**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

**ANS = B**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS =**

1. **Probabilty of employees greater than age of 44= Pr(X>44)**

**Pr(X > 44) = 1 - Pr(X ≤ 44).**

**Z = (X - )/ = (X - 38)/6**

**Thus the question can be answered by using the normal table to find**

**Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1)=84.1345%**

**Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86%**

**So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%**

**Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.**

**b) Probabilty of employees less than age of 30 = Pr(X<30).**

**Z = (X - )/ = (30 - 38)/6**

**Thus the question can be answered by using the normal table to find**

**Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS =**

**As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .**

**Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).**

**Therefore in the question**

**2X1~ N(2 u,4 σ^2) and**

**X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )**

**2X1-(X1+X2) = N( 4µ,6 σ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

**ANS =D**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company**.**

**ANS = a): Range containing 95% probability for profit of company is**

**(Rs. 99M, Rs. 1026M).**

**b): Rs. 170.1 Million.**

**c): First division of the company has larger probability of making a loss.**

**Given that:**

**$1 = Rs.45**

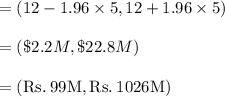
**https://tex.z-dn.net/?f=Profit_1%20%5Csim%20N(5%2C%203%5E2)%20%5C%5C%20Profit_2%20%5Csim%20N(7%2C%204%5E2)**

**Thus,**

**Company’s profit:**

**95% of the probability lies between 1.96 standard deviation of the mean.**

**Thus range is:**

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1. Specify the 5th percentile of profit (in Rupees) for the company

ANS **= Fifth percentile is calculated as :**

**https://tex.z-dn.net/?f=P(Z%20%5Cleq%20%5Cdfrac%7Bp-12%7D%7B5%7D)%20%3D%200.05**

**https://tex.z-dn.net/?f=%5Cdfrac%7Bp-12%7D%7B5%7D%20%3D%20-1.644%5C%5Cp%20%3D%2012%20-%208.22%20%3D%203.78%5C%5C**

**Thus at $3.78M dollars, or Rs. 170.1M amount, 5th  percentile of profit lies.**

**Or 5th percentile of profit is Rs. 170.1 Million.**

1. Which of the two divisions has a larger probability of making a loss in a given year?

ANS =

**Loss is when profit < 0**

**Thus: p< 0**

**The first division of company, thus have larger probability of making a loss in a given**

**year.**