P158 3.4

3(a) 检查纽结

$$S_1(1) = 4 - \frac{11}{4} + \frac{3}{4} = 2 \quad, \quad S_2(1) = 2 - 0 + 0 - 0 = 0 \quad, \quad S_1(1) = S_2(1)$$

$$S_1'(1) = -\frac{11}{4} + \frac{9}{4} = -\frac{1}{2} \quad, \quad S_2'(1) = -\frac{1}{2} + 0 - 0 = 0 \quad, \quad S_1'(1) = S_2'(1)$$

$$S_1''(1) = \frac{3}{4} \times 3 \times 2 = \frac{9}{2} \quad, \quad S_2''(1) = 2c - 0 = 2c \quad, \quad S_1''(1) = S_2''(1)$$

$$\therefore c = \frac{9}{4}$$

考虑自然样条

$$S_1''(0) = \frac{3}{4} \times 3 \times 2 \times 0 = 0 \quad, \quad S_2''(2) = \frac{9}{2} - \frac{9}{2} = 0 \quad, \quad S_1''(0) = S_2''(0) = 0$$

$$\therefore 满足自然样条$$

考虑抛物线端点

$$\therefore c_1 \neq c_2, \; d_1 \neq d_2 \neq 0$$

∴不满足抛物线端点

考虑 非纽结三次样条

$$S_1'''(2) = \frac{3}{4} \times 3 \times 2 \times 1 = \frac{9}{2} \qquad S_2'''(2) = -\frac{3}{4} \times 3 \times 2 \times 1 = -\frac{9}{2} \qquad S_1'''(2) \neq S_2'''(2)$$

$\therefore$ 不满足非纽结三次样条

8(a) $(0,1)$, $(2,3)$, $(3,2)$

$\therefore \delta_1 = 2$, $\delta_2 = 1$, $\Delta_1 = 2$, $\Delta_2 = -1$, $a_1 = 1$, $a_2 = 3$

$\therefore 2\delta_1 + 2\delta_2 = 6$, $3(\Delta_2/\delta_2 - \Delta_1/\delta_1) = -6$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 6 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -6 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$d_1 = \frac{C_2 - C_1}{3\delta_1} = -\frac{1}{6} \qquad d_2 = \frac{C_3 - C_2}{3\delta_2} = \frac{1}{3}$$

$$b_1 = \frac{\Delta_1}{\delta_1} - \frac{\delta_1}{3}(2C_1 + C_2) = 1 + \frac{2}{3} \cdot 1 = \frac{5}{3}$$

$$b_2 = \frac{\Delta_2}{\delta_2} - \frac{\delta_2}{3}(2C_2 + C_3) = -1 + \frac{1}{3} \cdot 2 = -\frac{1}{3}$$

$$\therefore S_1(x) = 1 + \frac{5}{3}x + 0 - \frac{1}{6}x^3 \qquad\qquad [0, 2]$$

$$S_2(x) = 3 - \frac{1}{3}(x-2) - (x-2)^2 + \frac{1}{3}(x-2)^3 \qquad [2, 3]$$

14. (a) $S_1''(2) = \frac{1}{2} \times 3 \times 2 \times 2 = 6$ $S_2''(2) = 2c$ $S_1''(2) = S_2''(2) \Rightarrow c = 3$

(b) $S_1''(0) = \frac{1}{2} \times 3 \times 2 \times 0 = 0$ $S_2''(3) = 3 \times 2 d(3-1) = 0 \Rightarrow d = 0$

$S_2''(3) = 2c + 6d(3-2) = 0 \Rightarrow d = -1$.

P163 3.5

8. $(x_1, y_1) = (0, 1)$, $(x_2, y_2) = (x_2, 1)$

$(x_3, y_3) = (1, y_3)$, $(x_4, y_4) = (1, 0)$

$b_x = 3(x_2 - x_1) = 3x_2$

$c_x = 3(x_3 - x_2) - b_x = 3 - 6x_2$

$d_x = x_4 - x_1 - b_x - c_x = 3x_2 - 2$

$b_y = 3(y_2 - y_1) = 0$

$c_y = 3(y_3 - y_2) - b_y = 3y_3 - 3$

$d_y = y_4 - y_1 - b_y - c_y = 2 - 3y_3$

$\therefore \begin{cases} x(t) = x_1 + b_x t + c_x t^2 + d_x t^3 = 3x_2 t + (3 - 6x_2) t^2 + (3x_2 - 2) t^3 \\ y(t) = y_1 + b_y t + c_y t^2 + d_y t^3 = 1 + (3y_3 - 3) t^2 + (2 - 3y_3) t^3 \end{cases}$

$\therefore \begin{cases} x(\frac{1}{3}) = \frac{1}{3} \Rightarrow x_2 + (3 - 6x_2)\frac{1}{9} + (3x_2 - 2)\frac{1}{27} = \frac{1}{3} \Rightarrow x_2 = \frac{1}{6} \\ y(\frac{1}{3}) = \frac{2}{3} \Rightarrow 1 + (3y_3 - 3)\frac{1}{9} + (2 - 3y_3)\frac{1}{27} = \frac{2}{3} \Rightarrow y_3 = -\frac{1}{3} \end{cases}$

$\therefore \begin{cases} x(t) = \frac{1}{2} t + 2t^2 - \frac{3}{2} t^3 \\ y(t) = 1 - 4t^2 + 3t^3 \end{cases}$