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P257 6.1

$$8(a) \begin{cases} y' = t \\ y(0) = y_0 \\ t \in [0, 1] \end{cases}$$

$$L = \max \left| \frac{\partial y'}{\partial y} \right| = 0$$

$\therefore f(t, y) = t$ 在 $0 \leq t \leq 1, -\infty < y < +\infty$ 是利普希茨连续

$\therefore y' = t$ 在 $[0, 1]$ 上存在唯一解

11(a) 当 $y(0) = 0$ 时, $y(t) = \frac{1}{2}t^2$

当 $z(0) = 1$ 时, $z(t) = \frac{1}{2}t^2 + 1$

$$|y(t) - z(t)| = |1| \leq e^{t-0} |1|$$

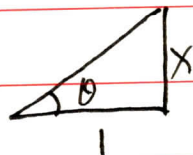
15. $\begin{cases} y' = \sin y \end{cases}$

(a) $\begin{cases} y(a) = y_a \\ t \in [a, b] \end{cases} \quad L = \max \left| \frac{\partial \sin y}{\partial y} \right| = 1$

$\therefore f(t, y) = \sin y$ 在 $a \leq t \leq b, -\infty < y < +\infty$ 是利普希茨连续

$\therefore y' = \sin y$ 在 $[a, b]$ 上存在唯一解

(b) $y'(t) = \frac{2e^{t-a} \tan(y_a/2)}{1 + [e^{t-a} \tan(y_a/2)]^2}$



$\therefore \sin(2 \arctan x) = \sin 2\theta = 2 \sin \theta \cos \theta = 2x / (1+x^2)$

$\therefore \sin y(t) = \sin(2 \arctan [e^{t-a} \tan(y_a/2)])$

$$= \frac{2e^{t-a} \tan(y_a/2)}{1 + [e^{t-a} \tan(y_a/2)]^2}$$

又 $y(a) = y_a$

$\therefore y(t)$ 是初值问题的解.