

Information Arrival in Financial Markets

Job Market Paper

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This article introduces a new high-frequency analysis of six years of data for options written on the S&P 500 and traded on the Chicago Board of Exchange. I quantify in real time the information contained in the probability measure implied by option prices, using concepts developed in information theory. Here information is analogous to a reduction in uncertainty surrounding the future price of the underlying security. A simple nonparametric estimator allows us to measure the amount of information gained as an option approaches maturity. I then test for jumps in the expectation of said future price. I find the intraday flow of information in a large and important market is not continuous, and often increases in discrete intervals. This fact is used to identify events in which a large amount of information is revealed to investors.

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How does information arrive in financial markets? In this paper, I confront the basic characterization of the process by which investors learn about the future value of an asset. The topic is of importance to much of financial economics, yet continues to be one of the least explored. Indeed, this paper is the first to quantify in real time how information drives price discovery in option markets. In doing so, the paper offers three methodological contributions to the literature on measuring the information found in option prices, and documents two empirical facts not explained by existing theoretical models. First, I find the arrival of information drives jumps in investor expectation of the future price, and second, that this process is not constant over the life of an option.

This paper joins a growing literature of high-frequency analysis of investor expectations, of which Birru and Figlewski (2012) offer another example. Both the literature and this paper estimate the distribution of future returns as implied by observed option prices. Following Cox and Ross (1976) and Cox, Ross, and Rubinstein (1979), this estimation relies on a representative investor's ability to arbitrage an option and its underlying asset. In that way, all risk except the underlying uncertainty surrounding the asset's future price may be hedged away. The resulting implied distribution is known as the "risk-neutral density". Following Harrison and Kreps (1979), if this distribution is known, then options may be priced as if all investors are "risk-neutral". In other words, the price of an option is independent of the individual risk-preferences of an investor. Figlewski (2018) offers a recent review of the key ideas

in this literature. Option prices therefore reflect investor beliefs over the probability the world will achieve some future state, and it is this information that is of vital interest to investors, researchers, and policymakers. This fact combined with the growth in derivative markets has inspired renewed interest in understanding how the beliefs of investors respond to new information.

In this paper, I provide a high-frequency analysis of the price discovery process in option markets. Using six years of data for options written on the S&P 500 and traded on the Chicago Board of Exchange, I characterize the intraday evolution of the density function implied by the price of options with the same maturity date. The analysis is done for the final 3 months of each option's life-cycle, as the density is shown to become more and more concentrated over time. This paper is the first to estimate the intraday dynamics of the risk-neutral density over the life-cycle, and offers the following three methodological contributions to the literature. First, I show how a simple nonparametric estimator can be used to approximate the implied density of future returns at high-frequencies. Second, I show how concepts developed in information theory can be used to quantify the amount of information contained in the estimated density. Third, I show how this novel approach permits a simple testing procedure for the presence of jumps in the evolution of the risk-neutral density, coinciding with the arrival of new information. The results of this testing procedure represent the paper's main contribution to the literature.

I find that information often arrives in discrete intervals. Even at high-frequencies the risk-neutral density can be shown to jump, a result not anticipated by existing theoretical models. The testing procedure reveals both the frequency and magnitude of these jumps in investor expectations. I identify at least one jump for a majority of days, and find days without jumps contribute little to the total information gained over the life-cycle. I then document two empirical facts new to the literature: First, the majority of information accrues only in the final month. I show investors learn little about the future price of an asset for much of an option's life. Second, jumps contribute a majority of information early in the life-cycle. Only in the final month does information arrive often enough to contribute more to the total gained.

The paper builds on earlier work in many ways, but several features distinguish the findings from previous results. These include; 1. a focus on the evolution of the risk-neutral density over an option's life cycle, 2. a fully non-parametric estimation technique, 3. a measure of information as a reduction in uncertainty, 4. a simple framework to test for jumps, and 5. the frequency and length of the sample of options data used. These features are discussed briefly in turn.

This paper targets the evolution of the risk-neutral density over time. In a departure from much of the earlier literature (Aït-sahalia and Lo, 1998), among others, the density is not estimated for a fixed maturity. Instead, for options with the same expiration date, the focus is on how the density becomes more concentrated as the maturity date approaches. The new

perspective is shown to permit a simple testing procedure for the presence of jumps in the evolution of the density, and does not require the estimated density to be interpolated over time.

The paper employs an estimation procedure that is fully nonparametric. Inspired by the original procedure proposed by Breeden and Litzenberger (1978), the estimator places no restrictions on the shape of the density function or the dynamics of the underlying asset's price. In contrast to semiparametric procedures, such as those following Shimko (1993), the estimation procedure does not require the interpolation or the extrapolation of observed option prices, or the need for significant tradeoffs in measures of goodness-of-fit and smoothness.

This paper introduces concepts from information theory to quantify the uncertainty investors face about the future value of an underlying asset. The basic insight is that information can be measured as the reduction in uncertainty over time. The basic quantity of information theory is entropy. The concept is not new to economics, and both Sims (2003) and Frankel and Kamienica (2019) use entropy to model rational inattention and the information in decision problems respectively. Stutzer (2000) and Buchen and Kelly (1996) go so far as to use the concept of maximum entropy to infer the risk-neutral density from observed prices. However, this paper represents the first application of entropy to the problem of quantifying the information gained in the density function over time.

The paper employs a framework and hypothesis test for jumps in the risk-neutral density. Here the evolution of the entropy of the density reflects the arrival of new information. The procedure is equivalent to testing for jumps in a nonstationary timeseries, and follows Zivot and Andrews (1992). The test derives from the literature on testing for unit roots in economic timeseries, beginning with Dickey and Fuller (1979), and generalized by Said and Dickey (1984).

The choice of procedure is threefold. First the test is straightforward and transparent. It is fast and simple to implement, and the results are easily interpreted. Second, the procedure reveals the frequency, magnitude, and timing of each jump, three items that are of immediate interest. Third, the procedure permits a statistical test for each identified jump. The analysis therefore differs in this respect from other high-frequency event-studies, such as Goldberg and Grisse (2013) and Andersen, Bollerslev, Diebold and Vega (2003), among others, who examine the response of interest rates and exchange rates to economic news announcements.

Finally, the paper uses a new dataset of intraday quotes for all options written on the S&P 500 and traded on the Chicago Board of Exchange. The data is novel in two respects, namely the high frequency and long calendar span of the sample of SPX options analyzed. The intraday analysis covers six years, or nearly 1,500 trading days, beginning in January 2009 and ending in December 2014. In comparison to other high-frequency studies, Jiang and Tian (2005) and Birru and Figlewski (2012), who focus on forecasting realized

variance and the change in the quantiles of the risk-neutral density during the fall of 2008, the data presented here represents a more complete picture of the intraday evolution of investor expectations.

The paper proceed as follows. In section 3, I describe the high-frequency options data sample. Section 4 characterizes the nonparametric estimator used to approximate the risk-neutral density. In section 4 I propose the use of information theory to quantify the information in the estimated density. Section 5 describes the framework to test for jumps in the arrival of information. Section 6 discusses the results of the hypothesis tests. In section 7 I present select case studies around events where there was a large jump in information. Finally, section 8 concludes.

II. Background

A. Options

A derivative is a financial instrument whose value depends on the price of an underlying asset. For example, the value of the index options used in this paper “derive” from the price of an underlying stock index, the S&P 500. Introduced in 1957, the S&P 500 was the first stock index weighted by market capitalization. Today over \$9.9 trillion dollars is benchmarked to the index, with indexed assets totaling \$3.4 trillion. The S&P 500 covers roughly 80% of total US market capitalization. Options written on the index trade worldwide, in both over-the-counter and exchange-traded markets. The largest and most

liquid exchange-traded contract is the SPX, traded on the Chicago Board of Exchange (CBOE). The CBOE is the largest exchange for trading stock options. An options exchange offers standardized contracts and manages credit risk between counterparties, typically through a centralized clearing house. The CBOE began trading standardized contracts in 1973. Today, the notional value of SPX options is roughly \$5.5 trillion, with the number of open contracts exceeding 20 million. In 2019, the average daily volume exceeded 1.28 million contracts.

The SPX contracts used here are European-style call options on the underlying S&P 500 Index. The buyer of a European call option has the right to buy the underlying asset on a predetermined date (T) and for a predetermined price (K). The predetermined date is known as the expiration or maturity date. The price is the exercise or strike price. A single SPX contract is for 100 times the index at the given strike price. Settlement for index options is always in cash. If the buyer chooses to exercise the call option, they receive $(X_T - K) \times 100$ from the seller where, X is the settlement value of the index. Strike prices for SPX options are defined by the exchange. Strike prices for options near the value of the underlying index are typically offered at \$5 intervals. For options far from the current value of the underlying index, strike prices may only be available at \$10, \$25, \$50, or \$100 intervals. Trading in SPX options ends on the business day before the day on which the final settlement value of the index is calculated.

SPX options trade on a March cycle. All US stock options trade on either a January, February, or March cycle. The SPX cycle consists of options expiring every 3 months; March, June, September, and December. The standard expiration date is the third Friday of the near expiration month. For example, the “June call” is the SPX call option expiring on the third Friday of June, and trading in the June call ends on the Thursday before the third Friday. When an option expires, trading in a new option with a maturity date set in the next expiration month in the cycle begins. Regular trading in SPX options occurs every business day from 8:30 am to 3:15 pm Central time.

B. Risk-Neutral Valuation

A Risk-neutral valuation is a general result in the pricing of derivatives. In theory, when valuing a derivative we may assume that investors are risk-neutral. This assumption means that the individual risk-preferences of investors do not enter into the pricing equation. Introduced by Cox and Ross (1976) and expanded on in Cox, Ross, and Rubinstein (1979), the concept arises from the relationship between a derivative and its underlying asset. Since both are affected by the same underlying source of uncertainty, a portfolio can be constructed to arbitrage any gain or loss in the asset with an equivalent loss or gain in the derivative. The resulting portfolio would have no risk, and its return would be the risk-free rate. The cost of constructing such a portfolio is then the price of the derivative. Harrison and Kreps (1979) show that in a market with no profitable arbitrage opportunities, the risk-neutral price is the

correct price, even under risk aversion. In a complete market their result can be extended to include the risk-neutral price being unique.

The idea that information could be extracted from the observed price of an option was introduced by Black and Scholes (1973). By inverting the valuation formula, Black and Scholes were able to estimate the future volatility of an asset, as implied by the current price of its derivative. Merton (1973) extends the Black-Scholes model to continuous time. In continuous time, the evolution of the price of the underlying asset is modelled as a diffusion process. The Black-Scholes-Merton model assumes this process to be a geometric Brownian motion. This assumption implies the shape of the terminal distribution of the asset's price is lognormal. If known, this distribution may be used to price an option independent of the risk preferences of investors. This is the basic idea of risk-neutral valuation. If it is possible to arbitrage an asset and its derivative, the derivative may be priced as if it were riskless. The terminal distribution required for obtaining the riskless price is known as the “risk-neutral” density.

In practice, the ability to continuously rebalance such a portfolio is limited by transaction costs and financing requirements. The resulting arbitrage is then unlikely to be riskless. Consequently, theoretical models do not always perform well empirically. Black and Scholes (1972) discovered this fact early on, noting that for options on the same stock the volatility implied by their model is not constant across strike prices, thus inventing the well-known Black- Scholes “volatility smile”. There now exists a large body of empirical

evidence that implied volatility is not constant across strike prices and maturities. Taken together, the evidence strongly suggests the assumption of a lognormal terminal distribution implied by diffusion process that is a geometric Brownian motion does not hold in practice. A phenomenon known as the Black-Scholes-Merton anomaly. Figlewski (2018) offers a more intensive introduction to the key ideas, issues, and finding introduced here.

The Black-Scholes-Merton anomaly has led to a large literature that attempts to estimate a risk-neutral density which better fits the observed option prices, and it is to this literature that the nonparametric estimator I propose contributes. The discussion of the estimator is therefore limited to a class of nonparametric and semiparametric methods. This class of methods includes other direct implementations of Breeden and Litzenberger (1978), (Shimko (1993), Malz (1997), Bliss and Panigirtzoglous (2002), Weinberg (2001), and Dumas, Fleming, and Whaley (1998)), and other methods based on such concepts as maximum entropy (Buchen and Kelly (1996), and Stutzer (1996)), kernel regression (Ait-shalia and Lo (1998)), and the binomial tree method of Cox, Ross, Rubinstein (1979), (Rubinstein (1994), Rubinstein (1996), Jackwerth (1997,1999)). For an extensive review of methods for estimating the density from observed option prices see Jackwerth (1999), Jondeau and Rockinger (2000), and Bliss and Panigirtzoglou (2002) Coutant, Jondeau, and Rockinger (2001), and Datta, Londono, and Ross (2017)).

III. Data

Throughout the paper I use data for a sample of options written on the S&P 500 and traded on the Chicago Board of Exchange. The data is novel in two respects, principally the high frequency and long calendar span of the sample of SPX options analyzed. The data represent the most complete sample of high-frequency investor expectations to date. Here I describe the data in detail.

Daily ‘Trade And Quote’ or TAQ data were obtained from the Chicago Board of Exchange (CBOE). The raw files contain all trades and quotes for options written on the S&P 500, and traded on the CBOE. The raw data consists of daily files for each of what is typically 252 trading days per year. The sample covers six years, or approximately 1,500 trading days beginning January 2, 2009 and ending December 31, 2014. Regular trading hours occur from 8:30 am to 3:15 pm Central time each day. Quotes are updated throughout the day for all available strikes. Trades in SPX options are sparse relative to quotes, and are rarely observed simultaneously across strikes. To avoid introducing additional pricing-errors, quotes are used in the analysis which follows. Each quote consists of bid and ask price, size, and the corresponding strike price, together with a timestamp, underlying index price, and flags indicating the class of option and market condition, either open or pre-open. Very few quotes are observed pre-open.

The traditional SPX options chain is AM-settled on the third Friday of every month. Options trading on the March cycle consist of those expiring every 3 months; March, June, September, and December. Nontraditional SPX options are PM-settled, with varying expiration dates, including the last trading day of the month and weekly options. ‘Long-term Equity Anticipation Securities’ or LEAPS are traditional SPX options with expiration dates up to five years in the future.

The sample consists of 24 nonoverlapping option chains, beginning with the options expiring March 21, 2009 and ending with those expiring December 20, 2014. Following convention, option chains are referred to by their expiration date. There are on average 62 trading days for each curve, with the March 2009 option having only 55 trading days observed. The sample consists of 83 5-minute intervals for each trading day, beginning at 8:25 am and ending at 3:15 pm. Intraday quotes are aggregated by strike to the corresponding 5-minute interval.

Only quotes for traditional SPX call options are used, those which are AM-settled on the third Friday of the near expiration month. Nontraditional SPX options and LEAPS are excluded from the sample. To create the 5-minute samples used in the analysis, the average of the best bid and offer were taken for the sample of options with quotes posted in the preceding 5-minute. The reported 9:00 am call price for a given strike is then the mid-price of the best bid and best offer observed from 8:55:00.000 am to 8:59:59.999 am. In the rare

instance where quotes were not observed for a particular strike, the average of the best bid and offer price were carried over from the previous interval.

There are two additional notes regarding the sample. First, trading in traditional SPX options ends on the business day before the settlement date, and before trading in a new option in the following cycle begins. No quotes for the current option chain are observed on these days, and they are excluded from the sample. Second, far out of the money options are often illiquid, and I exclude call options written on strikes issued at intervals greater than \$50.

The result is a sample of 1,479 days, each day containing 83 5-minute intervals. Each interval contains the average of the best bid and best offer price for call options written on strike prices at intervals less than \$50. Table 1 reports daily summary statistics for the sample of options and the underlying index. Columns 1-3 report the average number, minimum and maximum observed strikes, call and underlying index prices observed for each day in the sample. Columns 4 and 5 report the average and standard deviation of the sample of strike, call and underlying index prices for each day. Table 1 reports summary statistics for the entire six-year sample, as well as these statistics for each year. Table A in the appendix reports the separate summary statistics for each option chain.

The number of strikes observed each day is large. Looking at Table 1, there are between 117 and 187, and on average 146, strikes observed. Strike prices range from \$150 to \$2,250 and are traded at intervals of less than \$50. The options sampled cover a large degree of moneyness, with the average ratio

of the strike price to the current index price ranging from 0.3 to 1.4. Thus on a typical day, probabilities between a 70% decrease and 40% increase in the index over the following days are observed.

The average number of quotes observed each day is also large. For the options sampled, I observe on average approximately 718,000 quotes per day, ranging in price from \$0.30 to \$1,373, with an average mid-price of about \$243 and average daily standard deviation of \$219. The value of the underlying index ranges from \$674 to \$2,075, with an average price of around \$1,380, and a small average daily standard deviation of just \$0.18. Finally, the observed number of strikes and quotes increases consistently over the six years in the sample, as does the value of the underlying index. For simplicity and convenience, much of the analysis refers to options expiring in either the first or last year of the sample, 2009 and 2014. This is done to show the findings are not artifacts of any larger trends in the market. Figure 1 reports the daily level of both the underlying S&P500 Index and the CBOE Volatility Index or VIX Index for the sample period. For much of the sample, the value of the underlying index is increasing. Implied volatility as calculated by the CBOE is highest early in the sample, spikes in mid-2010 and late 2011, and declines steadily from the beginning of 2012. In Table 1 these trends are evident in the range of strike prices available and the average value of the underlying index.

[Insert Table 1 Here]

[Insert Figure 1 Here]

IV. Information

A. In Option Prices

Black and Scholes (1973) introduced the idea that information could be extracted from the price of traded options. In general, option prices reflect investor beliefs over the probability the underlying asset will take a particular value. Consider two options with adjacent strikes. Intuitively, any difference in their price must reflect the likelihood that the value of the underlying asset will fall between them. This holds across all strikes. Therefore the observed difference in option prices across strikes, or state-prices, contains information about how likely investors believe different states are. Breeden and Litzenberger (1978) show how to obtain the entire risk-neutral density from these differences. Here the information contained in option prices is extracted as a probability distribution over the future value of the underlying index. This information allows for a more complete view of investor beliefs; both how they evolve over time, and how they change in response to new information or events. To extract this information, option prices are combined across strikes to mimic a security which pays \$1 if an individual state is realized, and \$0 if not. The price of such security would then be proportional to the probability of a state occurring. This idea was first introduced in time-state preference model of Arrow (1964) and Debreu (1959). The result is known as an ‘Arrow-Debreu’ security, a type of elementary claim whose value is contingent on a state being realized.

Interpreting the information extracted from option prices requires summarizing the characteristics of the implied probability distribution of random process. There much of early the literature focused on using the risk-neutral density to estimate the implied volatility of the underlying process, often for the purpose of forecasting realized volatility (Canina and Figlewski (1993), Weinberg (2001), and Jiang and Tian (2005), among others). Later literature has expanded the focus to examining the response of higher-order moments, principally skewness and kurtosis, to key events, see for example Birru and Figlewski (2012). The approach has known limitations, including the sensitivity of the estimated risk-neutral density to the choice of estimation methods. As a result, quantile moments are often used and are found to be a more robust (Datta, Londono, and Ross (2014), Flamouris and Giamouridis (2002) and Campa, Chang, and Refalo (2002), Birru and Figlewski (2012)).

A growing literature studies the information contained in option prices in this manner, using the moments of the risk-neutral density to examine a wide range of markets; including but not limited to international stock indexes, (Kang and Kim (2006), Shiratsuka (2001), Glatzer and Scheicher (2005), Äijö (2008), Kim and Kim (2003), and Syrdal (2002)), exchange rates, (Weinberg (2001), Bahra (1997), Campa, Chang, and Refalo (2002), and Londono and Zhou (2012)), government bonds, (Neuhaus (1995), Shiratsuka (2001), and Cheng (2010)), inflation options, (Kitsul and Wright (2012)), and oil commodities, (Datta, Londono, and Ross (2014), Melick and Thomas (1997), Flamouris and Giamouridis (2002), and Askari and Krichene (2008)).

This paper contributes to the literature analyzing the information contained in the price of index options, but departs in the methodology employed. Similar to Weinberg (2001), Jiang and Tian (2005), Kang, Kim, and Yoon (2010), and Birru and Figlewski (2012), the focus here is for options written on the S&P 500. Two features however distinguish the analysis from earlier work. This include a departure from using both a fixed horizon risk-neutral density and the moments of the estimated density function. Instead, I propose the use of information theoretic concepts to quantify the amount of information investors gain about the likely value of the underlying index at maturity over time. It is to these concepts I now turn.

B. Information Theory

The basic insight of information theory is to measure information as a reduction in uncertainty, and the basic building block is entropy. Entropy quantifies the amount of uncertainty in a random variable, here the outcome of a stochastic process (Shannon 1948). Given a random variable X , with probability density function $p(X)$, entropy is simply $-E[\log(p(X))]$. It should be noted that the concept applies whether X is a discrete or continuous random variable. That is, whether $p(X)$ is a density with respect to a Lebesgue measure on \mathbf{R}^k , or a discrete measure on a countable set of points. By convention, $p \log p = 0$ for any value X such that $p = 0$. The base of the logarithm determines only a scaling factor for the amount of information. Base 2 is often used as intuitively the outcome of a fair coin toss contains one “bit” of

information. Here I provided a brief introduction to the basic ideas of information theory, for a complete technical introduction see Cover and Thomas (1991).

The amount of information or entropy in an event is calculated using the probability of that event. The more deterministic or certain an event is, the less surprising or informative its realization is. The ‘surprisal’ or ‘self-information’ of a discrete event x is defined as $h(x) = -\log_2(p(x))$, and is 0 when $p(x) = 1$. Here the intuitive is clear, since no information is learned from a certain outcome. The classic example is that of a fair coin toss, where the self-information of a single toss is $\log_2(1/2) = 1$. Compare this event to the outcome of a fair dice, and the intuition and idea behind entropy becomes even clearer, $-\log_2(1/6) \approx 2.58$. In a sentence; the more certain an event is, the less informative its outcome, and the lower its entropy.

Calculating the information in a random variable is the same as calculating the information of the probability distribution of the events of a random variable. In other words, the amount of information in a random variable is the surprisal of each event weighted by the probability of those events. In practice, estimating the entropy of a random variable is equivalent to estimating the information or surprisal for the probability distribution of events. Entropy then is the average amount of information of an event drawn from the probability distribution of a random variable.

In this paper, I propose the use of entropy to quantify the amount of information in the risk-neutral density implied by option prices. I employ two measures of information, the traditional Shannon (1948) entropy and the generalized Rényi (1961) entropy. Shannon entropy is the expected or average amount of information for an event drawn from its distribution. Rényi entropy is equivalent to the L_2 norm of the distribution. Shannon entropy can be shown to generalize to Rényi entropy. Both concepts measure how broadly distributed the outcome of a random variable is. Intuitively, the higher the entropy, the broader the distribution. For example, the uniform distribution is the ‘maximum entropy distribution’ on any given interval $[a, b]$.

Two final points of clarification, I break somewhat with convention in the analysis that follows by reporting $-H(x)$ not $H(x)$, and using the natural logarithm in place of log base 2 for both Shannon and Rényi entropies. The units are then known as “nats”. This is done only for convenience, and does not affect the results. The negative rescaling simply allows for both entropies to be displayed on a single positive axis, and for the magnitude and direction of the gains in information, trend, and jumps to be more readily compared. For the remainder of the paper, when uncertainty declines the reported measures entropy will be shown to increase! Finally, entropy is not variance, which is the measure of variation in a random variable. No, here entropy is defined as a measure of uncertainty in the distribution of probabilities, and is not equivalent to implied volatility.

[Insert Table 2 Here]

[Insert Figure 2 Here]

IV. Estimation

[Under Revision]

V. Testing

I now turn to the paper's main empirical contribution, a simple testing procedure for the presence of jumps in the arrival of information. The approach reveals both the frequency and magnitude of jumps in investor expectations, and permits the testing of several hypotheses. Building on the concept of information as a reduction in uncertainty surrounding the future price of the index, the arrival of information is reflected in the evolution of the risk-neutral density. This process can be quantified using entropy. For options with the same maturity date, the density becomes more concentrated over time. Hence both measures of entropy will increase, as investors become increasingly certain of the likely distribution of future prices. Intuitively, less weight is placed on options farther from the money, as the probability of large changes in the value of the underlying index declines near maturity.

The evolution of the entropy of the risk-neutral density reflects the arrival of information. If information arrives continuously, if investors gain information in small amounts, or if expectations of the future price evolve slowly,

then entropy will increase gradually. Alternatively, if information arrives discontinuously, if investors gain information in large amounts, or if expectations of the future price change abruptly, then entropy will be shown to jump. This intuition can be formalized by specifying the dynamics of the underlying diffusion process. For example, if the underlying price were to follow a geometric Brownian motion, then the entropy of the implied distribution would increase both continuously and at a constant rate across time.

The information theoretic framework introduces several testable hypotheses for how information drives the evolution of the risk-neutral density. First, information arrives continuously, and investors respond by constantly adjusting expectations; or information arrives discretely, and investors update expectations in response to new information. Second, investors learn at a constant rate and information accrues consistently across the lifecycle; or learning is highly variable, and information accrues at different rates across time. Third, information accrual is deterministic, and any shock to expectations will return to some trend; or information follows a random walk, and investors learn little day-to-day as shocks affect expectations indefinitely.

The results of these hypothesis tests have real-world implications. Consider a temporary negative shock to the expected future value of the index. If information follows a stochastic trend, then investors remain persistently less certain about the future value of the index. This uncertainty may persist until an option matures. Alternatively, if uncertainty is purely deterministic, investors may profit from betting information will rebound to normal levels.

Likewise, if information accrues at a constant rate, or if information is found to arrive continuously, investors may try to anticipate how uncertainty and the density function will evolve following such a shock. Here it is important to consider that any model which assumes the underlying price follows a continuous diffusion process also implicitly assumes that information arrives continuously, accrues at a constant rate, and evolves following a deterministic trend. This fact can be shown for many models including the Black-Scholes-Merton model.

Testing the three hypotheses is equivalent to testing for jumps in a nonstationary timeseries. Here I follow the procedure proposed by Zivot and Andrews (1992) to test for a single jump at an unknown time. The procedure derives from the literature on testing for unit roots in economic timeseries, beginning with Dickey and Fuller (1979), and generalized by Said and Dickey (1984). Perron (1989) found the proposed augmented Dickey–Fuller procedure to be biased against rejecting the null hypothesis in the presence of jumps or structural breaks in the timeseries. Zivot and Andrews (1992) later endogenize Perron’s procedure to test for jumps occurring at an unknown time.

The choice of procedure is threefold. First the test is simple and transparent. It is fast and easy to implement using standard econometric tools and software, and the results are easy to interpret. Second, the procedure reveals the frequency, magnitude, and timing of each jump, three items of immediate interest in our analysis. And finally, the procedure provides a statistical test

for each identified jump, a feature many event-type studies lack. I now turn to describing the procedure in detail.

Following Zivot and Andrews (1992), the null hypothesis (5.1) is a random walk with possible drift and no jump. The alternative hypothesis (5.2) is a trend-stationary process with a single jump in its trend occurring at an unknown time. The basic idea is to first estimate a sequence of alternative or trend-stationary models, one for each interval in the series. I then select the jump which gives the most weight to a single alternative model, and test that model against the null hypothesis. Here Zivot and Andrews follow Perron's (1989) augmented Dickey–Fuller testing strategy, and use a single regression equation to test for a unit root. Operationally, their procedure identifies both the timing and magnitude of the single largest jump in each timeseries. Each regression also includes an optimal number of lags, typically chosen by either Schwarz's (1978) Bayesian Information Criterion or a sequence of t-tests.

Null Hypothesis—Random Walk with Drift:

$$H_0: \quad y_t = \mu + y_{t-1} + e_t \quad (5.1)$$

Alternative Hypothesis—Trend Stationary Model + Jump:

$$H_1: \quad y_t = \mu_1 + \delta t + (\mu_2 - \mu_1)DU_t + e_t \quad (5.2)$$

The number of lags included in each regression is chosen to reduce the residuals to white noise. The optimal number included is allowed to vary for

each candidate model, and is selected for each regression by a sequence of t-tests, as suggested by Ng and Perron (1995) and Campbell and Perron (1991). Ng and Perron find their approach suffers fewer size distortions and retains comparable power to Schwarz’s (1978) Information Criteria. In practice both Zivot and Andrews (1992) and Perron (1989) follow such a procedure; working backward from $k = \bar{k}$ to $k = 0$, selecting the first k^* such that the t-stat on c_k for $l < k$ is significant. It is their procedure I follow here, setting $\bar{k} = \text{floor}[12\{(T+1)/100\}^{0.25}]$ as proposed by Schwert (1989), where $T = 83$ and in practice $\bar{k} = 11$.

Regression Model—Zivot and Andrews (1992):

$$y_t = \hat{\mu} + \hat{\theta}DU_t(\hat{\lambda}) + \hat{\delta}t + \hat{\alpha}y_{t-1} + \sum_{j=1}^k \hat{c}_j \Delta y_{t-j} + \hat{e}_t \quad (5.3)$$

The regression model (5.3) facilitates the testing of several hypotheses. In the model, $\{y_t\}$ represents the amount of information measured at each interval, where $t \in (0, 83)$ is the number of 5-minute intervals after 8:25 am Central time. The “trend” parameter $\hat{\delta}t$ measures the gain in information as the average change in entropy over each day. The intercept $\hat{\mu}$ captures the initial level of information for each trading day. The “jump” parameter $\hat{\theta}$ captures the magnitude of a one-time shift in level of entropy occurring at a single time, t , where DU_t is indicator variable such that $DU_t(\hat{\lambda}) = 1$ if $t > \lambda t$, and 0 otherwise. The Zivot and Andrews (1992) procedure is to first

estimate a sequence of models, one for each possible jump, and to then select the best alternative model from that class of candidate models. The ‘best’ being the model which gives the most weight to the alternative hypothesis. Finally, the null hypothesis, $\hat{\alpha}y_{t-1} = 1$, is tested against the best alternative model, using the asymptotic critical values derived by Zivot and Andrews (1992). In practice, the alternative model is selected from the group of candidate models as the single model having the smallest one-sided t-statistic against the null for the test of $\hat{\alpha} = 1$. Testing $\hat{\theta}DU_t(\hat{\lambda}) = 0$ effectively tests for a jump at a given 5-minute interval; where $\hat{\lambda} = t/T$ for $t \in (2, T - 1)$ and solves $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}}(\lambda)$.

In the application here, I begin with the two measures of information; Shannon and Rényi entropy. Both are measured at each of 83 5-minute intervals each day, over six years or 1,479 days in the sample, and representing 24 non-overlapping option chains. For each day, I estimate the regression model at each of 73 candidate intervals, one for 5-minute intervals between the hours of 9:00 and 3:00 pm Central time. From the resulting class of 73 candidate models I select the alternative model which best fits the data, here the model with the smallest t-statistic for the test of $\hat{\alpha} = 1$. Finally, the random walk model is tested against the alternative trend-stationary model. The $\hat{\theta}$ and $\hat{\delta}t$ parameters are recorded for each day, along with the number of lags k^* selected, and the result of the main hypothesis test. For days where the null is rejected, a secondary test of $\hat{\theta} = 0$ is performed to identify large jumps.

Much of the following analysis focuses on relative magnitude of the “trend” or $\hat{\delta}t$ and the “jump” or $\hat{\theta}$ parameters. It may be helpful then to review the possible outcomes of the testing procedure and the implication of each result for the analysis. First, for a given day the procedure may fail to reject the null hypothesis of a random walk. This would be the most likely outcome for days where the estimated time trend was very small, and no jump was identified. This result would indicate a day where little to no information was gained. Second, the procedure could reject the null hypothesis but detect no jump, such that $\hat{\delta}t > \hat{\theta}$. This result would indicate a day where the trend or continuous gain in information dominates the arrival of any single bit of information. This result is most consistent with the model of the evolution of an asset’s price as a continuous diffusion process. In practice, information is shown to arrive nearly continuously, and investors respond by making constant small adjustments in their expectations of the future price of the asset. Finally, for a given day, the procedure may reject the null hypothesis and detect a jump. This result would indicate a day with a large discrete gain in information, a jump which may dominate the more continuous gain in information, such that $\hat{\theta} > \hat{\delta}t$. Here information is shown to arrive discretely, and investors must respond by making fewer but larger revisions to their expectations.

VI. Results

The testing procedure reveals several surprising results. Information is shown to often arrive in discrete intervals. Even at high-frequencies the risk-neutral density can be shown to jump. In the data, the procedure identifies at least one jump for a majority of days, and finds days without jumps contribute little to the total gain in information. Additionally, the testing procedure reveals both the frequency and magnitude of jumps in investor expectations. The findings highlight two facts new to the literature. First, the majority of information accrues in the final month of an option's life-cycle. In fact, investors learn little about the future price of an asset for most of an option's life. Second, jumps contribute the majority of information gained during the first two months. Only in the final month does information arrive often enough for the trend to contribute more to the total gained. The size and frequency of jumps does not decline.

Information arrives in discrete intervals. The intraday analysis reveals the frequent occurrence of jumps in the risk-neutral density. Table 3 and Table 4 summarize the results of the daily Zivot and Andrews (1992) procedure for each of 24 option chains. Each table displays the results of the daily hypothesis test of $\hat{\alpha} = 1$. In columns 3-5, I report the total number of days for each chain where the null hypothesis of a random walk is rejected at the 10, 5, and 1

percent level. On average, the null hypothesis is rejected for a majority of days, and the alternative model of a trend-stationary process with a potential one-time jump in information is selected. This is true for between one-half and two-thirds of days depending on the measure of entropy and the level of significance. The result implies that investors 1. gain information about the future value of an asset over time, and 2. make large revisions to their expectations in response to new information. This is shown to occur even at high frequencies, as the procedure identifies at least one jump for a majority of days. The results also document several days where little to no information is gained. For days where the procedure fails to reject the null, no jumps are found to have occurred. Combined with only a small gain in the trend of information, the daily process for these days would resemble a random walk.

Tables 3 and 4 also report the average daily change in total, trend, and jump estimates of information, taken over all days in each option chain, in columns 6-8. On average, the day-to-day gain in information is small, and positive; with a trend component that is on average larger than either the jump or total gain in information. On average jumps appear to coincide more with increases in uncertainty, as they are negative and nonzero. This holds for both Shannon and Rényi entropies.

The final two columns report the average daily contribution of jump and trend estimates as a percentage of the total gain in information. Here it becomes apparent that jumps contribute to a large proportion of the total information gained, and are more often positive than their average over all days

appears. On average, jumps contribute to almost half of the total information gained over the cycle. Across all days, a single jump, over a single 5-minute interval, is nearly as large as the estimated trend across the entire day. This result implies that jumps occur for a majority of days, and that days without jumps likely contribute little to the total gain in information.

The testing procedure reveals the magnitude of jumps in investor expectations. Figures 5 panels 2 and 3 present histograms of the estimated daily trend and jump components for all days, and each measure of entropy. The trend component is unimodal, centered near zero, and negatively skewed. The trend in Shannon entropy is more broadly distributed and more skewed in comparison to the trend in Rényi entropy. The estimated jump component is bimodal, centered near zero, slightly positively skewed, and not symmetric. The jumps in Shannon entropy are more broadly distributed and skewed compared to the Rényi entropy. For both measures, there are few days where the estimated jump is zero, and a large number of days the jump component is larger than the trend. To understand how the trend and jump components vary day-to-day, Figure 5 panels 1 and 4 present heat maps of the estimated daily trend and jump components for all days. For each map, the bright yellow and deep blue correspond to estimates of the daily trend and jump in the 95th percentile of each distribution, with blue-green estimates near zero.

Several patterns emerge that require further consideration. First, the size and frequency of jump estimates appears evenly distributed over the life-cycle. Second, the magnitude of the trend estimate is not evenly distributed.

Looking at the shading in panels 1 and 4 of both figures, a large positive trend appears more likely in the final 20 to 30 days of an option's life. This result suggests information accrues at different rates across time, that is, gains in information are not constant across the life-cycle. Figure 6 panels 1 and 2 shows this in high contrast. Here, days with a strictly positive trend and jump are reported in white. Again the frequency of positive jumps appears evenly distributed over the life-cycle, while the frequency of days with a strictly positive trend seems to concentrate in the final 20 days of an option's life. The result is even more pronounced if we focus only on the last two weeks, or 10 days, of trading. Few days in the last two weeks of an option's life report a negative trend. In contrast, the magnitude and frequency of jumps does not appear to change.

[Insert Tables 3 and 4 Here]

[Insert Figures 5 and 6 Here]

When then is information gained? Figure 6 panel 3 reports frequency of days where the trend is larger than the estimated jump for both measures of entropy. Note that there are many days where the trend exceeds the estimated jump, and Figure 6 panel 3 does not distinguish days where both are negative or near zero from days where the trends is large and positive. However, when viewed in conjunction with Figure 6 panels 1 and 2 it becomes more apparent that information does not accrue consistently across the lifecycle. Only in the

final 10-20 days of an option's life cycle is the estimated trend persistently positive, and consistently larger than the estimated jumps. This explains many of the results in Tables 3 and 4. If for the majority of days the daily gain in information is small, then jumps may contribute much to the gain in information. This effect would be particularly acute if days without jumps were found to contribute little to the total gained.

Figure 6 panel 4 reports the days where the null hypothesis of a random walk is rejected, in a test of $\hat{\alpha} = 1$. Again it can be seen that jumps are quite common, and that the alternative model is selected more often in the final month of an option's life. Indeed, days where little to no information is learned (days where the test fails to reject the null) are concentrated in the first two month of the sample. Given that the trend is less pronounced during this period, days without jumps likely contribute little to the total gain in information during much of an option's early life. This is not to say that information commonly follows a random walk, or to suggest investors learn nothing day-to-day. No, information accrual is decidedly deterministic. The trend is small, but I find little evidence that shocks affect expectations or the level of information indefinitely.

Taken together, the initial results suggest learning is highly variable, and information about the final price of the asset accrues at different rates across time. The testing procedure makes this clear. The findings contradict the assumption that investors are less certain about the future price of an asset mostly as a function of time, something a continuous diffusion process

implicitly assumes. Instead, the arrival of information in discreet intervals results in jumps in the level of uncertainty. This fact, combined with a small trend for much of an options life cycle, and varying levels of initial uncertainty, explains the results of Tables 3 and 4. To verify this intuition, I turn to the question of quantifying these differences across the life-cycle.

The following findings highlight two facts new to the literature. First, the majority of information accrues in the final month of an option's life-cycle. Table 2 and Figure 2 show that on average nearly 60 percent of gain in entropy accrues in the final month. Given that this represents only one-third of an options life, the observed gain in information in the last month is both large and unexpected. The finding suggests that investors learn little about the future price of an asset for much of an option's life. This result runs contrary to the common assumption that investors become more certain about the likely final price of an asset at something close to a constant rate. The finding also raises new questions regarding what drives the differences in the rate of the accrual of information in the final month of an option's life.

Tables 5 and 6 document the source of the difference in the accrual rate of information. Each table reports the results of the daily Zivot and Andrews (1992) procedure for each of the 24 option chains. Here the sample is split into thirds, with the results of the unit root test and gains in information reported separately for the first 2 months and final 1 month of each option chain. On average there are 40 trading days in the first two months, and 21 trading days in the final month; a ratio 2:1, representing on average 66 and

34 percent of each life-cycle. Column 3 reports the fraction of days in each subsample for each option. Each table displays the results of the daily hypothesis test of $\hat{\alpha} = 1$. In columns 4-6, I report the total number of days for each chain where the null hypothesis of a random walk is rejected at the 10, 5, and 1 percent level. On average, the null hypothesis is rejected for a similar fraction of days in both subsamples, with the alternative model selected for between one-half and two-third of days depending on the measure of entropy and the level of significance.

Tables 5 and 6 report the average daily change in total, trend, and jump estimates of information for each period. Columns 7-9 report that the average day-to-day gain in information remains small and positive, but is noticeably larger during the final month of the sample. This holds for both Shannon and Rényi entropies. Indeed, the average trend estimate triples for the Shannon entropy and doubles for the Rényi entropy in the last month. In contrast, the average jump estimate does not change. Figure 7 demonstrate this result visually. Here I plot the average absolute trend and jump estimates for the first 2 months in white, and the final month in black. The results match those as reported in columns 8 and 9 respectively. In almost all cases the average trend for the last month of the life-cycle is large and positive, particularly early on in the sample when the initial level of uncertainty is highest. By comparison, the average jump estimate does not appear systematically larger in final month. This result holds for both measures of information.

Only in the final month does information arrive often enough for the trend to contribute more to the total gained. The final two columns of Tables 5 and 6 report the average daily contribution of jump and trend estimates as a percentage of the total gain in information, across the first two and final months of the sample. On average, the trend contributes to a larger percentage gain in total information in the final month. As a result, jumps are shown to contribute a smaller percentage gain. That is not to say the importance of jumps diminishes. On average, a single jump continues to contribute nearly an equivalent amount of information as the estimated trend across the entire day. This effect is simply more pronounced earlier in the sample when the trend estimates are on average smaller.

The testing procedure reveals the change in magnitude of the trend and jump estimates in the final month of an option's life. Figure 8 present histograms of the estimated daily trend and jump components for each subsample and measure of entropy. In panel 1, the trend and jumps in Shannon entropy are shown for all 3-months in white, and the final month in grey. Panel 2 reports the trend and jumps in Shannon entropy for the first 2-months in white and the final month in grey. Looking across both panels, the shape of the distribution of jumps does not change in the final month of the sample. The shaded subsample remains bimodal, centered near zero, and slightly asymmetric. By comparison, the shape of the distribution of estimated trends is noticeably more skewed in the final month. In all cases the distribution of trend estimates remains unimodal, with the final month being negatively

skewed and centered in the positive quadrant. This effect is most obvious in panel 2, where the occurrence or frequency of days with a large positive trend is near equal between the first two and final month of the sample, a counter-intuitive result given the relative lengths of the subsamples. The finding supports the conclusion that a large positive trend occurs predominately in the final 20 days of an option's life. Similar patterns emerge in the Rényi entropy. In panel 3 and 4, the contribution of the final month to the right tail of the histogram is pronounced. Both results suggest that the gain in information is not constant across the life-cycle. Indeed, Figure 8 clearly show that information accrues at different rates across the life-cycle. For both measures of entropy the estimated trend or continuous gain in information in the final month is on average larger, more frequent, and persistently more positive. Likewise, the estimated daily jumps or discreet gains in information are frequent, nonzero, and larger than the trend for many days. The result supports the conclusion that jumps contribute much to the total gain in information and, given the small trend, contribute the majority of information which accrues during the first two months.

[Insert Tables 5 and 6 Here]

[Insert Figures 7 and 8 Here]

VII. Case-Studies

[Under Revision]

VIII. Conclusions

[Under Revision]

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