P177 4.1

2(b)
$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 6 \\ 3 & 2 & 2 \\ 6 & 2 & 7 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 11 \end{bmatrix}$$

线性方程：
$$\begin{bmatrix} 7 & 3 & 6 \\ 3 & 2 & 2 \\ 6 & 2 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$r = b - Ax = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ RMSE = 0

8(b) (1, 2), (3, 2), (4, 1), (6, 3)

选择模型 $y = c_1 + c_2 x$

$$Ax=b \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 14 \\ 14 & 62 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 30 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 14 \\ 14 & 62 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 30 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 19 \\ 2 \end{bmatrix}$$

$$\therefore y = \frac{19}{13} + \frac{2}{13}x$$

$$r = b - Ax = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 5 \\ 1 \\ -14 \\ 8 \end{bmatrix}$$

$$RMSE = \sqrt{\frac{(5/13)^2 + (1/13)^2 + (-14/13)^2 + (8/13)^2}{4}} = \sqrt{\frac{11}{26}} \approx 0.650444$$

9(b) 选择模型 $y=c_1+c_2x+c_3x^2$

$$Ax=b \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \\ 1 & 9 & 16 & 36 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix} = \begin{bmatrix} 4 & 14 & 62 \\ 14 & 62 & 308 \\ 62 & 308 & 1634 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \\ 1 & 9 & 16 & 36 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 30 \\ 144 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 14 & 62 \\ 14 & 62 & 308 \\ 62 & 308 & 1634 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 30 \\ 144 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 77/26 \\ -79/78 \\ 1/6 \end{bmatrix}$$

$\therefore y = 77/26 - 79/78\, x + 1/6\, x^2$

$$r = b - Ax = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \frac{1}{26}\begin{bmatrix} -3 \\ 15 \\ -15 \\ 3 \end{bmatrix}$$

$$RMSE = \sqrt{\frac{(3/26)^2 + (15/26)^2 + (15/26)^2 + (3/26)^2}{4}} = \frac{3}{2\sqrt{13}} \approx 0.416025 < 0.650444$$

$\therefore$ 最优抛物线拟合的RMSE小于最优直线拟合的.

P186 4.2 $(0,0),\ (\frac{1}{6},2),\ (\frac{1}{3},0),\ (\frac{1}{2},-1),\ (\frac{2}{3},1),\ (\frac{5}{6},1)$

2(a) 选择模型 $F_3(t)=c_1+c_2\cos 2\pi t+c_3\sin 2\pi t$

$$Ax=b \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -1 & 0 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^TA=\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad A^Tb=\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

线性方程

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}=\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}=\begin{bmatrix} \frac{1}{2} \\ \frac{2}{3} \\ 0 \end{bmatrix}$$

$\therefore F_3(t)=\frac{1}{2}+\frac{2}{3}\cos 2\pi t$

$$\cancel{r=Ax-b=}\ b-Ax = -\begin{bmatrix} 1 & 1 & 0 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -1 & 0 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{1}{2} \\ \frac{2}{3} \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} -7/6 \\ \frac{7}{6} \\ -1/6 \\ -5/6 \\ 5/6 \\ 1/6 \end{bmatrix}$$

$$\|e\|_2\ \cancel{RMSE}=\sqrt{\frac{(7/6)^2+(7/6)^2+(1/6)^2+(5/6)^2+(5/6)^2+(1/6)^2}{1}}=\frac{5}{\sqrt{6}}$$

选择模型 $F_4(t)=c_1+c_2\cos 2\pi t+c_3\sin 2\pi t+c_4\cos 4\pi t$

$$Ax=b \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & -1 & 0 & 1 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^TA=\begin{bmatrix} 6 & & & \\ & 3 & & \\ & & 3 & \\ & & & 3 \end{bmatrix} \qquad A^Tb=\begin{bmatrix} 3 \\ 2 \\ 0 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & & & \\ & 3 & & \\ & & 3 & \\ & & & 3 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{2}{3} \\ 0 \\ -1 \end{bmatrix}$$

$\therefore F_4(t)=\frac{1}{2}+\frac{2}{3}\cos 2\pi t-\cos 4\pi t$

$$r=b-Ax=\begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & -1 & 0 & 1 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{2}{3} \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{6} \\ \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{6} \\ \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$\|e\|_2 \cancel{RMSE}=\sqrt{\frac{(1/6)^2+(2/3)^2+(2/3)^2+(1/6)^2+(1/3)^2+(1/3)^2}{1}}=\frac{\sqrt{7}}{\sqrt{6}}<\frac{5}{\sqrt{6}}$$

4(a) $(-2,4)$, $(-1,2)$, $(1,1)$, $(2,\frac{1}{2})$

选择模型 $y=c_1 e^{c_2 t} \Rightarrow \ln y = \ln c_1 + c_2 t = k + c_2 t$

$$Ax=b \Rightarrow \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} k \\ c_2 \end{bmatrix} = \begin{bmatrix} 2\ln 2 \\ \ln 2 \\ 0 \\ -\ln 2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 10 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2\ln 2 \\ \ln 2 \\ 0 \\ -\ln 2 \end{bmatrix} = \begin{bmatrix} 2\ln 2 \\ -7\ln 2 \end{bmatrix}$$

线性方程:

$$\begin{bmatrix} 4 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} k \\ c_2 \end{bmatrix} = \begin{bmatrix} 2\ln 2 \\ -7\ln 2 \end{bmatrix} \Rightarrow \begin{bmatrix} k \\ c_2 \end{bmatrix} = \begin{bmatrix} \frac{\ln 2}{2} \\ -\frac{7}{10}\ln 2 \end{bmatrix}$$

$\therefore \ln y = \frac{\ln 2}{2} - \frac{7}{10}\ln 2\, t \Rightarrow y = 2^{\frac{1}{2}} \cdot 2^{-\frac{7}{10}t}$

$\therefore y(-2) = 2^{\frac{19}{10}} \quad y(-1) = 2^{\frac{6}{5}} \quad y(1) = 2^{-\frac{1}{5}} \quad y(2) = 2^{-\frac{9}{10}}$

$\therefore \|e\|_2 = \sqrt{(4-2^{\frac{19}{10}})^2 + (2-2^{\frac{6}{5}})^2 + (2^{-\frac{1}{5}}-1)^2 + (2^{-\frac{9}{10}}-\frac{1}{2})^2} = 3.21636$

P188 4-3

2(a) $y_1 = A_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$, $r_1 = \|y_1\| = \sqrt{1+2^2+2^2} = 3$, $q_1 = \frac{y_1}{\|y_1\|} = \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$

$$y_2 = A_2 - q_1 q_1^T A_2 = \begin{bmatrix} 3 \\ -6 \\ -0 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}(2+4) = \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}$$

$r_{22} = \|y_2\| = \sqrt{1^2+2^2+2^2} = 3$

$$q_2 = \frac{y_2}{\|y_2\|} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}$$

$r_{12} = q_1^T A_2 = 2+4 = 6$

$$y_3 = A_3 - (q_1 q_1^T + q_2 q_2^T) A_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}\frac{2}{3} - \begin{bmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}\left(-\frac{1}{3}\right) = \cancel{\begin{bmatrix} \frac{4}{9} \\ \frac{2}{3} \\ 0 \end{bmatrix}} \begin{bmatrix} \frac{4}{9} \\ \frac{2}{9} \\ -\frac{4}{9} \end{bmatrix}$$

$$q_3 = \frac{y_3}{\|y_3\|} = \begin{bmatrix} \frac{4}{9} \\ \frac{2}{9} \\ -\frac{4}{9} \end{bmatrix} \Big/ \frac{2}{3} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix} = QR$$

6(a) 令 $x_1 = [2, -2, 1]^T$, $\|x_1\| = 3$, $w_1 = [\|x_1\|, 0, 0]^T = [3, 0, 0]^T$

$v_1 = w_1 - x_1 = [3, 0, 0]^T - [2, -2, 1]^T = [1, 2, -1]^T$

$$H_1 = I - 2\frac{v_1 v_1^T}{v_1^T v_1} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} - \frac{2}{6}\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

$$H_1 A = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 0 & 0 \\ 0 & -3 \end{bmatrix}$$

$x_2 = [0, -3]^T$, $\|x_2\| = 3$, $w_2 = [3, 0]^T$

$v_2 = w_2 - x_2 = [3, 0]^T - [0, -3]^T = [3, 3]^T$

$$H_2 = I - 2\frac{v_2 v_2^T}{v_2^T v_2} = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} - \frac{2}{18}\begin{bmatrix} 9 & 9 \\ 9 & 9 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$H_2 H_1 A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 0 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} = R$$

$$Q = H_1^{-1} H_2^{-1} = H_1 H_2 = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \end{bmatrix}$$

7(a) $\begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 6 \end{bmatrix} \Rightarrow$ $Rx = Q^T b$ $\begin{bmatrix} 3 & 6 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \end{bmatrix}\begin{bmatrix} 3 \\ -3 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ -1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix}$

P211 4.5

1(a) $r_1(x,y)=\sqrt{(x-x_1)^2+(y-y_1)^2}-R_1=\sqrt{x^2+y^2-2y+1}-1$

$r_2(x,y)=\sqrt{(x-x_2)^2+(y-y_2)^2}-R_2=\sqrt{x^2+y^2-2x-2y+2}-1$

$r_3(x,y)=\sqrt{(x-x_3)^2+(y-y_3)^2}-R_3=\sqrt{x^2+y^2+2y+1}-1$

$$Dr(x,y)=\begin{bmatrix} x/\sqrt{x^2+y^2-2y+1} & (y-1)/\sqrt{x^2+y^2-2y+1} \\ (x-1)/\sqrt{x^2+y^2-2x-2y+2} & (y-1)/\sqrt{x^2+y^2-2x-2y+2} \\ x/\sqrt{x^2+y^2+2y+1} & (y+1)/\sqrt{x^2+y^2+2y+1} \end{bmatrix}$$

1(b) $r_1(x,y)=\sqrt{(x-x_1)^2+(y-y_1)^2}-R_1=\sqrt{x^2+y^2+2x+1}-1$

$r_2(x,y)=\sqrt{(x-x_2)^2+(y-y_2)^2}-R_2=\sqrt{x^2+y^2-2x-2y+2}-1$

$r_3(x,y)=\sqrt{(x-x_3)^2+(y-y_3)^2}-R_3=\sqrt{x^2+y^2-2x+2y+2}-1$

$$Dr(x,y)=\begin{bmatrix} (x+1)/\sqrt{x^2+y^2+2x+1} & y/\sqrt{x^2+y^2+2x+1} \\ (x-1)/\sqrt{x^2+y^2-2x-2y+2} & (y-1)/\sqrt{x^2+y^2-2x-2y+2} \\ (x-1)/\sqrt{x^2+y^2-2x+2y+2} & (y+1)/\sqrt{x^2+y^2-2x+2y+2} \end{bmatrix}$$

5(a) $y=c_1t^{c_2}$ $\quad \partial y/\partial c_1=t^{c_2}$ $\quad \partial y/\partial c_2=c_1t^{c_2}\ln t$

$$r=\begin{bmatrix} c_1t_1^{c_2}-y_1 \\ c_1t_2^{c_2}-y_2 \\ c_1t_3^{c_2}-y_3 \end{bmatrix} \quad Dr=\begin{bmatrix} t_1^{c_2} & c_1t_1^{c_2}\ln t_1 \\ t_2^{c_2} & c_1t_2^{c_2}\ln t_2 \\ t_3^{c_2} & c_1t_3^{c_2}\ln t_3 \end{bmatrix}$$

5(b) $y=c_1te^{c_2t}$ $\quad \partial y/\partial c_1=te^{c_2t}$ $\quad \partial y/\partial c_2=c_1t^2e^{c_2t}$

$$r=\begin{bmatrix} c_1t_1e^{c_2t_1}-y_1 \\ c_1t_2e^{c_2t_2}-y_2 \\ c_1t_3e^{c_2t_3}-y_3 \end{bmatrix} \quad Dr=\begin{bmatrix} t_1e^{c_2t_1} & c_1t_1^2e^{c_2t_1} \\ t_2e^{c_2t_2} & c_1t_2^2e^{c_2t_2} \\ t_3e^{c_2t_3} & c_1t_3^2e^{c_2t_3} \end{bmatrix}$$