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Frobenius Frobenius [0] Frobenius 1980 B. Dubrovin 2 nius Frobenius Frobenius Frobenius potential Frobenius Frobenius potenial WDVV Frobenius WDVV Witten, Dijkgraaf, H. Verlinde, E. Verlinde

Frobenius Frobenius Frobenius Frobenius WDVV Frobenius potential [0, 0]

2 [0] Weierstrass $\wp$ Frobenius potential Frobenius potential WDVV $\mu \in \mathbb{Z}_{\geq 4}$ $M := (\mathbb{C} \times \mathbb{C}^*) \times (\mathbb{C}^*)^2 \times \mathbb{C}^{\mu-4}$ $(s_1, \cdots, s_\mu)$ ∞, 0 1 e^{s_3} $(\mu - 3)$ $\mathbb{P}^1$

$$F(z; s_1, s_2, \cdots, s_\mu) := z + s_1 + \frac{e^{s_2}}{z} + \sum_{i=1}^{\mu-3} \frac{z e^{(i-1)s_3} s_{i+3} s_\mu^{i-1}}{(z - e^{s_3})^i}$$

M $\frac{dz}{z}$ Frobenius Frobenius

$\mu = 4, 5, 6, 7, 8$ Frobenius $(M, \circ, \boldsymbol{e}, E, \eta)$ F Frobenius potential Mathematica Ma-Zuo [0] $z = \infty$ $\mu - 1$ $z = 0$ 1 $z = e^{s_\mu}$ $\mu - 3$ Frobenius potential $\mu = 4, 5$ [0] Frobenius potential $\mu = 6$

$\mu = 6$ $F(z; s_1, s_2, s_3, s_4, s_5, s_6) = z + \frac{e^{s_2}}{z} + \frac{z s_4}{z - e^{s_3}} + \frac{z e^{s_3} s_5 s_6}{(z - e^{s_3})^2} + \frac{z e^{2 s_3} s_6^3}{(z - e^{s_3})^3} + s_1$ $t_1 = s_1,\ t_2 = s_2,\ t_3 = s_3,\ t_4 = s_4,\ t_5 = s_5 - \frac{1}{2} s_6^2,\ t_6 = s_6$ $(t_1, t_2, t_3, t_4, t_5, t_6)$ Frobenius potential $\mathcal{F}$

. $\left(F, \frac{dz}{z}\right)$ $M := \mathbb{C}^3 \times (\mathbb{C}^*)^3$ *Frobenius* $(\circ, e = \partial_{t_1}, E = t_1 \partial_{t_1} + 2\partial_{t_2} + \partial_{t_3} + t_4 \partial_{t_4} + \frac{2}{3} t_5 \partial_{t_5} + \frac{1}{3} t_6 \partial_{t_6}, \eta)$ *Frobenius potential*
Frobenius potential $\mathcal{F}$

$$\begin{aligned}\mathcal{F}(t_1, t_2, t_3, t_4, t_5, t_6) =& \frac{1}{2} t_1^2 t_2 + t_1 t_3 t_4 + t_1 t_5 t_6 + \frac{1}{2} t_3 t_4^2 + \frac{t_4 t_5^2}{6 t_6} + \frac{1}{2} t_4 t_5 t_6 + \frac{1}{24} t_4 t_6^3 - \frac{t_5^4}{108 t_6^2} \\ & - \frac{1}{24} t_5^2 t_6^2 - \frac{1}{960} t_6^6 + \frac{1}{2} t_4^2 \log t_6 + e^{t_2} - e^{t_2 - t_3} \left(t_4 - t_5 t_6 + \frac{1}{2} t_6^3\right) \\ & + e^{t_3} \left(t_4 + t_5 t_6 + \frac{1}{2} t_6^3\right)\end{aligned}$$

. $\left(F, \frac{dz}{z}\right)$ $M := (\mathbb{C} \times \mathbb{C}^*) \times (\mathbb{C}^*)^2 \times \mathbb{C}^{\mu-4}$ *Frobenius* $(t_1, \cdots, t_\mu)$ *Frobenius potential* $f, g \in \mathbb{Q}[t_4, t_5, \cdots, t_\mu]$ $q \in \mathbb{Q}[t_\mu, t_\mu^{-1}][t_4, t_5, \cdots, t_{\mu-1}]$

$$\begin{aligned}\mathcal{F}(\boldsymbol{t}) = & t_1 \left(\frac{1}{2} t_2^2 + t_3 t_4 + t_5 t_\mu + \frac{1}{2(\mu - 3)} \sum_{k=6}^{\mu} t_k t_{\mu-k+5}\right) + q(t_3, t_4, \cdots, t_{\mu-1}, t_\mu) \\ & + e^{t_2 - t_3} \cdot f(t_4, t_5, \cdots, t_\mu) + e^{t_3} \cdot g(t_4, t_5, \cdots, t_\mu) + e^{t_2} + \frac{1}{2} t_4^2 \log t_\mu\end{aligned}$$

$f,\ g,\ q$

o1. f g

o2. q $() - () = 2$

t_1 e^{t_2}, $\frac{1}{2} t_4^2 \log t_\mu$ $\boldsymbol{s}$ $\boldsymbol{t}$ [0, 0, 0, 0] f g q 2

Frobenius Frobenius [0] Frobenius Frobenius

.1. $M = (M, \mathcal{O}_M)$ μ $\mathcal{T}_M$ Ω_M^1 M μ d Frobenius (Frobenius structure of rank μ and dimension d)

$\delta, \delta', \delta'' \in \mathcal{T}_M$

$$\nabla\!\!\!/_{\delta}(C_{\delta'}\delta'') - C_{\delta'}(\nabla\!\!\!/_{\delta}\delta'') - C_{\nabla\!\!\!/_{\delta}\delta'}\delta'' = \nabla\!\!\!/_{\delta'}(C_{\delta}\delta'') - C_{\delta}(\nabla\!\!\!/_{\delta'}\delta'') - C_{\nabla\!\!\!/_{\delta'}\delta}\delta'' \tag{4}$$

(iv) $\circ$ e $\nabla\!\!\!/$-

$$\nabla\!\!\!/ e = 0 \tag{5}$$

$(v) d$ Euler E Lie Lie_E $\circ$ η 1 $2-d$

$$Lie_E(\circ) = \circ, \quad Lie_E(\eta) = (2-d)\eta \tag{6}$$

$\delta, \delta' \in \mathcal{T}_M$

$$E(\eta(\delta, \delta')) - \eta([E, \delta], \delta') - \eta(\delta, [E, \delta']) = (2-d)\eta(\delta, \delta') \tag{7}$$

$$[E, \delta \circ \delta'] - [E, \delta] \circ \delta' - \delta \circ [E, \delta'] = \delta \circ \delta' \tag{8}$$

.2. Frobenius $(\eta, \circ, e, E)$ M Frobenius (Frobenius manifold)

.3 ([0])**.** $f : \mathbb{C}^3 \to \mathbb{C}$ ADE ADE

$$\begin{aligned}
f(x,y,z) =& x^{\mu+1} + yz & h = \mu + 1 \quad & A_\mu \\
f(x,y,z) =& x^2 y + y^{\mu-1} + z^2 & h = 2(\mu-1) \quad & D_\mu \\
f(x,y,z) =& x^3 + y^4 + z^2 & h = 12 \quad & E_6 \\
f(x,y,z) =& x^3 + xy^3 + z^2 & h = 18 \quad & E_7 \\
f(x,y,z) =& x^3 + y^5 + z^2 & h = 30 \quad & E_8
\end{aligned}$$

h f “ ” $S := \mathbb{C}^\mu$ (universal unfolding) $F : \mathbb{C}^3 \times S \to \mathbb{C}$

$$Jac(f) := \mathcal{O}_{\mathbb{C}^3} \Big/ \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) \tag{9}$$

$1 =: \phi_0(x,y,z), \ldots, \phi_\mu(x,y,z)$

$$F(x,y,z; s_1 \ldots, s_n) := f(x,y,z) + \sum_{i=1}^{\mu} s_i \cdot \phi_i(x,y,z) \tag{10}$$

$p : \mathbb{C}^3 \times S \to S$ F $\mathcal{C}$ F Jacobi $Jac(F)$

$$Jac(F) := p_* \mathcal{O}_{\mathcal{C}} = \mathcal{O}_{\mathbb{C}^3 \times S} \Big/ \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right) \tag{11}$$

$\mathcal{O}_S$-

$$\mathcal{T}_S \;\cong\; Jac(F), \quad \delta \mapsto \widehat{\delta} F|_{\mathcal{C}} \tag{12}$$

$\widehat{\delta}$ δ x, y, z $\mathcal{T}_{\mathbb{C}^3\times S}$ $\circ : \mathcal{T}_S \times \mathcal{T}_S \to \mathcal{T}_S$

$$\widehat{(\delta \circ \delta')}F|_{\mathcal{C}} := \widehat{\delta}F|_{\mathcal{C}} \cdot \widehat{\delta}'F|_{\mathcal{C}} \tag{13}$$

$e, E \in \mathcal{T}_S$

$$\widehat{e}F|_{\mathcal{C}} = 1, \quad \widehat{E}F|_{\mathcal{C}} = F|_{\mathcal{C}} \tag{14}$$

$\eta : \mathcal{T}_S \times \mathcal{T}_S \to \mathcal{O}_S$

$$\eta(\delta, \delta') := Res_{\mathbb{C}^3\times S/S} \begin{bmatrix} (\widehat{\delta}F \cdot \widehat{\delta}'F)dx \wedge dy \wedge dz \\ \frac{\partial F}{\partial x}\frac{\partial F}{\partial y}\frac{\partial F}{\partial z} \end{bmatrix} \tag{15}$$

$dx \wedge dy \wedge dz$ $(S, \eta, \circ, e, E)$ μ $1 - \frac{2}{h}$ Frobenius

Frobenius [0]
$\nabla\!\!\!/$

$$\mathcal{T}_M^f := \ker \nabla\!\!\!/ = \{\delta \in \mathcal{T}_M \mid \quad \delta' \in \mathcal{T}_M \quad \nabla\!\!\!/_{\delta'}\delta = 0 \quad \} \tag{16}$$

.4 $(i)\mathcal{T}_M^f$ M μ $\mathcal{T}_M^f$ M $\mathbb{C}_M$ $\bigoplus_{i=1}^{\mu}\mathbb{C}_M$

$(ii)\eta$ $\mathbb{C}_M$-

$$\eta : \mathcal{T}_M^f \times \mathcal{T}_M^f \longrightarrow \mathbb{C}_M \tag{17}$$

$(iii)\mathcal{O}_M$- $Q \in End_{\mathcal{O}_M}(\mathcal{T}_M)$ $Q := id_{\mathcal{T}_M} - \nabla\!\!\!/ E$ Q $End_{\mathbb{C}_M}(\mathcal{T}_M^f)$ $Qe = 0$

$$\eta(Q\delta, \delta') + \eta(\delta, Q\delta') = d \cdot \eta(\delta, \delta'), \quad \delta, \delta' \in \mathcal{T}_M^f \tag{18}$$

.5. M $(t_1, \ldots, t_\mu)$

$(i)e = \partial_1$

$(ii)\mathcal{T}_M^f = \bigoplus_{i=1}^{\mu}\mathbb{C}_M\partial_i$

(iii)Euler E

$$E = \sum_{i=1}^{\mu}\left\{(1 - q_i)t^i + c_i\right\}\partial_i \tag{19}$$

$q_i \neq 1$ $c_i = 0$

$\partial_i := \frac{\partial}{\partial t_i}$

Q

$$Q\frac{\partial}{\partial t^i} = \frac{\partial}{\partial t^i} - \nabla_{\frac{\partial}{\partial t^i}}E = \frac{\partial}{\partial t^i} - \left[E, \frac{\partial}{\partial t^i}\right] = q_i\frac{\partial}{\partial t^i} \tag{20}$$

.6. $(M, \eta, \circ, e, E)$ 2 d Frobenius (t^1, t^2) (20) 18 $q_1 = 0,\ q_1 + q_2 = d$

$$E = t^1\frac{\partial}{\partial t^1} + \{(1 - d) + r\}\frac{\partial}{\partial t^2} \tag{21}$$

.1 (iii)

.7. M $\mathcal{F}$

$$\eta(\partial_i \circ \partial_j, \partial_k) = \eta(\partial_i, \partial_j \circ \partial_k) = \partial_i\partial_j\partial_k\mathcal{F} \quad i,j,k = 1,\ldots,\mu \tag{22}$$

$$E\mathcal{F} = (3-d)\mathcal{F} + (\quad 2 \quad) \tag{23}$$

$$\eta_{ij} := \eta(\partial_i, \partial_j) = \partial_1\partial_i\partial_j\mathcal{F} \tag{24}$$

$\mathcal{F}$ Frobenius potential

.8. .5 .7 $(t^1,\ldots,t^\mu)$ Frobenius potential$\mathcal{F}$ Frobenius (23) Frobenius Frobenius potential

$\circ : \mathcal{T}_M \times \mathcal{T}_M \to \mathcal{T}_M$ Frobenius potential 3

.9. $\mathcal{F}$ Frobenius potential $i,j,k,\ell \in \{1,\ldots,\mu\}$ WDVV

$$\sum_{a,b=1}^{\mu} \partial_i\partial_j\partial_a\mathcal{F} \cdot \eta^{ab} \cdot \partial_b\partial_k\partial_\ell\mathcal{F} = \sum_{a,b=1}^{\mu} \partial_i\partial_k\partial_a\mathcal{F} \cdot \eta^{ab} \cdot \partial_b\partial_j\partial_\ell\mathcal{F} \tag{25}$$

$(\eta^{ab}) := (\eta_{ab})^{-1}$

.10 ([0, Example 1.1])**.** 2 d Frobenius Frobenius potential

$$\mathcal{F}(t^1,t^2) = \frac{1}{2}\eta_{12}(t^1)^2t^2 + c(t^2)^{\frac{3-d}{1-d}}, \quad d \neq -1,1,3 \tag{26}$$

$$\mathcal{F}(t^1,t^2) = \frac{1}{2}\eta_{12}(t^1)^2t^2 + c(t^2)^2\log t^2, \quad d = -1 \tag{27}$$

$$\mathcal{F}(t^1,t^2) = \frac{1}{2}\eta_{12}(t^1)^2t^2 + c\log t^2, \quad d = 3 \tag{28}$$

$$\mathcal{F}(t^1,t^2) = \frac{1}{2}\eta_{12}(t^1)^2t^2 + c\exp\left(\frac{2}{r}t^2\right), \quad d = 1,\ r \neq 0 \tag{29}$$

$$\mathcal{F}(t^1,t^2) = \frac{1}{2}\eta_{12}(t^1)^2t^2, \quad d = 1,\ r = 0 \tag{30}$$

(t^1,t^2) $\eta_{12} \in \mathbb{C}\backslash\{0\},\ c \in \mathbb{C}$ (1.1.27)

F μ 4 μ $M := (\mathbb{C}\times\mathbb{C}^*)\times(\mathbb{C}^*)^2\times\mathbb{C}^{\mu-4}$ M $(s_1,s_2,\cdots,s_\mu)$ $\mathbb{P}^1$ F ∞, 0 1 e^{s_3} $\mu-3$ F

$$F(z;s_1,s_2,\cdots,s_\mu) := z + s_1 + \frac{e^{s_2}}{z} + \sum_{i=1}^{\mu-3}\frac{ze^{(i-1)s_3}s_{i+3}s_\mu^{i-1}}{(z-e^{s_3})^i}. \tag{31}$$

$\circ\mathfrak{X} := \mathbb{P}^1 \times M \setminus F^{-1}(\infty)$ $p : \mathfrak{X} \to M$ F $\mathcal{C}$ F Jacobi $\mathrm{Jac}(F)$

$$\mathrm{Jac}(F) := p_*\mathcal{O}_\mathcal{C} = p_*\mathcal{O}_\mathfrak{X} \Big/ \left(\frac{\partial F}{\partial z}\right) \tag{32}$$

$\mathcal{O}_M$

$$\mathcal{T}_M \simeq \mathrm{Jac}(F),\ \delta \mapsto \widehat{\delta} F|_{\mathcal{C}} \tag{33}$$

$\widehat{\delta}$ δ z $\mathcal{T}_{\mathfrak{X}}$ $\circ : \mathcal{T}_M \times \mathcal{T}_M \to \mathcal{T}_M$

$$\widehat{(\delta \circ \delta')} F|_{\mathcal{C}} := \widehat{\delta} F|_{\mathcal{C}} \cdot \widehat{\delta'} F|_{\mathcal{C}} \tag{34}$$

$\boldsymbol{e}$ $\boldsymbol{e} \in \mathcal{T}_M$ $\widehat{\boldsymbol{e}} F|_{\mathcal{C}} = 1$ $\circ$

Euler E $E \in \mathcal{T}_M$ $\widehat{E} F|_{\mathcal{C}} = F|_{\mathcal{C}}$ Euler

$$E = s_1 \frac{\partial}{\partial s_1} + 2 \frac{\partial}{\partial s_2} + \frac{\partial}{\partial s_3} + \sum_{k=1}^{\mu-3} \frac{(\mu - 2 - k)}{\mu - 3} s_{k+3} \frac{\partial}{\partial s_{k+3}}$$

$\frac{dz}{z}$ [0, 0] 1 μ Frobenius M

$\mathcal{O}_M$ ηM $\mathcal{O}_M$ $\eta : \mathcal{T}_M \times \mathcal{T}_M \to \mathcal{O}_M$

$$\eta(\partial_{s_i}, \partial_{s_j}) = \frac{1}{2\pi\sqrt{-1}} \int_{|\frac{\partial F}{\partial z}| = \epsilon} \frac{\partial_{s_i} F \cdot \partial_{s_j} F}{z \partial_z F} \cdot \frac{dz}{z}$$

η_{ij}

.11. $\mathcal{O}_M$ η 2 $\left(\frac{dz}{z}\right)^2$ $Jac(F)$ $\frac{dz}{z}$ $\Omega_F := p_* \Omega^1_{\mathfrak{X}/M} / dF$ η Ω_F $\mathcal{O}_M$- $\mathcal{T}_M$

Frobenius potential $\mathcal{F} F$

$$\eta(\partial_{s_i}, \partial_{s_j}) = \frac{1}{2\pi\sqrt{-1}} \int_{|\frac{\partial F}{\partial z}| = \epsilon} \frac{\partial_{s_i} F \cdot \partial_{s_j} F}{z \partial_z F} \cdot \frac{dz}{z} \tag{35}$$

[0, Lemma 4.3]

.12. $\boldsymbol{s} = (s_1, s_2, \cdots, s_\mu)$

$$t_1 := s_1,\ t_2 := s_2,\ t_3 := s_3,\ t_4 := s_4,\ t_\mu := s_\mu, \tag{36}$$

$$t_i = q_i(s_i, s_{i+1}, \cdots, s_\mu)\ \left.\frac{\partial t_i}{\partial s_j}\right|_{s_5 = \cdots = s_{i-1} = s_{i+1} = \cdots = s_{\mu-1} = 0} = \delta^i_j$$

$\boldsymbol{t} = (t_1, t_2, \cdots, t_\mu)$ $i = 5, 6, \cdots, \mu - 1$ $q_i \in \mathbb{Q}[t_\mu, t_\mu^{-1}][t_i, \cdots, t_{\mu-1}]$ δ^i_j *Kronecker delta*

.13.

$$\frac{\partial^3 \mathcal{F}}{\partial t_i \partial t_j \partial t_k} = \eta(\partial_{t_i} \circ \partial_{t_j}, \partial_{t_k}) = \frac{1}{2\pi\sqrt{-1}} \int_{|\frac{\partial F}{\partial z}| = \epsilon} \frac{\partial_{t_i} F \cdot \partial_{t_j} F \cdot \partial_{t_k} F}{z \partial_z F} \cdot \frac{dz}{z} \tag{37}$$

$\mathcal{F}$ $\left(F, \frac{dz}{z}\right)$ Frobenius potential

$\frac{\partial^3 \mathcal{F}}{\partial t_i \partial t_j \partial t_k} = \eta(\partial_{t_i} \circ \partial_{t_j}, \partial_{t_k})$ c_{ijk}

$\mu = 4$

.14. $M := \mathbb{C}\times(\mathbb{C}^*)^3$ M (s_1,s_2,s_3,s_4) $\mathbb{P}^1$ F

$$F(z;s_1,s_2,s_3,s_4)=z+\frac{e^{s_2}}{z}+\frac{zs_4}{z-e^{s_3}}+s_1. \tag{38}$$

(s_1,s_2,s_3,s_4)

$.\eta$

$$(\eta(\partial_{s_i},\partial_{s_j}))=\begin{pmatrix}0100\\1000\\0001\\0010\end{pmatrix} \tag{39}$$

$\{s_i\}_i$ □

$t_1:=s_1$ $t_2:=s_2$ $t_3:=s_3$ $t_4:=s_4$ (t_1,t_2,t_3,t_4)

.15. $M := \mathbb{C}\times(\mathbb{C}^*)^3$ *Frobenius* $(\circ,\ e=\partial_{t_1},\ E=t_1\partial_{t_1}+2\partial_{t_2}+\partial_{t_3}+t_4\partial_{t_4},\ \eta)$ $\left(F,\dfrac{dz}{z}\right)$ *Frobenius potential*

$$\mathcal{F}(t_1,t_2,t_3,t_4)=\frac{1}{2}t_1^2t_2+t_1t_3t_4+\frac{1}{2}t_3t_4^2+e^{t_2}-t_4e^{t_2-t_3}+t_4e^{t_3}+\frac{1}{2}t_4^2\log t_4. \tag{40}$$

. c_{ijk} Frobenius potential A $(i,j,k);A$
$(1,1,2);\frac{1}{2}t_1^2t_2,(1,3,4);t_1t_3t_4,(2,2,2);e^{t_2}-t_4e^{t_2-t_3},(3,3,3);t_4e^{t_3},(3,4,4);\frac{1}{2}t_3t_4^2,(4,4,4);\frac{1}{2}t_4^2\log t_4$
Frobenius potental □

Frobenius potential $\mathcal{F}$ WDVV Mathematica
([0]). $F=z+\dfrac{e^{s_2}}{z}+s_1$ η

$$(\eta_{ij})=\begin{pmatrix}01\\10\end{pmatrix}.$$

s_1,s_2 t_1,t_2 *Frobenius potential*

$$\mathcal{F}=\frac{1}{2}t_1^2t_2+e^{t_2}$$

$\mu=4$ t_3,t_4
$\mu=5$

.16. $M := \mathbb{C}^2\times(\mathbb{C}^*)^3$ M (s_1,s_2,s_3,s_4,s_5) $\mathbb{P}^1$ F

$$F(z;s_1,s_2,s_3,s_4,s_5)=z+\frac{e^{s_2}}{z}+\frac{zs_4}{z-e^{s_3}}+\frac{ze^{s_3}s_5^2}{(z-e^{s_3})^2}+s_1. \tag{41}$$

(s_1,s_2,s_3,s_4,s_5)

$.\eta$

$$(\eta(\partial_{s_i},\partial_{s_j})) = \begin{pmatrix} 01000 \\ 10000 \\ 00010 \\ 00100 \\ 00001 \end{pmatrix} \tag{42}$$

$\{s_i\}_i$ □

$\mu = 4$ (s_1,s_2,s_3,s_4,s_5) (t_1,t_2,t_3,t_4,t_5)

.17. $M := \mathbb{C}^2 \times (\mathbb{C}^*)^3$ *Frobenius* $(\circ, e = \partial_{t_1}, E = t_1\partial_{t_1} + 2\partial_{t_2} + \partial_{t_3} + t_4\partial_{t_4} + \frac{1}{2}t_5\partial_{t_5}, \eta)$ $\left(F, \frac{dz}{z}\right)$ *Frobenius potential*

$$\begin{aligned}\mathcal{F}(t_1,t_2,t_3,t_4,t_5) =& \frac{1}{2}t_1^2t_2 + t_1t_3t_4 + t_1t_5^2 + \frac{1}{2}t_3t_4^2 + \frac{1}{2}t_4t_5^2 - \frac{1}{24}t_5^4 + e^{t_2} - t_4e^{t_2-t_3} + t_5^2e^{t_2-t_3} \\ &+ t_4e^{t_3} + t_5^2e^{t_3} + \frac{1}{2}t_4^2\log t_5.\end{aligned} \tag{43}$$

.

$(1,1,2); \frac{1}{2}t_1^2t_2,\ (1,3,4); t_1t_3t_4,\ (1,5,5); t_1t_5^2,\ (2,2,2); e^{t_2} - t_4e^{t_2-t_3} + t_5^2e^{t_2-t_3},\ (3,3,3); t_4e^{t_3} + t_5^2e^{t_3},$
$(3,4,4); \frac{1}{2}t_3t_4^2,\ (4,4,5); \frac{1}{2}t_4^2\log t_5,\ (4,5,5); \frac{1}{2}t_4t_5^2,\ (5,5,5); -\frac{1}{24}t_5^4$

Frobenius potental □

Frobenius potential $\mathcal{F}$ WDVV Mathematica

$\mu = 6$

.18. $M := \mathbb{C}^3 \times (\mathbb{C}^*)^3$ M $(s_1,s_2,s_3,s_4,s_5,s_6)$ $\mathbb{P}^1$ F

$$F(z;s_1,s_2,s_3,s_4,s_5,s_6) = z + \frac{e^{s_2}}{z} + \frac{zs_4}{z-e^{s_3}} + \frac{ze^{s_3}s_5s_6}{(z-e^{s_3})^2} + \frac{ze^{2s_3}s_6^3}{(z-e^{s_3})^3} + s_1. \tag{44}$$

$t_1 = s_1,\ t_2 = s_2,\ t_3 = s_3,\ t_4 = s_4,\ t_5 = s_5 - \frac{1}{2}s_6^2,\ t_6 = s_6$ $(t_1,t_2,t_3,t_4,t_5,t_6)$

. $(s_1,s_2,s_3,s_4,s_5,s_6)$ η

$$(\eta(\partial_{s_i},\partial_{s_j})) = \begin{pmatrix} 01000 & 0 \\ 10000 & 0 \\ 00010 & 0 \\ 00100 & 0 \\ 00000 & 1 \\ 00001 & -2s_6 \end{pmatrix} \tag{45}$$

$\{s_i\}_i$ s_5s_6 t_5, t_6 1 $s_5s_6 = t_5t_6 + at_6^3$, $(a \in \mathbb{C})$ $(6,6)$ $2(1-2a)t_6$ 0 $a = \frac{1}{2}$ F :

$$F(z;t_1,t_2,t_3,t_4,t_5,t_6) = z + \frac{e^{t_2}}{z} + \frac{zt_4}{z-e^{t_3}} + \frac{ze^{t_3}(t_5t_6 + \frac{1}{2}t_6^3)}{(z-e^{t_3})^2} + \frac{ze^{2t_3}t_6^3}{(z-e^{t_3})^3} + t_1. \tag{46}$$

F η

$$(\eta_{ij}) = \begin{pmatrix} 010000 \\ 100000 \\ 000100 \\ 001000 \\ 000001 \\ 000010 \end{pmatrix} \tag{47}$$

$\{t_i\}_i$ □

.19. $M := \mathbb{C}^3 \times (\mathbb{C}^*)^3$ *Frobenius* $(\circ, e = \partial_{t_1}, E = t_1\partial_{t_1} + 2\partial_{t_2} + \partial_{t_3} + t_4\partial_{t_4} + \frac{2}{3}t_5\partial_{t_5} + \frac{1}{3}t_6\partial_{t_6}, \eta)$ $\left(F, \frac{dz}{z}\right)$ *Frobenius potential*

$$\begin{aligned} &\mathcal{F}(t_1, t_2, t_3, t_4, t_5, t_6) \\ &= \frac{1}{2}t_1^2 t_2 + t_1 t_3 t_4 + t_1 t_5 t_6 + \frac{1}{2}t_3 t_4^2 + \frac{1}{6}\frac{t_4 t_5^2}{t_6} + \frac{1}{2}t_4 t_5 t_6 + \frac{1}{24}t_4 t_6^3 - \frac{1}{108}\frac{t_5^4}{t_6^2} \\ &- \frac{1}{24}t_5^2 t_6^2 - \frac{1}{960}t_6^6 + \frac{1}{2}t_4^2 \log t_6 + e^{t_2} - e^{t_2 - t_3}\left(t_4 - t_5 t_6 + \frac{1}{2}t_6^3\right) \\ &+ e^{t_3}\left(t_4 + t_5 t_6 + \frac{1}{2}t_6^3\right). \end{aligned} \tag{48}$$

.

$$\begin{aligned} &(1,1,2); \frac{1}{2}t_1^2 t_2,\ (1,3,4); t_1 t_3 t_4,\ (1,5,6); t_1 t_5 t_6, (2,2,2); e^{t_2} - t_4 e^{t_2 - t_3} + t_5 t_6 e^{t_2 - t_3} - \frac{1}{2}t_6^3 e^{t_2 - t_3}, \\ &(3,3,3); t_4 e^{t_3} + t_5 t_6 e^{t_3} + \frac{1}{2}t_6^3 e^{t_3},\ (3,4,4); \frac{1}{2}t_3 t_4^2,\ (4,4,6); \frac{1}{2}t_4^2 \log t_6,\ (4,5,5); \frac{1}{6}\frac{t_4 t_5^2}{t_6}, \\ &(4,5,6); \frac{1}{2}t_4 t_5 t_6,\ (4,6,6,); \frac{1}{24}t_4 t_6^3,\ (5,6,6); -\frac{1}{108}\frac{t_5^4}{t_6^2} - \frac{1}{24}t_5^2 t_6^2,\ (6,6,6); -\frac{1}{960}t_6^6 \end{aligned}$$

Frobenius potental □

Frobenius potential $\mathcal{F}$ WDVV Mathematica

$\mu = 7$

.20. $M := \mathbb{C}^4 \times (\mathbb{C}^*)^3$ M $(s_1, s_2, s_3, s_4, s_5, s_6, s_7)$ $\mathbb{P}^1$ F

$$F(z; s_1, s_2, s_3, s_4, s_5, s_6, s_7) = z + \frac{e^{s_2}}{z} + \frac{z s_4}{z - e^{s_3}} + \frac{z e^{s_3} s_5 s_7}{(z - e^{s_3})^2} + \frac{z e^{2s_3} s_6 s_7^2}{(z - e^{s_3})^3} + \frac{z e^{3s_3} s_7^4}{(z - e^{s_3})^4} + s_1. \tag{49}$$

$t_1 = s_1,\ t_2 = s_2,\ t_3 = s_3,\ t_4 = s_4,\ t_5 = s_5 + \frac{s_6^2}{8s_7} + \frac{1}{2}s_6 s_7 + \frac{1}{6}s_7^3,\ t_6 = s_6 - s_7^2,\ t_7 = s_7$ $(t_1, t_2, t_3, t_4, t_5, t_6, t_7)$

. $(s_1, s_2, s_3, s_4, s_5, s_6, s_7)$ η

$$
(\eta(\partial_{s_i}, \partial_{s_j})) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & -\frac{s_6 + 3s_7^2}{4s_7} \\ 0 & 0 & 0 & 0 & 1 & -\frac{s_6 + 3s_7^2}{4s_7} & -\frac{-s_6^2 + 2s_6 s_7^2 - 9s_7^4}{4s_7^2} \end{pmatrix} \tag{50}
$$

$\{s_i\}_i$ $s_6 s_7^2$ $t_6,\ t_7$ 1 $s_5 s_7$ $t_5,\ t_6,\ t_7$ 1 $s_6 s_7^2 = t_6 t_7^2 + c_1 t_7^4,\ s_5 s_7 = t_5 t_7 + c_2 t_6 t_7 + c_3 t_6^2 + c_4 t_7^4$ $(c_i \in \mathbb{C}, i = 1, 2, \cdots, 4)$ $(6,7),\ (7,7)$

$$
\begin{aligned}
&(6,7); -\frac{(1 - 8c_3)t_6 + (3 - c_1 - 4c_2)t_7^2}{4t_7}, \\
&(7,7); -\frac{(-1 + 8c_3)t_6^2 + 2(1 + c_1 - 4c_2)t_6 t_7^2 - (9 - 14c_1 + c_1^2 + 24c_4)}{4t_7^2}.
\end{aligned}
$$

0

$$
\begin{cases} 1 - 8c_3 = 0 \\ 3 - c_1 - 4c_2 = 0 \\ 1 + c_1 - 4c_2 = 0 \\ 9 - 14c_1 + c_1^2 + 24c_4 = 0 \end{cases}
$$

$c_1 = 1,\ c_2 = \frac{1}{2},\ c_3 = \frac{1}{8},\ c_4 = \frac{1}{6}$ F :

$$
\begin{aligned}
F(z; t_1, t_2, t_3, t_4, t_5, t_6, t_7) = &z + \frac{e^{t_2}}{z} + \frac{z t_4}{z - e^{t_3}} + \frac{z e^{t_3}(t_5 t_7 + \frac{1}{8}t_6^2 + \frac{1}{2}t_6 t_7^2 + \frac{1}{6}t_7^4)}{(z - e^{t_3})^2} \\
&+ \frac{z e^{2t_3}(t_6 t_7^2 + t_7^4)}{(z - e^{t_3})^3} + \frac{z e^{3t_3} t_7^4}{(z - e^{t_3})^4} + t_1.
\end{aligned} \tag{51}
$$

F η

$$
(\eta_{ij}) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{52}
$$

$\{t_i\}_i$ $\square$

.21. $M := \mathbb{C}^4 \times (\mathbb{C}^*)^3$ *Frobenius* $(\circ,\ e = \partial_{t_1},\ E = t_1\partial_{t_1} + 2\partial_{t_2} + \partial_{t_3} + t_4\partial_{t_4} + \frac{3}{4}t_5\partial_{t_5} + \frac{2}{4}t_6\partial_{t_6} +$

$\dfrac{1}{4}t_7\partial_{t_7},\ \eta)$ $\left(F, \dfrac{dz}{z}\right)$ *Frobenius potential*

$$\begin{aligned}\mathcal{F}(t_1,t_2,t_3,t_4,t_5,t_6,t_7) &= \frac{1}{2}t_1^2t_2 + t_1t_3t_4 + t_1t_5t_7 + \frac{1}{8}t_1t_6^2 + e^{t_2} + \frac{1}{2}t_4^2\log t_7 \\ &+ \frac{1}{2}t_3t_4^2 + \frac{1}{16}t_4t_6^2 - \frac{1}{96}\frac{t_4t_6^3}{t_7^2} + \frac{1}{24}t_4t_6t_7^2 + \frac{1}{24}\frac{t_5^3}{t_7} - \frac{1}{32}\frac{t_5^2t_6^2}{t_7^2} - \frac{1}{24}t_5^2t_7^2 \\ &+ \frac{1}{256}\frac{t_5t_6^4}{t_7^3} - \frac{1}{96}t_5t_6^2t_7 + \frac{1}{720}t_5t_7^5 - \frac{1}{1536}t_6^4 - \frac{1}{6144}\frac{t_6^6}{t_7^4} - \frac{1}{1152}t_6^2t_7^4 \\ &- \frac{1}{18144}t_7^8 + e^{t_2-t_3}\left(-t_4 + \frac{1}{8}t_6^2 + t_5t_7 - \frac{1}{2}t_6t_7^2 + \frac{1}{6}t_7^4\right) \\ &+ e^{t_3}\left(t_4 + \frac{1}{8}t_6^2 + t_5t_7 + \frac{1}{2}t_6t_7^2 + \frac{1}{6}t_7^4\right). \end{aligned} \tag{53}$$

.

$(1,1,2); \dfrac{1}{2}t_1^2t_2,\ (1,3,4); t_1t_3t_4,\ (1,5,7); t_1t_5t_7,\ (1,6,6); \dfrac{1}{8}t_1t_6^2,$
$(2,2,2); e^{t_2} + e^{t_2-t_3}\left(-t_4 + \dfrac{1}{8}t_6^2 + t_5t_7 - \dfrac{1}{2}t_6t_7^2 + \dfrac{1}{6}t_7^4\right), (3,3,3); e^{t_3}\left(t_4 + \dfrac{1}{8}t_6^2 + t_5t_7 + \dfrac{1}{2}t_6t_7^2 + \dfrac{1}{6}t_7^4\right),$
$(3,4,4); \dfrac{1}{2}t_3t_4^2, (4,4,7); \dfrac{1}{2}t_4^2\log t_7,\ (4,6,6); \dfrac{1}{16}t_4t_6^2 - \dfrac{1}{96}\dfrac{t_4t_6^3}{t_7^2},\ (4,6,7); \dfrac{1}{24}t_4t_6t_7^2,\ (5,5,5); \dfrac{1}{24}\dfrac{t_5^3}{t_7},$
$(5,5,7); -\dfrac{1}{32}\dfrac{t_5^2t_6^2}{t_7^2} - \dfrac{1}{24}t_5^2t_7^2,\ (5,6,6); \dfrac{1}{256}\dfrac{t_5t_6^4}{t_7^3} - \dfrac{1}{96}t_5t_6^2t_7,\ (5,7,7); \dfrac{1}{720}t_5t_7^5,\ (6,6,6); -\dfrac{1}{1536}t_6^4 - \dfrac{1}{6144}\dfrac{t_6^6}{t_7^4},$
$(6,6,7); -\dfrac{1}{1152}t_6^2t_7^4,\ (7,7,7); -\dfrac{1}{18144}t_7^8.$

Frobenius potental □

Frobenius potential $\mathcal{F}$ WDVV Mathematica

$\mu = 8$

.22. $M := \mathbb{C}^5 \times (\mathbb{C}^*)^3$ M $\boldsymbol{s} = (s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8)$ $\mathbb{P}^1$ F

$$F(z;\boldsymbol{s}) = z + \frac{e^{s_2}}{z} + \frac{zs_4}{z-e^{s_3}} + \frac{ze^{s_3}s_5s_8}{(z-e^{s_3})^2} + \frac{ze^{2s_3}s_6s_8^2}{(z-e^{s_3})^3} + \frac{ze^{3s_3}s_7s_8^3}{(z-e^{s_3})^4} + \frac{ze^{4s_3}s_8^5}{(z-e^{s_3})^5} + s_1. \tag{54}$$

$t_1 := s_1,\ t_2 := s_2,\ t_3 := s_3,\ t_4 := s_4,\ t_5 := s_5 + \dfrac{17}{10}s_7^2 + \dfrac{s_7^3}{5s_8^2} - \dfrac{s_6s_7}{s_8} + s_6s_8 - \dfrac{49}{20}s_7s_8^2 + \dfrac{17}{30}s_8^4$ $t_6 := s_6 - \dfrac{s_7^2}{5s_8} - \dfrac{2}{5}s_7s_8 + \dfrac{7}{12}s_8^3,\ t_7 := s_7 - \dfrac{3}{2}t_8^2,\ t_8 = s_8$ $\boldsymbol{t} = (t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8)$

. $\boldsymbol{s}$ η

$$(\eta(\partial_{s_i},\partial_{s_j}))=$$
$$\begin{pmatrix}
0&1&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0\\
0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&\frac{1}{5}&-\frac{s_7+4s_8^2}{5s_8}\\
0&0&0&0&0&\frac{1}{5}&-\frac{4s_7+4s_8^2}{25s_8}&-\frac{-4s_7^2+5s_6s_8-5s_7s_8^2-16s_8^4+4s_8^2}{25s_8^2}\\
0&0&0&0&1&-\frac{s_7+4s_8^2}{5s_8}&-\frac{-4s_7^2+5s_6s_8-5s_7s_8^2-16s_8^4+4s_8^2}{25s_8^2}&-\frac{2(2s_7^3-5s_6s_7s_8+3s_7^2s_8^2+5s_6s_8^3-12s_7s_8^4+32s_8^6)}{25s_8^3}
\end{pmatrix} \tag{55}$$

$\boldsymbol{s}$ s_7s_8 t_7, t_8 1 $s_6s_8^2$ t_6, t_7, t_8 1 $s_5s_8^3$ t_5, t_6, t_7, t_8 1 $s_7s_8^3=t_7t_8^3+c_1t_8^5$, $s_6s_8^2=t_6t_8^2+c_2t_7^2t_8+c_3t_7t_8^3+c_4t_8^5$, $s_5s_8=t_5t_8+c_5t_6t_7+c_6t_6t_8^2+c_7t_7^2t_8+c_8t_7t_8^3+c_9t_8^5$ ($c_i\in\mathbb{C}, i=1,2,\cdots,9$) $(6,8)$, $(7,7)$, $(7,8)$, $(8,8)$

$$\begin{aligned}
&(6,8);-\frac{(1-5c_5)t_7+(4-c_1-5c_6)t_8^2}{5t_8},\\
&(7,7);-\frac{4(1-5c_2)t_7+2(2+2c_1-5c_3)t_8^2}{25t_8},\\
&(7,8);-\frac{1}{25t_8^2}\{-4(1-5c_2)t_7^2+5(1-5c_5)t_6t_8-5(1-8c_2+2c_1c_2-c_3+10c_7)t_7t_8^2\\
&\quad+(-16+3c_1+4c_1^2+20c_3-5c_1c_3-10c_4-25c_8)t_8^4\},\\
&(8,8);-\frac{2}{25t_8^3}\{2(1-5c_2)t_7^3+5(-1+5c_5)t_6t_7t_8+(3-2c_1-15c_2+10c_1c_2)t_7^2t_8^2\\
&\quad+5(1+c_1-5c_6)t_6t_8^3+(-12-4c_1-2c_1^2+25c_3+10c_4-50c_8)t_7t_8^4\\
&\quad+(32-44c_1+c_1^2+2c_1^3+65c_4-10c_1c_4-100c_9)t_8^6\}.
\end{aligned}$$

0

$$\begin{cases}
1-5c_5=0\\
4-c_1-5c_6=0\\
1-5c_2=0\\
2+2c_1-5c_3=0\\
1-8c_2+2c_1c_2-c_3+10c_7=0\\
-16+3c_1+4c_1^2+20c_3-5c_1c_3-10c_4-25c_8=0\\
3-2c_1-15c_2+10c_1c_2=0\\
1+c_1-5c_6=0\\
-12-4c_1-2c_1^2+25c_3+10c_4-50c_8=0\\
32-44c_1+c_1^2+2c_1^3+65c_4-10c_1c_4-100c_9=0
\end{cases}$$

$c_1=\dfrac{3}{2}$, $c_2=\dfrac{1}{5}$, $c_3=1$, $c_4=\dfrac{7}{12}$, $c_5=\dfrac{1}{5}$, $c_6=\dfrac{1}{2}$, $c_7\dfrac{1}{10}$, $c_8=\dfrac{1}{6}$, $c_9=\dfrac{1}{24}$ F

:

$$
\begin{aligned}
F(z;\boldsymbol{t}) = z + \frac{e^{t_2}}{z} + \frac{zt_4}{z-e^{t_3}} + \frac{ze^{t_3}(t_5t_8 + \frac{1}{5}t_6t_7 + \frac{1}{2}t_6t_8^2 + \frac{1}{6}t_7t_8^3 + \frac{1}{10}t_7^2t_8 + \frac{1}{24}t_8^5)}{(z-e^{t_3})^2} \\
+ \frac{ze^{2t_3}(t_6t_8^2 + t_7t_8^3 + \frac{1}{5}t_7^2t_8 + \frac{7}{12}t_8^5)}{(z-e^{t_3})^3} + \frac{ze^{3t_3}(t_7t_8^3 + \frac{3}{2}t_8^5)}{(z-e^{t_3})^4} + \frac{ze^{4t_3}t_8^5}{(z-e^{t_3})^5} + t_1.
\end{aligned} \tag{56}
$$

F η

$$
(\eta_{ij}) = \begin{pmatrix}
0&1&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0\\
0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&\frac{1}{5}&0\\
0&0&0&0&0&\frac{1}{5}&0&0\\
0&0&0&0&1&0&0&0
\end{pmatrix} \tag{57}
$$

$\boldsymbol{t}$ $\square$

.23. $M := \mathbb{C}^5 \times (\mathbb{C}^*)^3$ *Frobenius* $(\circ,\ e = \partial_{t_1},\ E = t_1\partial_{t_1} + 2\partial_{t_2} + \partial_{t_3} + t_4\partial_{t_4} + \frac{4}{5}t_5\partial_{t_5} + \frac{3}{5}t_6\partial_{t_6} + \frac{2}{5}t_7\partial_{t_7} + \frac{1}{5}t_8\partial_{t_8},\ \eta)$ $\left(F,\ \frac{dz}{z}\right)$ *Frobenius potential*

$$
\begin{aligned}
\mathcal{F}(\boldsymbol{t}) &= \frac{1}{2}t_1^2t_2 + t_1t_3t_4 + t_1t_5t_7 + \frac{1}{5}t_1t_6t_7 \\
&+ \frac{1}{2}t_3t_4^2 + \frac{1}{5}\frac{t_4t_5t_7}{t_8} + \frac{1}{2}t_4t_5t_8 + \frac{1}{10}\frac{t_4t_6^2}{t_8} + \frac{1}{24}t_4t_6t_8^2 + \frac{1}{1500}\frac{t_4t_7^4}{t_8^3} + \frac{1}{120}t_4t_7^2t_8 - \frac{1}{2880}t_4t_8^5 \\
&- \frac{1}{50}\frac{t_5^2t_7^2}{t_8^2} + \frac{1}{10}\frac{t_5^2t_6}{t_8} - \frac{1}{24}t_5^2t_8^2 + \frac{1}{125}\frac{t_5t_6t_7^3}{t_8^3} - \frac{1}{25}\frac{t_5t_6^2t_7}{t_8^2} - \frac{1}{60}t_5t_6t_7t_8 - \frac{1}{3125}\frac{t_5t_7^5}{t_8^4} + \frac{1}{720}t_5t_7t_8^4 \\
&+ \frac{2}{375}\frac{t_6^3t_7^2}{t_8^3} - \frac{1}{200}\frac{t_6^4}{t_8^2} - \frac{1}{600}t_6^2t_7^2 - \frac{3}{2500}\frac{t_6^2t_7^4}{t_8^4} - \frac{1}{960}t_6^2t_8^4 + \frac{3}{31250}\frac{t_6t_7^6}{t_8^5} - \frac{1}{7200}t_6t_7^2t_8^3 \\
&+ \frac{1}{12096}t_6t_8^7 - \frac{1}{375000}\frac{t_7^8}{t_8^6} - \frac{1}{24000}t_7^4t_8^2 - \frac{1}{25920}t_7^2t_8^6 - \frac{1}{276480}t_8^{10} \\
&+ e^{t_2-t_3}\left(-t_4 + \frac{1}{5}t_6t_7 + t_5t_8 - \frac{1}{10}t_7^2t_8 - \frac{1}{2}t_6t_8^2 + \frac{1}{6}t_7t_8^3 - \frac{1}{24}t_8^5\right) \\
&+ e^{t_3}\left(t_4 + \frac{1}{5}t_6t_7 + t_5t_8 + \frac{1}{10}t_7^2t_8 + \frac{1}{2}t_6t_8^2 + \frac{1}{6}t_7t_8^3 + \frac{1}{24}t_8^5\right) + e^{t_2} + \frac{1}{2}t_4^2\log t_8.
\end{aligned} \tag{58}
$$

.

$$(1,1,2); \frac{1}{2}t_1^2t_2,\ (1,3,4); t_1t_3t_4,\ (1,5,8); t_1t_5t_7,\ (1,6,7); \frac{1}{5}t_1t_6t_7,$$
$$(2,2,2); e^{t_2}+e^{t_2-t_3}\left(-t_4+\frac{1}{5}t_6t_7+t_5t_8-\frac{1}{10}t_7^2t_8-\frac{1}{2}t_6t_8^2+\frac{1}{6}t_7t_8^3-\frac{1}{24}t_8^5\right),$$
$$(3,3,3); e^{t_3}\left(t_4+\frac{1}{5}t_6t_7+t_5t_8+\frac{1}{10}t_7^2t_8+\frac{1}{2}t_6t_8^2+\frac{1}{6}t_7t_8^3+\frac{1}{24}t_8^5\right),\ (3,4,4); \frac{1}{2}t_3t_4^2,$$
$$(4,4,8); \frac{1}{2}t_4^2\log t_8,\ (4,5,8); \frac{1}{5}\frac{t_4t_5t_7}{t_8}+\frac{1}{2}t_4t_5t_8,\ (4,6,6); \frac{1}{10}\frac{t_4t_6^2}{t_8},\ (4,6,8); \frac{1}{24}t_4t_6t_8^2,$$
$$(4,7,7); \frac{1}{1500}\frac{t_4t_7^4}{t_8^3}+\frac{1}{120}t_4t_7^2t_8,\ (4,8,8); -\frac{1}{2880}t_4t_8^5,\ (5,5,8); -\frac{1}{50}\frac{t_5^2t_7^2}{t_8^2}+\frac{1}{10}\frac{t_5^2t_6}{t_8}-\frac{1}{24}t_5^2t_8^2,$$
$$(5.6.7); \frac{1}{125}\frac{t_5t_6t_7^3}{t_8^3}-\frac{1}{25}\frac{t_5t_6^2t_7}{t_8^2}-\frac{1}{60}t_5t_6t_7t_8,\ (5,7,8); -\frac{1}{3125}\frac{t_5t_7^5}{t_8^4}+\frac{1}{720}t_5t_7t_8^4,$$
$$(6,6,6); \frac{2}{375}\frac{t_6^3t_7^2}{t_8^3}-\frac{1}{200}\frac{t_6^4}{t_8^2},\ (6,6,7); -\frac{1}{600}t_6^2t_7^2-\frac{3}{2500}\frac{t_6^2t_7^4}{t_8^4},\ (6,6,8); -\frac{1}{960}t_6^2t_8^4,$$
$$(6,7,8); \frac{3}{31250}\frac{t_6t_7^6}{t_8^5}-\frac{1}{7200}t_6t_7^2t_8^3, (6,8,8); \frac{1}{12096}t_6t_8^7,$$
$$(7,7,8); -\frac{1}{375000}\frac{t_7^8}{t_8^6}-\frac{1}{24000}t_7^4t_8^2-\frac{1}{25920}t_7^2t_8^6,\ (8,8,8); -\frac{1}{276480}t_8^{10}$$

Frobenius potental □

Frobenius potential $\mathcal{F}$ WDVV Mathematica

μ $\mu = 4,5,6,7,8$ F Frobenius potential

.24. (2.1.1) F $(F, \frac{dz}{z})$ $M := (\mathbb{C}\times\mathbb{C}^*)\times(\mathbb{C}^*)^2\times\mathbb{C}^{\mu-4}$ *Frobenius* *.12* *Frobenius potential* $f, g \in \mathbb{Q}[t_4, t_5, \cdots, t_\mu]$ $q_\mu \in \mathbb{Q}[t_\mu, t_\mu^{-1}][t_3, t_4, \cdots, t_{\mu-1}]$

$$\mathcal{F}(\boldsymbol{t}) = t_1\left(\frac{1}{2}t_2^2+t_3t_4+t_5t_\mu+\frac{1}{2(\mu-3)}\sum_{k=6}^{\mu}t_kt_{\mu-k+5}\right)+q(t_3,t_4,\cdots,t_{\mu-1},t_\mu)$$
$$+e^{t_2-t_3}\cdot f(t_4,t_5,\cdots,t_\mu)+e^{t_3}\cdot g(t_4,t_5,\cdots,t_\mu)+e^{t_2}+\frac{1}{2}t_4^2\log t_\mu \tag{59}$$

$q,\ f,\ g$

o1.$f,\ g$

o2.q $()-()=2$

$\mathcal{F}(\boldsymbol{t})$ $e^{t_2},\ \frac{1}{2}t_4^2\log t_\mu$

.25. (2.7.1) q $e^{t_2}, \frac{1}{2}t_4^2\log t_\mu$

(i)(2.1.1) *Frobenius potential* e^{t_2}

(ii)(2.1.1) *Frobenius potential* $\frac{1}{2}t_4^2\log t_\mu$

(i) .12 $e^{s_3} = s_1 = s_4 = s_5 = \cdots = s_{\mu-1} = 0$

$$\frac{\partial F}{\partial s_2} = \frac{\partial t_2}{\partial s_2}\cdot\frac{\partial F}{\partial t_2} = \frac{\partial F}{\partial t_2}$$

$e^{s_3} = s_1 = s_4 = s_5 = \cdots = s_{\mu-1} = 0$ $e^{t_3} = t_1 = t_4 = t_5 = \cdots = t_{\mu-1} = 0$ $e^{s_3} = s_1 = s_4 =$

$s_5 = \cdots = s_{\mu-1} = 0$ $\eta(\partial_{t_2} \circ \partial_{t_2}, \partial_{t_2}) = \eta(\partial_{s_2} \circ \partial_{s_2}, \partial_{s_2})$

$$\partial_{s_2} F|_{e^{s_3}=s_1=s_4=s_5=\cdots=s_{\mu-1}=0} = \frac{e^{s_2}}{z},$$
$$\partial_z F|_{e^{s_3}=s_1=s_4=s_5=\cdots=s_{\mu-1}=0} = 1 - \frac{e^{s_2}}{z^2}.$$

$\eta(\partial_{s_2} \circ \partial_{s_2}, \partial_{s_2})$ $e^{s_3} = s_1 = s_4 = s_5 = \cdots = s_{\mu-1} = 0$

$$\begin{aligned}\eta(\partial_{s_2} \circ \partial_{s_2}, \partial_{s_2}) &= \frac{1}{2\pi\sqrt{-1}} \int_{|\frac{\partial F}{\partial z}|=\epsilon} \frac{\partial_{s_2} F \cdot \partial_{s_2} F \cdot \partial_{s_2} F}{z \partial_z F} \cdot \frac{dz}{z} \\ &= -\sum_{k=0,\infty} \underset{z=k}{Res} \left(\frac{\partial_{s_2} F \cdot \partial_{s_2} F \cdot \partial_{s_2} F}{z \partial_z F} \cdot \frac{1}{z} \right) \\ &= e^{s_2} + 0 = e^{s_2}.\end{aligned}$$

$\eta(\partial_{t_2} \circ \partial_{t_2}, \partial_{t_2}) = \eta(\partial_{s_2} \circ \partial_{s_2}, \partial_{s_2}) = e^{s_2} = e^{t_2}$ c_{222} $\frac{\partial \mathcal{F}}{\partial t_2 \partial t_2 \partial t_2} = e^{t_2}$ $\mathcal{F}$ e^{t_2}

(ii)i .12 $e^{s_2} = s_1 = s_5 = s_6 = \cdots = s_{\mu-1} = 0$ $\eta(\partial_{t_4} \circ \partial_{t_4}, \partial_{t_\mu}) = \eta(\partial_{s_4} \circ \partial_{s_4}, \partial_{s_\mu})$ $e^{s_2} = s_1 = s_5 = s_6 = \cdots = s_{\mu-1} = 0$ $e^{t_2} = t_1 = t_5 = t_6 = \cdots = t_{\mu-1} = 0$

$$\partial_{s_4} F|_{e^{s_2}=s_1=s_5=s_6=\cdots=s_{\mu-1}=0} = \frac{z}{z - e^{s_3}},$$
$$\partial_{s_\mu} F|_{e^{s_2}=s_1=s_5=s_6=\cdots=s_{\mu-1}=0} = \frac{(\mu-3) z e^{(\mu-4)s_3} s_\mu^{\mu-4}}{(z - e^{s_3})^{\mu-3}},$$
$$\partial_z F|_{e^{s_2}=s_1=s_5=s_6=\cdots=s_{\mu-1}=0} = 1 - \frac{e^{s_3} s_4}{(z - e^{s_3})^2} - \frac{((\mu-2)z + e^{s_3}) e^{(\mu-4)s_3} s_\mu^{\mu-3}}{(z - e^{s_3})^{\mu-2}}.$$

$\eta(\partial_{s_4} \circ \partial_{s_4}, \partial_{s_\mu})$ $e^{s_2} = s_1 = s_5 = s_6 = \cdots = s_{\mu-1} = 0$

$$\begin{aligned}\eta(\partial_{s_4} \circ \partial_{s_4}, \partial_{s_\mu}) &= \frac{1}{2\pi\sqrt{-1}} \int_{|\frac{\partial F}{\partial z}|=\epsilon} \frac{\partial_{s_4} F \cdot \partial_{s_4} F \cdot \partial_{s_\mu} F}{z \partial_z F} \cdot \frac{dz}{z} \\ &= -\sum_{k=0,\infty,e^{s_3}} \underset{z=k}{Res} \left(\frac{\partial_{s_4} F \cdot \partial_{s_4} F \cdot \partial_{s_\mu} F}{z \partial_z F} \cdot \frac{1}{z} \right) \\ &= 0 + 0 + \frac{1}{s_\mu} = \frac{1}{s_\mu}.\end{aligned}$$

$\eta(\partial_{t_4} \circ \partial_{t_4}, \partial_{t_\mu}) = \eta(\partial_{s_4} \circ \partial_{s_4}, \partial_{s_8}) = \frac{1}{s_\mu} = \frac{1}{t_\mu}$ c_{448} $\frac{\partial \mathcal{F}}{\partial t_4 \partial t_4 \partial t_\mu} = \frac{1}{t_\mu}$ $\mathcal{F}$ $\frac{1}{2} t_4^2 \log t_\mu$

□

o1.

o1.1.

o1.1. $\mu = 8$ $t_4,\ t_7^2 t_8,\ t_6 t_8^2,\ t_8^5$ 1 3 3 5
[0]

[0] [0] $\mu = 5$ $\mu = 7$ .21 [0] $\mu = 5$ Frobenius potential

$$
\begin{aligned}
&\mathcal{F}(t_1,t_2,t_3,t_4,t_5)\\
&= \frac{1}{2}t_1^2t_5 + t_1t_2t_4 + \frac{1}{8}t_1^3 + \frac{1}{24}\frac{t_2^3}{t_4} - \frac{1}{32}\frac{t_3^2t_2^2}{t_4^2} + \frac{1}{256}\frac{t_2t_3^4}{t_4^3} - \frac{1}{6144}\frac{t_3^6}{t_4^4}\\
&- \frac{1}{24}E_2t_4^2t_2^2 - \frac{1}{96}E_2t_2t_3^2t_4 + \frac{1}{720}E_4t_2t_4^5 - \frac{1}{1536}E_2t_3^4 - \frac{1}{1152}E_4t_3^2t_4^4 - \frac{1}{18144}E_6t_4^8. \qquad (60)
\end{aligned}
$$

E_2, E_4, E_6 Eisenstein $\frac{z}{2} + \frac{z}{e^z-1} = \sum_{k=0}^{\infty}\frac{B_{2k}}{(2k)!}z^{2k}$ B_{2k} $q := e^{2\pi\sqrt{-1}\tau}$ $\sigma_{2k-1}(n) := \sum_{d|n} d^{2k-1}$

$$
E_{2k}(\tau) = 1 - \frac{4k}{B_{2k}}\sum_{n=1}^{\infty}\sigma_{2k-1}(n)q^n
$$

$\mu = 7$ Frobenius potential

$$
\begin{aligned}
\mathcal{F}(t_1,t_2,t_3,t_4,t_5,t_6,t_7) &= \frac{1}{2}t_1^2t_2 + t_1t_3t_4 + t_1t_5t_7 + \frac{1}{8}t_1t_6^2 + e^{t_2} + \frac{1}{2}t_4^2\log t_7\\
&+ \frac{1}{2}t_3t_4^2 + \frac{1}{16}t_4t_6^2 - \frac{1}{96}\frac{t_4t_6^3}{t_7^2} + \frac{1}{24}t_4t_6t_7^2 + \frac{1}{24}\frac{t_5^3}{t_7} - \frac{1}{32}\frac{t_5^2t_6^2}{t_7^2} - \frac{1}{24}t_5^2t_7^2\\
&+ \frac{1}{256}\frac{t_5t_6^4}{t_7^3} - \frac{1}{96}t_5t_6^2t_7 + \frac{1}{720}t_5t_7^5 - \frac{1}{1536}t_6^4 - \frac{1}{6144}\frac{t_6^6}{t_7^4} - \frac{1}{1152}t_6^2t_7^4\\
&- \frac{1}{18144}t_7^8 + e^{t_2-t_3}\left(-t_4 + \frac{1}{8}t_6^2 + t_5t_7 - \frac{1}{2}t_6t_7^2 + \frac{1}{6}t_7^4\right)\\
&+ e^{t_3}\left(t_4 + \frac{1}{8}t_6^2 + t_5t_7 + \frac{1}{2}t_6t_7^2 + \frac{1}{6}t_7^4\right) \qquad (61)
\end{aligned}
$$

$q \to 0$ $E_{2k} \to 1$ (60) (61) t_3, t_4 [0] $\mu = 4$ $\mu = 6$ [0] $\mu = 6$ $\mu = 8$ [0] orbifold 0 1

*

F 31 $\mu = 4,5,6,7,8$ Frobenius potential Frobenius potential [0] Fro potential

f, g q_μ ? f, g e t_2, t_3 c_{333} $e^{t_2-t_3}f$ $e^{t_3}g$ t_3 [0] ? [0, 2.1.1] F_T n $n-1$ F μ $z = e^{s_3}$ $\mu - 3$ n $\mu - 2$ $n = \mu - 2$ F_T $\mu - 3$ F $z = e^{s_3}$

f, g, q_μ [0]

* Mathematica $\mu = 8$

* .22

```
F = z + t[1] + E^t[2]/z + z * t[4]/(z - E^t[3]) + z * E^t[3](t[5]t[8] + c[5]t[6]t[7] + c[6]t[6]t[8]^2
+c[7]t[7]^2t[8] + c[8]t[7]t[8]^3 + c[9]t[8]^5)/(z - E^t[3])^2 + z * E^(2t[3])(t[6]t[8]^2 + c[2]t[7]^2t[8]+c[3]t[7]t[8]^3 + c[4]t[
+z * E^(4t[3])t[8]^5/(z - E^t[3])^5
```

$\frac{e^{t[2]}}{z} + z + t[1] + \frac{zt[4]}{-e^{t[3]}+z} + \frac{e^{4t[3]}zt[8]^5}{(-e^{t[3]}+z)^5} + \frac{e^{3t[3]}z\left(t[7]t[8]^3+c[1]t[8]^5\right)}{(-e^{t[3]}+z)^4} + \frac{e^{2t[3]}z\left(c[2]t[7]^2t[8]+t[6]t[8]^2+c[3]t[7]t[8]^3+c[4]t[8]^5\right)}{(-e^{t[3]}+z)^3} + \frac{e^{t[3]}z\left(c[5]t[6]t[7]+t[5]t[8]+c[7]t[7]^2t[8]+c[6]t[6]t[8]^2+c[8]t[7]t[8]^3+c[9]t[8]^5\right)}{(-e^{t[3]}+z)^2}$

```
Do[eta[i, j] = -Residue[(D[F, t[i]] * D[F, t[j]])/(z * D[F, z]) * 1/z, {z, ∞}]
-Residue[(D[F, t[i]] * D[F, t[j]])/(z * D[F, z]) * 1/z, {z, 0}]
-Residue[(D[F, t[i]] * D[F, t[j]])/(z * D[F, z]) * 1/z, {z, E^(t[3])}], {i, 1, 8}, {j, 1, 8}];
Table[eta[i, j], {i, 1, 8}, {j, 1, 8}]
```

{{0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1},

$\{0,0,0,0,0,0,\frac{1}{5},-\frac{t[7]-5c[5]t[7]+4t[8]^2-c[1]t[8]^2-5c[6]t[8]^2}{5t[8]}\},$
$\{0,0,0,0,0,\frac{1}{5},-\frac{2(2t[7]-10c[2]t[7]+2t[8]^2+2c[1]t[8]^2-5c[3]t[8]^2)}{25t[8]},-\frac{1}{25t[8]^2}(-4t[7]^2+20c[2]t[7]^2+5t[6]t[8]-25c[5]t[6]t[8]-$
$5t[7]t[8]^2+40c[2]t[7]t[8]^2-10c[1]c[2]t[7]t[8]^2+5c[3]t[7]t[8]^2-50c[7]t[7]t[8]^2-16t[8]^4+3c[1]t[8]^4+4c[1]^2t[8]^4+$
$20c[3]t[8]^4-5c[1]c[3]t[8]^4-10c[4]t[8]^4-25c[8]t[8]^4)\},$
$\{0,0,0,0,1,-\frac{t[7]-5c[5]t[7]+4t[8]^2-c[1]t[8]^2-5c[6]t[8]^2}{5t[8]},-\frac{1}{25t[8]^2}(-4t[7]^2+20c[2]t[7]^2+5t[6]t[8]-25c[5]t[6]t[8]-$
$5t[7]t[8]^2+40c[2]t[7]t[8]^2-10c[1]c[2]t[7]t[8]^2+5c[3]t[7]t[8]^2-50c[7]t[7]t[8]^2-16t[8]^4+3c[1]t[8]^4+4c[1]^2t[8]^4+$
$20c[3]t[8]^4-5c[1]c[3]t[8]^4-10c[4]t[8]^4-25c[8]t[8]^4),-\frac{1}{25t[8]^3}2(2t[7]^3-10c[2]t[7]^3-5t[6]t[7]t[8]+25c[5]t[6]t[7]t[8]+$
$3t[7]^2t[8]^2-2c[1]t[7]^2t[8]^2-15c[2]t[7]^2t[8]^2+10c[1]c[2]t[7]^2t[8]^2+5t[6]t[8]^3+5c[1]t[6]t[8]^3-25c[6]t[6]t[8]^3-$
$12t[7]t[8]^4-4c[1]t[7]t[8]^4-2c[1]^2t[7]t[8]^4+25c[3]t[7]t[8]^4+10c[4]t[7]t[8]^4-50c[8]t[7]t[8]^4+32t[8]^6-$
$44c[1]t[8]^6+c[1]^2t[8]^6+2c[1]^3t[8]^6+65c[4]t[8]^6-10c[1]c[4]t[8]^6-100c[9]t[8]^6)\}\}$

$\mathbf{Solve}[\{1-5c[5]==0,4-c[1]-5c[6]==0,1-5c[2]==0,2+2c[1]-5c[3]==0,$
$1-8c[2]+2c[1]c[2]-c[3]+10c[7]==0,-16+3c[1]+4c[1]\hat{}2+20c[3]-5c[1]c[3]-10c[4]-25c[8]==0,$
$3-2c[1]-15c[2]+10c[1]c[2]==0,1+c[1]-5c[6]==0,-12-4c[1]-2c[1]\hat{}2+25c[3]+10c[4]-50c[8]==0,$
$32-44c[1]+c[1]\hat{}2+2c[1]\hat{}3+65c[4]-10c[1]c[4]-100c[9]==0\},$
$\{c[1],c[2],c[3],c[4],c[5],c[6],c[7],c[8],c[9]\}]$

$\left\{\left\{c[1]\to\frac{3}{2},c[2]\to\frac{1}{5},c[3]\to 1,c[4]\to\frac{7}{12},c[5]\to\frac{1}{5},c[6]\to\frac{1}{2},c[7]\to\frac{1}{10},c[8]\to\frac{1}{6},c[9]\to\frac{1}{24}\right\}\right\}$

$F'=z+t[1]+E\hat{}t[2]/z+z*t[4]/(z-E\hat{}t[3])+z*E\hat{}t[3](t[5]t[8]+1/5*t[6]t[7]+1/2*t[6]t[8]\hat{}2$
$+1/10*t[7]\hat{}2t[8]+1/6*t[7]t[8]\hat{}3+1/24*t[8]\hat{}5)/(z-E\hat{}t[3])\hat{}2$
$+z*E\hat{}(2t[3])(t[6]t[8]\hat{}2+1/5*t[7]\hat{}2t[8]+t[7]t[8]\hat{}3+7/12*t[8]\hat{}5)/(z-E\hat{}t[3])\hat{}3$
$+z*E\hat{}(3t[3])(t[7]t[8]\hat{}3+3/2*t[8]\hat{}5)/(z-E\hat{}t[3])\hat{}4$
$+z*E\hat{}(4t[3])t[8]\hat{}5/(z-E\hat{}t[3])\hat{}5$

$\frac{e^{t[2]}}{z}+z+t[1]+\frac{zt[4]}{-e^{t[3]}+z}+\frac{e^{4t[3]}zt[8]^5}{(-e^{t[3]}+z)^5}+\frac{e^{t[3]}z\left(\frac{1}{5}t[6]t[7]+t[5]t[8]+\frac{1}{10}t[7]^2t[8]+\frac{1}{2}t[6]t[8]^2+\frac{1}{6}t[7]t[8]^3+\frac{t[8]^5}{24}\right)}{(-e^{t[3]}+z)^2}+\frac{e^{2t[3]}z\left(\frac{1}{5}t[7]^2t[8]+t[6]t[8]^2+t[7]t[8]^3\right.}{(-e^{t[3]}+z)^3}$
$\frac{e^{3t[3]}z\left(t[7]t[8]^3+\frac{3t[8]^5}{2}\right)}{(-e^{t[3]}+z)^4}$

$\mathbf{Do}[\mathrm{eta}'[i,j]=-\mathbf{Residue}[(D[F',t[i]]*D[F',t[j]])/(z*D[F',z])*1/z,\{z,\infty\}]$
$-\mathbf{Residue}[(D[F',t[i]]*D[F',t[j]])/(z*D[F',z])*1/z,\{z,0\}]$
$-\mathbf{Residue}[(D[F',t[i]]*D[F',t[j]])/(z*D[F',z])*1/z,\{z,E\hat{}(t[3])\}],\{i,1,8\},\{j,1,8\}];$
$\mathbf{Table}[\mathrm{eta}'[i,j],\{i,1,8\},\{j,1,8\}]$

$\{\{0,1,0,0,0,0,0,0\},\{1,0,0,0,0,0,0,0\},\{0,0,0,1,0,0,0,0\},\{0,0,1,0,0,0,0,0\},\{0,0,0,0,0,0,0,1\},$
$\{0,0,0,0,0,0,\frac{1}{5},0\},\{0,0,0,0,0,\frac{1}{5},0,0\},\{0,0,0,0,1,0,0,0\}\}$

*Frobenius potential .23

$P=\frac{1}{2}t[1]^2t[2]+t[1]t[3]t[4]+t[1]t[5]t[8]+\frac{1}{5}t[1]t[6]t[7]+$
$\frac{1}{120}e^{t[2]-t[3]}\left(120e^{t[3]}-120t[4]+24t[6]t[7]+120t[5]t[8]-12t[7]^2t[8]-60t[6]t[8]^2+20t[7]t[8]^3-5t[8]^5\right)+$
$\frac{1}{120}e^{t[3]}\left(120t[4]+12t[6]\left(2t[7]+5t[8]^2\right)+t[8]\left(120t[5]+12t[7]^2+20t[7]t[8]^2+5t[8]^4\right)\right)$
$+\frac{1}{2}t[3]t[4]^2+\frac{1}{2}\mathrm{Log}[t[8]]t[4]^2+\frac{1}{10}t[4]t[5]\left(\frac{2t[7]}{t[8]}+5t[8]\right)+$
$\frac{t[4]t[6]^2}{10t[8]}+\frac{1}{10}t[4]t[6]t[7]-\frac{t[4]t[6]t[7]^2}{50t[8]^2}+\frac{1}{24}t[4]t[6]t[8]^2+t[4]\left(\frac{t[7]^4}{1500t[8]^3}+\frac{1}{120}t[7]^2t[8]\right)-\frac{t[4]t[8]^5}{2880}$
$-\frac{1}{300}t[5]^2\left(\frac{6t[7]^2}{t[8]^2}-\frac{30t[6]}{t[8]}+\frac{25t[8]^2}{2}\right)+\frac{t[5]t[6]t[7]^3}{125t[8]^3}-\frac{t[5]t[6]^2t[7]}{25t[8]^2}$
$-\frac{1}{60}t[5]t[6]t[7]t[8]+t[5]\left(-\frac{t[7]^5}{3125t[8]^4}+\frac{1}{720}t[7]t[8]^4\right)+\frac{\frac{4}{3}t[6]^3t[7]^2-\frac{5}{4}t[6]^4t[8]}{250t[8]^3}+-\frac{1}{600}t[6]^2t[7]^2-\frac{3t[6]^2t[7]^4}{2500t[8]^4}-$
$\frac{1}{960}t[6]^2t[8]^4+t[6]\left(\frac{3t[7]^6}{31250t[8]^5}-\frac{t[7]^2t[8]^3}{7200}\right)+\frac{t[6]t[8]^7}{12096}-\frac{t[7]^8}{375000t[8]^6}-\frac{t[7]^4t[8]^2}{24000}-\frac{t[7]^2t[8]^6}{25920}-\frac{t[8]^{10}}{276480}$

$\mathbf{fnc}[\mathrm{i\_,j\_,k\_}]:=-\mathbf{Residue}[(D[F',t[i]]*D[F',t[j]]*D[F',t[k]])/(z*D[F',z])*1/z,\{z,\infty\}]$
$-\mathbf{Residue}[(D[F',t[i]]*D[F',t[j]]*D[F',t[k]])/(z*D[F',z])*1/z,\{z,0\}]-$
$\mathbf{Residue}[(D[F',t[i]]*D[F',t[j]]*D[F',t[k]])/(z*D[F',z])*1/z,\{z,E\hat{}t[3]\}]-D[P,t[k],t[j],t[i]]$
$\mathbf{int}[\mathrm{i\_,j\_,k\_}]:=\mathbf{Integrate}[\mathbf{Integrate}[\mathbf{Integrate}[\mathrm{fnc}[i,j,k],t[i]],t[j]],t[k]]$

$\frac{1}{2}t[1]^2t[2]+t[1]t[3]t[4]+\frac{1}{2}\mathrm{Log}[t[8]]t[4]^2+\frac{1}{2}t[3]t[4]^2+\frac{1}{5}t[1]t[6]t[7]+\frac{1}{10}t[4]t[6]t[7]-\frac{1}{600}t[6]^2t[7]^2-\frac{t[7]^8}{375000t[8]^6}-$
$\frac{3t[6]^2t[7]^4}{2500t[8]^4}+\frac{t[5]t[6]t[7]^3}{125t[8]^3}-\frac{t[5]t[6]^2t[7]}{25t[8]^2}-\frac{t[4]t[6]t[7]^2}{50t[8]^2}+\frac{t[4]t[6]^2}{10t[8]}+t[1]t[5]t[8]-\frac{1}{60}t[5]t[6]t[7]t[8]+\frac{1}{24}t[4]t[6]t[8]^2-$

$$\frac{t[7]^4t[8]^2}{24000}-\frac{1}{960}t[6]^2t[8]^4-\frac{t[4]t[8]^5}{2880}-\frac{t[7]^2t[8]^6}{25920}+\frac{t[6]t[8]^7}{12096}-\frac{t[8]^{10}}{276480}+\frac{1}{10}t[4]t[5]\left(\frac{2t[7]}{t[8]}+5t[8]\right)+\frac{\frac{4}{3}t[6]^3t[7]^2-\frac{5}{4}t[6]^4t[8]}{250t[8]^3}+$$
$$t[4]\left(\frac{t[7]^4}{1500t[8]^3}+\frac{1}{120}t[7]^2t[8]\right)-\frac{1}{300}t[5]^2\left(\frac{6t[7]^2}{t[8]^2}-\frac{30t[6]}{t[8]}+\frac{25t[8]^2}{2}\right)$$
$$+t[6]\left(\frac{3t[7]^6}{31250t[8]^5}-\frac{t[7]^2t[8]^3}{7200}\right)+t[5]\left(-\frac{t[7]^5}{3125t[8]^4}+\frac{1}{720}t[7]t[8]^4\right)$$
$$+\frac{1}{120}e^{t[2]-t[3]}\left(120e^{t[3]}-120t[4]+24t[6]t[7]+120t[5]t[8]-12t[7]^2t[8]-60t[6]t[8]^2+20t[7]t[8]^3-5t[8]^5\right)$$
$$+\frac{1}{120}e^{t[3]}\left(120t[4]+12t[6]\left(2t[7]+5t[8]^2\right)+t[8]\left(120t[5]+12t[7]^2+20t[7]t[8]^2+5t[8]^4\right)\right)$$

$\mathbf{Do[int[}\boldsymbol{i,j,k}\mathbf{],\{}\boldsymbol{i}\mathbf{,1,8\},\{}\boldsymbol{j}\mathbf{,1,8\},\{}\boldsymbol{k}\mathbf{,1,8\}];Table[int[}\boldsymbol{i,j,k}\mathbf{],\{}\boldsymbol{i}\mathbf{,1,8\},\{}\boldsymbol{j}\mathbf{,1,8\},\{}\boldsymbol{k}\mathbf{,1,8\}]}$

$\{\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$
$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$
$\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$
$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$
$\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$
$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$
$\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$
$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$
$\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$
$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$
$\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$
$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$
$\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$
$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$
$\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$
$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\}\}$

*WDVV

WDVV

$$\sum_{a,b=1}^{\mu}\partial_i\partial_j\partial_a\mathcal{F}\cdot\eta^{ab}\cdot\partial_b\partial_k\partial_\ell\mathcal{F}=\sum_{a,b=1}^{\mu}\partial_i\partial_k\partial_a\mathcal{F}\cdot\eta^{ab}\cdot\partial_b\partial_j\partial_\ell\mathcal{F}$$

$$\sum_{a,b=1}^{\mu}\partial_i\partial_j\partial_a\mathcal{F}\cdot\eta^{ab}\cdot\partial_b\partial_k\partial_\ell\mathcal{F}-\sum_{a,b=1}^{\mu}\partial_i\partial_k\partial_a\mathcal{F}\cdot\eta^{ab}\cdot\partial_b\partial_j\partial_\ell\mathcal{F}=0$$

i,j,k,ℓ 0 $\mu=8$ (η_{ij}) (η^{ij})

$$(\eta^{ij})=\begin{pmatrix}01000000\\10000000\\00010000\\00100000\\00000001\\00000050\\00000500\\00001000\end{pmatrix}.$$

$\mathbf{Do[wdvv[}\boldsymbol{i,j,k,l}\mathbf{]}=\boldsymbol{D[D[D[P,t[1]],t[j]],t[i]]D[D[D[P,t[l]],t[k]],t[2]]+D[D[D[P,t[2]],t[j]],t[i]]D[D[D[P,t[l]],t[k]],t[1]]}$
$\boldsymbol{+D[D[D[P,t[3]],t[j]],t[i]]D[D[D[P,t[l]],t[k]],t[4]]+D[D[D[P,t[4]],t[j]],t[i]]D[D[D[P,t[l]],t[k]],t[3]]}$
$\boldsymbol{+D[D[D[P,t[5]],t[j]],t[i]]D[D[D[P,t[l]],t[k]],t[8]]+D[D[D[P,t[8]],t[j]],t[i]]D[D[D[P,t[l]],t[k]],t[5]]}$
$\boldsymbol{+D[D[D[P,t[6]],t[j]],t[i]]*5*D[D[D[P,t[l]],t[k]],t[7]]+D[D[D[P,t[7]],t[j]],t[i]]*5*D[D[D[P,t[l]],t[k]],t[6]]}$
$\boldsymbol{-D[D[D[P,t[1]],t[k]],t[i]]D[D[D[P,t[l]],t[j]],t[2]]-D[D[D[P,t[2]],t[k]],t[i]]D[D[D[P,t[l]],t[j]],t[1]]}$
$\boldsymbol{-D[D[D[P,t[3]],t[k]],t[i]]D[D[D[P,t[l]],t[j]],t[4]]-D[D[D[P,t[4]],t[k]],t[i]]D[D[D[P,t[l]],t[j]],t[3]]}$
$\boldsymbol{-D[D[D[P,t[5]],t[k]],t[i]]D[D[D[P,t[l]],t[j]],t[8]]-D[D[D[P,t[8]],t[k]],t[i]]D[D[D[P,t[l]],t[j]],t[5]]}$

−D[D[D[P, t[6]], t[k]], t[i]] * 5 * D[D[D[P, t[l]], t[j]], t[7]] − D[D[D[P, t[7]], t[k]], t[i]] * 5 * D[D[D[P, t[l]], t[j]], t[6]],
{i, 1, 8}, {j, 1, 8}, {k, 1, 8}, {l, 1, 8}];
Table[wdvv[i, j, k, l], {i, 1, 8}, {j, 1, 8}, {k, 1, 8}, {l, 1, 8}]
{{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
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{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
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{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
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{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
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{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
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{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
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{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},

{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
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