I have put the term $2\chi_-(u_- - u_+)W_+$ into the right hand side (into $\mathcal{R}_+$) of equation $W_+' - 2u_+W_+ = \mathcal{R}_+$, based on our estimate $(1 < \eta < 2)$,

$$W_+ \sim (T_\infty - t)^{-\eta}, \quad \mathcal{R}_+ \sim (T_\infty - t)^{-(\eta+1)}$$

So for $\chi_-(u_- - u_+)W_+$ to be small we want

$$|\chi_-(u_- - u_+)(T_\infty - t)|_\infty$$

to be small.

I know that $(u_- - u_+)(t) = o(\frac{1}{T_\infty - t})$ as $t \to t_*$ since $u_+(t)$ is the leading order in the asymptotics of u_-, however I think this only hold true for t near the gluing point t_*, for t on the far left, for example when $t = 0$, comparing $u_-(t)$ and $u_+(t)$ gives

$$u_-(0) = \varepsilon^{\frac{1}{3}} u_R(-\varepsilon^{-\frac{2}{3}}\delta) \sim -\sqrt{\delta}$$

and

$$u_+(0) \sim T_\infty^{-1} = (\varepsilon^{-1}\delta + \varepsilon^{-\frac{1}{3}}\Omega_0)^{-1}$$

which is of different order in ε,