The University of New Mexico

School of Engineering

Electrical and Computer Engineering Department

ECE 535 Satellite Communications

Student Name: Alex Hostick

Student SN: 10201139

Module # 3-2: 2.19, 2.23, 2.24, 2.25, 2.27

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Prof. Tarief Elshafiey

2.19 Determine the Julian days for the following dates and times: midnight March 10, 1999; noon, February 23, 2000; 16:30h, March 1, 2003; 3 P.M., July 4, 2010.

Midnight March 10, 1999:

1999 = 2451178.5

Day of Year = 69

UT = 13 hours = 0.5416667

Julian Day = 2451178.5 + 69 + 0.5416667

= 2,451,248.5

Noon, February 23, 2000:

2000 = 2451543.5

Day of Year = 54

UT = 12 hours = 0.5

Julian Day = 2451543.5 + 54 + 0.5

= 2,451,598

16:30h, March 1, 2003:

2003 = 2452639.5

Day of Year = 60

UT = 16.5 = 0.6875

Julian Day = 2452639.5 + 60 + 0.6875

= 2,452,700.1875

3 P.M., July 4, 2010:

2010 = 2455196.5

Day of Year = 185

UT = 3 P.M. = 15:00h = 0.625

Julian Day = 2,455,382.125

2.23 The Molnya 3-(25) satellite has the following parameters specified: perigee height 462 km; apogee height 40,850 km; period 736 min; inclination 62.8°. Using an average value of 6371 km for the earth's radius, calculate (a) the semi-major axis and (b) the eccentricity. (c) Calculate the nominal mean motion n_0. (d) Calculate the mean motion. (e) Using the calculated value for a, calculate the anomalistic period and compare with the specified value. Calculate (f) the rate of regression of the nodes, and (g) the rate of rotation of the line of apsides.

h_p = 462km

h_a = 40,850km

P = 736 min

Inclination (i) = 62.8° (1.096 radian)

R = 6,371km

$\mu = 3.986005x10^{14} m^3/s^2$

(a) Semi-major axis

r_p = 40,850km + 6,371km = 47,221 km

r_a = 462km + 6,371 km = 6,833 km

$$a = \frac{r_a + r_p}{2} \xrightarrow{yields} \frac{47{,}221km + 6{,}833km}{2} = 27{,}027km$$

(b) Eccentricity

$$e = \frac{r_a}{a} - 1 \xrightarrow{yields} \frac{47{,}221km}{27{,}027km} - 1 = 0.747$$

(c) Nominal Mean Motion n_0

$$n_0 = \sqrt{\frac{\mu}{a^3}} \xrightarrow{yields} \sqrt{\frac{3.986005x10^{14}m^3/s^2}{(27{,}027x10^3m)^3}} = \sqrt{\frac{3.986005x10^{14}m^3/s^2}{19.742x10^{22}m}} = \sqrt{2.019x10^{-8}}$$

$$n_0 = 1.421x10^{-4}\ rad/\sec * 86400\ sec = 12.277\ rad/day$$

(d) Mean Motion

$$n = \frac{2\pi}{P} \xrightarrow{yields} \frac{2\pi}{736min * 60sec} = \frac{2\pi}{44160} = 1.423x10^{-4} rad/sec * 86400\sec = 12.276 rad/day$$

(e) With a = 27,027km, find anomalistic period

$$P_A = \frac{2\pi}{n_0} \xrightarrow{yields} \frac{2\pi}{1.421x10^{-4}rad/sec} = 44{,}218.88s/60s = 736.98min$$

(f) Rate of Regression of the Nodes

$$K = \frac{nK_1}{a^2(1-e^2)^2} \xrightarrow{yields} \frac{(1.423x10^{-4}rad/sec)(66{,}063.1704km^2)}{(27{,}027km)^2(1-(0.747)^2)^2} = \frac{9.394}{1.42705x10^8} = 6.583x10^{-8}$$

$$\frac{d\Omega}{dt} = -Kcos(i) \xrightarrow{yields} -6.583x10^{-8}(\cos(1.096rad)) = -3.0009x10^{-8}rad/sec = -0.149deg/day$$

(g) Rate of Rotation of the line of apsides

$$\frac{d\omega}{dt} = K\left(2 - 2.5sin^2(i)\right) \xrightarrow{yields} 6.583x10^{-8}\left(2 - 2.5sin^2(1.096rad)\right) = 1.448x10^{-9}rad/sec = 0.007deg/day$$

2.24 Repeat the Calculations in Problem 2.23 for tan inclination of 63.435

Inclination(i) = 63.435° or 1.107 rad

(a) Semi-major axis

r_p = 40,850km + 6,371km = 47,221 km

r_a = 462km + 6,371 km = 6,833 km

$$a = \frac{r_a + r_p}{2} \xrightarrow{yields} \frac{47{,}221km + 6{,}833km}{2} = 27{,}027km$$

(b) Eccentricity

$$e = \frac{r_a}{a} - 1 \xrightarrow{yields} \frac{47{,}221km}{27{,}027km} - 1 = 0.747$$

(c) Nominal Mean Motion n_0

$$n_0 = \sqrt{\frac{\mu}{a^3}} \xrightarrow{yields} \sqrt{\frac{3.986005x10^{14}m^3/s^2}{(27{,}027x10^3m)^3}} = \sqrt{\frac{3.986005x10^{14}m^3/s^2}{19.742x10^{22}m}} = \sqrt{2.019x10^{-8}}$$

$$n_0 = 1.421x10^{-4}\ rad/\sec * 86400\ sec\ =\ 12.277\ rad/day$$

(d) Mean Motion

$$n = \frac{2\pi}{P} \xrightarrow{yields} \frac{2\pi}{736min * 60sec} = \frac{2\pi}{44160} = 1.423x10^{-4}rad/sec * 86400\sec = 12.276rad/day$$

(e) With a = 27,027km, find anomalistic period

$$P_A = \frac{2\pi}{n_0} \xrightarrow{yields} \frac{2\pi}{1.421x10^{-4}rad/sec} =\ 44{,}218.88s/60s = 736.98min$$

(f) Rate of Regression of the Nodes

$$K = \frac{nK_1}{a^2(1-e^2)^2} \xrightarrow{yields} \frac{(1.423x10^{-4}rad/sec)(66{,}063.1704km^2)}{(27{,}027km)^2(1-(0.747)^2)^2} = \frac{9.394}{1.42705x10^8} = 6.583x10^{-8}$$

$$\frac{d\Omega}{dt} = -Kcos(i) \xrightarrow{yields} -6.583x10^{-8}(\cos(1.107rad)) =\ -2.944x10^{-8}rad/sec = -0.146deg/day$$

(g) Rate of Rotation of the line of apsides

$$\frac{d\omega}{dt} = K\left(2 - 2.5sin^2(i)\right) \xrightarrow{yields} 6.583x10^{-8}\left(2 - 2.5sin^2(1.107rad)\right) = 2.3037x10^{-9}rad/sec\ = 0.0114deg/day$$

2.25 Determine the orbital condition necessary for the argument of perigee to remain stationary in the orbital plane. The orbit for a satellite under this condition has an eccentricity of 0.001 and a semi-major axis of 27,000km. At a given epoch the perigee is exactly on the line of Aries. Determine the satellite position relative to this line after a period of 30 days from epoch.

e = 0.001

a = 27,000km

For the satellite to remain stationary in the orbital plane, Inclination(i) = 63.435° or 1.0715 rad

<u>Find mean motion:</u>

$$n = \sqrt{\frac{\mu}{a^3}} \xrightarrow{yields} \sqrt{\frac{3.986005x10^{14}m^3/s^2}{(27{,}000x10^3m)^3}} = \sqrt{\frac{3.986005x10^{14}m^3/s^2}{19.683x10^{22}m}} = \sqrt{2.025x10^{-8}}$$

$$n = 1.42306x10^{-4}\ rad/sec$$

<u>Find K:</u>

$$K = \frac{nK_1}{a^2(1-e^2)^2} \xrightarrow{yields} \frac{(1.42306x10^{-4}rad/sec)(66{,}063.1704km^2)}{(27{,}000km)^2(1-(0.001)^2)^2} = 1.289604x10^{-8}rad/sec$$

Satellite Position:

$$\frac{d\Omega}{dt} = -Kcos(i) \xrightarrow{yields} -1.289604x10^{-8}(\cos(1.0715rad)) = -6.1748rad/sec$$

$$\frac{d\Omega}{dt}_{30\ days} = -\frac{6.1748rad}{sec} * 2592000 = -0.016rad/day_{30} = -0.917°$$

Note: In MS Excel, the position is exactly 0.856°W most likely due to significant digits/rounding.

2.27 A satellite has an inclination of 90° and an eccentricity of 0.1. At epoch, which corresponds to the time of perigee passage, the perigee height is 2,643.24km directly over the north pole. Determine (a) the satellite mean motion. For 1 day after epoch determine (b) the true anomaly, (c) the magnitude of the radius vector to the satellite, and (d) the latitude of the subsatellite point.

Inclination (i) = 90° (polar orbit)

$\mu = 3.986005\text{x}10^{14}\ m^3/s^2$

$h_p = 2{,}643.24km$

$K_1 = 66{,}063.1704km^2$

(a) Mean Motion

$$r_p = 2{,}643.24km + 6{,}371km = 9{,}014.24km$$

$$a = \frac{r_p}{1-e} \xrightarrow{yields} \frac{9{,}014.24km}{1-0.1} = 10{,}015.82km$$

$$n = \sqrt{\frac{\mu}{a^3}} \xrightarrow{yields} \sqrt{\frac{3.986005\text{x}10^{14}\ m^3/s^2}{(10{,}015.82x10^3m)^3}} = 6.29x10^{-4}s^{-1} * 86400 = 54.42rad/day$$

(b) True Anomaly

Assume M ≈ E for small eccentricity of 0.1.

$$tan\left(\frac{v}{2}\right) = \sqrt{\frac{1+e}{1-e}}tan\frac{E}{2} \xrightarrow{yields} \sqrt{\frac{1.1}{0.9}}tan(\frac{54.42}{2}) = 1.074\tan(27.21rad) = -1.9361rad$$

$$\left(\frac{v}{2}\right) = tan^{-1}(-1.9361rad) = -1.094rad$$

$$v = 2 * -1.904 = -2.1881rad\ or - 125.363deg$$

(c) Magnitude of the Radius Vector

$$r = \frac{a(1-e^2)}{1+ecos(v)} \xrightarrow{yields} \frac{(10{,}015.82km)(1-0.1^2)}{1+0.1\cos(-125.37)} = \frac{9{,}915.6618km}{0.94212} = 10{,}524.8395km$$

(d) Latitude of the Subsatellite Point

$$\emptyset = sin^{-1}(\sin(v+\omega)\sin(i)) \xrightarrow{yields} sin^{-1}(\sin(-125.37)\sin(90))$$

$$\emptyset = sin^{-1}(-0.8154) * (1)) \cong -54.63°$$

$$\emptyset = 90° - 54.63° = 35.37°S\ lat\ (due\ to\ south\ to\ north\ orbit, passing\ the\ apogee)$$