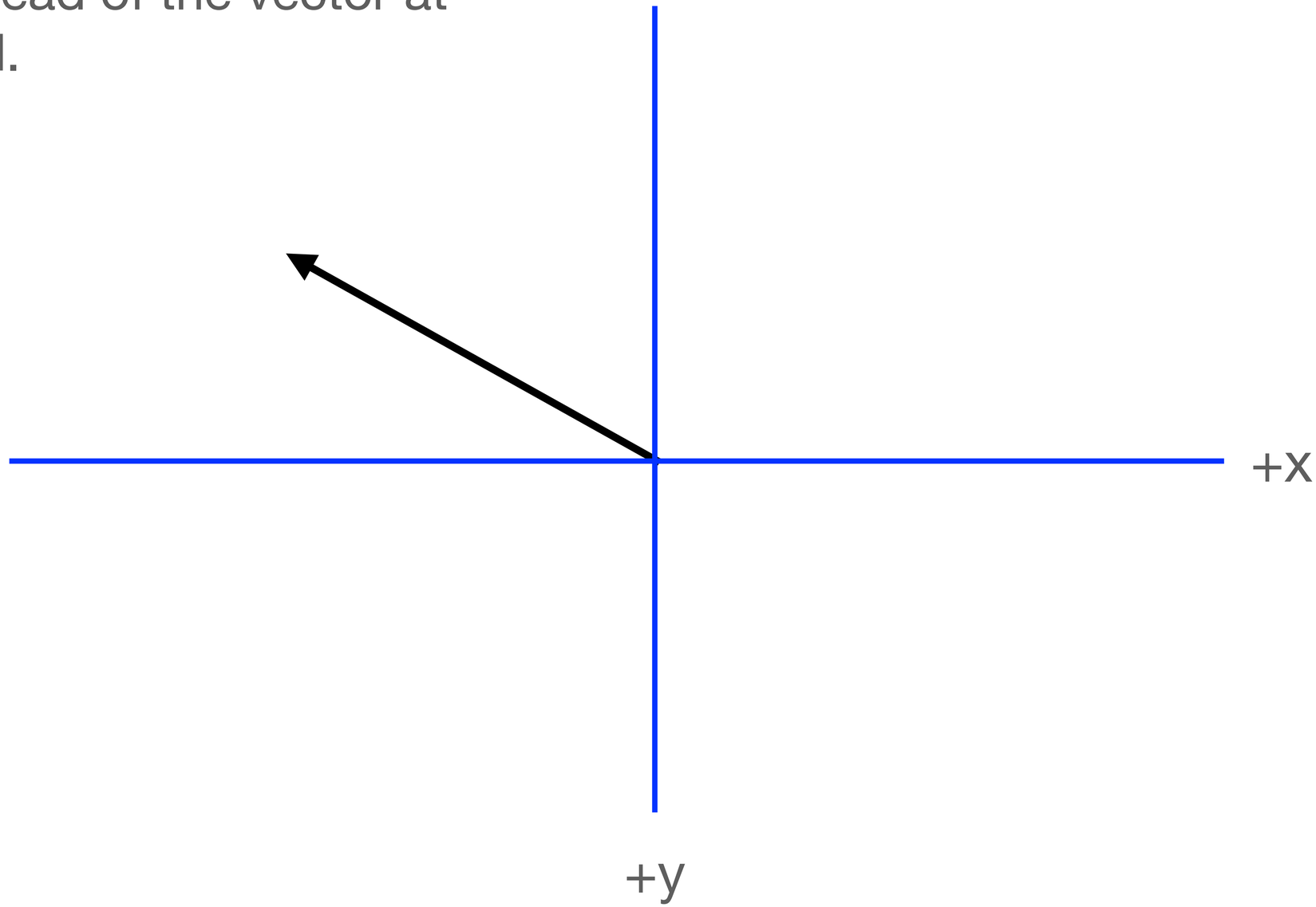


We can’t say that a line segment has a magnitude and a direction unless we have some coordinate system by which to quantify those two things.



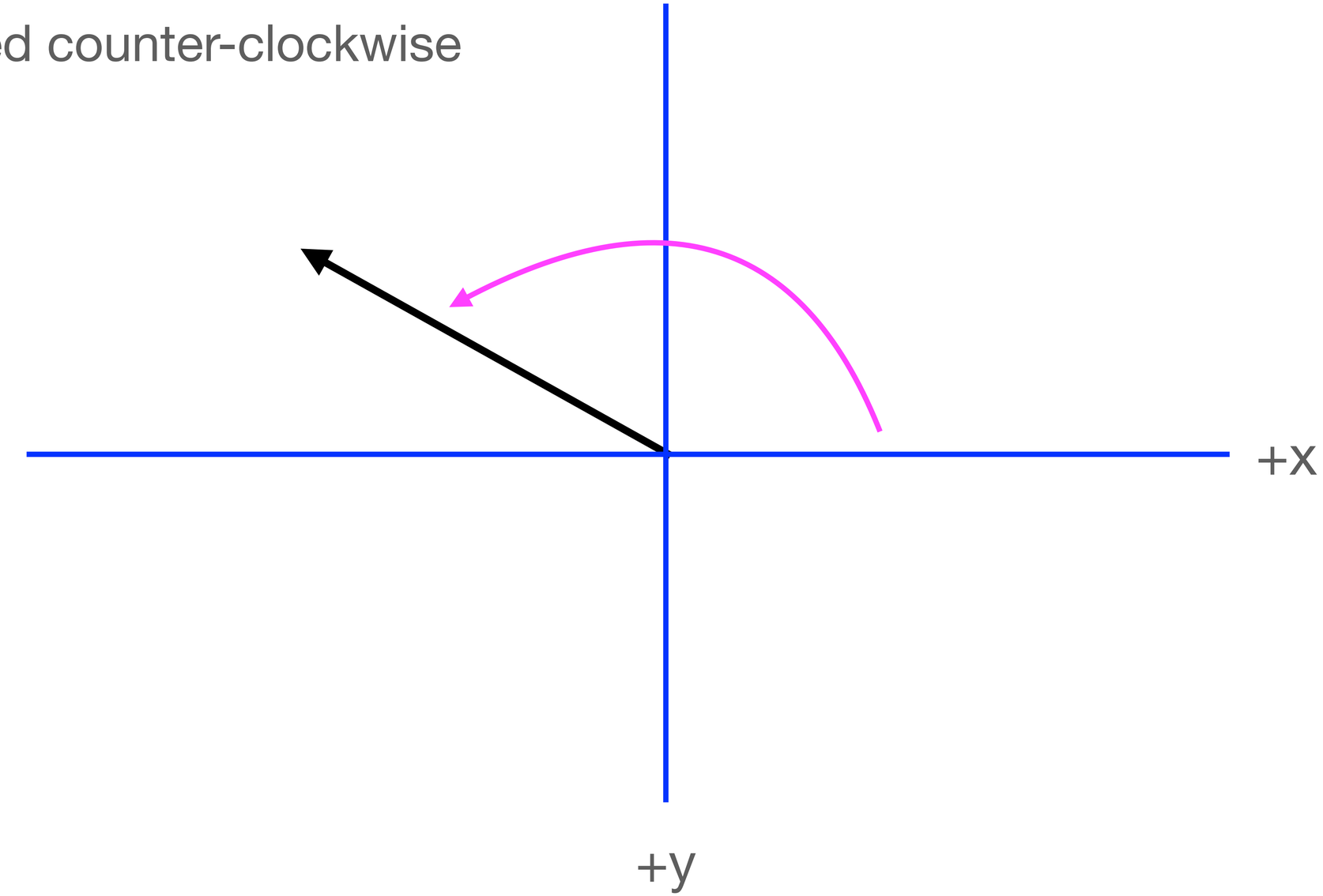
We have to presume that a vector comes with some coordinate system if we are to say that it has a magnitude and a direction.

The origin of a vector’s coordinate system will lie at the tail end of the vector with the head of the vector at the opposite end.

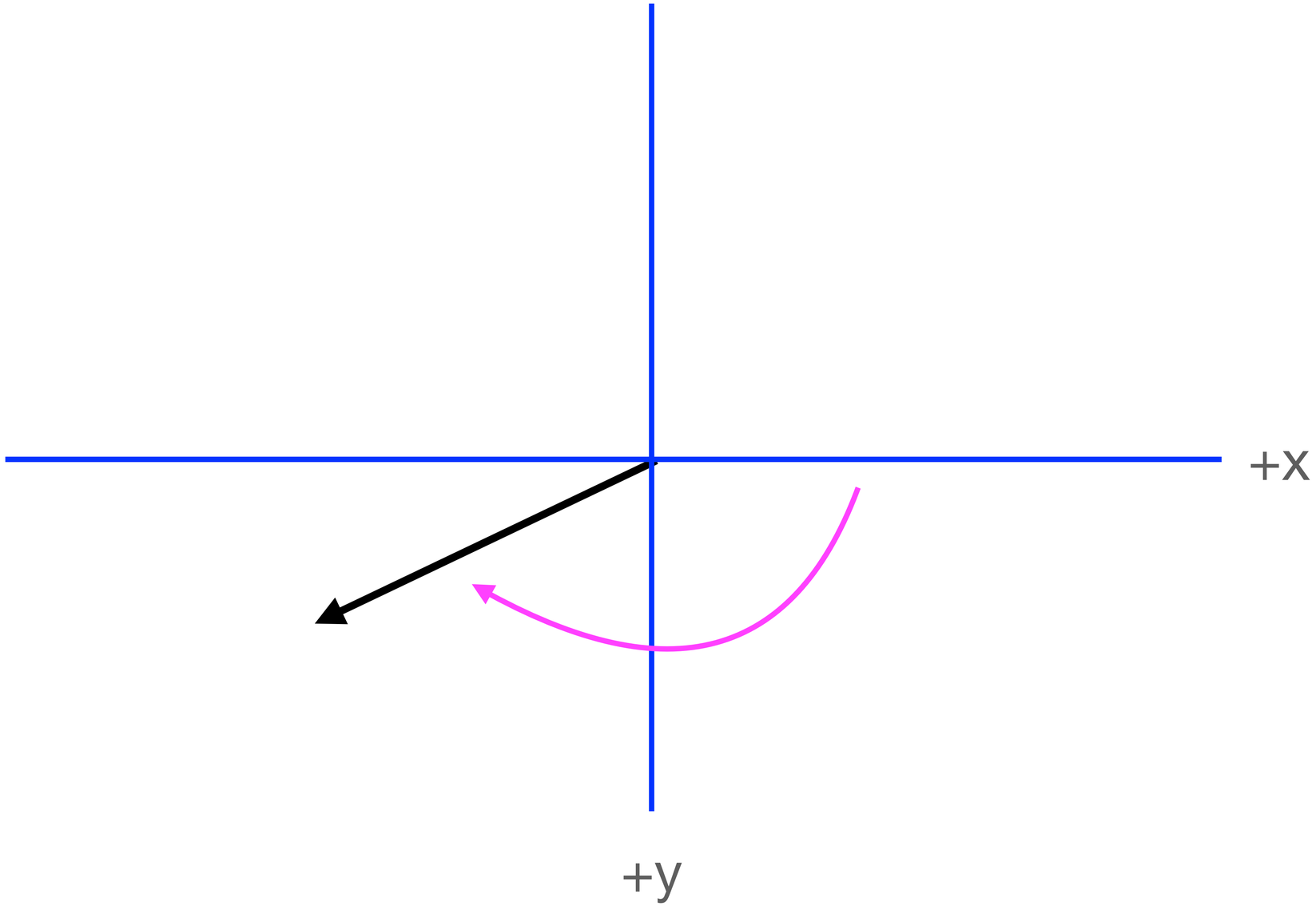


The direction of a vector is the angle as measured from the positive X axis.

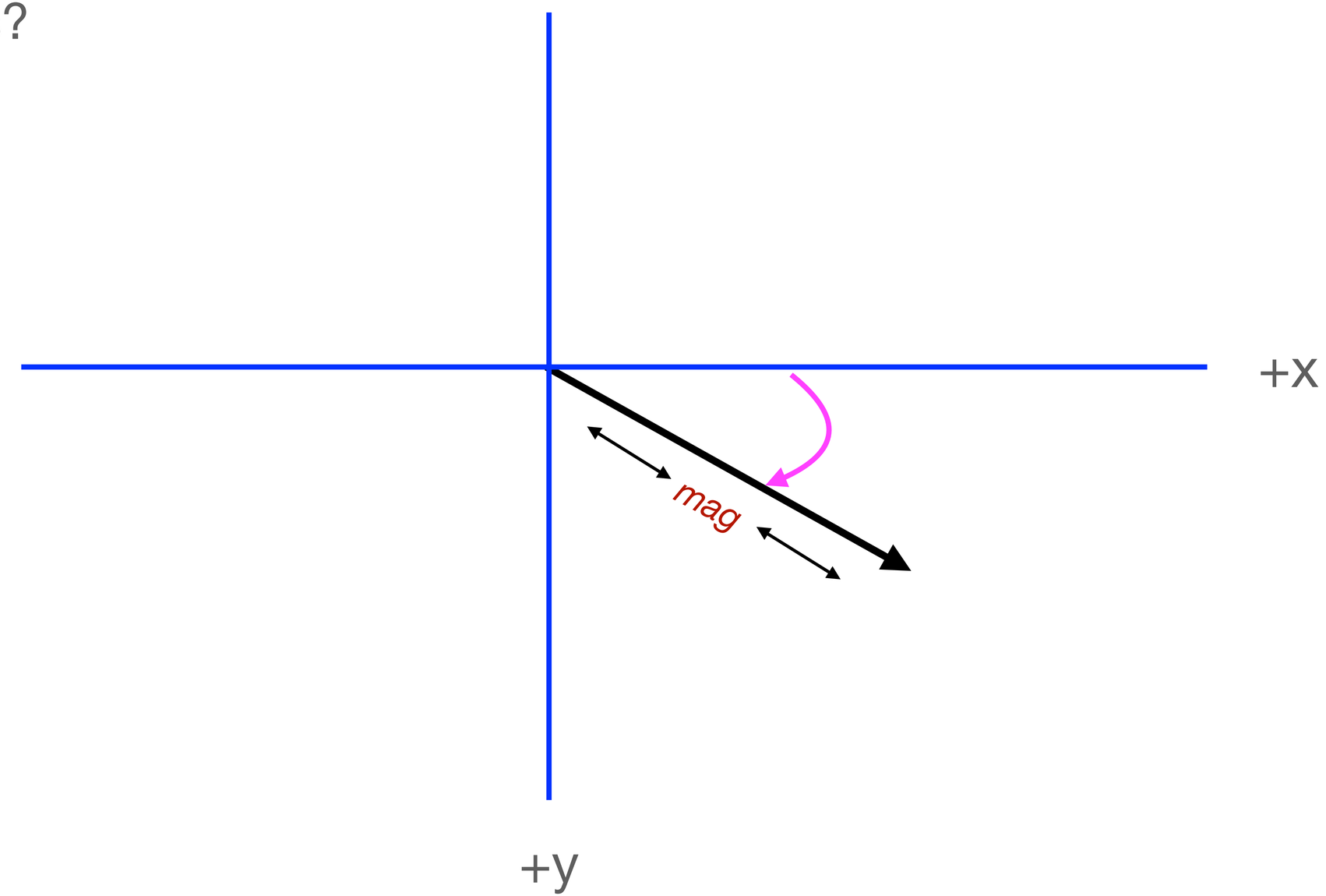
An angle measured counter-clockwise is negative.



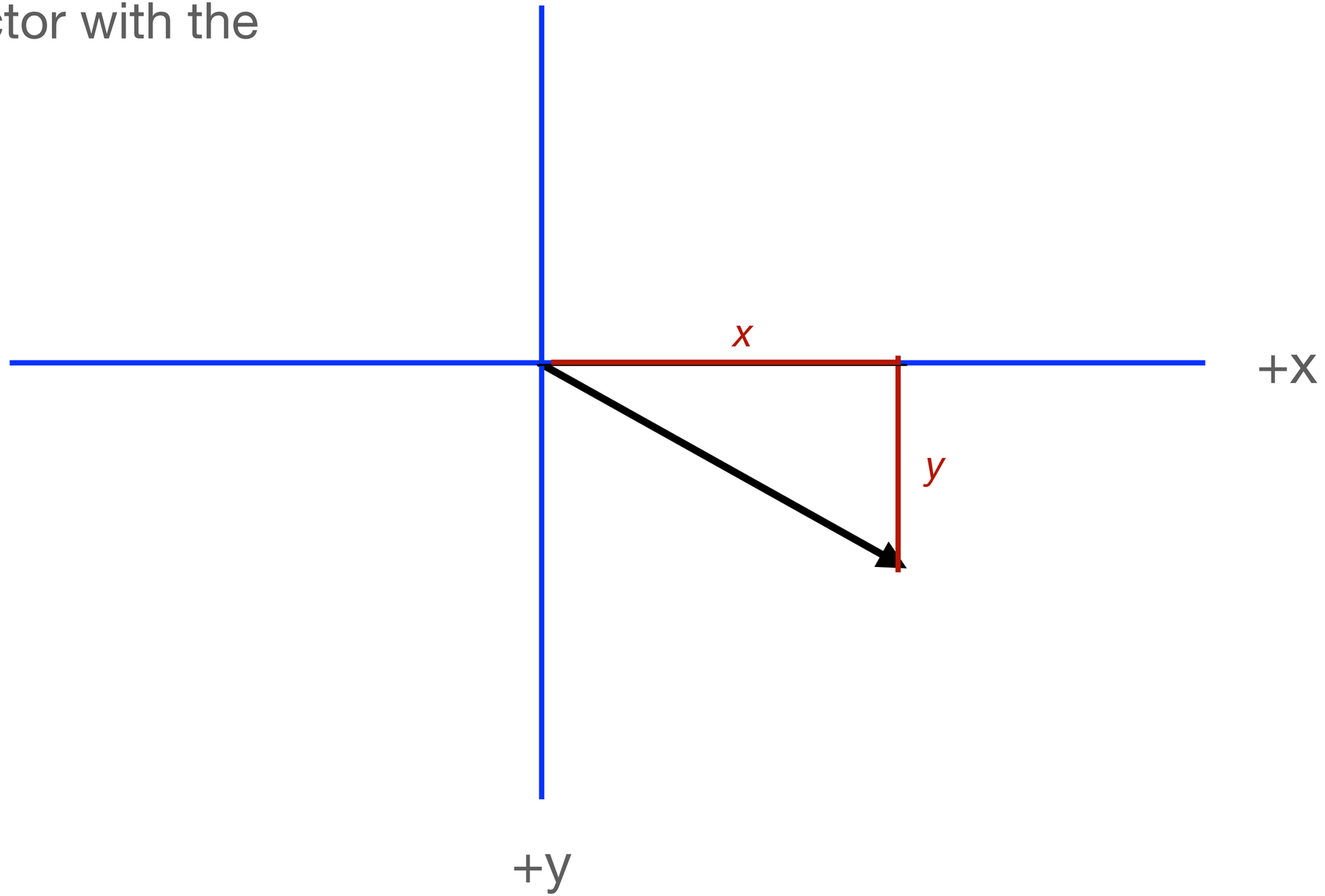
An angle measured clockwise is positive.



How do we calculate the magnitudes and directions of vectors?



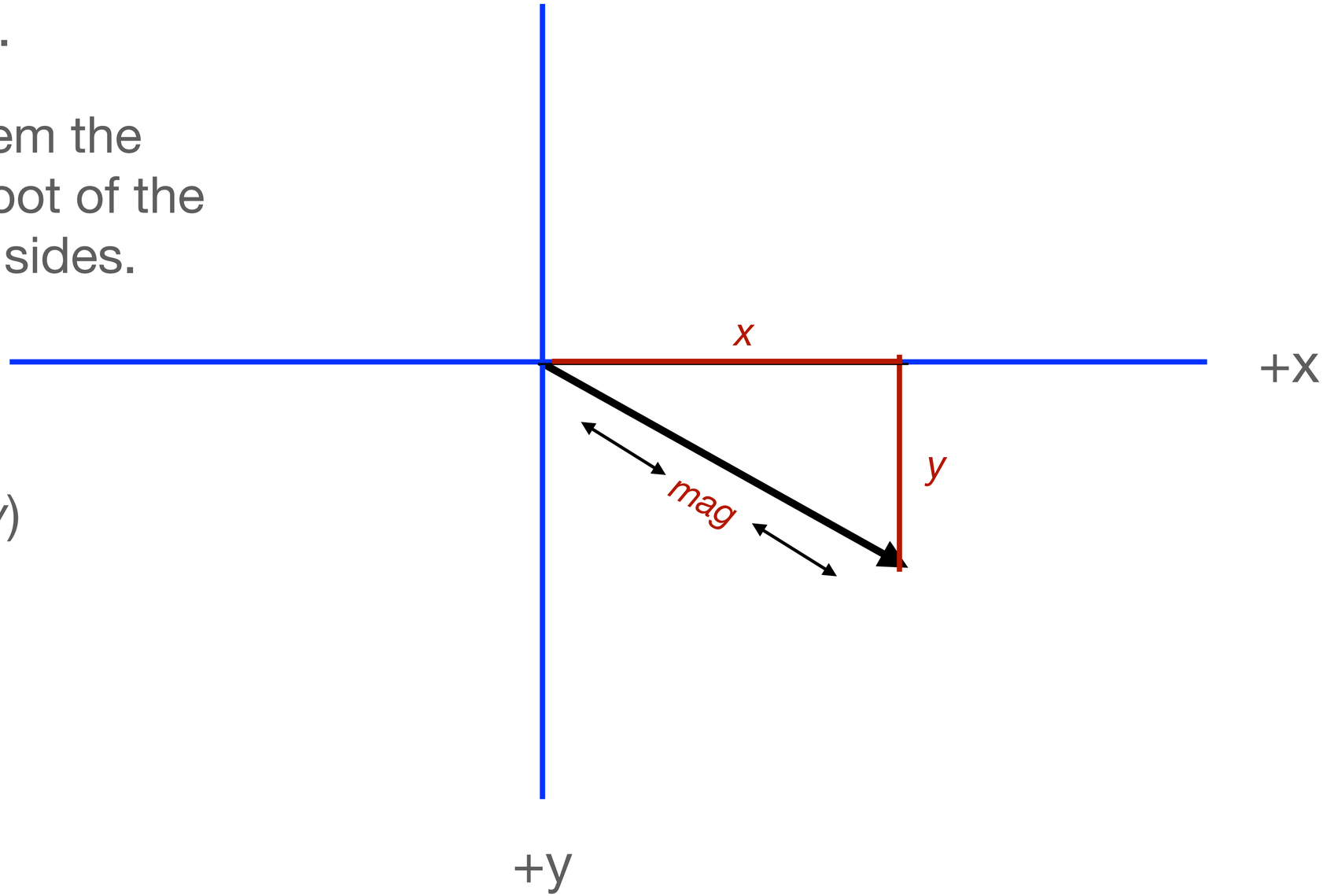
We must already know the dimensions of the sides of the right triangle formed by the vector with the X axis.



The magnitude of the vector is the length of the hypotenuse of the triangle with sides *x* and *y*.

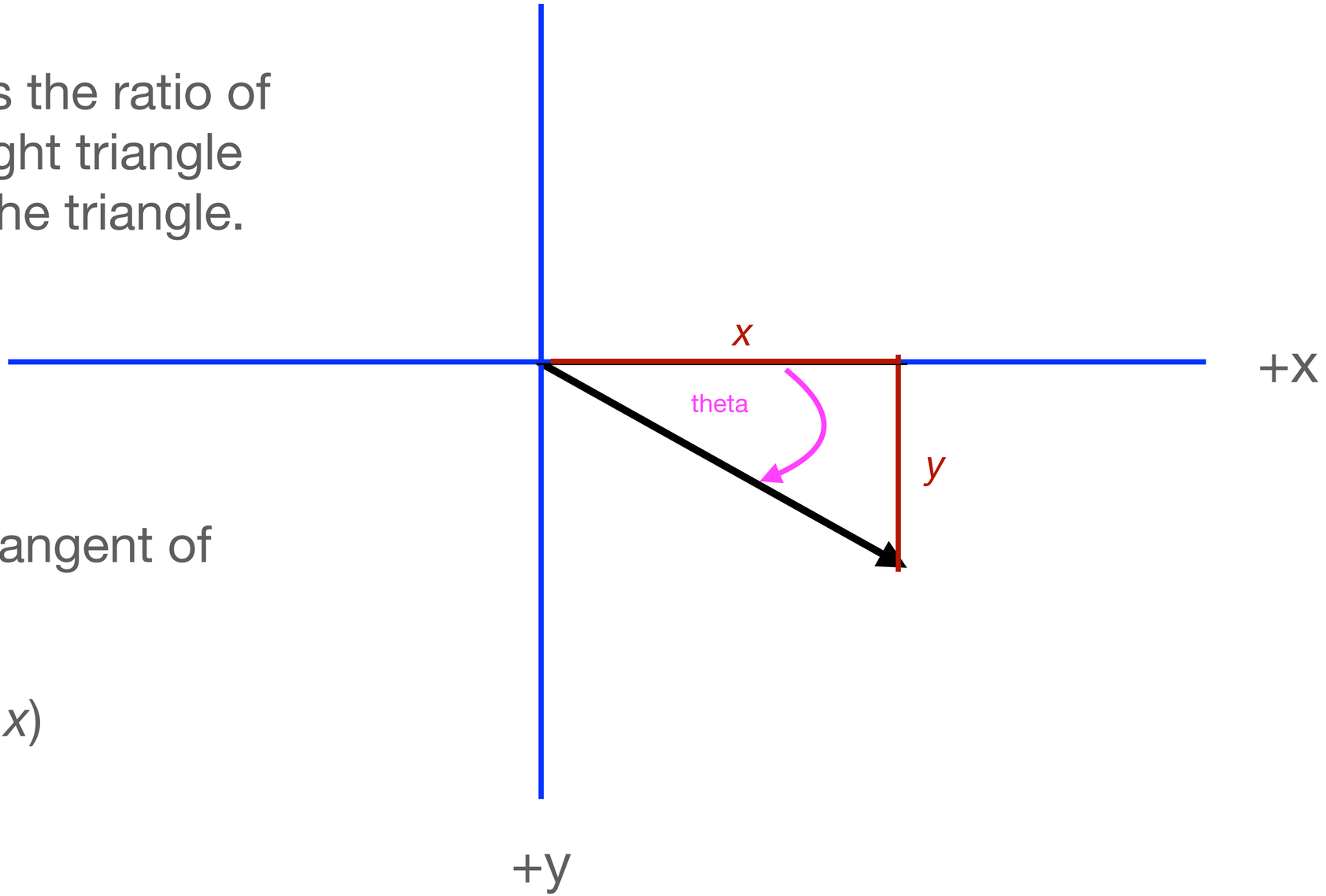
By the pythagorean theorem the magnitude is the square root of the sum of the squares of the sides.

mag = Math.sqrt(*x***x* + *y***y*)



The direction of the vector is the angle formed by the vector with the positive X axis.

The tangent of the angle is the ratio of the opposite side of the right triangle with the adjacent side of the triangle.

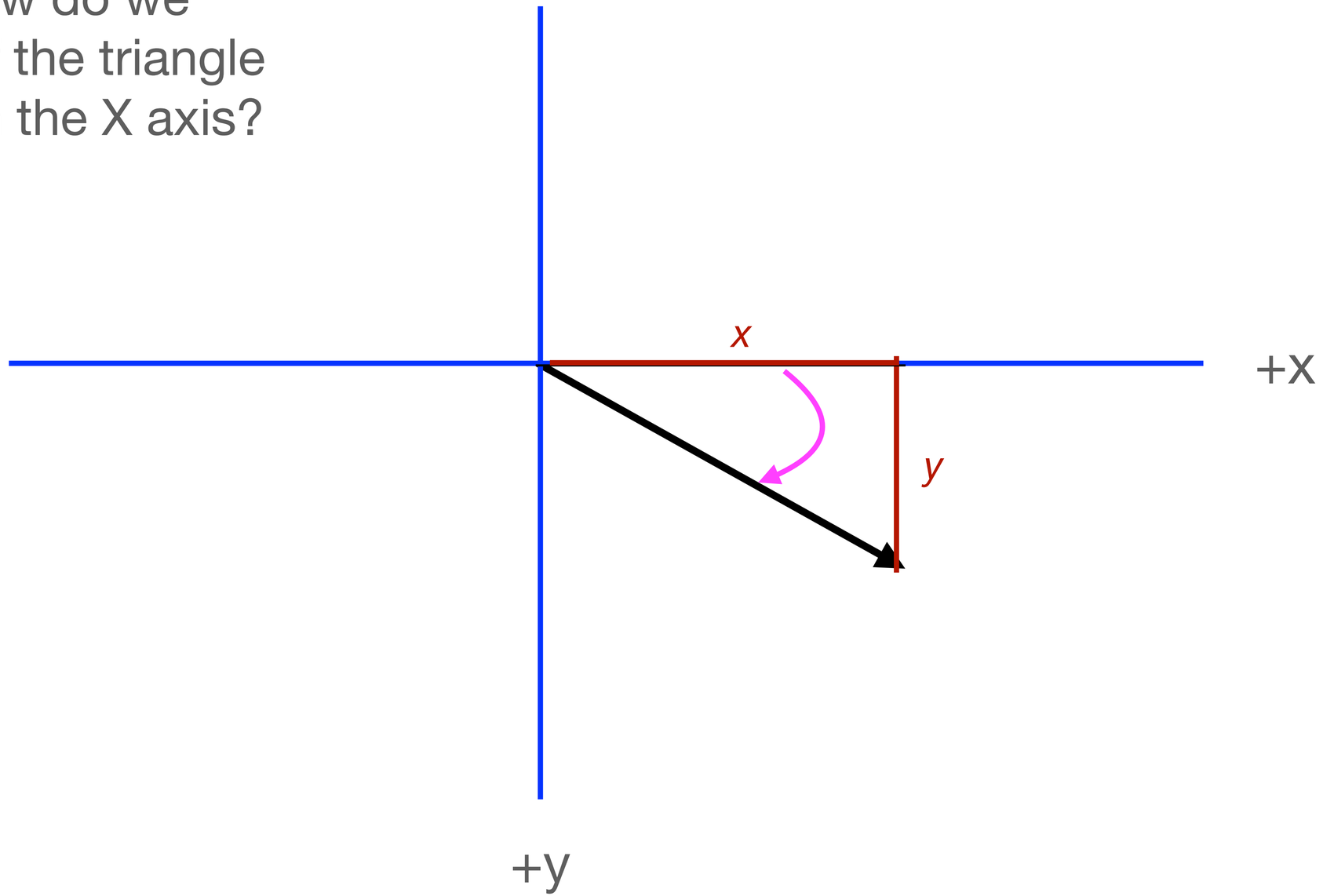


tan(theta) = opp/adj = *y*/*x*

The angle theta is the arctangent of the ratio.

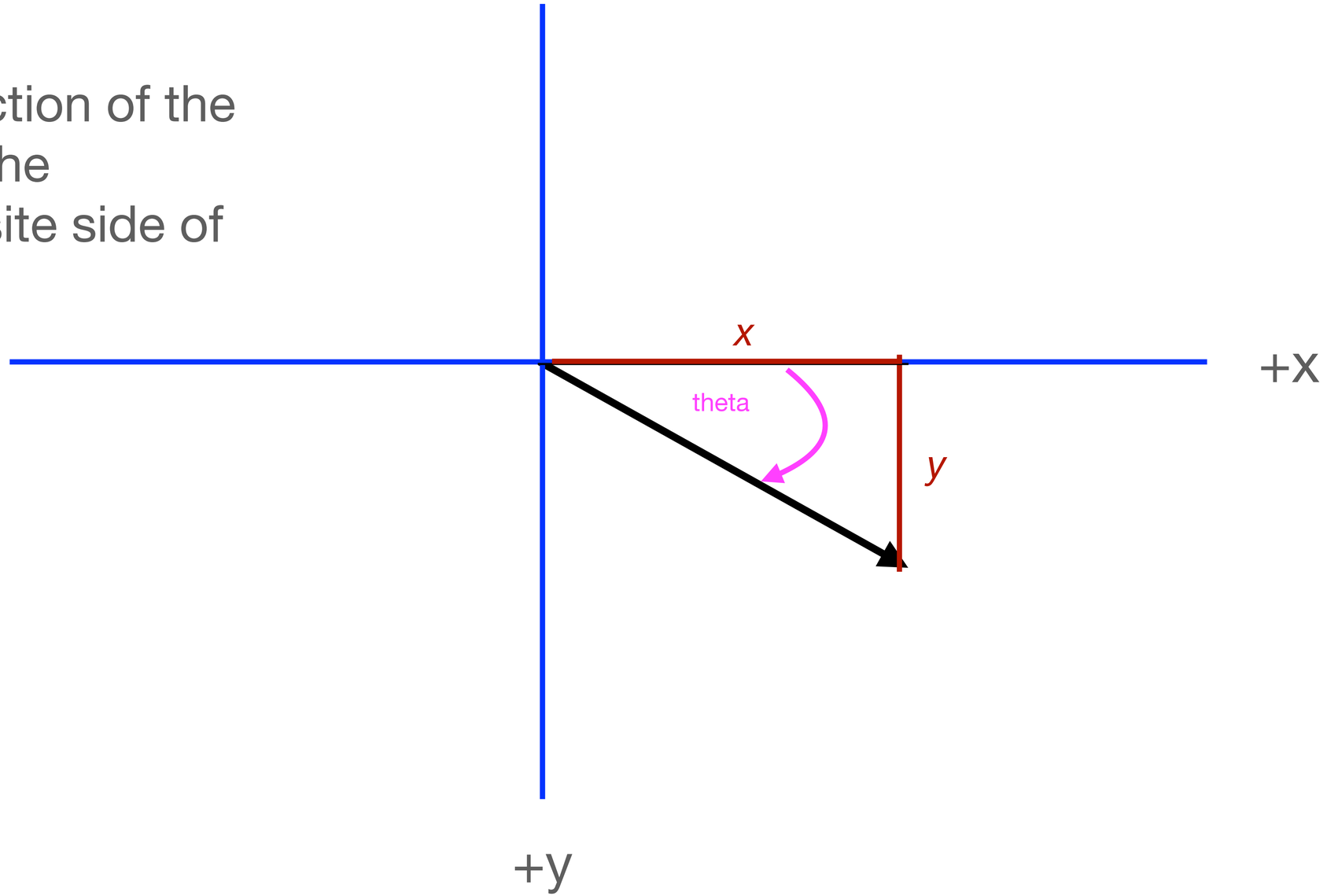
dir = theta = Math.atan2(*y*,*x*)

If we know the direction and the magnitude of a vector, how do we calculate the two sides of the triangle formed by the vector with the X axis?



The sine of an angle of a right triangle is the ratio of the opposite side of the triangle to the hypotenuse.

The angle theta is the direction of the vector. The magnitude is the hypotenuse and the opposite side of the angle is *y*.



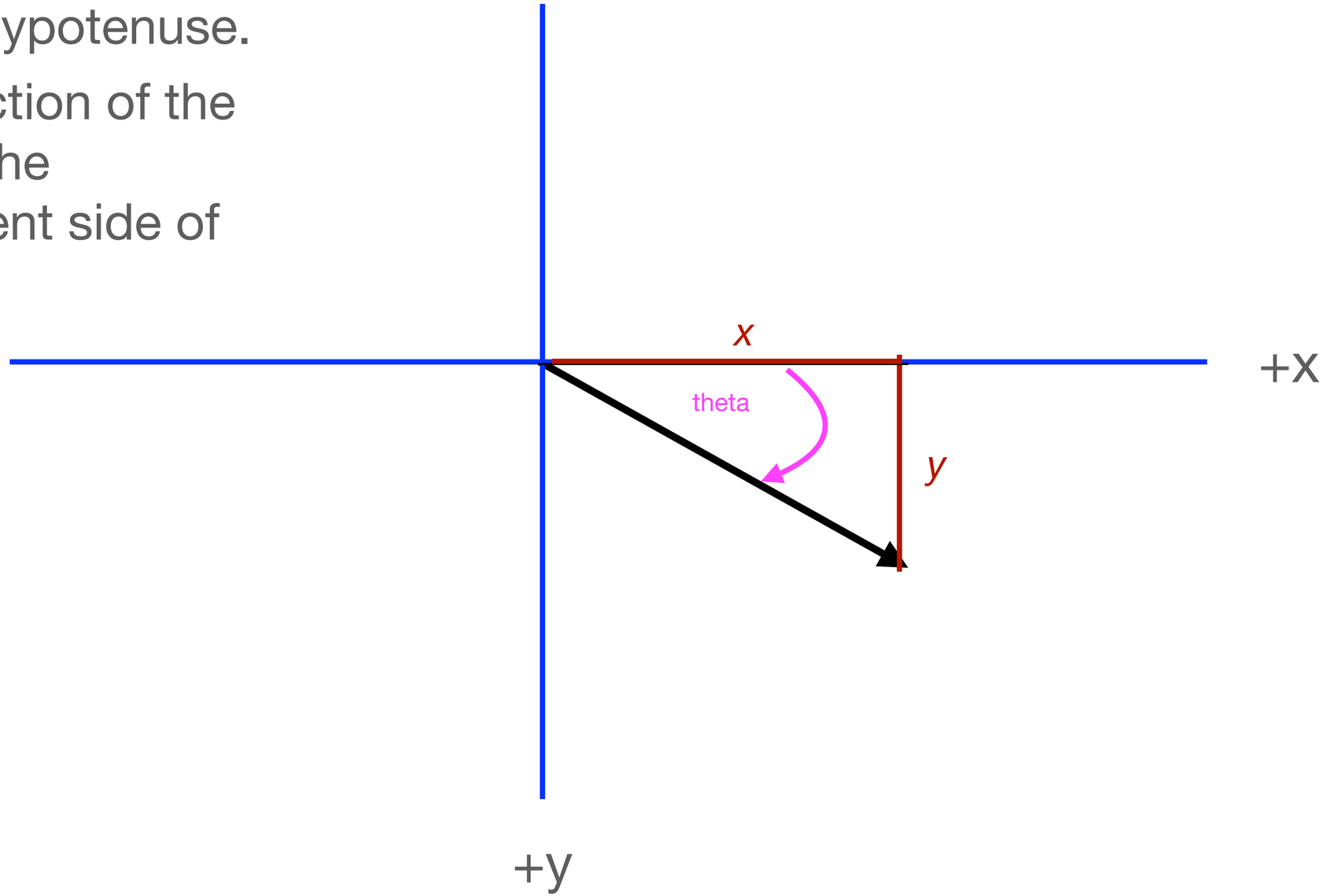
sin(theta) = opp/hyp

sin(dir) = *y*/mag

y = mag * Math.sin(dir)

The cosine of an angle of a right triangle is the ratio of the adjacent side of the triangle to the hypotenuse.

The angle theta is the direction of the vector. The magnitude is the hypotenuse and the adjacent side of the angle is *x*.



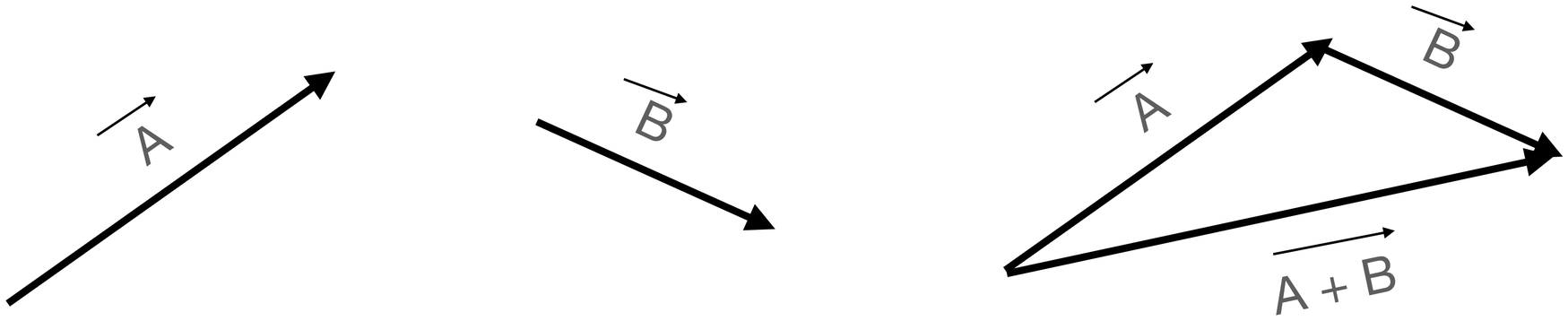
cos(theta) = adj/hyp

cos(dir) = *x*/mag

x = mag * Math.cos(dir)

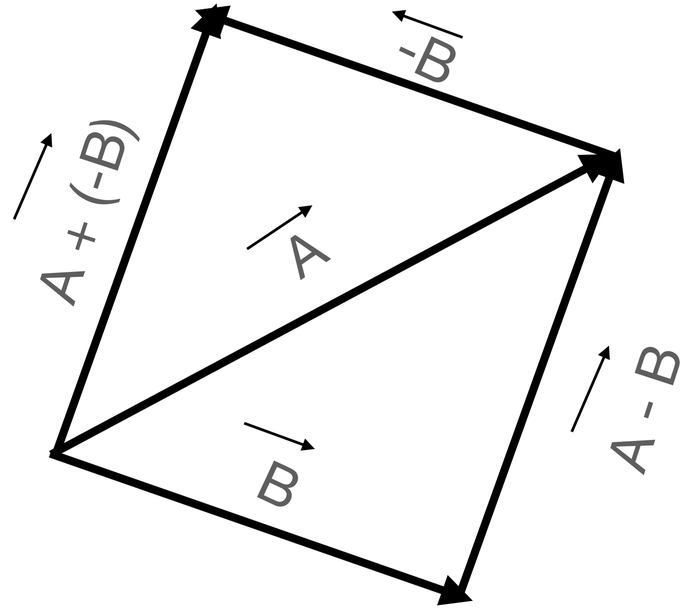
Vector Addition

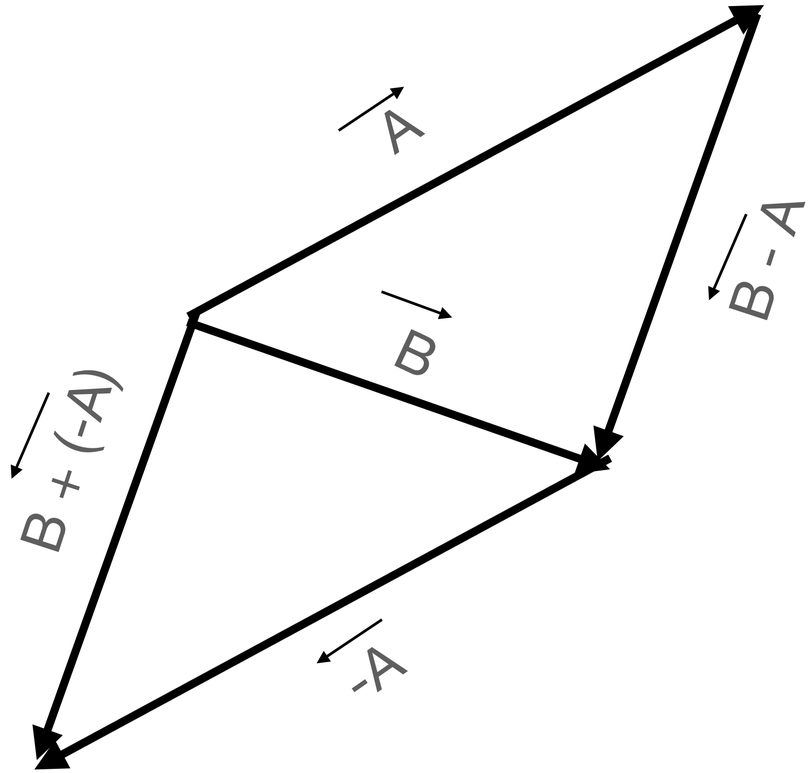
Adding two or more vectors together is equivalent to placing the tail of one vector at the head of another vector and the sum of the two is a vector from the tail of the first to the head of the second.



$$(A + B).x = A.x + B.x$$
$$(A + B).y = A.y + B.y$$

Vector Subtraction Finding the difference between two vectors is equivalent to adding the negative of one vector to the other. The negative of a vector is a vector pointing in the opposite direction with the same magnitude.





(A - B).x = A.x - B.x
(A - B).y = A.y - B.y

(B - A).x = B.x - A.x
(B - A).y = B.y - A.y

Vector Subtraction

When taking the difference between two vectors, what direction do you want the resulting vector to have?

The vector A - B is a vector which when added to B will give you A so “towards” A. The vector B - A is a vector which when added to A will give you B so “towards” B.