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1-4-7 $f(x)=x^4-7x^3+18x^2-20x+8$, $f(2)=0$

$f'(x)=4x^3-21x^2+36x$, $f'(2)=0$

$f''(x)=12x^2-42x+36$, $f''(2)=0$

$f'''(x)=24x-42$, $f'''(2)=6$

∴ $r=2$ 为3重根, $m=3$, 不会二次收敛

∴ $\lim\limits_{i\to\infty}\frac{e_{i+1}}{e_i}=S=\frac{(m-1)}{m}=\frac{2}{3}$

1-4-3(a) $f(x)=x^5-2x^4+2x^2-x$, $f(-1)=f(0)=f(1)=0$

$f'(x)=5x^4-8x^3+4x-1$, $f'(-1)=8$, $f'(0)=-1$, $f'(1)=0$

$f''(x)=20x^3-24x^2+4$, $f''(-1)=-40$, $f''(0)=4$, $f''(1)=0$

$f'''(x)=60x^2-48x$, $f'''(1)=12$

对于 $r=-1$, 收敛是~~线性~~的二次收敛.

$\lim\limits_{i\to\infty}\frac{e_{i+1}}{e_i^2}=M=\frac{f''(-1)}{2f'(-1)}=-2.5 \Rightarrow e_{i+1}\approx 2.5e_i^2$

对于 $r=0$, 收敛是二次收敛

$\lim\limits_{i\to\infty}\frac{e_{i+1}}{e_i^2}=M=\frac{f''(0)}{2f'(0)}=-\frac{1}{8} \Rightarrow e_{i+1}\approx 0.125e_i^2$

对于 $r=1$, 收敛是线性的

$\lim\limits_{i\to\infty}\frac{e_{i+1}}{e_i}=S=\frac{(m-1)}{m}=\frac{3-1}{3}=\frac{2}{3}\Rightarrow e_{i+1}\approx\frac{2}{3}e_i$

1.4-12 $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$ $f(x) = \frac{1}{x}$

$f'(x) = -\frac{1}{x^2}$

$\therefore x_{i+1} = x_i + \frac{x_i^2}{x_i} = 2x_i$

$\therefore x_{50} = x_0 \cdot 2^{50} = 2^{50}$

1.5-1(a) $f(x) = x^3 - 2x - 2$ $x_0 = 1, x_1 = 2$

$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)} = \frac{8}{5}$

$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)} = \frac{168}{97} = 1.74227$

1.5-2(a) $f(x) = x^3 - 2x - 2$ $x \in [1, 2]$

同上，得 $x_2 = \frac{8}{5}$

$\because f(2) f(\frac{8}{5}) = -\frac{276}{125} < 0$ $\therefore$ 取 $x \in [\frac{8}{5}, 2]$

$\therefore x_3 = \frac{2 f(\frac{8}{5}) - \frac{8}{5} f(2)}{\frac{8}{5} - 2} = \frac{169}{97} = 1.74227$

1.5-3(a) $f(x) = x^3 - 2x - 2$ $x_0 = 1, x_1 = 2, x_2 = 0$

$x_3 = x_2 - \frac{r(r-q)(x_2 - x_1) + (1-r)s(x_2 - x_0)}{(q-1)(r-1)(s-1)} = -0.2$

$$x_4 = x_3 - \frac{r(r-q)(x_3-x_2)+(1-r)s(x_3-x_1)}{(q-1)(r-1)(s-1)} = -\frac{241}{2009} = -0.11996$$

1.5-7 (A) 二分法为线性收敛 $\lim\limits_{i\to\infty}\frac{e_i}{e_{i-1}}=\frac{1}{2}$

(B) 割线法，由近似误差关系为

$$\frac{e_{i+1}}{e_i e_{i-1}} = \left|\frac{f''(r)}{2f'(r)}\right| = 1.26134$$

$$\frac{e_i}{e_{i-1}^{\alpha}} = \left|\frac{f''(r)}{2f'(r)}\right|^{\alpha-1} = 1.26134^{\frac{\sqrt{5}-1}{2}} = 1.1543$$

为超线性收敛

(C) 不动点迭代，$g_1(x)=\frac{x}{2}+\frac{1}{x^3}$，$g_1'(x)=\frac{1}{2}-\frac{3}{x^4}$

$$\left|\lim_{i\to\infty}\frac{e_i}{e_{i-1}}\right| = |g'(r)| = 1 \quad \text{无法收敛}$$

(D) 不动点迭代，$g_2(x)=\frac{x}{3}+\frac{1}{3x^3}$，$g_2'(x)=\frac{1}{3}-\frac{1}{x^4}$

$$\left|\lim_{i\to\infty}\frac{e_i}{e_{i-1}}\right| = |g'(r)| = \frac{1}{6}$$

∴ 速度 $B>D>A>C$

可以使用牛顿方法进行求解，对于 $f(x)=x^4-2$，$f'(x)=4x^3$

$f'(r)=f'(2^{\frac{1}{4}})=4\cdot 2^{\frac{3}{4}}\neq 0$

可以实现二次收敛.

2.1-2(a)

$$\left[\begin{array}{ccc|c} 2 & -2 & -1 & -2 \\ 4 & 1 & -2 & 1 \\ -2 & 1 & -1 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 2 & -2 & -1 & -2 \\ 0 & 5 & 0 & 5 \\ 0 & -1 & -2 & -5 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -0.5 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -2 & -4 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\therefore x=1, y=1, z=2$

2.1-3(a)

$$\left[\begin{array}{ccc|c} 3 & -4 & 5 & 2 \\ 0 & 3 & -4 & -1 \\ 0 & 0 & 5 & 5 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 3 & -4 & 0 & -3 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\therefore x=\frac{1}{3}, y=1, z=1$

2.1-6 $\quad \dfrac{2(5000)^3/3}{(5000)^2/0.005} = \dfrac{2\cdot 5000}{3\cdot 200} = \dfrac{50}{3}s = 16.6667s$

2.2-8 $\dfrac{100\cdot 2(8000)^3/3}{2(2000)^3/3/0.1} = 640s = 10min\,40s$

2.3-2(a)

$\|A\|_\infty = 2+4 = 6$

2.3-5(a)

$$\left[\begin{array}{cc|c} 1 & -2 & 3 \\ 3 & -4 & 7 \end{array}\right] \Rightarrow \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

相对前向误差

$$x - x_a = \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \frac{\|x-x_a\|_\infty}{\|x\|_\infty} = \frac{3}{1} = 3$$

相对后向误差

$$r = b - Ax_a = \begin{bmatrix} 3 \\ 7 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}, \quad \frac{\|r\|_\infty}{\|b\|_\infty} = \frac{3}{7}$$

$$EMF = 3 \Big/ \frac{3}{7} = 7$$

2.3-7

$$H(5) = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/5 \\ 1/2 & 1/3 & 1/4 & 1/5 & 1/6 \\ 1/3 & 1/4 & 1/5 & 1/6 & 1/7 \\ 1/4 & 1/5 & 1/6 & 1/7 & 1/8 \\ 1/5 & 1/6 & 1/7 & 1/8 & 1/9 \end{bmatrix} \quad \|H\|_\infty = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \frac{137}{60}$$