**Theorem 0.1.** *There exists the following functorial isomorphism*

$$\alpha_{X,Y} \colon \operatorname{Hom}_{\text{mod-}k}(X,Y) \simeq \operatorname{Hom}_{\text{mod-}k}(Y,X)^{\vee}.$$

*namely, Serre functor is identity in* mod-$k$*.*

*Proof.*

$$\begin{array}{rcc} \colon \operatorname{Hom}_{\text{mod-}k}(Y,X) \times \operatorname{Hom}_{\text{mod-}k}(X,Y) & \longrightarrow & k \\ \Psi & & \Psi \\ (g,f) & \longmapsto & \operatorname{tr}(g \circ f) \end{array} .$$

First we prove this mapping defines perfect bilinear forms. Let $a_1, a_2 \in k$.

$$\begin{aligned} \operatorname{tr}((a_1 g_1 + a_2 g_2) \circ f) &= \operatorname{tr}(a_1 g_1 \circ f + g_2 \circ f) = a_1 \operatorname{tr}(g_1 \circ f) + a_2 \operatorname{tr}(g_2 \circ f) \\ \operatorname{tr}(g \circ (a_1 f_1 + a_2 f_2)) &= \operatorname{tr}(a_1 g \circ f_1 + a_2 g \circ f_2) = a_1 \operatorname{tr}(g \circ f_1) + a_2 \operatorname{tr}(g \circ f_2) \end{aligned}$$

Next, we prove nondegeneracy. Let $f \neq 0 \colon X \to Y$. Choose $x \in X$ with $f(x) \neq 0$ and pick $\varphi \in Y^{\vee}$ with $\varphi(f(x)) \neq 0$.Define $g \colon Y \to X$ by

$$g(y) = \varphi(y)x.$$

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