



四川大學

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$$1.4-7 \quad f(x) = x^4 - 7x^3 + 18x^2 - 20x + 8, \quad f(2) = 0$$

$$f'(x) = 4x^3 - 21x^2 + 36x, \quad f'(2) = 0$$

$$f''(x) = 12x^2 - 42x + 36, \quad f''(2) = 0$$

$$f'''(x) = 24x - 42, \quad f'''(2) = 6$$

$\therefore x=2$ 为 3 重根, $m=3$, 收敛是二次收敛

$$\therefore \lim_{i \rightarrow \infty} \frac{e_{i+1}}{e_i} = 5 = \frac{(m-1)}{m} = \frac{2}{3}$$

$$1.4-3(a) \quad f(x) = x^5 - 2x^4 + 2x^2 - x, \quad f(-1) = f(0) = f(1) = 0$$

$$f'(x) = 5x^4 - 8x^3 + 4x - 1, \quad f'(-1) = 8, f'(0) = -1, f'(1) = 0$$

$$f''(x) = 20x^3 - 24x^2 + 4, \quad f''(-1) = -4, f''(0) = 4, f''(1) = 0$$

$$f'''(x) = 60x^2 - 48x, \quad f'''(1) = 12$$

对于 $r=-1$, 收敛是线性的 = 一次收敛

$$\lim_{i \rightarrow \infty} \frac{e_{i+1}}{e_i^2} = m = \frac{f''(-1)}{2f'(-1)} = -2.5 \Rightarrow e_{i+1} \approx 2.5 e_i^2$$

对于 $r=0$, 收敛是二次收敛

$$\lim_{i \rightarrow \infty} \frac{e_{i+1}}{e_i^2} = m = \frac{f''(0)}{2f'(0)} = -\frac{1}{8} \Rightarrow e_{i+1} \approx 0.125 e_i^2$$

对于 $r=1$, 收敛是线性的

$$\lim_{i \rightarrow \infty} \frac{e_{i+1}}{e_i} = 5 = \frac{(m-1)}{m} = \frac{3-1}{3} = \frac{2}{3} \Rightarrow e_{i+1} \approx \frac{2}{3} e_i$$



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$$1.4-12 \quad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad f(x) = \frac{1}{x} \\ f'(x) = -\frac{1}{x^2}$$

$$\therefore x_{i+1} = x_i + \frac{x_i^2}{x_i} = 2x_i$$

$$\therefore x_{50} = x_0 \cdot 2^{50} = 2^{50}$$

$$1.5-1(a) \quad f(x) = x^3 - 2x - 2 \quad x_0 = 1, x_1 = 2$$

$$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)} = \frac{8}{5}$$

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)} = \frac{169}{97} = 1.74227$$

$$1.5-2(a) \quad f(x) = x^3 - 2x - 2 \quad x \in [1, 2]$$

$$\text{同上, 得 } x_2 = \frac{8}{5}$$

$$\because f(2) f(\frac{8}{5}) = -\frac{276}{125} < 0 \quad \therefore \text{取 } x \in [\frac{8}{5}, 2]$$

$$\therefore x_3 = \frac{2 f(\frac{8}{5}) - \frac{8}{5} f(2)}{\frac{8}{5} - 2} = \frac{169}{97} = 1.74227$$

$$1.5-3(a) \quad f(x) = x^3 - 2x - 2 \quad x_0 = 1, x_1 = 2, x_2 = 0$$

$$x_3 = x_2 - \frac{p(r-q)(x_2 - x_1) + (1-r)s(x_2 - x_0)}{(q-1)(r-1)(s-1)} = -0.2$$



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$$x_4 = x_3 - \frac{r(r-9)(x_3-x_2) + (1-r)s(x_3-x_1)}{(9-1)(r-1)(s-1)} = -\frac{24}{2078} = -0.11886$$

1.5-7 (A) = 分法为线性收敛 $\lim_{i \rightarrow \infty} \frac{e_i}{e_{i-1}} = \frac{1}{2}$

(B) 割线法, 近似误差关系为

$$\frac{e_{i+1}}{e_i e_{i-1}} = \left| \frac{f''(r)}{2f'(r)} \right| = 1.26134$$

$$\frac{e_i}{e_{i-1}^2} = \left| \frac{f''(r)}{2f'(r)} \right|^{\frac{1}{2}} = 1.26134^{\frac{1}{2}} = 1.1543$$

为超线性收敛

(C) 不动点迭代, $g_1(x) = \frac{x}{2} + \frac{1}{x^3}$, $g_1'(x) = \frac{1}{2} - \frac{3}{x^4}$

$$\lim_{i \rightarrow \infty} \frac{e_i}{e_{i-1}} = |g_1'(r)| = \frac{1}{2} \neq 0 \quad \text{无法收敛}$$

(D) 不动点迭代, $g_2(x) = \frac{x}{3} + \frac{1}{3x^3}$, $g_2'(x) = \frac{1}{3} - \frac{1}{x^4}$

$$\lim_{i \rightarrow \infty} \frac{e_i}{e_{i-1}} = |g_2'(r)| = \frac{1}{6}$$

速度 $B > D > A > C$

可以使用牛顿方法进行求解, 对 $f(x) = x^4 - 2$, $f'(x) = 4x^3$

$$f'(r) = f'(2^{\frac{1}{4}}) = 4 \cdot 2^{\frac{3}{4}} \neq 0$$

可以实现二次收敛.



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2.1-2(a)

$$\begin{bmatrix} 2 & -2 & -1 & -2 \\ 4 & 1 & -2 & 1 \\ -2 & 1 & -1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -2 & -1 & -2 \\ 0 & 5 & 0 & 5 \\ 0 & -1 & -2 & -5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -0.5 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -2 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\therefore x=1, y=1, z=2$$

2.1-3(a)

$$\begin{bmatrix} 3 & -4 & 5 & 2 \\ 0 & 3 & -4 & -1 \\ 0 & 0 & 5 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -4 & 0 & -3 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\therefore x=\frac{1}{3}, y=1, z=1$$

$$2.1-6 \quad \frac{2(5000)^3/3}{(5000)^2/0.005} = \frac{2 \cdot 5000}{3 \cdot 200} = \frac{50}{3} \text{ s} = 16.6667 \text{ s}$$



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$$2.1-7 \quad \frac{2(8000)^3/3}{(4000)^2/2000} = 60.755$$

2.2-4(a)

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} \xrightarrow{\textcircled{2}-\textcircled{1} \times 2} \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 3 & 1 & 5 \end{bmatrix} \xrightarrow{\textcircled{3}-\textcircled{1} \times 1}$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = U \quad \therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

~~$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{cases} x_1 = -1 \\ x_2 = 3 \\ x_3 = 2 \end{cases}$$~~

$$\Rightarrow \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$2.2-7 \quad \frac{2(5000)^3/3}{(500)^2/1000} = 333.3335 = 5 \min 33.335$$



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$$2.2-8 \quad \frac{100 \cdot 2(8000)^3/3}{2(2000)^3/3 \cdot 10.1} = 6405 = 10 \text{ min } 40.5$$

2.3-2(a)

$$\|A\|_{\infty} = 2 + 4 = 6$$

2.3-5(a)

$$\begin{bmatrix} 1 & -2 & 3 \\ 3 & -4 & 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

相对前向误差

$$x - x_a = \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \frac{\|x - x_a\|_{\infty}}{\|x\|_{\infty}} = \frac{3}{1} = 3$$

相对后向误差

$$r = b - Ax_a = \begin{bmatrix} 3 \\ 7 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}, \quad \frac{\|r\|_{\infty}}{\|b\|_{\infty}} = \frac{3}{7}$$

$$\text{MF} = 3 / \frac{3}{7} = 7$$

2.3-7

$$H(s) = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/5 \\ 1/2 & 1/3 & 1/4 & 1/5 & 1/6 \\ 1/3 & 1/4 & 1/5 & 1/6 & 1/7 \\ 1/4 & 1/5 & 1/6 & 1/7 & 1/8 \\ 1/5 & 1/6 & 1/7 & 1/8 & 1/9 \end{bmatrix}$$

$$\|H\|_{\infty} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{137}{60}$$