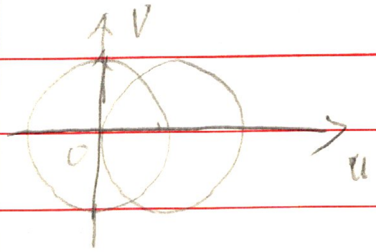
P122 2.7

2(a) $DF(x)=\begin{bmatrix} e^{u+2v} & 2e^{u+2v} \\ \cos(u+v) & \cos(u+v) \end{bmatrix}$ $DF(x_0)=\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

$F(x)=F(x_0)+DF(x_0)\cdot(x-x_0)+O(x-x_0)^2=\begin{bmatrix} 1 \\ 0 \end{bmatrix}+\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}+O(x^2)$

3(a) $\begin{cases} u^2+v^2=1 \\ (u-1)^2+v^2=1 \end{cases} \Rightarrow \begin{cases} u=\frac{1}{2} \\ v=\pm\frac{\sqrt{3}}{2} \end{cases}$



4(a) $F(x)=\begin{bmatrix} u^2+v^2-1 \\ (u-1)^2+v^2-1 \end{bmatrix}$, $DF(x)=\begin{bmatrix} 2u & 2v \\ 2u-2 & 2v \end{bmatrix}$, $DF^{-1}(x)=\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1-u}{2v} & \frac{u}{2v} \end{bmatrix}$

$x_1=x_0-DF^{-1}(x_0)F(x_0)=\begin{bmatrix} 1 \\ 1 \end{bmatrix}-\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}=\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$

$x_2=x_1-DF^{-1}(x_1)F(x_1)=\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}-\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}=\begin{bmatrix} \frac{1}{2} \\ \frac{7}{8} \end{bmatrix}$

5(a) $x_1=x_0-A_0^{-1}F(x_0)=\begin{bmatrix} 1 \\ 1 \end{bmatrix}-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$A_1=A_0+\frac{[F(x_1)-F(x_0)-A_0(x_1-x_0)](x_1-x_0)^T}{(x_1-x_0)^T(x_1-x_0)}=\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

$x_2=x_1-A_1^{-1}F(x_1)=\begin{bmatrix} 0 \\ 1 \end{bmatrix}-\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$A_2=A_1+\frac{[F(x_2)-F(x_1)-A_1(x_2-x_1)](x_2-x_1)^T}{(x_2-x_1)^T(x_2-x_1)}=\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

6(a) $x_1 = x_0 - B_0 F(x_0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$B_1 = B_0 + \frac{[x_1 - x_0 - B_0(F(x_1) - F(x_0))](x_1 - x_0)^T B_0}{(x_1 - x_0)^T B_0 [F(x_1) - F(x_0)]} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$x_2 = x_1 - B_1 F(x_1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$B_2 = B_1 + \frac{[x_2 - x_1 - B_1(F(x_2) - F(x_1))](x_2 - x_1)^T B_1}{(x_2 - x_1)^T B_1 [F(x_2) - F(x_1)]} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

P134 3.1

1(a) $P_2(x) = 1 \cdot \frac{(x-2)(x-3)}{(0-2)(0-3)} + 3 \cdot \frac{(x-0)(x-3)}{(2-0)(2-3)} + 0 \cdot \cdots = -\frac{4}{3}x^2 + \frac{11}{3}x + 1$

2(a)

0	1		
		1	
2	3		$-\frac{4}{3}$
		-3	
3	0		

$\therefore P(x) = 1 + 1(x-0) - \frac{4}{3}(x-0)(x-2)$

$= -\frac{4}{3}x^2 + \frac{11}{3}x + 1$

P140 2 (a)

1	0		
		$\ln 2$	
2	$\ln 2$		$-\frac{\ln 2}{6}$
		$\frac{\ln 2}{2}$	
4	$\ln 4$		

$\therefore P(x) = 0 + \ln 2 \cdot (x-1) - \frac{\ln 2}{6}(x-1)(x-2)$

$= -\frac{\ln 2}{6}x^2 + \frac{3\ln 2}{2}x - \frac{4\ln 2}{3}$

(b) $P(x) = -\frac{\ln 2}{6}x^2 + \frac{3\ln 2}{2}x - \frac{4\ln 2}{3}$

$P(3) = -\frac{9}{6}\ln 2 + \frac{9}{2}\ln 2 - \frac{4}{3}\ln 2 = \frac{5}{3}\ln 2$

(c) $|\ln x - P(x)| \le \frac{|(x-1)(x-2)(x-4)|}{3!} f^{(3)}_{max}(x) = \frac{|(x-1)(x-2)(x-4)|}{3}$

当 $x=3$ 时, $|\ln 3 - P(3)| \le \frac{2}{3} = 0.6667$

(d) 实际误差 $\ln 3 - P(3) = 0.056633$

P48 33

3 $x_i = \frac{-1+1}{2} + \frac{1+1}{2}\cos\frac{(2i-1)\pi}{2n} = \cos\frac{(2i-1)\pi}{2n}$

$\therefore x_1 = \cos\frac{\pi}{10}$ $x_2 = \cos\frac{3\pi}{10}$ $x_3 = \cos\frac{5\pi}{10}$ $x_4 = \cos\frac{7\pi}{10}$ $x_5 = \cos\frac{9\pi}{10}$

计算插值多项式得 $Q_5(x) = 1 + 0.997317x + 0.499556x^2 + 0.177335x^3 + 0.0434341x^4$

$|e^x - Q_5(x)| \le \frac{|(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)|}{5!} e = \frac{e}{5!\,2^4} = 0.00141577$

可以精确到小数点后两位.

4. $x_i = \frac{1+0.6}{2} + \frac{1-0.6}{2}\cos\frac{(2i-1)\pi}{2n} = 0.8+0.2\cos\frac{(2i-1)\pi}{2n}$

$x_1 = 0.8+0.2\cos\frac{\pi}{10}$ $x_2 = 0.8+0.2\cos\frac{3\pi}{10}$ $x_3 = 0.8+0.2\cos\frac{5\pi}{10}$

$x_4 = 0.8+0.2\cos\frac{7\pi}{10}$ $x_5 = 0.8+0.2\cos\frac{9\pi}{10}$

计算插值多项式得 $Q_5(x) = 1.004935 + 0.996827x + 0.577005x^2 + 0.074618x^3 + 0.0928855x^4$

$|e^x - Q_5(x)| \le \frac{|(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)|}{5!}e = \frac{0.2^5 e}{5!\,2^4} = 4.53047\times10^{-7}$

可以精确到小数点后6位.

P158 3.4

3(a) 检查组结

$s_1(1) = 4 - \frac{11}{4} + \frac{3}{4} = 2$, $s_2(1) = 2-0+0-0=0$, $s_1(1)=s_2(1)$

$s_1'(1) = -\frac{11}{4} + \frac{9}{4} = -\frac{1}{2}$, $s_2'(1) = -\frac{1}{2}+0-0=0$, $s_1'(1)=s_2'(1)$

$s_1''(1) = \frac{3}{4}\times3\times2 = \frac{9}{2}$, $s_2''(1) = 2c-0=2c$, $s_1''(1)=s_2''(1)$

$\therefore c = \frac{9}{4}$

考虑自然样条

$s_1''(0) = \frac{3}{4}\times3\times2\times0 = 0$, $s_2''(2) = \frac{9}{2}-\frac{9}{2}=0$, $s_1''(0)=s_2''(0)=0$

$\therefore$ 满足自然样条

考虑抛物线端点

$\because c_1 \ne c_2,\ d_1 \ne d_2 \ne 0$ $\therefore$ 不满足抛物线端点

考虑非纽结三次样条

$S_1'''(2)=\frac{3}{4}\times3\times2\times1=\frac{9}{2}$ $S_2'''(2)=-\frac{3}{4}\times3\times2\times1=-\frac{9}{2}$ $S_1'''(2)\neq S_2'''(2)$

∴不满足非纽结三次样条

8(a) $(0,1),(2,3),(3,2)$

$\therefore \delta_1=2,\ \delta_2=1,\ \Delta_1=2,\ \Delta_2=-1,\ a_1=1,\ a_2=3$

$\therefore 2\delta_1+2\delta_2=6,\ 3(\Delta_2/\delta_2-\Delta_1/\delta_1)=-6$

$$\begin{bmatrix}1&0&0\\1&6&1\\0&0&1\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}0\\-6\\0\end{bmatrix}\Rightarrow\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}0\\-1\\0\end{bmatrix}$$

$$d_1=\frac{c_2-c_1}{3\delta_1}=-\frac{1}{6}\qquad d_2=\frac{c_3-c_2}{3\delta_2}=\frac{1}{3}$$

$$b_1=\frac{\Delta_1}{\delta_1}-\frac{\delta_1}{3}(2c_1+c_2)=1+\frac{2}{3}\cdot1=\frac{5}{3}$$

$$b_2=\frac{\Delta_2}{\delta_2}-\frac{\delta_2}{3}(2c_2+c_3)=-1+\frac{1}{3}\cdot2=-\frac{1}{3}$$

$\therefore S_1(x)=1+\frac{5}{3}x-\frac{1}{6}x^3$ $[0,2]$

$S_2(x)=3-\frac{1}{3}(x-2)-(x-2)^2+\frac{1}{3}(x-2)^3$ $[2,3]$

14. (a) $S_1''(2)=\frac{1}{2}\times3\times2\times2=6$ $\quad S_2''(2)=2c \quad S_1''(2)=S_2''(2)\Rightarrow c=3$

(b) $S_1''(0)=\frac{1}{2}\times3\times2\times0=0$ $\quad \cancel{S_2''(3)=3\times2d(3-1)=0\Rightarrow d=0}$

$S_2''(3)=2c+6d(3-2)=0\Rightarrow d=-1$

P163 3.5

8. $(x_1,y_1)=(0,1)$, $(x_2,y_2)=(x_2,1)$

$(x_3,y_3)=(1,y_3)$, $(x_4,y_4)=(1,0)$

$b_x=3(x_2-x_1)=3x_2$

$c_x=3(x_3-x_2)-b_x=3-6x_2$

$d_x=x_4-x_1-b_x-c_x=3x_2-2$

$b_y=3(y_2-y_1)=0$

$c_y=3(y_3-y_2)-b_y=3y_3-3$

$d_y=y_4-y_1-b_y-c_y=2-3y_3$

$\therefore \begin{cases} x(t)=x_1+b_xt+c_xt^2+d_xt^3=3x_2t+(3-6x_2)t^2+(3x_2-2)t^3 \\ y(t)=y_1+b_yt+c_yt^2+d_yt^3=1+(3y_3-3)t^2+(2-3y_3)t^3 \end{cases}$

$\because \begin{cases} x(\frac{1}{3})=\frac{1}{3}\Rightarrow x_2+(3-6x_2)\frac{1}{9}+(3x_2-2)\frac{1}{27}=\frac{1}{3}\Rightarrow x_2=\frac{1}{6} \\ y(\frac{1}{3})=\frac{2}{3}\Rightarrow 1+(3y_3-3)\frac{1}{9}+(2-3y_2)\frac{1}{27}=\frac{2}{3}\Rightarrow y_2=-\frac{1}{3} \end{cases}$

$\therefore \begin{cases} x(t)=\frac{1}{2}t+2t^2-\frac{3}{2}t^3 \\ y(t)=1-4t^2+3t^3 \end{cases}$