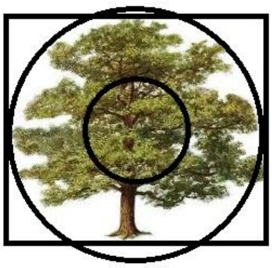
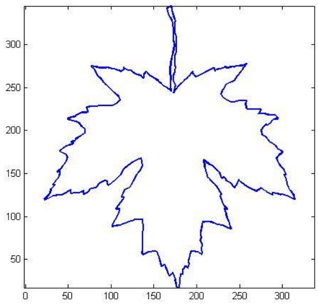




A CLOSE LOOK ON LEAVES

Team 14990



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1. Introduction

Leaf, which is responsible for photosynthesis and storage of food and water, is a very important organ of a plant. Leaf has so many different and interesting shapes and contours, which make it so catching and conspicuous in the plant kingdom. For centuries, people have always wondered why the leaf has so many splendid shapes. Is this just a gift from the Almighty God? Or is it the adjustment the leaf takes during its evolution and adaptation?

Another mystery of the leaf lies in the classification of different leaf types. Can we do so in a more reliable and scientific way instead of judging subjectively? We have tried our best to build a model on a quantitative base to increase the accuracy and efficiency of classifying different leaves.

The final mystery about the tree is how much the total leaf mass on a tree is. Just like people always want to figure out how much hair a person has, human beings always have a fancy to find out a solution from the seemingly infinity and incalculable.

We are deeply convinced that the true glamour of scientific and mathematical models is to unveil the intriguing and the changeable of the nature. Aiming to do so, we will generate models that can fit large quantities of data with highest accuracy within a quick flick.

2. Breaking Down the Problem

After carefully analyzing the problem, we conclude four main sub-problems to tackle in our paper:

1. Classification the different types of leaves

2. Relationship between the leaf distribution and leaf shape

3. Relationship between the tree profile and leaf shape

4. Calculation of the total leaf mass on a leaf

To tackle the first problem, we set a set of parameters to quantify the characters of the leaf shape and use the leaf shape as the main standard for our classification process.

As for the second question, we want to use the overlapping area that one leaf casts on the leaf directly under it as a medium to associate the leaf distribution and leaf shape, since the leaf shape will affect the overlapping and we assume the leaf distribution will try to minimize the overlapping area.

As for the third question, we want to refer to the process we take when tackling the first problem and also set some parameters for the tree profile. After that, we will compare their parameters and judge whether there is a relation between tree profile and leaf shape.

To deal with the total mass of the leaves, we want to use the age to link the size of tree and the total weight of leaves of it because the tree size has an obvious relationship with its age and the age will affect a tree's sequestration of carbon dioxide, which will reflect the weight of a tree's total leaves.

3. Assumptions

1. The trees being studied are all individual ("open grown") trees, such as trees typically planted along streets, in yards, and in parks. Our calculation does not apply to densely raised trees, as in typical reforestation projects where large numbers of trees are planted closely together.
2. Assume the shape of the leaves does not reflect special uses for the trees, such as to resist extremely windy, cold, parched, wet or dry conditions or to catch food.
3. Assume the type of the leaf distribution to be discussed (leaf length and internode distance relation) is only a reflection of the tree's natural tendency to sunlight.
4. Assume the tree profile we consider is the part above ground, including the trunk, the branches and leaves.
5. Assume all parts of leaf lay on a flat surface and the thickness or protrusion of veins are neglectable.
6. Assume leaves are the only part of the tree that reacts in photosynthesis and respiration so that the carbon dioxide sequestration of a tree is the sum of the sequestration of the leaves.
7. Assume the sequestration of a tree or a leaf is the net amount of CO_2 fixed in a tree, which is the difference between the CO_2 released in respiration and the CO_2 absorbed in photosynthesis.
8. Assume the trees are in healthy, mature and stable condition. The trees of the same species have same characteristics.

4. Nomenclatures

R	*Rectangularity*
A_{leaf}	*the area of leaf*
$A_{rectangle}$	*the area of minimum bounding rectangle*
AR	*the aspect ratio*
L_{short}	*the length of shorter side*
L_{long}	*the length of longer side*
C	*the circularity*
R_{in}	*the radius of in-circle*
R_{ex}	*the radius of ex-circle*
FF	*Form factor*
P_{leaf}	*the perimeter of leaf*
$ERAI$	*the edge regularity area index*
BPA	*The bounding polygon area*
$ERPI$	*the edge regularity perimeter index*
BPP	*the bounding polygon perimeter*
PI_i	*Proportional index*
I_D	*the Index of Deviation*
L_{major}	the length of major axis
$A_{overlapping}$	the overlapping area
α	the solar altitude
M_{leaf}	the total mass of leaves on a tree
$M_{carbon\ \ dioxide}$	mass of carbon dioxide sequestered (lbs)
A_s	ability to sequester carbon dioxide (lbs/g)
A	the age of the tree

5. Model One: Leaf Classification

5.1 Decisive Parameters

In order to classify the shapes of the given leaf, we want to set a number of parameters and establish a database for comparison. After carefully and thoroughly analyzing the leaves, we develop seven most significant parameters as shown below:

1. **Rectangularity**

Firstly, we define the ratio of the area of the leaf to the **Minimum Bounding Rectangle** as the leaf's **Rectangularity**, how much does the leaf resemble a rectangle (refer to figure 2.1). The maximum possible value of this parameter is 1.

$$R = \frac{A_{leaf}}{A_{rectangle}}$$

A_{leaf} stands for the area of leaf

$A_{rectangle}$ stands for the area of minimum bounding rectangle

2. **Aspect ratio**

After defining the rectangularity, now we define the **Aspect Ratio**, which describes the proportional relationship between the width of and height of a leaf's **Minimum Bounding Rectangle**, as another key character to classify general shape of a leaf. The bigger this ratio is, the more this leaf resembles a square (refer to figure 2.1). The maximum possible value of this parameter is 1.

Figure 2.1

$$\mathrm{AR} = \frac{L_{short}}{L_{\mathrm{long}}}$$

Where

AR stands for the aspect ratio

L_{short} stands for the length of shorter side

L_{long} stands for the length of longer side

3. **Circularity**

To evaluate how roundish a leaf is, we consider that the respective radius of in-circle and ex-circle. The ratio of the former to the latter, which we define as Circularity, may well reflect this characteristic. The greater the ratio of **Circularity** is, the closer the leaf is to a circle, (refer to figure 2.2) The maximum possible value of this parameter is 1.

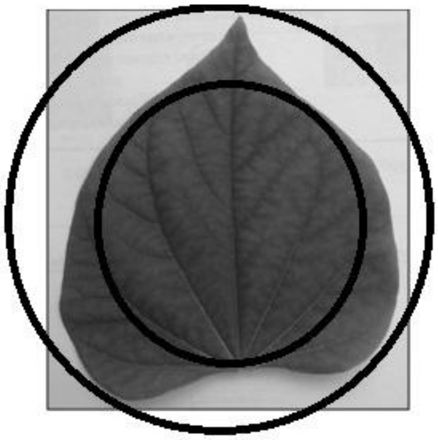


Figure 2.2

$$C = \frac{R_{in}}{R_{ex}}$$

Where

C stands for the circularity

R_{in} stands for the radius of in-circle

R_{ex} stands for the radius of ex-circle

4. **Form factor**

Form Factor, a famous shape description parameter, is another essential indicator of leaf classification. The maximum possible value of this parameter is 1.

$$\text{FF} = \frac{4\pi A_{leaf}}{{P_{leaf}}^2}$$

Where

P_{leaf} stands for the perimeter of leaf

5. **Edge regularity area index.**

Although the aspect ratio and the rectangularity of two leaves may resemble, the contour or the exact shape of two leaves may vary much.

Thus, In order to take the different contour of the leaf into consideration, we join every convex dot along the contour and develop a specific parameter, which we call **Bounding Polygon Area**. The ratio between the leaf area and this bounding polygon area is a good quantitative factor, defined as **Edge Regularity Area Index**.

The more this ratio is close to 1, the less jagged and the smoother this leaf's contour is (refer to figure 2.3). The maximum possible value of this parameter is 1.

$$\mathrm{ERAI} = \frac{A_{leaf}}{BPA}$$

Where

Figure 2.3

ERAI stands for the edge regularity area index

BPA stands for bounding polygon area

6. **Edge regularity perimeter index**

Similarly, we develop another parameter: **Bounding Polygon Perimeter,** the perimeter of the polygon when we join the convex dots of a leaf. We define the ratio of the convex dot perimeter and the perimeter of the leaf as **Edge Regularity Perimeter Index.** This time, the smaller this ratio is, the more jagged and irregular the contour of the leaf is (refer to figure 2.4). The maximum possible value of this parameter is 1.

$$\mathrm{ERPI} = \frac{BPP}{P_{leaf}}$$

Where

ERPI stands for the edge regularity perimeter index

BPP stands for the bounding polygon perimeter

7. **Proportional index**

Since it is also highly critical to capture the spatial distribution of different portions of a leaf along its vertical axis, we divide the minimum bounding rectangle into four blocks horizontally, and calculate the proportion of the leaf area in a particular region to the total leaf, which we refer to as the **Proportional Index**.

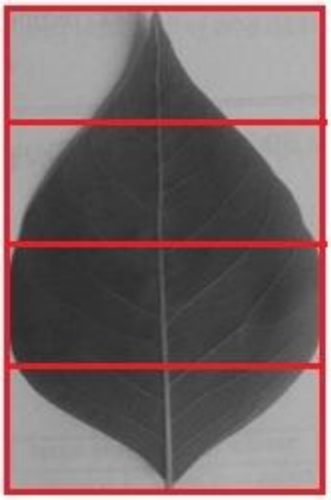
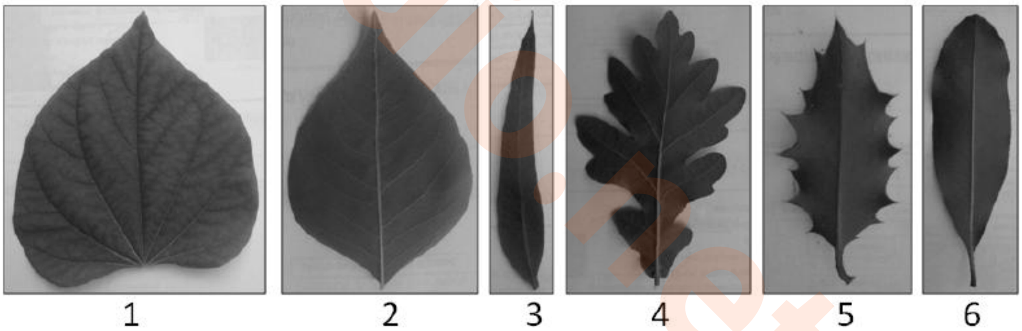


Figure 2.4

$$PI_i = \frac{area\ of\ block\ i}{A_{leaf}}$$

Now, we can develop a database of six most common leaves containing seven parameters discussed above.

These are six most commonly seen leaf types in North America:



Database:

	1	2	3	4	5	6
Rectangularity	0.6627	0.5902	0.6250	0.4772	0.4876	0.6576
Aspect Ratio	0.8615	0.6600	0.1800	0.6383	0.4792	0.3111
Circularity	0.8140	0.5432	0.4564	0.3454	0.3123	0.3311

Form Factor	0.9139	0.6206	0.2823	0.2470	0.3662	0.4956
ER Area Index	0.9322	0.8780	0.9091	0.8500	0.7880	0.8895
ER Perimeter Index	0.8727	0.8889	0.9384	0.8602	0.8231	0.9903
PI_1	0.0649	0.0769	0.1179	0.1909	0.1299	0.2920
PI_2	0.2958	0.3555	0.2208	0.3892	0.3606	0.4187
PI_3	0.3439	0.4243	0.4139	0.3047	0.4123	0.2677
PI_4	0.2954	0.1433	0.2474	0.1152	0.0970	0.0220

5.2 Comparison

When given a specific leaf, we can calculate seven characteristics of it and compare them with our database by calculating the squared deviation of each of parameter of the given leaf from the corresponding standard parameter of each category. We realize the fact that some of seven parameters are somehow more important than others. So in an effort to make our model more accurate and reliable, we induce the conception of the **Index of Deviation**, denoted as $\boldsymbol{I_D}$, which comes from the sum of the squared deviation between the database and the leaf –to–be–classified times the proper weight.

$$I_D = \sum_{i=1}^{7} I_i \cdot W_i$$

$\mathbf{I_1}$	$(\mathbf{R_{new}} - \mathbf{R})^2$
$\mathbf{I_2}$	$(\mathbf{AR_{new}} - \mathbf{AR})^2$
$\mathbf{I_3}$	$(\mathbf{C_{new}} - \mathbf{C})^2$
$\mathbf{I_4}$	$(\mathbf{FF_{new}} - \mathbf{FF})^2$
$\mathbf{I_5}$	$(\mathbf{ERAI_{new}} - \mathbf{ERAI})^2$

$\mathbf{I_6}$	$(\mathbf{ERPI_{new}} - \mathbf{ERPI})^2$
I_7	$\frac{1}{4} \cdot \sum_{i=1}^{4} (PI_{i_{new}} - PI_i)^2$

In order to decide the respective weight more scientifically, we resort to the help of Analytical Hierarchy Process.

AHP

First of all, we build a seven by seven matrix reciprocal matrix by pair comparison:

	R	AR	C	FF	ERAI	ERPI	PI
Rectangularity	1	1/3	1	1/4	1/2	1/2	1/7
Aspect Ratio	3	1	3	1	2	2	1/3
Circularity	1	1/3	1	1/4	1/2	1/2	1/7
Form Factor	4	1	4	1	3	3	1/2
ER Area Index	2	1/2	2	1/3	1	1	1/4
ER Perimeter Index	2	1/2	2	1/3	1	1	1/4
Proportional Index	7	3	7	2	4	4	1

**The number of each cell is explained in the following chart:*

Intensity of Value	Interpretation
1	Requirements i and j are of equal value.
3	Requirement i has a slightly higher value than j.
5	Requirement i has a strongly higher value than j.
7	Requirement i has a very strongly higher value than j.
9	Requirement i has an absolutely higher value than j.
2, 4, 6, 8	These are intermediate scales between two adjacent judgments.
Reciprocals	If Requirement i has a lower value than j

We then input Pair Ratio Matrix into computer program of MATLAB (Code available in appendix), and get the weight of each factor (W_i).

W1	0.0480
W2	0.1583
W3	0.0480
W4	0.2048
W5	0.0855
W6	0.0855
W7	0.3701

The following calculation is intended to test the consistency of the above AHP.

$$\lambda_{max} = 7.0512$$

$$CI = \frac{\lambda_{max} - n}{n - 1} = 0.0087$$

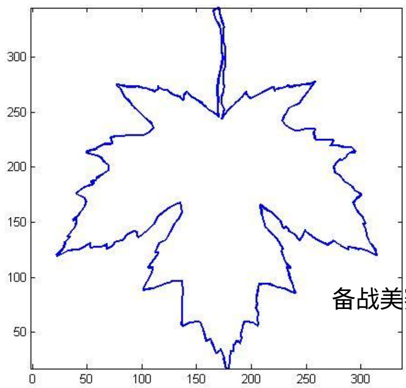
$$CR = \frac{CI}{RI} = 0.0064$$

$$CR < 0.01$$

So, the above method displays perfectly acceptable consistency and the weights are reasonable.

5.3 Model Testing

Now we will take the maple leaf as an example to test our leaf classification model. Before testing, we compare this maple leaf with the six categories in our database and find out that it resembles Category 4 most. Now we will test our hypothesis with our model based on



quantitative analysis.

Firstly, process the image of a given maple leaf:

Then, calculate the rectangularity, aspect ratio, circularity, form factor, edge regularity area index, edge regularity perimeter index and the proportional index of the given leaf. In this case, the seven parameters are shown below:

Parameter	Maple Leave
Rectangularity	0.3554
Aspect Ratio	0.9079
Circularity	0.2691
Form Factor	0.1572
ER Area Index	0.6248
ER Perimeter Index	0.3787
PI_1	0.0968
PI_2	0.4628
PI_3	0.4311
PI_4	0.0093

Finally, calculate the index of deviation by using the formula generated before.

$$\sum_{i=1}^{7} I_i \cdot W_i$$

The six indexes of deviation after comparing the parameters of a given maple leaf with the parameters of six categories in our database are shown below:

Maple Leave	1	2	3	4	5	6
I_1	0.0045	0.0026	0.0035	0.0007	0.0008	0.0044
I_2	0.0003	0.0097	0.0839	0.0115	0.0291	0.0564
I_3	0.0066	0.0043	0.0004	0.0001	0.0004	0.0000
I_4	0.1173	0.0440	0.0032	0.0017	0.0090	0.0235
I_5	0.0081	0.0055	0.0069	0.0043	0.0023	0.0060
I_6	0.0209	0.0223	0.0268	0.0198	0.0169	0.0320
I_7	0.1095	0.0277	0.1073	0.0384	0.1817	0.0619
$\mathbf{I_D}$	**0.2672**	**0.1161**	**0.2319**	**0.0765**	**0.2402**	**0.1841**

Since the index of deviation between the given maple leaf and Category 4 is smallest, the maple leaf falls into Category 4, which is consistent with our hypothesis.

5.4 Conclusion

So far, our model has solved the problem of classification very well based on seven well-rounded parameters and a functional database. Through this quantitative model, we can decide the general shape of any given leaf reliably and scientifically. Our model is robust under reasonable conditions, which can be seen through the testing part discussed above. However, since our database contains only the six commonly seen leaf types in North America, the variety of database itself still has room for improvement. With the more and more different leaf species added to the database, our model can apply to a wider and more geographically comprehensive scope.

6. Model Two: Leaf Distribution and Leaf Shape

6.1 Introduction

Leaf shape varies so greatly that it still remains a mystery why leaves display such a immense variety of shape. Leaf veins are the skeleton of a leaf, and ground tissues the muscles. Genetic and environmental factors contribute to the different patterns of leaf veins and ground tissue, thereby determining the leaf shape. In this model, we narrow the big picture into how leaf distribution act as an influence on leaf shape.

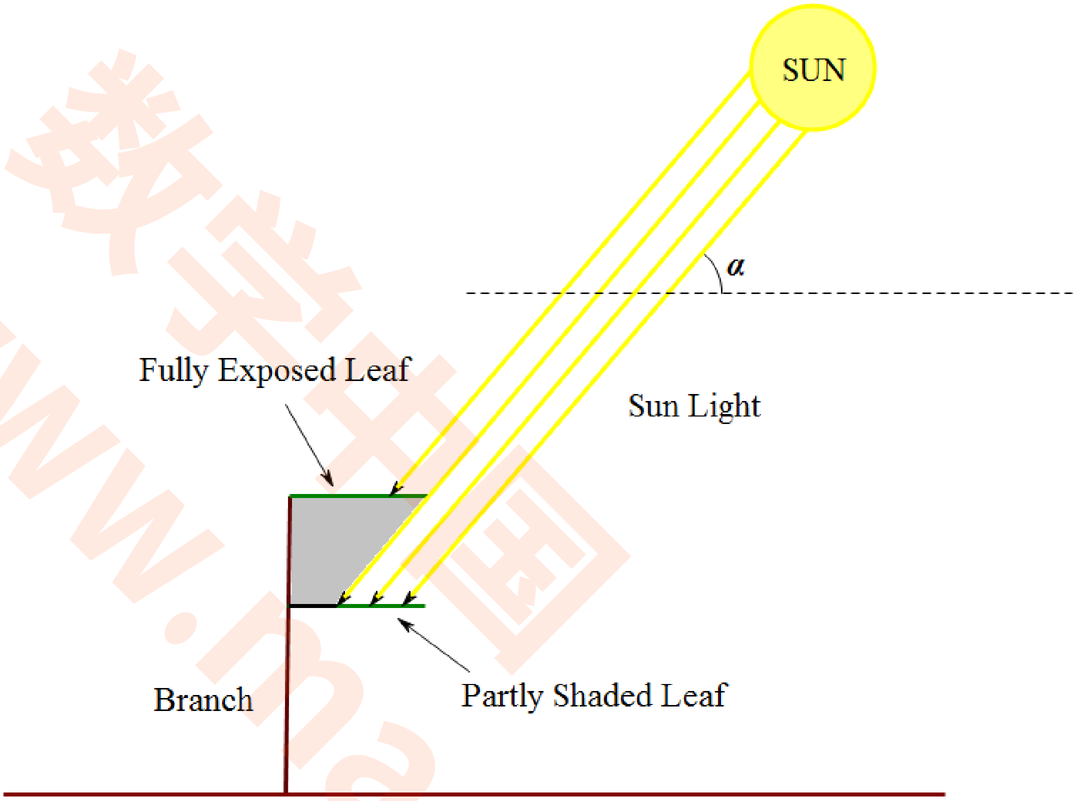
6.2 Idealized Leaf Distribution Model

To downplay the effects of miscellaneous factors that contribute to the relation between the distribution and shape of a leaf, we construct an idealized model that immensely simplifies the complex condition.

The tree is made up of a branch perpendicular to the ground surface and two identical leaves grown on the branch ipsilaterally and horizontally. The leaves face upward and point toward the sun in the sky.

We suppose the tree is placed on the latitude of L (Northern). So the greatest average solar altitude in a year, which is attained in the noon of Vernal Equinox, is defined as α.

The above information is explained in the following image:



6.3 Analysis of Overlapping Areas

Our key focus of the study is the partly shaded leaf. What proportion of the leaf (PL) is shaded is the output of the model. The solar attitude α is a critical control variable. According to the influence of the angle on PL, we divide α into three scenarios.

6.3.1 Solar Altitude trending toward 90°

The situation in which α is trending toward 90°usually takes place in tropical regions. And it is very clear through observation that, in tropical regions, leaf shapes are typically broad and wide with the tree crowns usually containing only one layer of leaves. This can be explained through the illustration above as the shaded part of the lower leaf would be too big with α trending toward 90°to supply enough solar energy for photosynthesis and the biggest absorption of energy can be achieved by a broad leaf shape.

6.3.2 Solar Altitude trending toward 0°

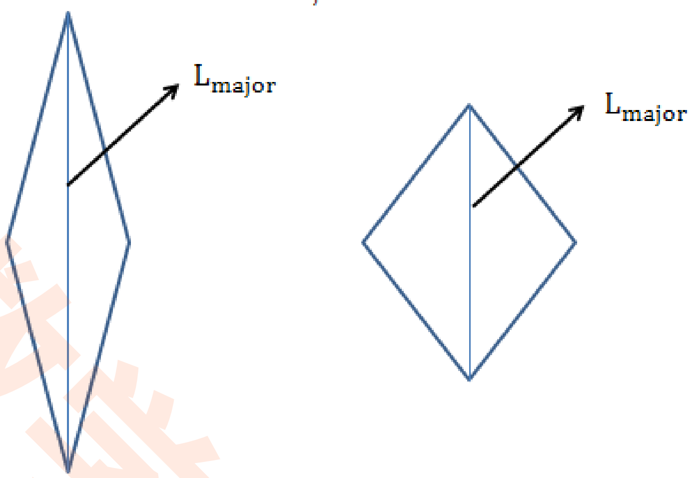
The situation in which α is trending toward 0°usually takes place in frigid zones. It is also very clear that in these regions, leaves are typically acicular with the tree crowns containing dense layers of closely grown leaves. This can also be explained through the illustration above as the shaded part of the lower leaf would approach zero with α trending toward 0°, allowing a much more concentrated distribution of leaves than in other situations. In addition, the maximum absorption of energy can be best achieved by needle-like leaves.

6.3.3 Solar Altitude within normal range

This scenario is typical in the temperate zone on earth, where sunlight irradiates the leaves in a tilted way. It is also the case in which our idealized model is the most suitable. Another crucial factor that we control in this case is the distance (h) between the two points connecting the leaves and the branch.

Our aim is to discuss the relationship between the leaf shape and the leaf distribution. We assume a tree's leaf distribution will try to minimize the overlapping area between leaves, so our model will try to find out the quantitative relationship between the overlapping area and the shape of leaf.

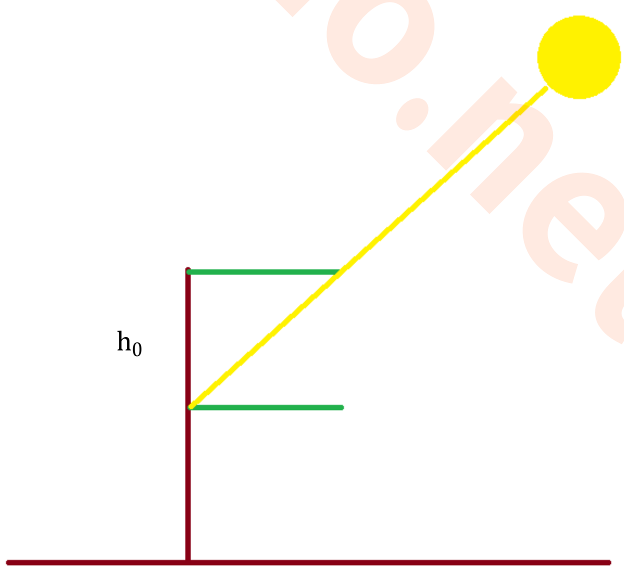
In order to simplify the model, we use the rhombus, with the length of major axis denoted as L_{major} and the length of minor axis denoted as L_{minor}, to replace the leaf shape. Also, we control the area of the first leaf to ensure its constant exposure area to the sun. Since the area is fixed, now we only need to change the length of the major axis to change the shape of the leaf. In other words, during our later study, we will use the change of L_{major} to symbolize the change of the leaf shape.



Also, since we have fixed the area of the leaf and just change its shape during our study, the smallest overlapping area, which we denote as $A_{overlapping}$, means the smallest ration of E, which is defined as follows:

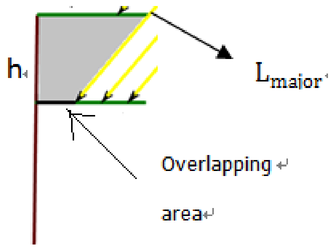
$$E = \frac{A_{overlapping}}{\text{area of one leaf}}$$

Obviously, the most efficient and effective way for two leaves to distribute is to be just totally exposed in sunlight as illustrated below, when h=h_0 and E=0. Thus, the tree can take whatever shape it wants since we control the leaf's surface area.



Now, we want to study what if $h<h_0$.

If $h< h_0$, there will be a shaded region $A_{overlapping}$. The stress of the following study is analyzing the relation between h, L_{major} and E when given a fixed solar altitude α.



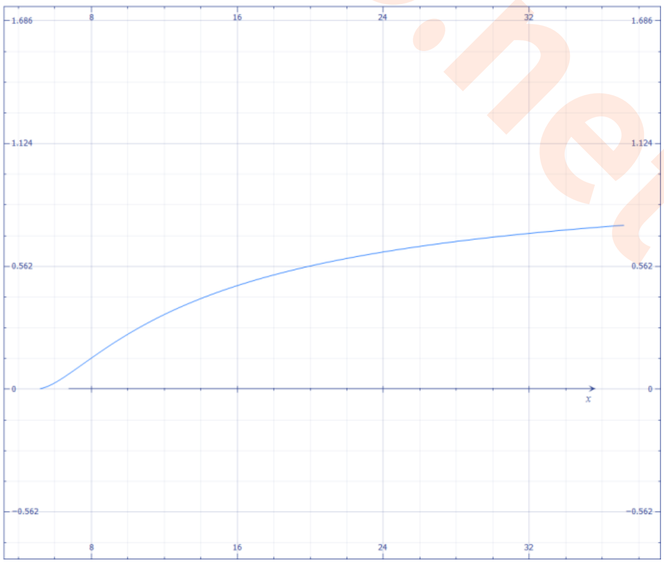
We can easily give the relationship between these factors according to their geometric features:

$$E = \left(\frac{L_{major} \cdot \tan\alpha - h}{L \cdot \tan\alpha}\right)^2$$

Namely
$$E = \left(1 - \frac{h}{L_{major} \cdot \tan\alpha}\right)^2$$

Where tanα is a constant.

When we fix the value of h and give α a particular value, we can get the relationship between the length of leaf and the overlapping area:



*x-axis refers to L_{major}

y-axis refers to $A_{overlapping}$

We discover from the function above that the closer the L_{major} approaches $L \cdot \tan\alpha$, which is equal to h_0, the smaller the overlapping area will be.

So far, we have considered the both situations when h=h_0 and h<h_0. From what we discussed above, the best leaf distribution of the leaf occurs when h=h_0, which means

$$h = L_{major} \cdot \tan\alpha$$

6.4 Model Testing

Now, we need to test whether this correlation between the leaf distribution and leaf shape is right. We collect the data of the length (L_{major}) and internode (h) of a variety of trees and use our formula to calculate the respective solar altitude of the trees. By converting the solar altitude into latitude, we can predict the origin of this particular tree. We choose Ligustrum quihoui Carr, Osmanthus fragrans and Camellia japonica as our test objects and here is the result of our testing:

Test Object	L_{major}	h	$\tan\alpha$	Predicted Latitude	True Lattitude
Ligustrum quihoui Carr.	2	2.5	1.25	38.65	35~35
Osmanthus fragrans	10	18.5	1.85	28.39	23~29
Camellia japonica	6	9	1.50	33.69	32~36

From the table, we can see that our predicted latitude of origin is always within the range of its true latitude, so it is safe to say that the correlation between leaf distribution and leaf shape generated by our model is reliable and correct. In other words, there does exist a relationship between the leaf distribution and leaf shape.

7. Model Three: Tree Profile and Leaf Shape

7.1 Introduction

In this part of our paper, we will explore the correlation between the tree profile and the leaf shape. When we classify the leaf in model one, we define a set of parameters to better understand the shape of leaf on a quantitative base.

Similarly, we will apply this method to explore the profile of tree.

7.2 Hypothesis

Since the vein structure determines the leaf shape and the branch structure determines the tree profile and, to some degree, the leaf veins resemble the branches（refer to figure 3.1）,we have a wild hypothesis that, leaf shape be a two-dimensional mimic of the tree profile. In other words, there does exist a correlation between the leaf shape and the tree profile.

7.3 Comparison of Leaf Shape and Tree Contour

In an effort to judge whether our hypothesis is true or not, our model's general idea is to compare some specific parameters between the tree profile and the leaf shape, thus determine whether a relation exists.

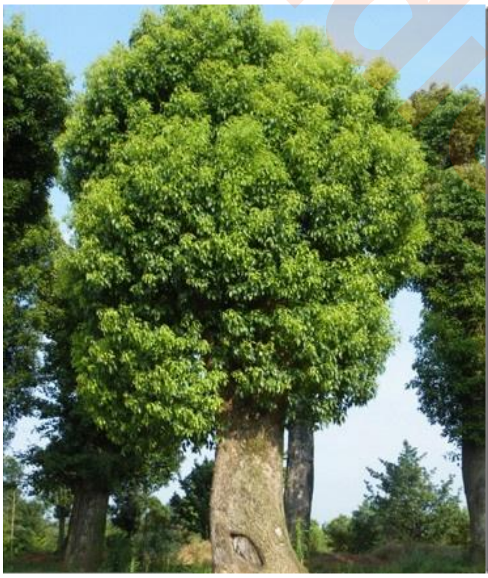
The leaf shape is two-dimensional, so it is relatively easy to study its parameters. However, the tree profile is three dimensional, so it is important to find out a two-dimensional characteristic of a tree to study its parameters. Since the longitudinal section of a particular tree reflects the general size characteristics of a grown tree, we choose to focus on it in our later study and analysis.

7.3.1 Tree Profile Classification

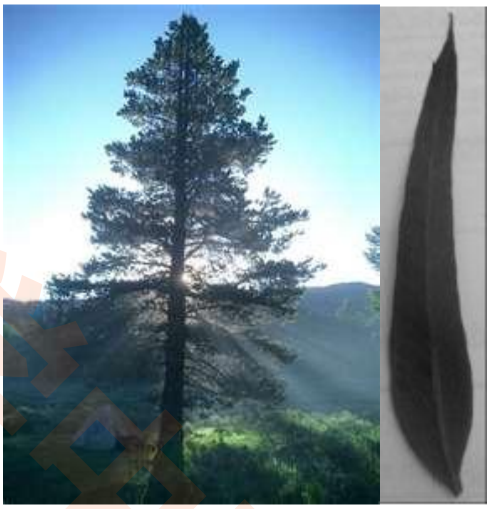
In the leaf classification model, there are 6 general classes of leaves. Since we only compare the general resemblance between leaf and tree, class five (elliptic leaf with serrated margin) is incorporated into class two. As a result, we get 5 classes of leaves and 5 respective types of trees.

Class 1 Cordate (Texas Redbud):


Class 2 Elliptic (Camphor Tree):




Class 3 Subulate (Pine Tree)：



Class 4 Palmate (Oak Tree)：

Class 5 Obovate (Mockernut Hickory):

7.3.2 Parameters of the tree

According to the data we get and referring to the parameters we set for the leaf, this time we appoint 3 parameters for the longitudinal section that can be compared with those of the leaf shape, namely *the rectangularity, the aspect ratio and circularity*. Since we have discussed these three parameters in detail before, we will just briefly go through them.

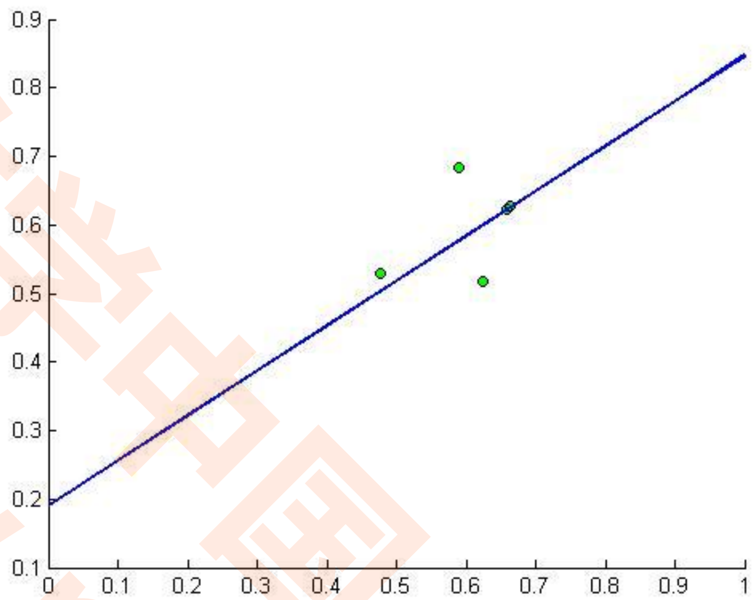
A. Rectangularity

Rectangularity is defined as the ratio between the longitudinal section area of a tree and the area of the smallest bounding rectangle area.

First we compare the rectangularity of the tree profile with that of leaf we calculated in model 1 in the following chart.

Class	Leaf	Tree
1	0.6627	0.6281
2	0.5902	0.6846
3	0.6250	0.5180
4	0.4772	0.5292
5	0.6576	0.6238

Second, we draw a scatter plot with the independent variable of leaf and the dependent variable of tree and use MATLAB to construct the linear regression of the data. The following plot shows the fitting relationship of the data.



Y = 0.3905x + 0.3614

Also, we use F-test to test the significance of the linear regression:

r^2=0.1783

F=0.63

P=0.4850

Obviously, the test shows that the linear relationship between the rectangularity of leaf and rectangularity of tree is insignificant.

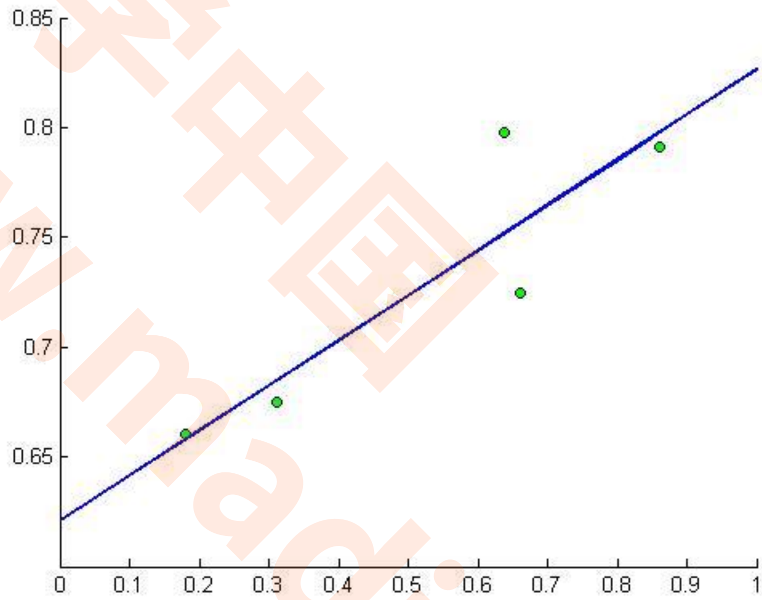
B. Aspect ratio

Aspect ratio is defined as the ration between the short side and the long side of the smallest bounding rectangle area.

The comparison chart is shown below.

Class	Leaf	Tree
1	0.8615	0.7914
2	0.6600	0.7243
3	0.1800	0.6601
4	0.6383	0.7980
5	0.3111	0.6750

Using the same statistical method, we construct the following plot.



Y = 0.2055x + 0.6208

This time the result of statistical inference is as follows:

r^2=0.7985

F=11.8857

P=0.041<0.05

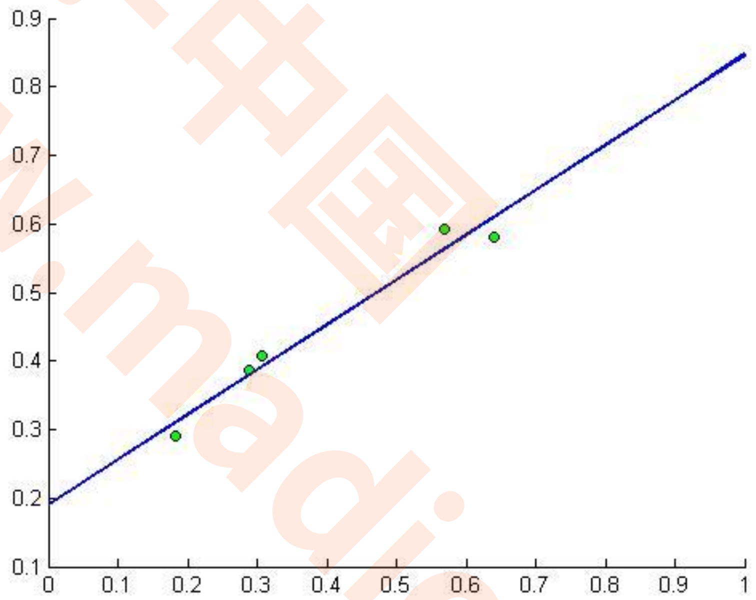
As P value is within acceptance range, the linear relationship between Aspect Ratio of leaf and that of tree.

C. Circularity

Circularity is defined as the ratio between the radius of in-circle and ex-circle of the area of interest.

Class	Leaf	Tree
1	0.6396	0.5800
2	0.5698	0.5928
3	0.1834	0.2895
4	0.3069	0.4070
5	0.2889	0.3866

Using Matlab, we construct the following plot.



Y= 0.6567x + 0.1900

The F-test shows the following.

r^2=0.9645

F=81.5595

P=0.0029<0.05

The P value small enough for us to say that there exists a linear relationship between the circularity of the tree and the leaf.

7.4 Conclusion

From the test of the above three parameters, we may conclude that leaf shape and tree profile have a possibly relationship between each other. Although the test of rectangularity does not support our hypothesis, the test of aspect ratio and the test of circularity well corroborate the theory that leaf shape is a two-dimensional mimic of the tree contour. Since, the tree shape and leaf shape both are affected by an great deal of genetic and environmental factors, the shape characteristics of tree may not well reflect the tree shape. But as far as we know, the shape of leaf resembles the shape of tree to some extent.

8. Model Four: Leaf Mass

8.1 Introduction

In order to estimate the mass of the leaves of a certain tree when given the size characteristics, we have developed a model based on a database.

One simple way to calculate the total leaf mass of a tree is to simply multiply the number of the leaves on a tree and the mass of a single leaf. Our method is more accurate and less demanding in that our model is independent of these two factors, namely the number of leaves and the net weight of one single leaf, but dependent on one more reliable factor of a grown tree --- photosynthesis.

Our methodology of estimating the leave mass of a tree is based on three variables: tree age and growth rate, which is determined by tree species, and general type (Hardwood or conifer). In other words, given the age and type of a tree, we are able to estimate the total mass of leaves. In this model, CO2 is used as a calculating media.

After the explanation of the model, we try to study if there is a correlation between leaf mass size characteristics of a tree. The answer is positive, which we use tree age as a link to prove.

8.2 Leaf Mass and Tree Age

8.2.1 Leaf Mass and CO2 Sequestration

First of all, according to the information we acquired from the past researches, we divide the tree into two main types based on their different ability of sequestering carbon dioxide from the atmosphere.

These two types and their respective ability are shown below:

	Hardwood	Conifer
Ability to sequester Carbon Dioxide (lbs of CO2/g of leaf)	0.00250	0.00165

**A tree's ability to sequester the carbon dioxide is different from its ability to absorb it since the tree also releases CO2 into the atmosphere all day round because of the respiration. In other words, CO2 sequestration = CO2 absorption – CO2 release.

Now we only need to decide the weight of carbon dioxide sequestered by the tree and use the converting method below to calculate the total mass of leaves of that tree.

$$M_{leaf} = \frac{M_{carbon\ \ dioxide}}{A_s}$$

In which,

M_{leaf} :total mass of leaves (g)

$M_{carbon\ \ dioxide}$:mass of carbon dioxide sequestered (lbs)

A_s:ability to sequester carbon dioxide (lbs/g)

8.2.2 CO2 Sequestration and Tree Age

The relationship between the amount of CO2 sequestered, the age of a tree and the type of tree is shown in a study published by U.S. Department of Energy in April 1998. The study below divided the growth rate of a certain tree into three categories: fast, moderate and slow. The corresponding chart that categorizes different tree species into 3 grow rates is enclosed in the Appendix 1.

Table 2: Survival Factors and Annual Carbon Sequestration Rates for Common Urban Trees

Tree Age (yrs)	Survival Factors by Growth Rate			Annual Sequestration Rates by Tree Type and Growth Rate (lbs. carbon/tree/year)					
				Hardwood			Conifer		
	Slow	Moderate	Fast	Slow	Moderate	Fast	Slow	Moderate	Fast
0	0.873	0.873	0.873	1.3	1.9	2.7	0.7	1.0	1.4
1	0.798	0.798	0.798	1.6	2.7	4.0	0.9	1.5	2.2
2	0.736	0.736	0.736	2.0	3.5	5.4	1.1	2.0	3.1
3	0.706	0.706	0.706	2.4	4.3	6.9	1.4	2.5	4.1
4	0.678	0.678	0.678	2.8	5.2	8.5	1.6	3.1	5.2
5	0.658	0.658	0.658	3.2	6.1	10.1	1.9	3.7	6.4
6	0.639	0.639	0.644	3.7	7.1	11.8	2.2	4.4	7.6
7	0.621	0.621	0.630	4.1	8.1	13.6	2.5	5.1	8.9
8	0.603	0.603	0.616	4.6	9.1	15.5	2.8	5.8	10.2
9	0.585	0.589	0.602	5.0	10.2	17.4	3.1	6.6	11.7
10	0.568	0.576	0.589	5.5	11.2	19.3	3.5	7.4	13.2
11	0.552	0.564	0.576	6.0	12.3	21.3	3.8	8.2	14.7
12	0.536	0.551	0.563	6.5	13.5	23.3	4.2	9.1	16.3
13	0.524	0.539	0.551	7.0	14.6	25.4	4.6	9.9	17.9
14	0.512	0.527	0.539	7.5	15.8	27.5	4.9	10.8	19.6
15	0.501	0.516	0.527	8.1	16.9	29.7	5.3	11.8	21.4
16	0.490	0.504	0.516	8.6	18.1	31.9	5.7	12.7	23.2
17	0.479	0.493	0.505	9.1	19.4	34.1	6.1	13.7	25.0
18	0.469	0.483	0.495	9.7	20.6	36.3	6.6	14.7	26.9
19	0.459	0.472	0.484	10.2	21.9	38.6	7.0	15.7	28.8
20	0.448	0.462	0.474	10.8	23.2	41.0	7.4	16.7	30.8
21	0.439	0.452	0.464	11.4	24.4	43.3	7.9	17.8	32.8
22	0.429	0.442	0.454	12.0	25.8	45.7	8.3	18.9	34.9
23	0.419	0.433	0.445	12.5	27.1	48.1	8.8	20.0	37.0
24	0.410	0.424	0.435	13.1	28.4	50.6	9.2	21.1	39.1
25	0.401	0.415	0.426	13.7	29.8	53.1	9.7	22.2	41.3
26	0.392	0.406	0.417	14.3	31.2	55.6	10.2	23.4	43.5
27	0.384	0.398	0.409	15.0	32.5	58.1	10.7	24.6	45.7
28	0.375	0.389	0.400	15.6	33.9	60.7	11.2	25.8	48.0
29	0.367	0.381	0.392	16.2	35.3	63.3	11.7	27.0	50.3
30	0.359	0.373	0.383	16.8	36.8	65.9	12.2	28.2	52.7
31	0.352	0.365	0.375	17.5	38.2	68.5	12.7	29.5	55.1
32	0.344	0.358	0.367	18.1	39.7	71.2	13.3	30.7	57.5
33	0.337	0.350	0.360	18.7	41.1	73.8	13.8	32.0	59.9
34	0.330	0.343	0.349	19.4	42.6	76.5	14.3	33.3	62.4
35	0.323	0.336	0.339	20.0	44.1	79.3	14.9	34.7	64.9

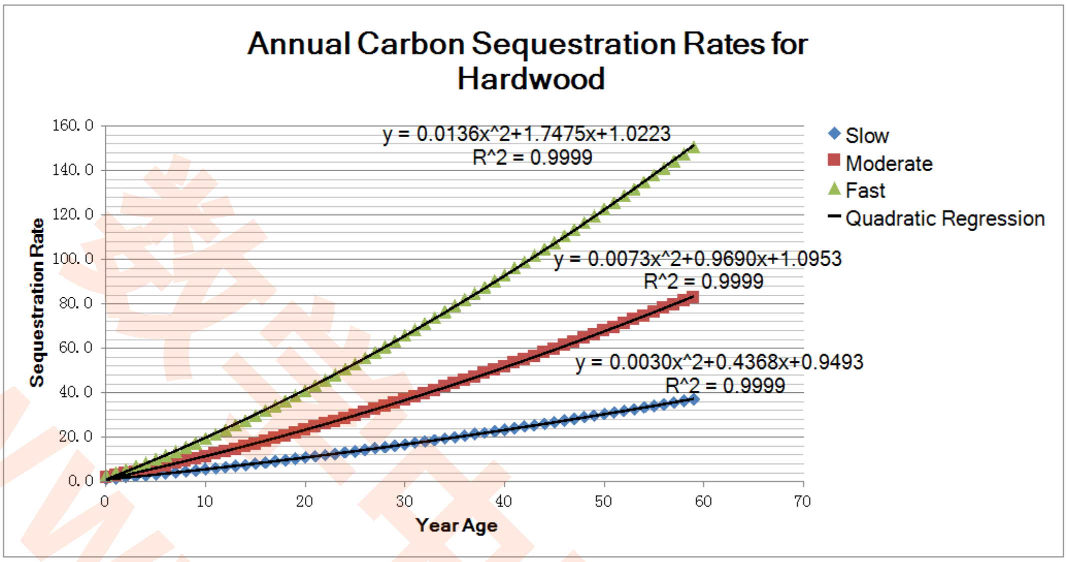
8 Method for Calculating Carbon Sequestration by Trees in Urban and Suburban Settings

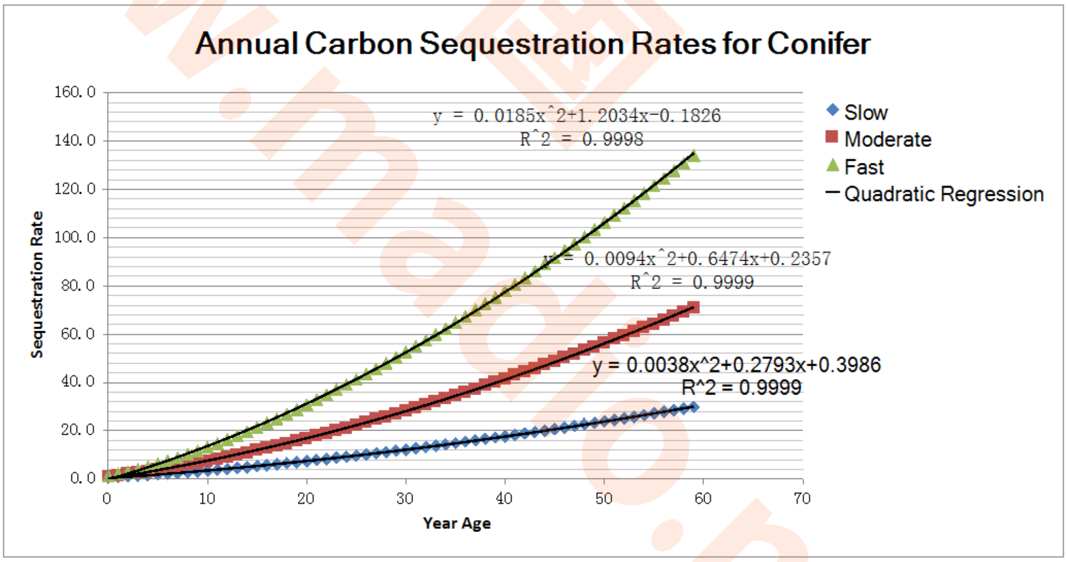
Table 2: Survival Factors and Annual Carbon Sequestration Rates for Common Urban Trees (Cont'd)

Tree Age (yrs)	Survival Factors by Growth Rate			Annual Sequestration Rates by Tree Type and Growth Rate (lbs. carbon/tree/year)					
				Hardwood			Conifer		
	Slow	Moderate	Fast	Slow	Moderate	Fast	Slow	Moderate	Fast
36	0.316	0.329	0.329	20.7	45.6	82.0	15.5	36.0	67.5
37	0.310	0.322	0.320	21.4	47.1	84.8	16.0	37.3	70.1
38	0.303	0.315	0.310	22.0	48.6	87.6	16.6	38.7	72.7
39	0.297	0.308	0.301	22.7	50.2	90.4	17.2	40.1	75.3
40	0.291	0.302	0.293	23.4	51.7	93.2	17.7	41.5	78.0
41	0.285	0.296	0.284	24.1	53.3	96.1	18.3	42.9	80.7
42	0.279	0.289	0.276	24.8	54.8	99.0	18.9	44.3	83.4
43	0.273	0.283	0.268	25.4	56.4	101.9	19.5	45.8	86.2
44	0.267	0.277	0.260	26.1	58.0	104.8	20.1	47.2	89.0
45	0.261	0.269	0.253	26.8	59.6	107.7	20.7	48.7	91.8
46	0.256	0.261	0.245	27.6	61.2	110.7	21.3	50.2	94.7
47	0.251	0.254	0.238	28.3	62.8	113.6	22.0	51.7	97.5
48	0.245	0.247	0.231	29.0	64.5	116.6	22.6	53.2	100.4
49	0.240	0.239	0.225	29.7	66.1	119.6	23.2	54.8	103.4
50	0.235	0.232	0.218	30.4	67.8	122.7	23.9	56.3	106.3
51	0.230	0.226	0.212	31.1	69.4	125.7	24.5	57.9	109.3
52	0.225	0.219	0.206	31.9	71.1	128.8	25.2	59.4	112.3
53	0.221	0.213	0.199	32.6	72.8	131.8	25.8	61.0	115.4
54	0.216	0.207	0.193	33.4	74.5	134.9	26.5	62.6	118.4
55	0.211	0.201	0.188	34.1	76.2	138.0	27.2	64.2	121.5
56	0.207	0.195	0.182	34.8	77.9	141.2	27.8	65.9	124.6
57	0.203	0.189	0.177	35.6	79.6	144.3	28.5	67.5	127.8
58	0.198	0.184	0.171	36.3	81.3	147.5	29.2	69.2	130.9
59	0.194	0.178	0.166	37.1	83.0	150.6	29.9	70.8	134.1

Method for Calculating Carbon Sequestration by Trees in Urban and Suburban Settings 9

From the chart, we can discover an obvious positive relationship between the age of the tree and the amount of CO2 sequestered if we fix the growth rate and the type of the tree. We fit the data into a set of quadratic regression functions shown below:





In each function next to the curve, such as $y=0.0073x^2+0.9690x+1.0953$, which describes Annual Sequestration Rate for fast-growing Hardwood, y is the sequestration rate and x is the tree age.

After fitting the data and conduct the statistical inference, we surprisingly find that the fitting curve can actually fit the data perfectly. With the fitting curve and the functions, we can easily estimate the CO_2 sequestered when given the age of a certain tree.

8.2.3 Leaf Mass and Tree Age

Since we have known the converting relation between the mass of leaves and the CO2 sequestered, now we can derive the correlation between the age of a tree and the total mass of leaves on that tree by using the CO2 sequestered as a media. After processing the data by the formula in 8.2.1, we get the corresponding relation table shown below. Each cell of the table is the leaf mass of a tree of a certain age, type and growth rate.

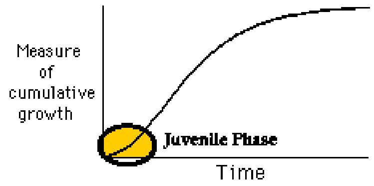
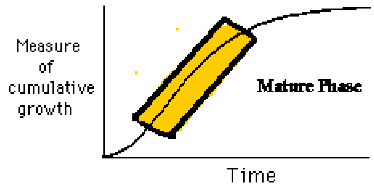
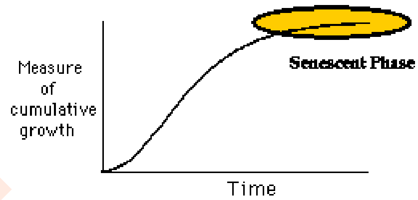
	Hardwood			Conifer		
Year Age	Slow	Moderate	Fast	Slow	Moderate	Fast
0	520	760	1080	424	606	848
1	640	1080	1600	545	909	1333
2	800	1400	2160	667	1212	1879
3	960	1720	2760	848	1515	2485
4	1120	2080	3400	970	1879	3152
5	1280	2440	4040	1152	2242	3879
6	1480	2840	4720	1333	2667	4606
7	1640	3240	5440	1515	3091	5394
8	1840	3640	6200	1697	3515	6182
9	2000	4080	6960	1879	4000	7091
10	2200	4480	7720	2121	4485	8000
11	2400	4920	8520	2303	4970	8909
12	2600	5400	9320	2545	5515	9879
13	2800	5840	10160	2788	6000	10848
14	3000	6320	11000	2970	6545	11879
15	3240	6760	11880	3212	7152	12970
16	3440	7240	12760	3455	7697	14061
17	3640	7760	13640	3697	8303	15152
18	3880	8240	14520	4000	8909	16303
19	4080	8760	15440	4242	9515	17455
20	4320	9280	16400	4485	10121	18667
21	4560	9760	17320	4788	10788	19879
22	4800	10320	18280	5030	11455	21152
23	5000	10840	19240	5333	12121	22424
24	5240	11360	20240	5576	12788	23697
25	5480	11920	21240	5879	13455	25030
26	5720	12480	22240	6182	14182	26364
27	6000	13000	23240	6485	14909	27697
28	6240	13560	24280	6788	15636	29091
29	6480	14120	25320	7091	16364	30485
30	6720	14720	26360	7394	17091	31939
31	7000	15280	27400	7697	17879	33394

32	7240	15880	28480	8061	18606	34848
33	7480	16440	29520	8364	19394	36303
34	7760	17040	30600	8667	20182	37818
35	8000	17640	31720	9030	21030	39333
36	8280	18240	32800	9394	21818	40909
37	8560	18840	33920	9697	22606	42485
38	8800	19440	35040	10061	23455	44061
39	9080	20080	36160	10424	24303	45636
40	9360	20680	37280	10727	25152	47273
41	9640	21320	38440	11091	26000	48909
42	9920	21920	39600	11455	26848	50545
43	10160	22560	40760	11818	27758	52242
44	10440	23200	41920	12182	28606	53939
45	10720	23840	43080	12545	29515	55636
46	11040	24480	44280	12909	30424	57394
47	11320	25120	45440	13333	31333	59091
48	11600	25800	46640	13697	32242	60848
49	11880	26440	47840	14061	33212	62667
50	12160	27120	49080	14485	34121	64424
51	12440	27760	50280	14848	35091	66242
52	12760	28440	51520	15273	36000	68061
53	13040	29120	52720	15636	36970	69939
54	13360	29800	53960	16061	37939	71758
55	13640	30480	55200	16485	38909	73636
56	13920	31160	56480	16848	39939	75515
57	14240	31840	57720	17273	40909	77455
58	14520	32520	59000	17697	41939	79333
59	14840	33200	60240	18121	42909	81273

8.3 Tree Age and Tree Size

As stated above, we utilize the age of a tree as a link between the two variables, leaf mass and the size characteristics of trees. Since we have found out the correlation between the age of tree and the total leaf mass on it, now we only need to work out the relation between the age of the tree and the size characteristics of it.

Tree size is the accumulation of growth, which is a biological phenomenon of increase with time. According the life cycle of a tree, it experiences the following phases:

Juvenile phase (youth) - accelerating rate of growth.	
Full vigor phase (maturity) - constant rate of growth.	
Senescent phase - decelerating rate of growth.	

As shown above, the cumulative growth curve, whose pattern is characteristic for the life span of an organism, resembles logistic growth function. Thus we construct the following model

$$\mathrm{P} = k_1 \cdot (1 - e^{k_2 \cdot A})^{k_3}$$

Transformation leads to the following

$$\mathrm{A} = k_4 \cdot \ln(1 - k_5 \cdot P^{k_6})$$

Where,

P is the general profile of a tree(volume, height, mass, diameter).

A is the age of the tree.

k1, k2, k3, k4, k5 and k6 are constants.

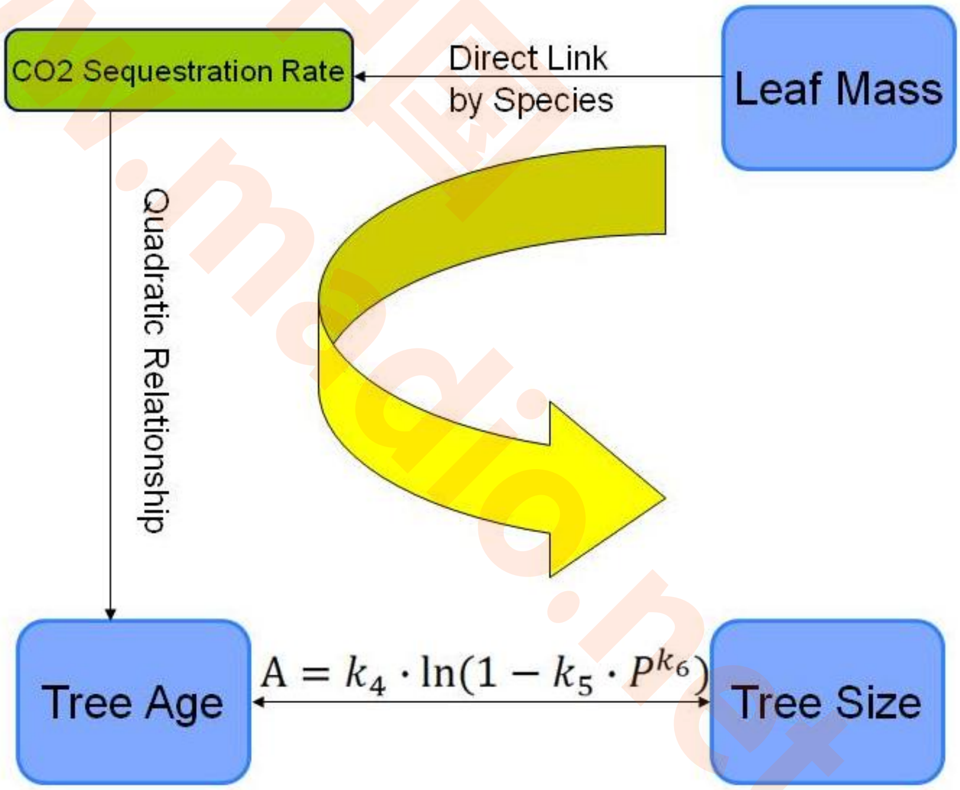
It is important to note that P, the general profile of a tree, is a variable to describe the size characteristics of a tree, which vary greatly in different species of trees.

Therefore it is not possible to work out an exact set of constants in the model that suits all kinds of trees. This correlation between the size characters and age is primarily qualitative.

8.4 Leaf Mass and Tree Size

Finally, we get to answer the question whether there exists a correlation between leaf mass and tree size characteristics.

According to our study in X.2, leaf mass and tree age are related with each other mathematically linked by CO2 Sequestration. Our model in X.3 determines a function between tree age and tree size. Through this relationship map shown below, it is clear that there must exist a correlation between leaf mass and tree size.



9. Strengths and Weaknesses

Now we will analyze the strengths and weaknesses of each sub-model in our paper:

Model 1

Strength:

1. Our model is based on quantitative analysis, so the classification process is both objective and efficient;
2. Our model chooses leaf types that are most typical and common as the categories.

Weakness:

1. Because of the limit of time and access to information, we only create six categories, which may not cover all the leaf types.

Model 2

Strength:

1. We have taken into consideration three conditions: tropical zone, temperate zone and frigid zone when discussing the relationship between the leaf distribution and the leaf shape;
2. The result of our model conforms to our observation and the data we found.

Weakness:

1. We consider the leaves distribution on a single branch, but have not considered the inner-influence between different leaves of different branches.

Model 3

Strength:

1. The whole process use data and quantitative analysis as foundations, so the output is objective and reasonable.

Weakness:

1. We have limited categories of tree profiles, so our data to fit the curve is not plenty enough.

Model 4

Strength:

1. We use the carbon sequestration rate and age as the mediums to calculate the mass of total laves, which are far more scientific than estimating the number of leaves and weight of each leaf.

Weakness:

1. Since the data we get are from the statistics a little bit outdated, the accuracy of our output may need some slight adjustment.

10. Reference

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[5] Hirokazu Tsukaya Graduate School of Science, University of Tokyo, "Mechanism of Leaf-Shape Determination," in Annu. Rev. Plant Biol. 2006.57:477-496.

[6] U.S. Department of Energy Energy Information Administration, "Method for Calculating Carbon Sequestration by Trees in Urban and Suburban Settings," April 1998

11. A Letter to the Science Journal Editor

Dear editor,

From the very ancient times, leaves have been an important part of human life as they were weaved into clothes, used as containers of water, and most of all, relied upon as the main converter of carbon dioxide into oxygen. However, due to the variety and large quantity of leaves, there have always remained a lot of questions and mysteries of them, such as the reason behind their different shapes and distribution pattern as well as the relationship between leaf shape, leaf mass and tree profile. So, out of the interest in discovering the wonders of the nature and exploring the potential practical use of the natural phenomena, we, a group of high school students from Shanghai, have done some researches and now we are very glad to present to you our key findings:

We first focus on the possible influence of leaf distribution on leaf shape. We discover that for survival reasons, the trees develop an optimal leaf distribution and shape pattern to best adjust to the specific region of its origin by maximizing the exposure area to sunshine, thereby gaining the most nutrients for photosynthesis. And after testing our findings, we are convinced there does exist a mathematical relationship between the solar altitude, leaf shape and leaf distribution. With the above finding, we may be able to determine the best location for replanting or assisted-migration of a tree species by observing its leaf distribution.

Our second key finding is a rough correlation between the tree profile and leaves growing on that tree. In fact, we make a hypothesis that a leaf be a two-dimensional mimic of the tree. After we set up the database and compare the shape of leaf and the contour of tree carefully, some similarities between certain characteristics have surfaced. This finding suggests that the natural world might contain many examples of self-similarity, mathematical concept which means that an object is approximately similar to a part of itself, such as the famous Koch Snowflake and Mandelbrot Set.

The third part of our study deals with the relationship between the tree size characteristics and the total mass of the leaves. The two ends are linked by two calculating mediums, the CO2 sequestration rates and the age of the tree. Therefore, we are able to estimate the total mass of the leaves given some profiles of a tree such as its height, diameter, volume, age and type, and vice versa. This finding might have great potential for agricultural and environmental uses, such as a new method to estimate tea production or wood production, or estimation of CO2 sequestration effect of a leaf, a tree and, most importantly, a forest, which is a great alleviator of global warming.

In hope of publishing our research in your outstanding science journal, we’ve enclosed our research paper for you to examine and judge. We are convinced that our research on leaves is promising to contribute to a variety of areas.

Yours faithfully

12. Appendix

12.1 Chart of Tree Species and Growth Rate

Species	Type	Growth Rate	Species	Type	Growth Rate
Ailanthus, *Ailanthus altissima*	H	F	Maple, bigleaf, *Acer macrophyllum*	H	S
Alder, European, Alnus glutinosa	H	F	Maple, Norway, *Acer platanoides*	H	M
Ash, green, Fraxinus pennsylvanica	H	F	Maple, red, *Acer rubrum*	H	M
Ash, mountain, American, *Sorbus americana*	H	M	Maple, silver, *Acer saccharinum*	H	M
Ash, white, *Fraxinus americana*	H	F	Maple, sugar, *Acer saccharum*	H	S
Aspen, bigtooth, *Populus grandidentata*	H	M	Mulberry, red, *Morus rubra*	H	F
Aspen, quaking, *Populus tremuloides*	H	F	Oak, black, *Quercus velutina*	H	M
Baldcypress, *Taxodium distichum*	C	F	Oak, blue, *Quercus douglasii*	H	M
Basswood, American, *Tilia americana*,	H	F	Oak, bur, *Quercus macrocarpa*	H	S
Beech, American, *Fagus grandifolia*	H	S	Oak, California black, *Quercus kelloggii*	H	S
Birch, paper (white), *Betula papyrifera*	H	M	Oak, California White, *Quercus lobata*	H	M
Birch, river, *Betula nigra*	H	M	Oak, canyon live, *Quercus chrysolepsis*	H	S
Birch, yellow, *Betula alleghaniensis*	H	S	Oak, chestnut, *Quercus prinus*	H	S
Boxelder, *Acer negundo*	H	F	Oak, Chinkapin, *Quercus muehlenbergii*	H	M
Buckeye, Ohio, *Aesculus glabra*	H	S	Oak, Laurel, *Quercus laurifolia*	H	F
Catalpa, northern, *Catalpa speciosa*	H	F	Oak, live, *Quercus virginiana*	H	F
Cedar-red, eastern, *Juniperus virginiana*	C	M	Oak, northern red, *Quercus rubra*	H	F
Cedar-white, northern, *Thuja occidentalis*	C	M	Oak, overcup , *Quercus lyrata*	H	S
Cherry, black, *Prunus serotina*	H	F	Oak, pin, *Quercus palustris*	H	F
Cherry, pin, *Prunus pennsylvanica*	H	M	Oak, scarlet, *Quercus coccinea*	H	F
Cottonwood, eastern, *Populus deltoides*	H	M	Oak, swamp white, *Quercus bicolor*	H	M
Crabapple, *Malus* spp.	H	M	Oak, water, *Quercus nigra*	H	M
Cucumbertree, *Magnolia acuminata*	H	F	Oak, white, *Quercus alba*	H	S
Dogwood, flowering, *Cornus florida*	H	S	Oak, willow, *Quercus phellos*	H	M
Elm, American, *Ulmus americana*	H	F	Pecan, *Carya illinoensis*	H	S
Elm, Chinese, *Ulmus parvifolia*	H	M	Pine, European black, *Pinus nigra*	C	S
Elm, rock, *Ulmus thomasii*	H	S	Pine, jack, *Pinus banksiana*	C	F
Elm, September, *Ulmus serotina*	H	F	Pine, loblolly, *Pinus taeda*	C	F
Elm, Siberian, *Ulmus pumila*	H	F	Pine, longleaf, *Pinus palustris*	C	F
Elm, slippery, *Ulmus rubra*	H	M	Pine, ponderosa, *Pinus ponderosa*	C	F
Fir, balsam, *Abies balsamea*	C	S	Pine, red, *Pinus resinosa*	C	F
Fir, Douglas, Pseudotsuga menziesii	C	F	Pine, Scotch, *Pinus sylvestris*	C	S
Ginkgo, *Ginkgo biloba*	H	S	Pine, shortleaf, *Pinus echinata*	C	F
Hackberry, *Celtis occidentalis*	H	F	Pine, slash, *Pinus elliottii*	C	F
Hawthorne, *Crataegus* spp.	H	M	Pine, Virginia, *Pinus virginiana*	C	M
Hemlock, eastern, *Tsuga canadensis*	C	M	Pine, white eastern, *Pinus strobus*	C	F
Hickory, bitternut, *Carya cordiformis*	H	S	Poplar, yellow, *Liriodendron tulipifera*	H	F
Hickory, mockernut, *Carya tomentosa*	H	M	Redbud, eastern, *Cercis canadensis*	H	M
Hickory, shagbark, *Carya ovata*	H	S	Sassafras, *Sassafras albidum*	H	M
Hickory, shellbark, Carya laciniosa	H	S	Spruce, black, *Picea mariana*	C	S
Hickory, pignut, *Carya glabra*	H	M	Spruce, blue, *Picea pungens*	C	M
Holly, American, *Ilex opaca*	H	S	Spruce, Norway, *Picea abies*	C	M
Honeylocust, *Gleditsia triacanthos*	H	F	Spruce, red, *Picea rubens*	C	S
Hophornbeam, eastern, *Ostrya virginiana*	H	S	Spruce, white, *Picea glauca*	C	M
Horsechestnut, common, *Aesculus hippocastanum*	H	F	Sugarberry, *Celtis laevigata*	H	F
Kentucky coffeetree, *Gymnocladus dioicus*	C	F	Sweetgum, *Liquidambar styraciflua*	H	F
Linden, little-leaf, *Tilia cordata*	H	F	Sycamore, *Platanus occidentalis*	H	F
Locust, black, *Robinia pseudoacacia*	H	F	Tamarack, *Larix laricina*	C	F
London plane tree *Platanus_X_acerifolia*	H	F	Walnut, black, *Juglans nigra*	H	F
Magnolia, southern, *Magnolia grandifolia*	H	M	Willow, black, *Salix nigra*	H	F

Type: H = Hardwood, C = Conifer Growth Rate: S = Slow, M = Moderate, F = Fast

12.2 MATLAB Program Code

How to use MATLAB to realize AHP

```
A=input('A=');
[n,n]=size(A);
%Input the pair ratio matrix as "A".
x=ones(n,100);
y=ones(n,100);
m=zeros(1,100);
m(1)=max(x(:,1));
y(:,1)=x(:,1);
x(:,2)=A*y(:,1);
```

```
m(2)=max(x(:,2));
y(:,2)=x(:,2)/m(2);
p=0.0001;i=2;k=abs(m(2)-m(1));
while k>p
    i=i+1;
    x(:,i)=A*y(:,i-1);
    m(i)=max(x(:,i));
    y(:,i)=x(:,i)/m(i);
    k=abs(m(i)-m(i-1));
end
a=sum(y(:,i));
w=y(:,i)/a;
%Find the Eigen Vector
t=m(i);
disp('EigenVector Wi=');
disp(w);
disp('LamdaMax=');
disp(t);
CI=(t-n)/(n-1);
%Calculate the Consistency Index (CI)
RI=[0 0 0.52 0.89 1.12 1.26 1.36 1.41 1.46 1.49 1.52
1.54 1.56 1.58 1.59];
%Random Consistency Index (RI)
disp('CI=');disp(CI);
disp('RI=');disp(RI(n));
if RI(n)>0
CR=CI/RI(n);
%Calculate the consistency ratio according to the
formula
    disp('CR=');disp(CR);
    if CR<0.10
        disp('Consistency Acceptable');
    else
        disp('Consistency Inacceptable');
end
%Consistency Check
end
if RI(n)<=0
```

```
disp('Consistency Acceptable');
%Consistency Check
end
```

How to calculate the area of a leaf

```
l=imread('leaftype1.jpg');
imshow(l);
%Import the picture
m=ginput;
%Pick the points on the perimeter of the figure and
record the coordinates into "m"
x(1,:)=m(:,1);
y(1,:)=m(:,2);
x=[x x(1)];
y=[y y(1)];
N=length(x);
t=1:N;
ti=1:(N-1)/100000:N;
xi=spline(t,x,ti);
yi=spline(t,y,ti);
area=polyarea(xi,yi);
%calculate the area of the figure
points=[x;y];
fnplt(cscvn(points));axis equal
%draw the estimated function and check the accuracy;
save the figure as '1-1.jpg' for further use
```

How to calculate the perimeter of a leaf

```
k=imread('1-1.jpg');
BW=im2bw(k);
%turn the picture into binary form
STATS=regionprops(BW,'Perimeter');
%calculate the perimeter of the binary figure
STATS.Perimeter;
%print the perimeter
```

Linear Regression with significance test

```
x=input('Input x');
```

```
% Input the parameters of the leaves as x
coordinates
y=input('Input y');
% Input the parameters of the trees as y coordinates
c=length(x);
xi=0:0.1:1;
a=polyfit(x,y,1);
% Find the coefficients of the fitting curve of the
points
yi=polyval(a,xi);
% Calculate the y coordinates of the fitting curve
hold on;
plot(x,y,'go','MarkerEdgeColor','k','MarkerFace
Color','g','MarkerSize',5);
% Draw the original input data
plot(xi,yi,'linewidth',2,'markersize',7);
% Draw the fitting curve
hold off;
x=x';
y=y';
x=[ones(c,1),x];
[b,bint,r,rint,stats]=regress(y,x);
% Significance test
```