$$\eta_{\mu\nu} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{1}$$

$$F_{\mu\nu} := \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix} \quad F^{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix} \tag{2}$$

This tensor is called Faraday tensor and staisfies the following equation.

$$\partial^\gamma F^{\beta\nu} + \partial^\beta F^{\nu\gamma} + \partial^\nu F^{\gamma\beta} = 0 \tag{3}$$

This means

$$\begin{cases} \nabla \times E = -\dfrac{\partial B}{\partial t} \\ \nabla \cdot B = 0 \end{cases} \tag{4}$$

$$\mathcal{J}^\nu := \frac{1}{\mu_0}\partial_\mu F^{\mu\nu} \tag{5}$$

If you denote $\mathcal{J} = {}^t(\rho, J) = {}^t(\rho, j_x, j_y, j_z)$

$$\begin{cases} \nabla \cdot E = \mu_0 \rho \\ \nabla \times B = \mu_0 J + \dfrac{\partial E}{\partial t} \end{cases} \tag{6}$$

$$\mathcal{T}^{\mu\nu} := \frac{1}{\mu_0}\left(F^{\mu\alpha}\eta_{\alpha\beta}F^{\beta\nu} + \frac{1}{4}\eta^{\mu\nu}F^{\beta\gamma}F_{\beta\gamma}\right) \tag{7}$$

This tensor is called energy-momentum tensor.

Thm

$$\partial_\mu \mathcal{T}^{\mu\nu} = \mathcal{J}_\mu F^{\mu\nu} \tag{8}$$

Proof. From (3)

$$\partial_\mu \mathcal{T}^{\mu\nu} = \frac{1}{\mu_0}\left(\partial_\mu F^{\mu\alpha}\eta_{\alpha\beta}F^{\beta\nu} + \frac{1}{4}\partial_\mu \eta^{\mu\nu}F^{\beta\gamma}F_{\beta\gamma}\right)$$

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