习题 6.3

1(a) $\begin{cases} y_1' = y_1 + y_2 \\ y_2' = -y_1 + y_2 \\ y_1(0) = 1 \\ y_2(0) = 0 \end{cases} \Rightarrow \begin{cases} y_1(t) = e^t \cos t \\ y_2(t) = -e^t \sin t \end{cases} \Rightarrow \begin{cases} y_1(1) = 1.46869 \\ y_2(1) = -2.28736 \end{cases}$

$w_{0,1} = 1$ $\quad w_{0,2} = 0$

$w_{1,1} = 1 + \frac{1}{4}(1+0) = \frac{5}{4}$ $\quad w_{1,2} = 0 + \frac{1}{4}(-1+0) = -\frac{1}{4}$

$w_{2,1} = \frac{5}{4} + \frac{1}{4}(\frac{5}{4} - \frac{1}{4}) = \frac{3}{2}$ $\quad w_{2,2} = -\frac{1}{4} + \frac{1}{4}(-\frac{5}{4} - \frac{1}{4}) = -\frac{5}{8}$

$w_{3,1} = \frac{3}{2} + \frac{1}{4}(\frac{3}{2} - \frac{5}{8}) = \frac{55}{32}$ $\quad w_{3,2} = -\frac{5}{8} + \frac{1}{4}(-\frac{3}{2} - \frac{5}{8}) = -\frac{37}{32}$

$w_{4,1} = \frac{55}{32} + \frac{1}{4}(\frac{55}{32} - \frac{37}{32}) = \frac{119}{64}$ $\quad w_{4,2} = -\frac{37}{32} + \frac{1}{4}(-\frac{55}{32} - \frac{37}{32}) = -\frac{15}{8}$

$\therefore e_1 = |w_{4,1} - y_1(1)| = 0.3807$, $e_2 = |w_{4,2} - y_2(1)| = 0.4124$

2(a) $w_{1,1} = 1 + \frac{1}{8}[1+0 + (1 + \frac{1}{4}(1+0)) + (0 + \frac{1}{4}(-1+0))] = \frac{5}{4}$

$w_{1,2} = 0 + \frac{1}{8}[-1+0 + (1 + \frac{1}{4}(1+0)) + (0 + \frac{1}{4}(-1+0))] = -\frac{5}{16}$

$w_{2,1} = \frac{5}{4} + \frac{1}{8}[\frac{5}{4} - \frac{5}{16} + \frac{5}{4} + \frac{1}{4}(\frac{5}{4} - \frac{5}{16}) + \frac{5}{16} + \frac{1}{4}(-\frac{5}{4} - \frac{5}{16})] = \frac{375}{256}$

$$w_{2,2}=-\frac{5}{16}+\frac{1}{8}\left[-\frac{5}{4}-\frac{5}{16}-\left(\frac{5}{4}+\frac{1}{4}\left(\frac{5}{4}-\frac{5}{16}\right)\right)+\left(-\frac{5}{16}+\frac{1}{4}\left(-\frac{5}{4}-\frac{5}{16}\right)\right)\right]=-\frac{25}{32}$$

$$w_{3,1}=\frac{375}{256}+\frac{1}{8}\left[\frac{375}{256}-\frac{25}{32}+\left(\frac{375}{256}+\frac{1}{4}\left(\frac{375}{256}-\frac{25}{32}\right)\right)+\left(\frac{25}{32}+\frac{1}{4}\left(-\frac{375}{256}-\frac{25}{32}\right)\right)\right]=\frac{1625}{1024}$$

$$w_{3,2}=-\frac{25}{32}+\frac{1}{8}\left[-\frac{375}{256}-\frac{25}{32}-\left(\frac{375}{256}+\frac{1}{4}\left(\frac{375}{256}-\frac{25}{32}\right)\right)+\left(-\frac{25}{32}+\frac{1}{4}\left(-\frac{375}{256}-\frac{25}{32}\right)\right)\right]=-\frac{5875}{4096}$$

$$w_{4,1}=\frac{1625}{1024}+\frac{1}{8}\left[\frac{1625}{1024}-\frac{5875}{4096}+\left(\frac{1625}{1024}+\frac{1}{4}\left(\frac{1625}{1024}-\frac{5875}{4096}\right)\right)+\left(-\frac{5875}{4096}+\frac{1}{4}\left(-\frac{1625}{1024}-\frac{5875}{4096}\right)\right)\right]=\frac{100625}{65536}$$

$$w_{4,2}=-\frac{5875}{4096}+\frac{1}{8}\left[-\frac{1625}{1024}-\frac{5875}{4096}-\left(\frac{1625}{1024}+\frac{1}{4}\left(\frac{1625}{1024}-\frac{5875}{4096}\right)\right)+\left(-\frac{5875}{4096}+\frac{1}{4}\left(-\frac{1625}{1024}-\frac{5875}{4096}\right)\right)\right]=-\frac{9375}{4096}$$

$\therefore e_1=|w_{4,1}-y_1(1)|=0.0667$, $e_2=|w_{4,2}-y_2(1)|=0.0015$

3(a) $y''-ty=0$

令 $y_1=y$, $y_2=y'$, 则

$$\begin{cases} y_1'=y_2 \\ y_2'=ty_1 \end{cases}$$