We identify SU(2) with the group of unit quaternions

$$\mathbb{H}_1 = \{\, a + bi + cj + dk \mid a,b,c,d \in \mathbb{R},\ a^2 + b^2 + c^2 + d^2 = 1 \,\},$$

via the homomorphism

$$\rho:\ \mathbb{H}_1 \longrightarrow \mathrm{SU}(2) \subset \mathrm{SL}_2(\mathbb{C}), \qquad \rho(a + bi + cj + dk) = \begin{pmatrix} a + bi & c + di \\ -\,c + di & a - bi \end{pmatrix}.$$

In particular,

$$\rho(i) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \rho(j) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \rho(k) = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

As unit quaternions,

$$T^* = \Big\{ \pm 1,\ \pm i,\ \pm j,\ \pm k,\ \tfrac{1}{2}\big(\pm 1 \pm i \pm j \pm k\big) \Big\}.$$

Matrix representatives are obtained via ρ. For example, generators can be chosen as

$$\rho(i) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \rho(j) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \rho\Big(\tfrac{1+i+j+k}{2}\Big) = \tfrac{1}{2}\begin{pmatrix} 1+i & 1+i \\ -1+i & 1-i \end{pmatrix}.$$

O^* is generated by T^* together with the additional unit quaternions

$$\tfrac{1}{\sqrt{2}}(\pm 1 \pm i), \quad \tfrac{1}{\sqrt{2}}(\pm 1 \pm j), \quad \tfrac{1}{\sqrt{2}}(\pm 1 \pm k),$$

with all sign choices. Thus one generating set is

$$\rho(i),\ \rho(j),\ \rho\Big(\tfrac{1+i+j+k}{2}\Big),\ \rho\Big(\tfrac{1+i}{\sqrt{2}}\Big),$$

where

$$\rho\Big(\tfrac{1+i}{\sqrt{2}}\Big) = \tfrac{1}{\sqrt{2}}\begin{pmatrix} 1+i & 0 \\ 0 & 1-i \end{pmatrix}.$$

Let $\varphi = \frac{1+\sqrt{5}}{2}$ be the golden ratio and $\varphi' = \frac{1-\sqrt{5}}{2} = -\varphi^{-1}$. Then I^* is generated by $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$ together with the unit quaternions

$$\tfrac{1}{2}\big(\pm \varphi \pm i \pm j \pm k\big), \qquad \tfrac{1}{2}\big(\pm \varphi' \pm i \pm j \pm k\big),$$

with sign choices constrained so that the norm equals 1. A convenient generating pair is

$$\rho(i) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \rho\Big(\tfrac{\varphi+i+j+k}{2}\Big) = \tfrac{1}{2}\begin{pmatrix} \varphi+i & 1+i \\ -1+i & \varphi-i \end{pmatrix}.$$

Let $T^*, O^*, I^* \subset \mathrm{SL}_2(\mathbb{C})$ be the binary tetrahedral, octahedral, and icosahedral groups. For the natural action on $\mathbb{C}[x,y]$, the invariant ring $\mathbb{C}[x,y]^G$ ($G \in \{T^*, O^*, I^*\}$) is a hypersurface: there exist homogeneous generators X, Y, Z (of suitable positive degrees) such that, after a linear rescaling of variables over $\mathbb{C}$, one has the weighted – homogeneous normal forms

$$\begin{aligned}
&\mathbf{E}_6\ (\textbf{for } G = T^*)\textbf{:} && \mathbb{C}[x,y]^{T^*} \simeq \mathbb{C}[X,Y,Z]/\big(X^2 + Y^3 + Z^4\big),\\
&\mathbf{E}_7\ (\textbf{for } G = O^*)\textbf{:} && \mathbb{C}[x,y]^{O^*} \simeq \mathbb{C}[X,Y,Z]/\big(X^2 + Y^3 + YZ^3\big),\\
&\mathbf{E}_8\ (\textbf{for } G = I^*)\textbf{:} && \mathbb{C}[x,y]^{I^*} \simeq \mathbb{C}[X,Y,Z]/\big(X^2 + Y^3 + Z^5\big).
\end{aligned}$$

These define the Kleinian (Du Val, simple) surface singularities of types E_6, E_7, E_8.

Weights. Each equation is weighted – homogeneous; a convenient choice of weights is

$$\begin{aligned}
E_6:&\quad 2\,w_X = 3\,w_Y = 4\,w_Z = 12 && \Rightarrow\ (w_X, w_Y, w_Z) = (6,4,3),\\
E_7:&\quad 2\,w_X = 3\,w_Y = w_Y + 3\,w_Z = 18 && \Rightarrow\ (w_X, w_Y, w_Z) = (9,6,4),\\
E_8:&\quad 2\,w_X = 3\,w_Y = 5\,w_Z = 30 && \Rightarrow\ (w_X, w_Y, w_Z) = (15,10,6).
\end{aligned}$$

Remark. Explicit Klein invariants (X, Y, Z) for T^*, O^*, I^* can be written down (classically via binary forms), but any such choice differs only by an invertible linear change and scalar rescaling; the resulting hypersurface is always one of the three normal forms displayed above.