Below we list some of the trajectories of diffusion/flow process. Here we consider the denoising process as $t : 1 \to 0$. Which is the reverse of [1].

I VE The VE path has the following perturbation kernel:

$$p_t(\mathbf{x}) = \mathcal{N}(\mathbf{x} \mid \mathbf{x}_0, \sigma_t \mathbf{I}) \tag{1}$$

Where $\sigma_0 \to 0^+$, $\sigma_1 \gg 1$. According to Theorem 3 of [1]. the conditional vector field has the following form.

$$u_t(\mathbf{x} \mid \mathbf{x}_1) = -\frac{\sigma_t'}{\sigma_t}(\mathbf{x} - \mathbf{x}_1). \tag{2}$$

For VE, there are two types of perturbation kernels.

SMLD [2] Here, $\sigma_t = \left(\frac{\sigma_1}{\sigma_0}\right)^t \sigma_0$. $\sigma_0 = 0.01, \sigma_1 = 2$ In actual case, $\sigma_1 = 100$, but we set it to 2 for our toy example.

EDM [3] Here, $\sigma_t = t$

II VP The perturbation kernel for VP [4] is

$$p_t(\mathbf{x} \mid \mathbf{x}_0) = \mathcal{N}(\mathbf{x} \mid \alpha_t \mathbf{x}_0, (1 - \alpha_t^2)I), \text{ where } \alpha_t = e^{-\frac{1}{2}T(t)}, \quad T(t) = \int_0^t \beta(s)\mathrm{d}s, \tag{3}$$

The conditional vector field has the following form:

$$u_t(\mathbf{x} \mid \mathbf{x}_0) = \frac{\alpha_t'}{1 - \alpha_t^2}(\alpha_t \mathbf{x} - \mathbf{x}_1) = -\frac{T'(t)}{2}\left[\frac{e^{-T(t)}x - e^{-\frac{1}{2}T(t)}x_1}{1 - e^{-T(t)}}\right] \tag{4}$$

III OT Optimal transport trajectory. The perturbation kernel has the following form:

$$p_t(\mathbf{x} \mid \mathbf{x}_0) = \mathcal{N}(\mathbf{x} \mid \alpha_t \mathbf{x}_0, \sigma_t^2 I) \text{ where } \alpha_t = 1 - t, \sigma_t = 1 - (1 - \sigma_{\min})t \tag{5}$$

The conditional vector field has the following form:

$$u_t(\mathbf{x} \mid \mathbf{x}_0) = \frac{\mathbf{x}_0 - (1 - \sigma_{\min})\mathbf{x}}{\sigma_{\min} + (1 - \sigma_{\min})t} \tag{6}$$

IV sub-VP sub-VP proposed in [5] has the following form of perturbation kernel.

$$p_t(\mathbf{x} \mid \mathbf{x}_0) = \mathcal{N}(\mathbf{x} | \alpha_t \mathbf{x}_0, (1 - \alpha_t^2)^2 I), \text{ where } \alpha_t = e^{-\frac{1}{2}T(t)}, T(t) = \int_0^t \beta(s)\mathrm{d}s \tag{7}$$

The conditional vector field has the following form:

$$u_t(\mathbf{x} \mid \mathbf{x}_0) = -\frac{1}{2}T'(t)\left[\frac{2e^{-T(t)}\mathbf{x} - \mathbf{x}_0(e^{-\frac{3}{2}T(t)} + e^{-\frac{1}{2}T(t)})}{1 - e^{-T(t)}}\right] \tag{8}$$

V Cosine Cosine noise schedules proposed in [6] has the following perturbation kernel:

$$p_t(\mathbf{x} \mid \mathbf{x}_0) = \mathcal{N}(\mathbf{x} \mid \alpha_t \mathbf{x}_0, (1 - \alpha_t^2)I), \text{ where } \alpha_t = \cos(t\pi/2) \tag{9}$$

The conditional vector field has the following form:

$$u_t(\mathbf{x} \mid \mathbf{x}_0) = \frac{\alpha_t'}{1 - \alpha_t^2}(\alpha_t \mathbf{x} - \mathbf{x}_1) = -\frac{\pi/2}{\sin(t\pi/2)}\left(\cos(t\pi/2)\mathbf{x} - \mathbf{x}_0\right) \tag{10}$$

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