## INSTITUTE OF CONTROL AND COMPUTATION ENGINEERING FACULTY OF ELECTRONICS AND INFORMATION TECHNOLOGY WARSAW UNIVERSITY OF TECHNOLOGY







#### MASTER OF SCIENCE THESIS

# STAR-TRACKER PROGRAM FOR CUBESAT SATELLITES

Szymon MICHALSKI

Supervisor: prof. dr hab. inż. Ryszard Romaniuk

## ${\bf Contents}$

N	omer	nclatur	e	4								
1	Intr	Introduction 5										
	1.1	Motiva	ation	5								
	1.2		e of thesis	5								
	1.3		at	6								
	1.4		of attitude estimation	6								
	1.5		ard computer	7								
2	Pre	Preliminaries 8										
	2.1	Coordi	inate frames	8								
		2.1.1	ECI frame	8								
		2.1.2	ECEF frame	8								
		2.1.3	NED frame	8								
		2.1.4	BODY frame	8								
	2.2	Attitud	de representations	8								
		2.2.1	Euler angles	8								
		2.2.2	Quaternions	8								
	2.3	Quater	rnion properties	8								
		2.3.1	Advantages of quaternions	8								
		2.3.2	Multiplication of quaternions	8								
		2.3.3	Quaternions and rotations	8								
	2.4	Wahba	a's problem $\dots$	8								
	2.5	Choles	ky factorization	9								
	2.6	Lyapui	nov analysis	9								
3	Star	r-track	er program	10								
	3.1	Centro	oid - start recognition	10								
	3.2	Star id	lentification	11								
		3.2.1	Angle Matching	12								
		3.2.2	Spherical Triangle Matching	12								
		3.2.3	Planar Triangle	12								
		3.2.4	Pyramid	12								
		3.2.5	Rate Matching	13								
		3.2.6	Voting	13								
		3.2.7	Grid	13								
	3.3	Star-ca	atalogue and searching for matching stars	13								
		3.3.1	Star Catalogue Generation	13								
		3.3.2	Candidate Matching	13								

Star-trac	lor for	Cuboget	gotollito	
Star_trac	ker tor	Cilhesat	satembe	S

#### MICHALSKI Sz.

		3.3.3	Result Verification		13			
		3.3.4	k-vector		13			
	3.4	Attitu	de Determination		14			
		3.4.1	q-method		14			
		3.4.2	QUEST		14			
		3.4.3	TRIAD		14			
		3.4.4	The Fast Optimal Attitude Matrix		14			
		3.4.5	DCM (direction cosine matrix) - (Singular Value De-					
			composition?)		14			
4	Pro	totype	,		16			
5	Complete program							
6	Testing of star-tracker							
	List of Tables 22 List of Figures 23 toc							

## Nomenclature

- c Speed of light in a vacuum inertial frame
- h Planck constant

http://ssl.mit.edu/publications/theses/SM-2006-HuffmanKara.pdf [1] [2] [3] [4] [5] [6] [7] [8]

#### 1 Introduction

#### 1.1 Motivation

The goal of this work is to make fully operational star-tracker program, that could be used on Cubesat satellites. Such program could be used on space missions and could start Polish state-of-the-art technology in growing space technology sector.

#### 1.2 Outline of thesis

This thesis consists of several chapters. Here they are shortly summarized:

Chapter 1 serves as introduction to this thesis and describest the motivation and goal of this work. It also describes the background of the topic.

Chapter 2 describes all the important foundations for the fully understanding given work.

Chapter 3 is the main part of this thesis. It describes how the star-tracker program works and goes through detailed comparison of different approaches.

Chapter 4 describes the created prototype of star-tracker in Python language.

Chapter 5 talks about the implementation of star-tracker on the existing prototype of on-board computer.

Chapter 6 describes how the finished program is performing.

Chapter 7 contains conclusons about this work and created star-tracker program.

#### 1.3 Cubesat

Cubesat was designed on Caltech in 1999[9]. Dimensions of satellite are measured in units. Each unit (often described simply as u) can be 10x10x10cm and can weight up to 1.33 kg. Satellites can be 1u, 2u, 3u, 6u or even 12u.

Such small satellites are suspectible to noise from densly packed electronics.

Zdjecie Cubesata

Directional antennas

CubeSat missions, goals, what can they be and are used for? Why is it innovative and important?

#### 1.4 Means of attitude estimation

There exist many different types of attitude estimation: sun sensors, star-trackers, magnetometers, etc. However star-tracker gives the best possible accuracy for nowadays and is not suspectible to electrical nor magnetic noise.

[6] Wiley J. Larson and James R. Wertz, editors. Space Mission Analysis and De- sign. Microcosm, Inc., Torrance, CA, second edition, 1995. [11]

Sensor Accuracy
Magnetometers  $1.0o~(5000 \mathrm{km~alt})~5.0~(200~\mathrm{km~alt})$  Attitude measured relative to Earth Sensors  $0.05~(\mathrm{GEO})~0.1~(\mathrm{LEO})$ Sun sensors 0.01Star sensors  $2~\mathrm{arc\text{-}sec}$ Gyroscopes  $0.001~\mathrm{deg/hr}$ 

Table 1: Sensor Accuracy Ranges. Adapted from [10]

0.01 to 0.5

### 1.5 On-board computer

This section will describe the on-board computer which was done as part of other thesis.

#### 2 Preliminaries

- 2.1 Coordinate frames
- 2.1.1 ECI frame
- 2.1.2 ECEF frame
- 2.1.3 NED frame
- 2.1.4 BODY frame
- 2.2 Attitude representations
- 2.2.1 Euler angles
- 2.2.2 Quaternions
- 2.3 Quaternion properties
- 2.3.1 Advantages of quaternions
- 2.3.2 Multiplication of quaternions
- 2.3.3 Quaternions and rotations
- 2.4 Wahba's problem

$$\sum_{j}^{n} ||r_j - Mb_j|| \tag{1}$$

- 2.5 Cholesky factorization
- 2.6 Lyapunov analysis

#### 3 Star-tracker program

Generally star-tracker is divided into three main parts[12]:

- recogiting stars on the image and converting the data into list of star vectors by calculating star centroids;
- identyfing which star vector represents which real star in catalogue. This is done by comparing star vectors from the image with data in star catalogue, which is generated before space mission;
- estimating the attitude by calculating the displacement between two frames.

#### 3.1 Centroid - start recognition

Due to limitations of camera there exists necessity of calculating star centroids. Each camera converts image into photo divided by pixels. As it is necessary to have high precision of star coordinates, the pixel accuracy is not enough. Subpixel accuracy is needed. Typically it is done by defocusing the lens of the camera and calculating the lumosity of all pixels around the lightest ones. The idea of how to calculate such centroids is adapted from [12].

If FOV is too small, one star will be considered by program as few stars, and if FOV is too large, few stars placed near each other will be considered as one star. Calculating star centroids is tradeoff between counting few stars as one and counting one star as a few. It seems however that it is worse to count one star as few than few stars as one.

$$x_{start} = x - \frac{a_{ROI} - 1}{2} \tag{2}$$

$$y_{start} = y - \frac{a_{ROI} - 1}{2} \tag{3}$$

$$x_{end} = x_{start} + a_{ROI} \tag{4}$$

$$y_{end} = y_{start} + a_{ROI} \tag{5}$$

$$I_{bottom} = \sum_{i=1}^{x_{end}-1} I(i, y_{start})$$
 (6)

$$I_{top} = \sum_{i=2}^{x_{end}} I(i, y_{end}) \tag{7}$$

$$I_{left} = \sum_{j=1}^{y_{end}-1} I(x_{start}, j)$$
(8)

$$I_{right} = \sum_{j=2}^{y_{end}} I(x_{start}, j)$$
 (9)

$$I_{border} = \frac{I_{top} + I_{bottom} + I_{left} + I_{right}}{4(a_{ROI} - 1)}$$

$$\tag{10}$$

$$\tilde{I}(x,y) = I(x,y) - I_{border} \tag{11}$$

$$B = \sum_{i=x_{start}+1}^{x_{end}-1} \sum_{j=y_{start}+1}^{y_{end}-1} \tilde{I}(i,j)$$
 (12)

$$x_{CM} = \sum_{i=x_{start}+1}^{x_{end}-1} \sum_{j=y_{start}+1}^{y_{end}-1} \frac{i \times \tilde{I}(i,j)}{B}$$

$$\tag{13}$$

$$x_{CM} = \sum_{i=x_{start}+1}^{x_{end}-1} \sum_{j=y_{start}+1}^{y_{end}-1} \frac{j \times \tilde{I}(i,j)}{B}$$

$$\tag{14}$$

$$u = \frac{\begin{bmatrix} \mu x_{CM} & \mu y_{CM} & f \end{bmatrix}^T}{\| \begin{bmatrix} \mu x_{CM} & \mu y_{CM} & f \end{bmatrix} \|}$$
(15)

#### 3.2 Star identification

all [13]

Brightness Independent 4-Star Matching Algorithm for Lost-in-Space 3-Axis Attitude Acquisition[14]

SP-Search: A New Algorithm for Star Pattern Recognition [15]

Star Identification using Neural networks [16] [17]

Star pattern recognition using neural networks [18]

#### 3.2.1 Angle Matching

#### 3.2.2 Spherical Triangle Matching

#### 3.2.3 Planar Triangle

[19]

$$s = \frac{1}{2}(a+b+c) \tag{16}$$

$$a = ||\boldsymbol{u_p} - \boldsymbol{u_q}|| \tag{17}$$

$$b = ||\boldsymbol{u_q} - \boldsymbol{u_r}|| \tag{18}$$

$$c = ||\boldsymbol{u_p} - \boldsymbol{u_r}|| \tag{19}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)} \tag{20}$$

$$J = A \frac{(a^2 + b^2 + c^2)}{36} \tag{21}$$

#### 3.2.4 Pyramid

[20]

#### 3.2.5 Rate Matching

Samaan, M. A., Mortari, D., Junkins, J. L., "Recursive Mode Star Identification Algorithms," AAS Paper No. 01-149, pp. 677-692.

#### **3.2.6** Voting

[21]

#### 3.2.7 Grid

http://dsp.ucsd.edu/kreutz/Publications/Padgett1997.pdf

#### 3.3 Star-catalogue and searching for matching stars

#### 3.3.1 Star Catalogue Generation

$$\boldsymbol{u} = \begin{bmatrix} \cos \alpha \cos \delta \\ \sin \alpha \cos \delta \\ \sin \delta \end{bmatrix} \tag{22}$$

$$m_i \le m_{max} \tag{23}$$

$$m_j \le m_{max} \tag{24}$$

$$\boldsymbol{u_a^T u_b} \ge \cos \theta_{FOV} \tag{25}$$

#### 3.3.2 Candidate Matching

#### 3.3.3 Result Verification

#### 3.3.4 k-vector

[22]

[23]

[24]

#### 3.4 Attitude Determination

[1]

AIM (Attitude estimation using Image Matching)[?]

all [10] [25]

#### 3.4.1 q-method

#### 3.4.2 **QUEST**

improvement to quest implementation [26]

kallman filtering [27]

#### 3.4.3 TRIAD

#### 3.4.4 The Fast Optimal Attitude Matrix

3.4.5 DCM (direction cosine matrix) - (Singular Value Decomposition?)

[12]

$$B = \sum_{i=1}^{n} b_i r_i^T \tag{26}$$

$$\boldsymbol{B} = \boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^T \tag{27}$$

$$\mathbf{U}_{+} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & det \mathbf{U} \end{bmatrix}$$

$$\mathbf{V}_{+} = \mathbf{V} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & det \mathbf{V} \end{bmatrix}$$

$$\mathbf{A} = \mathbf{U}_{+} \mathbf{V}_{+}^{T}$$
(30)

$$\mathbf{V}_{+} = \mathbf{V} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & det \mathbf{V} \end{bmatrix}$$
 (29)

$$\boldsymbol{A} = \boldsymbol{U}_{+} \boldsymbol{V}_{+}^{T} \tag{30}$$

## 4 Prototype

## 5 Complete program

## 6 Testing of star-tracker

[28]

#### References

- [1] K. L. Jenssen, K. H. Yabar, and J. T. Gravdahl, "A comparison of attitude determination methods: theory and experiments," in *proceedings* of the 62nd International Astronautical Congress, Cape Town, South Africa, pp. 3–7, 2011.
- [2] R. G. Valenti, I. Dryanovski, and J. Xiao, "Keeping a Good Attitude: A Quaternion-Based Orientation Filter for IMUs and MARGs," Sensors, vol. 15, no. 8, pp. 19302–19330, 2015.
- [3] T. Delabie, "A highly efficient attitude estimation algorithm for star trackers based on optimal image matching," in AIAA Guidance, Navigation and Control Conference, Minneapolis, Minnesota, 2012.
- [4] E. Jalabert, E. Fabacher, N. Guy, S. Lizy-Destrez, W. Rappin, and G. Rivier, "Optimization of star research algorithm for ESMO star tracker," 2011.
- [5] D. Felikson, J. Hahmall, M. F. Vess, and M. Ekinci, "On-Orbit Solar Dynamics Observatory (SDO) Star Tracker Warm Pixel Analysis," in AIAA Guidance, Navigation and Control Conference, Portland, Oregon, vol. 6728, 2011.
- [6] M. W. Knutson, Fast star tracker centroid algorithm for high performance CubeSat with air bearing validation. PhD thesis, Massachusetts Institute of Technology, 2012.
- [7] A. Rose, "STAR integrated tracker," arXiv preprint nucl-ex/0307015, 2003.
- [8] D. Mortari and A. Romoli, "StarNav III: a three fields of view star tracker," in *Aerospace Conference Proceedings*, 2002. *IEEE*, vol. 1, pp. 1–57, IEEE, 2002.
- [9] H. Heidt, J. Puig-Suari, A. Moore, S. Nakasuka, and R. Twiggs, "Cube-Sat: A new generation of picosatellite for education and industry low-cost space experimentation," 2000.
- [10] C. D. Hall, "Spacecraft attitude dynamics and control," *Lecture Notes posted on Handouts page [online]*, vol. 12, no. 2003, 2003.
- [11] S. M. R. C. P. Lima, "Comparison of small satellite attitude determination methods," 2000.

- [12] C. R. McBryde and E. G. Lightsey, "A star tracker design for CubeSats," in *Aerospace Conference*, 2012 IEEE, pp. 1–14, March 2012.
- [13] B. B. Spratling and D. Mortari, "A survey on star identification algorithms," *Algorithms*, vol. 2, no. 1, pp. 93–107, 2009.
- [14] Y. Dong, F. Xing, and Z. You, "Brightness independent 4-star matching algorithm for lost-in-space 3-axis attitude acquisition," *Tsinghua Science & Technology*, vol. 11, no. 5, pp. 543–548, 2006.
- [15] D. Mortari, "SP-search: A new algorithm for star pattern recognition," *Advances in the Astronautical Sciences*, vol. 102, no. Pt II, pp. 1165–1174, 1999.
- [16] S. S. Miri and M. E. Shiri, "Star identification using Delaunay triangulation and distributed neural networks," *International Journal of Modeling and Optimization*, vol. 2, no. 3, p. 234, 2012.
- [17] T. Lindblad, C. S. Lindsey, Å. Eide, Ö. Solberg, and A. Bolseth, "Star Identification using Neural Networks,"
- [18] C. Li, K. Li, L. Zhang, S. Jin, and J. Zu, "Star pattern recognition method based on neural network," *Chinese Science Bulletin*, vol. 48, no. 18, pp. 1927–1930, 2003.
- [19] C. L. Cole and J. L. Crassidis, "Fast star-pattern recognition using planar triangles," *Journal of guidance, control, and dynamics*, vol. 29, no. 1, pp. 64–71, 2006.
- [20] D. Mortari, M. A. Samaan, C. Bruccoleri, and J. L. Junkins, "The pyramid star identification technique," *Navigation*, vol. 51, no. 3, pp. 171–183, 2004.
- [21] M. Kolomenkin, S. Pollak, I. Shimshoni, and M. Lindenbaum, "Geometric voting algorithm for star trackers," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, no. 2, pp. 441–456, 2008.
- [22] D. Mortari and J. Rogers, "A k-vector Approach to Sampling, Interpolation, and Approximation," *The Journal of the Astronautical Sciences*, vol. 60, no. 3-4, pp. 686–706, 2013.
- [23] D. Mortari, "A fast on-board autonomous attitude determination system based on a new star-ID technique for a wide FOV star tracker," *Advances in the Astronautical Sciences*, vol. 93, pp. 893–904, 1996.

- [24] D. Mortari and B. Neta, "K-vector range searching techniques," Adv. Astronaut. Sci, vol. 105, pp. 449–464, 2000.
- [25] F. L. Markley and D. Mortari, "How to estimate attitude from vector observations," 1999.
- [26] Cheng Yang and Shuster Malcolm D., "Improvement to the Implementation of the QUEST Algorithm," *Journal of Guidance, Control, and Dynamics*, vol. 37, no. 1, pp. 301–305, 2013. doi: 10.2514/1.62549.
- [27] M. Shuster, "Kalman filtering of spacecraft attitude and the QUEST model," *Journal of the Astronautical Sciences*, vol. 38, pp. 377–393, 1990.
- [28] J.-J. Kim, J. Tappe, A. Jordan, and B. Agrawal, Star Tracker Attitude Estimation for an Indoor Ground-Based Spacecraft Simulator. Guidance, Navigation, and Control and Co-located Conferences, American Institute of Aeronautics and Astronautics, aug 2011. doi:10.2514/6.2011-6270.

## List of Tables

1 Sensor Accuracy Ranges. Adapted from [10] . . . . . . . . . . . 6

## List of Figures

Robot Learning Darmstadt Problems with Euler Angles: Not Unique: Many angles result in the same rotation Hard to quantify differences between two Euler Angles Unit-Quaternion Solves the problems of singularities with the Euler Angles Easier to compute differences of orientations Important if we want to control the orientation of the end-effector See Siciliano or Spong Textbook!

Polar moment