SP-Search: A New Algorithm for Star Pattern Recognition

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The new Spherical-Polygon (SP) Search approach, to identify stars observed by wide Field-Of-View (FOV) star tracker, which does not require initial guess of the spacecraft attitude and which does not use magnitude information, is presented. The principal idea of the proposed method is based on the fact that any star direction can always be expressed as a linear combination of two non-parallel star directions (star pair basis) together with their vector cross-product. The star identification problem is, therefore, transformed into the simpler problem of finding which stars, within a large star catalog, are admissible with a given direction. This is straightforwardly solved by approximating a cone with a spherical polygon and using the ***K***-vector technique applied to all of the three components of the given direction. Numerical and analytical studies are used to evaluate the main characteristics of the proposed method and to show how they could be improved (increasing the CCD precision and the star pair basis angular displacement). This new method to identify observed stars can be used within any general star pattern recognition technique based on stars triangles, especially for the multiple-FOVs star sensors (DIGISTAR II and III sensors) because they observe stars in orthogonal directions. Numerical plots validate this method, which is particularly suitable for the *lost in space* general case, by showing its very good probability of success (95% in average) to identify stars.

Introduction

Presently, the Charge-Coupled Device (CCD) star trackers are the sensors providing the most precise attitude data set. This implies a quite accurate spacecraft attitude estimation from vector observations. The precision of such instruments, which is continuously improving with time, is actually lower than 10 arcsec, especially thanks to the defocusing technique which is, at the present state of art, well tested and validated.

An additional and substantial improvement in the attitude data set precision is obtained with the multiple Field-Of-View (FOV) star trackers as the DIGISTAR II and III sensors, proposed in Reference 1. These star trackers, which use one/two mirrors deflecting the sensor FOV to two/three orthogonal directions, provide the attitude estimation algorithm with the optimal condition data set of almost orthogonal observed directions. This allows to obtain a substantial gain in precision (up to 28 times with respect to an equivalent-FOV star tracker) while the possibility of still using the sensor when one of its FOVs is not operating (because of the sun, the moon, the earth), that is, in a not fully operating mode, implies also a clear gain in terms of operating time. These multiple FOVs star sensors are proposed (and will be designed) by the Texas A&M University together with the University of Rome "La Sapienza", within the general DIGISTAR project, mainly devoted to the development of small star tracker technologies.[1,2]

During these two last decades many different approaches have been devised and presented to solve the problem to identify the star pattern observed by a wide-FOV star sensor, especially for the *lost*

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in space case, that is, when no attitude information is available. Some of these algorithms use angular separations,[3-5] others use angular separation with knowledge of an attitude initial guess,[6-8] others use star triangles,[9-10] but also fuzzy logic and neural network[11-13] have been used, and stochastic[14] approaches as well. Even though a serious comparison study among all of these different approaches is yet to be done, a great improvement in this field was given by the introduction of the ***K***-vector technique.[15] This technique has solved a problem common in almost all of the star identification techniques: the searching phase which, actually, represents the heaviest computational load of almost all of the existing star identification proposed procedures. This has allowed the development of a new star pattern recognition that does not require of any searching phase. The ***K***-vector technique, which was adapted[16] to the multiple FOVs star sensors DIGISTAR II and III, is a general procedure that could be adopted within almost any existing approach, by improving it in terms of speed. The method, therefore, makes very fast (since it does not require on-board searching phases) the problem to identify the indices of all the data x_i satisfying a given requirement $x_i \in [x_{\min}, x_{\max}]$, within any n-long data vector $\boldsymbol{x}$.

The proposed new SP-Search (Spherical Polygon-Search) approach to identify the stars as observed by a wide FOV star tracker, uses more than once the ***K***-vector technique, does not require any (accurate or not) initial guess of the spacecraft attitude, does not use the magnitude information (because inaccurate and varying with time), and is particularly suitable in the *lost in space* general case. The method uses a reference observed star pair as a basis on which the star identification process is accomplished for all of the other and remaining (n–2) observed stars. Since the performance of the proposed method increases with the star pair basis angular separation, it is clear that it will be more suitable for the new multiple FOVs star trackers DIGISTAR II and III. Due to lack of time, however, this paper only applies the proposed approach to an existing standard one-FOV star sensor leaving the application of the described method to DIGISTAR II and III sensors as the subject of a future paper.[18]

Star Identification Idea

Let us consider the problem to identify the n stars $\boldsymbol{s}_i$ (i=1-n) observed by a wide-FOV star tracker within a large star catalog. Using any two observed stars, say $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$, it will be always possible (with the only exception that stars $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$ are parallel or thereabouts, as for double stars) to express any other observed star $\boldsymbol{s}_k$ (where k=1-n, $k \neq i,j$) as

$$\boldsymbol{s}_k = a\boldsymbol{s}_i + b\boldsymbol{s}_j + c\boldsymbol{s}_i \times \boldsymbol{s}_j \tag{1}$$

The a, b, and c constants of Eq. (1) can be computed as

$$[a \quad b \quad c]^{\mathrm{T}} = [\boldsymbol{s}_i \quad \boldsymbol{s}_j \quad \boldsymbol{s}_i \times \boldsymbol{s}_j]^{-1} \boldsymbol{s}_k \tag{2}$$

Note that the spherical triangle defined by the three stars $\boldsymbol{s}_i$, $\boldsymbol{s}_j$, and $\boldsymbol{s}_k$, is completely identified by the a, b and c coefficients, which constitute an invariant set with respect the used system of coordinates. In particular, in the inertial system of coordinates, there will be three stars ($\boldsymbol{v}_i$, $\boldsymbol{v}_j$ and $\boldsymbol{v}_k$) satisfying

$$\boldsymbol{v}_k = a'\boldsymbol{v}_i + b'\boldsymbol{v}_j + c'\boldsymbol{v}_i \times \boldsymbol{v}_j \tag{3}$$

where, in the ideal case of using a star sensor with absolute precision, the conditions $a'=a$, $b'=b$ and $c'=c$, hold.

Now, the more precise the sensor is, the closer the a', b' and c' coefficients (*inertial* coefficients) used in Eq. (3) will be to the a, b and c *observed* coefficients of Eq. (1). The problem of how much the inertial coefficients will differ from the observed ones is a problem that might be analytically solved as a function of the angle between the star pair basis [$\boldsymbol{s}_i$, $\boldsymbol{s}_j$] and the sensor precision β. A numerical approach, however, is here adopted to quantify how far the true $\boldsymbol{v}_k$ is from the direction computed using the *observed* a, b and c coefficients.

Let us now use the $\boldsymbol{K}$-vector technique[15] for the [$\boldsymbol{s}_i$-$\boldsymbol{s}_j$] star pair basis. This technique leads to the construction of two integer vectors $\boldsymbol{I}$, and $\boldsymbol{J}$, containing the indices of all the admissible star pairs (for the sensor FOV aperture and the set magnitude threshold), and to a proper vector of indices, the $\boldsymbol{K}$-vector, the use of which allows an immediate identification of all of the n_{ij} admissible star pairs [$\boldsymbol{v}_{\boldsymbol{I}(k)}$, $\boldsymbol{v}_{\boldsymbol{J}(k)}$], (where $k \in [k_{\text{start}}\text{-}k_{\text{end}}]$, $k_{\text{end}} - k_{\text{start}} + 1 = n_{ij}$, and k_{end} and k_{start} are provided by the $\boldsymbol{K}$-vector), matching with the observed $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$.

Let us consider the kth admissible star pair [$\boldsymbol{v}_{\boldsymbol{I}(k)}$, $\boldsymbol{v}_{\boldsymbol{J}(k)}$]. Due to the ambiguity of whether $\boldsymbol{v}_{\boldsymbol{I}(k)}=\boldsymbol{v}_i$ and $\boldsymbol{v}_{\boldsymbol{J}(k)}=\boldsymbol{v}_j$, or $\boldsymbol{v}_{\boldsymbol{I}(k)}=\boldsymbol{v}_j$ and $\boldsymbol{v}_{\boldsymbol{J}(k)}=\boldsymbol{v}_i$, Eq. (3) provides two inertial directions associated with the considered star pair

$$\begin{cases} \boldsymbol{w}_k^{(1)} = a\boldsymbol{v}_{\boldsymbol{I}(k)} + b\boldsymbol{v}_{\boldsymbol{J}(k)} + c\boldsymbol{v}_{\boldsymbol{I}(k)} \times \boldsymbol{v}_{\boldsymbol{J}(k)} \\ \boldsymbol{w}_k^{(2)} = a\boldsymbol{v}_{\boldsymbol{J}(k)} + b\boldsymbol{v}_{\boldsymbol{I}(k)} + c\boldsymbol{v}_{\boldsymbol{J}(k)} \times \boldsymbol{v}_{\boldsymbol{I}(k)} \end{cases} \tag{4}$$

This implies that the true inertial $\boldsymbol{v}_k$ star, which identifies the observed $\boldsymbol{s}_k$ star, is close to one of the $2n_{ij}$ inertial directions provided by Eq. (4), with the index k ranging between the two values k_{start} and k_{end}.

Therefore, the proposed method can be applied with success only if it is demonstrated that:

1. the searched true inertial star $\boldsymbol{v}_k$ is assured to fall within a cone of axis $\boldsymbol{w}_k$ and aperture $h\beta$ (where h is a limited constant to be quantified), and
2. the probability to find wrong inertial star(s) within all of the $2n_{ij}$ cones of axes $\boldsymbol{w}_k$ [those evaluated using Eq. (4)] and aperture $h\beta$, is very small.

Next two sections deal with the demonstration of these two properties.

Numerical estimation of the error between w_k and v_k

As already said, due to sensor precision ($0 \leq \beta$), a difference between the *inertial* coefficients [a', b', c'] and the *observed* coefficients [a, b, c], arises. This is why, in the real case, the computed direction $\boldsymbol{w}_k$ will differ from the searched true direction $\boldsymbol{v}_k$. In this section this difference is quantified from a numerical point of view by providing the statistical angular distance between the computed direction $\boldsymbol{w}_k$, evaluated using Eq. (4), and the associated true direction $\boldsymbol{v}_k$, as a function of the sensor precision β and the angular distance between the basis star pair $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$.

Let us consider a reference circular-FOV star tracker with ϑ_{FOV}=10deg. of aperture (it observes stars with angular separation up to 20 deg.). For this reference star sensor Fig. 1 shows the average values obtained in 10,000 tests of the probability (in percentage) to find less than three stars as a function of the magnitude threshold m, because a minimum number of three stars is required to accomplish the star identification process. Therefore, according with Fig. 1, with a selected magnitude threshold of m=4.2, the number of the observed stars is greater or equal to three in the 99% of the cases.

Let us also consider that the sensor CCD provides star directions with uniform error distribution (maximum value of β=10 arcsec). With all of these data, which constitute our reference star sensor (ϑ_{FOV}=10deg., m=4.2, and β=10 arcsec), it is possible to evaluate the average values of the n_{ij} admissible star pairs as a function of the angle between $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$ (basis star pair angle).

Figure 2 plots the linear best fitting of the obtained results for a basis star pair step of 0.1 deg and for four different values of β=4, 7, 10 and 13 arcsec. The value of n_{ij}, for β=10", ranges as $1.59 \leq n_{ij} \leq 22.92$.

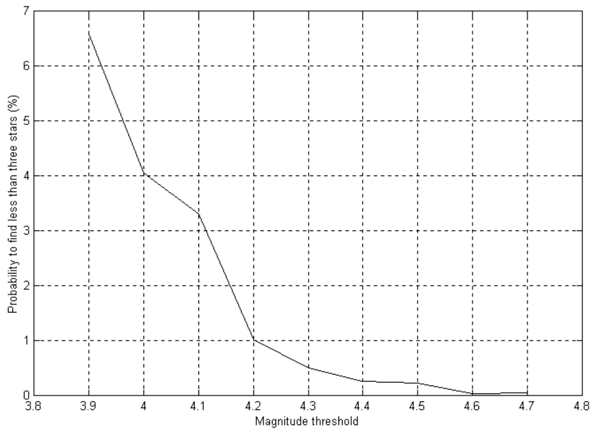

Fig. 1 Probability to observe n<3

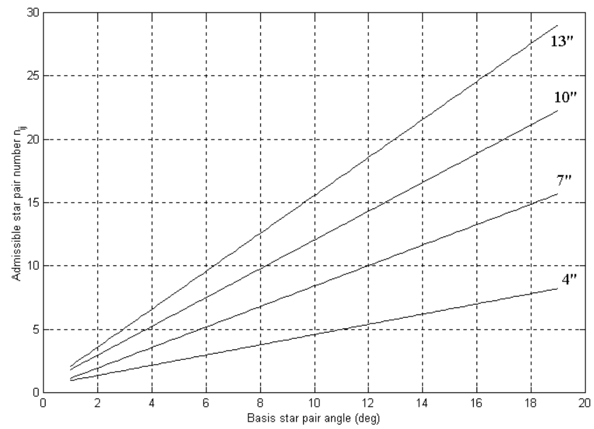

Fig. 2 Numerical evaluation of n_{ij}

Figure 3 shows the maximum and the mean values, obtained with 1000 random tests, of the angle between the computed $\boldsymbol{w}_k$ and its true direction $\boldsymbol{v}_k$, as a function of the basis star pair angle. This figure clearly shows that the basis star pair angle plays an important role in choosing the value of h and, consequently, in establishing the aperture $h\beta$ of the cone about $\boldsymbol{w}_k$ where it will be searched the true direction $\boldsymbol{v}_k$. The more the $\boldsymbol{s}_i$ is displaced from $\boldsymbol{s}_j$, the better the *inertial* coefficients [a', b', c'] approximate the *observed* coefficients [a, b, c], and therefore the closer $\boldsymbol{w}_k$ will be to $\boldsymbol{v}_k$.

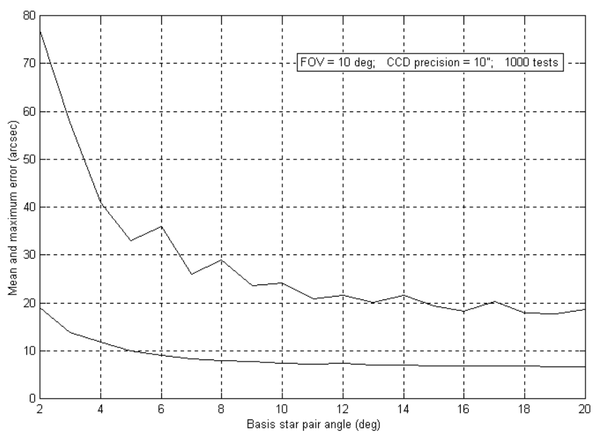

Fig. 3 Numerical evaluation of h

This implies that, for a single FOV star tracker, it will be convenient to choose the basis star pair as the most angularly separated. In particular, Fig. 3 suggests to choose h=8, for the used reference star sensor, provided that the angle between the star pair basis is, at least, of 2 degree.

The convenience of selecting the largest basis star pair reaches the maximum when the basis star pair consists of two orthogonal stars. Therefore, when a multiple-FOV star sensor (as DIGISTAR II or DIGISTAR III) is used, it will be convenient to select $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$ such that each one belongs to a different FOV.

In this way, the two stars constituting the basis star pair are kept close to being orthogonal to each other, condition that, not only maximizes the obtainable attitude accuracy, but also minimizes the error between the computed $\boldsymbol{w}_k$ and the true direction $\boldsymbol{v}_k$.

Figures 4 and 5 show the maximum and mean errors obtained in 1000 tests, between $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ for the multiple FOVs star trackers DIGISTAR II and DIGISTAR III, respectively. The CCD precision was set to β=10 arcsec and the DIGISTAR II and III sensors cover a sky spherical surface equivalent with that of the 10 deg. single-FOV star sensor. This implies a DIGISTAR II with two orthogonal FOVs of 7.07 deg. of aperture and a DIGISTAR III with three orthogonal FOVs of 5.77 deg. of aperture.

Figures 4 and 5 show two important aspects: 1) an error reduction (maximum and mean errors reduced by four times with respect to the equivalent one-FOV star sensor), and 2) an almost independent behavior with respect to the basis star pair angle. This fact confirms that the idea to identify stars as proposed will be better suitable for the DIGISTAR II and III multiple FOVs stars sensors. In fact, a future paper[18] will be dedicated to the extension of the proposed star pattern recognition to the multiple FOVs star trackers DIGISTAR II and DIGISTAR III.

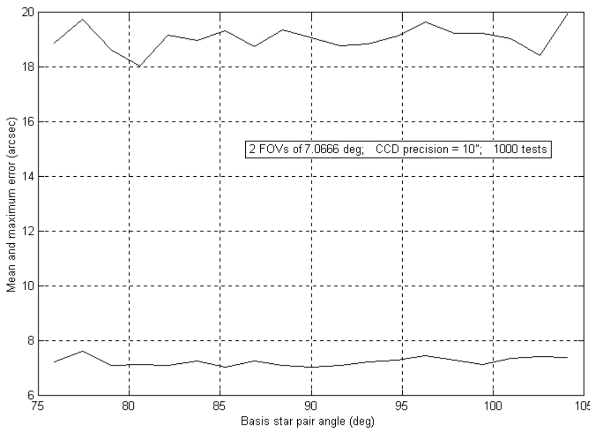

Fig. 4 w_k**-**v_k **error (DIGISTAR II)**

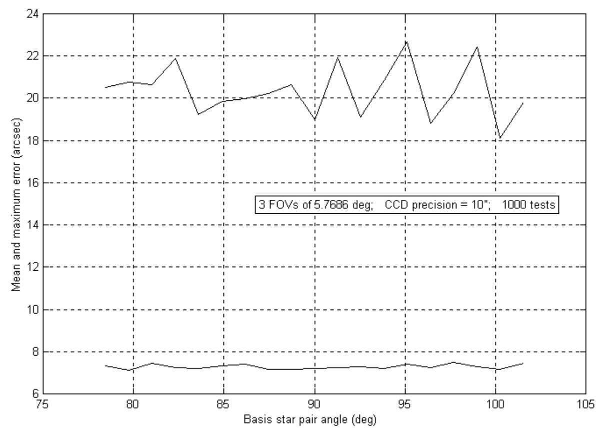

Fig. 5 w_k**-**v_k **error (DIGISTAR III)**

Probability to observe a wrong star

This section quantifies the probability to find a wrong inertial star within all of the $2n_{ij}$ cones of axes $\boldsymbol{w}_k$ and aperture $h\beta$. Figure 6 plots the number of all of the observable stars n as a function of the magnitude threshold m used. Note that the ordinate axis is logarithmic. Therefore a linear best fitting approximation of this curve is given by the function

$$\ln(n) \cong c_1 m + c_2 \tag{5}$$

where $\ln(n)$ provides the natural logarithm of the number of observable stars n and the coefficient values are approximated as $c_1 \cong 1.1132$, and $c_2 \cong 2.2789$. For our reference star sensor (magnitude threshold set to m=4.2), a number of n=1,047.7 stars become visible in the whole celestial sphere.

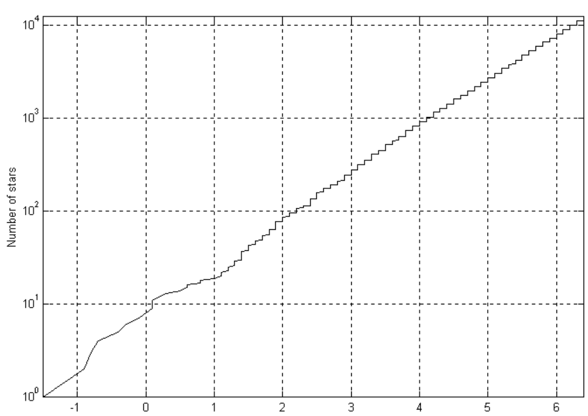

Fig. 6 Star number vs Magnitude

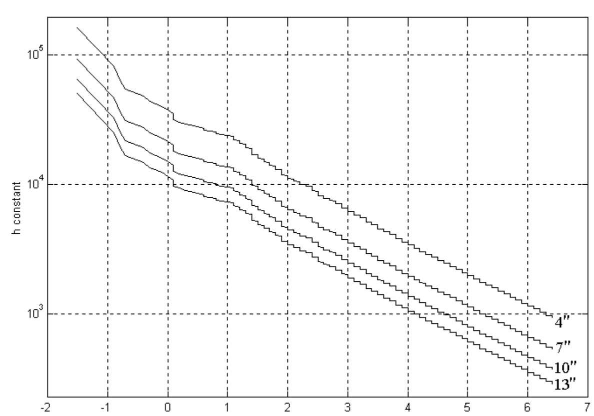

Fig. 7 Probability to observe one star

Let the star distribution be uniform over the celestial sphere. This implies a constant spherical star density of $\rho = n/(4\pi)$. With this assumption, the expected number of stars $E\{n\}$, falling within a cone of axis $\boldsymbol{w}_k$ [one of the two solutions given by Eq. (4)] and aperture $h\beta$ (where h is a constant to be defined), is

$$E\{n\} = \rho S(h\beta) = \frac{n}{4\pi} S(h\beta) \tag{6}$$

where $S(h\beta)$ indicates spherical area of a cone having aperture $h\beta$

$$S(h\beta) = \int_0^{2\pi} d\varphi \int_0^{h\beta} \sin\alpha \, d\alpha = 2\pi[1-\cos(h\beta)] \tag{7}$$

Thus, the expected number of stars becomes

$$E\{n\} = [1-\cos(h\beta)]n/2 \tag{8}$$

Figure 7 gives the values for h to obtain $E\{n\}=1$, as a function of the magnitude threshold m,

$$h = \frac{1}{\beta}\cos^{-1}[1-2e^{-(c_1 m+c_2)}] \tag{9}$$

and for the four different values of the maximum sensor error considered $\beta=[4, 7, 10, 13]$ arcsec. In particular for $m=4.2$ and $\beta=10$ arcsec, which are the values of the chosen reference star tracker, we obtain $h\approx 1{,}274.7$. Now, considering that n_{ij} ranges as $1.59 \le n_{ij} \le 22.92$, the probability $P\{n\}$ to find a wrong star within all the $2n_{ij}$ cones is

$$P\{n\} = 2n_{ij}E\{n\} = nn_{ij}[1-\cos(h\beta)] \tag{10}$$

which ranges between a minimum value of $1.25 \cdot 10^{-4}$ and a maximum value of $1.8 \cdot 10^{-3}$. In other words, a wrong star (in all of the $2n_{ij}$ cones) will be experienced once every 553.7 cases if $n_{ij}=22.92$ (worst case) and every 7,981.3 \cases if $n_{ij}=1.59$ (best case). These results give an idea on the reliability of the proposed method.

Finally, the remaining problem is how to find the admissible *inertial* stars $\boldsymbol{v}_k$ falling within the cone of axis $\boldsymbol{w}_k$ and aperture $h\beta$. To this end, the next section proposes a fast solution method, which approximates the observed cone area as an area identified by a spherical polygon.

Identification of all the admissible stars

The problem to find all the stars admissible with a given direction $\boldsymbol{w}_k \equiv \{x, y, z\}^{\mathrm{T}}$ with uncertainty $h\beta$, that is, all those falling within the cone of axis $\boldsymbol{w}_k$ and aperture $h\beta$, can be easily accomplished using the ***K***-vector technique applied to the three star components x, y, and z. In fact, the uncertainty cone of the $\boldsymbol{w}_k \equiv \{x, y, z\}^{\mathrm{T}}$ direction implies that the true component $x_{true}=\boldsymbol{v}_k(1)$ must fall within the range

$$x_{true} \in [x\cos(h\beta) - \sqrt{1-x^2}\sin(h\beta), \quad x\cos(h\beta) + \sqrt{1-x^2}\sin(h\beta)] \equiv [x_{\min}, \quad x_{\max}] \tag{11}$$

and, similarly for the other two components y_{true} and z_{true} of $\boldsymbol{v}_k$, within the ranges

$$\begin{cases} y_{true} \in [y\cos(h\beta) - \sqrt{1-y^2}\sin(h\beta), \quad y\cos(h\beta) + \sqrt{1-y^2}\sin(h\beta)] \equiv [y_{\min}, \quad y_{\max}] \\ z_{true} \in [z\cos(h\beta) - \sqrt{1-z^2}\sin(h\beta), \quad z\cos(h\beta) + \sqrt{1-z^2}\sin(h\beta)] \equiv [z_{\min}, \quad z_{\max}] \end{cases} \tag{12}$$

When using Eq. (11) to evaluate $x_{\min}$ and $x_{\max}$, it is important to check the following limit cases

$$\begin{cases} \text{if} \quad x \ge +\cos(h\beta) \quad \text{then} \quad x_{\max} = +1, \quad \text{and} \quad x_{\min} = x\cos(h\beta) - \sqrt{1-x^2}\sin(h\beta) \\ \text{if} \quad x \le -\cos(h\beta) \quad \text{then} \quad x_{\min} = -1, \quad \text{and} \quad x_{\max} = x\cos(h\beta) + \sqrt{1-x^2}\sin(h\beta) \end{cases} \tag{13}$$

occurring when $\boldsymbol{w}_k$ falls within the cone having as axis the $\boldsymbol{x}$ coordinate axis and aperture $h\beta$. Analogous conditions are similarly valid for the other two components y and z

$$\begin{gathered} \begin{cases} \text{if} \quad y \ge +\cos(h\beta) \quad \text{then} \quad y_{\max} = +1, \quad \text{and} \quad y_{\min} = y\cos(h\beta) - \sqrt{1-y^2}\sin(h\beta) \\ \text{if} \quad y \le -\cos(h\beta) \quad \text{then} \quad y_{\min} = -1, \quad \text{and} \quad y_{\max} = y\cos(h\beta) + \sqrt{1-y^2}\sin(h\beta) \end{cases} \\ \begin{cases} \text{if} \quad z \ge +\cos(h\beta) \quad \text{then} \quad z_{\max} = +1, \quad \text{and} \quad z_{\min} = z\cos(h\beta) - \sqrt{1-z^2}\sin(h\beta) \\ \text{if} \quad z \le -\cos(h\beta) \quad \text{then} \quad z_{\min} = -1, \quad \text{and} \quad z_{\max} = z\cos(h\beta) + \sqrt{1-z^2}\sin(h\beta) \end{cases} \end{gathered} \tag{14}$$

The stars satisfying the condition given in Eqs. (11,13) are distributed in an annular spherical surface identified as the area between the two cones having as axis the $\boldsymbol{x}$ coordinate axis and with aperture $x_{\min}$ and $x_{\max}$, respectively. Now the admissible stars are all those satisfying all the three different and orthogonal annular spherical surfaces (with respect to all of the three coordinate axes) identified by the Eqs. (11-14), respectively. Thus, the searched admissible stars are all those falling within the intersection among the three annular spherical surfaces depicted in Fig. 8. This spherical area is identified by the spherical polygon which approximates the searched cone of axis $\boldsymbol{w}_k$ and aperture $h\beta$. The spherical polygon is therefore given by the intersection of the spherical area

$$A(x,h\beta)=\begin{cases} = 2\pi[1-|x|\cos(h\beta)+\sqrt{1-x^2}\sin(h\beta)] & \text{if } |x|>\cos(h\beta) \\ = 4\pi\sqrt{1-x^2}\sin(h\beta) & \text{if } |x|\le\cos(h\beta) \end{cases} \tag{15}$$

together with the areas defined by $A(y,h\beta)$ and $A(z,h\beta)$. The maximum spherical area is achieved at $A(0,h\beta)$, which is that associated with a maximum annular radius. In this case, for the selected reference star tracker (m=4.2, h=8, and β=10 arcsec), the expected number of the undesired (wrong) stars, falling within the maximum spherical surface $A(0,h\beta)$, is

$$E\{n_{\max}\}=\rho A(0,h\beta)=e^{(c_1 m+c_2)}\sin(h\beta)\approx 0.406 \tag{16}$$

which represents an acceptable maximum value.

It is obvious that only two of the three annular spherical surfaces, shown in Fig. 8, can be *equatorial*. Moreover, the expected value $E\{n\}$ is less than $E\{n_{\max}\}$, the more accurate the star tracker is (smaller β), or, the more displaced the basis star pair is (smaller h), both of them reduce the value of $E\{n_{\max}\}$.

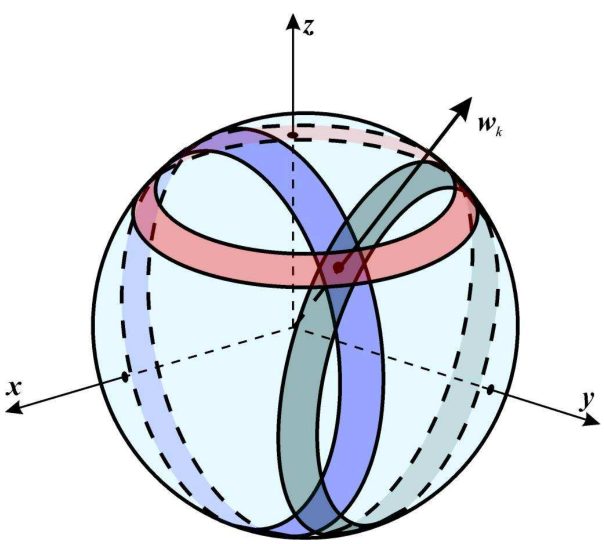

Fig. 8 The spherical polygon

Now, the problem to find all of the admissible stars, that is, all those falling within the spherical polygon defined by Eqs. (11-14), is solved using, once again, the $\boldsymbol{K}$-vector technique.[15]

Let us show, just for example, how to proceed with the first (the x) component.

Let $\boldsymbol{V}=[\boldsymbol{v}_1\ \boldsymbol{v}_2\ \ldots\ \boldsymbol{v}_{n-1}\ \boldsymbol{v}_n]$, where $\boldsymbol{v}_i\equiv\{x_i, y_i, z_i\}^{\mathrm{T}}$, be a reduced star catalog (it contains the stars with magnitude less and equal to the instrument magnitude threshold m) and $\boldsymbol{V}_x$ the same star catalog sorted by the x_i components in ascending order, that is, such that:

$$-1\le x_i\le x_{i+1}\le +1, \quad \forall i\in[1\text{-}(n-1)]$$

Let $\boldsymbol{I}_x$ be the associated integer vector containing the sorting indices, that is, $\boldsymbol{V}(\boldsymbol{I}_x(i))=\boldsymbol{V}_x(i)$, $\forall i\in[1\text{-}n]$.

The relevant $\boldsymbol{K}_x$-vector is then built as follows. Set the constants

$$a_1=\frac{2n}{(n-1)^2} \qquad \text{and} \qquad a_0=-\frac{n(n+1)}{(n-1)^2} \tag{17}$$

then, starting with $\boldsymbol{K}_x(1)=0$, the integer vector $\boldsymbol{K}_x$ is recursively constructed as

$$\boldsymbol{K}_x(i)=j \quad \text{where} \quad x_j\le a_1 i+a_0<x_{j+1} \qquad (i=1,\cdots,n) \tag{18}$$

From a practical point of view, $\boldsymbol{K}_x(i)$ gives the number of elements (they are j) below the value $a_1 i+a_0$.

Once the vectors $\boldsymbol{I}_x$ and $\boldsymbol{K}_x$ are built, the evaluation of the two indices identifying, in the star catalog $\boldsymbol{V}$, all of the data falling within the range $[x_{\min}, x_{\max}]$ (the admissible stars) becomes a straightforward and fast task. In fact, the extremes of the searched indices set $[k_{start} \leq k \leq k_{end}]$ are easily computed as

$$k_{start} = \boldsymbol{K}_x(bot\{(x_{\min} - a_0)/a_1\}) + 1 \quad \text{and} \quad k_{end} = \boldsymbol{K}_x(top\{(x_{\max} - a_0)/a_1\}) \tag{19}$$

where the function $top\{g\}$ rounds g to the nearest integer greater than g, and $bot\{g\}$ rounds g to the nearest integer lower than g. Then, with the k_{start} and k_{end} evaluated, all the admissible catalog stars $\boldsymbol{v}_i$ associated with the $\boldsymbol{w}_k$ are those identified by $\boldsymbol{v}_k=\boldsymbol{V}(\boldsymbol{I}_x(k))$, where $k\in[k_{start} - k_{end}]$ and where k_{start} and k_{end} are defined in Eq. (19).

The $[\boldsymbol{I}_y, \boldsymbol{K}_y]$ and the $[\boldsymbol{I}_z, \boldsymbol{K}_z]$ vectors, which are those associated with the y and the z components, are built and used similarly to the $[\boldsymbol{I}_x, \boldsymbol{K}_x]$ vectors. With all of these integer vectors, the identification of the stars belonging to the spherical polygon, that is, all of those satisfying Eqs. (11-14), becomes a fast and easy problem. If only one catalog star, say the $\boldsymbol{v}_k$ star, falls within this common area, then the $\boldsymbol{s}_k$ observed star is identified as the $\boldsymbol{v}_k$ star. The fact that only one star falls within the uncertainty cone occurs almost all the times, as confirmed by the numerical tests in the next section.

Numerical tests

In order to validate the above described procedure, a set of N=1000 numerical tests have been performed using the MATLAB[17] software. The reference star tracker used for the tests approximates the DIGISTAR I star sensor: it has one circular FOV with aperture of 10 deg, its CCD provides data with uniform error distribution and maximum error of β=10 arcsec, and the magnitude threshold is set to 4.2 (that is, m=4 with uncertainty of $\pm$0.2). For each test a random attitude was considered and the first two observed stars displaced by an angle greater than 2 deg (which justified the choice of h=8), are selected as the basis star pair. Using this star pair basis, the remaining (n–2) stars are identified with the procedure previously described.

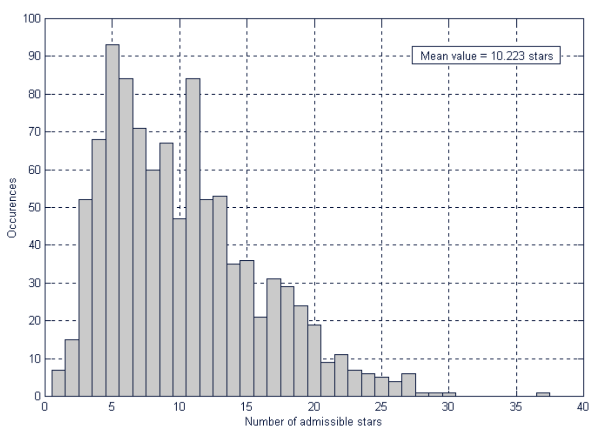

Fig. 9 Admissible star pairs number

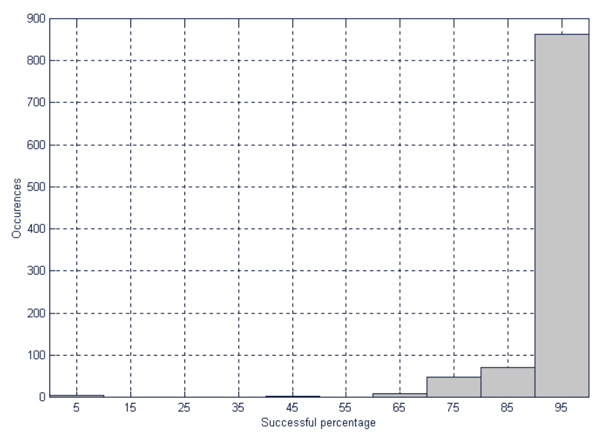

Fig. 10 Successful percentage histogram

Figure 9 shows the histogram of the number of the star pairs that are admissible with the star pair basis $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$. The average value of $n_{ij}\approx$10.223 is obtained. Figures 10 shows the overall histogram of the success (in percent) in identifying stars and Fig. 11 provides the successful percentage as a function of the number of the observed stars. In particular, the continuous line in Fig. 11, which indicates the average values of the obtained results, states that the proposed method works with success in more than 95% of the cases. This value increases with a higher precise sensor (lower β) and when adopting multiple FOVs star sensors. Finally, Figure 12 plots the percentage of success as a function of the angle between the stars of the star pair basis.

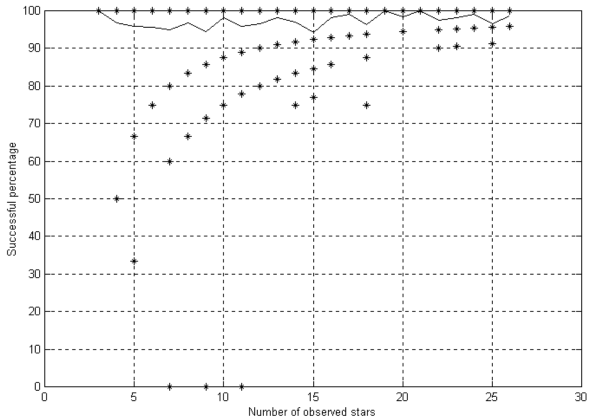

Fig. 11 Successful percentage results

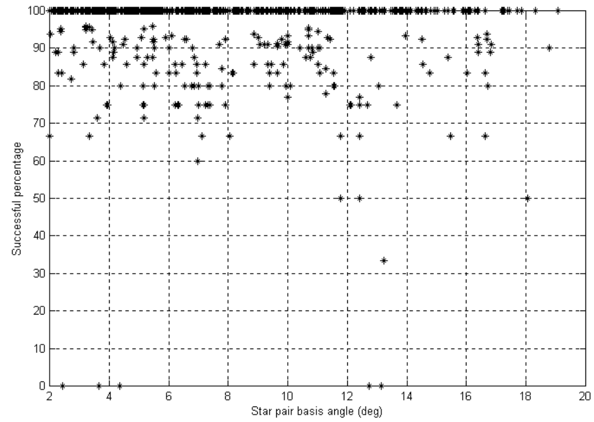

Fig. 12 Successful percentage as a function of the basis star pair angle.

The numerical tests performed are not exhaustive. They do not complete the analysis of the proposed method. They are provided only with the intention to show some of main characteristics of the proposed idea to identify observed stars. Comparisons with other competitive methods, in terms of speed, reliability (to discard spikes) and robustness (with respect to some uncertainties), should be done in a future paper as well as the adaptation (that is, the extension) of the proposed method to the multiple FOVs star sensors DIGISTAR II and III. [18]

Further researches and studies, completing this new approach to identify stars, should also be devoted to the star pattern recognition philosophy. In fact, the present paper does not suggest how the proposed method should be used within a general algorithm, leaving this important aspect waiting for future proposals.

Conclusions

This paper presents the new Spherical-Polygon (SP) Search approach to the problem of identifying stars observed by a wide Field-Of-View (FOV) star tracker which does not require initial guess of spacecraft attitude and does not use the magnitude information. The basic idea of the proposed method is originated from the fact that any direction, in particular an observed star direction, can be expressed as a linear combination of two non-parallel directions (star pair basis) together with their vector cross product. This moves the star pattern recognition problem into the problem of finding which stars, within a large star catalog, are admissible with respect to a given direction. The method extensively uses the ***K***-vector technique, which is applied to two observed stars (the star pair basis) first and then to all of the three components of the given direction.

The paper contains numerical and analytical studies investigating the principal characteristics of the proposed method. It is shown that the performances of the method improve when using higher precise CCD and for more displaced stars as the star pair basis. This fact makes this star pattern recognition technique suitable when applied to multiple-FOVs star sensors (DIGISTAR II and III star trackers), because they observe star directions approximately orthogonal to each other.

The method has a probability to identify observed stars of more than 95% (in average). It was originally devised to identify single stars (and it is used here only in this way). However, it could be the heart of a general star pattern identification algorithm, to be proposed. However, even though the method seems to be promising, further studies and researches should be done in order to validate it in terms of reliability, speed, and robustness. In particular, comparisons of the proposed method with other different algorithms - as well as the adaptation of the method to the multiple FOVs star sensor DIGISTAR II and III - are also expected to be done.

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References

1. MORTARI, D., POLLOCK, T.C., and JUNKINS, J.L., "Towards the Most Accurate Attitude Determination System Using Star Trackers," Paper AAS 98-159 of the 8th Annual AIAA/AAS Space Flight Mechanics Meeting, Monterey, CA, Feb. 9-11, 1998.
2. JU, G., and JUNKINS, J.L., "Comparative Study for Specifications of Small Star Trackers," Internal Document of the Center for Mechanics and Control, Texas A&M University, College Station, October 13, 1997.
3. RAPPAPORT, B., DUNNING, T., JORDAN, J., PHILLIPS, K., and STANTON, R., "Autonomous Star Identification for Spacecraft Attitude Control," Proceedings of the Conference in Astronomy from Large Databases: Scientific Objectives and Methodological Approaches, Garching, Germany, Oct. 1987, pp. 239-244.
4. DE ANTONIO, L., UDOMKESMALEE, S., ALEXANDER, J., BLUE, R., DENNISON, E., SEVASTON, G., and SCHOLL, M., "Star-Tracker Based, All-Sky, Autonomous Attitude Determination," SPIE Proceedings, Vol. 1949, 1993, pp. 204-215.
5. QUINE, B., and DURRANT-WHYTE, H.F., "A Fast Autonomous Star Acquisition Algorithm for Spacecraft". Paper T1-16 of the IFAC Conference on Intelligent Autonomous Control in Aerospace, IACA' 95, Aug. 14-16, 1995, Beijing, China.
6. JUNKINS, J.L., WHITE, C.C., and TURNER, J.D., "Star Pattern Recognition for Real Time Attitude Determination," *Journal of the Astronautical Sciences*, Vol. 25, 1977, pp. 251-270.
7. STRIKWERDA, T.E., and JUNKINS, J.L., "Star Pattern Recognition and Spacecraft Attitude Determination," U.S. Army Engineer Topographic Laboratories, ETL-0260, Fort Belvoir, VA, May 1981.
8. STRIKWERDA, T.E., FISHER, H.L., KILGUS, C.C., and FRANK, L.J., "Autonomous Star Identification and Spacecraft Attitude Determination with CCD Star Trackers," Spacecraft Guidance, Navigation and Control Systems - First International Conference held at ESTEC, Noordwijk, The Netherlands, 4-7 June, 1991, pp. 195-200.
9. SASAKI, T., and KOSAKA, M., "A Star Identification Method for Satellite Attitude Determination using Star Sensors," Proceedings of the Fifteenth International Symposium on Space Technology and Sciences, Tokyo, Japan, May 1986, pp. 1125-1130.
10. LIEBE, C.C., "Pattern Recognition of Star Constellations for Spacecraft Applications," IEEE AES Magazine, Vol. 7, 1992, pp. 34-41.
11. GUNDERSON, R.W., "Application of Fuzzy Isodata Algorithms to Star Tracker Pointing Systems," Proceedings of the Seventh Triennial World Congress, Helsinki, Finland, June 1978, pp. 1319-1323.
12. SINGLEY, M.E., "Pattern Recognition for Space Applications," DDF Final Report, NASA Marshall Space Flight Center, AL, NASA-TM-82586, May 1984.
13. ALVEDA, P., and SAN MARTIN, A.M., "Neural Network Star Pattern Recognition of Spacecraft Attitude Determination and Control," Advances in Neural Information Processing System I, Denver, 1988, pp. 314-322.
14. UDOMKESMALEE, S., ALEXANDER, J.W., and TOLIVAR, A.F., "Stochastic Star Identification," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 6, November-December 1994, pp. 1283-1286.
15. MORTARI, D., "Search-Less Algorithm for Star Pattern Recognition," *Journal of the Astronautical Sciences*, Vol. 45, No. 1, January-March, 1997.
16. MORTARI, D., and ANGELUCCI, M., "Star Pattern Recognition and Mirror Assemble Misalignment for DIGISTAR II and III Multiple FOVs Star Sensors," Paper AAS 99-182 of the 9th Annual AIAA/AAS Space Flight Mechanics Meeting, Breckenridge, CO, Feb. 7-10, 1999.
17. MATLAB Reference Guide. The MATH WORKS Inc., Natick, MA, October 1992.
18. MORTARI, D., JUNKINS, J.L., and ANGELUCCI, M., "SP-Search Star Pattern Recognition for Multiple Field-of-View Star Trackers," Submitted to the AAS/AIAA Astrodynamics Specialist Conference, Girdwood AK, August 15-19, 1999.