Let us consider the binary dihedral group

$$g = \begin{pmatrix} \zeta_n & 0 \\ 0 & \zeta_n^{-1} \end{pmatrix}, \qquad h = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad G = \langle g, h \rangle \subset \mathrm{SL}_2(\mathbb{C}),$$

where $\zeta_n = e^{2\pi i/n}$ is a primitive n-th root of unity. We study the invariant ring of G acting on $\mathbb{C}[x, y]$.

Define invariant polynomials

$$A := x^{2n} + y^{2n}, \qquad B := xy\,(x^{2n} - y^{2n}), \qquad C := (xy)^2.$$

These are invariant under both g and h. Using the identity

$$(x^{2n} - y^{2n})^2 = (x^{2n} + y^{2n})^2 - 4(xy)^{2n},$$

we obtain

$$B^2 \;=\; C\big(A^2 - 4C^{\,n}\big).$$

Hence

$$\mathbb{C}[x, y]^G \;\simeq\; \mathbb{C}[A, B, C] \;/\; \big(B^2 - C(A^2 - 4C^n)\big).$$

Now set

$$X := B, \qquad Y := A, \qquad Z := C.$$

Then the relation becomes

$$X^2 = ZY^2 - 4Z^{\,n+1}.$$

Since we may freely rescale the variables over $\mathbb{C}$, this equation can be normalized to

$$x^2 + y^2 z + z^{\,n+1} = 0,$$

which is the standard form.

Therefore the invariant ring of the binary dihedral group corresponds to the Kleinian surface singularity of type

$$D_{n+2}: \quad x^2 + y^2 z + z^{\,n+1} = 0.$$