P223 6.1 1. $f(x)=\ln x$ $f'(x)=\frac{1}{x}$ $f'(1)=1$

(a) $f'(1)=\frac{f(1.1)-f(1)}{0.1}=\frac{\ln 1.1}{0.1}=0.953101$ $e=|1-0.953101|=0.046898$

(b) $f'(1)=\frac{f(1.01)-f(1)}{0.01}=\frac{\ln 1.01}{0.01}=0.995033$ $e=|1-0.995033|=0.004967$

(c) $f'(1)=\frac{f(1.001)-f(1)}{0.001}=\frac{\ln 1.001}{0.001}=0.999500$ $e=|1-0.999500|=0.000500$

13. $f(x+3h)=f(x)+3hf'(x)+\frac{9h^2}{2}f''(x)+O(h^3)$

$f(x-h)=f(x)-hf'(x)+\frac{h^2}{2}f''(x)+O(h^3)$

$\Rightarrow f(x+3h)-9f(x-h)=-8f(x)+12hf'(x)+O(h^3)$

13 $\Rightarrow f'(x)=\frac{f(x+3h)+8f(x)-9f(x-h)}{12h}+O(h^2)$

14. (a) $F(x)=\frac{4f(h/2)-f(h)}{4-1}=\left[4\frac{f(x+\frac{3}{2}h)+8f(x)-9f(x-\frac{h}{2})}{6h}\right.$

$\left.-\frac{f(x+3h)+8f(x)-9f(x-h)}{12h}\right]/3=\frac{-f(x+3h)+8f(x+\frac{3}{2}h)+56f(x)-72f(x-\frac{h}{2})+9f(x-h)}{36h}$

P232 5.2

1(a) $I=\int_0^1 x^2dx=\frac{1}{3}x^3\Big|_0^1=\frac{1}{3}$

当 $m=1$, $h=1$ 时

$I'=\int_0^1 x^2dx=\frac{1}{2}(0+1)=\frac{1}{2}$ $\quad e=I'-I=\frac{1}{6}$

当 $m=2$, $h=\frac{1}{2}$ 时

$I'=\int_0^1 x^2dx=\frac{1}{4}(0+1+2\cdot\frac{1}{4})=\frac{3}{8}$ $\quad e=I'-I=\frac{1}{24}$

当 $m=4$, $h=\frac{1}{4}$ 时

$I'=\int_0^1 x^2dx=\frac{1}{8}[0+1+2(\frac{1}{16}+\frac{1}{4}+\frac{9}{16})]=\frac{11}{32}$ $\quad e=I'-I=\frac{1}{96}$

~~4.~~

3(a) 当 $m=1$, $h=\frac{1}{2}$ 时

$I'=\int_0^1 x^2dx=\frac{1}{6}[0+1+4(\frac{1}{4})+2\cdot 0]=\frac{1}{3}$ $\quad e=I'-I=0$

当 $m=2$, $h=\frac{1}{4}$ 时

$I'=\int_0^1 x^2dx=\frac{1}{12}[0+1+4(\frac{1}{16}+\frac{9}{16})+2(\frac{1}{4})]=\frac{1}{3}$ $\quad e=I'-I=0$

当 $m=4$, $h=\frac{1}{8}$ 时

$I'=\int_0^1 x^2dx=\frac{1}{24}[0+1+4(\frac{1}{64}+\frac{9}{64}+\frac{25}{64}+\frac{49}{64})+2(\frac{1}{16}+\frac{1}{4}+\frac{9}{16})]=\frac{1}{3}$ $\quad e=I'-I=0$

12. $\int_0^1 1\,dx = c_1+c_2+c_3 = 1$

$\int_0^1 x\,dx = 0+0.5c_2+c_3 = \frac{1}{2}$ $\Rightarrow \begin{cases} c_1 = \frac{1}{6} \\ c_2 = \frac{2}{3} \\ c_3 = \frac{1}{6} \end{cases}$

$\int_0^1 x^2\,dx = 0+0.25c_2+c_3 = \frac{1}{3}$

P237 5.3

1(a) $\int_0^1 x^2\,dx$ $\quad R_{11} = \frac{h_1}{2}[f(a)+f(b)] = \frac{1}{2}\cdot(0+1) = \frac{1}{2}$

$R_{21} = \frac{h_2}{2}[f(a)+f(b)+2f(\frac{a+b}{2})] = \frac{1}{4}(0+1+2\cdot\frac{1}{4}) = \frac{3}{8}$

$R_{22} = \frac{2^2R_{21}-R_{11}}{3} = \frac{4\cdot\frac{3}{8}-\frac{1}{2}}{3} = \frac{1}{3}$

$R_{31} = \frac{1}{2}R_{21} + h_3[f(a+h_3)+f(a+3h_3)] = \frac{3}{16}+\frac{1}{4}(\frac{1}{16}+\frac{9}{16}) = \frac{11}{32}$

$R_{32} = \frac{2^2R_{31}-R_{21}}{3} = \frac{4\cdot\frac{11}{32}-\frac{3}{8}}{3} = \frac{1}{3}$

$R_{33} = \frac{4^2R_{32}-R_{22}}{4^2-1} = \frac{16\cdot\frac{1}{3}-\frac{1}{3}}{15} = \frac{1}{3}$

$\therefore I = R_{33} = \frac{1}{3}$

P240 5.4 (b) $\int_0^{\frac{\pi}{2}} \cos x\,dx = 1$

对 $[0, \frac{\pi}{2}]$

$$S_{[0,\frac{\pi}{2}]} = \frac{\pi}{2} \cdot \frac{\cos 0 + \cos\frac{\pi}{2}}{2} = \frac{\pi}{4}$$

$$S_{[0,\frac{\pi}{4}]} = \frac{\pi}{4} \frac{\cos 0 + \cos\frac{\pi}{4}}{2} = 0.6704$$

$$S_{[\frac{\pi}{4},\frac{\pi}{2}]} = \frac{\pi}{4} \cdot \frac{\cos\frac{\pi}{4} + \cos\frac{\pi}{2}}{2} = 0.2777$$

$\therefore |S_{[0,\frac{\pi}{2}]} - S_{[0,\frac{\pi}{4}]} - S_{[\frac{\pi}{4},\frac{\pi}{2}]}| \approx 0.163 > 3TOL = 0.15$

对 $[0, \frac{\pi}{4}]$

$$S_{[0,\frac{\pi}{8}]} = \frac{\pi}{8} \frac{\cos 0 + \cos\frac{\pi}{8}}{2} \approx 0.3776$$

$$S_{[\frac{\pi}{8},\frac{\pi}{4}]} = \frac{\pi}{8} \frac{\cos\frac{\pi}{8} + \cos\frac{\pi}{4}}{2} \approx 0.3202$$

$\therefore |S_{[0,\frac{\pi}{4}]} - S_{[0,\frac{\pi}{8}]} - S_{[\frac{\pi}{8},\frac{\pi}{4}]}| \approx 0.0276 < 3TOL - \frac{1}{2} = 0.075$

对 $[\frac{\pi}{4}, \frac{\pi}{2}]$

$$S_{[\frac{\pi}{4},\frac{3\pi}{8}]} = \frac{\pi}{8} \frac{\cos\frac{\pi}{4} + \cos\frac{3\pi}{8}}{2} \approx 0.2140$$

$$S_{[\frac{3\pi}{8},\frac{\pi}{2}]} = \frac{\pi}{8} \frac{\cos\frac{3\pi}{8} + \cos\frac{\pi}{2}}{2} \approx 0.0752$$

$\therefore |S_{[\frac{\pi}{4},\frac{\pi}{2}]} - S_{[\frac{\pi}{4},\frac{3\pi}{8}]} - S_{[\frac{3\pi}{8},\frac{\pi}{2}]}| = 0.0114 < 3TOL - \frac{1}{2} = 0.075$

$\therefore S = S_{[0,\frac{\pi}{8}]} + S_{[\frac{\pi}{8},\frac{\pi}{4}]} + S_{[\frac{\pi}{4},\frac{3\pi}{8}]} + S_{[\frac{3\pi}{8},\frac{\pi}{2}]} = 0.8871$ $\therefore e_4 - S = 0.0129$

P245 5.5

1(a) $\int_{-1}^{1}(x^3+2x)dx = -\frac{1}{3\sqrt{3}} - \frac{2}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \frac{2}{\sqrt{3}} = 0 \quad \therefore e = 0$

4(a) $\int_0^4 \frac{x\,dx}{\sqrt{x^2+9}} = \int_{-1}^{1} 2\cdot\frac{[(4-0)t+4+0]/2\cdot dt}{\sqrt{[(4-0)t+4+0]^2/2^2+9}} = \int_{-1}^{1}\frac{(4t+4)dt}{\sqrt{4t^2+8t+13}}$

$\approx \frac{-4/\sqrt{3}+4}{\sqrt{4/3-8/\sqrt{3}+13}} + \frac{4/\sqrt{3}+4}{\sqrt{4/3+8/\sqrt{3}+13}} \approx 1.99172$

$\therefore e = 2 - 1.99172 = 0.00828$

P257 6.1

3(b) $\frac{dy}{dt} = t^2y \Rightarrow \frac{dy}{y} = t^2dt \Rightarrow y = e^{\frac{1}{3}t^3} + C$

$\because y(0) = 1 \quad \therefore C = 0, \quad y = e^{\frac{1}{3}t^3}, \quad y(1) = e^{\frac{1}{3}} \approx 1.3956$

5(b) $w_1 = w_0 + ht_0^2w_0 = 1+0 = 1 \quad w_2 = w_1 + ht_1^2w_1 = \frac{65}{64}$

$w_3 = w_2 + ht_2^2w_2 = \frac{1105}{1024} \quad w_4 = w_3 + ht_3^2w_3 = \frac{80665}{65536} \approx 1.2308$

$e = |e^{\frac{1}{3}} - w_4| \approx 0.16476$

P268 6.2

1(b) $w_1 = w_0 + \frac{h}{2}[t_0^2 w_0 + (t_0+h)^2(w_0 + h[t_0^2 w_0])] = 1 + \frac{1}{8}[0 + \frac{1}{16}[1+0]] \approx 1.0078$

$w_2 = w_1 + \frac{h}{2}[t_1^2 w_1 + (t_1+h)^2(w_1 + h[t_1^2 w_1])] \approx 1.04767$

$w_3 = w_2 + \frac{h}{2}[t_2^2 w_2 + (t_2+h)^2(w_2 + h[t_2^2 w_2])] \approx 1.15868$

$w_4 = w_3 + \frac{h}{2}[t_3^2 w_3 + (t_3+h^2)(w_3 + h[t_3^2 w_3])] \approx 1.46646$

$e = |e^{\frac{1}{3}} - w_4| = 0.07084$

4(b) $f(t,w) = t^2 w \quad f'(t,w) = 2wt + t^2(t^2 w)$

$\therefore w_{i+1} = w_i + hf(t_i, w_i) + \frac{h^2}{2}f'(t_i, w_i) = [1 + ht^2 + \frac{h^2}{2}(2t+t^4)]w_i$

$w_1 = [1 + ht_0^2 + \frac{h^2}{2}(2t_0 + t_0^4)]w_0 = 1 \quad w_2 = [1 + ht_1^2 + \frac{h^2}{2}(2t_1 + t_1^4)]w_1 \approx 1.03137$

$w_3 = [1 + ht_2^2 + \frac{h^2}{2}(2t_2 + t_2^4)]w_2 \approx 1.23823$

$w_4 = [1 + ht_3^2 + \frac{h^2}{2}(2t_3 + t_3^4)]w_3 \approx 1.48264$

$e = |e^{\frac{1}{3}} - w_4| = 0.08702$