A Comparison of Attitude Determination Methods for Small Satellites

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***Abstract*—This paper aims a first order comparative study between two different attitude determination methods. Study is accomplished in terms of accuracy and time required for computation. A test case satellite is chosen. Gravity gradient torque is taken into consideration as an external moment. Earth oblateness J_2 is taken. Sensors are magnetometer and Sun sensor.**

***Keywords*—small satellites; attitude determination; TRIAD; q-method**

I. Introduction (*Heading 1*)

Attitude determination is the process of obtaining attitude parameters from the measurements of minimum two kinds of physical quantities [1]. These physical quantities can be magnetic field, sun vector, stars, etc. Magnetometers, Sun sensors or star trackers can be used to measure these physical sensors that are used in small satellites because of the limitations of small satellite missions in terms of power and structure.

Process of attitude determination is sometimes referred deterministic method or point-to-point approach [2]. There are two methods proposed for attitude determination purposes which are TRIAD and q-method. Other methods which are described in last decades are derivation of those methods. Attitude determination methods are very advantageous for some purposes, such as low computational efforts, and independence of attitude motion models [1]. However, minimum two sensor measurements are required to apply methods.

Today's tendency on attitude is about estimation algorithms, such Kalman filter, and sometimes attitude determination algorithms are disregarded. However, estimation algorithms are required good initial guesses and attitude determination methods can provide useful information.

There is not much information in literature about which method should be used or when. This work addresses a need to answer questions about determination methods. A quantitative results are provided on accuracy and computational load. While accomplish this, only considered disturbance is gravity gradient torque because of negligible contribution of aerodynamic torque, magnetic dipole moment and solar radiation pressure. Earth oblateness J_2 is consideredin orbit modelling. Two-Line Elements (TLE) data of ITUpSAT-I is used.

II. Attitude Determination Methods

A. TRIAD

Let denote w_1 and w_2 two vectors in one coordinate frame (typically in body frame), v_1 and v_2 is in another coordinate frame (typically in reference frame. First step of the algorithm as follows:

$$\boldsymbol{r}_1 = \boldsymbol{w}_1 / |\boldsymbol{w}_1| \tag{1a}$$

$$\boldsymbol{r}_2 = (\boldsymbol{r}_1 \text{ x } \boldsymbol{w}_2) / |\boldsymbol{r}_1 \text{ x } \boldsymbol{w}_2| \tag{1b}$$

$$\boldsymbol{r}_3 = \boldsymbol{r}_1 \text{ x } \boldsymbol{r}_2 \tag{1c}$$

and the following matrices in reference frame:

$$\boldsymbol{s}_1 = \boldsymbol{v}_1 / |\boldsymbol{v}_1| \tag{2a}$$

$$\boldsymbol{s}_2 = (\boldsymbol{s}_1 \text{ x } \boldsymbol{v}_2) / |\boldsymbol{s}_1 \text{ x } \boldsymbol{v}_2| \tag{2b}$$

$$\boldsymbol{s}_3 = \boldsymbol{s}_1 \text{ x } \boldsymbol{s}_2 \tag{2c}$$

Then the transformation matrix between body and reference frame, namely attitude matrix is computed as follows [3]:

$$A = \boldsymbol{r}_1 .\, \boldsymbol{s}_1^T + \boldsymbol{r}_2 .\, \boldsymbol{s}_2^T + \boldsymbol{r}_3 .\, \boldsymbol{s}_3^T \tag{3}$$

B. q-Method

For a given vector r_i in reference frame and b_i in body frame, an orthogonal 3 x 3 matrix D that minimizes cost function L given by [4];

$$L(D) = 1/2 \sum a_i |\boldsymbol{b}_i - D\boldsymbol{r}_i|^2 \tag{4}$$

Here, a_i is weight of the measurement. It can be represented in quaternion form:

$$J(\boldsymbol{q}) = 1/2 \sum a_i |\boldsymbol{b}_i - D(\boldsymbol{q})\boldsymbol{r}_i|^2 \tag{5}$$

Equation (1), it is searched that quaternion $\boldsymbol{q}$ that minimizes cost function J. Here, q is the eigenvector corresponding the largest eigenvalue of K matrix [5], which generated as follows:

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$$\sigma = \sum a_i \boldsymbol{b}_i^{T} \boldsymbol{r}_i \quad (6a)$$

$$B = \sum a_i \boldsymbol{b}_i \boldsymbol{r}_i^{T} \quad (6b)$$

$$S = B + B^{T} \quad (6c)$$

$$\boldsymbol{z} = \sum a_i \boldsymbol{b}_i \times \boldsymbol{r}_i \quad (6d)$$

where T denotes the transpose, and

$$K = \begin{bmatrix} S - \sigma I_3 & z \\ z^T & \sigma \end{bmatrix} \quad (7)$$

I_3 is 3 x 3 identity matrix.

Because of the mathematical definition of quaternions, one can obtain negative quaternion. This could be explained by the relation between quaternion and the Euler axis rotation [6]. This problem could be solve by choosing the quaternion set which is scalar part is positive [7].

III. Test Parameters and Simulation

ITUpSAT-I is selected as test case spacecraft, which is first cubesat of Istanbul Technical University and was launched in 2009 [8]. Two-Line Elements (TLE) data is used for orbit propagation. The World Magnetic Model is used for magnetic field modelling [9]. Sun vector is modelled from previous studies [10]. Time step for the attitude dynamics simulation is taken as 0.1 sec and simulation is carried on along 600 steps. Gaussian white noise is added to the magnetic field and sun models to make the simulation more realistic. Initial quaternion values, angular velocities and moments of inertia in every axes for the satellite for simulation is selected as follows:

$$q_0 = \begin{bmatrix} 0.1768 \\ 0.3062 \\ 0.1768 \\ 0.9185 \end{bmatrix} \quad (8)$$

$$\omega_0 = \begin{bmatrix} 0.25 \\ -0.15 \\ 0.15 \end{bmatrix} \text{ rad/sec} \quad (9)$$

$$I = \begin{bmatrix} 2.1 & 0 & 0 \\ 0 & 2.0 & 0 \\ 0 & 0 & 1.9 \end{bmatrix} \times 10^{-3} \text{ kgm}^2 \quad (10)$$

Results are shown below.

IV. Results

A. TRIAD

After simulation of attitude dynamics, TRIAD algorithm is applied. Results are shown in Fig. 1:

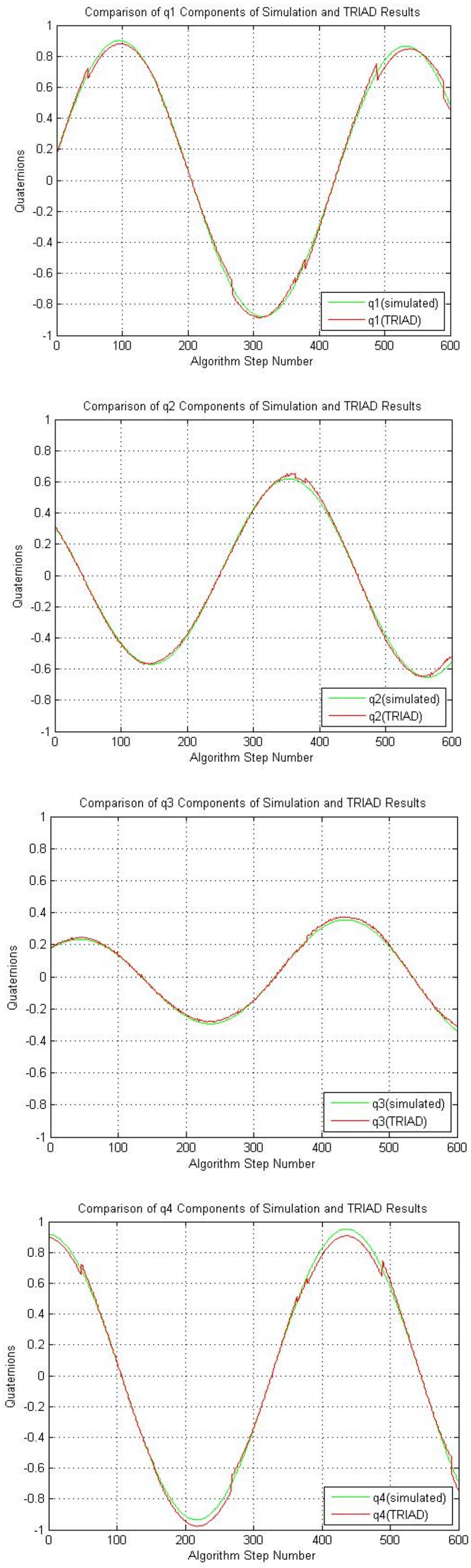

Fig. 1. TRIAD results for quaternion attitude

B. q-Method

In Fig. 2, q-method results are shown.

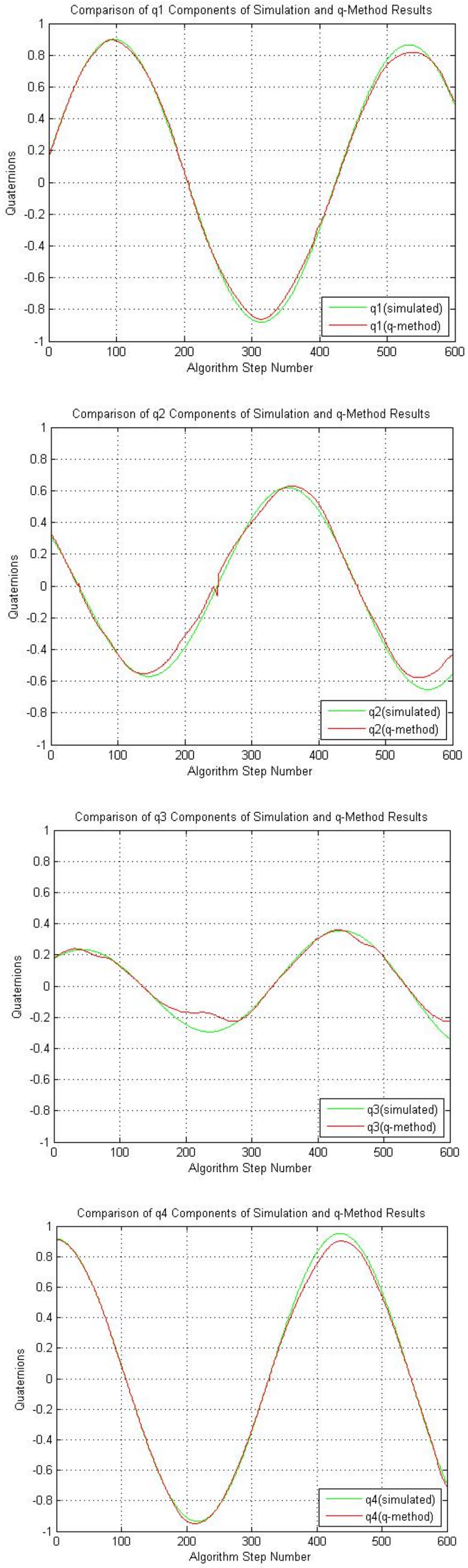

Fig. 2. q-Method results for quaternion attitude

C. Error Results

Absolute errors for both methods are shown in Fig. 3.

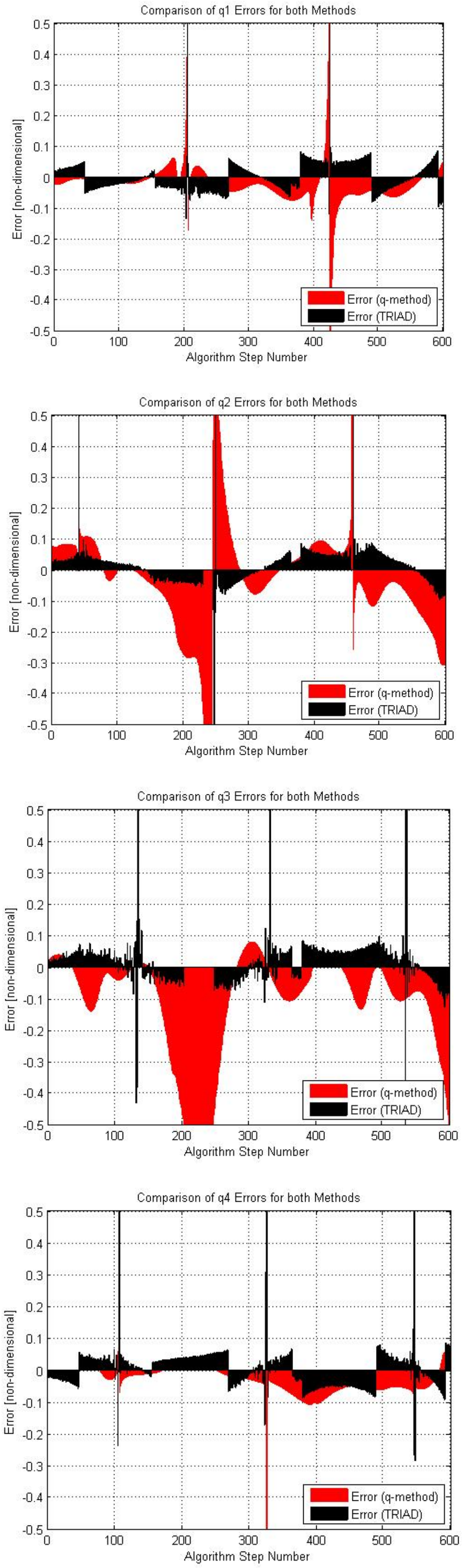

Fig. 3. Comparison of absolute errors of TRIAD and q-method

RMS values of error results are calculated for more detailed analysis. It is shown Table I.

TABLE I. RMS VALUES OF ABSOLUTE ERRORS

Quaternion	RMS Values	
	TRIAD	***q-Method***
q_1	0,024105638	0,023824791
q_2	0,016914358	0,044105982
q_3	0,011714810	0,039012602
q_4	0,030958420	0,028253846

D. Computational Load

Computational load analysis aims to provide that first order information about execution times for both methods. Rough values of execution times are calculated in computer programming environment. It is shown in Table II.

TABLE II. EXECUTION TIMES

Method	***TRIAD***	***q-Method***
Time (s)	0.079867	2.748969

Fig. 1, Fig. 2, Fig. 3 shows that performance of TRIAD and q-method algorithm is very close to each other except some regions. As seen in Table I, rms values of errors of each component of quaternion attitude point out that q-method has better performance for q1 and q4 component of attitude and TRIAD has better performance for others. Absolute errors of each methods does not exceed 0.1 except a region for q-method. TRIAD has less execution time than q-Method, as expected.

V. CONCLUSION

Today's tendency on satellite attitude determination mostly lies on attitude estimation algorithms, especially Kalman filtering. However, attitude determination algorithms still holds their position due to their low computational effort, relatively high accuracy and easily application. Since filtering algorithms need good first estimations for estimating attitude more accurate than those attitude determination algorithms, attitude determination algorithms can provide good first estimations for filtering algorithms.

There are not much work on attitude determination algorithms in literature. When or under which conditions attitude determination algorithms should be used? This question can be replied in terms of accuracies and computational load. Here, TRIAD and q-method have almost same accuracy, except some regions for q-Method. q-Method can deal with more than two measurements, and this is the main advantage of it. Low accuracy of q-Method in some regions can be caused by dealing with only two measurements. Because of optimality of q-Method, accuracy of q-Method is expected to be more than TRIAD.

Low computational effort is the main advantage for both methods. TRIAD requires least execution time. However, q-Method's need for execution time is not significant at all.

Attitude determination methods can be used under conditions of low computational capacity or under restrictions of small satellites' hardware. This work aims to provide to cost-benefit approach for the attitude determination and control system designer which method to be beneficial for satellite. Today, technological improvements overcome can overcome the problem of restrictions, and filtering algorithms take place of attitude determination algorithms. Nevertheless, their huge computational effort for estimation, and requirement for good estimation makes the attitude determination algorithms still valuable.

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