P257 6.1

9(a) $\begin{cases} y' = t \\ y(0) = y_0 \\ t \in [0,1] \end{cases}$

$L = \max\left|\frac{\partial y'}{\partial y}\right| = 0$

∴ $f(t,y) = t$ 在 $0 \le t \le 1, -\infty < y < +\infty$ 是利普希茨连续

∴ $y' = t$ 在 $[0,1]$ 上存在唯一解

11(a) 当 $Y(0) = 0$ 时, $Y(t) = \frac{1}{2}t^2$

当 $z(0) = 1$ 时, $z(t) = \frac{1}{2}t^2 + 1$

$|Y(t) - z(t)| = 1 \le e^{t-0} \cdot |1|$

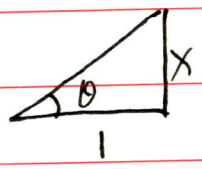
15.

(a) $\begin{cases} y' = \sin y \\ y(a) = y_a \\ t \in [a,b] \end{cases}$ $L = \max\left|\frac{\partial \sin y}{\partial y}\right| = 1$

∴ $f(t,y) = \sin y$ 在 $a \le t \le b, -\infty < y < \infty$ 是利普希茨连续

∴ $y' = \sin y$ 在 $[a,b]$ 上存在唯一解

(b) $y'(t) = \frac{2e^{t-a}\tan(y_a/2)}{1 + [e^{t-a}\tan(y_a/2)]^2}$



∵ $\sin(2\arctan X) = \sin 2\theta = 2\sin\theta\cos\theta = 2X/(1+X^2)$

∴ $\sin[y(t)] = \sin(2\arctan[e^{t-a}\tan(y_a/2)])$

$= \frac{2e^{t-a}\tan(y_a/2)}{1 + [e^{t-a}\tan(y_a/2)]^2}$

又∵ $y(a) = y_a$

∴ $y(t)$ 是初值问题的解.

习题 6.3

1(a) $\begin{cases} y_1' = y_1 + y_2 \\ y_2' = -y_1 + y_2 \\ y_1(0) = 1 \\ y_2(0) = 0 \end{cases} \Rightarrow \begin{cases} y_1(t) = e^t \cos t \\ y_2(t) = -e^t \sin t \end{cases} \Rightarrow \begin{cases} y_1(1) = 1.46869 \\ y_2(1) = -2.28736 \end{cases}$

$w_{0,1} = 1$ $\quad w_{0,2} = 0$

$w_{1,1} = 1 + \frac{1}{4}(1+0) = \frac{5}{4}$ $\quad w_{1,2} = 0 + \frac{1}{4}(-1+0) = -\frac{1}{4}$

$w_{2,1} = \frac{5}{4} + \frac{1}{4}(\frac{5}{4} - \frac{1}{4}) = \frac{3}{2}$ $\quad w_{2,2} = -\frac{1}{4} + \frac{1}{4}(-\frac{5}{4} - \frac{1}{4}) = -\frac{5}{8}$

$w_{3,1} = \frac{3}{2} + \frac{1}{4}(\frac{3}{2} - \frac{5}{8}) = \frac{55}{32}$ $\quad w_{3,2} = -\frac{5}{8} + \frac{1}{4}(-\frac{3}{2} - \frac{5}{8}) = -\frac{37}{32}$

$w_{4,1} = \frac{55}{32} + \frac{1}{4}(\frac{55}{32} - \frac{37}{32}) = \frac{119}{64}$ $\quad w_{4,2} = -\frac{37}{32} + \frac{1}{4}(-\frac{55}{32} - \frac{37}{32}) = -\frac{15}{8}$

$\therefore e_1 = |w_{4,1} - y_1(1)| = 0.3807$, $e_2 = |w_{4,2} - y_2(1)| = 0.4124$

2(a) $w_{1,1} = 1 + \frac{1}{8}[1+0 + (1 + \frac{1}{4}(1+0)) + (0 + \frac{1}{4}(-1+0))] = \frac{5}{4}$

$w_{1,2} = 0 + \frac{1}{8}[-1+0 + (1 + \frac{1}{4}(1+0)) + (0 + \frac{1}{4}(-1+0))] = -\frac{5}{16}$

$w_{2,1} = \frac{5}{4} + \frac{1}{8}[\frac{5}{4} - \frac{5}{16} + \frac{5}{4} + \frac{1}{4}(\frac{5}{4} - \frac{5}{16}) + \frac{5}{16} + \frac{1}{4}(-\frac{5}{4} - \frac{5}{16})] = \frac{375}{256}$

$$w_{2,2}=-\frac{5}{16}+\frac{1}{8}\left[-\frac{5}{4}-\frac{5}{16}-\left(\frac{5}{4}+\frac{1}{4}\left(\frac{5}{4}-\frac{5}{16}\right)\right)+\left(-\frac{5}{16}+\frac{1}{4}\left(-\frac{5}{4}-\frac{5}{16}\right)\right)\right]=-\frac{25}{32}$$

$$w_{3,1}=\frac{375}{256}+\frac{1}{8}\left[\frac{375}{256}-\frac{25}{32}+\left(\frac{375}{256}+\frac{1}{4}\left(\frac{375}{256}-\frac{25}{32}\right)\right)+\left(\frac{25}{32}+\frac{1}{4}\left(-\frac{375}{256}-\frac{25}{32}\right)\right)\right]=\frac{1625}{1024}$$

$$w_{3,2}=-\frac{25}{32}+\frac{1}{8}\left[-\frac{375}{256}-\frac{25}{32}-\left(\frac{375}{256}+\frac{1}{4}\left(\frac{375}{256}-\frac{25}{32}\right)\right)+\left(-\frac{25}{32}+\frac{1}{4}\left(-\frac{375}{256}-\frac{25}{32}\right)\right)\right]=-\frac{5875}{4096}$$

$$w_{4,1}=\frac{1625}{1024}+\frac{1}{8}\left[\frac{1625}{1024}-\frac{5875}{4096}+\left(\frac{1625}{1024}+\frac{1}{4}\left(\frac{1625}{1024}-\frac{5875}{4096}\right)\right)+\left(-\frac{5875}{4096}+\frac{1}{4}\left(-\frac{1625}{1024}-\frac{5875}{4096}\right)\right)\right]=\frac{100625}{65536}$$

$$w_{4,2}=-\frac{5875}{4096}+\frac{1}{8}\left[-\frac{1625}{1024}-\frac{5875}{4096}-\left(\frac{1625}{1024}+\frac{1}{4}\left(\frac{1625}{1024}-\frac{5875}{4096}\right)\right)+\left(-\frac{5875}{4096}+\frac{1}{4}\left(-\frac{1625}{1024}-\frac{5875}{4096}\right)\right)\right]=-\frac{9375}{4096}$$

$\therefore e_1=|w_{4,1}-y_1(1)|=0.0667$, $e_2=|w_{4,2}-y_2(1)|=0.0015$

3(a) $y''-ty=0$

令 $y_1=y$, $y_2=y'$, 则

$$\begin{cases} y_1'=y_2 \\ y_2'=ty_1 \end{cases}$$

P282 6.4

1(d) $\begin{cases} y' = 5t^4 y \\ y(0) = 1 \end{cases} \Rightarrow y(t) = e^{t^5} \Rightarrow y(1) = e = 2.71828$

$w_0 = 1$

$w_1 = 1 + \frac{1}{4} \cdot 5 \cdot (\frac{1}{8})^4 (1 + \frac{1}{8} \cdot 5 \cdot 0^4 \cdot 1) = \frac{16389}{16384}$

$w_2 = \frac{16389}{16384} + \frac{1}{4} \cdot 5 \cdot (\frac{1}{4} + \frac{1}{8})^4 (\frac{16389}{16384} + \frac{1}{8} \cdot 5 \cdot (\frac{1}{4})^4 \frac{16389}{16384} = \frac{563\ 880\ 465\ 833}{549\ 755\ 813\ 888}$

$w_3 = w_2 + \frac{1}{4} \cdot 5 \cdot (\frac{1}{2} + \frac{1}{8})^4 (w_2 + \frac{1}{8} \cdot 5 \cdot (\frac{1}{2})^4 w_2) \approx 1.2283$

$w_4 = w_3 + \frac{1}{4} \cdot 5 \cdot (\frac{3}{4} + \frac{1}{8})^4 (w_3 + \frac{1}{8} \cdot 5 \cdot (\frac{3}{4})^4 w_3) \approx 2.3062$

$e = |w_4 - y(1)| \approx 0.412$

3(d) ~~$w_1 = 1 + \frac{1}{24}(5 \cdot 0^4 \cdot 1 + 2 \cdot 5 \cdot (0 + \frac{1}{8})^4$~~ $k_i = 5 t_i^4 w_i$ $S_2 = 5(t_i + \frac{1}{8})^4 (w_i + \frac{1}{8} S_1)$

$S_3 = 5(t_i + \frac{1}{8})^4 (w_i + \frac{1}{8} S_2)$ $S_4 = 5(t_i + \frac{1}{4})^4 (w_i + \frac{1}{4} S_3)$

$\therefore w_1 = w_0 + \frac{1}{24}(S_1 + 2S_2 + 2S_3 + S_4) \approx 1.0010$

$w_2 = w_1 + \frac{1}{24}(S_1 + 2S_2 + 2S_3 + S_4) \approx 1.0318$

$w_3 = w_2 + \frac{1}{24}(S_1 + 2S_2 + 2S_3 + S_4) \approx 1.2678$

$w_4 = w_3 + \frac{1}{24}(S_1 + 2S_2 + 2S_3 + S_4) \approx 2.7033$

$e = |w_4 - y(1)| \approx 0.00785$

P311 7-1

3. $\begin{cases} y''=-4y \\ y(a)=y_a \\ y(b)=y_b \end{cases}$ (a) $y(t)=c_1\cos 2t+c_2\sin 2t$

(b) 当 $a=0$ 时, $b=\pi$ 时

$\begin{cases} y(0)=c_1=y_a \\ y(\pi)=c_1=y_b \end{cases} \Rightarrow y_a=y_b$

(c) 当 $a=0$ 时, $b=\frac{\pi}{2}$ 时

$\begin{cases} y(0)=c_1=y_a \\ y(\frac{\pi}{2})=-c_1=y_b \end{cases} \Rightarrow y_a+y_b=0$

(d) 当 $a=0$ 时, $b=\frac{\pi}{4}$ 时

$\begin{cases} y(0)=c_1=y_a \\ y(\frac{\pi}{4})=c_2=y_b \end{cases}$ $\Rightarrow$ 无条件, y_a 与 y_b 取任意值, 解都存在.