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P186 4.2 $(0,0),(\frac{1}{6},2),(\frac{1}{3},0),(\frac{1}{2},-1),(\frac{2}{3},1),(\frac{5}{6},1)$

2(a) 选择模型 $F_3(t)=c_1+c_2\cos 2\pi t+c_3\sin 2\pi t$

$$Ax=b \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -1 & 0 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^TA=\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad A^Tb=\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

线性方程

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}=\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}=\begin{bmatrix} \frac{1}{2} \\ \frac{2}{3} \\ 0 \end{bmatrix}$$

$\therefore F_3(t)=\frac{1}{2}+\frac{2}{3}\cos 2\pi t$

$$\cancel{r=Ax-b=} \; b-Ax = -\begin{bmatrix} 1 & 1 & 0 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -1 & 0 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{1}{2} \\ \frac{2}{3} \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} -7/6 \\ \frac{7}{6} \\ -1/6 \\ -5/6 \\ 5/6 \\ 1/6 \end{bmatrix}$$

$$\|e\|_2 \cancel{RMSE=} = \sqrt{\frac{(7/6)^2+(7/6)^2+(1/6)^2+(5/6)^2+(5/6)^2+(1/6)^2}{1}}=\frac{5}{\sqrt{6}}$$

选择模型 $F_4(t) = c_1 + c_2\cos 2\pi t + c_3\sin 2\pi t + c_4\cos 4\pi t$

$$Ax=b \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & -1 & 0 & 1 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 6 & & & \\ & 3 & & \\ & & 3 & \\ & & & 3 \end{bmatrix} \qquad A^Tb = \begin{bmatrix} 3 \\ 2 \\ 0 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & & & \\ & 3 & & \\ & & 3 & \\ & & & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{2}{3} \\ 0 \\ -1 \end{bmatrix}$$

$\therefore F_4(t) = \frac{1}{2} + \frac{2}{3}\cos 2\pi t - \cos 4\pi t$

$$r = b - Ax = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & -1 & 0 & 1 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{2}{3} \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{6} \\ \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{6} \\ \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$\|e\|_2 \cancel{RMSE} = \sqrt{\frac{(1/6)^2+(2/3)^2+(2/3)^2+(1/6)^2+(1/3)^2+(1/3)^2}{1}} = \frac{\sqrt{7}}{\sqrt{6}} < \frac{5}{\sqrt{6}}$$

4(a) $(-2,4),\ (-1,2),\ (1,1),\ (2,\frac{1}{2})$

选择模型 $y=c_1 e^{c_2 t} \Rightarrow \ln y = \ln c_1 + c_2 t = k + c_2 t$

$$Ax=b \Rightarrow \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} k \\ c_2 \end{bmatrix} = \begin{bmatrix} 2\ln 2 \\ \ln 2 \\ 0 \\ -\ln 2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 10 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2\ln 2 \\ \ln 2 \\ 0 \\ -\ln 2 \end{bmatrix} = \begin{bmatrix} 2\ln 2 \\ -7\ln 2 \end{bmatrix}$$

线性方程:

$$\begin{bmatrix} 4 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} k \\ c_2 \end{bmatrix} = \begin{bmatrix} 2\ln 2 \\ -7\ln 2 \end{bmatrix} \Rightarrow \begin{bmatrix} k \\ c_2 \end{bmatrix} = \begin{bmatrix} \frac{\ln 2}{2} \\ -\frac{7}{10}\ln 2 \end{bmatrix}$$

$\therefore \ln y = \frac{\ln 2}{2} - \frac{7}{10}\ln 2\, t \Rightarrow y = 2^{\frac{1}{2}} \cdot 2^{-\frac{7}{10}t}$

$\therefore y(-2) = 2^{\frac{19}{10}} \quad y(-1) = 2^{\frac{6}{5}} \quad y(1) = 2^{-\frac{1}{5}} \quad y(2) = 2^{-\frac{9}{10}}$

$\therefore \|e\|_2 = \sqrt{(4-2^{\frac{19}{10}})^2 + (2-2^{\frac{6}{5}})^2 + (2^{-\frac{1}{5}}-1)^2 + (2^{-\frac{9}{10}}-\frac{1}{2})^2} = 3.21636$

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2(a) $y_1 = A_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$, $r_1 = \|y_1\| = \sqrt{1+2^2+2^2} = 3$, $q_1 = \frac{y_1}{\|y_1\|} = \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$

$$y_2 = A_2 - q_1 q_1^T A_2 = \begin{bmatrix} 3 \\ -6 \\ -0 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}(2+4) = \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}$$

$r_{22} = \|y_2\| = \sqrt{1^2+2^2+2^2} = 3$

$$q_2 = \frac{y_2}{\|y_2\|} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}$$

$r_{12} = q_1^T A_2 = 2+4 = 6$

$$y_3 = A_3 - (q_1 q_1^T A_3 + q_2 q_2^T A_3) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}\frac{2}{3} - \begin{bmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}\left(-\frac{1}{3}\right) = \cancel{\begin{bmatrix} \frac{4}{9} \\ \frac{2}{3} \\ 0 \end{bmatrix}} \begin{bmatrix} \frac{4}{9} \\ \frac{2}{9} \\ -\frac{4}{9} \end{bmatrix}$$

$$q_3 = \frac{y_3}{\|y_3\|} = \begin{bmatrix} \frac{4}{9} \\ \frac{2}{9} \\ -\frac{4}{9} \end{bmatrix} \Big/ \frac{2}{3} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix} = QR$$

6(a) 令 $x_1 = [2,-2,1]^T$, $\|x_1\| = 3$, $w_1 = [\|x_1\|, 0, 0]^T = [3,0,0]^T$

$v_1 = w_1 - x_1 = [3,0,0]^T - [2,-2,1]^T = [1,2,-1]^T$

$$H_1 = I - 2\frac{v_1 v_1^T}{v_1^T v_1} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} - \frac{2}{6}\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

$$H_1 A = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 0 & 0 \\ 0 & -3 \end{bmatrix}$$

$x_2=[0,-3]^T$, $\|x_2\|=3$, $w_2=[3,0]^T$

$v_2=w_2-x_2=[3,0]^T-[0,-3]^T=[3,3]^T$

$$H_2=I-2\frac{v_2v_2^T}{v_2^Tv_2}=\begin{bmatrix}1&\\&1\end{bmatrix}-\frac{2}{18}\begin{bmatrix}9&9\\9&9\end{bmatrix}=\begin{bmatrix}0&-1\\-1&0\end{bmatrix}$$

$$H_2H_1A=\begin{bmatrix}1&0&0\\0&0&-1\\0&-1&0\end{bmatrix}\begin{bmatrix}3&6\\0&0\\0&-3\end{bmatrix}=\begin{bmatrix}3&6\\0&3\\0&0\end{bmatrix}=R$$

$$Q=H_1^{-1}H_2^{-1}=H_1H_2=\begin{bmatrix}\frac{2}{3}&-\frac{2}{3}&\frac{1}{3}\\-\frac{2}{3}&-\frac{1}{3}&\frac{2}{3}\\\frac{1}{3}&\frac{2}{3}&\frac{2}{3}\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&-1\\0&-1&0\end{bmatrix}=\begin{bmatrix}\frac{2}{3}&-\frac{1}{3}&\frac{2}{3}\\-\frac{2}{3}&-\frac{2}{3}&\frac{1}{3}\\\frac{1}{3}&-\frac{2}{3}&-\frac{2}{3}\end{bmatrix}$$

7(a) $\begin{bmatrix}2&3\\-2&-6\\1&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}3\\-3\\6\end{bmatrix}\Rightarrow$ $Rx=Q^Tb$ $\begin{bmatrix}3&6\\0&3\\0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}\frac{2}{3}&-\frac{1}{3}&\frac{2}{3}\\-\frac{2}{3}&-\frac{2}{3}&\frac{1}{3}\\\frac{1}{3}&-\frac{2}{3}&-\frac{2}{3}\end{bmatrix}\begin{bmatrix}3\\-3\\6\end{bmatrix}=\begin{bmatrix}7\\2\\-1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}3&6\\0&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}7\\2\end{bmatrix}\Rightarrow\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\\frac{2}{3}\end{bmatrix}$