

The University of New Mexico
School of Engineering
Electrical and Computer Engineering Department

ECE 535 Satellite Communications

Student Name: Alex Hostick

Student SN: 201

Module # 6: 5.3, 5.5, 5.7, 5.9, 5.13, 5.14, 5.17, 5.21, 5.24, 5.25, 5.26, 5.27

Fall 2023

Prof. Tarief Elshafiey

5.3 Two electric fields with an amplitude ratio of 3:1 and in time phase, act at right angles to one another in space. On a set of x-y axes draw the path traced by the tip of the resultant. Given that the total power developed across a $50\ \Omega$ load is 10 W , find the peak voltage corresponding to the unity amplitude.

$$P = 10\text{ W}, \Omega = 50, \text{ ratio } 3:1$$

Find peak voltage at unity amplitude:

Per Ohm's law and average RMS power in a sinusoidal wave...

$$V_{RMS} = \frac{V}{\sqrt{2}}$$

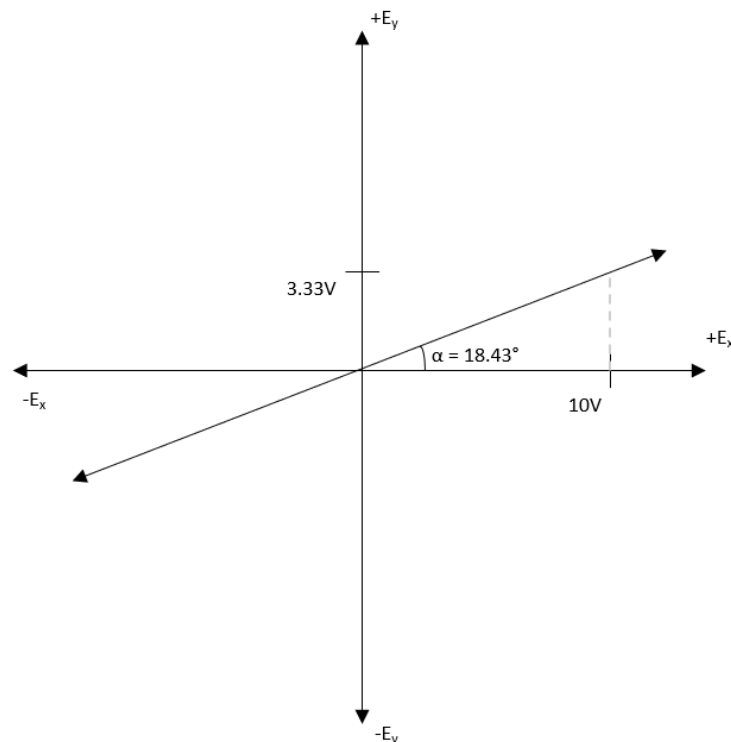
$$P = \frac{\left(\frac{V}{\sqrt{2}}\right)^2}{R} \xrightarrow{\text{yields}} P = \frac{(V)^2}{2R} \xrightarrow{\text{yields}} V = \sqrt{P * 2R} \xrightarrow{\text{yields}} = \sqrt{10\text{ W} * 2(50\text{ ohm})} = 31.62\text{ V}$$

$$V_{peak}^2 = E_x^2 + (E_y^2 = E_x^2, \text{ due to equal time phase})$$

$$31.62\text{ V}^2 = E_x^2 + (3E_x)^2 \xrightarrow{\text{yields}} 1000\text{ V} = E_x^2 + 9E_x^2 \xrightarrow{\text{yields}} 1000\text{ V} = 10E_x^2$$

$$E_x = \sqrt{100} = 10\text{ V}$$

$$\alpha = \tan^{-1} \frac{E_y = \frac{1}{3}E_x}{E_x} \xrightarrow{\text{yields}} \tan^{-1} \frac{3.33\text{ V}}{10\text{ V}} = 18.45^\circ$$



5.5 Two electric field vectors of amplitude ratio 3:1, are 90° out of time phase with one another. On a set of x-y axes draw the path traced by the tip of the resultant vector. If the peak voltages are 3 V and 1 V determine the average power developed in a 10 Ω load.

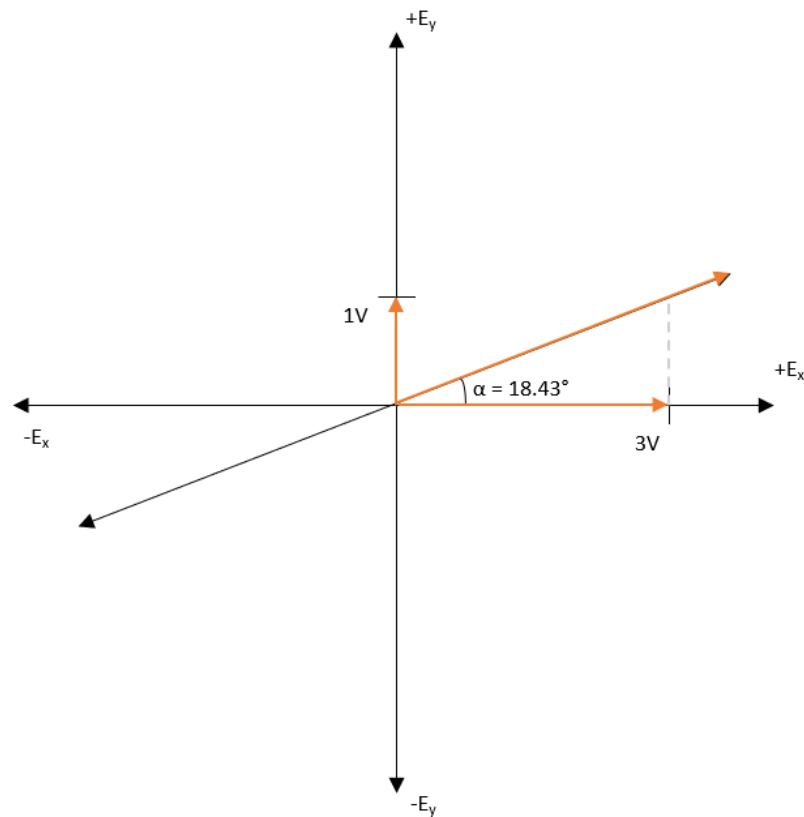
$$V_{RMS} = \frac{V}{\sqrt{2}}; E_x(t) = 3V\cos(\omega t); 1V\cos(\omega t + 90^\circ) = 1V\sin(\omega t)$$

$$V_{rms_x} = \frac{3V}{\sqrt{2}} = 2.12V; V_{rms_y} = \frac{1V}{\sqrt{2}} = 0.707V$$

$$V_{RMS} = \sqrt{V_{rms_x}^2 + V_{rms_y}^2} \xrightarrow{\text{yields}} = \sqrt{2.12^2 + 0.707^2} = 2.33V$$

$$P = \frac{V^2}{R} \xrightarrow{\text{yields}} P = \frac{(2.33)^2}{10} \xrightarrow{\text{yields}} P = 0.54W$$

$$V_{peak}^2 = E_x^2 + (E_y^2 = E_x^2, \text{due to equal time phase})$$



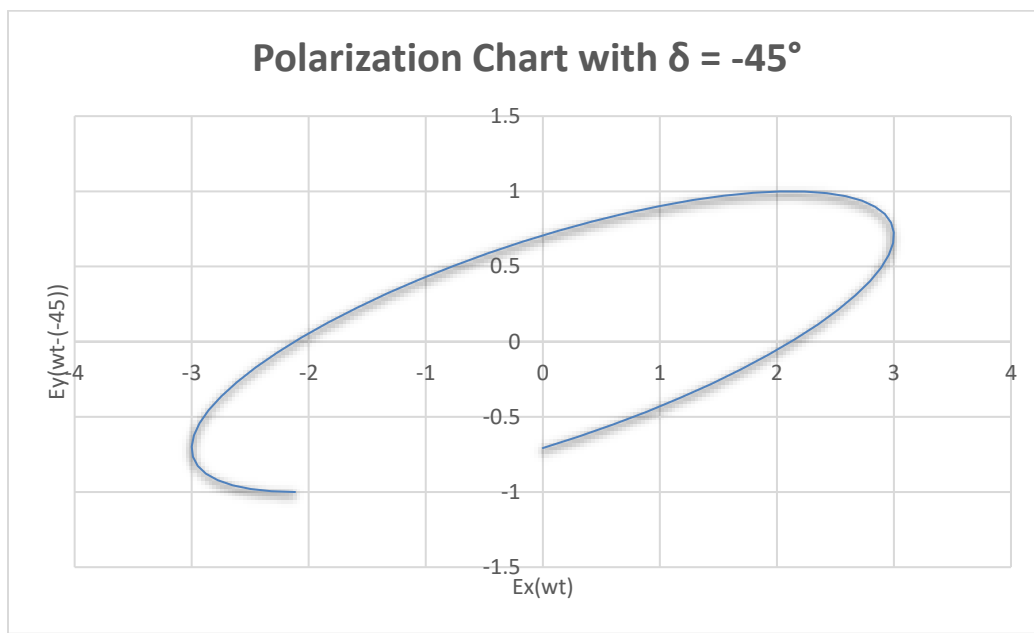
5.7 With $\delta = -45^\circ$ and equal amplitude components, determine the sense of polarization of a wave represented by Eq. (5.6).

$$E_x = 3V; E_y = 1V; -45^\circ \text{ phase angle}$$

$$E_x(t) = 3V\cos(\omega t)$$

$$E_y(t) = 1V\cos(\omega t - 45^\circ) = 1V\sin(\omega t)$$

Plot ωt from 0 to $\frac{\pi}{2}$



LH Elliptical Polarization

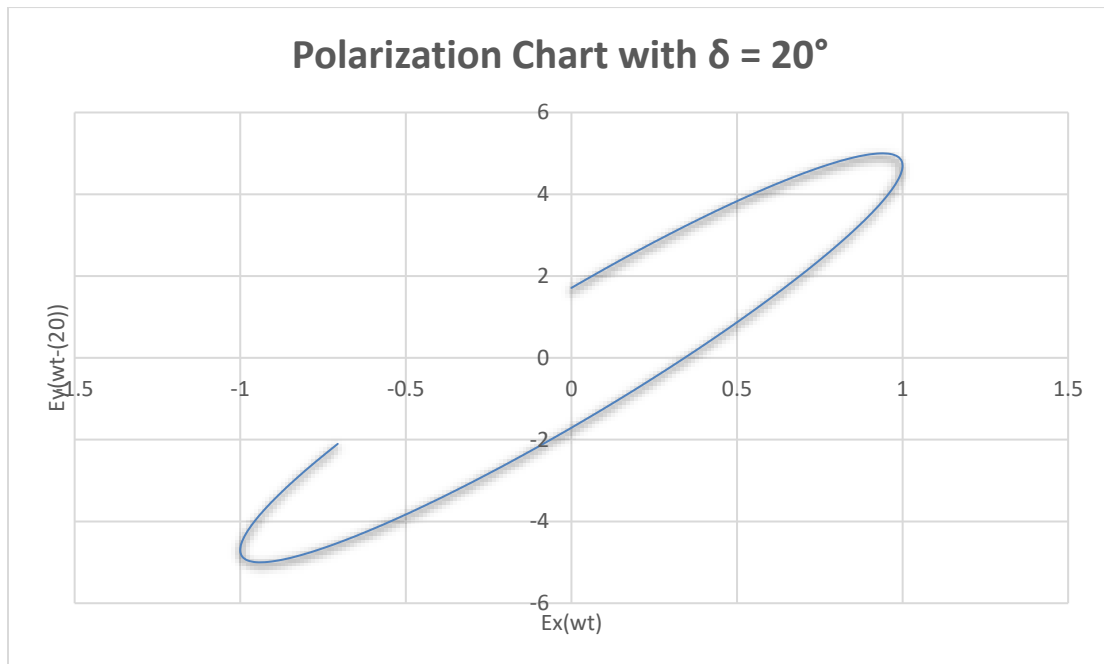
5.9 A plane TEM wave has a horizontal (+x directed) component of electric field of amplitude 3 V/m and a vertical (+y directed) component of electric field of amplitude 5 V/m. The horizontal component lags the vertical component by a phase angle of 20° . Determine the sense of polarization.

$$E_x = 3V; E_y = 5V; 20^\circ \text{ phase angle}$$

$$E_x(t) = 3V\cos(\omega t)$$

$$E_y(t) = 5V\cos(\omega t + 20^\circ)$$

Plot ωt from 0 to $\frac{\pi}{2}$



LH Elliptical Polarization

5.13 A plane TEM wave has a horizontal (+x-directed) component of electric field of amplitude 3 V/m and a vertical (+y-directed) component of electric field of amplitude 5 V/m. The components are in time phase with one another. Determine the angle a linearly polarized antenna must be at with reference to the x axis to receive maximum signal.

(a) At 0.01 percent:

$$\alpha = \tan^{-1} \frac{E_y}{E_x}$$

$$\alpha = \tan^{-1} \frac{5V/m}{3V/m}$$

$$\alpha = 59.04^\circ$$

5.14 For Prob. 5.13, what would be the reduction in decibels of the received signal if the antenna is placed along the x axis?

(a) At 0.01 percent:

$$E = \sqrt{E_x^2 + E_y^2}$$

$$E = \sqrt{3^2 + 5^2} = 5.83V/m$$

$$\text{Reduction of decibels} = dB = 20\log\left(\frac{F_1}{F_2}\right)$$

$$\text{Reduction of decibels} = 20\log\left(\frac{E_{received}}{E_{max}}\right) = 20\log\left(\frac{3}{5.83}\right) = -6.76dB$$

5.17 A geostationary satellite stationed at 90°W transmits a vertically polarized wave. Determine the polarization of the resulting signal received at an earth station situated at 70°W, 45°N.

$$\Phi_{SS} = -90^\circ; \lambda_E = 45^\circ; \Phi_E = -70^\circ; \text{vertically polarized \& geostationary}$$

$$a_{GSO} = 42,164\text{km}; R = 6,371\text{km}$$

$$B = \Phi_E - \Phi_{SS} = (-70^\circ) - (-90^\circ) = 20^\circ$$

$$R_x = R \cos \lambda \cos B \xrightarrow{\text{yields}} (6,371\text{km})(\cos(45^\circ))(\cos(20^\circ)) = 4,233.29\text{km}$$

$$R_y = R \cos \lambda \sin B \xrightarrow{\text{yields}} (6,371\text{km})(\cos(45^\circ))(\sin(20^\circ)) = 1,540.79\text{km}$$

$$R_z = R \sin \lambda \xrightarrow{\text{yields}} (6,371\text{km})(\sin(45^\circ)) = 4,504.30\text{km}$$

$$\mathbf{r} = -\mathbf{R} = -\begin{bmatrix} 4,233.29\text{km} \\ 1,540.79\text{km} \\ 4,504.30\text{km} \end{bmatrix} \quad \mathbf{k} = \begin{bmatrix} R_x - a_{GSO} \\ R_y \\ R_z \end{bmatrix} = \begin{bmatrix} -37,929\text{km} \\ 1,540.79\text{km} \\ 4,504.30\text{km} \end{bmatrix} \quad \mathbf{e} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{f} = \mathbf{k} \times \mathbf{r}$$

$$\mathbf{f} = \begin{bmatrix} a_x & a_y & a_z \\ -37,930.71\text{km} & 1,540.79\text{km} & 4,504.98\text{km} \\ -4233.29\text{km} & -1540.79\text{km} & -4504.98\text{km} \end{bmatrix} = a_x 0 - a_y 1.899 \times 10^8 + a_z 6.497 \times 10^7$$

$$\mathbf{g} = \mathbf{k} \times \mathbf{e}$$

$$\mathbf{g} = \begin{bmatrix} a_x & a_y & a_z \\ -37,930.71\text{km} & 1,540.79\text{km} & 4,504.98\text{km} \\ 0 & 0 & 1 \end{bmatrix} = a_x 1540.79 + a_y 3.79 \times 10^4 + a_z 0$$

$$\mathbf{h} = \mathbf{g} \times \mathbf{k}$$

$$\mathbf{h} = \begin{bmatrix} a_x & a_y & a_z \\ 1540.79\text{km} & 3.79 \times 10^4\text{km} & 0\text{km} \\ -37,930.71\text{km} & 1,540.79\text{km} & 4,504.98\text{km} \end{bmatrix} = a_x 1.709 \times 10^8 + a_y 6.94 \times 10^6 + a_z 1.436 \times 10^9$$

$$|h| = \sqrt{1.709 \times 10^8 + 6.94 \times 10^6 + 1.436 \times 10^9} = 1.447 \times 10^9$$

$$p = \frac{h}{|h|} = a_x 0.118 + a_y 0.005 + a_z 0.993$$

$$p \cdot f = 6.36 \times 10^7$$

$$|f| = 0 + 1.899 \times 10^8 + 6.497 \times 10^7 = 2.01 \times 10^8 \text{km}^2$$

$$\xi = \sin^{-1} \frac{p \cdot f}{|f|} \xrightarrow{\text{yields}} \sin^{-1} \frac{6.36}{2.01} = 0.322 \text{ rad or } 18.46^\circ$$

5.21 A linearly polarized wave traveling through the ionosphere suffers a Faraday rotation of 9° . Calculate (a) the polarization loss and (b) the cross polarization discrimination.

$$\theta = 9^\circ$$

(a) Polarization Loss

$$PL = 20 \log(\cos \theta_F) = 20 \log(\cos(9^\circ)) = -0.11 \text{ dB loss}$$

(b) Cross Polarization Discrimination

$$XPD = 20 \log \frac{E_{11}}{E_{12}} \text{ or } 20 \log(\cot(9^\circ)) = 16 \text{ dB}$$

5.24 A transmission path between an earth station and a satellite has an angle of elevation of 32° with reference to the earth. The transmission is circularly polarized at a frequency of 12 GHz. Given that rain attenuation on the path is 1 dB, calculate the cross-polarization discrimination.

$$EL = 32^\circ; f = 12\text{GHz}; A = 1\text{dB}; \text{Circular Polarized: } \tau = 45^\circ$$

$$XPD = U - V \log A$$

$$U = 30 \log(f) - 10 \log(0.5 - 0.4697 \cos 4\tau) - 40 \cos \theta$$

$$\text{Identity: } \cos 4(\tau) = 8 \cos^2(\tau) - 8 \cos^2(\tau + 1)$$

$$U = 30 \log(f) - 10 \log(0.5 - 0.4697(8 \cos^2(\tau) - 8 \cos^2(\tau + 1))) - 40 \cos \theta$$

$$U = 30 \log(12\text{GHz}) - 10 \log(0.5 - 0.4697(8 \cos^2(45) - 8 \cos^2(45 + 1))) - 40 \log(\cos(32))$$

$$U = 31.08 + 0.13 + 2.86$$

$$U = 34.07$$

$$V = \begin{cases} 20 & \text{for } 8 \leq f \leq 15 \text{ GHz} \\ 23 & \text{for } 15 \leq f \leq 35 \text{ GHz} \end{cases}$$

$$XPD = 34.07 - (20 \log(1)) = 34.07\text{dB}$$

5.25 Repeat Prob. 5.24 for a linearly polarized signal where the electric field vector is parallel to the earth at the earth station.

$$EL = 32^\circ; f = 12\text{GHz}; A = 1\text{dB}; \text{Parallel to Earth Station } \tau = 0^\circ$$

$$XPD = U - V \log A$$

$$U = 30 \log(f) - 10 \log(0.5 - 0.4697 \cos 4\tau) - 40 \cos \theta$$

$$U = 30 \log(12\text{GHz}) - 10 \log(0.5 - 0.4697(8 \cos^2(0) - 8 \cos^2(0 + 1))) - 40 \log(\cos(32))$$

$$U = 32.38 + 15.19 + 2.86$$

$$U = 50.42$$

$$V = \begin{cases} 20 & \text{for } 8 \leq f \leq 15 \text{ GHz} \\ 23 & \text{for } 15 \leq f \leq 35 \text{ GHz} \end{cases}$$

$$XPD = 50.42 - (20 \log(1)) = 50.4\text{dB}$$

5.26 Repeat Prob. 5.24 for a linearly polarized signal where the electric field vector lies in the plane containing the direction of propagation and the local vertical at the earth station.

$$EL = 32^\circ; f = 12\text{GHz}; A = 1\text{dB}; \text{Parallel to Earth Station: } \tau = 90 - \theta^\circ$$

$$XPD = U - V \log A$$

$$U = 30 \log(f) - 10 \log(0.5 - 0.4697 \cos 4\tau) - 40 \cos \theta$$

$$U = 30 \log(12\text{GHz}) - 10 \log(0.5 - 0.4697 \cos 4(90 - 32^\circ)) - 40 \log(\cos(32^\circ))$$

$$U = 32.38 + 1.02 + 2.86$$

$$U = 36.26$$

$$V = \begin{cases} 20 & \text{for } 8 \leq f \leq 15 \text{ GHz} \\ 23 & \text{for } 15 \leq f \leq 35 \text{ GHz} \end{cases}$$

$$XPD = 36.26 - (20 \log(1)) = 36.26\text{dB}$$

5.27 Repeat Prob. 5.24 for a signal frequency of 18 GHz and an attenuation of 1.5 dB.

$$EL = 32^\circ; f = 18\text{GHz}; A = 1.5\text{dB}; \text{Parallel to Earth Station: } \tau = 45^\circ$$

$$XPD = U - V \log A$$

$$U = 30 \log(f) - 10 \log(0.5 - 0.4697 \cos^4 \tau) - 40 \cos \theta$$

$$U = 30 \log(18\text{GHz}) - 10 \log(0.5 - 0.4697 \cos^4(45^\circ)) - 40 \log(\cos(32))$$

$$U = 3.77 + 0.13 + 2.86$$

$$U = 6.76$$

$$V = \begin{cases} 20 & \text{for } 8 \leq f \leq 15 \text{ GHz} \\ 23 & \text{for } 15 \leq f \leq 35 \text{ GHz} \end{cases}$$

$$XPD = 6.76 - (20 \log(1.5)) = 3.24\text{dB}$$