1.4-12 $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$ $f(x) = \frac{1}{x}$ $f'(x) = -\frac{1}{x^2}$

$\therefore x_{i+1} = x_i + \frac{x_i^2}{x_i} = 2x_i$

$\therefore x_{50} = x_0 \cdot 2^{50} = 2^{50}$

1.5-1(a) $f(x) = x^3 - 2x - 2$ $x_0 = 1$, $x_1 = 2$

$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)} = \frac{8}{5}$

$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)} = \frac{169}{97} = 1.7422$

1.5-2(a) $f(x) = x^3 - 2x - 2$ $x \in [1, 2]$

同上，得 $x_2 = \frac{8}{5}$

$\because f(2) f(\frac{8}{5}) = -\frac{276}{125} < 0$ $\therefore$ 取 $x \in [\frac{8}{5}, 2]$

$\therefore x_3 = \frac{2 f(\frac{8}{5}) - \frac{8}{5} f(2)}{\frac{8}{5} - 2} = \frac{169}{97} = 1.7422$

1.5-3(a) $f(x) = x^3 - 2x - 2$ $x_0 = 1$, $x_1 = 2$, $x_2 = 0$

$x_3 = x_2 - \frac{r(r-q)(x_2 - x_1) + (1-r)s(x_2 - x_0)}{(q-1)(r-1)(s-1)} = -0.2$

$$x_4 = x_3 - \frac{r(r-q)(x_3-x_2)+(1-r)s(x_3-x_1)}{(q-1)(r-1)(s-1)} = -\frac{241}{2029} = -0.11886$$

1.5-7 (A) 二分法为线性收敛 $\lim_{i\to\infty}\frac{e_i}{e_{i-1}} = \frac{1}{2}$

(B) 割线法，渐近误差关系为

$$\frac{e_{i+1}}{e_i e_{i-1}} = \left|\frac{f''(r)}{2f'(r)}\right| = 1.26134$$

$$\frac{e_i}{e_{i-1}^{\alpha}} = \left|\frac{f''(r)}{2f'(r)}\right|^{\alpha-1} = 1.26134^{\frac{\sqrt{5}-1}{2}} = 1.1543$$

为超线性收敛

(C) 不动点迭代，$g_1(x) = \frac{x}{2} + \frac{1}{x^3}$，$g_1'(x) = \frac{1}{2} - \frac{3}{x^4}$

$$\left|\lim_{i\to\infty}\frac{e_i}{e_{i-1}}\right| = |g'(r)| = 0$$ 无法收敛

(D) 不动点迭代，$g_2(x) = \frac{x}{3} + \frac{1}{3x^3}$，$g_2'(x) = \frac{1}{3} - \frac{1}{x^4}$

$$\left|\lim_{i\to\infty}\frac{e_i}{e_{i-1}}\right| = |g'(r)| = \frac{1}{6}$$

∴ 速度 B > D > A > C

可以使用牛顿方法进行求解，对于 $f(x) = x^4 - 2$，$f'(x) = 4x^3$

$f(r) = f'(2^{\frac{1}{4}}) = 4\cdot 2^{\frac{3}{4}} \neq 0$

可以实现二次收敛。