Building a 3D printed instrument for measuring surface tension
Project Plan

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1 Introduction

Surface tension is a measure of the energy required to form a unit of area between two fluids. Measurement of surface tension is critical to many applications in industry and commercial products [1–3]. Consequently, accurate measurement of surface tension is of critical importance and there are many methods of measuring the surface tension including the Wilhelmy balance, the pendant drop, and the Langmuir trough. A figure of a few of these methods is shown in Fig. 1. These in general fall into force-based methods and shape-based methods. Each method has its advantages and disadvantages for different systems of interest, but overall these methods have an accuracy of $\sim 0.1mNm^{-1}$[4]. A detailed analysis of surface tension can be found at [5].

The fundamental idea behind this project is to build and test a new cheap way and accessible way to measure the surface tension of liquids. To do this we will focus on the apparatus originally proposed by Moleai and Crocker[6]. This used 3D-printing techniques and an inverted microscope to image the curved meniscus generated by a tilted disk and and use the shape of this interface to calculate the surface tension of the fluid interface. Since the original publication, several improvements have been proposed to the apparatus and methodology. Through this project, we aim to explore some of these improvements and demonstrate that the original results can be replicated and ideally improved upon.

2 Theory

Here we follow a discussion adapted from [7] with a detailed analysis conducted in [5]. We have defined surface tension as a measure of the energy required to form a unit of area between two fluids. This can also be expressed in terms of the force to change the area of the interface. Therefore methods to measure the surface tension can attempt to measure the force required to change the area, this principle is used in Du Nouy ring experiments. An alternative is the shape of the interface can be measured because the interface shape is determined by the minimisation of energy this means surface tension can be calculated if the forces on the interface are known.

For a static surface the balance of forces at the surface leads to the Laplace-Young equation:

$$\Delta p = \gamma \nabla \cdot \mathbf{n} \tag{1}$$

where p is the pressure difference, γ is the surface tension and $\mathbf{n}$ is the surface normal. In the presence of a gravitational field $\Delta p = \Delta \rho g z$. Here $\Delta \rho$ is the difference in densities of the two fluids. If we define a function $f(r, \theta) = z - h(r, \theta)$ that vanishes on the surface then:

$$\mathbf{n} = \frac{\nabla \mathbf{f}}{|\nabla \mathbf{f}|} \tag{2}$$

The analytic form of $\nabla \cdot \mathbf{n}$ in terms of $h(r, \theta)$ is complicated, nevertheless an equation can be derived for the surface if the boundary conditions are known[5]. This is often difficult or impossible to do analytically in 2D and in general must be solved numerically. This is why all of the current methods of measuring surface tension employ highly symmetric fluid boundaries. The analysis discussed here is what is employed in many of the shape-based methods such as the pendant drop and is the basis of this project.

This can be further developed into the field of interfacial rheology. Which involves the investigation of anisotropic surface tension as well as shear properties of surfaces. These properties will not be the main focus of this investigation so will not be discussed further here.

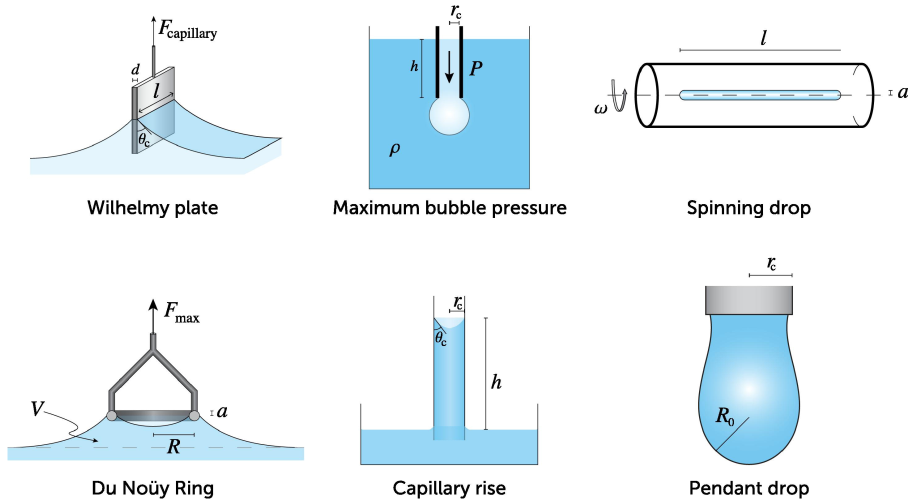

Figure 1: Diagrams of various methods currently used to measure the surface tensions of liquids. Taken from [8].

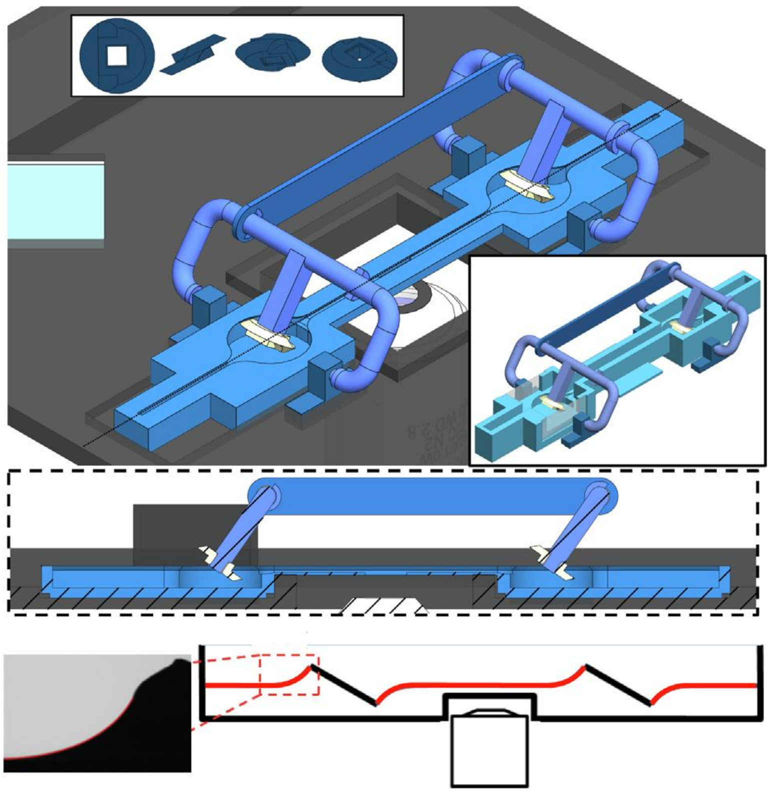

Figure 2: Diagram of the apparatus used by Moleai and Crocker taken from their paper[6]. This includes a diagram of interface formed.

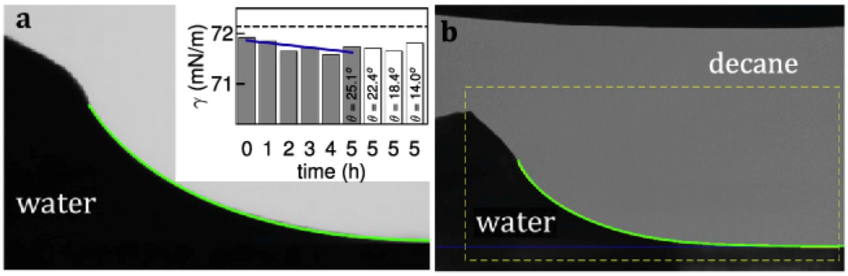

Figure 3: 'Meniscus of a-w, a, and oil-water, b, interfaces formed in the trough. Inset to (a) Gray bars show the average values of 100 surface tension measurements separated by one hour, with standard error of $0.04mNm^{-1}$. At the end of hour 5 the measurement was repeated at three different tilting angle to confirm the repeatability of the data'. Figure and caption taken from [6].

3 Previous Work

The original method is discussed in detail in Moleai and Crocker[6]. Here I will present a summary of their method and results. The paper presented a measurement device (shown in Fig. 2) using sharp disks placed in the plane of the fluid interface. These disks were then rotated along an axis that is also in the plane of the interface. This created a symmetric meniscus profile that can then be imaged with a camera. 'Because of the symmetric design of the trough, there is no volume change or bulk or interfacial flow'. This curve was then compared to numerical solutions of the curve in order to calculate the surface tension.

To image, the meniscus Crocker illuminated the meniscus with a telecentric light source directed to pass through the measurement apparatus. A photo of the light transmitted through the meniscus was then taken, examples of this are shown in Fig. 3. The Canny algorithm was then used to detect the meniscus edge in these images and extract the curve. This curve was then compared to numerically generated interfaces to calculate the surface tension.

Crocker discovered that the sensitivity is limited by both the tilting angle of the disk and the bond number. Where the bond number is defined as $B_0 = \frac{g\Delta\rho b^2}{\gamma}$. Changes in bond numbers correspond to changes in the meniscus profile. The smallest change in the surface tension that results in a measurable change in the meniscus profile varies with the bond number. With small changes in large bond numbers being more detectable than small bond numbers.

This paper also highlighted some of the important features when embarking on this experiment. These include appropriately washing the 3D prints to remove any impurities that may leach into the fluids and therefore spoil the results. To measure this Moleai measured the surface tension multiple times over a few hours to investigate how the measurement changed. This is likely something that will need to be considered in this project. Moleai used a different printer filament to the one that will be used through this project therefore it will be important to measure this effect again which will provide information on the robustness of this apparatus for other groups. Moleai also discussed that the temperature and purity of water also affected the measurement of surface tension. These are variables will be considered during this project.

This original paper used two wells each with a rotatable disk, these were connected with a narrow channel that was imaged with a microscope. This allowed additional measurements of dynamic surface tension and adsorbed protein layers to be made. This additional feature is not one we aim to explore in this project instead we will focus on improving and experimenting with the measurement of static surface tension.

4 Proposed Improvements and Scope of the Project

Possibly the biggest change to the methodology in the original paper is that the meniscus shapes were compared against numerical solutions in order to calculate the surface tension. Since then an analytical solution has been calculated by Dominic Vella. During this project, this solution will be used and compared to the data. This will provide both a new way of calculating the surface tension as well as experimental justification (or not) of this analytical solution.

Dominic Vella's solution is:

$$h(r,\theta) = \alpha R_{in} \frac{K_1(r/l_c)}{K_1(R_{in}/l_c)} cos(\theta) \tag{3}$$

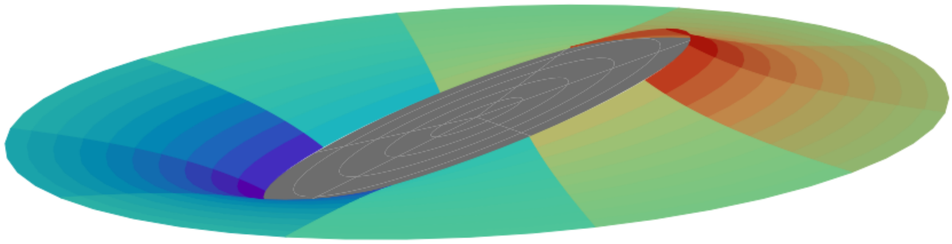

Figure 4: Plot of Vella's solution of the interface for $l_c = 2$ and $\alpha = 0.4rad$. The rotatable disk is shown in grey.

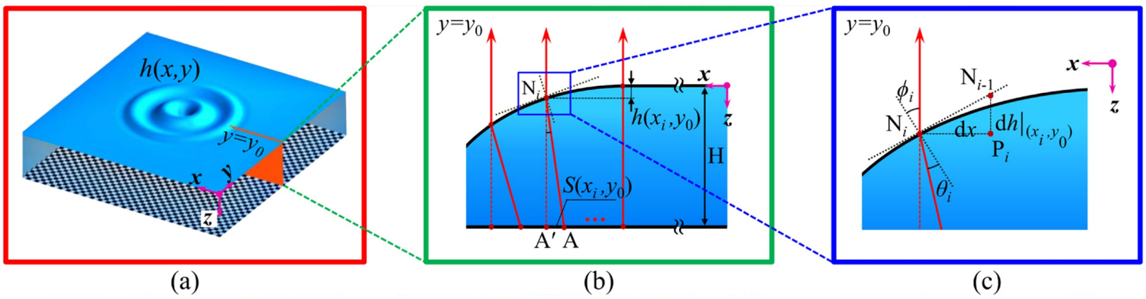

Figure 5: Schematic of how transmission-lattice phase analysis works. Taken from [9].

Where α is the angle of the perturbing disk relative to the unperturbed plane of the interface. l_c is the capillary length and is equal to $\sqrt{\frac{\gamma}{\Delta\rho g}}$. R_{in} is the radius of the disk. r and θ are defined from the centre of the disk. $K_n(z)$ is a modified Bessel function of the second kind. This solution is only valid in the limit of small α. A plot of this solution is shown in Fig. 4.

Another change is that the original group used 3D-printed disks as their surface deflection device. This limits the type of 3d printer that can be used for creating such an instrument, therefore, restricting the accessibility of the device. During this experiment machines, steel disks are to be used as the interface. This is not expected to have a dramatic improvement but does add some complications hence this would show that more accessible 3d printers could be used.

How the meniscus profile is measured represents one of the most important aspects of the experiment. It is therefore one of the areas that we aim to investigate further. Moleai presents possibly the simplest, though effective, means of measuring the meniscus however there are alternatives. There is some uncertainty on exactly how the interface will be measured during this project. While there may be other methods the currently proposed techniques are:

4.1 Transmission-Lattice Phase Analysis

This involves placing a periodic lattice at the bottom of the fluid container. The surface is then imaged from above. The lattice as imaged by the camera is distorted due to the reflection of light as shown in Fig. 5. By calculating the inverse of this distortion surface the shape of the fluid surface can be calculated. This method has been shown to be effective for different scales and with an accuracy of $\sim 0.1\mu m$ in vertical displacement when using a lattice with a periodicity of 1 line/mm as shown in [9, 10]. This method can be very easily adapted to this project with the possibility of using the 3D printer to create the lattice in the first place.

4.2 Laser Techniques

One possibility would be to reflect a laser off the interface. This has been done experimentally using many different methods [11–16]. These fall into a few categories where either the interference between different reflections is measured or the angle of reflection is directly measured. In general, these methods determine the contact angle of the fluid interface with a surface such as in the Wihlmey balance experiment [16]. By varying these methods these could be adapted to Moleai and Crocker's device, particularly the method discussed in [14] may be very applicable to this project due to its simplicity. This involves measuring the reflection angle of the laser off the surface when using a translation stage to move the laser across the surface to measure the angle of the interface. This paper does also demonstrate many of the limitations of such a method. For instance how the accuracy depends on the quality of the translation stage to which the device is mounted, and the difficulty of expanding this technique to 'closed systems' where air is not used as the second fluid.

4.3 Other Methods

An alternative method is using a wavefront sensing camera. This would require a laser to be transmitted through the interface hence acquiring some phase distortion and allowing the calculation of the meniscus shape similar to the method described in [17]. Another possibility would be the interference technique discussed in [18]. Both of these methods would provide accurate measures of the meniscus but may be overly complicated within the time allocated for the project.

Some other aspects of the device could also be experimented with. These include force sensors, varying the size and or shape of the disks, and motorising the tilt of the disk. These will be considered as extensions to the main focus of this project and will only be attempted if it is believed there will be time.

5 Timetable of Project

The precise timeline and methodology of this project is difficult to predetermine because some details are yet to be determined. Following a detailed research review and preliminary reading done in advance the first thing that must be decided is the primary method that will be used to measure the meniscus as this will dictate the rest of the project. This can only be determined after preliminary research and conversation with the department.

The during the initial stages of the project the main focus will be to replicate the apparatus designed by Moleai and Crocker and produce a working system that includes the changes proposed above. This will likely involve an iterative approach to designing the 3D prints and altering the methodology. Towards the end of this process is hoped that similar results to Moleai and Crocker can be obtained. This stage will likely take the bulk of the project time.

Once a working prototype has been made then data collection will begin. Initially, we will aim to measure the surface tension of water due to the simplicity that this system will provide. This will take time as again due to the novel nature of the experiment this will likely involve some iteration to collect quality data.

Once some good measured of surface tension for water are taken the project has the scope to continue in different directions which may depend on how the experiment has progressed up to that point. If a system can be created that is accurate and robust enough then this project will extend to more interesting interfaces, such as a water and oil. There are also other possibilities depending on how much time is left before the deadline.

Fig. 6 shows a Gantt diagram of an estimated timeline of the project.

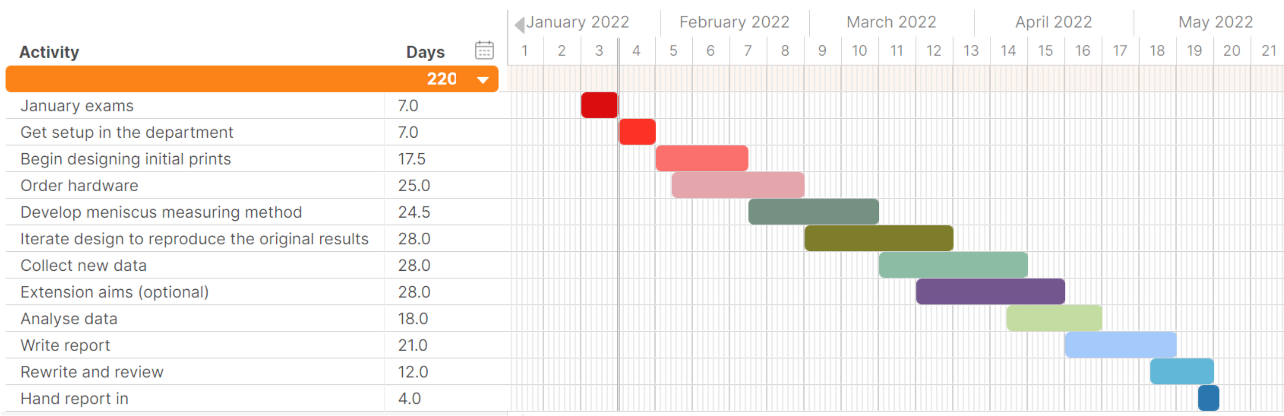

Figure 6: Gantt chart showing the rough timetable of the project. Created at 'https://plan.tomsplanner.com'.

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