[257] 6.1

3(b) $\frac{dy}{dt} = t^2 y \Rightarrow \frac{dy}{y} = t^2 dt \Rightarrow y = e^{\frac{1}{3}t^3} + c$

$\because y(0) = 1 \quad \therefore c = 0, \quad y = e^{\frac{1}{3}t^3}, \quad y(1) = e^{\frac{1}{3}} \approx 1.3956$

5(b) $w_1 = w_0 + ht_0^2 w_0 = 1 + 0 = 1 \quad w_2 = w_1 + ht_1^2 w_1 = \frac{65}{64}$

$w_3 = w_2 + ht_2^2 w_2 = \frac{1105}{1024} \quad w_4 = w_3 + ht_3^2 w_3 = \frac{80665}{65536} \approx 1.2308$

$e = |e^{\frac{1}{3}} - w_4| \approx 0.16476$

P268 6.2

1(b) $w_1 = w_0 + \frac{h}{2}[t_0^2 w_0 + (t_0+h)^2(w_0 + h[t_0^2 w_0])] = 1 + \frac{1}{8}[0 + \frac{1}{16}[1+0]] \approx 1.0078$

$w_2 = w_1 + \frac{h}{2}[t_1^2 w_1 + (t_1+h)^2(w_1 + h[t_1^2 w_1])] \approx 1.04767$

$w_3 = w_2 + \frac{h}{2}[t_2^2 w_2 + (t_2+h)^2(w_2 + h[t_2^2 w_2])] \approx 1.15868$

$w_4 = w_3 + \frac{h}{2}[t_3^2 w_3 + (t_3+h^2)(w_3 + h[t_3^2 w_3])] \approx 1.46646$

$e = |e^{\frac{1}{3}} - w_4| = 0.07084$

4(b) $f(t,w) = t^2 w \quad f'(t,w) = 2wt + t^2(t^2 w)$

$\therefore w_{i+1} = w_i + hf(t_i, w_i) + \frac{h^2}{2}f'(t_i, w_i) = [1 + ht^2 + \frac{h^2}{2}(2t+t^4)]w_i$

$w_1 = [1 + ht_0^2 + \frac{h^2}{2}(2t_0 + t_0^4)]w_0 = 1 \quad w_2 = [1 + ht_1^2 + \frac{h^2}{2}(2t_1 + t_1^4)]w_1 \approx 1.03137$

$w_3 = [1 + ht_2^2 + \frac{h^2}{2}(2t_2 + t_2^4)]w_2 \approx 1.23823$

$w_4 = [1 + ht_3^2 + \frac{h^2}{2}(2t_3 + t_3^4)]w_3 \approx 1.48264$

$e = |e^{\frac{1}{3}} - w_4| = 0.08702$