Set Theory

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Definitions

- It's a **well-defined** collection of **objects**.
- The **elements** of a set are the **objects** in a set.
- Usually sets are denoted with uppercase letters while elements are denoted with lowercase letters.

Notations

a. Is a member of
$x \in A$ → x is a member of set A.

b. Is not a member of
$y \notin A$ → y is not a member of set A.

c. Is a subset of
$A \subseteq B$ → A is a subset of set B.

d. Is a proper subset of
$A \subset B$ → A is a proper subset of B.

e. Null Set
$A = \emptyset$ → Set A is null (empty).

f. Intersection
$A \cap B$ → All elements that are in **both** A and B.

g. Union
$A \cup B$ → All elements in A or B or both.

h. Universal Set
$\mathbb{U}$ or U or Ω → The set of all elements currently under consideration

i. Complement
A^c → Compliment of A (Elements that are NOT inside set A)

j. Cartesian Product of sets.
$\{(a, b) | a \in A \; and \; b \in B\}$ → Set of all ordered pairs formed by taking one element from each set.

k. Power set
   $P(A)$ → All the possible subsets of set A.

Describing Sets

1. List elements
   $A = \{2,4,6,8\}$

2. Verbal Description
   "Even integers from 1 to 10. Exclusive."

3. Give a mathematical inclusion rule
   $A = \{Integer\ even\ x | \ 1 < x < 10\}$

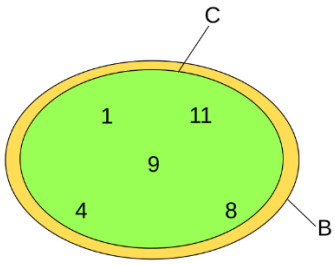
Special Sets

1. Null set (**∅**)
   A set with no element (An empty set).
   $A = \emptyset$ → A = {}

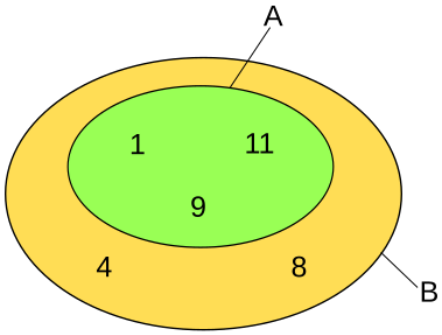
2. Universal Set (**U**, **𝕌**, **Ω**, **∑**)
   A set of all the elements under consideration.
   $A = \{2,4,6,8,10\}$
   $B = \{1,3,5,7\}$
   $\mathbb{U} = \{1,2,3,4,5,6,7,8,9,10\}$

Membership Relationships

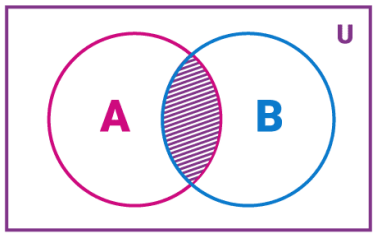
1. Subset of
   >> If all the members of a set are also members in another set.
   >> C is a subset of B
   >>$C \subseteq B$



2. Proper subset of
   >> If all the members of a set are also members of another set, but that another group has more elements.
   >> A is a proper subset of B
   >> $A \subseteq B$



3. Intersection
   >> The set of all elements that belong to both selected sets.
   >> $A \cap B$ means all the elements that belong to both A and B sets.



4. Union
   >> Set of all elements in selected sets.
   >> $A \cup B$ means all elements in A, or B or both.

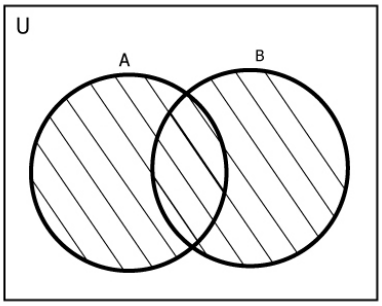
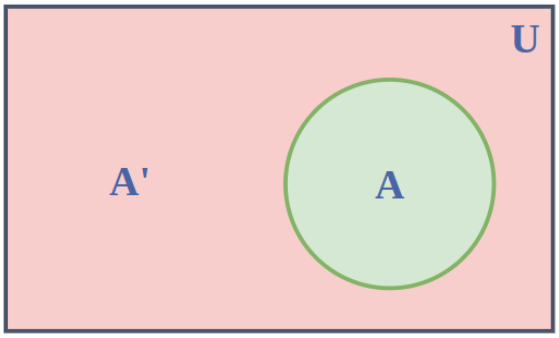
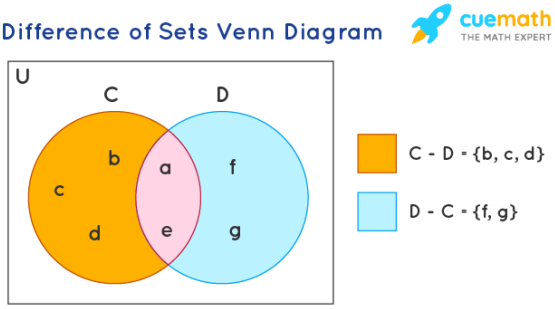


Figure: Union of Two Sets

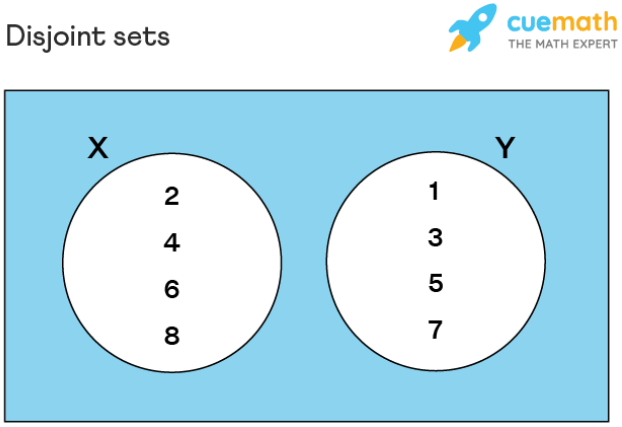
5. Complement
   >> The art of the universal set except the selected set.
   >> A^c means all the other elements under consideration except the elements in A.



6. Difference
   >> The set of elements that are in one set but not in the other.
   >> if
   >>> $A = \{1,2,3,4,5,6,7,8,9,10\}$
   >>> $B = \{2,4,6,8,10\}$
   >> $A - B = \{1,3,5,7,9\}$
   >> Also equals to $A \cap B'$



7. Disjoint sets
   >> Two sets that share **no** common elements.
   >> The intersection of disjoint sets is null.



   >> $X \cap Y = \emptyset$

8. Cartesian products of sets
   >> Set of all ordered pairs formed by taking one element from each set.
   >>> $A = \{a, b, c\}$
   >>>$B = \{2,4,6\}$

   >>$A \times B = \{(a,2), (a,4), (a,6), (b,2), (b,4), (b,6), (c,2), (c,4), (c,6)\}$

9. Power set
   >>All the possible subsets of given set.
   >> $A = \{3,4,5\}$
   >> $P(A) = \{\emptyset,\ \ \{3\},\ \{4\},\ \{5\},\ \{3,4\},\ \ \{4,5\},\ \{3,4,5\}\}$

Set Identities

1. Commutative Laws
   $A \cap B = B \cap A$
   $A \cup B = B \cup A$

2. Associative Laws
   $(A \cap B) \cap C = A \cap (B \cap C)$
   $(A \cup \mathrm{B}) \cup \mathrm{C} = \mathrm{A} \cup (\mathrm{B} \cup \mathrm{C})$

3. Distributive Laws
   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. Intersection with U
   $A \cap \mathbb{U} = A$

5. Double Complement Laws
   $(A')' = A$

6. Idempotent Laws
   $A \cap A = A$
   $A \cup A = A$

7. De Morgan's Laws
   $(A \cup B)' = A' \cap B'$
   $(A \cap B)' = A' \cup B'$

8. Union with U
   $A \cup \mathbb{U} = \mathbb{U}$

9. Absorption Laws
   $A \cup (A \cap B) = A$
   $A \cap (A \cup B) = A$

10. Alternate Representation for Set Difference
    $A - B = A \cap B'$