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Guest Editorial

Network Science: What's Math Got to Do with It? [1]

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Introduction

This year's ICM® problem involved network science, or more precisely, a component of network science—social network analysis. My post-contest reflections have led me to believe it is time for the mathematics community to engage in this emerging subject to build a rigorous mathematical foundation for this important science and to join in performing mathematical modeling and interdisciplinary problem solving.

Some people call network science a "new" emerging discipline, yet, as we know, mathematicians have been developing graph (network) theory for centuries, and scientists and engineers have been modeling networks for decades. What is new is that the traditional techniques have been replaced by an entirely new arsenal of mathematics, science, and modeling associated with networks.

Others call network science the "new" operations research in that it connects quantitative concepts and elements from several disciplines such as mathematics, computer science, and information science with the qualitative models from sociology and other social sciences. By its very nature, network science is interdisciplinary and involves emerging areas of science such as complex adaptive systems, cooperative game theory, agent-based modeling, data analytics, and social network analysis.

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[1] With both appreciation and apologies to Tina Turner and her emotional song "What's Love Got To Do With It" and full tongue-in-cheek realization that unlike "love," mathematics is certainly not a second-hand emotion.

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From another, perhaps simpler, perspective, you could merely call network science a form of applied mathematics—applied dynamic graph theory with additional data elements and attributes. Or perhaps from a modeling perspective, it is simply modeling with a highly structured, entity-linked complex adaptive system framework.

I do not pretend to know exactly what network science is, or where it fits in with today's scientific world, or what it will become. However, I believe that much of the strength of network modeling is in its ability to embrace the complexity of the real world. For me, that makes network science an important and empowering form of interdisciplinary modeling and problem solving—worthy of ICM problems and much more.

In particular, I hope that the mathematics community does not ignore it. Network science needs the engagement of the mathematics community to produce its underlying framework and to invent new mathematics and computational techniques for analysis of its complex structures, development of its synergistic processes, and organizing of its overwhelming data. Likewise, mathematics needs network science to establish the relevance of mathematics in the modern information-based world. As the ICM teams discovered, network science is exciting, relevant, enjoyable, and modern—elements that mathematics currently desperately needs to bolster its future place in society.

Mathematical Elements

What are the mathematical elements of network science? One way to define a network is to establish its

- components (nodes, links, data, processes);
- properties (dynamic, functional, layered); and
- applications (logistics, flow, transportation, Internet, metabolic networks, social networks, organizational networks—perhaps there are just too many categories to list!).

Another way is to use the concept of a mathematical graph (the nodal-link structure) with its nontrivial topological features and then classify the various types of graphs that occur (random, scale free, small world, scale rich) and the data (often heavy-tailed) that need to be mined and analyzed.

A foundational research management report on network science offered a layered approach of network roles—physical, communicative, informational, biological and social / cognitive—that connect together to produce the overall web-like network framework [National Research Council 2006].

No matter what definition or theoretical framework is used, network science is inherently and essentially mathematical at its core; there is plenty for applied mathematicians to do. Most networks are sufficiently complex that simply relying on visualization produces erroneous intuitive conceptions—or, worse yet, complete misunderstanding. Defining, computing, and measuring well-defined properties can counter those misguided perceptions and improve network modeling and analysis. Indispensable roles for mathematical modelers in network science are

- working with social scientists to build explicative and empirical models,
- creating appropriate measures for important applications, and
- finding appropriate properties and formalizing their measurement systems and calculations. Like many other mathematical modeling constructs, these properties can be classified as structural or functional; local, global, or regional; discrete or continuous; dynamic or static; and deterministic or stochastic.

The network science world needs mathematicians to help sort out these characteristics and do even more.

Significance

Network science has become a major global research thrust (with funding potential equal to or exceeding that of mathematics and many sciences) in the research agenda of governments, societies, militaries, businesses, and organizations. Its publication and citation qualities and quantities are significant. Just look at the remarkable citation record of the works by someone such as Albert-László Barabási to see this subject's influence in science. New societies, new conferences, and new journals with a network science theme are emerging at a dazzling pace. Network science is reaching the popular press and also entering the business world with tremendous fanfare.

Mathematicians' Engagement

One could argue that all the Mathematics Awareness Month themes since 1997, when the theme was "Mathematics and the Internet," have been related to networks in some way. The theme for 2004, "The Mathematics of Networks," established a firm connection, and I still use the myriad networks on that year's poster as examples to students of the variety and beauty of networks. However, while mathematicians are certainly aware of network science, I still do not see much real engagement by the U.S. mathematics community. Recent meetings of the Mathematical Association of America and the American Mathematical Society show only minimal mathematically-connected network research. The Society for Industrial and

Applied Mathematics and the Institute for Operations Research and Management Science and their members are slightly more engaged, although "when it comes to the research agenda now popularized by network science, [operations research] has been an underutilized resource" [Alderson 2008].

In my opinion, mathematics is a vastly underutilized and unfortunately often missing part of network science. In a comment on Alderson [2008], Nagurney adds that "it is not just the network topology and associated statistical aspects of networks that matter but flows that must be incorporated into network modeling as well as behavior" [Nagurney 2008]. Alderson and colleagues also wrote about mathematics and its engagement in Internet research that "surprising little attention has been paid in the mathematics and physics communities ... in the Internet research arena" [Willinger et al. 2009]. Of course, there is much more to network science than the Internet, but it is a significant network that most of the world confronts many times and in many ways every day.

Family Tree

It may be worth looking at a family tree for network science. While these relationships are subjective and incomplete, one can see in **Figure 1** a flow that brings together many elements of mathematics. The mathematics community should not miss out on an opportunity as rich and stimulating as network science. Ultimately, the mathematical elements of this discipline will be accomplished somehow and by someone. I suggest that this work be done by mathematicians—and the sooner, the better.

Network Science for Undergraduates

Does network science extend to the undergraduate curriculum? Based on their interest in this year's ICM problem, I believe that undergraduates would respond in the affirmative. In some institutions, network science is even making inroads in establishing undergraduate programs, and courses and programs traditionally offered at the graduate level are entering the undergraduate realm. My mathematics students tell me that they want to learn network science. As analysis of social media and online games are beginning to be seen to be parts of network science modeling, student interest in network science is growing in leaps and bounds. The bottom line is that network science is highly popular with students. I have taught three different network science courses over the past four semesters and am designing a fourth for the Fall semester of 2012. I usually have to turn away students or add more sections. This past Spring, I team-taught social network analysis with a sociologist and enjoyed the interdisciplinary modeling aspects of

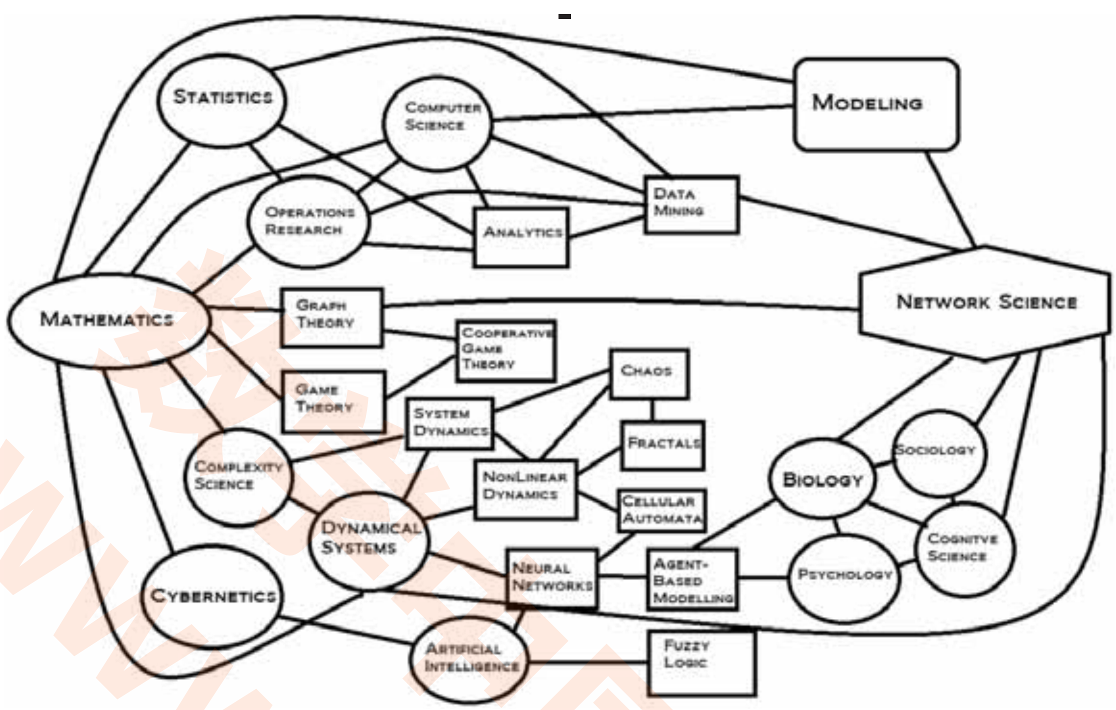

Figure 1. Disciplinary connection network model showing some of the links between mathematics and network science.

this exciting subject. Last year's award-winning publication for INFORMS was a network science book by Easley and Kleinberg from Cornell written for undergraduates [Easley and Kleinberg 2010]. Network science is on the map of undergraduate education.

Worldwide Interest

The ICM data show there may be differences among nations in perceptions or interests in interdisciplinary modeling and network modeling. The U.S. has always been slightly behind the rest the rest of the world in ICM / MCM® interest ratio, as measured by the proportion of teams who choose the ICM problem rather than one of the MCM problems. Usually, there is about half as much interest in the ICM from U.S. teams compared to teams from the rest of the world; this year, that ratio dropped to about one-third. I do not know why this is so or if this phenomenon has any real significance. Perhaps American students have a more disciplinary focus on mathematics or just haven't been exposed to as many network or interdisciplinary ideas. Whatever the reason, I personally hope that all students (high school, undergraduate and graduate) in every nation have the opportunity to study some aspects of networks, and that the mathematics they learn in doing so goes to excellent use.

Exhortation

Mathematicians, let's not miss this opportunity. Take another look at network science and see where you can contribute. Talk to colleagues in other disciplines and form teams to learn, study, research, teach, and engage in this enjoyable and important field of network science.

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About the Author

Chris Arney graduated from West Point and served as an intelligence officer in the U.S. Army. His academic studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his 30-year military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris then moved to RTP (Research Triangle Park), NC, where he served for various durations as chair of the Mathematical Sciences Division, of the Network Sciences Division, and of the Information Sciences Directorate of the Army Research Office. Chris has authored 22 books, written more than 120 technical articles, and given more than 250 presentations and 40 workshops. His technical interests include mathematical modeling, cooperative systems, pursuit-evasion modeling, robotics, artificial intelligence, military operations modeling, and network science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula. He is the founding director of COMAP's Interdisciplinary Contest in Modeling (ICM)®. In August 2009, he rejoined the faculty at West Point as the Network Science Chair and Professor of Mathematics.

Editor's Note
About This Issue

This year we had 5,000 (!) participating teams in the two contests combined; the 18 (!) Outstanding papers ran to over 500 manuscript pages. Editing and publishing all the Outstanding papers, which we once did, is simply not possible any more.

Hence, as in 2010 and 2011, we are able to present in the pages of the *Journal* only one Outstanding entry for each of the MCM and ICM problems. The selection of which papers to publish reflected editorial considerations and was done blind to the affiliations of the teams.

All of the 18 Outstanding papers appear in their original form on the 2012 MCM-ICM CD-ROM, which also has the press releases for the two contests, the results, the problems, unabridged versions of the Outstanding papers, and some of the commentaries. Information about ordering is at `http://www.comap.com/product/cdrom/index.html` or at (800) 772-6627.

MCM Modeling Forum

Results of the 2012 Mathematical Contest in Modeling

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Introduction

A total of 3,697 teams of undergraduates from hundreds of institutions and departments in 16 countries spent a weekend in February working on applied mathematics problems in the 28th Mathematical Contest in Modeling (MCM)®.

The 2012 MCM began at 8:00 P.M. EST on Thursday, February 9, and ended at 8:00 P.M. EST on Monday, February 13. During that time, teams of up to three undergraduates researched, modeled, and submitted a solution to one of two open-ended modeling problems. Students registered, obtained contest materials, downloaded the problem and data, and entered completion data through COMAP's MCM Website. After a weekend of hard work, solution papers were sent to COMAP on Monday. Two of the top papers appear in this issue of *The UMAP Journal*, together with commentaries.

In addition to this special issue of *The UMAP Journal*, COMAP offers a supplementary *2012 MCM-ICM CD-ROM* containing the press releases for the two contests, the results, the problems, unabridged versions of the Outstanding papers, and judges' commentaries. Information about ordering is at http://www.comap.com/product/cdrom/index.html or at (800) 772–6627.

Results and winning papers from the first 27 contests were published in special issues of *Mathematical Modeling* (1985–1987) and *The UMAP Journal* (1985–2011). The 1994 volume of *Tools for Teaching*, commemorating the tenth anniversary of the contest, contains the 20 problems used in the first 10 years of the contest and a winning paper for each year. That volume and the special

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MCM issues of the *Journal* for the last few years are available from COMAP. The 1994 volume is also available on COMAP's special *Modeling Resource* CD-ROM. Also available is *The MCM at 21* CD-ROM, which contains the 20 problems from the second 10 years of the contest, a winning paper from each year, and advice from advisors of Outstanding teams. These CD-ROMs can be ordered from COMAP at `http://www.comap.com/product/cdrom/index.html`.

This year, the two MCM problems represented significant challenges:

- Problem A, "The Leaves of a Tree," asked teams to model the leaves on a tree, classifying their shapes, investigating the effect of leaf shape on tree profile and branching, and estimating the leaf mass of a tree.
- Problem B, "Camping Along the Big Long River," asked teams to design a management plan for scheduling recreational multi-day rafting tours down a long stretch of a river. The goal was to maximize the number of trips, optimize campsite usage, and offer an optimal mix of trip lengths.

COMAP also sponsors:

- The MCM / ICM Media Contest (see p. 202).
- The Interdisciplinary Contest in Modeling (ICM)®, which runs concurrently with MCM and next year will offer a modeling problem involving network science. Results of this year's ICM are on the COMAP Website at `http://www.comap.com/undergraduate/contests`. The contest report, an Outstanding paper, and commentaries appear in this issue.
- The High School Mathematical Contest in Modeling (HiMCM)®, which offers high school students a modeling opportunity similar to the MCM. Further details are at `http://www.comap.com/highschool/contests`.

2012 MCM Statistics

- 3,697 teams participated (with 1,329 more in the ICM)
- 8 high school teams (<0.5%)
- 341 U.S. teams (9%)
- 2,428 foreign teams (91%), from Canada, China, Finland, Germany, India, Indonesia, Ireland, Malaysia, Mexico, Palestine, Singapore, South Africa, South Korea, Spain, Turkey, and the United Kingdom
- 10 Outstanding Winners (<0.5%)
- 17 Finalist Winners (<0.5%)
- 405 Meritorious Winners (11%)
- 1,048 Honorable Mentions (28%)
- 2,211 Successful Participants (60%)

Problem A: The Leaves of a Tree

"How much do the leaves on a tree weigh?" How might one estimate the actual weight of the leaves (or for that matter any other parts of the tree)? How might one classify leaves? Build a mathematical model to describe and classify leaves. Consider and answer the following:

- Why do leaves have the various shapes that they have?
- Do the shapes "minimize" overlapping individual shadows that are cast, so as to maximize exposure? Does the distribution of leaves within the "volume" of the tree and its branches effect the shape?
- Speaking of profiles, is leaf shape (general characteristics) related to tree profile / branching structure?
- How would you estimate the leaf mass of a tree? Is there a correlation between the leaf mass and the size characteristics of the tree (height, mass, volume defined by the profile)?

In addition to your one-page summary sheet, prepare a one-page letter to an editor of a scientific journal outlining your key findings.

Problem B: Camping along the Big Long River

Visitors to the Big Long River (225 miles) can enjoy scenic views and exciting white water rapids. The river is inaccessible to hikers, so the only way to enjoy it is to take a river trip that requires several days of camping.

River trips all start at First Launch and exit the river at Final Exit, 225 miles downstream. Passengers take either oar- powered rubber rafts, which travel on average 4 mph or motorized boats, which travel on average 8 mph. The trips range from 6 to 18 nights of camping on the river, start to finish.

The government agency responsible for managing this river wants every trip to enjoy a wilderness experience, with minimal contact with other groups of boats on the river.

Currently, X trips travel down the Big Long River each year during a six-month period (the rest of the year it is too cold for river trips). There are Y camp sites on the Big Long River, distributed fairly uniformly throughout the river corridor. Given the rise in popularity of river rafting, the park managers have been asked to allow more trips to travel down the river. They want to determine how they might schedule an optimal mix of trips, of varying duration (measured in nights on the river) and propulsion (motor or oar) that will utilize the campsites in the best way possible. In other words, how many more boat trips could be added to the Big Long River's rafting season?

The river managers have hired you to advise them on ways in which to develop the best schedule and on ways in which to determine the carrying capacity of the river, remembering that no two sets of campers can occupy the same site at the same time.

In addition to your one-page summary sheet, prepare a one-page memo to the managers of the river describing your key findings.

The Results

The solution papers were coded at COMAP headquarters so that names and affiliations of the authors would be unknown to the judges. Each paper was then read preliminarily by two "triage" judges at either Appalachian State University (Leaf Problem) or at the National Security Agency (River Problem) or at Carroll College (River Problem). At the triage stage, the summary and overall organization are the basis for judging a paper. If the judges' scores diverged for a paper, the judges conferred; if they still did not agree, a third judge evaluated the paper.

Additional Regional Judging sites were created at the U.S. Military Academy, the Naval Postgraduate School, and Carroll College to support the growing number of contest submissions.

Final judging took place at the Naval Postgraduate School, Monterey, CA. The judges classified the papers as follows:

	Outstanding	Finalist	Meritorious	Honorable Mention	Successful Participation	Total
Leaf Problem	4	8	226	482	862	1,582
River Problem	6	9	179	566	1,349	2,109
	10	17	405	1,048	2,211	3,691

We list here the 10 teams that the judges designated as Outstanding; the list of all participating schools, advisors, and results is at the COMAP Website.

Outstanding Teams

Institution and Advisor — **Team Members**

Leaf Problem

"How to Measure the Weight of Leaves on a Tree"
Hong Kong Baptist University — Xiaotian Wu
Kowloon, Hong Kong — Qingran Li
Alex Wing Kee Mok — Jin Liang

"A Close Look at Leaves"
Shanghai Foreign Language School
Shanghai, China
YiJung Wang
Bo Zhang
Yi Zhang
TianKun Lu

"Geometrical Tree"
National University of Singapore
Singapore
Weizhu Bao
Wenji Xu
Jing Zhang
Jingyi Lu

"The Secrets of Leaves"
Zhejiang University
Hangzhou, China
Zhiyi Tan
Cheng Fu
Danting Zhu
Hangqi Zhao

River Problem

"Best Schedule to Utilize the Big Long River"
Peking University
Beijing, China
Liu Xu Feng
Nan Bi
Chenwei Tian
Yuan Liu

"Computing Along the Big Long River"
Western Washington University
Bellingham, WA
Edoh Y. Amiran
Chip Jackson
Lucas Bourne
Travis Peters

"Optimization of Seasonal Throughput and Campsite Utilization on the Big Long River"
University of Colorado
Boulder, CO
Anne M. Dougherty
Stephen M. Kissler
Christopher Corey
Sean Wiese

"Getting Our Priorities Straight"
Bethel University
Arden Hills, MN
Nathan Gossett
Michael D. Tetzlaff
Blaine Goscha
Jacob Smith

"Optimal Scheduling for the Big Long River"
University of Colorado
Boulder, CO
Anne M. Dougherty

Tracy Babb
Christopher V. Aicher
Daniel J. Sutton

"C.A.R.S.: Cellular Automaton Rafting Simulation"
University of Louisville
Louisville, KY
Changbing Hu

Suraj Kannan
Joshua Mitchell
James Jones

Awards and Contributions

Each participating MCM advisor and team member received a certificate signed by the Contest Director and the appropriate Head Judge.

INFORMS, the Institute for Operations Research and the Management Sciences, recognized the team from the Shanghai Foreign Language School, China (Leaf Problem) and the team of Babb, Aicher, and Sutton from the University of Colorado (River Problem) as INFORMS Outstanding teams and provided the following recognition:

- a letter of congratulations from the current president of INFORMS to each team member and to the faculty advisor;
- a check in the amount of $300 to each team member;
- a bronze plaque for display at the team's institution, commemorating team members' achievement;
- individual certificates for team members and faculty advisor as a personal commemoration of this achievement; and
- a one-year student membership in INFORMS for each team member, which includes their choice of a professional journal plus the *OR/MS Today* periodical and the INFORMS newsletter.

The Society for Industrial and Applied Mathematics (SIAM) designated one Outstanding team from each problem as a SIAM Winner. The SIAM Award teams were from Zhejiang University (Leaf Problem) and the University of Louisville (River Problem). Each team member was awarded a $300 cash prize, and the teams received partial expenses to present their results in a special Minisymposium at the SIAM Annual Meeting in Minneapolis, MN in July. Their schools were given a framed hand-lettered certificate in gold leaf.

The Mathematical Association of America (MAA) designated one North American team from each problem as an MAA Winner. The Winner for the

Leaf Problem was a Finalist team from Cornell University with members Dennis Chua, Jessie Lin, and Alvin Wijaya, and advisor John R. Callister. The winner for the River Problem was the Outstanding team of Kissler, Corey, and Wiese from the University of Colorado. With partial travel support from the MAA, the teams presented their solution at a special session of the MAA Mathfest in Madison, WI in August. Each team member was presented a certificate by an official of the MAA Committee on Undergraduate Student Activities and Chapters.

Ben Fusaro Award

One Meritorious or Outstanding paper is selected for the Ben Fusaro Award, named for the Founding Director of the MCM and awarded for the ninth time this year. It recognizes an especially creative approach; details concerning the award, its judging, and Ben Fusaro are in Vol. 25 (3) (2004): 195–196. The Ben Fusaro Award Winner, for the Leaf Problem, was the Outstanding team from the National University of Singapore. A commentary about it appears in this issue.

Frank Giordano Award

For the first time, the MCM is designating a paper with the Frank Giordano Award. This award goes to a paper that demonstrates a very good example of the modeling process in a problem featuring discrete mathematics—this year, the River Problem. Having worked on the contest since its inception, Frank Giordano served as Contest Director for 20 years. The Frank Giordano Award for 2012 went to the Outstanding team from Western Washington University in Bellingham, WA.

Judging

Director
William P. Fox, Dept. of Defense Analysis, Naval Postgraduate School, Monterey, CA
Associate Directors
Patrick J. Driscoll, Dept. of Systems Engineering, U.S. Military Academy, West Point, NY
Kelly Black, Mathematics Dept., Clarkson University, Potsdam, NY

Leaf Problem

Head Judge
Patrick Driscoll, Dept. of Systems Engineering, U.S. Military Academy, West Point, NY

Associate Judges

William C. Bauldry, Chair-Emeritus, Dept. of Mathematical Sciences, Appalachian State University, Boone, NC (Head Triage Judge)

Karen Bolinger, Dept of Mathematics, Clarion University, Clarion, PA

Tim Elkins, Dept. of Systems Engineering, U.S. Military Academy, West Point, NY (INFORMS Judge)

J. Douglas Faires, Youngstown State University, Youngstown, OH (MAA Judge)

Ben Fusaro, Dept. of Mathematics, Florida State University, Tallahassee, FL (SIAM Judge)

Michael Jaye, Dept. of Defense Analysis, Naval Postgraduate School, Monterey, CA

Mario Juncosa, RAND Corporation, Santa Monica, CA (retired)

Peter Olsen, Johns Hopkins Applied Physics Laboratory, Baltimore, MD

John Scharf, Dept. of Mathematics, Engineering, and Computer Science, Carroll College, Helena, MT (Fusaro Award Judge)

Michael Tortorella, Dept. of Industrial and Systems Engineering, Rutgers University, Piscataway, NJ (Problem Author)

Dan Zwilliger, Raytheon, Boston, MA

Regional Judging Session at the U.S. Military Academy

Head Judge

Patrick J. Driscoll, Dept. of Systems Engineering

Associate Judges

Tim Elkins, James Enos, Kenny McDonald, and Elizabeth Schott, Dept. of Systems Engineering

Paul Steve Horton, Dept. of Mathematical Sciences

Jack Picciuto, Office of Institutional Research

—all from the United States Military Academy at West Point, NY

Paul Heiney, Dept of Mathematics, U.S. Military Academy Preparatory School, West Point, NY

Ed Pohl, Dept. of Industrial Engineering

Tish Pohl, Dept. of Civil Engineering

—both from University of Arkansas, Fayetteville, AR

Triage Session at Appalachian State University

Head Triage Judge

William C. Bauldry, Chair, Dept. of Mathematical Sciences

Associate Judges

Bill Cook, Ross Gosky, Jeffry Hirst, Katie Mawhinney, Trina Palmer, Greg Rhoads, René Salinas, Tracie McLemore Salinas, Kevin Shirley, and Nate Weigl

—all from the Dept. of Mathematical Sciences, Appalachian State University, Boone, NC

Amy H. Erickson and Keith Erickson
—Dept. of Mathematics, Georgia Gwinnett College, Lawrenceville, GA

Steven Kaczkowski and Douglas Meade
—Dept. of Mathematics, University of South Carolina, Columbia, SC

River Problem

Head Judge
Maynard Thompson, Mathematics Dept., University of Indiana, Bloomington, IN
Associate Judges
Peter Anspach, National Security Agency, Ft. Meade, MD (Head Triage Judge)
Robert Burks, Operations Research Dept., Naval Postgraduate School, Monterey, CA
Jim Case, Baltimore, MD (SIAM Judge)
Veena Mendiratta, Lucent Technologies, Naperville, IL
Greg Mislick, Operations Research Dept., Naval Postgraduate School, Monterey, CA
Scott Nestler, Operations Research Dept., Naval Postgraduate School, Monterey, CA
Jack Picciuto, Office of Institutional Research, U.S. Military Academy, West Point, NY
Kathleen M. Shannon, Dept. of Mathematics and Computer Science, Salisbury University, Salisbury, MD (MAA Judge)
Dan Solow, Case Western Reserve University, Cleveland, OH (INFORMS Judge)
Marie Vanisko, Dept. of Mathematics, Engineering, and Computer Science, Carroll College, Helena, MT (Giordano Award Judge)
Richard Douglas West, Francis Marion University, Florence, SC (Giordano Award Judge)

Regional Judging Session at the Naval Postgraduate School
Head Judges
William P. Fox, Dept. of Defense Analysis
Frank R. Giordano, Dept. of Defense Analysis
Associate Judges
Michael Jaye, Dept. of Defense Analysis
Robert Burks, Greg Mislick, and Scott Nestler, Operations Research Dept.
—all from the Naval Postgraduate School, Monterey, CA

Joanna Bieri, University of Redlands, Redlands, CA
Rich West, (retired) PA

Triage Session at Carroll College
Head Judge
Marie Vanisko
Associate Judges
Terry Mullen and Kelly Cline
—all from Dept. of Mathematics, Engineering, and Computer Science, Carroll College, Helena, MT

Triage Session at the National Security Agency
Head Triage Judge
Peter Anspach, National Security Agency (NSA), Ft. Meade, MD
Associate Judges
Jim Case, Dean McCullough, and judges from within NSA

Sources of the Problems

The Leaf Problem was contributed by Lee Zia (Program Director, National Science Foundation Division of Undergraduate Education). The River Problem was contributed by Catherine Roberts (Dept. of Mathematics and Computer Science, College of the Holy Cross, Worcester, MA).

Acknowledgments

Major funding for the MCM is provided by the National Security Agency (NSA) and by COMAP. Additional support is provided by the Institute for Operations Research and the Management Sciences (INFORMS), the Society for Industrial and Applied Mathematics (SIAM), and the Mathematical Association of America (MAA). We are indebted to these organizations for providing judges and prizes.

We also thank for their involvement and unflagging support the MCM judges and MCM Board members, as well as

- **Two Sigma Investments.** "This group of experienced, analytical, and technical financial professionals based in New York builds and operates sophisticated quantitative trading strategies for domestic and international markets. The firm is successfully managing several billion dollars using highly-automated trading technologies. For more information about Two Sigma, please visit `http://www.twosigma.com`."

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each paper here is the result of undergraduates working on a problem over a weekend. Editing (and usually substantial cutting) has taken place; minor errors have been corrected, wording has been altered for clarity or economy, and style has been adjusted to that of *The UMAP Journal*. The student authors have proofed the results. Please peruse these students' efforts in that context.

To the potential MCM advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

COMAP's Mathematical Contest in Modeling and Interdisciplinary Contest in Modeling are the only international modeling contests in which students work in teams. Centering its educational philosophy on mathematical modeling, COMAP serves the educational community as well as the world of work by preparing students to become better-informed and better-prepared citizens.

Editor's Note

The complete roster of participating teams and results has become too long to reproduce in the *Journal*. It can now be found at the COMAP Website, in separate files for each problem:

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http://www.comap.com/undergraduate/contests/mcm/contests/
   2012/results/2012-Problem-A.pdf
http://www.comap.com/undergraduate/contests/mcm/contests/
   2012/results/2012-Problem-B.pdf
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Media Contest

This year, COMAP again organized an MCM / ICM Media Contest.

Over the years, contest teams have increasingly taken to various forms of documentation of their activities over the grueling 96 hours—frequently in video, slide, or presentation form. This material has been produced to provide comic relief and let off steam, as well as to provide some memories days, weeks, and years after the contest. We *love* it, and we want to encourage teams (outside help is allowed) to create media pieces and share them with us and the MCM / ICM community.

The media contest is *completely separate* from MCM and ICM. No matter how creative and inventive the media presentation, it has *no* effect on the judging of the team's paper for MCM or ICM. We do not want work on the media project to detract or distract from work on the contest problems in any way. This is a separate competition, one that we hope is fun for all.

Further information about the contest is at

`http://www.comap.com/undergraduate/contests/mcm/media.html.`

There were 11 entries, from Zhejiang University, United States Military Academy, Dalian Maritime University, and Beijing Institute of Technology.

Outstanding Winners:

- United States Military Academy, joint entry from three teams (Nolan Miles, Andrew Lopez, Benjamin Garlick, Brian Kloiber, Calla Glavin, Kailee Kunst, Samuel Ellis, Tanner Robertson, Robert Hume)
- Zhejiang University (Jiajun Chen, Yuchen Lei, Canyang Jin)

Finalists:

- Dalian Maritime University (Chengcheng Bi, Xuefu Bai, Bo Han)
- Dalian Maritime University (Zuchen Tang, Zihao Yu, Bowen Zhang)

The remaining entries were awarded Honorable Mention.

Complete results, including links to the Outstanding and Finalist videos, are at

`http://www.comap.com/undergraduate/contests/mcm/contests/2012/results/media/media.html.`

A Close Look at Leaves

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Abstract

We construct four models to study leaf classification, relationships between leaf shape and leaf distribution, correlations between leaf shape and tree profile, and total leaf mass of a tree.

Model 1 deals with the classification of leaves. We focus primarily on the most conspicuous characteristic of leaves, namely, shape. We create seven geometric parameters to quantify the shape. Then we select six common types of leaves to construct a database. By calculating the deviation index of the parameters of a sample leaf from those of typical leaves, we can classify the leaf. To illustrate this classification process, we use a maple leaf as a test case.

Model 2 studies the relationship between leaf shape and leaf distribution. First, we simplify a tree into an idealized model and then introduce the concept of solar altitude. By analyzing the overlapping individual shadows through considering the relationship between leaf length and internode length under different solar altitudes, we find that the leaf shape and distribution are optimized to maximize sunlight exposure according to the solar altitude. We apply the model to three test types of trees.

Model 3 discusses the possible association between tree profile and leaf shape. Based on the similarity between the leaf veins and branch structure of trees, we propose that leaf shape is a two-dimensional mimic of the tree profile. Employing the method of Model 1, we set several parameters reflecting the general shape of each tree and compare them with those of its leaves. With the help of statistical tools, we demonstrate a rough association between tree profile and leaf shape.

Model 4 estimates the total leaf mass of a tree given size characteristics. Carbon dioxide (CO_2) sequestration rate and tree age are introduced to establish the link between leaf mass and tree size. Since a unit mass of a leaf sequesters CO_2 at a constant rate, the CO_2 sequestration rate has a quadratic relationship with the age of the tree, and the size the tree experiences logistic growth.

The UMAP Journal 33 (3) (2012) 205–222.

Introduction

We tackle four main subproblems:

- classification of leaves,
- the relationship between leaf distribution and leaf shape,
- the relationship between the tree profile and the leaf shape, and
- calculation of the total leaf mass of a tree.

To tackle the first problem, we select a set of parameters to quantify the characters of the leaf shape and use the leaf shape as the main standard for our classification process.

For the second question, we use the overlapping area that one leaf's shadow casts on the leaf directly under it as the link between the leaf distribution and the leaf shape, since the leaf shape affects the overlapping. We assume that the leaf distribution tries to minimize the overlapping area.

As for the third question, we set parameters for the tree profile and compare those with the parameters for the tree's leaf shape to judge whether there is a relation between tree profile and leaf shape.

We use age to link the size of tree and the total weight of its leaves, because the tree size has an obvious relationship with its age and the age affects a tree's sequestration of carbon dioxide, which affects the total weight of a tree's leaves.

Assumptions

- The trees are all individual ("open grown") trees, such as are typically planted along streets, in yards, and in parks. Our calculation does not apply to densely raised trees, as in typical reforestation projects where large numbers of trees are planted close together.
- The shape of the leaves does not reflect special uses for the trees, such as to resist extremely windy, cold, parched, wet, or dry conditions, or to produce food.
- The type of the leaf distribution (leaf length and internode distance relation) reflects the tree's natural tendency to sunlight.
- The tree profile that we consider is the part above ground, including the trunk, the branches, and leaves.
- All parts of a leaf can lie flat, and the thickness or protrusion of veins can be neglected.
- Leaves are the only part of the tree that reacts in photosynthesis and respiration, so that the carbon dioxide sequestration of a tree is the sum of the sequestration of the leaves.

- The sequestration of a tree or a leaf is the net amount of CO_2 fixed in a tree, which is the difference between the CO_2 released in respiration and the CO_2 absorbed in photosynthesis.
- The trees are in healthy, mature, and stable condition. Trees of the same species have same characteristics.

Model 1: Leaf Classification

Decisive Parameters

To classify the shape of a leaf, we set seven parameters and establish a database for comparison.

Rectangularity

We define the ratio of the area of the leaf to the area of its minimum bounding rectangle as the leaf's *rectangularity* (**Figure 1**).

Figure 1.

Figure 2.

Figure 3.

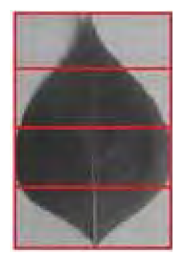

Figure 4.

The photographs of leaves in **Figures 1–4** are reproduced (with overlays by the authors of this paper) from Knight et al. [2010], by kind permission of that paper's authors.

Aspect Ratio

The aspect ratio is the ratio of the height of the minimum bounding rectangle to its width. (**Figure 2**).

Circularity

To evaluate how round a leaf is, we consider that ratio if the ex-circle to the in-circle. (**Figure 3**).

Form Factor

Form factor, a famous shape description parameter, is calculated as

$$\mathrm{FF} = \frac{4\pi A}{P^2},$$

where A is the area of the leaf and P is its perimeter.

Edge Regularity Area Index

Although the aspect ratio and the rectangularity of two leaves may be similar, the contour or the exact shape of two leaves may vary greatly. To take the different contour of the leaf into consideration, we join every convex dot along the contour and develop what we call the *bounding polygon area*. The ratio between the leaf area and this bounding polygon area is the *edge regularity area index*. The closer this ratio is to 1, the less jagged and smoother the leaf's contour is (**Figure 3**).

Edge Regularity Perimeter Index

Similarly, we develop another parameter, the *bounding polygon perimeter*, the perimeter of the polygon when we join the convex dots of a leaf. We define the ratio of the bounding polygon perimeter to the perimeter of the leaf to be the *edge regularity perimeter index*. The smaller this ratio, the more jagged and irregular the contour of the leaf is (**Figure 3**).

Proportional Index

Since it is also highly critical to capture the spatial distribution of different portions of a leaf along its vertical axis, we divide the minimum bounding rectangle into four horizontal blocks of equal height, and then calculate the proportion of the leaf area in a particular region to the total leaf, which we refer to as the *proportional index (PI)* for that region (**Figure 4**). Hence, the PI is a vector of length four.

Common Types of Leaves

We develop a database of the six most common leaf types in North America (**Figure 6**), using the seven parameters discussed above. **Table 2** gives the values of the parameters for each leaf type, as measured from scans of photos of leaves in Knight et al. [2010].

Comparison

Given a specific leaf, we calculate the seven characteristics of it and compare them with our database by calculating the squared deviation of each parameter of the given leaf from the corresponding standard parameter of each category. We realize that some of the parameters are somehow more important than others. So in an effort to make our model more accurate

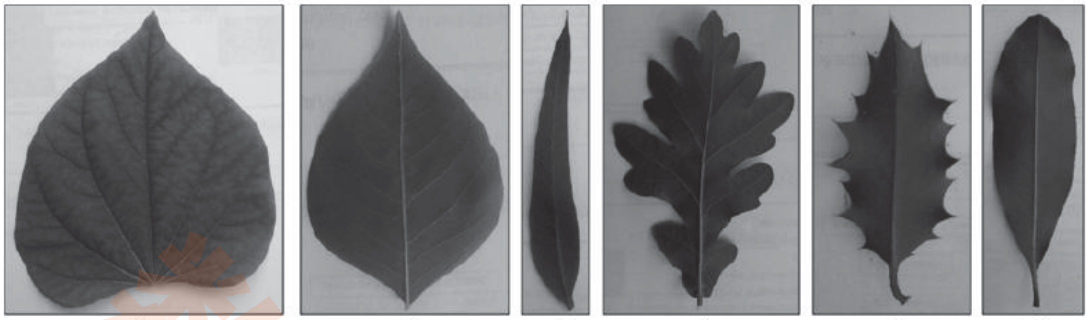
1 2 3 4 5 6

Figure 5. The six most common seen leaf types in North America. (The photos, from Knight et al. [2010], are reproduced by kind permission of that paper's authors.)

Table 1.
Parameter values for the six leaf types.

Type	1	2	3	4	5	6
Rectangularity	0.6627	0.5902	0.6250	0.4772	0.4876	0.6576
Aspect Ratio	0.8615	0.6600	0.1800	0.6383	0.4792	0.3111
Circularity	0.8140	0.5432	0.4564	0.3454	0.3123	0.3311
Form Factor	0.9139	0.6206	0.2823	0.2470	0.3662	0.4956
ER Area Index	0.9322	0.8780	0.9091	0.8500	0.7880	0.8895
ER Perimeter Index	0.8727	0.8889	0.9384	0.8602	0.8231	0.9903
PI_1	0.0649	0.0769	0.1179	0.1909	0.1299	0.2920
PI_2	0.2958	0.3555	0.2208	0.3892	0.3606	0.4187
PI_3	0.3439	0.4243	0.4139	0.3047	0.4123	0.2677
PI_4	0.2954	0.1433	0.2474	0.1152	0.0970	0.0220

and reliable, we introduce a weighted *index of deviation* I_D, with

$$I_D = \sum_{i=1}^{7} w_i I_i,$$

where each I_i is the squared deviation, except that

$$I_7 = \frac{1}{4}\sum_{j=1}^{4}(\mathrm{PI}_j - \mathrm{PI}_{\mathrm{new},i})^2.$$

We determine the weights via the Analytical Hierarchy Process (AHP) [Saaty 1982]. We build a 7×7 matrix reciprocal matrix by pair comparison:

$$\begin{array}{c} \\ \text{R} \\ \text{AR} \\ \text{C} \\ \text{FF} \\ \text{ERAI} \\ \text{ERPI} \\ \text{PI} \end{array} \begin{array}{c} \begin{array}{ccccccc} \text{R} & \text{AR} & \text{C} & \text{FF} & \text{ERAI} & \text{ERPI} & \text{PI} \end{array} \\ \begin{pmatrix} 1 & 1/3 & 1 & 1/4 & 1/2 & 1/2 & 1/7 \\ 3 & 1 & 3 & 1 & 2 & 2 & 1/3 \\ 1 & 1/3 & 1 & 1/4 & 1/2 & 1/2 & 1/7 \\ 4 & 1 & 4 & 1 & 3 & 3 & 1/2 \\ 2 & 1/2 & 2 & 1/3 & 1 & 1 & 1/4 \\ 2 & 1/2 & 2 & 1/3 & 1 & 1 & 1/4 \\ 7 & 3 & 7 & 2 & 4 & 4 & 1 \end{pmatrix} \end{array}.$$

The meaning of the number in each cell is explained in **Table 2**. The numbers themselves are based on our own subjective decisions.

Table 2.
The multiplication table of D_{10}.

Intensity of Value	Interpretation
1	Requirements i and j have equal value.
3	Requirement i has a slightly higher value than j.
5	Requirement i has a strongly higher value than j.
7	Requirement i has a very strongly higher value than j.
9	Requirement i has an absolutely higher value than j.
2, 4, 6, 8	Intermediate scales between two adjacent judgments.
Reciprocals	Requirement i has a *lower* value than j.

We then input the matrix into a Matlab program that calculates the weight w_i of each factor, as given in **Table 3**.

Table 3.
AHP-derived weights.

Factor	R	AR	C	FF	ERAI	ERPI	PI
Weight	0.0480	0.1583	0.0480	0.2048	0.0855	0.0855	0.3701

We test the consistency of the preferences for this instance of the AHP. For good consistency [Alonso and Lamata 2006, 446–447]:

- The principal eigenvalue $\lambda_{\max}$ of the matrix should be close to the number n of alternatives, here 7; we get $\lambda_{\max} = 7.05$.
- The consistency index CI = $(\lambda_{\max} - n)/(n-1)$ should be close to 0; we get CI $= 0.009$.
- The consistency ratio CR = CI/RI (where RI is the average value of CI for random matrices) should be less than 0.01; we get CR $= 0.006$.

Hence, our decision method displays perfectly acceptable consistency and the weights are reasonable.

Model Testing

We use a maple leaf of **Figure 6** to test our classification model. Visually, it resembles Category 4 most.

Figure 6. Test maple leaf.

Now we test this hypothesis with our model. First, we process the image of the leaf, calculating rectangularity, aspect ratio, circularity, form factor, edge regularity area index, edge regularity perimeter index, and the proportional index, with values as in **Table 2**. The values of the seven parameters are shown in **Table 4**.

Table 4.
Parameter values for the sample maple leaf.

Factor	R	AR	C	FF	ERAI	ERPI	PI_1	PI_2	PI_3	PI_4
Measured value	0.355	0.908	0.269	0.157	0.625	0.379	0.097	0.463	0.431	0.009

Finally, we use our weights to calculate the index of deviation I_D of the maple leaf from each of the six categories of leaves considered earlier. We show the results in **Table 5**.

Table 5.
Index of deviation of maple leaf from six common leaf categories.

Category	1	2	3	4	5	6
Index of deviation I_D	0.27	0.12	0.23	**0.08**	0.24	0.18

Since the index of deviation between the given maple leaf and Category 4 is smallest, the model predicts that the maple leaf falls into Category 4—which conclusion is consistent with our initial hypothesis.

Conclusion

Our model is robust under reasonable conditions, as can be seen from the testing above. However, since our database contains only the six commonly-seen leaf types in North America, the variety in the database has room for improvement.

Model 2: Leaf Distribution and Leaf Shape

Introduction

Genetic and environmental factors contribute to the pattern of leaf veins and tissue, thereby determining leaf shape. In this model, we investigate how leaf distribution influences leaf shape.

Idealized Leaf Distribution Model

We construct an idealized model that immensely simplifies the complex situation: The tree is made up of a branch perpendicular to the ground surface, and two identical leaves grown on the branch ipsilaterally (on the same side) and horizontally. The leaves face upward and point toward the sun in the sky. We suppose that the tree is at latitude L (Northern Hemisphere). Let the greatest average solar altitude in a year, which is attained at noon on the vernal equinox, be α.

Figure 7 illustrates our primitive model of a tree at noon on the vernal equinox.

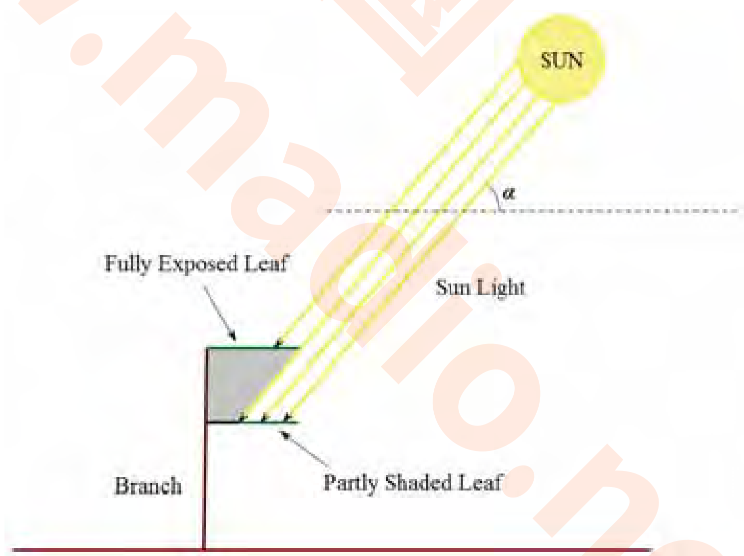

Figure 7. Primitive model of a tree, at noon on the vernal equinox.

Analysis of Overlapping Areas

Our focus is the partly shaded leaf in **Figure 7**. The output of the model is what proportion of the leaf (PL) is shaded. We divide the situation into three scenarios, depending on the influence of the angle α on PL.

Solar Altitude Near 90°

This situation usually takes place in tropical regions, where leaf shapes are typically broad and wide and the tree crown usually contains only one layer of leaves. This can be explained in terms of **Figure 7**: With α near 90°, the shaded part of the lower leaf would be too big to supply enough solar energy for photosynthesis, and the greatest absorption of energy can be achieved by a broad leaf shape.

Solar Altitude Near 0°

This situation usually takes place in frigid zones, where leaves are typically acicular (needle-shaped) and the tree crown contains dense layers of closely-grown leaves. In terms of **Figure 7**: With α near 0°, the shaded part of the lower leaf would approach zero, allowing a much more concentrated distribution of leaves than in other situations. In addition, the maximum absorption of energy can be best achieved by needle-like leaves.

Solar Altitude within Normal Range

This scenario is typical in the temperate zone on earth, where sunlight irradiates the leaves in a tilted way. It is also the case in which our idealized model is the most suitable. Another crucial factor that we control in this case is the distance h between the two points connecting the leaves and the branch. We assume that a tree's leaf distribution tries to minimize the overlapping area between leaves, so our model investigates the quantitative relationship between the overlapping area and the shape of the leaf.

To simplify the model, we model the leaf as a rhombus, whose major axis has length L_{major} and whose minor axis has length L_{minor}. Also, we fix the area of the leaf as A, to ensure constant exposure area to the sun. With area fixed, now we only need to change the length of the major axis to change the shape of the leaf (see **Figure 8**).

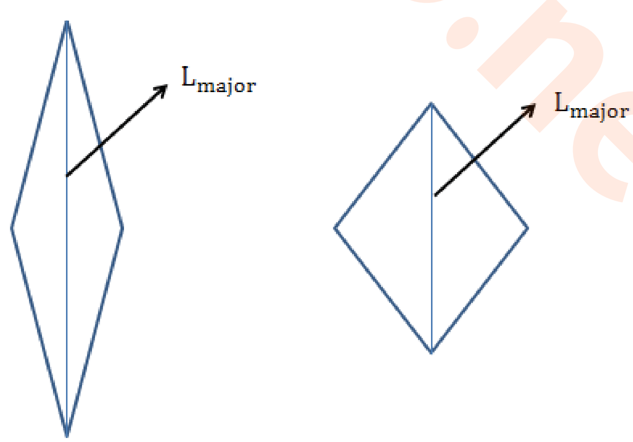

Figure 8. Two leaves of the same area but different lengths of major axis.

Also, since we have fixed the area of the leaf and just adjust its shape, the minimum proportion of the lower leaf shaded is

$$E = \frac{A_{\text{overlapping}}}{A},$$

where $A_{\text{overlapping}}$ is the smallest overlapping area.

The most efficient situation is for both leaves to be totally exposed to sunlight, as in **Figure 9a**: For some value $h = h_0$, we achieve $E = 0$.

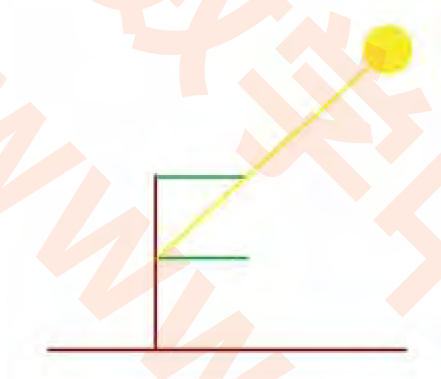

Figure 9a. Upper leaf does not overlap lower one.

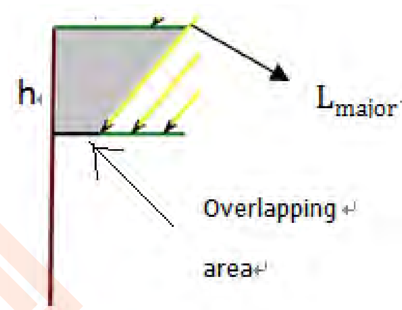

Figure 9b. Upper leaf overlaps lower one.

What if $h < h_0$, as in **Figure 9b**? We can easily give the relationship among h, L_{major}, and E for a given fixed solar altitude α:

$$E = \left(\frac{L_{\text{major}} \tan\alpha - h}{L_{\text{major}} \tan\alpha}\right)^2 = \left(1 - \frac{h}{L_{\text{major}} \tan\alpha}\right)^2.$$

For fixed h and α, the overlap area increases as the length of the leaf increase. The closer L_{major} is to $h/\tan\alpha$, the smaller the overlap.

From our discussion, the best leaf distribution occurs when $h = h_0$, which means $h = L_{\text{major}} \tan\alpha$.

Model Testing

We need to test whether this relation between leaf distribution and leaf shape is right. We offer data on leaf length L_{major} and internode distance h of several kinds of trees and use our formula to calculate the respective solar altitudes of the trees. By converting the solar altitude into latitude, we can predict the origin of a tree! We choose species native to China:

- *Ligustrum quihoui Carr.* (waxy-leaf privet or Quihou privet, a semi-evergreen to evergreen shrub);

- *Osmanthus fragrans* (sweet olive, tea olive, or fragrant olive, an evergreen shrub or small tree that is the city flower of Hangzhou, China); and
- *Camellia japonica* (Japanese camellia)

as our test trees. **Table 6** shows the results.

Table 6.
Test of model for leaf shape as a function of latitude.

Tree kind	L_{major}	h	Calculated $\tan\alpha$	Latitude Predicted	Latitude True
Ligustrum quihoui Carr.	2	2.5	1.25	38.7°	35–35°
Osmanthus fragrans	10	18.5	1.85	28.4°	23–29°
Camellia japonica	6	9	1.50	33.7°	32–36°

The predicted latitudes of origin are close to the true latitudes, confirming our hypothesis of a relationship between leaf distribution and leaf shape.

Model 3: Tree Profile and Leaf Shape

Hypothesis

Since

- the vein structure determines the leaf shape;
- the branch structure determines the tree profile; and
- to some degree, the leaf veins resemble branches,

we have a wild hypothesis that the leaf shape is two-dimensional mimic of the tree profile.

Comparison of Leaf Shape and Tree Contour

The leaf shape is two-dimensional, so it is relatively easy to study its parameters. However, the tree profile is three-dimensional, so it is important to find a two-dimensional characteristic of a tree to use for comparison. Since the longitudinal section of a particular tree reflects its general size characteristics, we focus on that.

Tree Profile Classification

In the leaf classification model, there are 6 general classes of leaves. Since we are comparing only the general resemblance between leaf and tree, we incorporate Class 5 (elliptic leaf with serrated margin) into Class 2 (elliptic leaf, smooth margin). As a result, we get 5 classes of leaves and 5 respective types of trees:

- Class 1: Cordate (Texas redbud)
- Class 2 and Class 5: Elliptic (camphor tree)
- Class 3: Subulate (pine)
- Class 4: Palmate (oak)
- Class 6: Obovate (mockernut hickory)

Parameters of the Tree

We appoint three parameters for the longitudinal section that can be compared with those of the leaf shape, namely, rectangularity, aspect ratio, and circularity.

Table 7 shows the measurements for both trees and leaves.

Table 7.
Comparison of leaf parameters and tree parameters.

Class	1	2 and 5	3	4	6
Rectangularity (R)					
Leaf	0.6627	0.5902	0.6250	0.4772	0.6576
Tree	0.6281	0.6846	0.5180	0.5292	0.6238
Aspect Ratio (AR)					
Leaf	0.8615	0.6600	0.1800	0.6383	0.3111
Tree	0.7914	0.7243	0.6601	0.7980	0.6750
Circularity (C)					
Leaf	0.6396	0.5698	0.1834	0.3069	0.2889
Tree	0.5800	0.5928	0.2895	0.4070	0.3866

For each of the parameter types, we drew a scatterplot, calculated the correlation, and investigated the statistical significance of the resulting line of best fit. Aspect ratio (AR) and circularity (C) were each statistically significant, pointing to linear relationships; rectangularity (R) was not.

Conclusion

The tests of aspect ratio and circularity support the theory that leaf shape is a two-dimensional mimic of the tree contour. Thus, the shape of leaf resembles the shape of tree to some extent.

Model 4: Leaf Mass

Introduction

A simple way to calculate the total leaf mass is to multiply the number of leaves by the mass of a single leaf. Our method is more accurate and less demanding, in that our model is (surprisingly!) independent of these two factors but dependent on a more reliable factor of a grown tree: photosynthesis. Our methodology of estimating the leaf mass of a tree is based on three variables:

- tree age;
- growth rate, which is determined by tree species; and
- general type (hardwood or conifer).

In other words, given the age and type of a tree, we can estimate the total mass of leaves. In this model, CO_2 is used as a calculating medium.

Leaf Mass and Tree Age

Leaf Mass and CO_2 Sequestration

Trees sequester CO_2 from the atmosphere in their leaves but mostly elsewhere in the tree. A tree's ability to sequester CO_2 is measured in terms of mass A_S of CO_2 (in pounds) per gram of leaf. Hardwood trees sequester more CO_2 per gram of leaf than conifers.

A tree's ability to sequester CO_2 is different from its ability to absorb it, since the tree also releases CO_2 into the atmosphere as part of its respiration. In other words,

$$\text{CO}_2 \text{ sequestration} = \text{CO}_2 \text{ absorption} - \text{CO}_2 \text{ release}.$$

Now we need only to estimate the weight of CO_2 sequestered by the tree and then calculate the total mass of the leaves as the ratio of the mass of CO_2 sequestered to the mass of CO_2 sequestered per leaf:

$$m_{\text{leaves}} = \frac{m_{\text{CO}_2}}{A_S}.$$

CO_2 Sequestration and Tree Age

The relationship between the amount of CO_2 sequestered, the age of a tree, and the type of tree is given in a table by the Energy Information Administration [1998, Table 2, 8–9], which also divides trees based on their growth rate: fast, moderate, or slow.

For each growth rate, we graphed the annual sequestration rate vs. age of the tree and fitted a quadratic model (see **Figure 10** for conifer example).

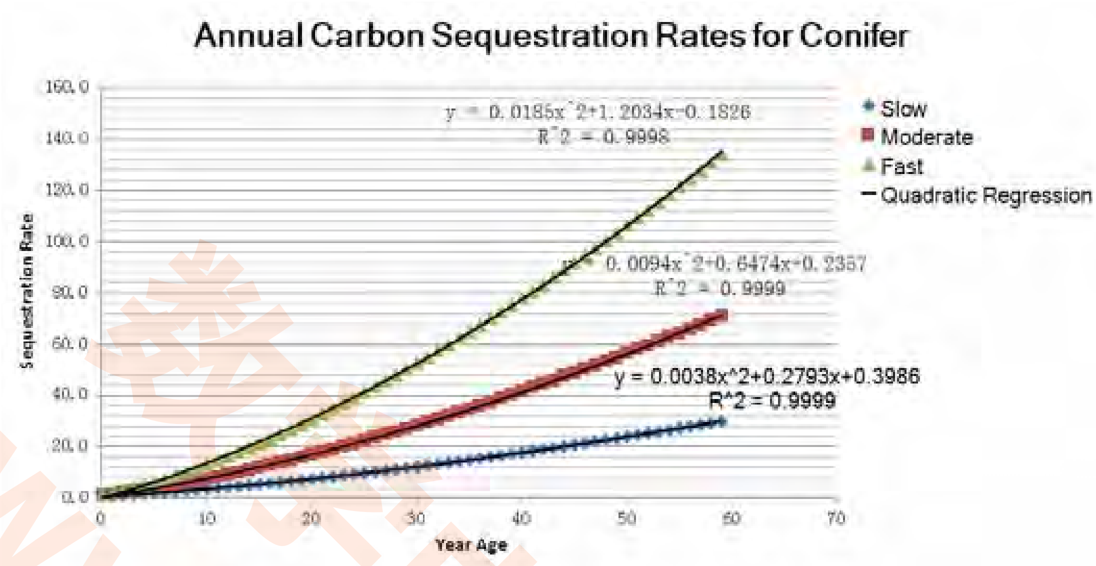

Figure 10. Annual CO_2 sequestration rates, in pounds of carbon per tree per year, for three rates of growth of conifer trees of increasing age.

We were surprised to find that the curves fit the data perfectly! (This fact strongly suggests that the original table values were not measured but calculated from such a model.) From the equations of the fitted curves, we can easily estimate the CO_2 sequestered for a tree of a given age and growth rate and consequently calculate the mass of the leaves.

Tree Age and Tree Size

Above, we used the age of a tree as a link between the two leaf mass and the size characteristics of the tree. Since we now know the relationship between the age of a tree (of a particular growth rate) and its total leaf mass, now we only need to work out the relationship between the age of the tree and the size characteristics of it. Tree size is the accumulation of growth, which is a biological phenomenon of increase with time.

In its life cycle, a tree experiences logistic growth, leading to a model for its "size," or profile, P (height, mass, diameter) as

$$P = k_1 \left(1 - e^{k_2 A}\right)^{k_3}, \qquad \text{hence} \qquad A = k_4 \ln\left(1 - k_5 P^{k_6}\right),$$

where A is the age of the tree and the k_i are constants that depend on the species of tree.

Leaf Mass and Tree Size

Finally, we get to answer the question of whether there is a relationship between leaf mass and tree size characteristics. Putting together our earlier models, we have the relationships in **Figure 11**.

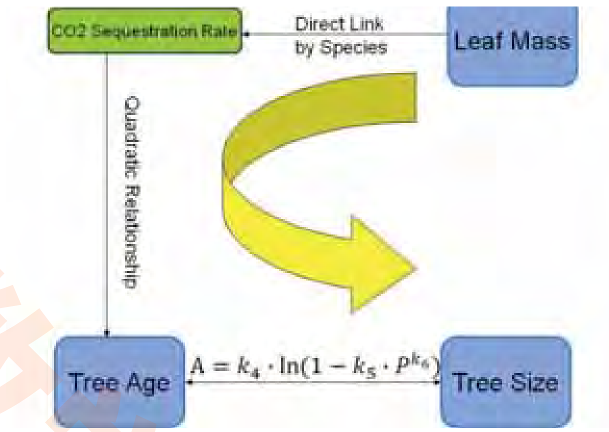

Figure 11.

According to our earlier results, leaf mass and tree age are related to each other through CO_2 sequestration, and we have just determined a function between tree age and tree size.

Strengths and Weaknesses

Model 1

Strengths:

Our model is based on quantitative analysis, so the classification process is both objective and efficient.

Our model is based on categories of leaf types that are the most typical and common.

Weakness:

We divide leaves into only six categories, which may not cover all leaf types.

Model 2

Strengths:

We have taken into consideration three climate conditions (tropical zone, temperate zone, and frigid zone) in discussing the relationship between the leaf distribution and the leaf shape.

The results of our model conform to the data that we found.

Weakness:

We consider the leaf distribution on a single branch but have not considered the inner-influence between different leaves of different branches.

Model 3

Strength:

The whole process uses data and quantitative analysis as foundations, so the output is objective and reasonable.

Weakness:

We have limited categories of tree profiles.

Model 4

Strength:

We use the carbon sequestration rate and age as the media to calculate the total mass of leaves, which is better than trying to estimate the number of leaves and the average weight of each.

Weakness:

The data are from a source that does not refer to the method of arriving at the data.

Letter to a Science Journal Editor

Dear Editor:

We present to you our key findings.

We first focus on the possible influence on leaf shape of the leaf distribution on the tree. For survival reasons, a tree should develop an optimal leaf distribution and shape pattern that adjust to the specific region of its origin, thereby gaining the most nutrients for photosynthesis by maximizing the exposure area to sunshine. We demonstrate a mathematical relationship among solar altitude, leaf shape, and leaf distribution. Based on this finding, we may be able to determine the best location for replanting or assisted-migration of a tree species by observing its leaf distribution.

Our second key finding is a rough relationship between the tree's profile and its leaves. In fact, we hypothesize that a leaf is a two-dimensional mimic of the tree. For several trees, we compared the shape of the leaf and the contour of the tree, finding similarities between certain characteristics.

This finding is another instance of the natural world containing examples of self-similarity, a mathematical concept that means that an object is approximately similar to a part of itself, as is the case for the mathematical objects of the Koch snowflake and the Mandelbrot set.

The third part of our study deals with the relationship between tree size characteristics and the total mass of the leaves. The two are linked by the CO_2 sequestration rate and the age of the tree. Hence, we can estimate the total mass of the leaves given some profile parameters of a tree, such as its height, diameter, volume, age, and type. This finding might have potential for agricultural and environmental uses, such as a new method to estimate tea production or wood production, or estimation of the CO_2 sequestration effect of a forest as an alleviator of global warming.

In hope of publishing our research in your journal, we enclose our research paper for you to examine and judge. We are convinced that our research on leaves promises to contribute to a variety of areas.

Sincerely yours,

Team 14990

Acknowledgment

The authors thank David Knight, James Painter, and Matthew Potter of the Dept. of Electrical Engineering at Stanford University for permission to reproduce photos of leaves from their paper Knight et al. [2010].

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Team members Tiankun Lu, Bo Zhang, and Yi Zhang.

Judges' Commentary: The Outstanding Leaf Problem Papers

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A manager would rather live with a problem he cannot solve than accept a solution he does not understand. —Robert E.D. "Gene" Woolsey [2003]

Introduction

Problem A of the 2012 Mathematical Contest in Modeling (MCM)™ was written by Lee Zia, who posed a challenging problem, "How can you measure the weight of leaves on a tree?" and several equally challenging subproblems. The problems were easy to state, but there were no traditional approaches. Successful teams would have to combine existing models, data, and new ideas in creative and original ways.

The results were gratifying. The judges were impressed by the variety of approaches submitted by the teams. The approaches were creative and the models showed each team's ability to use their own new ideas to refine and extend work that had gone before.

No two of the Outstanding papers shared the same model. Some share parts and data; but those are emphasized, combined, and used in different ways. Existing work, often quickly findable on Google, forms the scaffold on which each team built their own model. The final structures were a pleasure to behold.

Problem Statement

"How much do the leaves on a tree weigh?" How might one estimate the actual weight of the leaves (or for that matter any other parts of the tree)?

The UMAP Journal 33 (3) (2012) 223–229.

How might one classify leaves? Build a mathematical model to describe and classify leaves. Consider and answer the following:

- Why do leaves have the various shapes that they have?
- Do the shapes "minimize" overlapping individual shadows that are cast, so as to maximize exposure? Does the distribution of leaves within the "volume" of the tree and its branches effect the shape?
- Speaking of profiles, is leaf shape (general characteristics) related to tree profile / branching structure?
- How would you estimate the leaf mass of a tree? Is there a correlation between the leaf mass and the size characteristics of the tree (height, mass, volume defined by the profile)?

In addition to your one-page summary sheet, prepare a one-page letter to an editor of a scientific journal outlining your key findings.

Data

For some of the subproblems, such as the leaf classification, real data could be obtained easily. For others, such as the calculation of the mass of the leaves of the tree, it was difficult or impossible to obtain real data. In these latter cases, the teams showed creativity finding and using secondary sources.

Criteria for Judging

Here are some of the issues that kept papers from the final rounds:

- Errors in mathematics, which quickly took them out of further consideration.
- Including mathematics that didn't fit the flow of the presentation. In a few cases, mathematics appears to have been inserted to make a paper look more credible or to take the place of other work that had led to a dead end.
- Changing notation, sometimes even within a single section.
- Using undefined or poorly defined symbols, or using symbols before defining them.
- Incomplete expressions, either because the team made an error or because the expression did not survive the word-processor. (One of the Outstanding papers addressed in this commentary had a few incomplete expressions, probably because they didn't survive the word-processor,

but the coherence of its model and the strength of its presentation overcame that defect.)

Modeling Issues

This problem required two different types of model:

- *Increasing abstraction:* The leaf classification model abstracted from an immense number of natural leaf characteristics a set of artificial ones small enough to be useful for classification.
- *Decreasing abstraction:* The leaf mass problem took abstract models, applied them to data, and got concrete numerical results

Some models were difficult to understand; poor writing was the most common cause. Another cause was the use of inapposite mathematics. If the mathematics was a result of a "drive-by" insertion, fitting it into the model could be difficult.

Here are a few of the modeling issues that hurt some papers' chance of entering the final rounds:

- Questionable, conflicting, or unjustifiably speculative assumptions. Good papers did not assume any spherical cows ("a metaphor for highly simplified scientific models of complex real life phenomena" [Wikipedia 2012]).
- Dependence on *deus ex machina*: an assumption, equation, reference, or procedure invoked without explanation or context. Often the invocation would start with the phrase "It is well-known that..." It may be well-known to those who know it well, but that is unlikely to be the customer or client.
- Confusing, missing, or misplaced model definitions; model definitions are more complex and more important than mathematical ones, since they must not only name the *definiendum* but also specify what it is and what it is to be used for.
- Failure to reach a conclusion.
- Conflicting subproble models with unexplained conflicts between assumptions.
- Unexplained inconsistencies in data.
- An unclear, incomplete, or unrepresentative letter to the journal editor.
- A poor abstract:
  - too much detail, so much that it was difficult to see the overall structure of the model or the strategy for using it; or

  - too little detail, so that it was difficult for the reader to what was actually to be done; or
  - an incomplete abstract, presenting only part of the problem.
- Poor presentation, including bad prose style, poor vocabulary, and disorganized explanations. Good presentation won't get a bad paper into the finals, but poor presentation may keep a good one out. (The weight given to this criterion varies among the judges.)

Letter to a Journal Editor

The one-page letter to a journal editor was an important part of the problem. Its goal was to give insight into whether or not the teams could explain their results clearly, simply, and directly. The most important criterion of modeling is whether or not the models are used, either to increase understanding directly (through use) or indirectly (through publications, conferences, or professional tools such as software). A model that cannot be understood will not be used (see the quotation from Woolsey [2003] at the head of this commentary). A good letter should present an overview of the problem, technique, and results in a single page. The clarity of each team's letter is one indication of how their model might fare in the real world.

The Outstanding Papers

Hong Kong Baptist University

This team's entry was nicely laid out and easy to follow. The tree-classification models appeared to be traceable back to the first principles of physics.

Each model's development began with a clear description of the approach the team intended to follow. For example, in the leaf classification subproblem the approach was to reduce all leaf structures to a one of several polar coordinate functional shapes. These easily can be distinguished.

The team's solution to the problem of finding the mass of leaves on a tree was unique. The team used the structural properties of the tree, not properties of the tree canopy directly. The advantage of this approach is that the team did not need any information about the size or density of the canopy, the properties of individual leaves, or the number or distribution of the leaves. Knowing each branch's modulus of elasticity and its deflection under load provided enough information so that its leaf load could be inferred from the branch's deflection. Conceptually, this solution was much simpler than most of the others. As a practical matter, users of this solution might find difficulty in obtaining some of the data, such as the deflection of an unloaded branch; but if they could, this would an efficient and elegant solution.

The presentation was excellent for all models. The prose, graphics, and equations flowed seamlessly throughout the paper.

The team's letter to the editor was the paper's one weakness. The team employed a very high-level approach, laying out the overall goals for the problem, but without giving insight into the models' operational details.

Shanghai Foreign Language School

This paper had a particularly strong beginning. Within the space of three pages, the team

- reorganized the problem into four consolidated subproblems,
- stated their assumptions clearly and succinctly, and
- provided a table listing their model's parameters and their symbols.

The team's leaf classification model used seven simple measurement procedures involving 10 parameters, the most complicated of which is area. The measurements can be conducted on-site using only a sheet of fine-ruled graph paper. Only one parameter requires calculation: division of the area of a fractional leaf segment by the leaf's entire area. (In times past, this could have been done by eye with a simple nomograph. Now people will stop and key data into calculators.)

As with the team from Hong Kong Baptist University, the model for estimating leaf mass has an unusual approach. The model does not rely on direct measurements of leaf characteristics or tree size. This can be used to show that leaf-mass and tree size are correlated. The challenge in using this model is determination of the rate of sequestration of carbon-dioxide. The model uses sequestration data from a U.S. Department of Energy document.

The last section of the paper contained a clear and well-organized summary list of each problem's strengths and weaknesses.

The team's letter to the editor was clear and concise. It covered the high-level statement of the problem, then gave enough detail of the solution plan that an knowledgeable but non-expert reader could feel conversant with the approach.

National University of Singapore

This team's leaf classification algorithm is the simplest of the four described in this commentary. It has four steps:

- project the leaf onto a grid;
- determine the grid squares covered by the projection to determine if the leaf has convexities:

- If it convex, it is a palmate leaf, exit;
- if it is not convex, then perform further classification.

The leaf mass is calculated based on the team's vector tree model of tree-structure and their insolation model. The vector tree model represents a tree as three-dimensional vectors; daughter branches are obtained by applying a linear transform to the parents.

This entry made excellent use of graphics in presenting their models and results.

This team's letter to the editor successfully wove their research, their results, and their ideas about further research into a single clear narrative.

Zhejiang University

This paper presented a neural-net-based leaf classifier that was most sophisticated of all of the leaf classification schemes. The input layer had 4 nodes, the middle layer 10 nodes, and the output layer had 1 node.

The team divided a sample of leaves into four classes. They trained the network on 32 exemplars of each class, then tested the network on 8 other leaves drawn at random from the entire ensemble.

The network misclassified 1 of the 8. In general, it's impossible to tell how a back-propagation reaches its results; but it's reasonable to hypothesize that more training data might have corrected the one misclassification.

The leaf mass estimation was the most traditional of these four papers. It was based directly on the leaf mass constant, a known value that varies with tree species, and an estimate of the volume of an approximating regular solid.

Summary

These four solutions had strong similarities—importantly, not in the solutions themselves. Models work when they provide understandable bases for reasonable decisions. All four solutions met that criterion and several others:

- They were presented clearly.
  - The descriptive text was clear. There were comparatively few errors in grammar, vocabulary, or style; and these didn't interfere with the reader's understanding.
  - Graphics were appropriate and clear. They supported the argument being made. The appropriate text referred to them.
- The models were appropriate to the problem to be solved, in that
  - the assumptions and goals were clearly stated;

- the physics was correct and appropriate—there were no *dei ex machina* or spherical cows;
- there was no extraneous mathematics air-dropped into the model—the solution was organized in sections; and
- the graphics were easy to find.

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This paper benefitted from insights in the Judges' Commentary by Chris Arney and Kathryn Coronges [2012] in this issue.

About the Author

A graduate of the U.S. Coast Guard Academy, Peter Olsen retired from the Coast Guard Reserve as a Commander in 1997, after 27 years service, active and reserve. His most challenging assignment was to build the quantitative model used to allocate resources for the *Exxon Valdez* oil-spill cleanup. Of the model, Vice Admiral Robbins, the on-scene coordinator, wrote that it was completed on time, it was used by the people who paid for it, and its predictions were borne out by events.

Computing Along the Big Long River

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Abstract

We develop a model to schedule trips down the Big Long River. The goal is to optimally plan boat trips of varying duration and propulsion so as to maximize the number of trips over the six-month season.

We model the process by which groups travel from campsite to campsite. Subject to the given constraints, our algorithm outputs the optimal daily schedule for each group on the river. By studying the algorithm's long-term behavior, we can compute a maximum number of trips, which we define as the river's carrying capacity.

We apply our algorithm to a case study of the Grand Canyon, which has many attributes in common with the Big Long River.

Finally, we examine the carrying capacity's sensitivity to changes in the distribution of propulsion methods, distribution of trip duration, and the number of campsites on the river.

Introduction

We address scheduling recreational trips down the Big Long River so as to maximize the number of trips. From First Launch to Final Exit (225 mi), participants take either an oar-powered rubber raft or a motorized boat. Trips last between 6 and 18 nights, with participants camping at designated campsites along the river. To ensure an authentic wilderness experience, at most one group at a time may occupy a campsite. This constraint limits the number of possible trips during the park's six-month season.

We model the situation and then compare our results to rivers with similar attributes, thus verifying that our approach yields desirable results.

Our model is easily adaptable to find optimal trip schedules for rivers of varying length, numbers of campsites, trip durations, and boat speeds.

The UMAP Journal 33 (3) (2012) 231–246.

Defining the Problem

- How should trips of varying length and propulsion be scheduled to maximize the number of trips possible over a six-month season?
- How many new groups can start a river trip on any given day?
- What is the carrying capacity of the river—the maximum number of groups that can be sent down the river during its six-month season?

Model Overview

We design a model that

- can be applied to real-world rivers with similar attributes (i.e., the Grand Canyon);
- is flexible enough to simulate a wide range of feasible inputs; and
- simulates river-trip scheduling as a function of a distribution of trip lengths (either 6, 12, or 18 days), a varying distribution of propulsion speeds, and a varying number of campsites.

The model predicts the number of trips over a six-month season. It also answers questions about the carrying capacity of the river, advantageous distributions of propulsion speeds and trip lengths, how many groups can start a river trip each day, and how to schedule trips.

Constraints

The problem specifies the following constraints:

- Trips begin at First Launch and end at Final Exit, 225 miles downstream.
- There are only two types of boats: oar-powered rubber rafts and motorized boats.
- Oar-powered rubber rafts travel 4 mph on average.
- Motorized boats travel 8 mph on average.
- Group trips range from 6 to 18 nights.
- Trips are scheduled during a six-month period of the year.
- Campsites are distributed uniformly along the river.
- No two groups can occupy the same campsite at the same time.

Assumptions

- We can prescribe the ratio of oar-powered river rafts to motorized boats that go onto the river each day.

  There can be problems if too many oar-powered boats are launched with short trip lengths.

- The duration of a trip is either 12 days or 18 days for oar-powered rafts, and either 6 days or 12 days for motorized boats.

  This simplification still allows our model to produce meaningful results while letting us compare the effect of varying trip lengths.

- There can only be one group per campsite per night.

  This agrees with the desires of the river manager.

- Each day, a group can only move downstream or remain in its current campsite—it cannot move back upstream.

  This restricts the flow of groups to a single direction, greatly simplifying how we can move groups from campsite to campsite.

- Groups can travel only between 8 A.M. and 6 P.M., a maximum of 9 hours of travel per day (one hour is subtracted for breaks / lunch / etc.).

  This implies that per day, oar-powered rafts can travel at most 36 miles, and motorized boats at most 72 miles. This assumption allows us to determine which groups can reasonably reach a given campsite.

- Groups never travel farther than the distance that they can feasibly travel in a single day: 36 miles per day for oar-powered rafts and 72 miles per day for motorized boats.

- We ignore variables that could influence maximum daily travel distance, such as weather and river conditions.

  There is no way of accurately including these in the model.

- Campsites are distributed uniformly so that the distance between campsites is the length of the river divided by the number of campsites.

  We can thus represent the river as an array of equally-spaced campsites.

- A group must reach the end of the river on the final day of its trip:
  - A group will not leave the river early even if able to.
  - A group will not have a finish date past the desired trip length.

  This assumption fits what we believe is an important standard for the river manager and for the quality of the trips.

Table 1.
Notation.

Symbol	Meaning
g_i	group i
t_i	trip length for group i, measured in nights; $6 \leq t_i \leq 18$
d_i	number of nights group i has spent on the river
Y	number of campsites on the river
c_Y	location of campsite Y in miles downstream; $0 < c_Y < 225$
c_0	campsite representing First Launch (used to construct a waitlist of groups)
c_{final}	campsite (which is always "open") representing Final Exit
l_i	location of group i's current campsite in miles down the river; $0 < l_i < 225$
a_i	average distance that group i should move each day to be on schedule; $a_i = 225/t_i$
m_i	maximum distance that group i can travel in a single day
P_i	priority of group i; $P_i = (d_i/t_i)(l_i/225)$
G_c	set of groups that can reach campsite c
R	ratio of oar-powered rafts to motorized boats launched each day
X	current number of trips down Big Long River each year
M	peak carrying capacity of the river (maximum number of groups that can be sent down the river during its six-month season)
D	distribution of trip durations of groups on the river

Methods

We define some terms and phrases:

Open campsite: A campsite is open if there is no group currently occupying it: Campsite c_n is open if no group g_i is assigned to c_n.

Moving to an open campsite: For a group g_i, its campsite c_n, moving to some other open campsite $c_m \neq c_n$ is equivalent to assigning g_i to the new campsite. Since a group can move only downstream, or remain at their current campsite, we must have $m \geq n$.

Waitlist: The waitlist for a given day is composed of the groups that are not yet on the river but will start their trip on the day when their ranking on the waitlist and their ability to reach a campsite c includes them in the set G_c of groups that can reach campsite c, and the groups are deemed "the highest priority." Waitlisted groups are initialized with a current campsite value of c_0 (the zeroth campsite), and are assumed to have priority $P = 1$ until they are moved from the waitlist onto the river.

Off the River: We consider the first space off of the river to be the "final campsite" c_{final}, and it is always an open campsite (so that any number of groups can be assigned to it. This is consistent with the understanding that any number of groups can move off of the river in a single day.

The Farthest Empty Campsite

Our scheduling algorithm uses an array as the data structure to represent the river, with each element of the array being a campsite. The algorithm begins each day by finding the open campsite c that is farthest down the river, then generates a set G_c of all groups that could potentially reach c that night. Thus,

$$G_c = \{g_i \mid l_i + m_i \geq c\},$$

where l_i is the group's current location and m_i is the maximum distance that the group can travel in one day.

- The requirement that $m_i + l_i \geq c$ specifies that group g_i must be able to reach campsite c in one day.
- G_c can consist of groups on the river and groups on the waitlist.
- If $G_c = \emptyset$, then we move to the next farthest empty campsite—located upstream, closer to the start of the river. The algorithm always runs from the end of the river up towards the start of the river.
- If $G_c \neq \emptyset$, then the algorithm attempts to move the group with the highest priority to campsite c.

The scheduling algorithm continues in this fashion until the farthest empty campsite is the zeroth campsite c_0. At this point, every group that was able to move on the river that day has been moved to a campsite, and we start the algorithm again to simulate the next day.

Priority

Once a set G_c has been formed for a specific campsite c, the algorithm must decide which group to move to that campsite. The *priority* P_i is a measure of how far ahead or behind schedule group g_i is:

- $P_i > 1$: group g_i is behind schedule;
- $P_i < 1$: group g_i is ahead of schedule;
- $P_i = 1$: group g_i is precisely on schedule.

We attempt to move the group with the highest priority into c.

Some examples of situations that arise, and how priority is used to resolve them, are outlined in **Figures 1** and **2**.

Priorities and Other Considerations

Our algorithm always tries to move the group that is the most behind schedule, to try to ensure that each group is camped on the river for a

Downstream ⟶

Campsite	1	2	3	4	5	6
Group Priority	A $P_A = 1.1$	B $\mathbf{P_B = 1.5}$	C $P_C = 0.8$	Open	Open	Farthest open campsite

Figure 1. The scheduling algorithm has found that the farthest open campsite is Campsite 6 and Groups A, B, and C can feasibly reach it. Group B has the highest priority, so we move Group B to Campsite 6.

Downstream ⟶

Campsite	1	2	3	4	5	6
Group Priority	A $\mathbf{P_A = 1.1}$	Open	C $P_C = 0.8$	Open	Farthest open campsite	B

Figure 2. As the scheduling algorithm progresses past Campsite 6, it finds that the next farthest open campsite is Campsite 5. The algorithm has calculated that Groups A and C can feasibly reach it; since $P_A > P_C$, Group A is moved to Campsite 5.

number of nights equal to its predetermined trip length. However, in some instances it may not be ideal to move the group with highest priority to the farthest feasible open campsite. Such is the case if the group with the highest priority is *ahead* of schedule ($P < 1$).

We provide the following rules for handling group priorities:

- If g_i is *behind* schedule, i.e. $P_i > 1$, then move g_i to c, its farthest reachable open campsite.
- If g_i is *ahead* of schedule, i.e. $P_i < 1$, then calculate $d_i a_i$, the number of nights that the group has already been on the river times the average distance per day that the group should travel to be on schedule. If the result is greater than or equal (in miles) to the location of campsite c, then move g_i to c. Doing so amounts to moving g_i only in such a way that it is no longer ahead of schedule.
- Regardless of P_i, if the chosen $c = c_{\text{final}}$, then do not move g_i unless $t_i = d_i$. This feature ensures that g_i's trip will not end before its designated end date.

The one case where a group's priority is disregarded is shown in **Figure 3**.

Scheduling Simulation

We now demonstrate how our model could be used to schedule river trips.

In the following example, we assume 50 campsites along the 225-mile river, and we introduce 4 groups to the river each day. We project the trip

Downstream ⟶

Campsite	170	171	...	223	224	Off
Group Priority	D $\mathbf{P_D = 1.1}$ $t_D = 12$ $d_D = 11$	Open	Open	Open	Open	Farthest open campsite

Figure 3. The farthest open campsite is the campsite off the river. The algorithm finds that Group D could move there, but Group D has $t_D > d_D$—that is, Group D is supposed to be on the river for 12 nights but so far has spent only 11—so Group D remains on the river, at some campsite between 171 and 224 inclusive.

schedules of the four specific groups that we introduce to the river on day 25. We choose a midseason day to demonstrate our model's stability over time. The characteristics of the four groups are:

- g_1: motorized, $t_1 = 6$;
- g_2: oar-powered, $t_2 = 18$;
- g_3: motorized, $t_3 = 12$;
- g_4: oar-powered, $t_4 = 12$.

Figure 5 shows each group's campsite number and priority value for each night spent on the river. For instance, the column labeled g_2 gives campsite numbers for each of the nights of g_2's trip. We find that each g_i is off the river after spending exactly t_i nights camping, and that $P \to 1$ as $d_i \to t_i$, showing that as time passes our algorithm attempts to get (and keep) groups on schedule. **Figures 6** and **7** display our results graphically. These findings are consistent with the intention of our method; we see in this small-scale simulation that our algorithm produces desirable results.

Case Study

The Grand Canyon

The Grand Canyon is an ideal case study for our model, since it shares many characteristics with the Big Long River. The Canyon's primary river rafting stretch is 226 miles, it has 235 campsites, and it is open approximately six months of the year. It allows tourists to travel by motorized boat or by oar-powered river raft for a maximum of 12 or 18 days, respectively [Jalbert et al. 2006].

Using the parameters of the Grand Canyon, we test our model by running a number of simulations. We alter the number of groups placed on the water each day, attempting to find the carrying capacity for the river—the

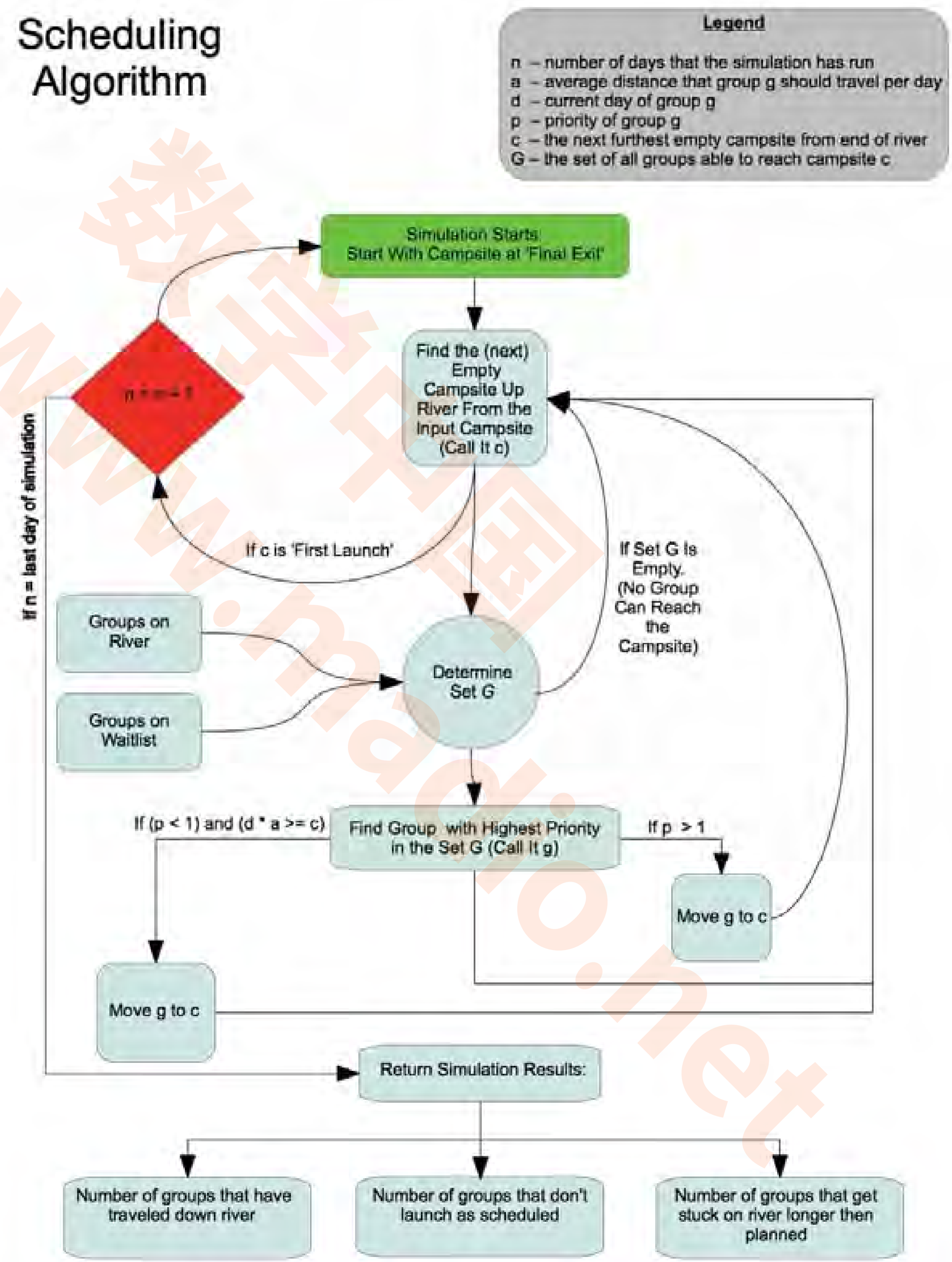

Figure 4. Visual depiction of scheduling algorithm.

Campsite numbers and priority values for each group

Number of nights spent on river	g_1	P_1	g_2	P_2	g_3	P_3	g_4	P_4
1	5	1.43	2	1.32	3	1.28	1	3.85
2	20	0.71	6	0.87	19	0.4	8	0.96
3	20	1.07	6	1.32	19	0.61	8	1.44
4	36	0.79	13	0.81	19	0.81	15	1.02
5	36	0.99	13	1.01	19	1.01	15	1.28
6	49	0.87	13	1.21	35	0.66	23	1
7	OFF	1	13	1.42	35	0.77	25	1.08
8			13	1.66	35	0.88	32	0.96
9			15	1.58	35	0.99	32	1.08
11			23	1.14	35	1.1	39	0.99
12			30	0.96	49	0.86	39	1.08
13			30	1.05	49	0.94	46	1
14			30	1.14	OFF	1	OFF	1
15			36	1.02				
16			36	1.1				
17			44	0.96				
18			44	1.02				
19			44	1.01				
20			OFF	1				

Figure 5. Schedule for example of groups launched on Day 25.

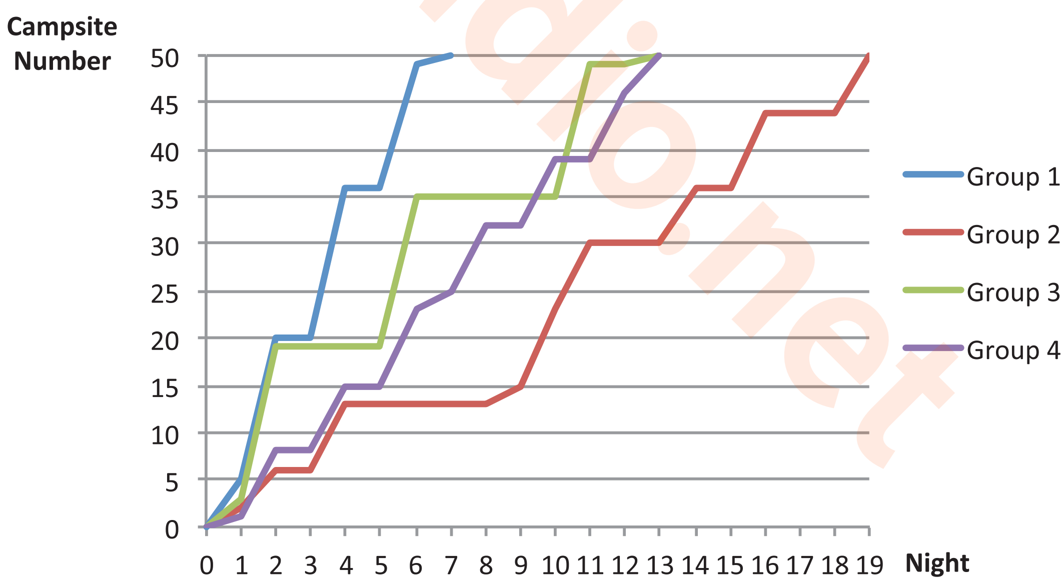

Figure 6. Movement of groups down the river based on **Figure 5**. Groups reach the end of the river on different nights due to varying trip-duration parameters.

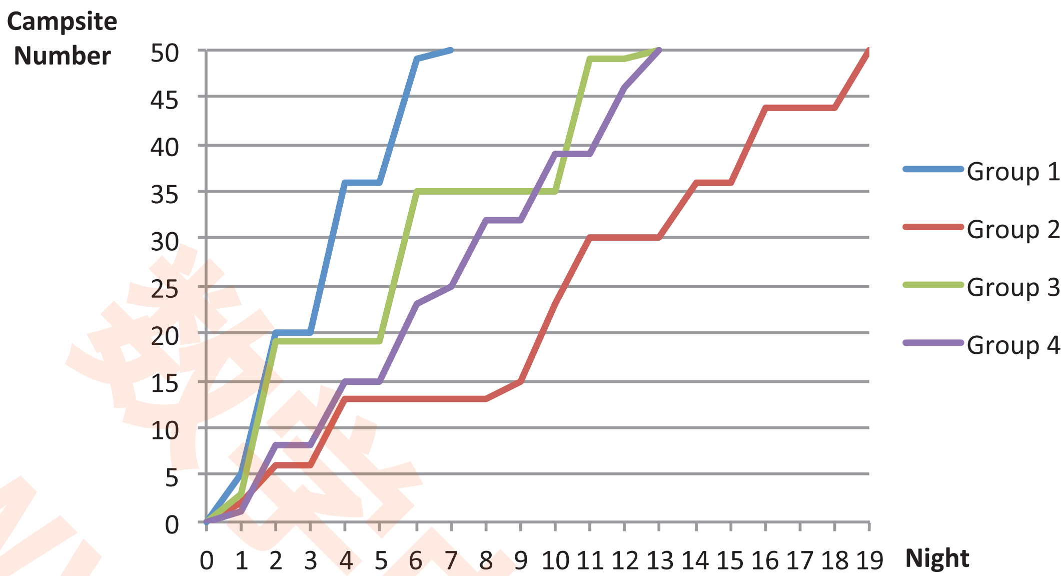

Figure 7. Priority values of groups over the course of each trip. Values converge to $P = 1$ due to the algorithm's attempt to keep groups on schedule.

maximum number of possible trips over a six-month season. The main constraint is that each trip must last the group's planned trip duration. During its summer season, the Grand Canyon typically places six new groups on the water each day [Jalbert et al. 2006], so we use this value for our first simulation. In each simulation, we use an equal number of motorized boats and oar-powered rafts, along with an equal distribution of trip lengths.

Our model predicts the number of groups that make it off the river (completed trips), how many trips arrive past their desired end date (late trips), and the number of groups that did not make it off the waitlist (total left on waitlist). These values change as we vary the number of new groups placed on the water each day (groups / day).

Table 2.
Results of simulations for the number of groups to launch each day.

Simulation	Groups / day	Trips Completed	Late	Left on waitlist
1	6	996		
2	8	1328		
3	10	1660		
4	12	1992		
5	14	2324		
6	16	2656		
7	17	2834		
8	18	2988		
9	19	3154	5	
10	20	3248	10	43
11	21	3306	14	109

Table 1 indicates that a maximum of 18 groups can be sent down the river each day. Over the course of the six-month season, this amounts to nearly 3,000 trips. Increasing groups / day above 18 is likely to cause late trips (some groups are still on the river when our simulation ends) and long waitlists. In Simulation 1, we send 1,080 groups down river (6 groups / day $\times$ 180 days) but only 996 groups make it off; the other groups began near the end of the six-month period and did not reach the end of their trip before the end of the season. These groups have negligible impact on our results and we ignore them.

Sensitivity Analysis of Carrying Capacity

Managers of the Big Long River are faced with a similar task to that of the managers of the Grand Canyon. Therefore, by finding an optimal solution for the Grand Canyon, we may also have found an optimal solution for the Big Long River. However, this optimal solution is based on two key assumptions:

- Each day, we put approximately the same number of groups onto the river; and
- the river has about one campsite per mile.

We can make these assumptions for the Grand Canyon because they are true for the Grand Canyon, but we do not know if they are true for the Big Long River.

To deal with these unknowns, we create **Table 3**. Its values are generated by fixing the number Y of campsites on the river and the ratio R of oar-powered rafts to motorized boats launched each day, and then increasing the number of trips added to the river each day until the river reaches peak carrying capacity.

Table 3.

Capacity of the river as a function of the number of campsites and the ratio of oarboats to motorboats.

		Number of campsites on the river				
		100	150	200	250	300
	1:4	1360	1688	2362	3036	3724
Ratio	1:2	1181	1676	2514	3178	3854
oar : motor	1:1	1169	1837	2505	3173	3984
	2:1	1157	1658	2320	2988	3604
	4:1	990	1652	2308	2803	3402

The peak carrying capacities in **Table 3** can be visualized as points in a three-dimensional space, and we can find a best-fit surface that passes (nearly) through the data points. This best-fit surface allows us to estimate the peak carrying capacity M of the river for interpolated values. Essentially, it gives M as a function of Y and R and shows how sensitive M is to changes in Y and / or R. **Figure 7** is a contour diagram of this surface.

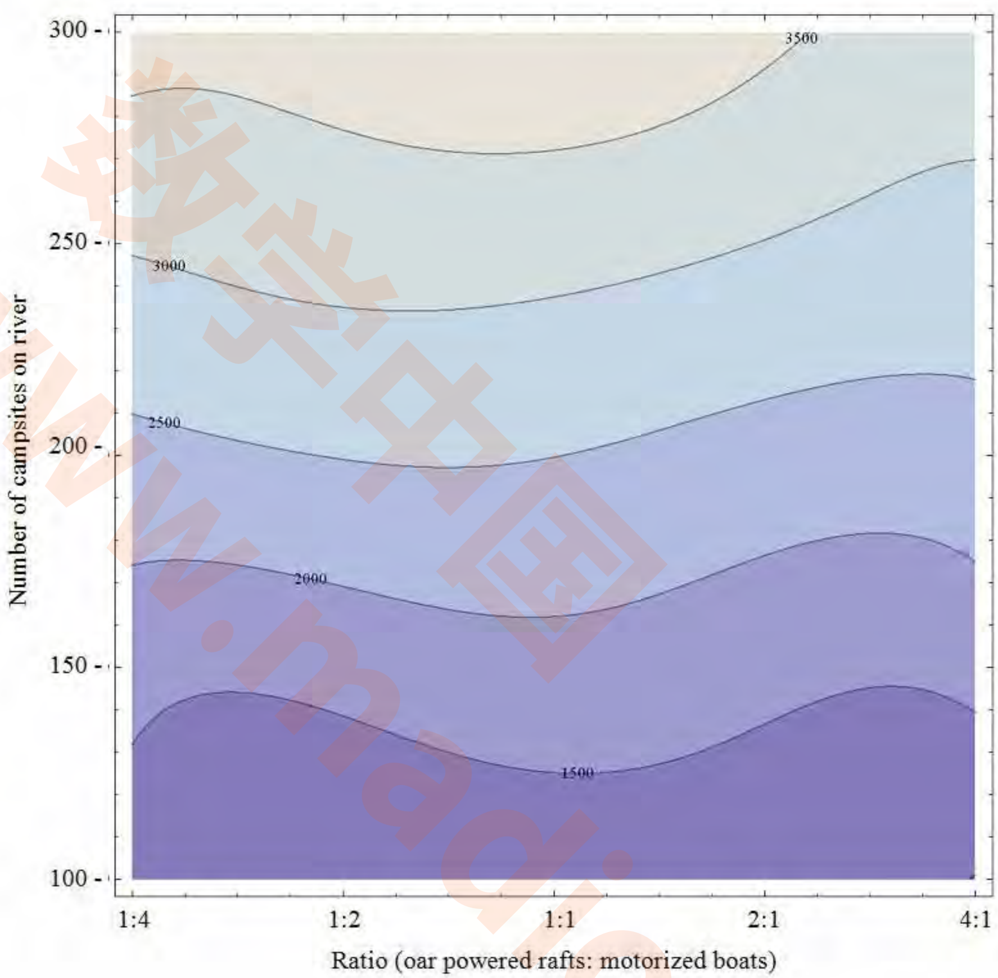

Figure 7. Contour diagram of the best-fit surface to the points of **Table 3**.

The ridge along the vertical line $R = 1 : 1$ predicts that for any given value of Y between 100 and 300, the river will have an optimal value of M when $R = 1 : 1$. Unfortunately, the formula for this best-fit surface is rather complex, and it doesn't do an accurate job of extrapolating beyond the data of **Table 3**; so it is not a particularly useful tool for the peak carrying capacity for other values of R. The best method to predict the peak carrying capacity is just to use our scheduling algorithm.

Sensitivity Analysis of Carrying Capacity re R and D

We have treated M as a function of R and Y, but it is still unknown to us how M is affected by the mix of trip durations of groups on the river (D).

For example, if we scheduled trips of either 6 or 12 days, how would this affect M? The river managers want to know what mix of trips of varying duration and speed will utilize the river in the best way possible.

We use our scheduling algorithm to attempt to answer this question. We fix the number of campsites at 200 and determine the peak carrying capacity for values of R and D. The results of this simulation are displayed in **Table 4**.

Table 4.
Carrying capacity of the river by trip lengths and boat type.

		Distribution of trip lengths			
		12 only	12 or 18	6 or 12	6, 12, or 18
Ratio oar : motor	1:4	2004	1998	2541	2362
	1:2	2171	1992	2535	2514
	1:1	2171	1986	2362	2505
	2:1	1837	2147	2847	2320
	4:1	2505	2141	2851	2308

Table 4 is intended to address the question of what mix of trip durations and speeds will yield a maximum carrying capacity. For example: If the river managers are currently scheduling trips of length

- 6, 12, or 18: Capacity could be increased either by increasing R to be closer to 1:1 or by decreasing D to be closer to "6 or 12."
- 12 or 18: Decrease D to be closer to "6 or 12."
- 6 or 12: Increase R to be closer to 4:1.

Conclusion

The river managers have asked how many more trips can be added to the Big Long River's season. Without knowing the specifics of how the river is currently being managed, we cannot give an exact answer. However, by applying our model to a study of the Grand Canyon, we found results which could be extrapolated to the context of the Big Long River. Specifically, the managers of the Big Long River could add approximately $(3{,}000 - X)$ groups to the rafting season, where X is the current number of trips and 3,000 is the capacity predicted by our scheduling algorithm.

Additionally, we modeled how certain variables are related to each other; M, D, R, and Y. River managers could refer to our figures and tables to see how they could change their current values of D, R, and Y to achieve a greater carrying capacity for the Big Long River.

We also addressed scheduling campsite placement for groups moving down the Big Long River through an algorithm which uses priority values to move groups downstream in an orderly manner.

Limitations and Error Analysis

Carrying Capacity Overestimation

Our model has several limitations. It assumes that the capacity of the river is constrained only by the number of campsites, the trip durations, and the transportation methods. We maximize the river's carrying capacity, even if this means that nearly every campsite is occupied each night. This may not be ideal, potentially leading to congestion or environmental degradation of the river. Because of this, our model may overestimate the maximum number of trips possible over long periods of time.

Environmental Concerns

Our case study of the Grand Canyon is evidence that our model omits variables. We are confident that the Grand Canyon could provide enough campsites for 3,000 trips over a six-month period, as predicted by our algorithm. However, since the actual figure is around 1,000 trips [Jalbert et al. 2006], the error is likely due to factors outside of campsite capacity, perhaps environmental concerns.

Neglect of River Speed

Another variable that our model ignores is the speed of the river. River speed increases with the depth and slope of the river channel, making our assumption of constant maximum daily travel distance impossible [Wikipedia 2012]. When a river experiences high flow, river speeds can double, and entire campsites can end up under water [National Park Service 2008]. Again, the results of our model don't reflect these issues.

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Memo to Managers of the Big Long River

In response to your questions regarding trip scheduling and river capacity, we are writing to inform you of our findings.

Our primary accomplishment is the development of a scheduling algorithm. If implemented at Big Long River, it could advise park rangers on how to optimally schedule trips of varying length and propulsion. The optimal schedule will maximize the number of trips possible over the six-month season.

Our algorithm is flexible, taking a variety of different inputs. These include the number and availability of campsites, and parameters associated with each tour group. Given the necessary inputs, we can output a daily schedule. In essence, our algorithm does this by using the state of the river from the previous day. Schedules consist of campsite assignments for each group on the river, as well those waiting to begin their trip. Given knowledge of future waitlists, our algorithm can output schedules months in advance, allowing management to schedule the precise campsite location of any group on any future date.

Sparing you the mathematical details, allow us to say simply that our algorithm uses a priority system. It prioritizes groups who are behind schedule by allowing them to move to further campsites, and holds back groups who are ahead of schedule. In this way, it ensures that all trips will be completed in precisely the length of time the passenger had planned for.

But scheduling is only part of what our algorithm can do. It can also compute a maximum number of possible trips over the six-month season. We call this the carrying capacity of the river. If we find we are below our carrying capacity, our algorithm can tell us how many more groups we could be adding to the water each day. Conversely, if we are experiencing river congestion, we can determine how many fewer groups we should be adding each day to get things running smoothly again.

An interesting finding of our algorithm is how the ratio of oar-powered river rafts to motorized boats affects the number of trips we can send downstream. When dealing with an even distribution of trip durations (from 6 to 18 days), we recommend a 1:1 ratio to maximize the river's carrying capacity. If the distribution is skewed towards shorter trip durations, then our model predicts that increasing towards a 4:1 ratio will cause the carrying capacity to increase. If the distribution is skewed the opposite way, towards longer trip durations, then the carrying capacity of the river will always be less than in the previous two cases—so this is not recommended.

Our algorithm has been thoroughly tested, and we believe that it is a powerful tool for determining the river's carrying capacity, optimizing daily schedules, and ensuring that people will be able to complete their trip as planned while enjoying a true wilderness experience.

Sincerely yours,

Team 13955




Team members Chip Jackson, Lucas Bourne, and Travis Peters, and team advisor Edoh Amiran.

Judges' Commentary: The Outstanding River Problem Papers

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Problem Overview and General Remarks

This year's problem dealt with scheduling variable-length river trips down a 225-mile stretch of a particular river, using either oar-powered rubber rafts (at 4 mph) or motor boats (at 8 mph). A fixed starting point and a fixed ending point were specified for all trips, with campsites distributed fairly uniformly down the river corridor. Minimal contact between groups of visitors was desired, and no two groups could share the same campsite. The goal was to maximize the number of trips over a six-month period, utilizing both types of transportation and allowing for trip lengths of 6 to 18 nights on the river. In addition to the executive summary, teams were required to write a memo to the managers of the river trips, advising them on the optimal scheduling of trips of various lengths over the six-month period, and taking the carrying capacity of the river into account.

The teams' approaches varied greatly, especially regarding the number of campsites available—a factor that had a significant impact on the number of trips that could be scheduled. Many teams found that the "Big Long River" greatly resembled a stretch of the Colorado River in the Grand Canyon, and some used that as a case study for their models. Simulations are available for scheduling trips on that river, but teams had to address all of the issues raised in the problem statement and come up with a solution that demonstrated their own creativity. The judges looked for that and for carefully-explained mathematical model-building with sensitivity analysis that went beyond what is found in the literature.

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Executive Summary and Memo

The executive summary is of critical importance, especially in early judging. It should

- motivate the reader;
- be polished, with a good synopsis of key results;
- give an overview of the model(s) used, together with the rationale for using such a model and the primary results obtained from that model; and
- state specific results obtained for the optimal solution.

Teams were also asked to write a memo appropriate for the manager of the Big River boat tours. Whereas the executive summary usually contains technical details, this memo was intended for a nontechnical person who wanted to know how best to schedule trips. Hence, the memo was supposed to give specifics on how to schedule trips of various types to best accommodate as many groups as possible. Vague generalizations were of little or no value.

Documentation

In comparison with previous contests, the judges were pleased to observe a noticeable improvement in how references were identified and in the specific precision of their documentation. Considering the online resources available, proper documentation was an especially important factor in this year's problem. Despite the improvement, many papers contained charts and graphs from Web sources with no documentation.

All graphs and tables should

- have labels and / or legends;
- provide information about what is discussed in the paper;
- be "called out" in the text of the paper, so as to refer the reader to them; and
- be explained in the text, including their significance.

The best papers used graphs to help clarify their results, and those papers also documented trustworthy resources whenever used.

Assumptions

Teams made many assumptions about travel along the river. Some were appropriate and played integral roles in the models used; others were superfluous. Some teams assumed that there would always be enough customers to fill any trips scheduled; other teams used probability distributions to describe the demand for different trips at different times of the season. Either approach could be used, but each led to different results. The carrying capacity of the river was dependent on the number of campsites available and the types of trips to be scheduled.

Since this is a modeling contest, much weight is put on whether or not the model could be used (with modification) in the real world. Therefore, assumptions required for simplification could not be totally unrealistic. Also, clear writing and exposition is essential to motivate and explain assumptions, and to derive and test models based on those assumptions.

The Model(s)

One can arrive at a fairly complete solution to this problem with pencil and paper alone. Problem solvers should at least consider this possibility before launching a simulation! Some teams began with a simple model, then improved it to accommodate the requirements better. Teams should be aware that it is not the quantity of models considered that is important, but rather the quality of the model selected and its applicability to the case at hand.

At a minimum, the solutions should try to come up with a mix of trips that seem reasonable. Most teams recognized that for a 225-mile river, a motor boat could run the entire distance in 6 to 8 nights, whereas a raft powered by oars would need 12 to 18 nights. While it is true that requiring only the shortest trip lengths would permit the most boats to get down the river, it was important to consider that not all groups would choose to travel that way.

Some teams considered a profit incentive when scheduling trips of varying duration on the river and used selected numbers from the Grand Canyon boat trips as a guide. For example, the cost of the trip might be a constant fixed cost plus an amount based on the number of nights on the river. In that case, shorter trips might allow more boats to launch and be optimal in terms of profit. Or perhaps it would be more valuable to people to get more time in this pristine wilderness, so they would pay a premium for the longer trips—in which case it might be worth sending fewer boats down the river. Many teams ignored cost / profit issues.

Teams assigned campsites so as to ensure that no two sets of campers occupied the same site at the same time. At the end of each night, the teams had to be sure that all crafts camped in reasonable locations and that the model did not require a boat to travel too far in any one day. Many teams measured the percentage of campsites occupied each night as a help in determining an optimal number of campsites and how good the solution was from a manager's perspective.

In addition to having no two groups at the same campsite, minimum contact between groups also implies minimizing crafts passing one another on the river. Teams that took this into account showed true diligence. Some teams even measured the average number of such contacts. Although neglecting this aspect was not a fatal flaw, proper consideration of the crossings gave the model added value.

Testing the Model—Simulations and Sensitivity Analysis

MCM teams are getting better at carrying out simulations, and this technique was of great value for the Big River problem. However, to carry out a simulation properly, criteria had to be specified for scheduling trips of varying length. A good flowchart with examples was very powerful in clarifying how a simulation was to be carried out. Some teams used a well-defined prioritization scheme that assured that no two groups stayed at the same campsite on any given night and rejected assignments that violated that criterion.

Sensitivity analysis was an essential ingredient. The better papers considered how their solution was impacted by changing the number of campsites and by changing the types of trips. This included varying the ratio of motor boats to oar-powered rafts and varying the ratio of trip durations. The graphical demonstration of the results of such sensitivity analysis was a powerful way to communicate the outcomes and to check for patterns of optimality.

Although sensitivity analysis could have included issues associated with boating accidents, inclement weather, and flash floods, most papers only alluded to such possibilities. Few teams considered anything but constant speeds for the river flow and the boats. Some teams considered extending the hours of travel.

Strengths and Weaknesses

A strong paper must assess its strengths and its weaknesses. One of the greatest strengths of any model is how well it reflects the real world situation. Hence, using a case study to validate a model is a powerful means to make that case. Most papers recognized the limitations of their models in failing to consider weather, river, and individual camper issues. A strong solution might mention among weaknesses that assigning campsites is something of a limitation, because an accident that prevents a boat from reaching its assigned campsite could mess up the model. A more realistic model would say that a given boat will go at most—rather than exactly—n

miles per day; and a flexible model would ensure that a boat could find an open campsite if it didn't make it to its goal campsite.

Concluding Remarks

Mathematical modeling is an art. It is an art that requires considerable skill and practice in order to develop proficiency. The big problems that we face now and in the future will be solved in large part by those with the talent, the insight, and the will to model these real-world problems and continuously refine those models. The judges are very proud of all participants in this Mathematical Contest in Modeling and we commend you for your hard work and dedication.

About the Author

Marie Vanisko is a Mathematics Professor Emerita from Carroll College in Helena, Montana, where she taught for more than 30 years. She was also a Visiting Professor at the U.S. Military Academy at West Point and taught for five years at California State University, Stanislaus. She chairs the Board of Directors at the Montana Learning Center on Canyon Ferry Lake and serves on the Engineering Advisory Board at Carroll College. She has been a judge for the MCM for 17 years and for the HiMCM for eight years.

Author's Commentary: The Outstanding River Problem Papers

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This MCM problem was inspired by a research project for the Grand Canyon National Park in Arizona, U.S.A. My collaborators and I developed a mathematical model to simulate white-water rafting traffic along the 225-mile Colorado River corridor within the national park. The National Park Service manages access to the river, guided by a document called the Colorado River Management Plan (CRMP). This research program began with efforts to revise the 1989 CRMP in the late 1990s. Our model was used as a tool by river managers at the National Park Service to explore options for scheduling rafting traffic.

At the time, every year (almost entirely over the summer months) more than 19,000 people rafted the river on trips guided by 16 licensed commercial companies, while approximately 3,500 private boaters paddled themselves down the river. Demand for access to the river far exceeded supply—a waiting list for a private river trip pass had over 7,000 names on it, and a quarter of those people had already waited over a dozen years.

The hope was that this mathematical model would provide insight into alternative management scenarios so that park managers could make smart decisions that would enable as many visitors as possible to enjoy the river, while at the same time maintaining standards for a wilderness experience.

Some simplifications were built into the MCM Problem, compared to the actual situation on the Colorado River.

- The campsites on the Colorado River are not distributed evenly throughout the river corridor. Indeed, there's a big congestion problem in a reach of the river with few campsites and many popular attraction sites. Some campsites are not suitable for motorized boats.

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- It is permissible to have more than one group camping at the same site, although the Colorado River Management Plan dictates that the schedule should minimize any camping within sight or sound of another party.

- There are two fixed-points on the river corridor—places where passengers are exchanged via hiking in-and-out of the canyon or traveling via helicopter. A trip with an exchange must make it to their site at a predetermined date and time. Otherwise, there are no assigned campsites—it's really impossible to assign a rafting trip a specified set of campsite locations because so much (flash floods, boat spills, accidents, health problems) can interfere with a party's ability to reach a certain location at a fixed time. Moreover, the river culture is such that assigned campsites would be anathema.

The model uses a Geographical Information System (GIS) to divide the river into 90-meter cells. We assigned each cell specific attributes (campsite, lunch spot, dangerous rapid, hiking trail, waterfall, etc.). We used hundreds of trip diaries and personal interviews with river guides to determine appropriate weights for the popularity of camping and attraction sites along the river corridor. Trip diaries also helped us estimate the average rate of travel of motor and oar boats through various reaches of the river (when the river corridor narrows, the water's velocity increases and so boats travel through faster). The model captures the complex dynamics of human visitors interacting with the environment and each other. It is both temporal and spatial as it carefully tracks every move that every trip makes.

Our model, titled the Grand Canyon River Trip Simulator (GCRTSim), was programmed in VisualBasic. A user can build any imaginable launch schedule and "run" the season down the virtual river. The results are then analyzed and judged against criteria established by the Park Service.

Our model leveraged a number of mathematical theories and ideas.

- *Intelligent agent theory*: Each trip has an assigned "personality" and makes all of its decisions consistent with that personality to optimize each day. Thus, a short commercial trip would be less likely to choose a long hike when it needs more time just to paddle down the river. Each trip is an intelligent agent operating within a complex system.

- *Decision theory*: Each trip makes decisions based on a fixed set of choices (e.g., to stop to camp or to continue to the next campsite). The model measures the utility gained from each choice and seeks to maximize the total utility for each trip (e.g., best campsites, key attractions, low crowds).

- *Game theory*: Strategic behavior and bargaining rules come into play as each trip seeks to influence the decisions of other trips. For example, can one trip claim a downstream campsite earlier in the day by communicating its desire with the other trips that it encounters?

- Essentially, the GCRTSim model boils down to a *constrained optimization* problem where the success of the entire season depends on individual decisions made by all of the trips, and the outcome depends on the combined strategies. For the National Park Service to manage the Grand Canyon rafting season successfully, the sum of all the individual decisions over the course of the entire season contributes to an overall utility that must be maximized.

The GCRTSim model suggested that the best solution was to expand the rafting season into the shoulder months in the spring and fall. The new CRMP was authorized in 2006, and the new approach to scheduling river trips has been in place since 2008. The number of private launches was dramatically increased without lowering the commercial use. The waiting list was converted to a lottery system that appears to be in favor with the private boaters. Yet, even with more trips being sent down the river each year, the overall crowding at any particular moment was reduced because the trips were spread out over additional months. The number of trips on the river at any one time was reduced from a high of 70 to a high of 60, so the perception of visitors is that the river is less crowded now than it used to be. It is also quieter, since the number of months in which motorized rafts and helicopter exchanges are allowed have been cut in half. A rafter going through the Grand Canyon National Park on the Colorado River will enjoy a genuine wilderness experience.

Photo Credit: Catherine A. Roberts.

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Judges' Commentary: The Giordano Award for the River Problem

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Introduction

For the first time in its history, the MCM is designating a paper with the Frank Giordano Award. This designation goes to a paper that demonstrates a very good example of the modeling process in a problem involving discrete mathematics.

Having worked on the contest since its inception, Frank Giordano served as Contest Director for 20 years. As Frank says:

> It was my pleasure to work with talented and dedicated professionals to provide opportunities for students to realize their mathematical creativity and whet their appetites to learn additional mathematics. The enormous amount of positive feedback I have received from participants and faculty over the years indicates that the contest has made a huge impact on the lives of students and faculty, and also has had an impact on the mathematics curriculum and supporting laboratories worldwide. Thanks to all who have made this a rewarding and pleasant experience!

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The Frank Giordano Award for 2012 went to the Outstanding team from Western Washington University (WWU) in Bellingham, WA. This solution paper was characterized by

- a high-quality application of the complete modeling process, including assumptions with clear justifications, a well-defined simulation, a case study, and sensitivity analysis;
- originality and creativity in the modeling effort to solve the problem as given; and
- clear and concise writing, making it a pleasure to read.

The River Problem

This year's problem dealt with scheduling variable-length river trips down a 225-mile stretch of a particular river, using either oar-powered rubber rafts (at 4 mph) or motor boats (at 8 mph). Fixed starting and ending points were specified for all trips, with campsites distributed fairly uniformly down the river corridor. Minimal contact between groups of visitors was desired and no two groups could share the same campsite. The goal was to maximize the number of trips over a six-month period utilizing both types of transportation and allowing for trip lengths of 6 to 18 nights on the river. In addition to the executive summary, teams were required to write a memo to the managers of the river trips, advising them of the carrying capacity of the river and how to schedule trips of various lengths over the six-month period.

The approaches that teams took varied greatly, especially with regard to the number of campsites available. That factor had a significant impact on the number of trips that could be scheduled. Many teams found that the "Big Long River" greatly resembled a stretch of the Colorado River in the Grand Canyon, and some looked at this as a case study for their models.

Simulations for scheduling trips on the Colorado River were available, but teams had to address all the issues raised and come up with a solution that demonstrated their own creativity

The Western Washington University Paper

Executive Summary Sheet and Memo

Although well written, this team's one-page sheet at the start was an abstract rather than a one-page executive summary. Typically, an executive summary contains more information (and often more sensitive information) than the abstract does. This team's one-page summary was too short and did not state results, but, to the team's credit, it did motivate the reader to read on.

Although it should have contained more specifics with regard to the scheduling, the team's memo, written in an appropriate nontechnical manner, was done much better.

Assumptions

One of the first things that made this paper stand out from the others was that assumptions were not merely listed but each one was justified. Assumptions were reasonable, and it was noted how the assumptions were to be used in the algorithm. This is something that is most important in the modeling process, but something that is frequently overlooked, so the team is to be commended for their thoroughness in this regard.

The Model and Methods

The team used a scheduling algorithm. The variables were well-defined; and it was clear how they arrived at their constraints, based on the stipulations stated in the problem. This was one of the few papers that allowed for groups to stay at a camp for more than one night, but that worked well for their algorithm and did not conflict with the problem statement. Using a very specific definition for the priority that one group would have over another group, the team was able to assign campsites in a successful manner. Interestingly, they started at the end of the river; and using the priority list, they placed the groups in campsites each night. One drawback with their model was that they did not consider crossings of groups while on the river.

Testing Their Models

The flowchart for the team's scheduling algorithm was clarified by the use of examples and simulations. The case study, using data from the Grand Canyon, enabled them to validate their model. They considered many different numbers of campsites, ranging from 50 to 235. With regard to the ratio of the types of boats and lengths of trips, they carried out sensitivity analysis, although they limited their trip lengths to 6, 12, or 18 nights on the river. The use of contour maps to demonstrate their results and to observe the "ridge" representing the 1:1 ratio of motor boats to oar-powered rafts was particularly noteworthy.

Recognizing Limitations of the Model

Recognizing the limitations of a model is an important last step in the completion of the modeling process. The students recognized that their algorithm would have to be modified if the river speed were taken into account. They also acknowledged that their carrying capacity for trips might be overestimated and that they had not considered environmental concerns.

References and Bibliography

The list of references was fairly thorough, and it was very good to see specific documentation of where those references were used in the paper.

Conclusion

The careful exposition in the development of the mathematical model made this paper one that the judges felt was worthy of the Frank Giordano Award. The team is to be congratulated on their analysis, their clarity, and using the mathematics that they knew to create and justify their own creative model for scheduling camping trips along the Big Long River.

About the Authors

Rich West is a Mathematics Professor Emeritus from Francis Marion University in Florence, South Carolina, where he taught for twelve years. Prior to his time at Francis Marion, he served in the U.S. Army for 30 years, 14 of which were spent teaching at the U.S. Military Academy. He is currently working on a National Science Foundation Grant on freshmen placement tests. He also serves as a Reading Consultant for AP Calculus and as a developmental editor for CLEP (College Level Equivalency Program) Calculus Exam. He has judged for both the MCM and HiMCM for over 10 years.

Marie Vanisko is a Mathematics Professor Emerita from Carroll College in Helena, Montana, where she taught for more than 30 years. She was also a Visiting Professor at the U.S. Military Academy at West Point and taught for five years at California State University, Stanislaus. She chairs the Board of Directors at the Montana Learning Center on Canyon Ferry Lake and serves on the Engineering Advisory Board at Carroll College. She has been a judge for the MCM for seventeen years and for the HiMCM for eight years.

ICM Modeling Forum

Results of the 2012 Interdisciplinary Contest in Modeling

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Introduction

In the 14th Interdisciplinary Contest in Modeling (ICM)®, 1,329 teams from six countries spent a weekend in February working on an applied modeling problem involving a criminal network. This year's contest began on Thursday, February 9, and ended on Monday, February 14, 2012. During that time, teams of up to three undergraduate or high school students researched, modeled, analyzed, solved, wrote, and submitted their solutions to an open-ended interdisciplinary modeling problem involving a criminal conspiracy network. After the weekend of challenging and productive work, the solution papers were sent to COMAP for judging. Seven of the papers were judged to be Outstanding by the expert panel of judges.

COMAP's Interdisciplinary Contest in Modeling (ICM) involves students working in teams to model and analyze an open interdisciplinary problem. Centering its educational philosophy on mathematical modeling, COMAP supports the use of mathematical tools to explore real-world problems. It serves society by developing students as problem solvers in order to become better informed and prepared as citizens, contributors, consumers, workers, and community leaders. The ICM is an example of COMAPs efforts in working towards these goals.

This year's problem was challenging in its demand for teams to utilize many aspects of science, mathematics, and analysis in their modeling and problem solving. The problem required teams to investigate the relationships of the

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members of a criminal conspiracy network within a business organization through social network analysis of their message traffic. It required teams to understand concepts from the informational and social sciences to build effective network and statistical models to analyze more than 400 messages, categorized into 15 topics, among 83 people. To accomplish their tasks, the students had to consider many difficult and complex disciplinary and interdisciplinary issues. The problem also included the customary requirements in the ICM to perform thorough analysis and research, exhibit creativity, and demonstrate effective communication. All members of the 1,329 competing teams are to be congratulated for their excellent work and dedication to interdisciplinary modeling and problem solving.

Instructions to the teams included:

- Your ICM submission should consist of a 1-page Summary Sheet and your solution cannot exceed 20 pages for a maximum of 21 pages.
- As modelers, you have to deal with the data you have and through valid assumptions decide what to do with holes, irregularities, redundancies, and errors.

Next year, we will continue the network science theme for the contest problem. Teams preparing for the 2013 contest should consider reviewing interdisciplinary topics in the areas of network science and social network analysis and assemble teams accordingly.

The Problem Statement: The Crime-Busting Problem

Your organization, the Intergalactic Crime Modelers (ICM), is investigating a conspiracy to commit a criminal act. The investigators are highly confident they know several members of the conspiracy, but hope to identify the other members and the leaders before they make arrests. The conspirators and the possible suspected conspirators all work for the same company in a large office complex. The company has been growing fast and making a name for itself in developing and marketing computer software for banks and credit card companies. ICM has recently found a small set of messages from a group of 82 workers in the company that they believe will help them find the most likely candidates for the unidentified co-conspirators and unknown leaders. Since the message traffic is for all the office workers in the company, it is very likely that some (maybe many) of the identified communicators in the message traffic are not involved in the conspiracy. In fact, they are certain that they know some people who are not in the conspiracy. The goal of the modeling effort will be to identify people in the office complex who are the most likely conspirators. A priority list would be ideal so ICM could investigate, place under surveillance, and/or interrogate the most likely candidates. A discriminate line separating conspirators from non-conspirators would also be helpful to distinctly categorize the people in each group. It would also be helpful to the district attorney (DA) if the model nominated the conspiracy leaders.

Before the data of the current case are given to your crime modeling team, your supervisor gives you the following scenario (called Investigation EZ) that she worked on a few years ago in another city. Even though she is very proud of her work on the EZ case, she says it is just a very small, simple example, but it may help you understand your task. Her data follow:

The 10 people whom she was considering as conspirators were: Anne#, Bob, Carol, Dave*, Ellen, Fred, George*, Harry, Inez, and Jaye# (* indicates prior known conspirators, # indicate prior known non-conspirators).

Here is the chronology of the 28 messages that she had for her case, with a code number for the topic of each message that she assigned based on her analysis of the message:

Anne to Bob: Why were you late today? (1)

Bob to Carol: That darn Anne always watches me. I wasn't late. (1)

Carol to Dave: Anne and Bob are fighting again over Bob's tardiness. (1)

Dave to Ellen: I need to see you this morning. When can you come by? Bring the budget files. (2)

Dave to Fred: I can come by and see you anytime today. Let me know when it is a good time. Should I bring the budget files? (2)

Dave to George: I will see you later — lots to talk about. I hope the others are ready. It is important to get this right. (3)

Harry to George: You seem stressed. What is going on? Our budget will be fine. (2) (4)

Inez to George: I am real tired today. How are you doing? (5)

Jaye to Inez: Not much going on today. Wanna go to lunch today? (5)

Inez to Jaye: Good thing it is quiet. I am exhausted. Can't do lunch today — sorry! (5)

George to Dave: Time to talk — now! (3)

Jaye to Anne: Can you go to lunch today? (5)

Dave to George: I can't. On my way to see Fred. (3)

George to Dave: Get here after that. (3)

Anne to Carol: Who is supposed to watch Bob? He is goofing off all the time. (1)

Carol to Anne: Leave him alone. He is working well with George and Dave. (1)

George to Dave: This is important. Darn Fred. How about Ellen? (3)

Ellen to George: Have you talked with Dave? (3)

George to Ellen: Not yet. Did you? (3)

Bob to Anne: I wasn't late. And just so you know — I am working through lunch. (1)

Bob to Dave: Tell them I wasn't late. You know me. (1)

Ellen to Carol: Get with Anne and figure out the budget meeting schedule for next week and help me calm George. (2)

Harry to Dave: Did you notice that George is stressed out again today? (4)

Dave to George: Darn Harry thinks you are stressed. Don't get him worried or he will be nosing around. (4)

George to Harry: Just working late and having problems at home. I will be fine. (4)

Ellen to Harry: Would it be OK, if I miss the meeting today? Fred will be there and he knows the budget better than I do. (2)

Harry to Fred: I think next year's budget is stressing out a few people. Maybe we should take time to reassure people today. (2) (4)

Fred to Harry: I think our budget is pretty healthy. I don't see anything to stress over. (2)

END of MESSAGE TRAFFIC

Your supervisor points out that she assigned and coded only 5 different topics of messages:

- 1) Bob's tardiness,
- 2) the budget,
- 3) important unknown issue but assumed to be part of conspiracy,
- 4) George's stress, and
- 5) lunch and other social issues.

As seen in the message coding, some messages had two topics assigned because of the content of the messages.

The way that your supervisor analyzed her situation was with a network that showed the communication links and the types of messages. **Figure 1** is a model of the message network that resulted, with the code for the types of messages annotated on the network graph.

Your supervisor points out that in addition to known conspirators George and Dave, Ellen and Carol were indicted for the conspiracy based on your supervisor's analysis, and later Bob self-admitted his involvement in a plea bargain for a reduced sentence, but the charges against Carol were later dropped. Your supervisor is still pretty sure that Inez was involved, but the case against her was never established. Your supervisor's advice to your team is identify the guilty parties so that people like Inez don't get off, people like Carol are not falsely accused, and ICM gets the credit so people like Bob do not have the opportunity to get reduced sentences.

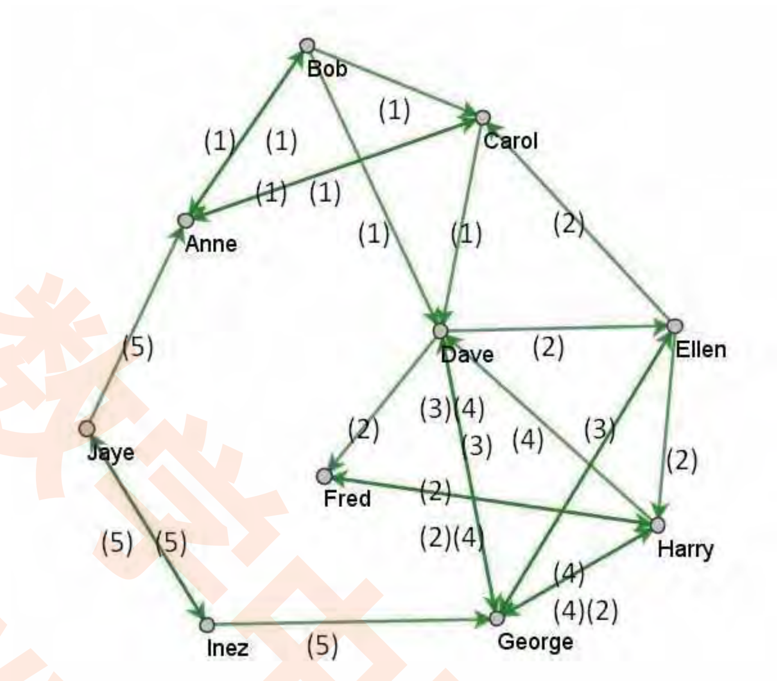

Figure 1. Network of messages from EZ Case.

The Current Case

Your supervisor has put together a network-like database for the current case, which has the same structure but is a bit larger in scope. The investigators have some indications that a conspiracy is taking place to embezzle funds from the company and use Internet fraud to steal funds from credit cards of people who do business with the company. The small example that she showed you for case EZ had only 10 people (nodes), 27 links (messages), 5 topics, 1 suspicious / conspiracy topic, 2 known conspirators, and 2 known non-conspirators.

So far, the new case has 83 nodes, 400 links (some involving more than 1 topic), over 21,000 words of message traffic, 15 topics (3 have been deemed to be suspicious), 7 known conspirators, and 8 known non-conspirators. These data are given in the attached spreadsheet files: `Names.xls`, `Topics.xls`, and `Messages.xls`[1]:

- `Names.xls` contains the key of node number to the office workers' names.
- `Topics.xls` contains the code for the 15 topic numbers to a short description of the topics. Because of security and privacy issues, your team will not have direct transcripts of all the message traffic.
- `Messages.xls` provides the links of the nodes that transmitted messages and the topic code numbers that the messages contained. Several messages contained up to three topics.

[1]These files were available to contestants at `http://www.comap.com/undergraduate/contests/mcm/contests/2012/problems/2012_ICM.zip`.

To help visualize the message traffic, a network model of the people and message links is provided in **Figure 2**. In this case, the topics of the messages are not shown in the figure as they were in **Figure 1**. These topic numbers are given in the file `Messages.xls` and described in `Topics.xls`.

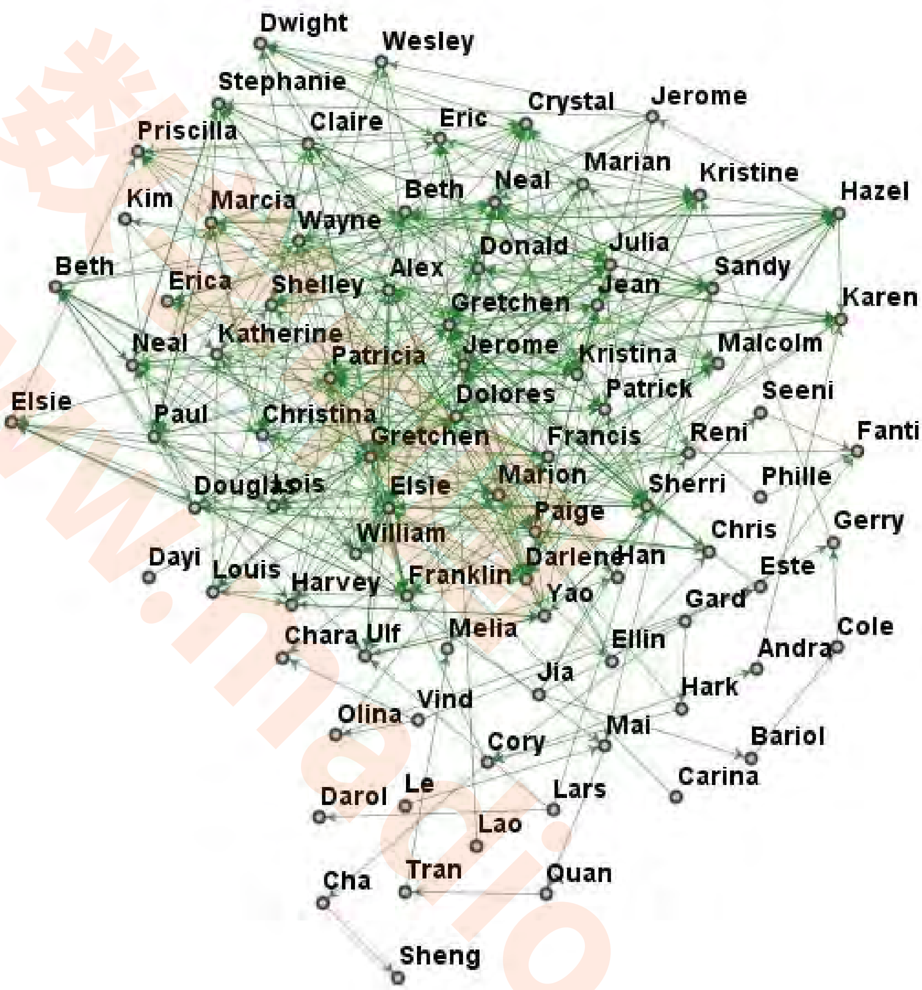

Figure 2. Visual of the network model of the 83 people (nodes) and 400 messages between these people (links).

Requirements:

Requirement 1: So far, it is known that Jean, Alex, Elsie, Paul, Ulf, Yao, and Harvey are conspirators. Also, it is known that Darlene, Tran, Jia, Ellin, Gard, Chris, Paige, and Este are not conspirators. The three known suspicious message topics are 7, 11, and 13. There is more detail about the topics in file Topics.xls. Build a model and algorithm to prioritize the 83 nodes by likelihood of being part of the conspiracy and explain your model and metrics. Jerome, Delores, and Gretchen are the senior managers of the company. It would be very helpful to know if any of them are involved in the conspiracy.

Requirement 2: How would the priority list change if new information comes to light that Topic 1 is also connected to the conspiracy and that Chris is one of the conspirators?

Requirement 3: A powerful technique to obtain and understand text information similar to this message traffic is called semantic network analysis where as a methodology in artificial intelligence and computational linguistics it provides a structure and process for reasoning about knowledge or language. Another computational linguistics capability in natural language processing is text analysis. For our crime busting scenario, explain how semantic and text analyses of the content and context of the message traffic, if you could obtain the original messages, could empower your team to develop even better models and categorizations of the office personnel. Did you use any of these capabilities on the topic descriptions in file Topics.xls to enhance your model?

Requirement 4: Your complete report will eventually go to the DA, so it must be detailed and clearly state your assumptions and methodology; but it cannot exceed 20 pages of write-up. You may include your programs as appendices in separate files that do not count in your page restriction, but including these programs is not necessary. Your supervisor wants ICM to be the world's best in solving white-collar, high-tech conspiracy crimes and wants your methodology will contribute to solving important cases around the world, especially those with very large databases of message traffic (thousands of people with tens of thousands of messages and possibly millions of words). She specifically asked you to include a discussion on how more thorough network, semantic, and text analyses of the message contents could help with your model and recommendations. As part of your report to her, explain the network modeling techniques you have used and why and how they can be used to identify, prioritize, and categorize similar nodes in a network database of any type, not just crime conspiracies and message data. For instance, could your method find the infected or diseased cells in a biological network where you had various kinds of image or chemical data for the nodes indicating infection probabilities and already identified some infected nodes?

The Results

The 1,329 solution papers were coded at COMAP headquarters so that names and affiliations of the authors were unknown to the judges. Each paper was then read preliminarily by triage judges at the U.S. Military Academy at West Point, NY. At the triage stage, the summary, the model description, and overall organization are the primary elements in judging a paper. Final judging by a team of modelers, analysts, and subject-matter experts took place in late March. The judges classified the 1,329 submitted papers as follows:

	Outstanding	Finalist	Meritorious	Honorable Mention	Successful Participant	Total
Crime-Busting	7	4	125	640	553	1,329

Outstanding Teams

Institution and Advisor	Team Members
"Social Network Analysis in Crime Busting"	
Northwesteren Polytechnical University	Chen Dong
Xi'an, China	Cunle Qian
Bingchang Zhou	Jianjun Ma
"Message Network Modeling for Crime Busting"	
Nanjing Univ. of Information Science and Technology	Yizhou Zhuang
Nanjing, Jiangsu, China	Senfeng Liu
Guosheng Cheng	Liusi Xiao
"Crime Busting by an Iterative Two-Phase Propagation Method"	
Shanghai Jiaotong University	Xilun Chen
Shanghai, China	Hang Qiu
Zulin Sun	Chunzhi Yang
"Finding Conspirators in the Network: Machine Learning with Resource-Allocation Dynamics"	
Univ. of Electronic Science and Technology of China	Fangjian Guo
Chengdu, Sichuan, China	Jiang Su
Tao Zhou	Jian Gao
"iRank Model: A New Approach to Criminal Network Detection"	
Mathematical Modeling Innovative Practice Base,	
Zhuhai College of Jinan University	Yi Zheng
Zhuhai, Guangdong, China	Yi Zeng
Jianwen Xie	You Tian
"Extended Criminal Network Analysis Model Allows Conspirators Nowhere to Hide"	
Huazhong University of Science and Technology	Dekang Zhu
Wuhan, Hebei, China	Junmin Yang
Zhengyang Mei	Xiang Chen
"Crime Ring Analysis with Electric Networks"	
Cornell University	Michael Luo
Ithaca, NY	Anirvan Mukherjee
Alexander Vladimirsky	Myron Zhang

Awards and Contributions

Each participating ICM advisor and team member received a certificate signed by the Contest Director. Additional awards were presented to the team from Cornell University by the Institute for Operations Research and the Management Sciences (INFORMS).

Judging

Contest Directors
Chris Arney, Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY
Joseph Myers, Computing Sciences Division, Army Research Office, Research Triangle Park, NC

Associate Director
Rodney Sturdivant, Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY

Judges
Dimitris Christopoulos, University of the West of England, Bristol, United Kingdom
Kathryn Coronges, Dept. of Behavioral Sciences and Leadership, U.S. Military Academy, West Point, NY
Kayla de la Haye, RAND Corporation, Santa Monica, CA
Tina Hartley, Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY
Brian Macdonald, Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY
Christopher Marcum, RAND Corporation, Santa Monica, CA
Robert Ulman, Network Sciences Division, Army Research Office, Research Triangle Park, NC

Triage Judges

Chris Arney, John Bacon, Jocelyn Bell, Kevin Blaine, Nicholas Clark, Gabe Costa, Michelle Craddock, Kevin Cummiskey, Chris Eastburg, Michael Findlay, James Gatewood, Andy Glen, Tina Hartley, Alex Heidenberg, Steven Horton, Nicholas Howard, John Jackson, Bill Kaczynski, Phil LaCasse, Bill Pulleyblank, Elizabeth Russell, Mick Smith, James Starling, Rodney Sturdivant, Andrew Swedberg, Csilla Szabo, Ben Thirey, Johan Thiel, Chris Weld, and Shaw Yoshitani.

—all of Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY; and

Joseph Myers, Army Research Office, Research Triangle Park, NC
Michelle Isenhour, George Mason University, VA
Hise Gibson and Chris Farrell, U.S. Army; and
Amanda Beecher, Dept. of Mathematics, Ramapo College of New Jersey, Mahwah, NJ.

Acknowledgments

We thank:

- the Institute for Operations Research and the Management Sciences (INFORMS) for its support in judging and providing prizes for a winning team, and
- all the ICM judges for their valuable and unflagging efforts.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the team papers here is the result of undergraduates working on a problem over a weekend. Editing (and usually substantial cutting) has taken place; minor errors have been corrected, wording has been altered for clarity or economy, and style has been adjusted to that of *The UMAP Journal*. The student authors have proofed the results. Please peruse these students' efforts in that context.

To the potential ICM advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

Editor's Note

The complete roster of participating teams and results has become too long to reproduce in the *Journal*. It can now be found at the COMAP Website:

`http://www.comap.com/undergraduate/contests/mcm/contests/2012/results/2012_ICM_Results.pdf`

About the Author

Chris Arney graduated from West Point and served as an intelligence officer in the U.S. Army. His academic studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his 30-year military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris then moved to RTP (Research Triangle Park), NC, where he served for various durations as chair of the Mathematical Sciences Division, of the Network Sciences Division, and of the Information Sciences Directorate of the Army Research Office. Chris has authored 22 books, written more than 120 technical articles, and given more than 250 presentations and 40 workshops. His technical interests include mathematical modeling, cooperative systems, pursuit-evasion modeling, robotics, artificial intelligence, military operations modeling, and network science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula. He is the founding director of COMAP's Interdisciplinary Contest in Modeling (ICM)®. In August 2009, he rejoined the faculty at West Point as the Network Science Chair and Professor of Mathematics.

Finding Conspirators in the Network via Machine Learning

Fangjian Guo
Jiang Su
Jian Gao

Web Sciences Center
University of Electronic Science and Technology of China
Chengdu, Sichuan, China

Advisor: Tao Zhou

KEY CONCEPTS

Machine learning

Logistic regression

Semantic diffusion

Bipartite graph

Resource-allocation dynamics

Kendall's tau

Problem Clarification: A conspiracy network is embedded in a network of employees of a company, with each edge representing a message sent from one employee (node) to another and categorized by topics. Given a few known criminals, a few known non-criminals, and suspicious topics, we seek to estimate the probability of criminal involvement for other individuals and to determine the leader of the conspirators.

Assumptions

- Conspirators and non-conspirators are linearly separable in the space spanned by local features (necessary for machine learning).
- A conspirator is reluctant to mention to an outsider topics related to crime.
- Conspirators tend not to talk frequently with each other about irrelevant topics.
- The leader of the conspiracy tries to minimize risk by restricting direct contacts.
- A non-conspirator has no idea of who are conspirators, hence treats conspirators and non-conspirators equally.

The UMAP Journal 33 (3) (2012) 275–292.

KEY TECHNIQUES

Gradient Descent

Revised LeaderRank

Model Design and Justification For an unidentified node (an employee not identified as a conspirator or non-conspirator), we model the probability of conspiracy as a sigmoid function of a linear combination of the node's features (logistic regression). Those features are formulated from local topological measures and the node's semantic messaging patterns. Parameters of the model are trained on a subset of identified conspirators and non-conspirators. The performance of the model is enhanced by discovering potential similarities among topics via topic-word diffusion dynamics on a bipartite graph. We also perform resource-allocation dynamics to identify the leader of the conspirators; the identification is supported by empirical evidence in criminal network research.

Strengths and Weaknesses The combination of topological properties and semantic affinity among individuals leads to good performance. The time complexity of the algorithm is linear, so the method is suitable for large amounts of data. However, our model requires assistance from semantic network analysis to form an expert dictionary. Also, intrinsic differences among networks may hinder portability of the model's features.

Introduction

As shown in **Figure 1**, criminals and conspirators tend to form organizational patterns, interconnected with one another for collaboration, while still maintaining social ties with the outside, thus providing a natural context for description and analysis via networks [Baker and Faulkner 1993].

Criminal networks can be captured from various information, resulting in different types of networks, where each node represents a person, and an edge is present when two nodes collaborate in the same task, share the same family name, or (as in this case) exchange messages [Krebs 2002].

Since the nodes in the graph can be a mixture of both criminals and non-criminals, it is desirable to determine suspected criminals from topological properties of the network and other prior knowledge, which includes known criminals, known non-criminals, and information related to their interactions. Moreover, we desire a priority list of descending criminal likelihood so as to identify the primary leader of the organization.

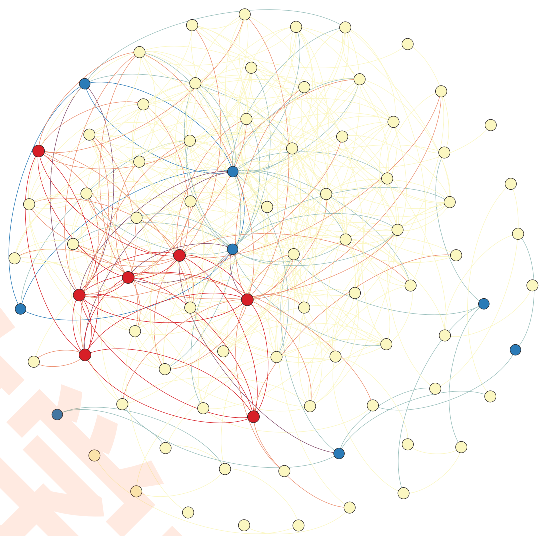

Figure 1. The 83-employee network. Red (darker gray) nodes are known conspirators and the blue (lighter gray) nodes are known non-conspirators.

Many authors have adopted centrality measures of the graph for analyzing the characteristics of criminals. Criminals with high betweenness-centrality are usually brokers, while those with high degree-centrality enjoy better profit by taking higher risk [Krebs 2002]. Morselli [2010] proposed that leaders of a criminal organization tend to balance profit and risk by making a careful trade-off between degree-centrality and betweenness-centrality.

However, centrality approaches, which utilize local properties, tend to overlook the complex topology of the whole network. Therefore, social network analysis (SNA) methods, including subgroup detection and blockmodeling, have been introduced, which try to discover the hidden topological patterns by partitioning the big network into small closely-connected cliques [Xu and Chen 2005]. Despite the light that they shed on the internal structures of criminal networks, these methods still suffer from intimidating complexity with large databases [Wheat 2007].

We carefully combine the local-feature-based methods with approaches related to global topology of conspiracy networks. We propose a machine learning scheme to leverage local features, so as to estimate each node's likelihood of conspiracy involvement. We adopt dynamics-based methods, which are less computationally expensive than most other topology-based approaches, to help identify the lead conspirator and to discover semantic connections between topics.

We start with the formulation of useful local features of a node in the network, which then lead to the machine learning scheme. We feed a subset of known conspirators and non-conspirators as a training samples into the learning algorithm. We then use the algorithm to estimate the probability of being a conspirator for every other individual in the network.

Since highly suspicious topics are essential to the performance of machine learning, we then try to discover similarities between topics, by performing simple source-allocation dynamics on the bipartite semantic network made up of topics and sensitive words. Those findings expand our knowledge on suspicious topics, thus enhancing the accuracy of our machine learning model.

To find the leader of the conspirators, we apply a dynamics-based ranking algorithm on a subgraph extracted from the network. Our findings are in agreement with empirical knowledge about the centrality balance of criminal leaders.

Finally, we perform sensitivity analysis to test the robustness of our approach.

A Machine Learning Solution

We use machine learning mainly because of its adaptiveness and reorganization, which simulate humans' actions to obtain fresh knowledge.

We describe the construction of our machine learning framework in detail, including feature formulation, core learning methods, and experimental results. Through statistical analysis on the results, we propose an enhancement based on semantic diffusion.

We commence with several necessary assumptions:

- All the data and information about the EZ case network and the 83-node network are relatively stable over a long period.
- The contents of the communication among conspirators tends to be relevant about suspicious topics or some formal issues, rather than gossip.
- The two networks feature similar core mechanisms for communication transmission.

Feature formulation

- **Centrality**

  We exploit three types of centrality—degree centrality, betweenness centrality, and closeness centrality—to determine the center of the suspicious network from different aspects:

  - ▶ *Degree centrality*. Degree centrality [Freeman 1979] indicates activeness of a member, and a member who tends to have more links to others may be the leader. However, as explained in Xu and Chen [2003], degree centrality is not quite reliable to indicate the team leader in a criminal network. For a graph $G(V, E)$, the normalized degree centrality of node i is

$$C_D(i) = \frac{\sum_{j=1}^{|V|} \nu(i,j)}{|V|-1}, \qquad i \neq j, \tag{1}$$

where ν is a binary indicator showing whether there exists a link between two nodes. Since our graph is directed, we calculate separately the in-degree and out-degree of every node.

- *Betweenness centrality.* Betweenness centrality [Freeman 1979] describes how much a node tends to be on the shortest path between other nodes. A node with large betweenness centrality does not necessarily have large degree but illustrates the role of "gatekeeper"—someone who is more likely to be a intermediary when two other members exchange information. The normalized betweenness centrality is

$$C_B(i) = \frac{\sum_{j=1}^{|V|} \sum_{k<j}^{|V|} \omega_{j,k}(i)}{|V|-1}, \qquad k \neq i, \tag{2}$$

where $\omega_{j,k}(i)$ indicates whether the shortest path between node j and node k passes through node i.

- *Closeness centrality.* Closeness centrality [Sabidussi 1966] is usually utilized to measure how far away one node is from the others. Closeness of a node is defined as the inverse of the sum of its distances to all other nodes and can be treated as a measure of efficiency when spreading information from itself to all other nodes sequentially. It indicates how easily an individual connects with other members. The normalized closeness centrality is

$$C_c(i) = \frac{\sum_{j=1}^{|V|} \rho(i,j) - C_{c\,\min}}{C_{c\,\max} - C_{c\,\min}}, \qquad i \neq j, \tag{3}$$

where $\rho(i,j)$ is the length of the shortest path connecting nodes i and j. $C_{c\,\min}$ and $C_{c\,\max}$ are the minimum and maximum lengths of the shortest paths.f

- **Number of known neighboring conspirators**
We consider as a significant feature the number of known neighboring conspirators of a node. The interaction among conspirators in a message network suggests a much stronger connectivity than that among non-conspirators: A conspirator is more likely to communicate with an accomplice. As shown in **Figure 2**, we calculate the ratio of known conspirators among one's adjacent neighbors, which measures proximity with known accomplices: The value is 1 if the individual connects with all the known conspirators, and 0 means that no conspirators connect to the individual. The known suspicious clique obviously represents a more compact connectivity. Therefore, the more known conspirators among

an individual's neighbors, the greater the possibility that the individual is an accomplice.

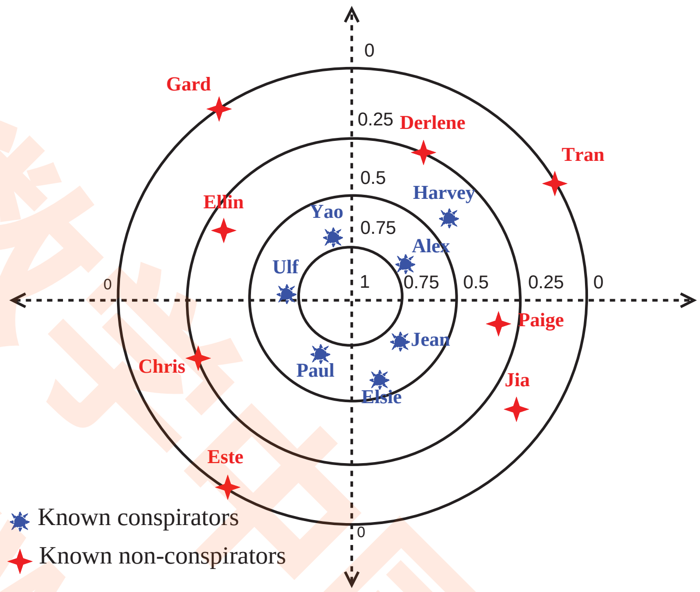

Figure 2. Ratio of known conspirators among adjacent neighbors. To avoid the overlapping of names with a linear scale, we adopt a topographic map type of diagram, with a peak at the center and symmetric contour circles around it. The closer a person is to the center, the more likely that the person is a conspirator.

- **Number of current non-suspicious messages from known conspirators**
  Table 1 shows the topics mentioned between known conspirators.[1] A known conspirator rarely talks with accomplices about topics irrelevant to their conspiracy, though a very small proportion of unknown topics appear. If most of the information received from a known conspirator is irrelevant, the receiver is probably not a conspirator.

Table 1.
Topics among known conspirators. Known conspiratorial topics have an asterisk and are highlighted in blue (light gray).

	Jean	Alex	Elsie	Poul	Ulf	Yao	Harvey
Jean		11*			8		14
Alex			1	13*	11*	3, 7*	
Elsie		11*			13*		
Poul	11*		7*		7*		4
Ulf		7*, 11*, 13*				13*	
Yao	13*	7*, 11*, 13*	7*, 9		13*		2, 7*
Harvey						13*	

[1] Topic 16 in the raw data is regarded as wrong and thus discarded.

Methods

We use logistic regression to model the probability of a node being involved in the conspiracy. We obtain the parameters of the model by using a gradient descent algorithm to solve an optimization problem on a training set.

Logistic Regression

We consider a training set $\{(x^{(i)}, y^{(i)})\}$ of size m, where $x^{(i)}$ is an n-dimensional feature vector and $y^{(i)}$ indicates the classification of the node, i.e., $y^{(i)} = 1$ for conspirators and $y^{(i)} = 0$ for non-conspirators. The nodes in the training set are drawn from the 15 known conspirators and non-conspirators.

As a specialization of a generalized linear model for Bernoulli distribution, logistic regression estimates the probability of being a conspirator as

$$P(y = 1|x; \theta) = h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}, \tag{4}$$

where $\theta \in \mathbb{R}^n$ is the parameter vector.

Under the framework of the generalized linear model, the *maximum a posteriori* (MAP) estimate of the parameter θ is

$$\min_\theta J(\theta), \tag{5}$$

where the cost function is given by

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^{(i)} \log\left(h_\theta(x)^{(i)}\right) - \left(1 - y^{(i)}\right)\log\left(1 - h_\theta(x^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2, \tag{6}$$

with λ being a regularization parameter.

Gradient Descent

The cost function $J(\theta)$ is minimized by gradient descent, which drives θ down the locally steepest slope, in hope of reaching the global minimum of the cost function.

At every iteration before convergence, a new θ replaces the old θ via

$$\theta := \theta - \alpha\nabla_\theta J(\theta), \tag{7}$$

where α is a small positive constant.

Leave-One-Out Cross Validation

Since we are informed of the correct classification of only N_0 nodes ($N_0 = 15$ in our case), in a given round we only use $(N_0 - 1)$ of them as the training set, while leaving one out for cross validation (C-V). At every round, the next correctly classified node is left out and the others serve as the training set; then the trained hypothesis is tested on the left-out node. In this way, by averaging N_0 rounds without overlapping, the errors for both the training set and the cross validation set can be evaluated.

Suppose, for example, that in the j-th round sample $(x^{(j)}, y^{(j)})$ is left out and the training set is given by

$$S_j = \{(x^{(l)}, y^{(l)}) \mid l = 1, 2, \cdots, j-1, j+1, \cdots, N_0\}. \qquad \textbf{(8)}$$

Using this training set, parameter vector $\theta^{(j)}$ is obtained, and the corresponding hypothesis is tested on both S_j and the left-out $(x^{(j)}, y^{(j)})$, obtaining this round's training error ε_{S_j} and C-V error ε_j.

Hence, by averaging over j, the training error and C-V error are

$$\varepsilon_S = \frac{1}{N_0}\sum_{j=1}^{N_0} \varepsilon_{S_j}, \qquad \varepsilon = \frac{1}{N_0}\sum_{j=1}^{N_0} \varepsilon_j. \qquad \textbf{(9)}$$

Setting the Regularization Parameter

The regularization parameter $\lambda > 0$ is selected to minimize the cross validation error, i.e.,

$$\lambda = \arg\min_{\lambda>0} \varepsilon. \qquad \textbf{(10)}$$

Results

By training the logistic regression model with our leave-one-out cross validation strategy, λ is optimally set to 1.9 and the overall C-V error is $\varepsilon = 0.27$ (with training error $\varepsilon_S = 0$). Then, while fixing the chosen λ, we retrain the hypothesis on the maximum training set, making full use of known conspirators and non-conspirators.

Table 2.

Top 10 in the priority list (known conspirators excluded).

Name	Dolores*	Crystal	Jerome*	Sherri	Neal	Christina	Jerome	William	Dwight	Beth
Node No.	10	20	34	3	17	47	16	50	28	38
Probability of conspiracy	.56	.51	.39	.32	.30	.27	.25	.25	.24	.23

The trained hypothesis gives the estimated probability for node i being a conspirator, resulting in a priority list of suspects, ranked in descent order of criminal likelihood. The top 10 suspects are shown in **Table 2**, with managers marked by an asterisk.

Figure 3 illustrates the probability of criminal involvement estimated by $h_\theta(x)$ versus the corresponding rank in the priority list, where three managers (Jerome, Dolores, and Gretchen)[2] are marked by circles.

Dolores (manager) is indeed the person deserving highest suspicion, and Jerome (manager) is also likely to be involved in conspiracy.

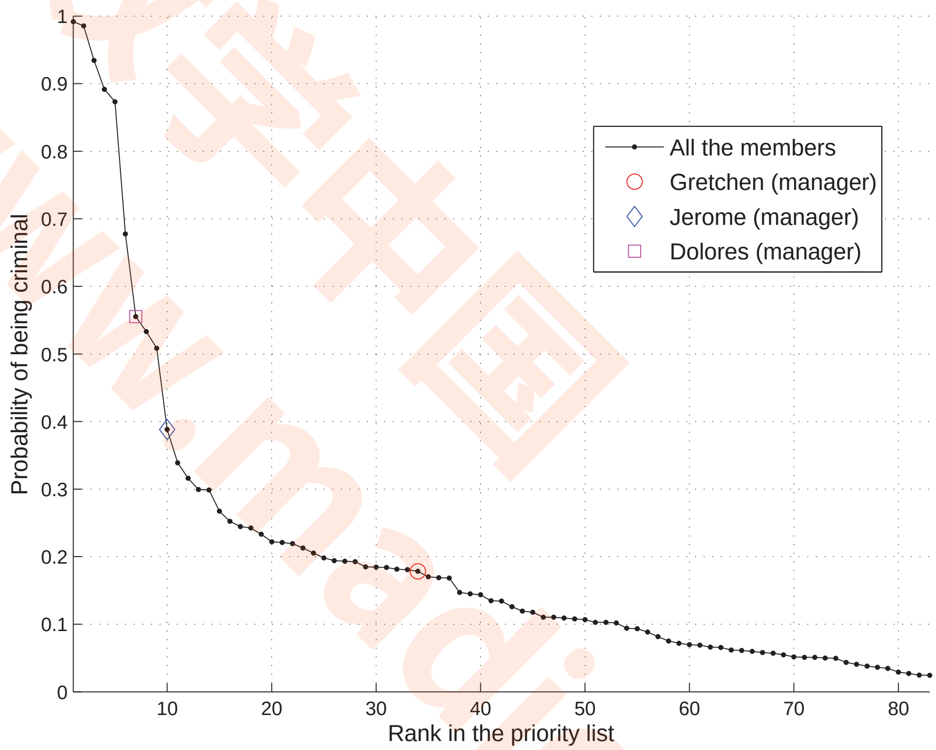

Figure 3. Probability of conspiracy vs. corresponding rank in the priority list

Semantic Model Enhancement

Semantic information is more important to humans than the complicated topology structure. For example, similar text information in their messages motivates us to conclude in the EZ case that Inez is similar to George, who is definitely a conspirator (for instance,“tired” when describing Inez and “stressed” when describing George). Similar cases can be also found in the 83-people network case: The word “Spanish” from known conspiratorial topic 7 is highly suspicious and appears repeatedly in other unknown topics (e.g., topic 2 and 12). The contents about “computer security,” which is

[2]Since several nodes are named either Gretchen or Jerome, we select those with bigger out-degrees to be managers, that is, Node 32 is Gretchen (manager) and Node 34 is Jerome (manager).

treated as part of the key in the whole conspiracy, is also active in many other unknown topics, such as 5 and 15. Hence, it is natural to train a computer to measure similarities among topics so as to reveal potential information.

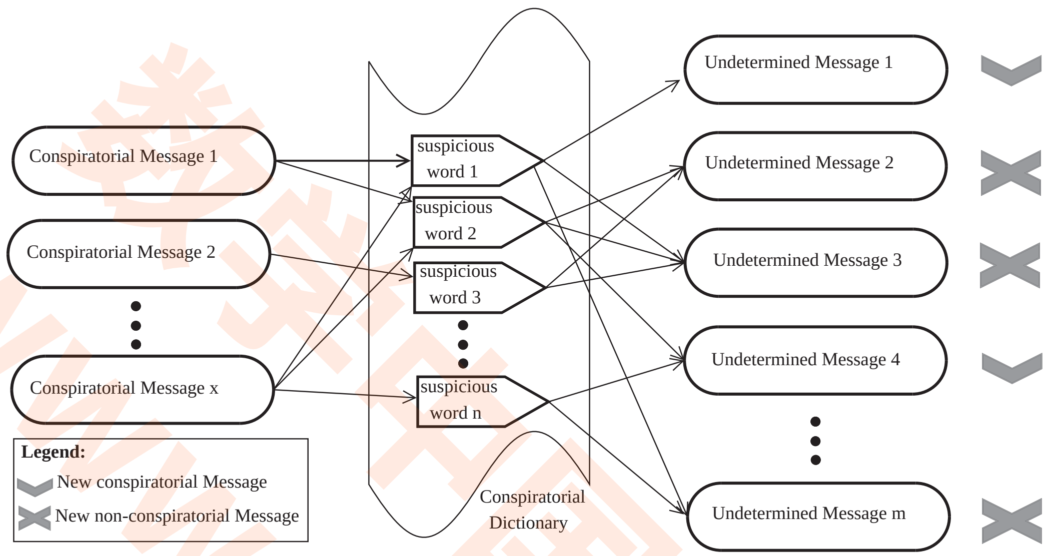

Figure 4. Framework of topic semantic diffusion.

Lexical ambiguity exists widely among words, which can have different meanings depending on context. So it is not wise to abandon human intelligence and depend only on algorithms. Therefore, we draw the problem of topic semantic diffusion into a topic-similarity measurement task based on an expert dictionary. We form the bipartite network illustrated in **Figure 4**, between the conspiratorial dictionary constructed from the conspiratorial messages about known suspicious topics, and all of the information in the message traffic. We exploit a resource allocation mechanism to extract the hidden information of networks [Zhou et al. 2007] and unfold the similarity among different topics.

The bipartite network is modeled as the bipartite graph $G = (D, T, E)$, where

- $D = \{d_i\}$ is the dictionary of suspicious words, shown in the middle column in **Figure 4**;
- $T = \{t_l\}$ is the message set, which is divided into two categories:
  - messages with known status (left column in **Figure 4**), and
  - undetermined messages (right column in **Figure 4**);
- E is an edge set, with $(d_i, t_l) \in E$ indicating that word d_i in the conspiratorial dictionary D occurred in message t_l of the message set T;

We initially give 1 unit of resource to each known conspiratorial message in T and 0 to the remaining messages. The process of semantic diffusion includes two steps, namely the redistribution of resource from message topics to keywords, and that from keywords back to topics.

We commence with the first allocation from set T to set D:

$$f(d_i) = \sum_{i=1}^{n} \frac{a_{il} f(t_l)}{K(t_l)}. \tag{11}$$

Equation **(11)** expresses the calculation of the resource held by t_l after the first step, where $K(t_l)$ denotes the degree of the node t_l, $f(x)$ denotes the resource carried by x, and a_{il} is defined as

$$a_{il} = \begin{cases} 1, & (d_i, t_l) \in E; \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

The intuitive explanation of Step 1 is simply the process of redistributing resource from T to D, with the transferred amount regulated by the degree of nodes in T.

This is followed by Step 2, which is to reflect the resource flow back to T from D obeying the same rule but in the inverse direction, as shown from the middle column to the right column in **Figure 4**. So the resource finally locates on t_i and satisfies

$$f^{'}(t_i) = \sum_{l=1}^{m} \frac{a_{il} f(d_l)}{K(d_l)} = \sum_{l=1}^{m} \frac{a_{il}}{K(d_l)} \sum_{j=1}^{n} \frac{a_{ji} f(t_j)}{K(t_j)}. \tag{13}$$

After this two-fold process, the amount of resource held by every element in T can be interpreted as a score of similarity. The rank of a topic according to its score represents the degree of its similarity to the information in the dictionary—that is, the higher the score, the more likely the topic is a newly-found conspiratorial topic.

We set $D = \{$'Spanish', 'system', 'network', 'computer', 'meeting'$\}$ as the conspiratorial dictionary, and **Table 3** illustrates the final result for all 15 topics in the 83-people network case. The known suspicious topics are 7, 11, and 13. They are naturally the top three, and topic 5 is more suspicious than other unknown topics. Topics 2, 12, and 15 are among the group with the second highest possibility in unknowns; and the remaining topics tend to be irrelevant to the conspiracy.

We then append topic 5 to the set of known conspiratorial topics and train the model again; the overall C-V error decreases from 0.27 to 0.13. Since Since topics 2, 12, and 15 are less similar to known suspicious topics, as shown in **Table 3**, appending them to model training does not evidently influence the correctness. The enhanced correctness here indicate that with enough keywords in the conspiratorial dictionary and enough topics with abundant contents, such a method is likely to perform very well.

On the other hand, if we utilize the speaker instead of the keywords to construct a bipartite graph with the topics, we will also get similarity

Table 1.
Rank of all topics based on similarity to known suspicious topics (known conspiratorial topics have an asterisk and are highlighted in blue).

Rank	Topic Number	Similarity to known suspicious topics
1	**11***	0.750
2	**7***	0.667
3	**13***	0.667
4	5	0.417
5	2	0.167
6	12	0.167
7	15	0.167
8	1,3,4,6,8,9,10,14	0

among topics based on the transmitting speaker. However, the determination of the relationship between differing results under these two standards is definitely beyond this paper.

The resource allocation method is also highly efficient: Its time complexity of computation is linear in the number of edges of the graph, which enables good performance with huge amounts of data.

Identifying the Leader of the Conspiracy

Our machine learning scheme tries to estimate the likelihood of a node committing conspiracy. However, the likelihood does not proportionally indicate leadership inside the network, because the identification of leaders is further complicated by the network's topology.

We adopt LeaderRank, a node-ranking algorithm closely related to network topology, to find the leader. We extract from the network the subgraph connected by known suspicious topics. Because of its robustness against random noise, LeaderRank is appropriate for addressing criminal network problems, which usually suffer from incompleteness and incorrectness.

LeaderRank

The LeaderRank algorithm is a state-of-the-art achievement on node ranking that is more tolerant of noisy data and robust against manipulations than traditional algorithms such as HITS and PageRank [Lü et al. 2011]. This method is mathematically equivalent to a random-walk mechanism on the directed network with adaptive probability, leading to a parameter-free algorithm readily applicable to any type of graph.

We insert a ground node, which connects with every node through newly-added bidirectional links, in order to make the entire network strongly connected, so that the random walk will definitely converge.

For a graph $G = (V, E)$, every node in the graph obtains 1 unit of resource except the ground node. After the beginning of the voting process, node i at step t will get an adaptive voting score $\nu(t)$ according to the voting from its neighbors:

$$\nu_i(t+1) = \sum_{j=1}^{|V|+1} \frac{\mu_{ij}}{D_{\text{out}}(j)}\, \nu_i(t), \tag{14}$$

where μ_{ij} is a binary indicator with value 1 if node i points to j and 0 otherwise. $D_{\text{out}}(j)$ denotes the out-degree of node j. The quotient of the above two arguments could be considered as the probability that a random walker at i goes to j in the next step. Finally, the leadership score of node i is $\nu_i(T_c) + \nu_{\text{gn}}(T_c)/|V|$, where $\nu_{\text{gn}}(T_c)$ is the score of the ground node at steady state.

Suspicious Topic Subnetwork Extraction

We extract from the network of company employees the subnetwork G_{T_S} connected by suspicious topics only, so as to minimize the coupling of the company's hierarchical structure to the conspiracy relations.

Suppose that T_{ij} denotes the set of topics mentioned by messages from node i to node j, and T_S denotes the set of known suspicious topics ($T_S = \{7, 11, 13\}$). Then G_{T_S} is the maximum subgraph of the original graph G, whereas

$$T_{ij} \subseteq T_S, \text{for all } (i, j) \subseteq E_{T_S}. \tag{15}$$

Edge Reverse

The original LeaderRank algorithm deals with finding leaders in Internet social networks, where the direction of an edge has a dissimilar meaning from our case: If A points to (follows) B in Twitter, then B is considered to be a leader of A. However, in our communication network, an edge from A to B suggests that A has sent B a message. Therefore, assuming that a leader in a criminal network tends to be a sender rather than a receiver, each edge in G_{T_S} has to be reversed to be compatible with LeaderRank's design. We denote by G'_{T_S} the reversed subnetwork induced by suspicious topics.

Results

By running LeaderRank on G'_{T_S}, a ranking score is assigned to every node in this subgraph, which generates a list of possible leaders ranked in descent order, as shown in **Table 4**.

Yao (node number 67) is ranked as the chief leader of the conspiracy.

Table 4.
Partial results of LeaderRank on G'_{T_S}.

Name	LeaderRank score
Yao	2.67
Alex	2.21
Paul	1.92
Elsie	1.62

Empirical Support

Empirical analysis of criminal networks finds that a leader of a criminal organization tends to carefully balance degree-centrality and betweenness-centrality. It has been proposed that the leader usually maintains a high betweenness-centrality but a relatively low degree-centrality, for enhancing efficiency while ensuring safety [Morselli 2010].

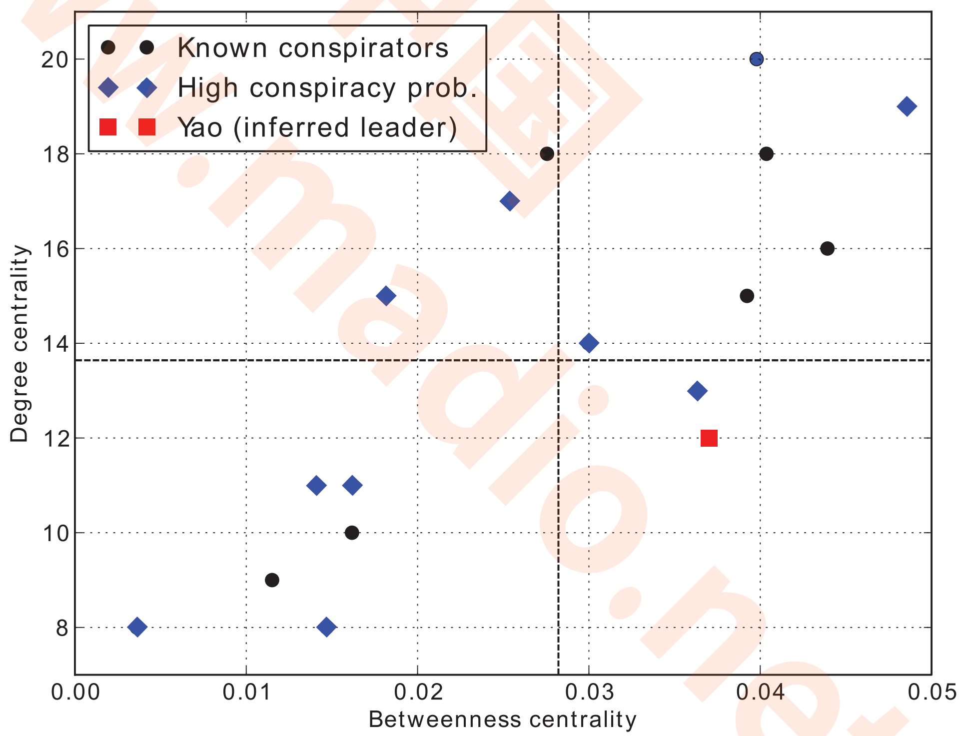

Figure 5. The joint distribution of betweenness centrality and degree centrality. Yao is at the lower right.

Figure 5 illustrates the joint distribution of betweenness centrality C_B and degree centrality ($D_{\text{in}} + D_{\text{out}}$) for the 7 known conspirators and 10 other nodes with high conspiracy likelihood, where two dashed lines mark average values of the displayed nodes. Yao's high betweenness-centrality with relatively low degree-centrality accord with the identity of a leader. Our conclusion that Yao is the leader is thus empirically supported.

Discussion

We identify the leader of the criminal network by performing the LeaderRank algorithm on the extracted, edge-reversed, suspicious-topic-connected subgraph; and our findings are strengthened by empirical research results.

Evaluating the Model

Sensitivity Analysis

Considering the usual incompleteness, imprecision, and even inconsistency in mapping criminal social networks [Xu and Chen 2005], the method for deducing criminality should be robust enough to cope with minor alternations of the network. Otherwise, there could be mistaken accusations. Therefore, we perform a sensitivity analysis on our approach.

Requirement 2 of the problem statement provides an appropriate scenario for such a test: While other conditions remain unchanged, new information indicates that Topic 1 is also connected to criminal activity, and Chris, who was considered innocent before, is now proven guilty.

Priority List

By applying our methods to these altered conditions, we find that among the top 10 of the previous priority list (the 7 known conspirators excluded), 7 of them are still in the new top 10, while the remaining 3 find their new places at 12th, 14th, and 16th.

A more sophisticated measurement of the sensitivity of the priority list is *Kendall's tau* coefficient τ [Sen 1968]. Given two priority lists $\{p_k\} = \{p_1, p_2, \cdots, p_n\}$ and $\{q_k\} = \{q_1, q_2, \cdots, q_n\}$—for example, $p_2 = 5$ means node 2 is ranked 5th in the $\{p_k\}$ list—then

- (i, j) (for $i \neq j$) is a *concordant pair* if their relative rankings agree in the two lists, i.e., $p_i > p_j$ and $q_i > q_j$, or $p_i < p_j$ and $q_i < q_j$;
- otherwise, if $p_i < p_j$ but $q_i > q_j$, or $p_i > p_j$ but $q_i < q_j$ (i, j) is a *discordant pair*.

Kendall's tau is defined as

$$\tau = \frac{\text{(number of concordant pairs)} - \text{(number of discordant pairs)}}{\frac{1}{2}n(n-1)}, \quad \textbf{(16)}$$

which lies in $[-1, 1]$, with 1 for perfect ranking agreement and -1 for utter disagreement.

The value of *Kendall's tau* for the two priority lists of Requirement 1 and Requirement 2 is $\tau = 0.86$, justifying the robustness of the machine learning approach.

Let us assume that known conspirators and non-conspirators are independently wrongly classified with the same specific probability. **Figure 6** depicts the expected Kendall's tau vs. the misclassification probability, separately for conspirators and non-conspirators. Even if the misclassification probability is as high as 0.5, Kendall's tau does not drop below 0.8, substantially proving the inherent stability of our methods.

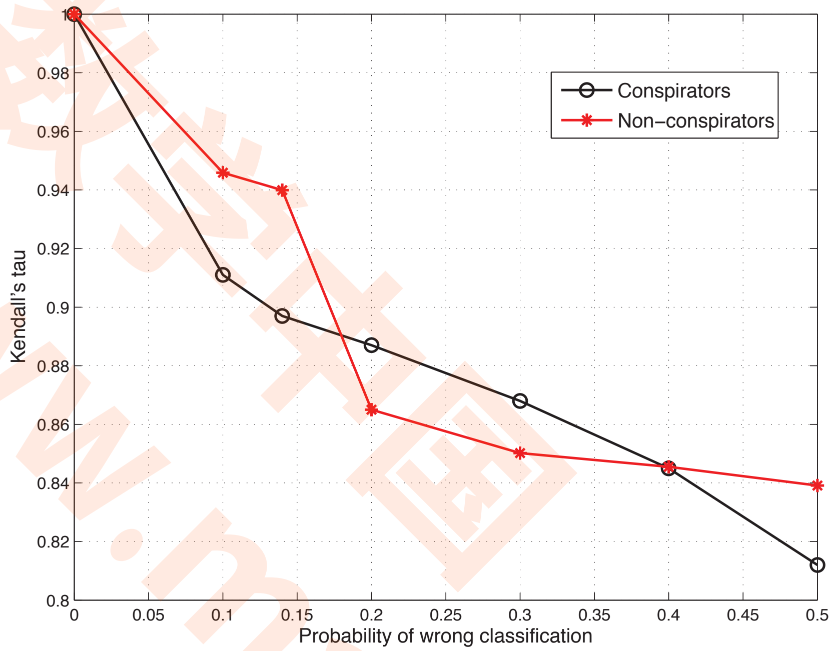

Figure 6. The expected Kendall's tau declines as misclassification probability increases.

Probability Inflation

Figure 7 illustrates the change of estimated conspiracy probability due to the modified conditions of Requirement 2, with the previous value as x-axis, and the new as y-axis. Generally, most nodes exhibit a small "inflation" in criminal probability, as indicated by the distribution of dots skewed from the diagonal line. The augmented probability is compatible with the new information that expands both the set of suspicious topics and known conspirators.

The analysis suggests that our machine learning method is insensitive to minor alterations and can still produce reasonable results with new information.

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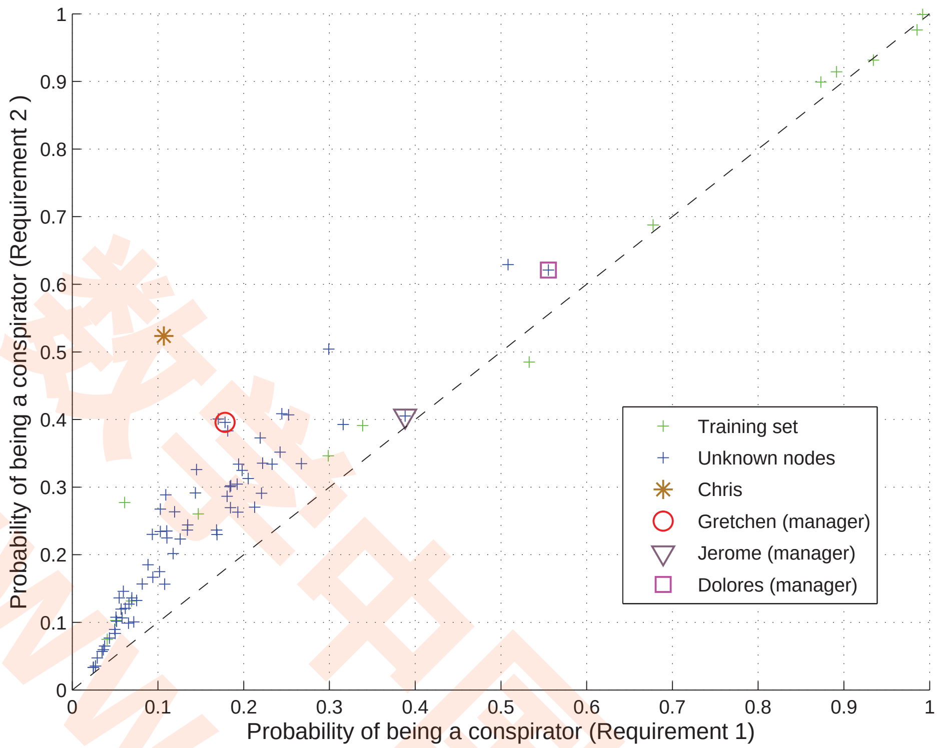

Figure 7. Criminal probabilities before and after the change of conditions.

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Judges' Commentary: Modeling for Crime Busting

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Introduction

The new topic area for this year's Interdisciplinary Contest in Modeling (ICM) was network science. The shift was popular with the student teams, since a record 1,329 teams submitted papers in solution to a "crime-busting" problem. Network science and / or social network analysis will continue to be the topic area for next year's problem as well. So, for teams that enjoyed this year's problem or want to prepare early for next year's contest, prepare by studying network modeling and assemble a team with that subject in mind.

The ICM continues to be an opportunity for teams of students to tackle challenging, real-world problems that require a wide breadth of understanding in multiple academic subjects. These elements are practically part of the definition of network science—an emerging subject that blends concepts, theories, structures, processes, and applications from mathematics, computer science, operations research, sociology, information science, and several other fields. ICM problems are often open-ended and challenging. Some, like the one this year, could be termed "wicked," in that there is not one correct answer nor a set or established method to model such a problem.

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The complex nature of the ICM problems and the short time limit require effective communication and coordination of effort among team members. One of the most challenging issues for the team is how to best organize and collaborate to use each team member's skills and talents. Teams that solve this organizational challenge often submit solutions that can rise to the final rounds of judging.

The Criminal Network Analysis Problem

The Information Age, along with its information-laden and highly-linked Internet, has brought us many amazing capabilities, along with new ways to commit crimes. This year's problem focused on potential conspirators within a company's communication network plotting to commit a crime. Some people were already identified either as known conspirators or as known non-conspirators. The goal of the model was to identify the most likely conspirators from the remaining people in the network through the analysis of confiscated and categorized message traffic. The many connections and links between the people and the messages made this an especially appropriate topic for network modeling. The main tasks expected of the students were to:

- **Requirement 1:** Build a model to prioritize the 83 people by likelihood of being part of the conspiracy and explain your model and metrics. Are any senior managers of the company involved in the conspiracy?
- **Requirement 2:** As new information comes to light, use your model to analyze this changing situation. A good network model is flexible and able to handle the changing nature, structure and information in a dynamic network setting.
- **Requirement 3:** If you could obtain the original messages, explain how semantic and text analyses of the message traffic could help you develop even better models.
- **Requirement 4:** Explain the network modeling techniques you developed and how they can be used to identify, prioritize, and categorize nodes in a network involving other kinds of data sources, not just crime and message data. Does your model generalize to other important problems in society? Again, this is the mark of strong models within network science and their potential to impact society.

Judges' Criteria

The panel of expert judges were impressed both by the strength of many of the submissions of individual teams, and fascinated by the variety of innovative approaches that students used to address the issues, challenges, and questions that were posed by the problem. The papers were rich in modeling methodology and creativity. In order to ensure that the individual judges assessed submissions on the same criteria, a rubric was developed. The framework used to evaluate submissions is described below.

Executive Summary

It was important that students succinctly and clearly explained the highlights of their submissions. The executive summary should contain brief descriptions of both the modeling approach and the bottom-line results. The remaining report provides a more detailed statement of the contents of the executive summary. One mark of an Outstanding paper is a summary with a well-connected and concise description of the approach used, the results obtained and any recommendations.

Modeling

Models and measures were needed to classify the people in the organization to identify conspirators. Many teams used probability or likelihood measures for criminal-like behavior of the people within the context of the known data. Other used decision-making criteria as their basic modeling framework. Some teams used the explicit structures of networks or graphs to determine classic local or global network metrics, properties, node clusters, or performance outcomes. For such a structure, critical assumptions, such as the directionality of influence and connection within the graph, lead to viable network models. Other teams ignored some of the aspects of the network structure and performed data mining, element classification, and discrimination. Those teams often found prioritization and ordering easier than discrimination.

Where to draw the line and commit to predict a conspirator was sometimes difficult. No matter the modeling framework, the assumptions needed for these models and the careful and appropriate development of these models were important in evaluating the quality of the solutions. The better submissions explicitly discussed why key assumptions were made and how assumptions affected the model development. Stronger submissions presented a balanced mix of mathematics and prose rather than a series of equations and parameter values without explanation. One major discriminator was the use or misuse of arbitrary parameters without any explanation or analysis. Establishing and explaining parameter values in models are at least as significant as making and validating assumptions.

Science

Semantic and text analysis are elements of the science of computational linguistics or natural language processing involving many challenging scientific and technological issues related to the nature, value and understanding of information and the production of knowledge or intelligence. Currently, many information-rich systems and organizations are facing data deluge and overload. Vast amounts of unstructured textual data are often collected and held for practically impossible human analysis. The magnitude of data makes this potentially valuable information at best a worthless distraction. Through natural language processing using semantic and text analysis the potentially valuable but hidden information can become visible, understandable, organized, and useful.

The ultimate goals of semantic and text analysis are to identify context, meaning, categorization, and entity attributes, and thereby produce human-ready synopses and standardized, interconnected, structured data (information networks). These highly sophisticated and complex processes are exactly what would be needed to model and solve this network conspiracy problem. Some teams did effective research and insightful analysis that tackled the complexity of the problem and included elements of text or semantic analysis in their model or described how their model could accommodate such capability had the raw message data been available. No matter what modeling was performed by the teams, the interdisciplinary nature of this problem was fully revealed in this requirement. These areas of information science and analytics will experience significant scientific and technological improvements in the future, and the ICM teams were exposed to this developing field in the context of their interdisciplinary science research.

Data/Validity/Sensitivity

Once the model was created, the use of test data and checks on the accuracy and robustness of the solution help to build confidence in the modeling approach. Sensitivity analysis of models to determine the effects of changing data and errors can often be more meaningful than specific output values. This is especially true for highly-structured and powerful data-rich models like networks. Some network structures are highly robust and flexible while others are fragile and highly sensitive to data. While this is a challenging element of network modeling, it was important to address this issue in the report.

Strengths/Weaknesses

A discussion of the strengths and weaknesses of the models is often where students demonstrate their understanding of what they have created.

The ability of a team to make useful recommendations fades quickly if team members do not understand the limitations or constraints of their assumptions or the implications of their modeling methodology. Networks are complex structures and, therefore, the strengths and weaknesses are often hidden from direct view or control of the modeler. Again, the better teams were able to discuss these elements despite these challenges.

Communication/Visuals/Charts

To clearly explain solutions, teams must use multiple modes of expression including diagrams and graphs, and, in the case of this competition, English. A solution that could not be understood did not progress to the final rounds of judging. The judges were delighted by the amazing array of powerful charts and graphs that explained both models and results. **Figures 1–3** on the this page and the next are intended as samples to show the richness of this kind of graphical analysis and reporting.

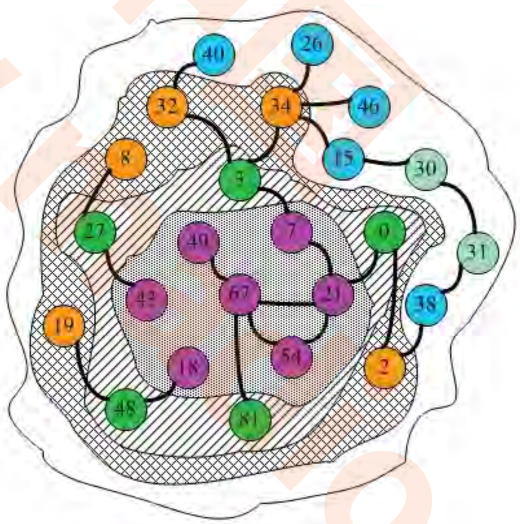


Figure 1. Teams provided informative graphic schematics to show the relationship and connections uncovered by their models. This graphic is from Team 12460 from Harbin Institute of Technology in Harbin, Heilongjiang, China.

Recommendations

Teams were specifically asked to discuss their conspiratorial priorities and the potential involvement of senior managers in their report that would be read by the district attorney. The ability of teams to evaluate the results of their analysis and make recommendations was important in identifying strong submissions.

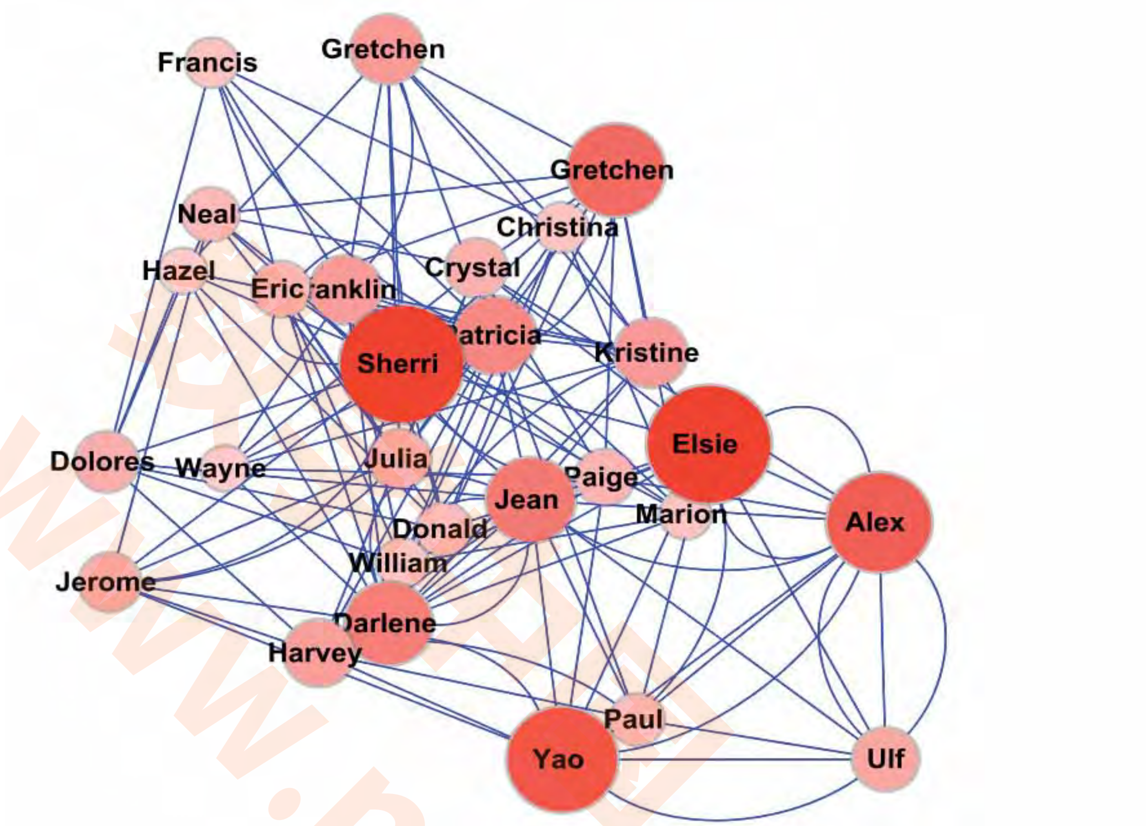


Figure 2. This network portrayal vividly showing the likelihood of conspirators is from Team 16075 from Huazhong University of Science and Technology in Wuhan, Hubei, China.

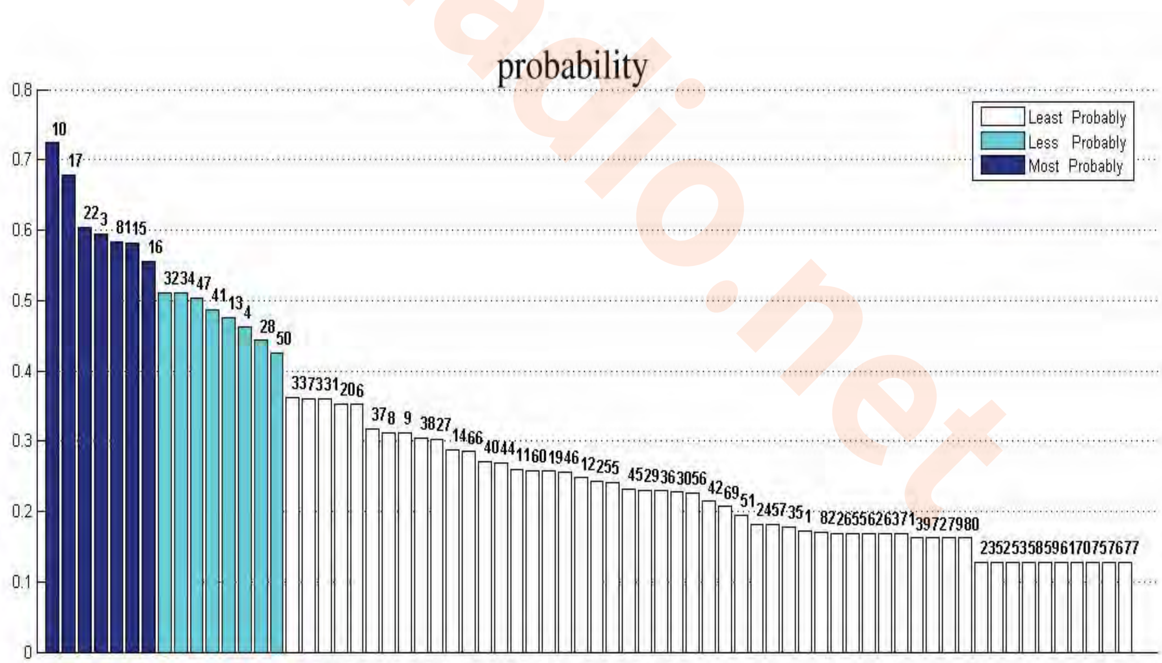


Figure 3. Teams that performed data analysis often used probability charts like this one from Team 13104 from Southeast University, Jiulonghu Campus, Nanjin, Jiangsu, China, to demonstrate their results.

Discussion of the Outstanding Papers

As you will discover in this section, many different approaches were used by ICM teams to model various aspects of the problem. Some teams used the basic structure of networks and their properties and computed classic centrality measures to tackle the issues. Some chose to model using a data mining framework. The Analytic Hierarchy Process (AHP) was a common method for addressing discrimination in the identification of a potential conspirator. As a result, the submissions this year were diverse and interesting to read. Overall, the basic modeling was often sound, creative, and sometimes quite powerful. Those that did not reach final judging generally suffered from two shortcomings. Some lacked clear explanation or evidence to support their conclusions and recommendations. They seemed to jump from their modeling directly to the results without sufficient analysis. Others failed to connect their mathematical models to the aspects and basic elements of information science. In general, poor communication was the most significant discriminator in determining which papers reached the final judging stage. Although the outstanding papers used different methodologies, they all addressed the problem in a comprehensive way by embracing the complexity of the issues, data, questions, and team objectives. These papers were generally well written and presented explanations of their modeling procedures. In several outstanding papers, a unique or innovative approach distinguished them from the rest of the finalists. Others were noteworthy for either the thoroughness of their modeling or the power of their communicated results.

Huazhong University of Science and Technology

The ICM team from Huazhong University of Science and Technology, Wuhan, China performed a thorough network analysis of the information flow and relationships of employees in the organization. In their paper, "Extended Criminal Network Analysis Model Allows Conspirators Nowhere to Hide," they provided an in-depth analysis of the relationships between people and the way the criminal network operated and expanded. This report presented their framework, models, analysis, and results in powerful visual formats that enabled readers to understand their work and feel confident in their results. In many ways, this paper is an excellent example of the potential of network modeling and the power of social network analysis to sort out nodal, edge, and data attributes through use of network measures and data analysis.

Mathematical Modeling Innovative Practice Base

The report entitled "iRank Model: A New Approach to Criminal Network Detection" was submitted by a team from the Mathematical Modeling

Innovative Practice Base, China. The Mathematical Modeling Innovative Practice Base, China, established in 2008, is an institute that promotes interdisciplinary research and educational activities, integrating mathematical modeling and computational approaches to address problems arising in various areas of science and engineering. Their report contained creative analysis of the available data from several perspectives, starting with basic analysis as shown by:

> Carefully examining into the patterns of information exchanges and social connections in the network, we can see that only 24% messages carry conspiratorial information, which seems not systematically significant given that 20% of all the topics are conspiratorial. Therefore, two patterns can be inferred from the statistical results:
>
> - Although conspirators are generally more active than the known innocent people, they exchange irrelevant information with each other. Conspiratorial messages only take a small portion in their message traffic.
> - Since the existing 7 conspirators have already involved in spreading about 40% of the total conspiratorial messages, it is very likely that the total number of conspirators is less than 20.

They also performed a very thorough social network analysis of the message network. This report contained excellent visualizations to explain their algorithm, analysis and results.

Nanjing University of Information Science and Technology

The ICM team from Nanjing University of Information Science and Technology, Nanjing, China, built three different models for finding and separating conspirators and then merged these for their best-case solution. A fourth model was used to identify the conspiracy leaders. Their paper, "Message Network Modeling for Crime Busting," was an excellent synopsis of the diverse methods one could use to approach this problem. Their emphasis was in classical network analysis and data mining algorithms. Once again, this team did a thorough job analyzing semantic analysis and its utility for information and network modeling.

Northwestern Polytechnical University

Finding the hidden features of a network was the theme of the paper entitled "Social Network Analysis in Crime Busting," by the ICM team from Northwestern Polytechnical University, Shaanxi, China. This paper started with the foundations of graphs and networks and built the concept of cooperation within the network. This concept was a fundamentally sound and deeper approach than those of many of the other models. The resulting model was a powerful one for understanding a conspiracy and the team did an excellent job in their creative modeling and analysis. Their discussion on semantics and text analysis was thorough and insightful in finding ways for possible inclusion of these more powerful methodologies in their models.

Shanghai Jiaotong University

"Crime Busting by an Iterative 2-phase Propagation Method," was submitted by a team from Shanghai Jiaotong University, Shanghai, China. Their classic propagation model of performing iterative and alternating computation of person suspiciousness and topic suspiciousness from each other was creative and powerful. Upon convergence of their model, they produced a priority list of conspirators and performed a thorough analysis. This team's model was both mathematically and scientifically simple yet elegant.

University of Electronic Science and Technology of China

The report and work entitled "Finding Conspirators in the Network: Machine learning with Resource-allocation Dynamics" from the University of Electronic Science and Technology of China, Chengdu, China, was strong from start to finish. This team made careful and thorough assumptions:

> (i) Two classes, conspirators and non-conspirators, are linearly separable in the space spanned by local features of a node, which is necessary to machine learning. (ii) A conspirator is reluctant to mention topics related to crime when talking with an outsider. (iii) Conspirators tend not to talk about irrelevant topics frequently with each other. (iv) The leader of conspiracy tries to minimize risk by restricting direct contacts. (v) A non-conspirator has no idea of who are conspirators, thus treating conspirator and non-conspirators equally.

Then they used machine learning and logistic regression to build their model. They were careful to show their analysis of leader selection and other problem requirements. They followed up their modeling and analysis with sensitivity analysis and a careful discussion of the strengths and weaknesses of their model and its approach. Most impressive was their ability to discuss the incorporation of semantic analysis into their model and the discussion of the power of information modeling to the future.

Cornell University

The team from Cornell University, Ithaca, NY, took a very different approach than the other Outstanding papers. Their paper "Crime Ring Analysis with Electric Networks" presented a model using an electrical circuit analogy for the conspiracy where the interactions between people, represented as circuit nodes, were considered a conductance term. This model was creative in its structure and enabled the team to perform an interesting analysis of the conspiracy factors. This team was selected as the INFORMS winner.

Conclusion

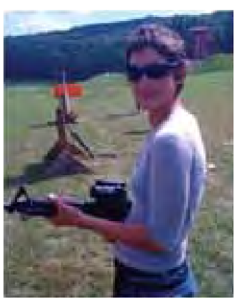
Among the 1,329 papers, there were many strong submissions, which made judging difficult. However, it was gratifying to see so many students with the ability to combine modeling, science and effective communication skills in order to understand such a complex problem and recommend solutions. We look forward to next year's competition, which will involve another problem in network science and hopefully, the participation of many teams of competent and passionate interdisciplinary modelers.

Recommendations for Future Participants

- **Answer the problem.** Weak papers sometimes do not address a significant part of the problem. Outstanding teams often cover all the bases and then go beyond.
- **Time management is critical.** Every year there are submissions that do an outstanding job on one aspect of the problem, then "run out of gas" and are unable to complete their solution. Outstanding teams have a plan and adjust as needed to submit a complete solution.
- **Coordinate your plan.** It is obvious in many weak papers how the work and writing was split between group members, then pieced together into the final report. For example, the output from one model doesn't match the input for the next model or a section appears in the paper that does not fit with the rest of the report. The more your team can coordinate the efforts of its members, the stronger your final submission will be.
- **The model is not the solution.** Some weak papers present a strong model, and then stop. Outstanding teams use their models to understand the problem and recommend or produce a solution.
- **Explain what you are doing and why.** Weak teams tend to use too many equations and too few words. Problem approaches appear out of nowhere. Outstanding teams explain what they are doing and why.

About the Authors

Chris Arney graduated from West Point and served as an intelligence officer in the U.S. Army. His academic studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his 30-year military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris then moved to RTP (Research Triangle Park), NC, where he served for various durations as chair of the Mathematical Sciences Division, of the Network Sciences Division, and of the Information Sciences Directorate of the Army Research Office. Chris has authored 22 books, written more than 120 technical articles, and given more than 250 presentations and 40 workshops. His technical interests include mathematical modeling, cooperative systems, pursuit-evasion modeling, robotics, artificial intelligence, military operations modeling, and network science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula. He is the founding director of COMAP's Interdisciplinary Contest in Modeling (ICM)®. In August 2009, he rejoined the faculty at West Point as the Network Science Chair and Professor of Mathematics.



Kate Coronges is an Assistant Professor in the Department of Behavioral Sciences and Leadership and a research fellow in the Network Science Center at the U.S. Military Academy. She has a Master's in Public Health and a Ph.D. in Health Behavior Research from the University of Southern California. Kate teaches courses in social network analysis and public policy, working with cadets to apply analytic tools to understand and model complex systems, particularly as they relate to public policy issues such as energy, education, information security, and health care. Her primary research effort involves a social network study of leadership and organizational performance. She also is working on an analysis of social acceptability of automatic biometric authentication tools, social determinants of phishing security vigilance, and modeling social media data to understand how protests turn to riots. Her publications in network science include the study of education, drug addiction, DADT ("Don't ask, don't tell") policy, coalition building, and security.

Reviews

Maasz, Juergen, and John O'Donoghue (eds.). 2011. *Real-World Problems for Secondary School Mathematics Students: Case Studies*. Rotterdam, The Netherlands: Sense Publishers; ix + 281 pp, $49.99 (P). ISBN 978–94–6091–541–3.

Secondary school mathematics teachers seek resources for bringing relevant applications of mathematics to their students, and this book is described as being "full of ideas for introducing real world problems into mathematics classrooms." The collection of 16 papers promises to provide teachers with a wealth of applications from a wide variety of school content areas (e.g., statistics, geometry, and calculus), and to focus on topics that should appeal to a student audience with diverse interests (e.g., energy issues, traveling to Mars, rugby and snooker, lotteries, logistic growth, worldwide oil reserves, and even Dirk Nowitzki).

That is, in fact, what this collection does provide. Many of the authors offer suggestions on how to format the material into classroom lessons, yet they also encourage teachers to individualize the lessons for their own students and their own circumstances. There is also an international flavor to the collection, a fact that will appeal to many teachers and many students. There are several cautions, however:

- Although the content is timely, class time will be needed for students to profit from these lessons; most lessons are not one- or two-period explorations. Teachers who are already short on time will have to weigh whether the advantage of providing students with interesting, nontrivial real world applications is enough to warrant requisite class days.
- Unless a teacher has a multiple-course assignment, he or she will not find a variety of lessons from which to select if one of the requirements is to illustrate applications of the mathematical topics covered in a particular course.
- Since the mathematics is accessible, but definitely nontrivial, much of the content may be daunting for students who are not already mathematically proficient.

For these reasons, this collection may best be seen as an excellent resource for a mathematics department rather than for one particular teacher. It

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would also be a fine resource for an independent study course or for an upper-level course in mathematical modeling.

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Thomson, Brian S. 2010. *The Calculus Integral*. North Charleston, SC: CreateSpace. x + 291 pp, $14.95 (P). ISBN 978–1–442180956. Free download at http://classicalrealanalysis.info/documents/T-CalculusIntegral-AllChapters-Portrait.pdf .

The study of integration within a first course of calculus is always problematic. The standard approach is to begin with the problem of determining areas under curves, create approximating sums, move on to the general Riemann sums, define the definite integral as a limit of these sums, and then prove the Fundamental Theorem of Calculus that links these limits of Riemann sums to what I shall refer to as *antiderivatives*, also known as *primitives* or *indefinite integrals*.

Thomson is one in a long line of mathematicians dissatisfied with this approach. It has many flaws. The definition of the definite integral as a limit of Riemann sums is incredibly sophisticated. For most students, the formal definition is quickly forgotten and the working definition of integration becomes antidifferentiation. The problem with this is that many students lose the link between antidifferentiation and Riemann sums. The reason that the standard textbook approach is problematic is that it is seriously ahistorical. Riemann created his definition in the 1850s for the specific purpose of determining how discontinuous a function might be yet still be integrable. His formulation is ideally suited for this purpose, a purpose that bears no relevance for the first year of university calculus.

The fact is that from the time that Newton first recognized the power of reversing differentiation as a tool for computing areas until Cauchy sought a characterization of integration that would enable him to assert that every continuous function is integrable, integration was defined as antidifferentiation. Thomson embraces this natural and historical definition of the integral, what he calls "The Calculus Integral," and uses it as the starting point for an exploration into our modern understanding of integration.

This book is described as appropriate for a course of honors calculus or a first course in real analysis. In either context, it would be challenging but do-able with the right students. The development is elegant and extremely original. After a dense first chapter that introduces the basic theorems needed to work with limits, sequences and series, continuity, and differentiability, Thomson begins by defining the indefinite integral of f on an open interval as a continuous function whose derivative coincides with f except possibly at finitely many points. Definite integrals are defined in terms of indefinite integrals. The Fundamental Theorem is introduced in two steps: First is the use of the Mean Value Theorem to establish the existence of a sequence of tags ζ_i such that

$$\int_a^b f(x)\,dx = \sum_{i=1}^{n} f(\zeta_i)(x_i - x_{i-1}).$$

Second comes the theorem that the definite integral can be uniformly approximated by Riemann sums with arbitrary tags. The emphasis has switched in a pedagogically significant way from defining the definite integral as a limit of Riemann sums to demonstrating that it can be approximated arbitrarily closely by Riemann sums, simply by controlling the length of the subintervals in the partition. This approach opens the door to the result that definite integrals are also uniformly approximated by Robbins sums, an interesting variation on the Riemann sum that was described by Herbert E. Robbins [1943].

The text continues through the study of sequences and series of integrals and the monotone convergence theorem, then into Cantor sets, sets of measure zero, functions with zero variation, and absolute continuity. The most original aspect of this text is the definition of the Lebesgue integral. Parallel to the Calculus Integral, the indefinite Lebesgue integral of f on an open interval is defined as an absolutely continuous function in the Vitali sense whose derivative coincides with f except possibly on a set of measure zero. The Lebesgue integral of f over the interval $[a, b]$ is then defined as $F(b) - F(a)$, where F is an indefinite Lebesgue integral of f on this interval. Connecting this definition to Riemann sums leads naturally into a discussion of the Henstock-Kurzweil integral, where the text ends. Traditional measure theory is nowhere to be found.

One of the most distinctive features of this book is that none of the theorems or corollaries is proven in the text. Instead, Thomson leads the reader through a series of exercises that build to each proof. The actual text is quite short, only 150 pages. It is followed by an almost equally long presentation of the solutions to the exercises. Just the text, leaving the scaffolded proofs to the students without the option of looking them up, would provide an excellent inquiry-based introduction to real analysis or a challenging senior seminar.

Thomson has given us a rich introduction to the complexities of integration with many historical references and intriguing asides. I agree with his use of the Calculus Integral and his approach to Riemann sums. Defining integration as a limit of Riemann sums makes no sense for first-year calculus. I am not convinced that his approach to Lebesgue integration makes better pedagogical sense than a more traditional route, but it does form part of a coherent and consistent approach to integration. The student who completes this book will be very well versed in real analysis and fully ready to tackle measure theory.

Reference

Robbins, Herbert E. 1943. Note on the Riemann integral. *American Mathematical Monthly* 50 (10) (December 1943): 617–618.

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