

**The University of New Mexico**  
**School of Engineering**  
**Electrical and Computer Engineering Department**  
  
**ECE 535 Satellite Communications**

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Module # 4-1: 3.1, 3.3, 3.6, 3.7, 3.8, 3.9, 3.14, 3.15, 3.16

Fall 2023

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### **3.1 Explain what is meant by the geostationary orbit. How do the geostationary orbit and geosynchronous orbit differ.**

Satellites in geostationary orbit appear to be stationary with respect to the earth. Three conditions are required for an orbit to be geostationary:

1. The satellite must travel eastward at the same rotational speed as the earth.
2. The orbit must be circular.
  - a. From Kepler's second law. Constant speed means equal areas must be swept out in equal times which can only occur in a circular orbit.
3. The inclination of the orbit must be zero.
  - a. Any inclination would have the satellite moving north and south and the orbit would not be geostationary.

Satellites in geosynchronous orbit do not appear stationary as they rotate in synchronism with the rotation of the earth. The satellite also does not have to be near-geostationary and not positioned over the equator. Geosynchronous satellites may also have an inclination.

**3.3 Determine the latitude and longitude of the farthest north earth station which can link with any given geostationary satellite. The longitude should be given relative to the satellite longitude, and the minimum elevation angle of  $5^\circ$  should be assumed for the earth station antenna. A spherical earth of mean radius 6371 km may be assumed.**

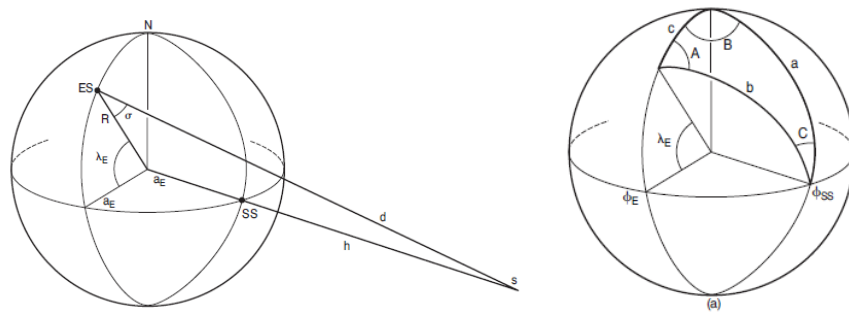
Elevation Angle =  $5^\circ$

$R = 6371\text{km}$

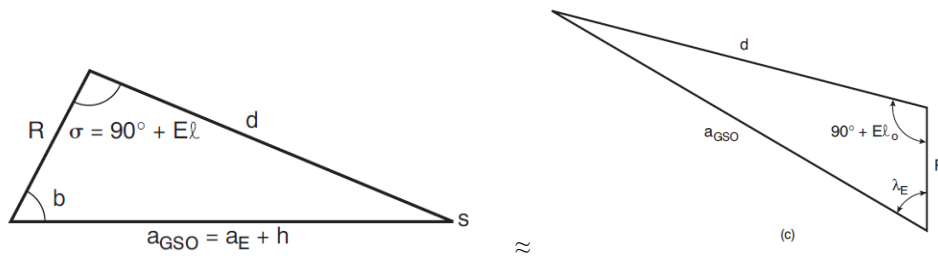
$\Phi_e = S$

$a_{\text{GSO}} = 42,164\text{km}$ ,  $h_{\text{GSO}} = 37,786\text{km}$

Base on the geometry for determining the angles to a spherical triangle from the book...



And the derived spherical plane triangle:



$$\sigma = 90^\circ + El = 90 + 5 = 95^\circ$$

$$S = \sin^{-1}\left(\frac{R}{a_{\text{GSO}}} * \sin(El)\right) \xrightarrow{\text{yields}} \sin^{-1}\left(\frac{6371\text{km}}{42164\text{km}} * \sin(95^\circ)\right) = 8.7^\circ$$

$$b = 180 - El - S = 180 - 95 - 8.7 = 76.3^\circ$$

$$\lambda_E = b = 76.3^\circ$$

If the longitude is relative to the satellite longitude, then  $\Phi_e = S$  as it would also be  $0^\circ$  for geostationary orbit.

**3.6 An earth station is located at latitude 35°N and longitude 100°W. Calculate the antenna-look angle for a satellite at 67°W.**

$$\lambda_E = 35^\circ$$

$$\Phi_E = -100^\circ$$

$$\Phi_{SS} = -67^\circ$$

Calculate b...

$$B = \Phi_E - \Phi_{SS} = -100^\circ - (-67^\circ) = -33^\circ$$

$$b = \cos^{-1}(\cos(B) * \cos(\lambda_E)) \xrightarrow{\text{yields}} \cos^{-1}(\cos(55^\circ) * \cos(35^\circ)) = 46.6^\circ$$

Calculate a...

$$A = \sin^{-1}\left(\frac{\sin|B|}{\sin(b)}\right) \xrightarrow{\text{yields}} \sin^{-1}\left(\frac{\sin|33^\circ|}{\sin(46.6^\circ)}\right) = 48.56^\circ$$

Azimuth

$$A_z = 180^\circ - A \xrightarrow{\text{yields}} 180^\circ - 48.56^\circ = 131.44$$

Elevation

$$d = \sqrt{R^2 + a_{GSO}^2 - (2 * R * a_{GSO} * \cos(b))} \xrightarrow{\text{yields}} d = \sqrt{6371km^2 + 42164km^2 - (2 * 6371km * 42164km * \cos(46.6^\circ))}$$

$$d = 38,069km$$

$$EL = \cos^{-1}\left(\frac{a_{GSO}}{d} * \sin(b)\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{42,164km}{38,069km} * \sin(46.6^\circ)\right) = 36.33^\circ$$

Antenna Look Angle:

$$\text{Azimuth} = 131.44^\circ$$

$$\text{Elevation} = 36.33^\circ$$

**3.7 An earth station is located at latitude 12°S and longitude 52°W. Calculate the antenna-look angle for a satellite at 70°W.**

$$\lambda_E = -12^\circ$$

$$\Phi_E = -52^\circ$$

$$\Phi_{SS} = -70^\circ$$

Calculate b...

$$B = \Phi_E - \Phi_{SS} = -52^\circ - (-70^\circ) = 18^\circ$$

$$b = \cos^{-1}(\cos(B) * \cos(\lambda_E)) \xrightarrow{\text{yields}} \cos^{-1}(\cos(18^\circ) * \cos(-12^\circ)) = 21.6^\circ$$

Calculate a...

$$A = \sin^{-1}\left(\frac{\sin|B|}{\sin(b)}\right) \xrightarrow{\text{yields}} \sin^{-1}\left(\frac{\sin|18^\circ|}{\sin(21.6^\circ)}\right) = 57.1^\circ$$

Azimuth

$$A_z = 360^\circ - A \xrightarrow{\text{yields}} 360^\circ - 57.1^\circ = 302.9^\circ$$

Elevation

$$d = \sqrt{R^2 + a_{GSO}^2 - (2 * R * a_{GSO} * \cos(b))} \xrightarrow{\text{yields}} d = \sqrt{6371km^2 + 42164km^2 - (2 * 6371km * 42164km * \cos(21.6^\circ))}$$

$$d = 36,314km$$

$$EL = \cos^{-1}\left(\frac{a_{GSO}}{d} * \sin(b)\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{42,164km}{36,314km} * \sin(21.6^\circ)\right) = 64.7^\circ$$

Antenna Look Angle:

$$\text{Azimuth} = 302.9^\circ$$

$$\text{Elevation} = 64.7^\circ$$

**3.8 An earth station is located at latitude 35°N and longitude 65°E. Calculate the antenna-look angle for a satellite at 19°E.**

$$\lambda_E = 35^\circ$$

$$\Phi_E = 65^\circ$$

$$\Phi_{SS} = 19^\circ$$

Calculate b...

$$B = \Phi_E - \Phi_{SS} = 65^\circ - 19^\circ = 46^\circ$$

$$b = \cos^{-1}(\cos(B) * \cos(\lambda_E)) \xrightarrow{\text{yields}} \cos^{-1}(\cos(46^\circ) * \cos(35^\circ)) = 55.3^\circ$$

Calculate a...

$$A = \sin^{-1}\left(\frac{\sin|B|}{\sin(b)}\right) \xrightarrow{\text{yields}} \sin^{-1}\left(\frac{\sin|46^\circ|}{\sin(55.3^\circ)}\right) = 61^\circ$$

Azimuth

$$A_z = 180^\circ + A \xrightarrow{\text{yields}} 180^\circ + 57.1^\circ = 241.04^\circ$$

Elevation

$$d = \sqrt{R^2 + a_{GSO}^2 - (2 * R * a_{GSO} * \cos(b))} \xrightarrow{\text{yields}} d = \sqrt{6371km^2 + 42164km^2 - (2 * 6371km * 42164km * \cos(55.3^\circ))}$$

$$d = 38,893km$$

$$EL = \cos^{-1}\left(\frac{a_{GSO}}{d} * \sin(b)\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{42,164km}{38,893km} * \sin(55.3^\circ)\right) = 27.4^\circ$$

Antenna Look Angle:

$$\text{Azimuth} = 241^\circ$$

$$\text{Elevation} = 27.4^\circ$$

**3.9 An earth station is located at latitude 30°S and longitude 130°E. Calculate the antenna-look angle for a satellite at 156°E.**

$$\lambda_E = -30^\circ$$

$$\Phi_E = 130^\circ$$

$$\Phi_{SS} = 156^\circ$$

Calculate b...

$$B = \Phi_E - \Phi_{SS} = 130^\circ - 156^\circ = -26^\circ$$

$$b = \cos^{-1}(\cos(B) * \cos(\lambda_E)) \xrightarrow{\text{yields}} \cos^{-1}(\cos(-26^\circ) * \cos(-30^\circ)) = 38.9^\circ$$

Calculate a...

$$A = \sin^{-1}\left(\frac{\sin|B|}{\sin(b)}\right) \xrightarrow{\text{yields}} \sin^{-1}\left(\frac{\sin|-26^\circ|}{\sin(38.9^\circ)}\right) = 44.2^\circ$$

Azimuth

$$A_z = A \xrightarrow{\text{yields}} = 44.2^\circ$$

Elevation

$$d = \sqrt{R^2 + a_{GSO}^2 - (2 * R * a_{GSO} * \cos(b))} \xrightarrow{\text{yields}} d = \sqrt{6371km^2 + 42164km^2 - (2 * 6371km * 42164km * \cos(38.9^\circ))}$$

$$d = 37,126km$$

$$EL = \cos^{-1}\left(\frac{a_{GSO}}{d} * \sin(b)\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{42,164km}{37,126km} * \sin(38.9^\circ)\right) = 49.3^\circ$$

Antenna Look Angle:

$$\text{Azimuth} = 44.2^\circ$$

$$\text{Elevation} = 49.3^\circ$$

**3.14 (a) An earth station is located at latitude 35°N. Assuming a polar mount antenna is used, calculate the angle of tilt. (b) Would the result apply to polar mounts used at the earth station specified in Probs. 3.6 and 3.8?**

$$\lambda_E = 35^\circ$$

**(a) Antenna Tilt Angle:**

$$d = \sqrt{R^2 + a_{GSO}^2 - (2 * R * a_{GSO} * \cos(\lambda_E))} \xrightarrow{\text{yields}} d = \sqrt{6371km^2 + 42164km^2 - (2 * 6371km * 42164km * \cos(35^\circ))}$$

$$d = 37,126km$$

$$EL = \cos^{-1}\left(\frac{a_{GSO}}{d} * \sin(\lambda_E)\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{42,164km}{37,126km} * \sin(35^\circ)\right) = 49.3^\circ$$

$$\delta = 90^\circ - EL - \lambda_E \xrightarrow{\text{yields}} 90^\circ - 49.3 - 35 = 5.7^\circ$$

**(b1) Would this work for a polar mount used at the earth station for problem 3.6?**

$$\delta = 90^\circ - (\sin^{-1}\left(\frac{a_{GSO}}{d} * \sin\lambda_E\right) - \lambda_E) \xrightarrow{\text{yields}} 90^\circ - (\sin^{-1}\left(\frac{42,164km}{37,126km} * \sin(35^\circ)\right) - 35^\circ) = 14.35^\circ$$

$$B = \cos^{-1}\left(\frac{\cos b}{\cos\lambda_E}\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{\cos(46.6)}{\cos(35)}\right) = 33^\circ$$

$$\Phi_E + B \xrightarrow{\text{yields}} -100 + 33 = -67^\circ E$$

$$\Phi_E - B \xrightarrow{\text{yields}} -100 - 33 = -132.98^\circ W$$

Satellite position at 67°W which is < 132.98°W, so yes, it would work.

**(b1) Would this work for a polar mount used at the earth station for problem 3.8?**

$$\delta = 90^\circ - (\sin^{-1}\left(\frac{a_{GSO}}{d} * \sin\lambda_E\right) - \lambda_E) \xrightarrow{\text{yields}} 90^\circ - (\sin^{-1}\left(\frac{42,164km}{38,893km} * \sin(35^\circ)\right) - 35^\circ) = 16.5^\circ$$

$$B = \cos^{-1}\left(\frac{\cos b}{\cos\lambda_E}\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{\cos(55.3)}{\cos(35)}\right) = 46^\circ$$

$$\Phi_E + B \xrightarrow{\text{yields}} 65 + 46 = 111^\circ E$$

$$\Phi_E - B \xrightarrow{\text{yields}} 65 - 46 = 19^\circ W$$

Satellite position at 19°W and = 19°W. If a 5° minimum elevation is needed, it would not work.



**3.15 Repeat Prob. 3.14 (a) for an earth station located at latitude 12°S. Would the result apply to a polar mount used at the earth station specified in Prob. 3.7?**

$$\lambda_E = -12^\circ$$

**(a) Antenna Tilt Angle:**

$$d = \sqrt{R^2 + a_{GSO}^2 - (2 * R * a_{GSO} * \cos(\lambda_E))} \xrightarrow{\text{yields}} d = \sqrt{6371km^2 + 42164km^2 - (2 * 6371km * 42164km * \cos(-12^\circ))}$$

$$d = 35,959km$$

$$EL = \cos^{-1}\left(\frac{a_{GSO}}{d} * \sin(\lambda_E)\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{42,164km}{35,959km} * \sin(-12^\circ)\right) = 104.12^\circ$$

$$\delta = 90^\circ - EL - \lambda_E \xrightarrow{\text{yields}} 90^\circ - 104.12 - (-12) = -2.12^\circ$$

**(b1) Would this work for a polar mount used at the earth station for problem 3.7?**

$$\delta = 90^\circ - (\sin^{-1}\left(\frac{a_{GSO}}{d} * \sin\lambda_E\right) - \lambda_E) \xrightarrow{\text{yields}} 90^\circ - (\sin^{-1}\left(\frac{42,164km}{35,959km} * \sin(-12^\circ)\right) - (-12^\circ)) = 116^\circ$$

$$B = \cos^{-1}\left(\frac{\cos b}{\cos\lambda_E}\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{\cos(21.6)}{\cos(-12)}\right) = 18^\circ$$

$$\Phi_E + B \xrightarrow{\text{yields}} 65 + 18 = 83^\circ E$$

$$\Phi_E - B \xrightarrow{\text{yields}} 65 - 18 = 47^\circ W$$

Satellite position at 70°W and > 47°W, so no, it would not work.

**3.16 Repeat Prob. 3.14 (a) for an earth station located at latitude 30°S. Would the result apply to a polar mount used at the earth station specified in Prob. 3.7?**

$$\lambda_E = -30^\circ$$

**(a) Antenna Tilt Angle:**

$$d = \sqrt{R^2 + a_{GSO}^2 - (2 * R * a_{GSO} * \cos(\lambda_E))} \xrightarrow{\text{yields}} d = \sqrt{6371km^2 + 42164km^2 - (2 * 6371km * 42164km * \cos(-30^\circ))}$$

$$d = 36,785km$$

$$EL = \cos^{-1}\left(\frac{a_{GSO}}{d} * \sin(\lambda_E)\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{42,164km}{36,785km} * \sin(-30^\circ)\right) = 124.96^\circ$$

$$\delta = 90^\circ - EL - \lambda_E \xrightarrow{\text{yields}} 90^\circ - 124.96 - (-30) = -4.96^\circ$$

**(b1) Would this work for a polar mount used at the earth station for problem 3.7?**

$$\delta = 90^\circ - (\sin^{-1}\left(\frac{a_{GSO}}{d} * \sin\lambda_E\right) - \lambda_E) \xrightarrow{\text{yields}} 90^\circ - (\sin^{-1}\left(\frac{42,164km}{36,785km} * \sin(-30^\circ)\right) - (-30^\circ)) = 155^\circ$$

$$B = \cos^{-1}\left(\frac{\cos b}{\cos\lambda_E}\right) \xrightarrow{\text{yields}} \cos^{-1}\left(\frac{\cos(44.79)}{\cos(-30)}\right) = 35^\circ$$

$$\Phi_E + B \xrightarrow{\text{yields}} 130 + 35 = 165^\circ E$$

$$\Phi_E - B \xrightarrow{\text{yields}} 130 - 35 = 95^\circ W$$

Satellite position at 156°E and < 165°E, so yes, it would work.