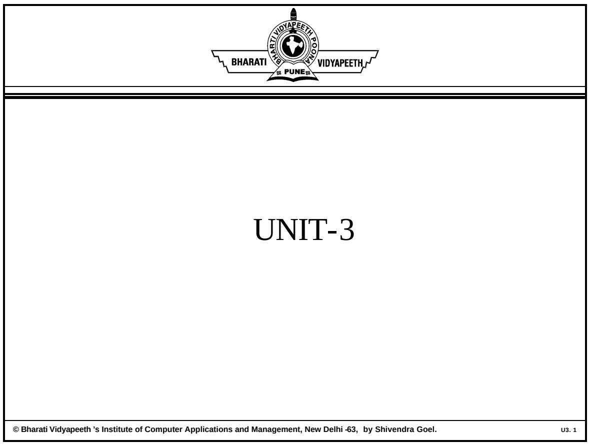
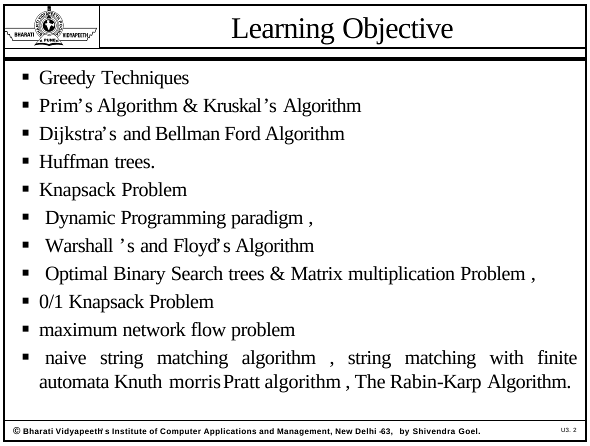
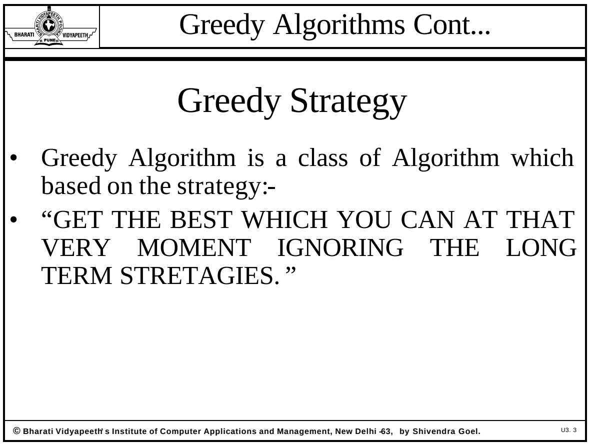


UNIT-3



Learning Objective

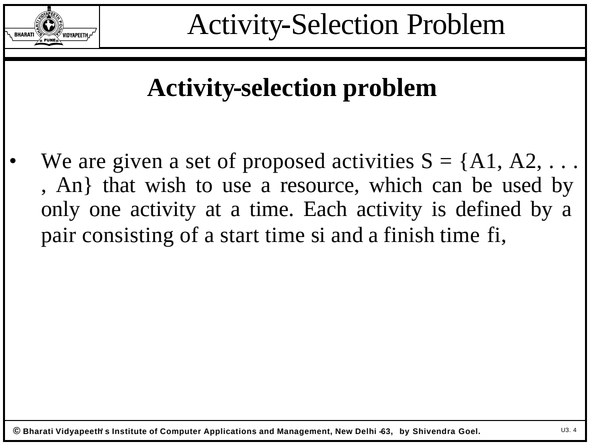
- Greedy Techniques
- Prim's Algorithm & Kruskal's Algorithm
- Dijkstra's and Bellman Ford Algorithm
- Huffman trees.
- Knapsack Problem
- Dynamic Programming paradigm ,
- Warshall 's and Floyd's Algorithm
- Optimal Binary Search trees & Matrix multiplication Problem ,
- 0/1 Knapsack Problem
- maximum network flow problem
- naive string matching algorithm , string matching with finite automata Knuth morris Pratt algorithm , The Rabin-Karp Algorithm.



Greedy Algorithms Cont...

Greedy Strategy

- Greedy Algorithm is a class of Algorithm which based on the strategy:-
- "GET THE BEST WHICH YOU CAN AT THAT VERY MOMENT IGNORING THE LONG TERM STRETAGIES. "

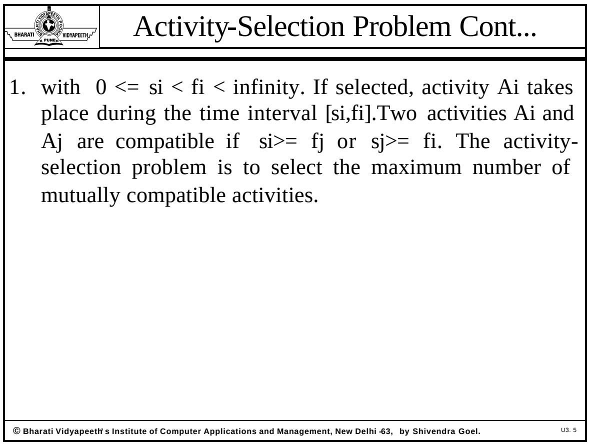


Activity-Selection Problem

Activity-selection problem

- We are given a set of proposed activities S = {A1, A2, . . . , An} that wish to use a resource, which can be used by only one activity at a time. Each activity is defined by a pair consisting of a start time si and a finish time fi,

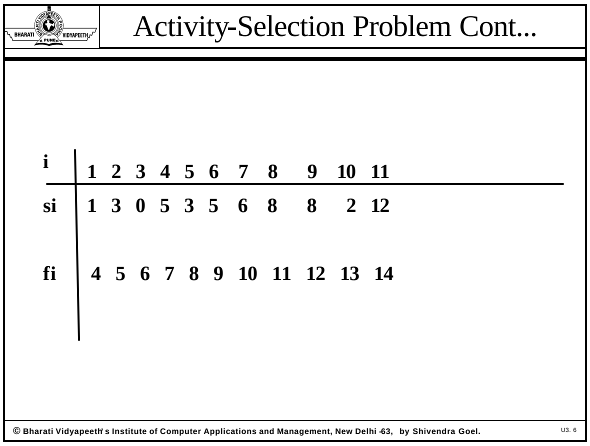
© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 4



Activity-Selection Problem Cont...

1. with 0 <= si < fi < infinity. If selected, activity Ai takes place during the time interval [si,fi].Two activities Ai and Aj are compatible if si>= fj or sj>= fi. The activity-selection problem is to select the maximum number of mutually compatible activities.

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 5

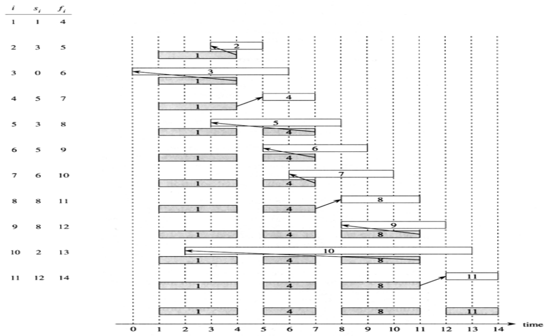


Activity-Selection Problem Cont...

i	1	2	3	4	5	6	7	8	9	10	11
si	1	3	0	5	3	5	6	8	8	2	12
fi	4	5	6	7	8	9	10	11	12	13	14

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 6

Activity-Selection Problem Cont...



i	s_i	f_i
1	1	4
2	3	5
3	0	6
4	5	7
5	3	8
6	5	9
7	6	10
8	8	11
9	8	12
10	2	13
11	12	14

Dynamic Programming

Dynamic Programming is a technique used for algorithm Design this technique is useful for designing algorithm for optimization Problems.

Dynamic Programming can be divided into four steps

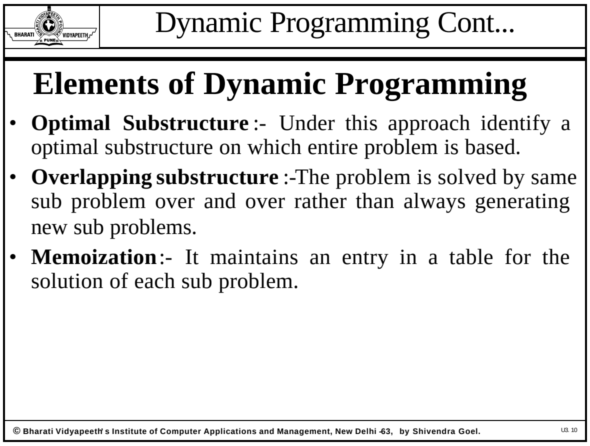
1)Characterization of suboptimal problems.

2)Recursively define the solution to the sub optimal problems.

Dynamic Programming Cont...

3)To compute an optimal solution for one of the sub optimal problems.

4)To combine the solutions for the suboptimal problems to reach the solution of the given problem.

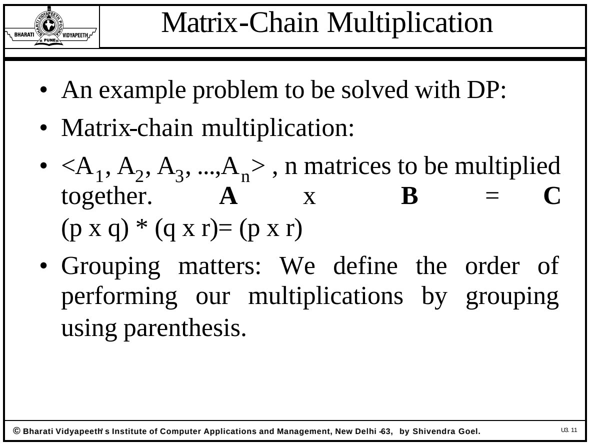


Dynamic Programming Cont...

Elements of Dynamic Programming

- **Optimal Substructure** :- Under this approach identify a optimal substructure on which entire problem is based.
- **Overlapping substructure** :-The problem is solved by same sub problem over and over rather than always generating new sub problems.
- **Memoization**:- It maintains an entry in a table for the solution of each sub problem.

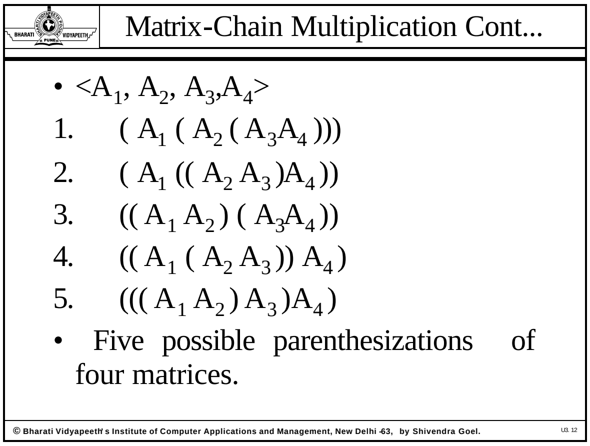
© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 10



Matrix-Chain Multiplication

- An example problem to be solved with DP:
- Matrix-chain multiplication:
- $<A_1, A_2, A_3, ..., A_n>$, n matrices to be multiplied together. **A** x **B** = **C** (p x q) * (q x r)= (p x r)
- Grouping matters: We define the order of performing our multiplications by grouping using parenthesis.

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 11



Matrix-Chain Multiplication Cont...

- $<A_1, A_2, A_3,A_4>$

1. $(A_1 (A_2 (A_3A_4)))$
2. $(A_1 ((A_2 A_3)A_4))$
3. $((A_1 A_2) (A_3A_4))$
4. $((A_1 (A_2 A_3)) A_4)$
5. $(((A_1 A_2) A_3)A_4)$

- Five possible parenthesizations of four matrices.

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 12

Matrix-Chain Multiplication Cont...

- **How much can different parenthesizations cost?**
- $<A_1, A_2, A_3>$
- A_1 : 10 x 100
- A_2 : 100 x 5
- A_3 : 5 x 50

Matrix-Chain Multiplication Cont...

- $((A_1 A_2) A_3)$
- $(A_1 A_2)$: 10 x 100 x 5 = 5000
- $((A_1 A_2) A_3)$: 10 x 5 x 50 = 2500

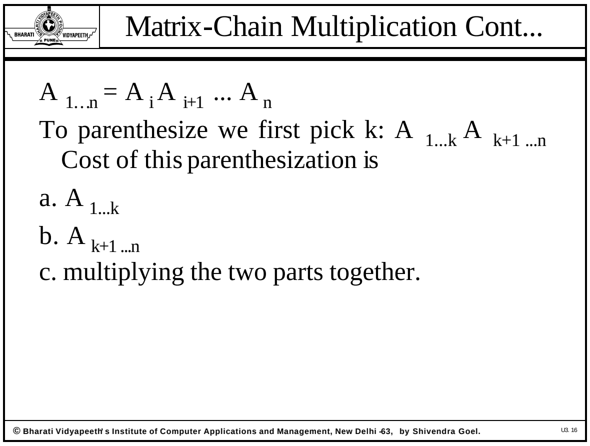
7500 multiplications

- $(A_1 (A_2 A_3))$
- $(A_2 A_3)$: 100 x 5 x 50 = 25000
- $(A_1 (A_2 A_3))$: 10 x 100 x 50 = 50000

75000 multiplications

Matrix-Chain Multiplication Cont...

- Matrix-Chain multiplication problem can be stated as follows: Given a chain of $<A_1, A_2, A_3, ..., A_n>$ of n matrices ,where i=1,2,3…….n .matrix Ai has dimension $P_{i-1} * P_i$, fully parenthesize the product $A_1\ A_2\ A_3 \ldots A_n$ in a way that minimizes the number of multiplications. Our main goal is only to determine an order for multiplying matrices that has the lowest cost.



Matrix-Chain Multiplication Cont...

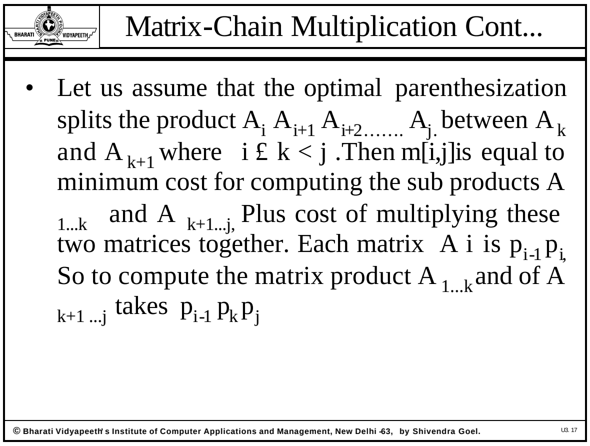
$A_{1…n} = A_i A_{i+1} ... A_n$

To parenthesize we first pick k: $A_{1...k}$ $A_{k+1 ...n}$
Cost of this parenthesization is

a. $A_{1...k}$

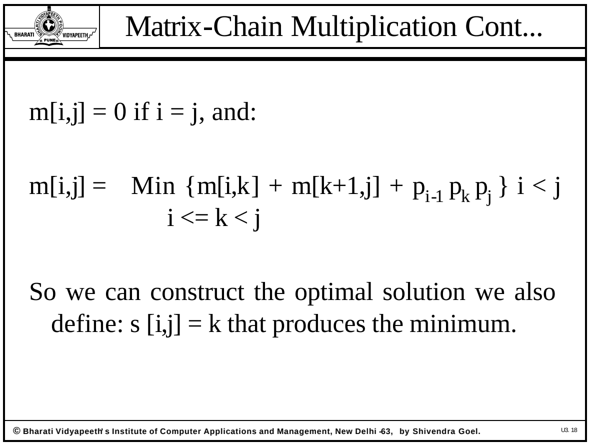
b. $A_{k+1 ...n}$

c. multiplying the two parts together.



Matrix-Chain Multiplication Cont...

- Let us assume that the optimal parenthesization splits the product A_i A_{i+1} A_{i+2}……. $A_{j.}$ between A_k and A_{k+1} where i £ k < j .Then m[i,j]is equal to minimum cost for computing the sub products $A_{1...k}$ and $A_{k+1...j,}$ Plus cost of multiplying these two matrices together. Each matrix A i is $p_{i-1} p_{i,}$ So to compute the matrix product $A_{1...k}$ and of $A_{k+1 ...j}$ takes $p_{i-1} p_k p_j$



Matrix-Chain Multiplication Cont...

m[i,j] = 0 if i = j, and:

$$m[i,j] = \min_{i <= k < j} \{m[i,k] + m[k+1,j] + p_{i-1} p_k p_j\} \quad i < j$$

So we can construct the optimal solution we also define: s [i,j] = k that produces the minimum.

Matrix-Chain Multiplication Cont...

A1=10X5

A2=5X3

A3=30X20

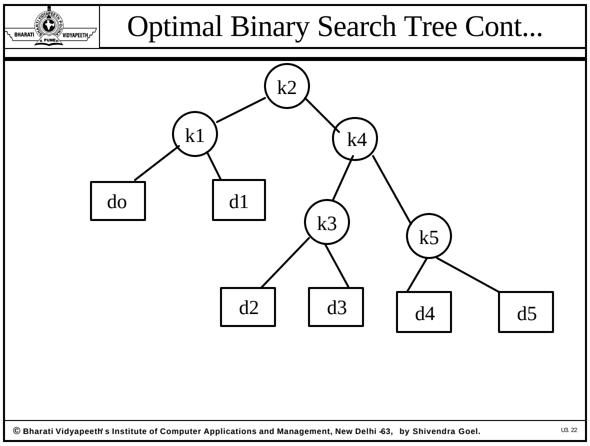
A4=20X15

Optimal Binary Search Tree

- In representation of BST there are various variation in representing a BST with that much number of nodes, Optimal BST is the one which has lowest search cost.

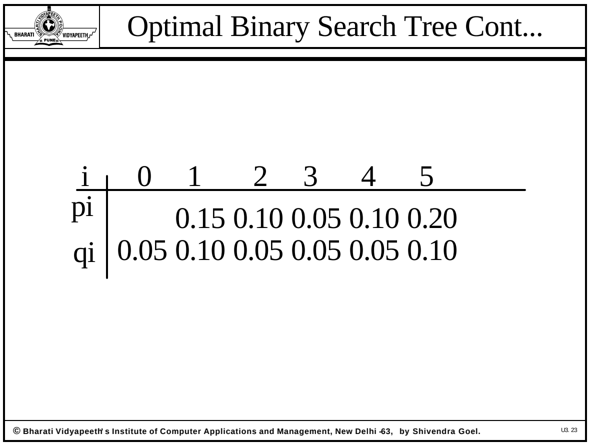
Optimal Binary Search Tree Cont...

? pi+ ? qi=1
search cost ki=(dept+1)*pi
search cost di=(dept+1)*qi
total search cost= search cost ? ki + search cost ? di



Optimal Binary Search Tree Cont...

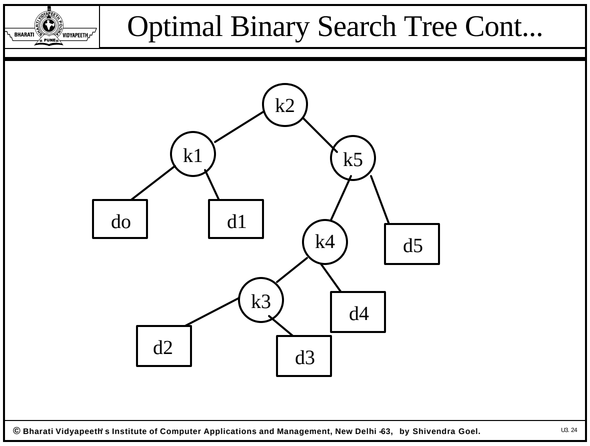
© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 22



Optimal Binary Search Tree Cont...

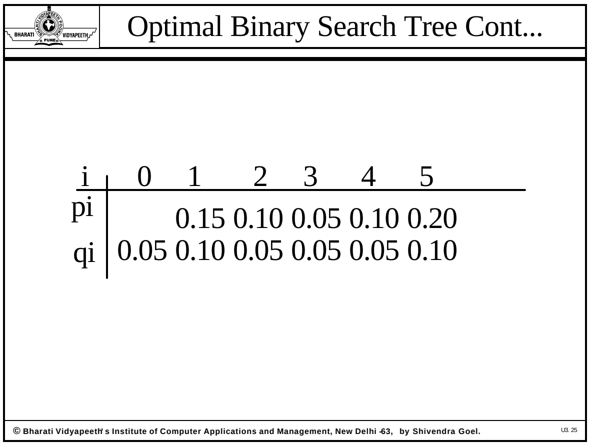
i	0	1	2	3	4	5
pi		0.15	0.10	0.05	0.10	0.20
qi	0.05	0.10	0.05	0.05	0.05	0.10

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 23



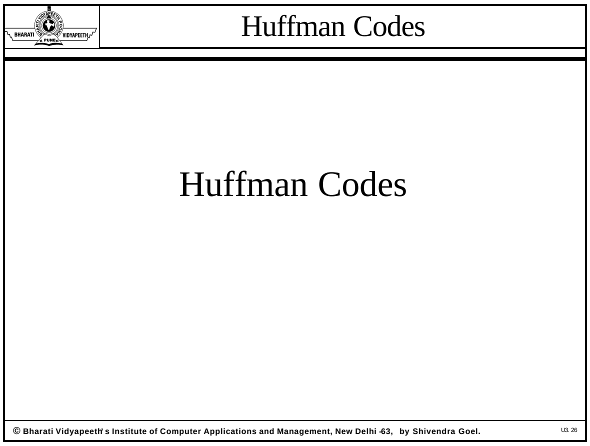
Optimal Binary Search Tree Cont...

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 24



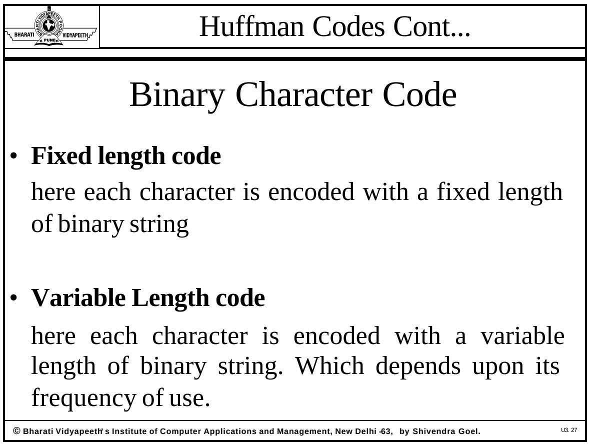
Optimal Binary Search Tree Cont...

i	0	1	2	3	4	5
pi		0.15	0.10	0.05	0.10	0.20
qi	0.05	0.10	0.05	0.05	0.05	0.10



Huffman Codes

Huffman Codes



Huffman Codes Cont...

Binary Character Code

- **Fixed length code**

  here each character is encoded with a fixed length of binary string

- **Variable Length code**

  here each character is encoded with a variable length of binary string. Which depends upon its frequency of use.

Huffman Codes Cont...

Example

- Suppose we have a 100 character data file that we wish to store compactly. We observe that the character in the file occur with the frequencies as

  F-5, E-9, C-12, B-13, D-16, A-45

- Now Using Fix Length encoding we have to take 3 bits for storing a single character so these 100 characters require 300 bits of storage

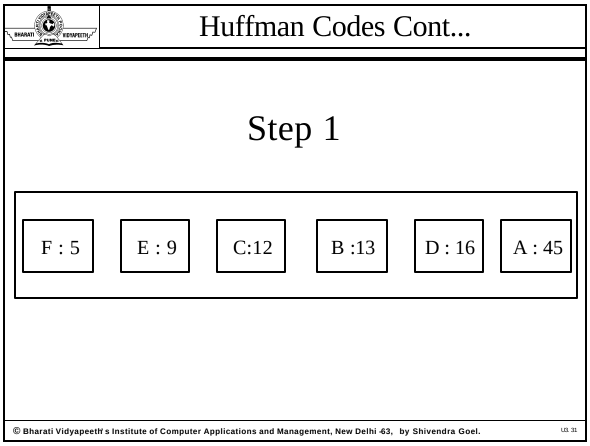
Huffman Codes Cont...

- But if we use a variable length code such as a requires 1 bit storage , c, b, d requires 3 bits and f, e requires 4 bit each then total storage space required is

  1*45 + 12*3 + 13*3 + 16* 3 + 5*4 + 9*4 = 224

Huffman Codes Cont...

Letter →	A	B	C	D	E	F
Frequency	45	13	12	16	9	5
FLC	000	001	010	011	100	101
VLC	0	101	100	111	1101	1100

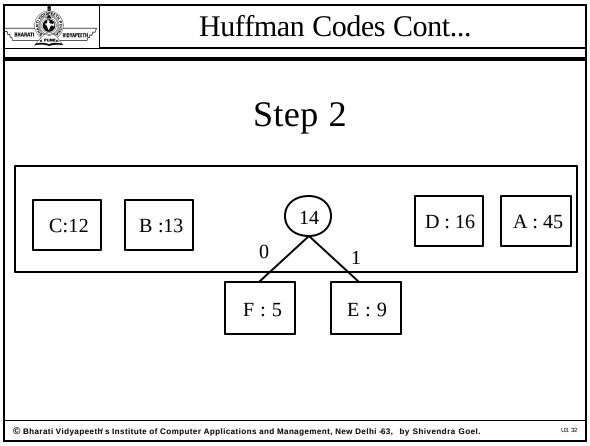


Huffman Codes Cont...

Step 1

F : 5 E : 9 C:12 B :13 D : 16 A : 45

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 31



Huffman Codes Cont...

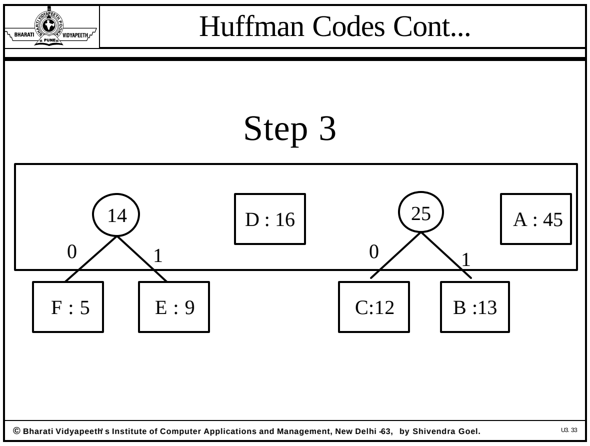
Step 2

C:12 B :13 14 D : 16 A : 45

0 1

F : 5 E : 9

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 32



Huffman Codes Cont...

Step 3

14 D : 16 25 A : 45

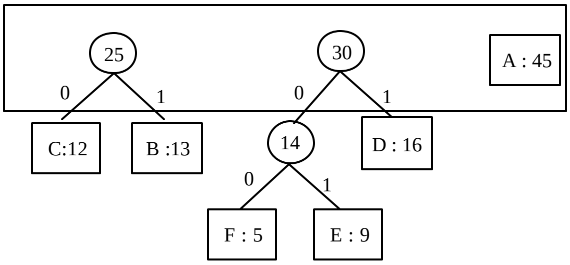
0 1 0 1

F : 5 E : 9 C:12 B :13

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 33

Huffman Codes Cont...

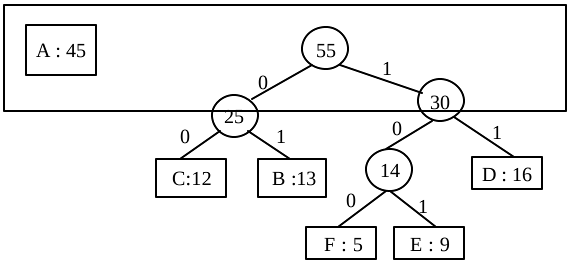
Step 4



© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 34

Huffman Codes Cont...

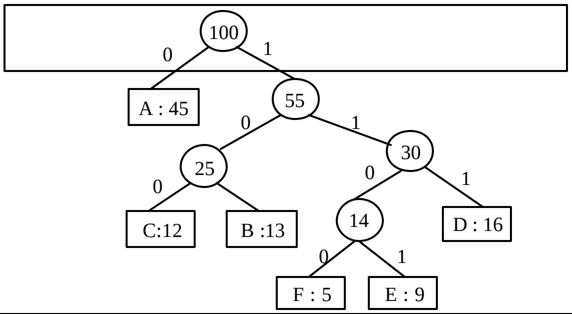
Step 5



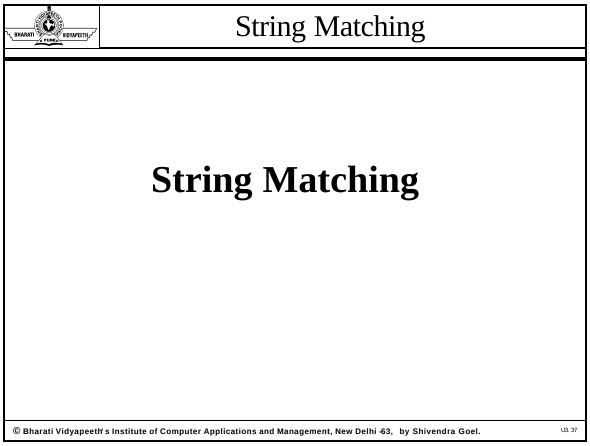
© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 35

Huffman Codes Cont...

Step 6

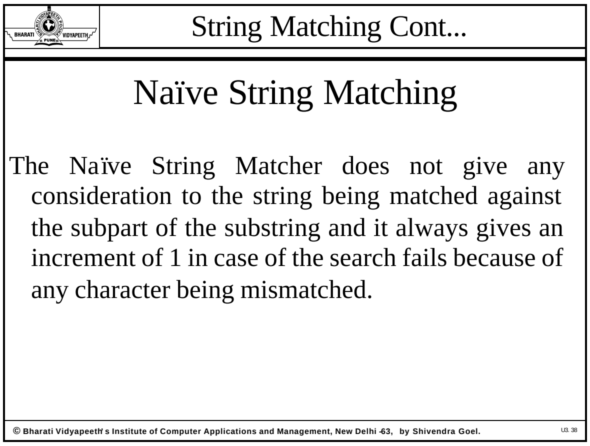


© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 36



String Matching

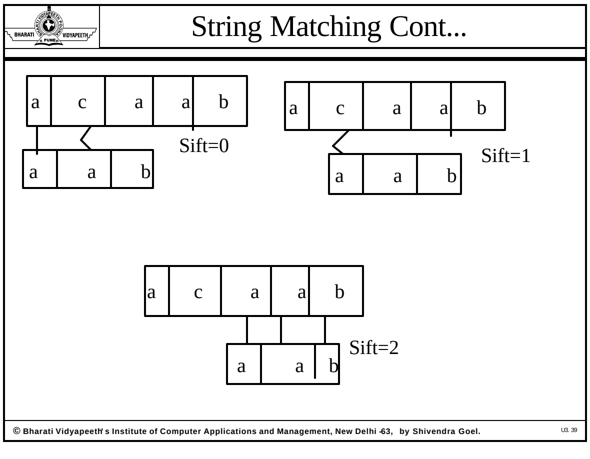
String Matching



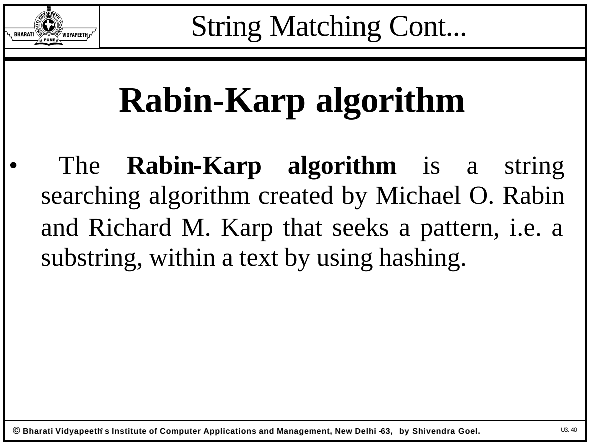
String Matching Cont...

Naïve String Matching

The Naïve String Matcher does not give any consideration to the string being matched against the subpart of the substring and it always gives an increment of 1 in case of the search fails because of any character being mismatched.



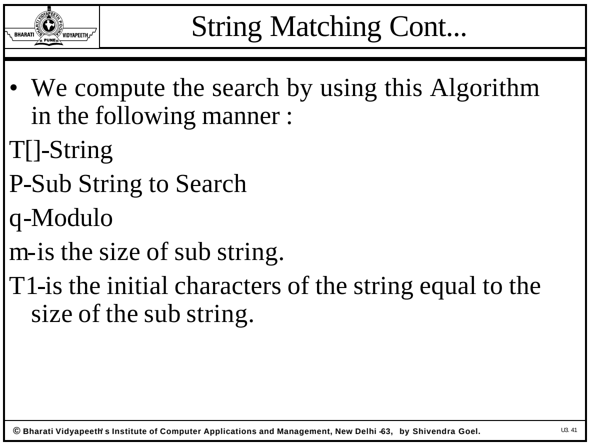
String Matching Cont...



String Matching Cont...

Rabin-Karp algorithm

- The **Rabin-Karp algorithm** is a string searching algorithm created by Michael O. Rabin and Richard M. Karp that seeks a pattern, i.e. a substring, within a text by using hashing.



String Matching Cont...

- We compute the search by using this Algorithm in the following manner :

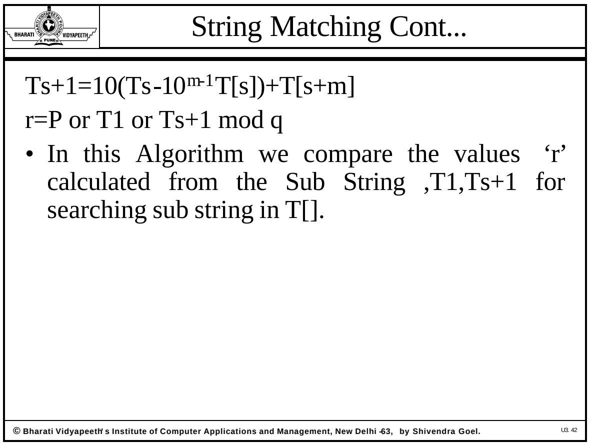
T[]-String

P-Sub String to Search

q-Modulo

m-is the size of sub string.

T1-is the initial characters of the string equal to the size of the sub string.



String Matching Cont...

$Ts+1=10(Ts-10^{m-1}T[s])+T[s+m]$

r=P or T1 or Ts+1 mod q

- In this Algorithm we compare the values 'r' calculated from the Sub String ,T1,Ts+1 for searching sub string in T[].

String Matching Cont...

Q. Find the String match in the following string

T=14321542

Where q=5 and

Substring to be match is 154

Using Rabin-Karp methodology.

String Matching Cont...

String Matching With
Finite Automata

String Matching Cont...

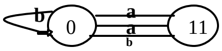
FINITE AUTOMATA

String Matching Cont...

Let us take an Example

- A simple 2-state finite automata
- State set Q = {0,1}
- Start state q0 = 0
- Input alphabet ？ = {a,b}

String Matching Cont...



State Transition Diagram

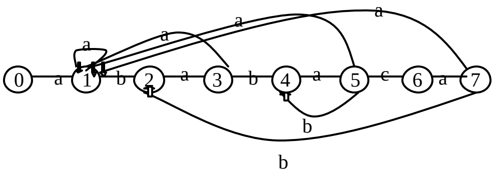
	input	
state	a	b
0	1	0
1	0	0

String Matching Cont...

String - Matching Automata

String Matching Cont...

Sub String: ababababaca



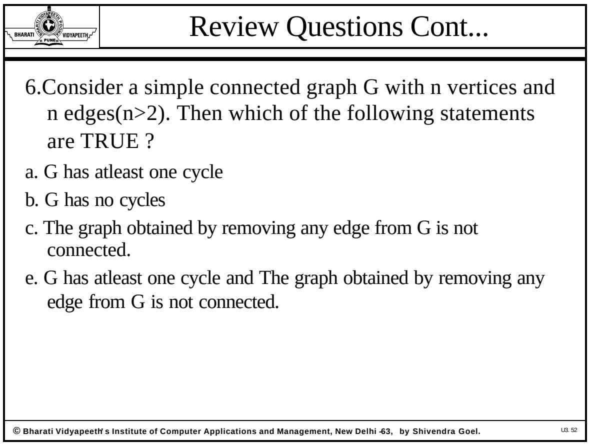
State Transition Diagram

Conclusion

- Graphs can be used to Courses in a university-list courses such that pre-requisites appear before the course itself
- Applications of Shortest Path
  - Finding shortest route from location X to location Y
  - Finding communication paths with least cost in a computer network
  - Finding transit paths with least cost in a railway network

Review Questions

1. ___________ algorithm is used to detect the presence of negative edge cycle in the given graph.
2. _____________ and __________ are the algorithms for finding Minimum cost Spanning Trees.
3. __________ and __________ Algorithm for finding Single source shortest paths
4. Graph is a _____________Data Structure
5. ____________algorithm is used for finding all pair shortest paths



Review Questions Cont...

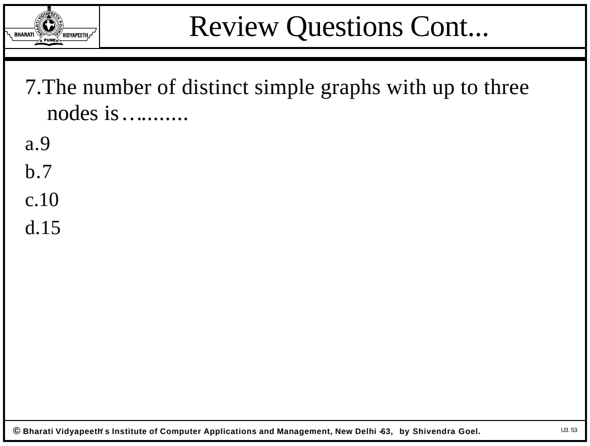
6.Consider a simple connected graph G with n vertices and n edges(n>2). Then which of the following statements are TRUE ?

a. G has atleast one cycle

b. G has no cycles

c. The graph obtained by removing any edge from G is not connected.

e. G has atleast one cycle and The graph obtained by removing any edge from G is not connected.



Review Questions Cont...

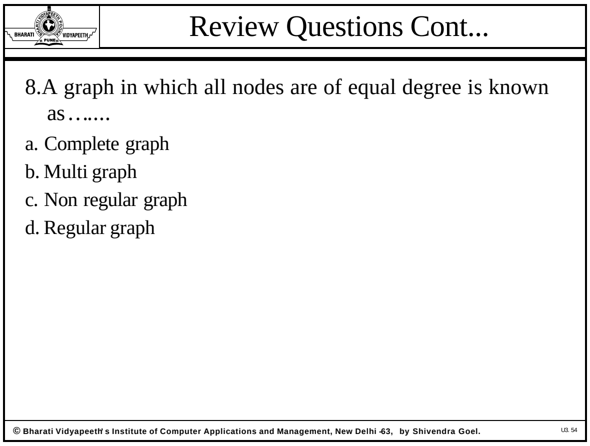
7.The number of distinct simple graphs with up to three nodes is…........

a.9

b.7

c.10

d.15



Review Questions Cont...

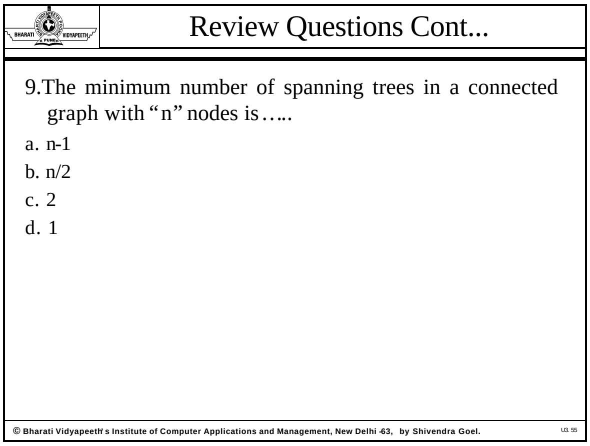
8.A graph in which all nodes are of equal degree is known as…....

a. Complete graph

b. Multi graph

c. Non regular graph

d. Regular graph



Review Questions Cont...

9.The minimum number of spanning trees in a connected graph with "n" nodes is…...

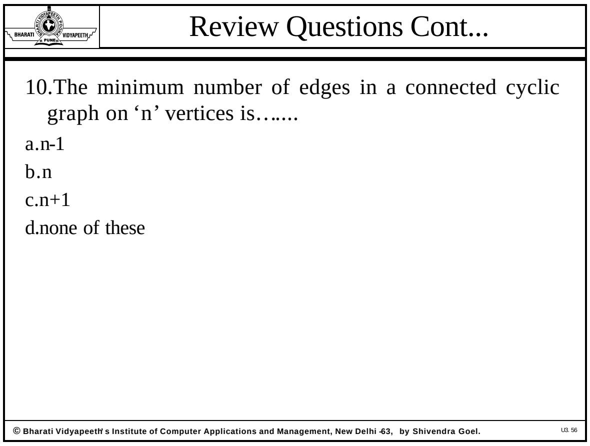
a. n-1

b. n/2

c. 2

d. 1

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 55



Review Questions Cont...

10.The minimum number of edges in a connected cyclic graph on 'n' vertices is…....

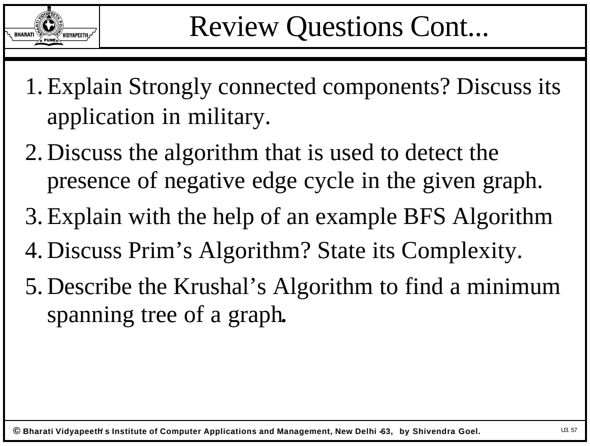
a.n-1

b.n

c.n+1

d.none of these

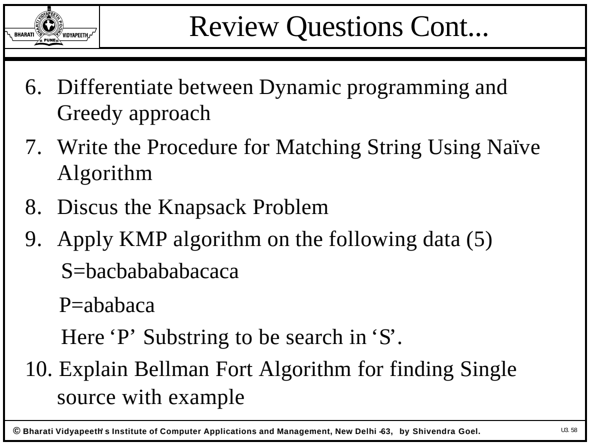
© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 56



Review Questions Cont...

1. Explain Strongly connected components? Discuss its application in military.
2. Discuss the algorithm that is used to detect the presence of negative edge cycle in the given graph.
3. Explain with the help of an example BFS Algorithm
4. Discuss Prim's Algorithm? State its Complexity.
5. Describe the Krushal's Algorithm to find a minimum spanning tree of a graph.

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 57



Review Questions Cont...

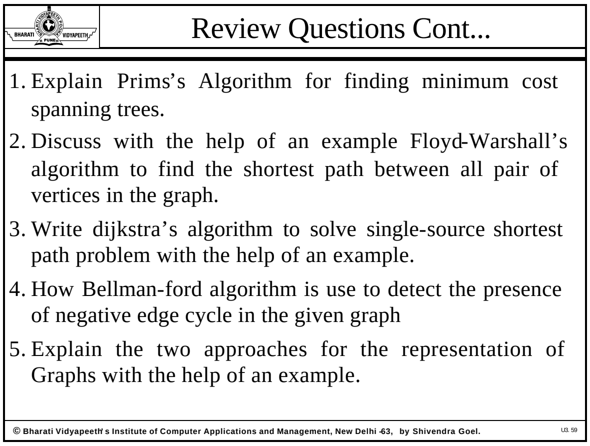
6. Differentiate between Dynamic programming and Greedy approach
7. Write the Procedure for Matching String Using Naïve Algorithm
8. Discus the Knapsack Problem
9. Apply KMP algorithm on the following data (5)

   S=bacbabababacaca

   P=ababaca

   Here 'P' Substring to be search in 'S'.
10. Explain Bellman Fort Algorithm for finding Single source with example

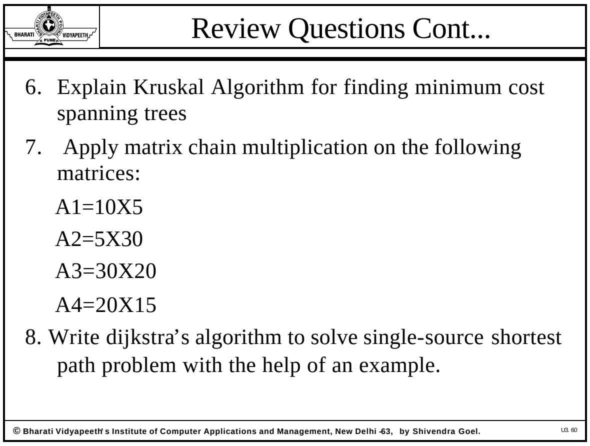
© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 58



Review Questions Cont...

1. Explain Prims's Algorithm for finding minimum cost spanning trees.
2. Discuss with the help of an example Floyd-Warshall's algorithm to find the shortest path between all pair of vertices in the graph.
3. Write dijkstra's algorithm to solve single-source shortest path problem with the help of an example.
4. How Bellman-ford algorithm is use to detect the presence of negative edge cycle in the given graph
5. Explain the two approaches for the representation of Graphs with the help of an example.

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 59



Review Questions Cont...

6. Explain Kruskal Algorithm for finding minimum cost spanning trees
7. Apply matrix chain multiplication on the following matrices:

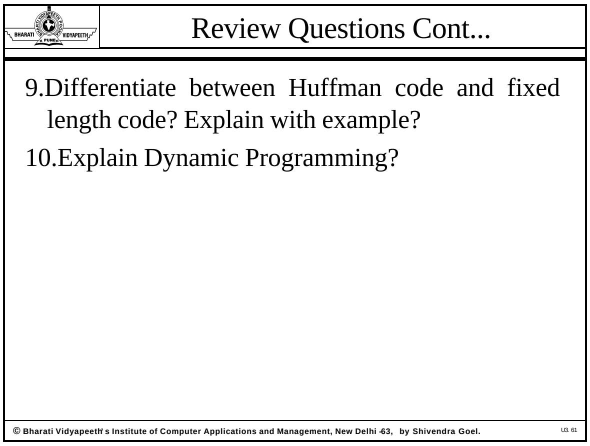
   A1=10X5

   A2=5X30

   A3=30X20

   A4=20X15
8. Write dijkstra's algorithm to solve single-source shortest path problem with the help of an example.

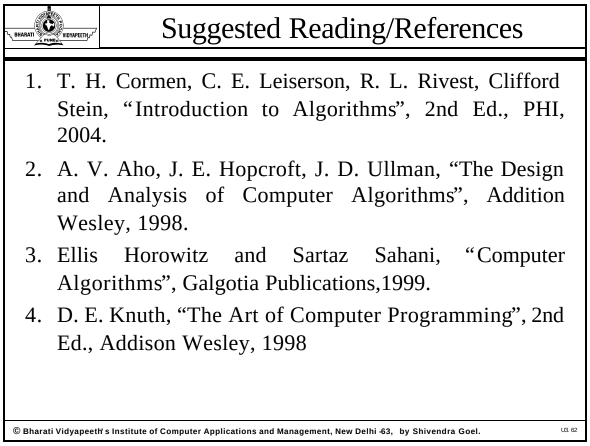
© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 60



Review Questions Cont...

9. Differentiate between Huffman code and fixed length code? Explain with example?
10. Explain Dynamic Programming?

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 61



Suggested Reading/References

1. T. H. Cormen, C. E. Leiserson, R. L. Rivest, Clifford Stein, "Introduction to Algorithms", 2nd Ed., PHI, 2004.
2. A. V. Aho, J. E. Hopcroft, J. D. Ullman, "The Design and Analysis of Computer Algorithms", Addition Wesley, 1998.
3. Ellis Horowitz and Sartaz Sahani, "Computer Algorithms", Galgotia Publications,1999.
4. D. E. Knuth, "The Art of Computer Programming", 2nd Ed., Addison Wesley, 1998

© Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi-63, by Shivendra Goel. U3. 62