2009 Contest Problems
MCM PROBLEMS
PROBLEM A: Designing a Traffic Circle

Many cities and communities have traffic circles—from large ones with many lanes in the circle (such as at the Arc de Triomphe in Paris and the Victory Monument in Bangkok) to small ones with one or two lanes in the circle. Some of these traffic circles position a stop sign or a yield sign on every incoming road that gives priority to traffic already in the circle; some position a yield sign in the circle at each incoming road to give priority to incoming traffic; and some position a traffic light on each incoming road (with no right turn allowed on a red light). Other designs may also be possible.

The goal of this problem is to use a model to determine how best to control traffic flow in, around, and out of a circle. State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice. Include a Technical Summary of not more than two double-spaced pages that explains to a Traffic Engineer how to use your model to help choose the appropriate flow-control method for any specific traffic circle. That is, summarize the conditions under which each type of traffic-control method should be used. When traffic lights are recommended, explain a method for determining how many seconds each light should remain green (which may vary according to the time of day and other factors). Illustrate how your model works with specific examples.

MCM2009 问题 A：设计一个交通环岛

在许多城市和社区都建立有交通环岛，既有多条行车道的大型环岛（例如巴黎的凯旋门和曼谷的胜利纪念碑路口），又有一至两条行车道的小型环岛。有些环岛在进入口设有"停车"标志或者让行标志，其目的是给已驶入环岛的车辆提供行车优先权；而在一些环岛的进入口的逆向一侧设立的让行标志是为了向即将驶入环岛的车辆提供行车优先权；还有一些环岛会在入口处设立交通灯（红灯会禁止车辆右转）；也可能会有其他的设计方案。

这一设计的目的在于利用一个模型来决定如何最优地控制环岛内部，周围以及外部的交通流。该设计的目的在于可利用模型做出最佳的方案选择以及分析影响选择的众多因素。解决方案中需要包括一个不超过两页纸，双倍行距打印的技术摘要，它可以指导交通工程师利用你们模型对任何特殊的环岛进行适当的流量控制。该模型可以总结出在何种情况之下运用哪一种交通控制法为最优。当考虑使用红绿灯的时候，给出一个绿灯的时长的控制方法（根据每日具体时间以及其他因素进行协调）。找一些特殊案例，展示你的模型的实用性。

PROBLEM B: Energy and the Cell Phone

This question involves the "energy" consequences of the cell phone revolution. Cell phone usage is mushrooming, and many people are using cell phones and giving up their landline telephones. What is the consequence of this in terms of electricity use? Every cell phone comes with a battery and a recharger.

Requirement 1

Consider the current US, a country of about 300 million people. Estimate from available data the number H of households, with m members each, that in the past were serviced by landlines. Now,

suppose that all the landlines are replaced by cell phones; that is, each of the m members of the household has a cell phone. Model the consequences of this change for electricity utilization in the current US, both during the transition and during the steady state. The analysis should take into account the need for charging the batteries of the cell phones, as well as the fact that cell phones do not last as long as landline phones (for example, the cell phones get lost and break).

Requirement 2

Consider a second "Pseudo US"—a country of about 300 million people with about the same economic status as the current US. However, this emerging country has neither landlines nor cell phones. What is the optimal way of providing phone service to this country from an energy perspective? Of course, cell phones have many social consequences and uses that landline phones do not allow. A discussion of the broad and hidden consequences of having only landlines, only cell phones, or a mixture of the two is welcomed.

Requirement 3

Cell phones periodically need to be recharged. However, many people always keep their recharger plugged in. Additionally, many people charge their phones every night, whether they need to be recharged or not. Model the energy costs of this wasteful practice for a Pseudo US based upon your answer to Requirement 2. Assume that the Pseudo US supplies electricity from oil. Interpret your results in terms of barrels of oil.

Requirement 4

Estimates vary on the amount of energy that is used by various recharger types (TV, DVR, computer peripherals, and so forth) when left plugged in but not charging the device. Use accurate data to model the energy wasted by the current US in terms of barrels of oil per day.

Requirement 5

Now consider population and economic growth over the next 50 years. How might a typical Pseudo US grow? For each 10 years for the next 50 years, predict the energy needs for providing phone service based upon your analysis in the first three requirements. Again, assume electricity is provided from oil. Interpret your predictions in term of barrels of oil.

MCM2009 问题 B：能源和手机
这个问题涉及到手机革命的能源问题。手机使用率迅速增加，许多人使用手机并放弃了固定电话。这方面的电能使用会带来什么后果？每个手机都配备了电池和充电器。
要求 1
考虑现在的美国，人口约为 3 亿，从现有数据估计美国有 H 个家庭，每个家庭有 M 个成员，以前是使用固定电话的。现在，假设所有的座机被手机取代，也就是说每个家庭成员都有一部手机。建立当前美国在手机使用的过渡和稳定两个阶段用电改变的模型，分析应该考虑到对移动电话充电的需要，同时移动电话不能像固定电话那样长期使用也是一个现实问题（比

如说移动电话可能会丢失或者损坏）

要求 2

考虑"伪美国"--一个约 3 亿人口，跟当前美国具有相同的经济状况的国家。然而，这个新兴国家既没有固定电话也没有移动电话，从能源角度看，为这个国家提供电话服务的最佳方式是什么？当然，手机有很多固定电话所不具有的用途和社会影响。这个讨论要涉及单独使用固定电话或者单独使用移动电话，或者混合使用二者所带来的广泛和潜在的影响。

要求 3

手机需要定期充电。但是许多人在不考虑手机是否要充电的情况下，总是将充电器一直插在电器插槽上，有的甚至整晚都在给手机充电。在你的要求 2 解决方案的基础上，针对"伪美国"，建立上述浪费方式的能源消耗的数学模型。另外，假定"伪美国"以石油作为电力来源，以原油桶为单位计算浪费量。

要求 4

估计各种需要充电的电器设备（电视、DVR、电脑外围设备等）所使用能源的数量，考虑设备没有使用，但插头仍然插在插座上的情况。要求用精确的数据建立模型，估计当前美国每天所浪费的能源数量，以原油（桶/天）计量。

要求 5

考虑人口及经济增长在未来的 50 年内的情况。如何使"伪美国"发展壮大？对于今后 50 年内的每一个 10 年进行电话服务的能源需求预测，前提是在你前三个要求的分析基础上进行。另外，假定以石油作为电力来源，以原油桶为单位计算。

2010

PROBLEM A: The Sweet Spot

Explain the "sweet spot" on a baseball bat.

Every hitter knows that there is a spot on the fat part of a baseball bat where maximum power is transferred to the ball when hit. Why isn't this spot at the end of the bat? A simple explanation based on torque might seem to identify the end of the bat as the sweet spot, but this is known to be empirically incorrect. Develop a model that helps explain this empirical finding.

Some players believe that "corking" a bat (hollowing out a cylinder in the head of the bat and filling it with cork or rubber, then replacing a wood cap)

enhances the "sweet spot" effect. Augment your model to confirm or deny this effect. Does this explain why Major League Baseball prohibits "corking"?

Does the material out of which the bat is constructed matter? That is, does this model predict different behavior for wood (usually ash) or metal (usually aluminum) bats? Is this why Major League Baseball prohibits metal bats?

问题一：甜蜜点 解释"甜蜜"对棒球球棒。

每个击球手都知道有一个地方在球棒的粗的部分是在最大功率转移到球击中时。为什么这个位置不是在球棒的顶端？一个简单的解释的基础上扭矩似乎可以确定最后的击球甜蜜点,但这是经验主义。开发一个模型来帮助解释这一实证调查。

一些球员认为"塞住"球棒（掏空出一个圆柱体在球棒的头部和填充软木或橡胶，然后盖上木盖）提高"甜蜜"效应。补充你的模型来证实或否认这种影响。这是否能解释为什么美国职棒大联盟禁止"塞住"球棒？

该材料在这里是球拍的事？就是说，这个模型预测不同的行为（通常为木灰）或金属（通常

是铝）蝙蝠？这就是为什么美国职棒大联盟禁止使用金属球棒？

PROBLEM B: Criminology

In 1981 Peter Sutcliffe was convicted of thirteen murders and subjecting a number of other peopl e to vicious attacks. One of the methods used to narrow the search for Mr. Sutcliffe was to find a "center of mass" of the locations of the attacks. In the end, the suspect happened to live in the sa me town predicted by this technique. Since that time, a number of more sophisticated technique s have been developed to determine the "geographical profile" of a suspected serial criminal bas ed on the locations of the crimes.

Your team has been asked by a local police agency to develop a method to aid in their investigatio ns of serial criminals. The approach that you develop should make use of at least two different sc hemes to generate a geographical profile. You should develop a technique to combine the results of the different schemes and generate a useful prediction for law enforcement officers. The predi ction should provide some kind of estimate or guidance about possible locations of the next crim e based on the time and locations of the past crime scenes. If you make use of any other evidenc e in your estimate, you must provide specific details about how you incorporate the extra informa tion. Your method should also provide some kind of estimate about how reliable the estimate will be in a given situation, including appropriate warnings.

In addition to the required one-page summary, your report should include an additional two-pag e executive summary. The executive summary should

provide a broad overview of the potential issues. It should provide an overview of your approach and describe situations when it is an appropriate tool and situations in which it is not an appropri ate tool. The executive summary will be read by a chief of police and should include technical det ails appropriate to the intended audience

问题：犯罪学

1981 彼得萨克利夫被判谋杀和对十三人的一些恶毒攻击。一种方法来缩小搜索克利夫先生是找到一个"重心"的地点的攻击。最终，犯罪嫌疑人恰好住在同一镇的预测技术。自那时以来，一些更先进的技术已经开发，以确定"地理"一个连续作案的犯罪根据犯罪地点。你的队伍已被当地警方要求制定一个方法帮助他们调查连环罪犯。该方法，你应该利用至少 2 种不同方案产生的地理分布。你应该建立一个技术结合两种方案的结果，并产生一个有用的预测为执法人员。预测应该提供估计和指导可能位置的下一个犯罪的时间和地点，过去的犯罪现场。如果你使用任何其他证据，你估计，你必须提供具体的细节你如何把额外信息。你还要对模型可靠估计将在一个给定的情况，包括适当的警告。

除了要求的一页摘要，你的报告应该包括一个额外的两页的执行摘要。执行状况应提供一个广阔的潜在问题。它提供了一个概述的方法，并描述的情况下，它是一个适当的工具和情况下，它不是一个适当的工具。执行摘要将读取由警察局长和应包括技术细节适当的目标受众

2011 Contest Problems

MCM PROBLEMS
PROBLEM A: Snowboard Course

Determine the shape of a snowboard course (currently known as a "halfpipe") to maximize the production of "vertical air" by a skilled snowboarder.

"Vertical air" is the maximum vertical distance above the edge of the halfpipe.

Tailor the shape to optimize other possible requirements, such as maximum twist in the air.

What tradeoffs may be required to develop a "practical" course?

MCM-A 题中文翻译

确定单板滑雪比赛的场地的形状，也叫 U 池（半管道），使得选手能达到最大的垂直高度。调整一下其形状，使之可以最优化地满足其他的要求，如在空中最大的旋转。建一个实用的场地，哪些因素是必须的？

PROBLEM B: Repeater Coordination

The VHF radio spectrum involves line-of-sight transmission and reception. This limitation can be overcome by "repeaters," which pick up weak signals, amplify them, and retransmit them on a different frequency. Thus, using a repeater, low-power users (such as mobile stations) can communicate with one another in situations where direct user-to-user contact would not be possible. However, repeaters can interfere with one another unless they are far enough apart or transmit on sufficiently separated frequencies.

In addition to geographical separation, the "continuous tone-coded squelch system" (CTCSS), sometimes nicknamed "private line" (PL), technology can be used to mitigate interference problems. This system associates to each repeater a separate subaudible tone that is transmitted by all users who wish to communicate through that repeater. The repeater responds only to received signals with its specific PL tone. With this system, two nearby repeaters can share the same frequency pair (for receive and transmit); so more repeaters (and hence more users) can be accommodated in a particular area.

For a circular flat area of radius 40 miles radius, determine the minimum number of repeaters necessary to accommodate 1,000 simultaneous users. Assume that the spectrum available is 145 to 148 MHz, the transmitter frequency in a repeater is either 600 kHz above or 600 kHz below the receiver frequency, and there are 54 different PL tones available.

How does your solution change if there are 10,000 users?

Discuss the case where there might be defects in line-of-sight propagation caused by mountainous areas.

MCM-B 题中文翻译

高频无线电频谱涉及到视线距离之内的传输与接收。我们可以使用"中继器"来克服这个限制。中继器可以接收微弱的信号并放大，再用不同的频率将其再次发送。因此，利用中继器，低功耗的用户（比如移动电台）能够在直接的用户到用户连接不可能实现的情况下与其他用户通信。但是，除非中继器互相之间的距离足够远或是以相差很大的频率发送，否则中继器会造成相互之间的影响。

除去地理上的隔离，"连续单音控制静噪制"（CTCSS），俗称"专线"（PL）技术可被用于缓和干扰问题。系统通过一种独特的次声频单音与每个讯号放大器（也称中继器）联系。每一个希望通过特定的放大器联系的用户都会发送这种独特的信号。放大器只、回应从它专线上转来的单音。通过该系统，两个相邻的放大器可以共同发送和接收相同的频率；相应的更多的放大器（也就是更多的用户）可以设置在一个特定的地区。

考虑一块半径为 40 英里的圆形平地,讨论适应 1000 个同时存在的用户所需要的最少的放大器的数量。假设可以接收到的电磁波谱从 145 至 148MHZ,在一个中继器内,发射频率可能高于也可能低于接收器 600KHZ,并且一共有 54 条不同的专线可用。

考虑一下 10000 人的情况以及在山区的时候视线距离内传输的缺点

2012 Contest Problems MCM PROBLEMS

PROBLEM A: The Leaves of a Tree
"How much do the leaves on a tree weigh?" How might one estimate the actual weight of the leaves (or for that matter any other parts of the tree)? How might one classify leaves? Build a mathematical model to describe and classify leaves. Consider and answer the following: • Why do leaves have the various shapes that they have?

• Do the shapes "minimize" overlapping individual shadows that are cast, so as to maximize exposure? Does the distribution of leaves within the "volume" of the tree and its branches effect the shape?

• Speaking of profiles, is leaf shape (general characteristics) related to tree profile/branching structure?

• How would you estimate the leaf mass of a tree? Is there a correlation between the leaf mass and the size characteristics of the tree (height, mass, volume defined by the profile)?
In addition to your one page summary sheet prepare a one page letter to an editor of a scientific journal outlining your key findings.

2012 美赛 A 题:一棵树的叶子

"一棵树的叶子有多重?"怎么能估计树的叶子(或者树的任何其它部分)的实际重量?怎样对叶子进行分类?建立一个数学模型来对叶子进行描述和分类。模型要考虑和回答下面的问题:

•为什么叶子具有各种形状?

叶子之间是要将相互重叠的部分最小化,以便可以最大限度的接触到阳光吗?树叶的分布以及树干和枝权的体积影响叶子的形状吗?就轮廓来讲,叶形(一般特征)是和树的轮廓以及分枝结构有关吗?

•你将如何估计一棵树的叶子质量?叶子的质量和树的尺寸特征(包括和外形轮廓有关的高度、质量、体积)有联系吗?除了你的一页摘要以外,给科学杂志的编辑写一封信,阐述你的主要发现。

PROBLEM B: Camping along the Big Long River
Visitors to the Big Long River (225 miles) can enjoy scenic views and exciting white water rapids. The river is inaccessible to hikers, so the only way to enjoy it is to take a river trip that requires several days of camping. River trips all start at First Launch and exit the river at Final Exit, 225 miles downstream. Passengers take either oar- powered rubber rafts, which travel on average 4 mph or motorized boats, which travel on average 8 mph. The trips range from 6 to 18 nights of camping on the river, start to finish.. The government agency responsible for managing this river wants every trip to enjoy a wilderness experience, with minimal contact with other groups of boats on the river. Currently, X trips travel down the Big Long River each year during a six month period (the rest of the year it is too cold for river trips). There are Y camp sites on the Big Long River, distributed f

airly

uniformly throughout the river corridor. Given the rise in popularity of river rafting, the park man agers have been asked to allow more trips to travel down the river. They want to determine how t hey might schedule an optimal mix of trips, of varying duration (measured in nights on the river) and propulsion (motor or oar) that will utilize the campsites in the best way possible. In other wo rds, how many more boat trips could be added to the Big Long River's rafting season? The river m anagers have hired you to advise them on ways in which to develop the best schedule and on wa ys in which to determine the carrying capacity of the river, remembering that no two sets of cam pers can occupy the same site at the same time. In addition to your one page summary sheet, pre pare a one page memo to the managers of the river describing your key findings.

2012 美赛 B 题：沿着"大长河"露营

　　游客在"大长河"(225 英里)可以享受到秀丽的风光和令人兴奋的白色湍流。这条河对于背包客来说是进不去的， 因此畅游这条长河的唯一办法就是在这条河上露营上几天。这次旅行从开始的下水点到最终结束点，共 225 英里，且是顺流而下的。乘客可以选择平均 4 英里/小时的以浆作为动力的橡胶筏或者平均 8 英里/小时的机动帆船旅行。整个旅行从开始到结束会经历 6 至 18 个夜晚。负责管理这条河的政府机构希望到这里的每一次旅行都能够享受到野外经历，以最少的接触到在河上其它的船只。目前， 每年在六个月期间(一年的其余部分的天气对于河流旅行来说太冷)，共有 X 次旅行，有 Y 处露营地，露营地均匀的分布整个河道。由于漂流的受欢迎程度的上升，公园管理者已经被要求允许更多的旅行次数。所以他们想确定怎样可能安排一个最优的混合的旅行方案，不同的时间(单位为夜)和推动方式(马达或浆)，最大限度的利用露营地。换句话说，在长河的漂流季，将会有多少更多的乘船旅行可以加进来?河流的管理者现在雇佣你，为他们提出最佳排程方式和河流承载能力的建议，记住两个露营者不能在同一时间内占据同一个露营地。除了你的一页摘要，准备一页备忘录，对河流的管理者描述你的主要发现。

2013 Contest Problems MCM PROBLEMS

PROBLEM A: The Ultimate Brownie Pan

When baking in a rectangular pan heat is concentrated in the 4 corners and the product gets ov ercooked at the corners (and to a lesser extent at the edges). In a round pan the heat is distribute d evenly over the entire outer edge and the product is not overcooked at the edges. However, sin ce most ovens are rectangular in shape using round pans is not efficient with respect to using the space in an oven.

Develop a model to show the distribution of heat across the outer edge of a pan for pans of diff erent shapes - rectangular to circular and other shapes in between.

Assume

1. A width to length ratio of W/L for the oven which is rectangular in shape. 2. Each pan must h ave an area of A.

3. Initially two racks in the oven, evenly spaced.

Develop a model that can be used to select the best type of pan (shape) under the following co nditions:

1. Maximize number of pans that can fit in the oven (N) 2. Maximize even distribution of heat (H) for the pan

3. Optimize a combination of conditions (1) and (2) where weights p and (1- p) are assigned to illustrate how the results vary with different values of W/L and p.

In addition to your MCM formatted solution, prepare a one to two page advertising sheet for the new Brownie Gourmet Magazine highlighting your design and results.

问题 A：终极布朗尼潘

当在一个矩形的锅烘烤时热量会在 4 个角落集中，并使产品在拐角处（以及在较小程度上在边缘处）加热过度。在一个圆形盘的热量 被均匀地分布在整个外缘并且在边缘处的产品不会加热过度。然而，相对于使用的空间的烘箱中，由于大多数烤炉是矩形的形状使用 圆形平底锅是效率不高。

开发一个模型来显示不同形状的平底锅锅的外边缘之间的热分布- 矩形到圆形和在两者之间的其它形状。

假设

1。烘箱是矩形的形状，宽度长度比为 W / L，定名为 A；

2。在 A 中，每盘必须有一个区域。 3。最初两个烘烤架均匀地间隔开。

开发模型，使其在下列条件下可用于选择的最佳类型的盘（形状）：

1。锅数量最大情况并适合烤箱的锅形状（N） 2。使锅的热量最大化均匀分布（H）的锅形状

3。优化的组合的条件（1）和（2）式中的权重 p 和（1 - p）被分配的结果来说明，W / L 和 p 的 不同的值的变化。

在除了您的 MCM 格式化论文外，制备一到两页的广告单，发到布朗尼美食杂志，注意突出您的设计和结果。

PROBLEM B: Water, Water, Everywhere

Fresh water is the limiting constraint for development in much of the world. Build a mathematical model for determining an effective, feasible, and cost-efficient water strategy for 2013 to meet the projected water needs of [pick one country from the list below] in 2025, and identify the best water strategy. In particular, your mathematical model must address storage and movement; de-salinization; and conservation. If possible, use your model to discuss the economic, physical, and environmental implications of your strategy. Provide a non-technical position paper to governmental leadership outlining your approach, its feasibility and costs, and why it is the "best water strategy choice."

Countries: United States, China, Russia, Egypt, or Saudi Arabia

问题 B：水，水，无处不在

新鲜的水资源限制约束世界大部分地区的发展。建立一个数学模型，制定有效的、可行的和具有成本效益的 2013 年水资源战略，以预计满足（从下面的列表中选择一个国家）到 2025 年的用水需求，并确定最佳的水战略。特别是，您的数学模型必须解决存储和运动，去盐碱化和保护。如果可能的话，用你的模型，探讨经济、物理和环境你的战略的影响。提供一个非技术性的立场文件向政府领导介绍你的方法以及其可行性和成本，并为什么它是"最好的水战略的选择。"

国家：美国，中国，俄罗斯，埃及，沙特阿拉伯