P257 6.1

9(a) $\begin{cases} y' = t \\ y(0) = y_0 \\ t \in [0,1] \end{cases}$

$$L = \max \left| \frac{\partial y'}{\partial y} \right| = 0$$

$\therefore f(t,y) = t$ 在 $0 \le t \le 1, -\infty < y < +\infty$ 是利普希茨连续

$\therefore y' = t$ 在 $[0,1]$ 上存在唯一解

11(a) 当 $Y(0) = 0$ 时，$Y(t) = \frac{1}{2}t^2$

当 $Z(0) = 1$ 时，$Z(t) = \frac{1}{2}t^2 + 1$

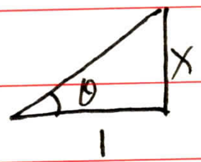
$$|Y(t) - Z(t)| = 1 \le e^{t-0} |1|$$

15. (a) $\begin{cases} y' = \sin y \\ y(a) = y_a \\ t \in [a,b] \end{cases}$ $\quad L = \max \left| \frac{\partial \sin y}{\partial y} \right| = 1$

$\therefore f(t,y) = \sin y$ 在 $a \le t \le b, -\infty < y < \infty$ 是利普希茨连续

$\therefore y' = \sin y$ 在 $[a,b]$ 上存在唯一解

(b) $$y'(t) = \frac{2e^{t-a} \tan(y_a/2)}{1 + [e^{t-a} \tan(y_a/2)]^2}$$



$\because \sin(2\arctan x) = \sin 2\theta = 2\sin\theta\cos\theta = 2x/(1+x^2)$

$\therefore \sin[y(t)] = \sin(2\arctan[e^{t-a} \tan(y_a/2)])$

$$= \frac{2e^{t-a} \tan(y_a/2)}{1 + [e^{t-a} \tan(y_a/2)]^2}$$

又 $\because y(a) = y_a$

$\therefore y(t)$ 是初值问题的解.