The University of New Mexico

School of Engineering

Electrical and Computer Engineering Department

ECE 535 Satellite Communications

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Module # 3-1: 2.1, 2.4, 2.5, 2.6, 2.8, 2.10, 2.11

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2.1 State Kepler’s three laws of planetary motion. Illustrate in each case their relevance to artificial satellites orbiting the earth.

Kepler’s First Law

The path followed by a satellite around the primary will be an ellipse. An ellipse has two focal points. The center of mass of the two-body system, termed the *barycenter*, is always centered on one of the foci. For artificial satellite systems, the center of the earth is one of the foci due to its enormous size. The eccentricity and the semimajor axis are two of the orbital parameters specified for satellites orbiting the earth.

Kepler’s Second Law

Kepler’s second law states that, for equal time intervals, a satellite will sweep out equal areas in its orbital plane, focused on the barycenter. Due to the equal area law, satellites take longer to travel a given distance when they are farther away from the earth. This property is used to increase the length of time the satellites can be seen from geographic regions of earth.

Kepler’s Third Law

Kepler’s third law states that the square of the periodic time of orbit is proportional to the cube of the mean distance between two bodies. The mean distance is equal to the semimajor axis and shows the fixed relationship between period and semimajor axis. This also implies that the period of a satellite orbiting the earth increases rapidly with the radius of its orbit. Geostationary satellites travel at the same rotational speed as the earth with a circular orbit and zero inclination.

2.4. A satellite orbit has an eccentricity of 0.2 and a semimajor axis of 10,000 km. Find the values of (a) the latus rectum; (b) the apogee height; (c) the perigee height. Assume a mean value of 6371 km for the earth's radius.

Apogee Height: $h_a = a(1+e) - R$

Perigee Height: $h_p = a(1-e) - R$

Latus Rectum: $LR = a(1-e^2)$

Eccentricity (e) = 0.2

Semi-major Axis (a) = 10,000 km

Earth's Radius (R) = 6,371 km

(a) Latus Rectum:

Find semi-minor axis first:

$LR = a(1-e^2)$

$LR = 10{,}000km(1-0.2^2)$

$LR = 10{,}000km(0.96)$

$LR = \ 9{,}600\ km$

(b) Apogee Height

$h_a = a(1+e) - R$

$h_a = 10{,}000km(1+0.2) - 6{,}371km$

$h_a = 12{,}000km - 6{,}371km$

$h_a = 5{,}629km$

(c) Perigee Height

$h_p = a(1-e) - R$

$h_p = 10{,}000km(1-0.2) - 6{,}371km$

$h_p = 8{,}000km - 6{,}371km$

$h_p = 1{,}629km$

2.5 For the satellite in Prob 2.4, find the length of the position vector when the true anomaly is 130°.

Position Vector(r): $r = \frac{LR}{1+e(\cos(v))}$

Latus Rectum (LR) = 9,600 km

Eccentricity(e) = 0.2

True Anomaly (v) = $130° * {}^{\pi}/_{180} = 2.26893\ radians$

Position Vector:

$$r = \frac{LR}{1 + e(\cos(v))}$$

$$r = \frac{9{,}600km}{1 + 0.2(\cos(2.26893))}$$

$$r = \frac{9{,}600km}{0.87144218}$$

$r = 11{,}016\ km$

2.6 The orbit for an earth-orbiting satellite has an eccentricity of 0.15 and a semi-major axis of 9,000km. Determine (a) its periodic time; b) the apogee height; (c) the perigee height. Assume a mean value of 6,371km for the earth's radius.

Apogee Height: $h_a = a(1+e) - R$

Perigee Height: $h_p = a(1-e) - R$

Kepler's Third Law: $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

Semi-major Axis(a) = 9,000 km

Eccentricity (e) = 0.15

$\mu = 3.986005\text{x}10^{14}\ m^3/s^2$

(a) Periodic Time:

Solve for mean motion with earth's oblateness:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$T = 2\pi\sqrt{\frac{(9x10^6 m)^3}{3.986005x10^{14}\ m^3/s^2}}$$

$$T = 2\pi\sqrt{\frac{7.29x10^{20} m^3}{3.986005\text{x}10^{14}\ m^3/s^2}}$$

$$T = 2\pi\sqrt{1828901.154 s^2}$$

$T \approx 8492.88\ s$ or 141.55 minutes or 2.36 hours

(b) Apogee Height

$$h_a = a(1+e) - R$$

$$h_a = 9{,}000km(1+0.15) - 6{,}371km$$

$$h_a = 10{,}350km - 6{,}371km$$

$$h_a = 3{,}979km$$

(c) Perigee Height

$$h_p = a(1-e) - R$$

$$h_p = 9{,}000km(1-0.15) - 6{,}371km$$

$$h_p = 7{,}650km - 6{,}371km$$

$$h_p = 1{,}279\ km$$

2.8 The semi-major axis for the orbit of an earth-orbiting satellite is found to be 9,500km. Determine the anomaly 10 minutes after passage of perigee.

Mean Motion: $n_0 = \sqrt{\frac{\mu}{a^3}}$

Mean Anomaly: $M = n_0 * t$

Semi-major Axis (a) = 9,500 km

Time (t) = 10 minutes or 600 seconds

$\mu = 3.986005\text{x}10^{14}\ m^3/s^2$

Solve for mean motion (n_0)

$$n_0 = \sqrt{\frac{\mu}{a^3}}$$

$$n_0 = \sqrt{\frac{3.986005\text{x}10^{14}\ m^3/s^2}{(9.5x10^6)^3}}$$

$$n_0 = \sqrt{\frac{3.986005\text{x}10^{14}\ m^3/s^2}{8.57375\text{x}10^{20} m^3}}$$

$$n_0 = \sqrt{4.649x10^{-7}s^2}$$

$$n_0 = 0.00216s^{-1}$$

Solve for Mean Anomaly:

$$M = n_0 * t$$

$$M = (0.00216s^{-1}) * (600\ s)$$

$$M = 1.296\ radians$$

2.10 Explain what is meant by apogee height and perigee height. The Cosmos 1675 satellite has an apogee height of 39,342km and a perigee height of 613km. Determine the semi-major axis and the eccentricity of its orbit. Assume a mean earth radius of 6371km.

Apogee Height: The point in orbit farthest from the earth.

Perigee Height: The point in orbit closest approach to earth.

Apogee Height (h_a) = 39,342 km

Earth Radius (R) = 6,371 km

Perigee Height (h_p) = 613 km

Apogee Height: $h_a = r_a - R$

Perigee Height: $h_p = r_p - R$

Distance from Earth's center for apogee/perigee:

$h_a = r_a - R \xrightarrow{yields} r_a = h_a + R$

$r_a = 39{,}342km + 6{,}371\ km = 45{,}713km$

$r_p = 613km + 6{,}371\ km = 6{,}984km$

Semi-major Axis:

$$a = \frac{r_a + r_p}{2}$$

$$a = \frac{45{,}713km + 6{,}984km}{2} = 26{,}348.5km$$

Eccentricity:

$r_a = a(1+e) \xrightarrow{yields} e = \frac{r_a}{a} - 1$

$e = \frac{45{,}713km}{26{,}348.5km} - 1$

$e = 1.734937 - 1$

$e \cong 0.734937$

$r_p = a(1-e) \xrightarrow{yields} e = -\left(\frac{r_p}{a}\right) + 1$

$e = -\left(\frac{6{,}984km}{26{,}348.5km}\right) + 1$

$e = (-0.2650625) + 1$

$e \cong 0.734937$

$e \cong 0.734937$

2.11 The Aussat 1 satellite in geostationary orbit has an apogee height of 35,795km and a perigee height of 35,779km. Assuming a value of 6,378km for earth's equatorial radius, determine the semi-major axis and the eccentricity of the satellite's orbit.

Apogee Height (h_a) = 35,795 km

Perigee Height (h_p) = 35,778 km

Earth Radius (R) = 6,378 km

Apogee Height: $h_a = r_a - R$

Perigee Height: $h_p = r_p - R$

Distance from Earth's center for apogee/perigee:

$$h_a = r_a - R \xrightarrow{yields} r_a = h_a + R$$

$$r_a = 35{,}795km + 6{,}378\ km = 42{,}173km$$

$$r_p = 35{,}779km + 6{,}378\ km = 42{,}157km$$

Semi-major Axis:

$$a = \frac{r_a + r_p}{2}$$

$$a = \frac{42{,}173km + 42{,}157km}{2} = 42{,}165km$$

Eccentricity:

$$r_a = a(1+e) \xrightarrow{yields} e = \frac{r_a}{a} - 1$$

$$e = \frac{42{,}173km}{42{,}165km} - 1$$

$$e = 1.00018973 - 1$$

$$e \cong 0.00018973$$

$$r_p = a(1-e) \xrightarrow{yields} e = -\left(\frac{r_p}{a}\right) + 1$$

$$e = -\left(\frac{42{,}157km}{42{,}165km}\right) + 1$$

$$e = (-0.9997984) + 1$$

$$e \cong 0.00018973$$

$$e \cong 1.8973x10^4$$