TSINGHUA SCIENCE AND TECHNOLOGY
ISSN 1007-0214 08/18 pp543-548
Volume 11, Number 5, October 2006

Brightness Independent 4-Star Matching Algorithm for Lost-in-Space 3-Axis Attitude Acquisition*

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Abstract: A star identification algorithm was developed for a charge-coupled device (CCD) or complementary metal-oxide-semiconductor (CMOS) autonomous star tracker to acquire 3-axis attitude information for a lost-in-space spacecraft. The algorithm took advantage of an efficient on-board database and an original "4-star matching" pattern recognition strategy to achieve fast and reliable star identification. The on-board database was composed of a brightness independent guide star catalog (mission catalog) and a *K*-vector star pair catalog. The star pattern recognition method involved direct location of star pair candidates and a simple array matching procedure. Tests of the algorithm with a CMOS active pixel sensor (APS) star tracker result in a 99.9% success rate for star identification for lost-in-space 3-axis attitude acquisition when the angular measurement accuracy of the star tracker is at least 0.01°. The brightness independent algorithm requires relatively higher measurement accuracy of the star apparent positions that can be easily achieved by CCD or CMOS sensors along with subpixel centroiding techniques.

Key words: star identification; attitude acquisition; star tracker; lost-in-space; active pixel sensor (APS)

Introduction

Star referencing gives the highest accuracy for spacecraft attitude determination. Such instruments are known as star trackers or star sensors[1]. The fourth generation star trackers, charge-coupled device (CCD)-based autonomous star trackers, are indispensable for most space systems nowadays[2]. However, it is predicted that active pixel sensors (APS) will replace the CCDs in the next generation of star trackers, which is expected to be compact, power saving, and inexpensive, and therefore, better for micro or nano spacecraft applications[3-5].

Autonomous star trackers have two basic operating modes. When *a priori* attitude information is unavailable, i.e., in the "lost-in-space" circumstance, the star tracker utilizes star patterns in the instantaneous field of view (FOV), with pattern recognition to identify stars and determine the initial attitude of the spacecraft. This is called the acquisition mode, where star pattern recognition is critically important. For a 3-axis stable spacecraft, once the attitude is acquired, star identification is no longer necessary and the star tracker turns to its normal tracking mode of tracking star positions and rapidly updating attitude information. The tracking mode will switch to the acquisition mode when the stable attitude is broken due to intentional manipulation or an unpredictable accident.

Star identification is, therefore, essential for attitude acquisition. Various star identification algorithms have been designed to this purpose[6-9]. Star patterns are based on a set of star information detected by the star

Received: 2005-05-18; revised: 2005-06-30
* Supported by the National Key Basic Research and Development (973) Program of China (No. G2000077606)
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tracker, i.e., star brightness and apparent position. Star trackers based on CCD or complementary metal-oxide-semiconductor (CMOS) sensors are better for position measurement than for brightness measurement[6,7]. Although CCD and CMOS performances have been continually improved, the brightness measurement is inherently unstable and nonstandard because the radiation sensitivity varies with the spectral response of the detector and optics and with time and environmental conditions. Therefore, only relative star magnitudes can be relied on. However, the CCD cameras provide the best position measurements. With widely implemented image processing techniques for centroiding, the apparent position of the observed stars can now be determined with subpixel accuracy[10].

This paper discusses a CMOS APS-based autonomous star tracker (AAST)[11,12]. The star identification algorithm for the initial attitude acquisition in the AAST system involves compilation of a brightness independent mission catalog and a *K*-vector star pair catalog, with a "4-star matching" pattern recognition procedure.

1 On-Board Database Development

The on-board database and its facilitation for pattern recognition are essential for an autonomous star tracker. However, the on-board database is developed based on the star identification algorithm and the hardware used for the star tracker imaging system[7]. The hardware parameters for the star tracker system used in the present work are listed in Table 1.

Table 1 Star tracker hardware parameters

Parameters	Value
Sensor	1024×1024@15 μm×15 μm
FOV	24.5° × 24.5°
Optical aperture	23 mm
Visual magnitude threshold	5.0
Defocus	3 × 3 pixels
Exposure	0.1-1.0 s

The database must be created on the ground, with the database developed for the present work based on the following four steps:

- Guide stars selection;
- Mission catalog compilation;
- Star patterns generation;
- *K*-vector star pair catalog organization.

1.1 Guide stars selection

For this algorithm, the guide stars were selected from the well-known SKY2000 astronomical catalog (master catalog)[13]. SKY2000 contains 299 485 stars up to the 10th visible magnitude and characterizes each star with abundant information, among which the celestial position and the brightness are the most important for star identification.

Even in a brightness-independent star identification procedure, star brightness is taken into account for the on-board database generation[7]. The star brightness recorded as the visual magnitude (M_v) in the astronomical catalog does not take into account the star detection of the CCD or CMOS sensors. In a brightness-dependent algorithm, the magnitude observed by the sensor needs to be estimated for each cataloged star and carefully calibrated for the star tracker. Errors in these magnitude estimates are an important source of uncertainty in the recognition process. In brightness independent algorithms, only a detection limit or magnitude threshold needs to be determined. Stars weaker than the magnitude threshold are unlikely to be detected by the star tracker so they should not be selected as guide stars.

Most star trackers opt to use stars with $M_v \leqslant 6$ for attitude determination. In the present algorithm, we used $M_v \leqslant 5$ as the brightness limit for guide stars. Lower sensitivities lead to smaller optical systems, smaller databases, less memory, quicker identification, and improved accuracy (because bright stars are imaged with higher SNR than faint ones, which results in more accurate estimates of their apparent positions), but lower sensitivities also require larger fields of view as a compromise to guarantee sufficient sky coverage for lost-in-space attitude acquisition.

In addition to sufficient brightness, guide stars must also not have anomalous characteristics, such as optical doubles, considerable proper motion, variable brightness, or large position uncertainty[7]. The magnitude threshold reduced the number of usable guide stars to 1631. Figure 1 shows the celestial distribution of these guide stars created by the website Java program.

1.2 Mission catalog compilation

The brightness independent algorithm mission catalog

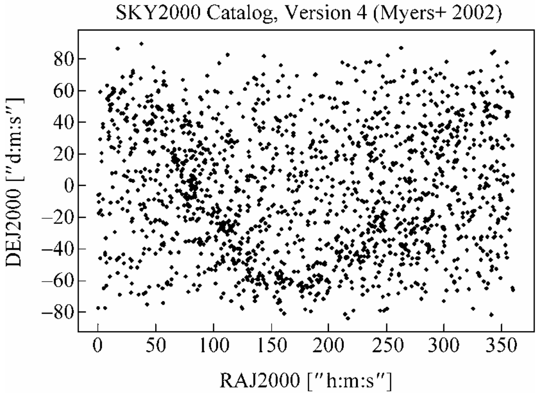

Fig. 1 Celestial distribution of guide stars from the SKY2000 master catalog

needs to contain the guide star position information. The star tracker uses the star positions as references to determine its boresight direction that is usually represented by a unit vector $\boldsymbol{v}\,(v_x,\ v_y,\ v_z)$ in the earth-centered inertial coordinates. However, the celestial star positions are described by the right ascension and declination (α, δ) in the master catalog. Therefore, to compile the mission catalog, the star positions must be transformed from celestial coordinates into unit vectors using

$$\boldsymbol{v}=\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}=\begin{bmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{bmatrix} \quad (1)$$

The guide stars should then be indexed so as to facilitate pattern generation and recognition. The current database uses the angular separation between each pair of stars in an arbitrary FOV, so the guide stars are listed in ascending order according to their right ascension (and declination for the stars at the same longitude) and numbered from 1 to 1631. In this way, stars appearing in any FOV will have sequential index numbers. Table 2 shows a part of the compiled mission catalog.

Table 2 Mission catalog

Star ID	v_x	v_y	v_z
1	0.226 06	0.001 573	–0.974 11
2	0.994 51	0.008 507	–0.104 28
3	0.957 88	0.015 632	–0.286 76
4	0.986 12	0.019 374	–0.164 88
5	0.996 92	0.023 214	–0.074 85
…	…	…	…
1627	0.905 22	–0.008 850	0.424 85
1628	0.999 07	–0.005 790	–0.042 64
1629	0.562 73	–0.002 430	0.826 64
1630	0.992 83	–0.002 980	0.119 50
1631	0.431 73	–0.000 160	–0.902 00

1.3 Star pattern generation

The cosine of the separation angular between two vectors $\boldsymbol{v}_i$ and $\boldsymbol{v}_j$ is

$$\cos\theta_{ij}=\boldsymbol{v}_i^{\mathrm{T}}\boldsymbol{v}_j=v_{ix}v_{jx}+v_{iy}v_{jy}+v_{iz}v_{jz} \quad (2)$$

If i and j represent the mission catalog ID of two stars and $\boldsymbol{v}_i$ and $\boldsymbol{v}_j$ represent their unit vectors, then $\cos\theta_{ij}$ is the angular separation between star i and star j. The angular separation recorded as (i, j, $\cos\theta_{ij}$) is the pattern used for star identification in the algorithm.

Calculating the angular separation of each star pair from the mission catalog would result in $\dfrac{1631\times(1631-1)}{2}=1\,329\,265$ patterns. In fact, not all of these patterns need to be calculated because of the star tracker angular measurement limit:

$$\theta_{\min}\leqslant\theta\leqslant\theta_{\max} \quad (3)$$

where $\theta_{\min}$ and $\theta_{\max}$ are the minimum and maximum angles that can be measured by the star tracker. Therefore, if the separation angle between two stars is larger than the star tracker's field of view, it cannot be measured by the star tracker. So $\theta_{\max}=\theta_{\mathrm{FOV}}$. With defocused optics which is used with subpixel techniques, if two stars are very close to each other (such as some optical doubles), their defocused spots will overlap significantly. The optics in the present APS system was designed to produce a defocused spot with 90% of the star spot energy converging in 3×3 pixels. Two spots can then be distinguished if their centers are separated by at least two pixels which gives $\theta_{\min}\approx 0.05°$. Therefore, only angular separations with separation angles smaller than $\theta_{\max}$ and larger than $\theta_{\min}$ need to be calculated which results in 65 890 patterns.

If there is no error or noise in the system, a measured angular separation $\cos\theta'$ between two observed stars will give a unique match. However, since errors are unavoidable, the actual pattern recognition procedure identifies any cataloged star pairs with separation angle θ that fall within the angular measurement accuracy ε of the star tracker:

$$|\theta'-\theta|<\varepsilon \quad (4)$$

as candidates for the observed star pair.

1.4 *K*-vector star pair catalog organization

To facilitate the locating of candidates, a star pair catalog was constructed with the star pairs arranged in ascending order of their angular separation values. Table 3 represents a part of the star pair catalog.

Table 3 Star pair catalog

Star pair ID	i	j	$\cos\theta_{ij}$
1	987	1097	0.909 961 953
2	5	11	0.909 964 505
3	818	853	0.909 965 190
4	818	852	0.909 965 592
5	263	344	0.909 965 681
6	621	672	0.909 969 465
7	178	331	0.909 969 630
8	486	560	0.909 971 784
…	…	…	…
65 888	810	811	0.999 999 429
65 889	217	219	0.999 999 493
65 890	855	860	0.999 999 609

The measured angular separation, $\cos\theta'_{ij}$, is then compared with each cosine value in the catalog to identify matches. A commonly-used method for this matching is the binary search technique (BST), which needs $2\log_2\left(N_\mathrm{p}\right)$ logic operations between real numbers, where N_p is the number of cataloged star pairs[6]. To improve the matching speed, Mortari proposed a novel search-less method based on an index vector named the K-vector[6]. In this method, an integer k is calculated from a linear equation fitting the measured angular separation $\cos\theta'$,

$$z(x) = mx + q = \cos\theta', \text{ and } k = \mathrm{int}\ (x) \tag{5}$$

where int (x) is some rounding function to get an integer from x. The K-vector element $K(k)$ represents the number of cataloged star pairs whose angular separations are smaller than $\cos\theta'$, which is the catalog ID of the star pair whose angular separation is equal to or just smaller than $\cos\theta'$. This method then does not use a search procedure to find the candidate star pairs. Mortari[6] described in detail the construction of the linear equation and the K-vector. Figure 2 presents the first eight star pairs in Table 3 and their corresponding K-vector elements. The symbols are the angular separation cosine values, while the solid line is the curve fit $z(x)$. The meaning of $K(k)$ can be understood from the figure.

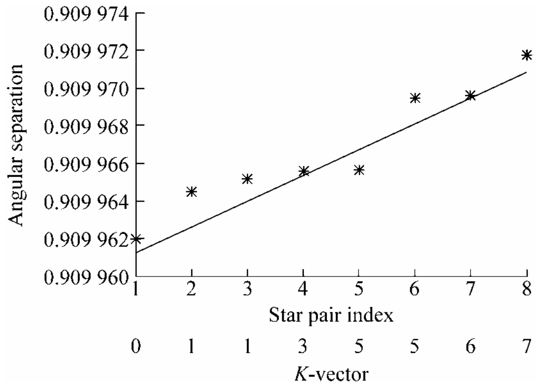

Fig. 2 *K*-vector elements of the first 8 star pairs

The calculation of $K(k)$ for each cataloged star pair gives a K-vector star pair catalog. Table 4 shows a part of the catalog. The cosine value no longer needs to be included in this catalog.

Table 4 *K*-vector star pair catalog

$K(k)$	i	j
0	987	1097
1	5	11
1	818	853
3	818	852
5	263	344
5	621	672
6	178	331
7	486	560
…	…	…
65 855	810	811
65 877	217	219
65 890	855	860

The on-board database then includes the K-vector star pair catalog and the mission catalog as reference for the star identification.

2 "4-Star Matching" Strategy

Once the K-vector has been generated, for a measured angular separation, $\cos\theta'_{ij}$, and the star tracker's accuracy, ε, two integer numbers can be obtained from

$$k_\mathrm{bot} = \mathrm{floor}\left(\frac{\cos\ (\theta_{ij}' + \varepsilon) - q}{m}\right),$$
$$k_\mathrm{top} = \mathrm{ceil}\left(\frac{\cos\ \left(\theta_{ij}' - \varepsilon\right) - q}{m}\right) \tag{6}$$

where floor(x) and ceil(x) are the rounding functions in Matlab. floor(x) gives the nearest integer less than or equal to x and ceil(x) gives the nearest integer greater than or equal to x. The ID range of candidate star pairs is then obtained from the K-vector:

$$\mathrm{ID}_{\mathrm{start}} = K(k_{\mathrm{bot}}) + 1, \quad \mathrm{ID}_{\mathrm{end}} = K(k_{\mathrm{top}}) \tag{7}$$

Many criteria and pattern recognition algorithms have been proposed to identify a unique star pair[9]. Our algorithm uses a 4-star matching strategy.

The 4-star matching procedure is:

(1) Pick four stars from the FOV as a group and label them as S_1, S_2, S_3, and S_4. The first 4-star group should be selected from the center of the FOV because star positions near the center are more precise than those at the edge due to optical distortions.

(2) Calculate the angular separations between each pair of the 4-star group and label the angular separation of star pair S_1 and S_2 as p_1, S_1 and S_3 as p_2, S_1 and S_4 as p_3, S_2 and S_3 as p_4, S_2 and S_4 as p_5, and S_3 and S_4 as p_6.

(3) Determine the array of candidate star pairs for each angular separation using the K-vector method.

(4) Define an $N \times 6$ zero matrix $\boldsymbol{M}$, where N equals the number of guide stars. The matrix rows, correspond to the guide stars in the mission catalog. The columns correspond to the star pairs p_1-p_6. If the star pair (i, j) $(i, j = 1,\cdots,N)$ appears in the candidate array of p_r $(r = 1,\cdots,6)$, then $M(i, r) = M(i, r) + 1$, $M(j, r) = M(j, r) + 1$.

(5) If and only if there is one row that is exactly [111000], the index of this row is identified as the catalog ID of star S_1. If there are no rows or more than one row with this value, then S_1 cannot be identified. For S_2, the row should be [100110], for S_3 [010101], and for S_4 [001011].

(6) Once a star is identified, it is not used again. The remaining stars are used with additional stars to make a new 4-star group. The above procedure is then repeated. If none of the four stars are identified in a matching procedure, at least one star should still be removed to build a new 4-star group. If there are no more than four stars left, then already identified stars must be used to form a 4-star group.

3 4-Star Matching Test

The star identification algorithm was tested using numerical simulations to validate its feasibility for lost-in-space 3-axis attitude acquisition with the AAST star tracker. For lost-in-space applications, full sky coverage and reliability are most important. At least, two stars must be identified to acquire the 3-axis attitude information.

The whole sky was surveyed through the star tracker's FOV (=24.5°) with its boresight rotation of 5° in each step in the right ascension direction from 0-360° and in the declination direction from −90°-+90° to produce 2664 observations. In practice, the star tracker will take one picture which may coincide with any of these 2664 observations. Any guide stars with separation angles less than 12.25° were included in each FOV. Figure 3 shows the number of guide stars in each FOV. The average number is 17.5 (the upper line in the figure), with only two FOVs containing less than four stars (the lower line in the figure).

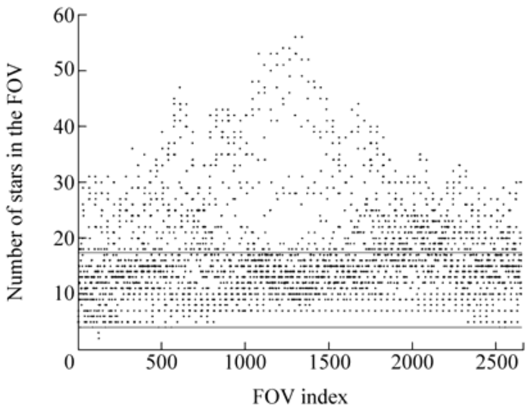

Fig. 3 Number of guide stars in each FOV (24.5°)

The 4-star matching procedure was performed for each of the remaining 2662 FOVs. A star is identified if and only if a unique and correct ID is identified for a star. If none or more than one ID is found, the star is reported to be "unidentified".

When the angular measurement error ε is set to 0.005°, each FOV had at least one successful 4-star matching with all four stars in a 4-star group being identified. When the measurement error was increased to 0.01°, there were occasionally "unidentified" results for one or two of the four stars, but all the unique outputs were correct. So the identification result for each star is either "identified" or "unidentified", but never "misidentified", which confirms the reliability of the 4-star matching strategy. For each FOV, the 4-star matching can identify at least two stars. With only two FOVs with less than four stars, the 4-star matching approach can achieve a $2662/2664 \approx 99.9\%$ success rate for 2-star identification for lost-in-space 3-axis attitude estimation.

With larger measurement errors, 5-star matching should be used to guarantee successful 2-star identification. However, the number of FOVs with less than five guide stars would increase, which would reduce the sky coverage. Therefore, the best strategy is to use 4-star matching and to improve the star tracker angular measurement accuracy to better than 0.01°, which can be easily achieved by CCD or CMOS sensors.

4 Conclusions

Reliability and speed are the most important attributes of a lost-in-space star identification. This paper presents a reliable, fast star identification algorithm for CCD or CMOS-based autonomous star trackers. The main characteristics of the algorithm are:

(1) Use a brightness-independent star recognition algorithm that not only eliminates the need for magnitude estimation and calibration, but also avoids the pattern recognition uncertainty caused by the magnitude measurement errors in the CCD or CMOS sensors.

(2) Match candidates are directly located through an indexing strategy that eliminates searches through large numbers of real numbers, which reduces onboard memory and improves recognition speed. Onboard resource economization is important for space applications.

(3) Use a star group matching strategy based on a very simple, reliable criterion. The number of stars in the matching group is related to the star tracker's precision. For a star tracker with $5.0M_v$ threshold 24.5° FOV, and 0.01° measurement accuracy, 4-star matching can achieve a 99.9% success rate for the 2-star identification needed for 3-axis attitude acquisition in lost-in-space circumstances.

Although 2-star identification can confirm a 3-axis attitude acquisition, more identified stars are expected to improve the attitude estimation accuracy. To this purpose, subpixel centroding techniques are usually used to improve the position measurement accuracy.

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