B. Tech. / IDD / Part II, III/Sem III, V/2022-23
Probability & Statistics : MA-202

Final Assignment Sheet

1. If X is a $\Gamma\left(\frac{n}{2}\right)$ variate, then prove that $Y = 2X$ follows χ^2- distribution with *n*-degrees of freedom. Hence prove that if X_i , $i = 1,2,...,n$ are standard normal variate, then $\sum_{i=1}^{n} X_i^2$ represents χ^2 – distribution with *n*-degrees of freedom.

2. The joint density function of the random variables X and Y is given by

$$f(x,y) = \begin{cases} x+y; \ 0 \le x \le 1, 0 \le y \le 1 \\ \ 0 \ ; \qquad \quad otherwise \end{cases}$$

Find the distribution of XY.

3. If X is an $F(m,n)$ variate, then show that $\frac{1}{X}$ is an $F(n,m)$ variate.

4. A population is defined by the density function

$$f(x;\theta) = \frac{x^{p-1} e^{-\frac{x}{\theta}}}{\theta^p \Gamma(p)}, \qquad 0 < x < \infty ,$$

where p is known and $p > 0$. Find the maximum likelihood estimate of $\theta (> 0)$ drawing a sample $x_1, x_2, \ldots\ldots, x_n$ from the population. Show that the estimate is constant and unbiased.

5. Find the maximum likelihood estimate of the parameter μ of the population whose probability density function. is given by

$$P(x = i) = \frac{1}{1+\mu}\left(\frac{\mu}{1+\mu}\right)^i, \ \mu > 0 \text{ for } i = 0,1,2.......;$$

Show that the estimate is consistent and unbiased to the population parameter.