Task 1:

a. Because A have 8 values, and $B_1, \cdots, B_{10}$ have 5 possible values.

$\therefore$ the number of the joint distribution is $8^1 \cdot 5^{10} - 1 = 78,124,999$

(these parameters need to sum to one, so we can drop one parameter.)

b. Because each B_i is conditionally independent of all other 9 B_j variables given A.

And the use of conditional independence reduce the size of the representation of the joint distribution from exponential in n to linear in n.

$\therefore P(B_j|A, B_i) = P(B_j|A) \Rightarrow P(A, B_1, B_2, \cdots, B_{10}) = P(A) \cdot P(B_1|A) \cdot P(B_2|A) \cdots P(B_{10}|A)$

After saving space via conditional independence relations, we need $8 \cdot 10 \cdot 5 - 1 = 399$ numbers.

c. Yes, because given A, then each B has the property of conditionally independent. We obtain

$P(B_j|A, B_i) = P(B_j|A) \Rightarrow P(A, B_1, B_2, \cdots, B_{10}) = P(A) \cdot P(B_1|A) \cdot P(B_2|A) \cdots P(B_{10}|A)$

$= P(A) \cdot \prod_i P(B_i|A)$, which is a naive bayes model.

Task 2:

a. 0. $p_0(h_1) = 0.1$, $p_0(h_2) = 0.2$, $p_0(h_3) = 0.4$, $p_0(h_4) = 0.2$, $p_0(h_5) = 0.1$

$p_0(q_1 = C) = \sum_{i=1}^{5} (P(q_1 = C|h_0) \cdot P(h_i) = 1 \cdot 0.1 + 0.75 \cdot 0.2 + 0.4 \cdot 0.5 + 0.2 \cdot 0.25 + 0 = 0.5$

C 1. $p_1(h_1) = \frac{P(q_1 = C|h_1) \cdot P_0(h_1)}{P_0(q_1 = C)} = \frac{1 \cdot 0.1}{0.5} = 0.2$ $p_1(h_2) = \frac{0.75 \cdot 0.2}{0.5} = 0.3$

$p_1(h_3) = \frac{0.5 \cdot 0.4}{0.5} = 0.4$ $p_1(h_4) = \frac{0.25 \cdot 0.2}{0.5} = 0.1$ $p_1(h_5) = 0$

C $p_1(q_2 = C) = 1 \cdot 0.2 + 0.75 \cdot 0.3 + 0.5 \cdot 0.4 + 0.25 \cdot 0.1 = 0.65$

2. $p_2(h_1) = \frac{1 \cdot 0.2}{0.65} = 0.308$ $p_2(h_2) = \frac{0.75 \cdot 0.3}{0.65} = 0.346$ $p_2(h_3) = \frac{0.5 \cdot 0.4}{0.65} = 0.308$

$p_2(h_4) = 0.25 \cdot 0.1 / 0.65 = 0.038$ $p_2(h_5) = 0$

C $p_2(q_3 = C) = 1 \cdot 0.308 + 0.75 \cdot 0.346 + 0.5 \cdot 0.308 + 0.25 \cdot 0.038 + 0 = 0.731$

3. $p_3(h_1) = 1 \cdot 0.308 / 0.731 = 0.421$ $p_3(h_2) = 0.75 \cdot 0.346 / 0.731 = 0.355$

$p_3(h_3) = 0.5 \cdot 0.308 / 0.731 = 0.211$ $p_3(h_4) = 0.25 \cdot 0.038 / 0.731 = 0.013$ $p_3(h_5) = 0$

$p_3(q_4 = C) = 1 \cdot 0.421 + 0.75 \cdot 0.355 + 0.5 \cdot 0.211 + 0.25 \cdot 0.013 = 0.796.$

C 4. $p_4(h_1) = 1 \cdot 0.421 / 0.796 = 0.529$ $p_4(h_2) = 0.75 \cdot 0.355 / 0.796 = 0.334$

$p_4(h_3) = 0.5 \cdot 0.241 / 0.796 = 0.133$ $p_4(h_4) = 0.25 \cdot 0.013 / 0.796 = 0.004$ $p_4(h_5) = 0$

$p_4(q_5 = C) = 1 \cdot 0.529 + 0.75 \cdot 0.334 + 0.5 \cdot 0.133 + 0.25 \cdot 0.004 = 0.847$

C 5. $p_5(h_1) = 1 \cdot 0.529 / 0.847 = 0.625$ $p_5(h_2) = 0.75 \cdot 0.334 / 0.847 = 0.296$ $p_5(h_3) = 0.5 \cdot 0.133 / 0.847 = 0.079$

$p_5(h_4) = 0.25 \cdot 0.013 / 0.847 = 0.004$ $p_5(h_5) = 0$

$p_5(q_6 = C) = 1 \cdot 0.625 + 0.75 \cdot 0.296 + 0.5 \cdot 0.079 + 0.25 \cdot 0.004 = 0.8875$

$\therefore p_5(q_6 = L) = 0 \cdot 0.625 + 0.25 \cdot 0.296 + 0.5 \cdot 0.079 + 0.75 \cdot 0.004 + 0 = 0.1165$

L 6. $p_6(h_1) = 0$ $p_6(h_2) = 0.25 \cdot 0.296 / 0.1165 = 0.635$ $p_6(h_3) = 0.5 \cdot 0.079 / 0.1165 = 0.339$

$p_6(h_4) = 0.75 \cdot 0.004 / 0.1165 = 0.026$ $p_6(h_5) = 0$

a. 0. $p_0(h_1) = 0.1$, $p_0(h_2) = 0.2$, $p_0(h_3) = 0.4$, $p_0(h_4) = 0.2$, $p_0(h_5) = 0.1$

$p_0(q_1 = C) = \sum_{i=1}^{5} (p(q_1 = C | h_0) \cdot P(h_i) = 1 \cdot 0.1 + 0.75 \cdot 0.2 + 0.4 \cdot 0.5 + 0.2 \cdot 0.25 + 0 = 0.5$

C 1. $p_1(h_1) = \dfrac{P(q_1 = C | h_1) \cdot P_0(h_1)}{P_0(q_1 = C)} = \dfrac{1 \cdot 0.1}{0.5} = 0.2$ $p_1(h_2) = \dfrac{0.75 \cdot 0.2}{0.5} = 0.3$

$p_1(h_3) = \dfrac{0.5 \cdot 0.4}{0.5} = 0.4$ $p_1(h_4) = \dfrac{0.25 \cdot 0.2}{0.5} = 0.1$ $p_1(h_5) = 0$

$p_1(q_2 = L) = 0 \cdot 0.2 + 0.25 \cdot 0.3 + 0.5 \cdot 0.4 + 0.75 \cdot 0.1 + 1 \cdot 0 = 0.35$

L 2. $p_2(h_1) = \dfrac{0 \cdot 0.2}{0.35} = 0$ $p_2(h_2) = \dfrac{0.25 \cdot 0.3}{0.35} = 0.214$ $p_2(h_3) = \dfrac{0.5 \cdot 0.4}{0.35} = 0.571$

$p_2(h_4) = 0.75 \cdot 0.1 / 0.35 = 0.214$ $p_2(h_5) = 0$

$p_2(q_3 = C) = 1.0 + 0.75 \cdot 0.214 + 0.5 \cdot 0.571 + 0.25 \cdot 0.214 + 0 = 0.5$

C 3. $p_3(h_1) = 1 \cdot 0 / 0.5 = 0$ $p_3(h_2) = 0.75 \cdot 0.214 / 0.5 = 0.321$

$p_3(h_3) = 0.5 \cdot 0.571 / 0.5 = 0.571$ $p_3(h_4) = 0.25 \cdot 0.214 / 0.5 = 0.107$ $p_3(h_5) = 0$

$p_3(q_4 = L) = 0.0 + 0.25 \cdot 0.321 + 0.5 \cdot 0.571 + 0.75 \cdot 0.107 + 1 \cdot 0 = 0.446$

L 4. $p_4(h_1) = 0.0 / 0.446 = 0$ $p_4(h_2) = 0.25 \cdot 0.321 / 0.446 = 0.18$

$p_4(h_3) = 0.5 \cdot 0.571 / 0.446 = 0.64$ $p_4(h_4) = 0.75 \cdot 0.107 / 0.446 = 0.18$ $p_4(h_5) = 0$

$p_4(q_5 = C) = 1.0 + 0.75 \cdot 0.18 + 0.5 \cdot 0.64 + 0.25 \cdot 0.18 = 0.5$

C 5. $p_5(h_1) = 1.0/0.5 = 0$ $p_5(h_2) = 0.75 \cdot 0.18/0.5 = 0.27$ $p_5(h_3) = 0.5 \cdot 0.14/0.5 = 0.64$

$p_5(h_4) = 0.25 \cdot 0.18/0.5 = 0.09$ $p_5(h_5) = 0$

$\therefore p_5(q_6 = L) = 0.0 + 0.25 \cdot 0.27 + 0.5 \cdot 0.64 + 0.75 \cdot 0.09 + 0 = 0.455$

L 6. $p_6(h_1) = 0$ $p_6(h_2) = 0.25 \cdot 0.27/0.455 = 0.148$ $p_6(h_3) = 0.5 \cdot 0.64/0.455 = 0.703$

$p_6(h_4) = 0.75 \cdot 0.09/0.455 = 0.148$ $p_6(h_5) = 0$

C. 0. $p_0(h_1) = 0.1$, $p_0(h_2) = 0.2$, $p_0(h_3) = 0.4$, $p_0(h_4) = 0.2$, $p_0(h_5) = 0.1$

$p_0(q_1 = C) = \sum_{i=1}^{5} (p(q_1 = C|h_0) \cdot P(h_i) = 1 \cdot 0.1 + 0.75 \cdot 0.2 + 0.4 \cdot 0.5 + 0.2 \cdot 0.25 + 0 = 0.5$

C 1. $p_1(h_1) = \frac{P(q_1 = C|h_1) \cdot P_0(h_1)}{P_0(q_1 = C)} = \frac{1 \cdot 0.1}{0.5} = 0.2$ $p_1(h_2) = \frac{0.75 \cdot 0.2}{0.5} = 0.3$

$p_1(h_3) = \frac{0.5 \cdot 0.4}{0.5} = 0.4$ $p_1(h_4) = \frac{0.25 \cdot 0.2}{0.5} = 0.1$ $p_1(h_5) = 0$

$p_1(q_2 = C) = 1 \cdot 0.2 + 0.75 \cdot 0.3 + 0.5 \cdot 0.4 + 0.25 \cdot 0.1 = 0.65$

C 2. $p_2(h_1) = \frac{1 \cdot 0.2}{0.65} = 0.308$ $p_2(h_2) = \frac{0.75 \cdot 0.3}{0.65} = 0.346$ $p_2(h_3) = \frac{0.5 \cdot 0.4}{0.65} = 0.308$

$p_2(h_4) = 0.25 \cdot 0.1/0.65 = 0.038$ $p_2(h_5) = 0$

$p_2(q_3 = C) = 1 \cdot 0.308 + 0.75 \cdot 0.346 + 0.5 \cdot 0.308 + 0.25 \cdot 0.038 + 0 = 0.731$

C 3. $p_3(h_1) = 1 \cdot 0.308/0.731 = 0.421$ $p_3(h_2) = 0.75 \cdot 0.346/0.731 = 0.355$

$p_3(h_3) = 0.5 \cdot 0.308/0.731 = 0.211$ $p_3(h_4) = 0.25 \cdot 0.038/0.731 = 0.013$ $p_3(h_5) = 0$

$p_3(q_4 = L) = 0 \cdot 0.421 + 0.25 \cdot 0.355 + 0.5 \cdot 0.211 + 0.75 \cdot 0.013 + 1 \cdot 0 = 0.204$

L 4. $p_4(h_1) = 0 \cdot 0.421/0.204 = 0$ $p_4(h_2) = 0.25 \cdot 0.355/0.204 = 0.435$

$p_4(h_3) = 0.5 \cdot 0.211/0.204 = 0.517$ $p_4(h_4) = 0.75 \cdot 0.013/0.204 = 0.048$ $p_4(h_5) = 0$

$p_4(q_5 = L) = 0 \cdot 0 + 0.25 \cdot 0.435 + 0.5 \cdot 0.517 + 0.75 \cdot 0.048 + 0 \cdot 0 = 0.403$

L 5. $p_5(h_1) = 0 \cdot 0/0.403 = 0$ $p_5(h_2) = 0.25 \cdot 0.435/0.403 = 0.270$ $p_5(h_3) = 0.5 \cdot 0.517/0.403 = 0.641$

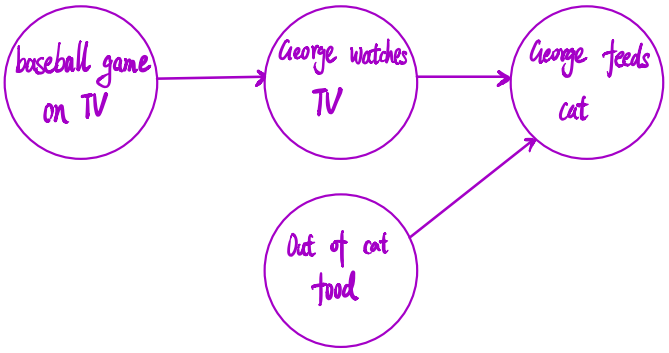
$p_5(h_4) = 0.75 \cdot 0.048/0.403 = 0.089$ $p_5(h_5) = 0$

$\therefore P_5(q_6=L) = 0.0 + 0.25 \cdot 0.270 + 0.5 \cdot 0.641 + 0.75 \cdot 0.089 + 0 = 0.455$

6. $P_6(h_1) = 0$ $P_6(h_2) = {}^{0.25 \cdot 0.270}/_{0.455} = 0.148$ $P_6(h_3) = {}^{0.5 \cdot 0.641}/_{0.455} = 0.704$

$P_6(h_4) = {}^{0.75 \cdot 0.089}/_{0.455} = 0.146$ $P_6(h_5) = 0$

Task 3:



Task 4:

P(baseball_game_on_TV)
0.30419

: $\frac{111}{365}$

P(out_of_cat_food)
0.16986

: $\frac{62}{365}$

basket game on TV	P(George_watches_TV)	
T	0.92793	: $\frac{103}{111}$
F	0.11811	: $\frac{30}{254}$

out_of_cat_food	George_watches_TV	P(George_feeds_cat)	
T	T	0.04167	24/
T	F	0.31579	38/
F	T	0.70642	109/
F	F	0.95876	194/

```
The probability of P(baseball_game_on_TV) is 0.304109589041
(62L, 365.0)
The probability of P(out_of_cat_food) is 0.169863013699
The probability of P(Geogrge_watches_TV|basket_game_on_TV) is 0.927927927928
The probability of P(Geogrge_watches_TV|not(basket_game_on_TV)) is 0.405511811024
The probability of P(Geogrge_feeds_cat|out_of_cat_food, Geogrge_watches_TV) is 0.0416666666667
The probability of P(Geogrge_feeds_cat|out_of_cat_food, not(Geogrge_watches_TV)) is 0.315789473684
The probability of P(Geogrge_feeds_cat|not(out_of_cat_food), Geogrge_watches_TV) is 0.706422018349
The probability of P(Geogrge_feeds_cat|not(out_of_cat_food), not(Geogrge_watches_TV)) is 0.958762886598
```

```python
import pandas as pd
data = pd.read_csv('./training_data.txt', names=['game', 'watch', 'wtoFood', 'feed'], sep='     ')

denom1 = float(data.shape[0])
pGame = data['game'].sum() / denom1
pWtoFood = data['wtoFood'].sum() / denom1
print(data['game'].sum(), denom1) # 111, 365
print('The probability of P(baseball_game_on_TV) is {}'.format(pGame)) # 0.30410
print(data['wtoFood'].sum(), denom1) # 62, 365
print('The probability of P(out_of_cat_food) is {}'.format(pWtoFood)) # 0.16986

denom2t = float(data[(data['game']==1)].shape[0]) # 111
denom2f = float(data[(data['game']==0)].shape[0]) # 254
watch = data[(data['game']>0)&(data['watch']>0)].sum()['watch'] # 103
pWatchTrue = watch / denom2t # 103/111
pWatchFalse = watch / denom2f # 103/254
print('The probability of P(Geogrge_watches_TV|basket_game_on_TV) is {}'.format(pWatchTrue))
print('The probability of P(Geogrge_watches_TV|not(basket_game_on_TV)) is {}'.format(pWatchFalse))

denom3tt = float(data[(data['wtoFood']==1)&(data['watch']==1)].shape[0]) # 24
denom3tf = float(data[(data['wtoFood']==1)&(data['watch']==0)].shape[0]) # 38
denom3ft = float(data[(data['wtoFood']==0)&(data['watch']==1)].shape[0]) # 109
denom3ff = float(data[(data['wtoFood']==0)&(data['watch']==0)].shape[0]) # 194
feedTT = data[(data['wtoFood']==1)&(data['watch']==1)&(data['feed']==1)].sum()['feed'] # 1
feedTF = data[(data['wtoFood']==1)&(data['watch']==0)&(data['feed']==1)].sum()['feed'] # 12
feedFT = data[(data['wtoFood']==0)&(data['watch']==1)&(data['feed']==1)].sum()['feed'] # 77
feedFF = data[(data['wtoFood']==0)&(data['watch']==0)&(data['feed']==1)].sum()['feed'] # 186
pFeedTT = feedTT / denom3tt
pFeedTF = feedTF / denom3tf
pFeedFT = feedFT / denom3ft
pFeedFF = feedFF / denom3ff
print('The probability of P(Geogrge_feeds_cat|out_of_cat_food, Geogrge_watches_TV) is {}'.format(pFeedTT))
print('The probability of P(Geogrge_feeds_cat|out_of_cat_food, not(Geogrge_watches_TV)) is {}'.format(pFeedTF))
print('The probability of P(Geogrge_feeds_cat|not(out_of_cat_food), Geogrge_watches_TV) is {}'.format(pFeedFT))
print('The probability of P(Geogrge_feeds_cat|not(out_of_cat_food), not(Geogrge_watches_TV)) is {}'.format(pFeedFF))
```

Task 5:

Let P(not(George Feeds Cat) | Baseball Game on TV), take 'g', 'w', 'wto', 'f' as abbreviation. We obtain:

$$P(\neg f|g) = P(\neg f, g)/P(g)$$

$$= \frac{count(\neg f, g)}{count(days)} \Big/ \frac{count(g)}{count(days)} = \frac{45}{365} \Big/ \frac{111}{365} = \frac{0.12328}{0.30411} = 0.40541$$

Task 6:

a. Markovian Blanket of node L is: G, P, Q, K, M.

(parents + children + children's parents)

b. $\because P(C) = 0.6,\ P(H|C) = 0.6,\ P(C,H) = P(H|C) \cdot P(C)$

$\therefore P(C,H) = 0.6 \cdot 0.6 = 0.36$

c. $P(O \mid \text{not}(J), E) = \frac{P(O, \text{not}(J), E)}{P(\text{not}(J), E)} = \frac{P(O \mid \text{not}(J)) \cdot P(\text{not}(J) \mid E) \cdot P(E)}{P(E) \cdot P(\text{not}(J) \mid E)}$

$= P(O \mid \text{not}(J)) = 0.8$