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MASTER OF SCIENCE THESIS

# STAR-TRACKER PROGRAM FOR CUBESAT SATELLITES

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## Nomenclature

$c$       Speed of light in a vacuum inertial frame

$h$       Planck constant

[1] [2] [3] [4] [5] [6] [7] [8]

# 1 Introduction

## 1.1 Motivation

The goal of this work is to make fully operational star-tracker program, that could be used on Cubesat satellites. Such program could be used on space missions and could start Polish state-of-the-art technology in growing space technology sector.

## 1.2 Outline of thesis

This thesis consists of several chapters. Here they are shortly summarized:

**Chapter 1** serves as introduction to this thesis and describes the motivation and goal of this work. It also describes the background of the topic.

**Chapter 2** describes all the important foundations for the fully understanding given work.

**Chapter 3** is the main part of this thesis. It describes how the star-tracker program works and goes through detailed comparison of different approaches.

**Chapter 4** describes the created prototype of star-tracker in Python language.

**Chapter 5** talks about the implementation of star-tracker on the existing prototype of on-board computer.

**Chapter 6** describes how the finished program is performing.

**Chapter 7** contains conclusions about this work and created star-tracker program.

### 1.3 Cubesat

Cubesat was designed on CalPoly in 1999[9]. Dimensions of satellite are measured in units. Each unit (often described simply as u) can be 10x10x10cm and can weight up to 1.33 kg. Satellites can be 1u, 2u, 3u, 6u or even 12u.

Such small satellites are susceptible to noise from densly packed electronics.

Zdjecie Cubesata

CubeSat missions, goals, what can they be and are used for? Why is it innovative and important?

### 1.4 Means of attitude estimation

There exist many different types of attitude estimation: sun sensors, star-trackers, magnetometers, etc. However star-tracker gives the best possible accuracy for nowadays and is not susceptible to electrical nor magnetic noise.

#### 1.4.1 Megnetometers

#### 1.4.2 Sun sensors

#### 1.4.3 Earth sensors

#### 1.4.4 GPS

#### 1.4.5 Star trackers

[10] [12]

Sensor	Accuracy		
Magnetometers	1.0o (5000km alt)	5.0 (200 km alt)	Attitude measured relative to Earth
Earth sensors	0.05 (GEO) 0.1 (LEO)		
Sun sensors	0.01		
Star sensors	2 arc-sec		
Gyroscopes	0.001 deg/hr		
Directional antennas	0.01 to 0.5		

Table 1: Sensor Accuracy Ranges. Adapted from [11]

## 1.5 On-board computer

This section will describe the on-board computer which was done as part of other thesis.

## **2 Preliminaries**

### **2.1 Coordinate frames**

#### **2.1.1 ECI frame**

[10]



### 2.1.2 ECEF frame

### 2.1.3 NED frame

### 2.1.4 BODY frame

## 2.2 Space environment

## 2.3 Attitude representations

### 2.3.1 Euler angles

### 2.3.2 Quaternions

## 2.4 Quaternion properties

### 2.4.1 Advantages of quaternions

### 2.4.2 Multiplication of quaternions

### 2.4.3 Quaternions and rotations

## 2.5 Wahba's problem

[13]

$$\sum_j^n ||r_j - Mb_j|| \tag{1}$$

## **2.6 Cholesky factorization**

## **2.7 Lyapunov analysis**

### 3 Star-tracker program

[14]

Generally star-tracker is divided into three main parts[15]:

- recogiting stars on the image and converting the data into list of star vectors by calculating star centroids;
- identyfing which star vector represents which real star in catalogue. This is done by comparing star vectors from the image with data in star catalogue, which is generated before space mission;
- estimating the attitude by calculating the displacement between two frames.

#### 3.1 Centroid - start recognition

[16]

Due to limitations of camera there exists necessity of calculating star centroids. Each camera converts image into photo divided by pixels. As it is necessary to have high precision of star coordinates, the pixel accuracy is not enough. Subpixel accuracy is needed. Typically it is done by defocusing the lens of the camera and calculating the lumosity of all pixels around the lightest ones. The idea of how to calculate such centroids is adapted from[15].

If FOV is too small, one star will be considered by program as few stars, and if FOV is too large, few stars placed near each other will be considered as one star. Calculating star centroids is tradeoff between counting few stars as one and counting one star as a few. It seems however that it is worse to count one star as few than few stars as one.

$$x_{start} = x - \frac{a_{ROI} - 1}{2} \quad (2)$$

$$y_{start} = y - \frac{a_{ROI} - 1}{2} \quad (3)$$

$$x_{end} = x_{start} + a_{ROI} \quad (4)$$

$$y_{end} = y_{start} + a_{ROI} \quad (5)$$

$$I_{bottom} = \sum_{i=1}^{x_{end}-1} I(i, y_{start}) \quad (6)$$

$$I_{top} = \sum_{i=2}^{x_{end}} I(i, y_{end}) \quad (7)$$

$$I_{left} = \sum_{j=1}^{y_{end}-1} I(x_{start}, j) \quad (8)$$

$$I_{right} = \sum_{j=2}^{y_{end}} I(x_{start}, j) \quad (9)$$

$$I_{border} = \frac{I_{top} + I_{bottom} + I_{left} + I_{right}}{4(a_{ROI} - 1)} \quad (10)$$

$$\tilde{I}(x, y) = I(x, y) - I_{border} \quad (11)$$

$$B = \sum_{i=x_{start}+1}^{x_{end}-1} \sum_{j=y_{start}+1}^{y_{end}-1} \tilde{I}(i, j) \quad (12)$$

$$x_{CM} = \sum_{i=x_{start}+1}^{x_{end}-1} \sum_{j=y_{start}+1}^{y_{end}-1} \frac{i \times \tilde{I}(i, j)}{B} \quad (13)$$

$$x_{CM} = \sum_{i=x_{start}+1}^{x_{end}-1} \sum_{j=y_{start}+1}^{y_{end}-1} \frac{j \times \tilde{I}(i, j)}{B} \quad (14)$$

$$u = \frac{\begin{bmatrix} \mu x_{CM} & \mu y_{CM} & f \end{bmatrix}^T}{\| \begin{bmatrix} \mu x_{CM} & \mu y_{CM} & f \end{bmatrix} \|} \quad (15)$$

### 3.2 Star identification

all [17]

Brightness Independent 4-Star Matching Algorithm for Lost-in-Space  
3-Axis Attitude Acquisition[18]

SP-Search: A New Algorithm for Star Pattern Recognition [19]

Star Identification using Neural networks [20] [21]

Star pattern recognition using neural networks [22]

#### 3.2.1 Angle Matching

[23]

$$\theta = \cos^{-1}(\mathbf{r}_1 \cdot \mathbf{r}_2) \quad (16)$$

$$\mathbf{b}_i = A\mathbf{r}_i \quad (17)$$

$$\tilde{\mathbf{b}}_i = A\mathbf{r}_i + \mathbf{v}_i, \quad \mathbf{v}_i^T A\mathbf{r}_i = 0 \quad (18)$$

$$\begin{aligned} E\{\mathbf{v}_i\} &= 0 \\ E\{\mathbf{v}_i \mathbf{v}_i^T\} &= \sigma_i^2 [\mathbf{I} - (A\mathbf{r}_i)(A\mathbf{r}_i)^T] \\ \mathbf{b}_i^T \mathbf{b}_j &= \mathbf{r}_i^T A^T A \mathbf{r}_j = \mathbf{r}_i^T \mathbf{r}_j \end{aligned} \quad (20)$$

$$\tilde{\mathbf{b}}_i = A\mathbf{r}_i + \mathbf{v}_i$$

$$\tilde{\mathbf{b}}_j = A\mathbf{r}_j + \mathbf{v}_j$$

$$z \equiv \tilde{\mathbf{b}}_i^T \tilde{\mathbf{b}}_j = \mathbf{r}_i^T \mathbf{r}_j + \mathbf{r}_i^T A^T \mathbf{v}_j + \mathbf{r}_j^T A^T \mathbf{v}_i + \mathbf{v}_i^T \mathbf{v}_j \quad (22)$$

$$E\{z\} = \mathbf{r}_i^T \mathbf{r}_j \quad (23)$$

$$p \equiv z - E\{z\} = \mathbf{r}_i^T A^T \mathbf{v}_j + \mathbf{r}_j^T A^T \mathbf{v}_i + \mathbf{v}_i^T \mathbf{v}_j \quad (24)$$

$$\sigma_p^2 \equiv E\{p\} = \mathbf{r}_1^T A^T R_2 A \mathbf{r}_1 + \mathbf{r}_2^T A^T R_a A \mathbf{r}_2 + \text{Trace}(R_1 R_2) = \text{Trace}(A \mathbf{r}_1 \mathbf{r}_1^T R_2) + \text{Trace}(A \mathbf{r}_2 \mathbf{r}_2^T R_1) + \text{Trace}(R_1 R_2) \quad (25)$$

### 3.2.2 Spherical Triangle Matching

[24]

$$A = 4 \tan^{-1} \sqrt{\tan \frac{s}{2} \tan \frac{s-a}{2} \tan \frac{s-b}{2} \tan \frac{s-c}{2}} \quad (26)$$

$$\begin{aligned} s &= \frac{1}{2}(a + b + c) \\ a &= \cos^{-1} \left( \frac{b_1 \cdot b_2}{|b_1||b_2|} \right) \\ b &= \cos^{-1} \left( \frac{b_2 \cdot b_3}{|b_2||b_3|} \right) \\ c &= \cos^{-1} \left( \frac{b_3 \cdot b_1}{|b_3||b_1|} \right) \end{aligned}$$

$$I_p = \sum \theta^2 dA \quad (28)$$

### 3.2.3 Planar Triangle

[25]

$$s = \frac{1}{2}(a + b + c) \quad (29)$$

$$a = ||\mathbf{u}_p - \mathbf{u}_q|| \quad (30)$$

$$b = ||\mathbf{u}_q - \mathbf{u}_r|| \quad (31)$$

$$c = ||\mathbf{u}_p - \mathbf{u}_r|| \quad (32)$$

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad (33)$$

$$J = A \frac{(a^2 + b^2 + c^2)}{36} \quad (34)$$

Derivatives

$$H = [\mathbf{h}_1^T \quad \mathbf{h}_2^T \quad \mathbf{h}_3^T] \quad (35)$$

$$\begin{aligned} \mathbf{h}_1^T &\equiv \frac{\delta A}{\delta a} \frac{\delta a}{\delta \mathbf{b}_1} + \frac{\delta A}{\delta c} \frac{\delta c}{\delta \mathbf{b}_1} \\ \mathbf{h}_2^T &\equiv \frac{\delta A}{\delta a} \frac{\delta a}{\delta \mathbf{b}_2} + \frac{\delta A}{\delta b} \frac{\delta b}{\delta \mathbf{b}_2} \\ \mathbf{h}_3^T &\equiv \frac{\delta A}{\delta b} \frac{\delta b}{\delta \mathbf{b}_3} + \frac{\delta A}{\delta c} \frac{\delta c}{\delta \mathbf{b}_3} \end{aligned}$$

$$\begin{aligned} \frac{\delta A}{\delta a} &= \frac{u_1 - u_2 + u_3 + u_4}{4A} \\ \frac{\delta A}{\delta b} &= \frac{u_1 + u_2 - u_3 + u_4}{4A} \\ \frac{\delta A}{\delta c} &= \frac{u_1 + u_2 + u_3 - u_4}{4A} \end{aligned}$$

$$\begin{aligned} u_1 &= (s - a)(s - b)(s - c) \\ u_2 &= s(s - b)(s - c) \\ u_3 &= s(s - a)(s - c) \\ u_4 &= s(s - a)(s - b) \end{aligned}$$

$$\begin{aligned} \frac{\delta a}{\delta \mathbf{b}_1} &= (\mathbf{b}_1 - \mathbf{b}_2)^T / a, & \frac{\delta a}{\delta \mathbf{b}_2} &= -\frac{\delta a}{\delta \mathbf{b}_1} \\ \frac{\delta b}{\delta \mathbf{b}_2} &= (\mathbf{b}_2 - \mathbf{b}_3)^T / b, & \frac{\delta b}{\delta \mathbf{b}_3} &= -\frac{\delta b}{\delta \mathbf{b}_2} \\ \frac{\delta c}{\delta \mathbf{b}_1} &= (\mathbf{b}_1 - \mathbf{b}_3)^T / c, & \frac{\delta c}{\delta \mathbf{b}_3} &= -\frac{\delta c}{\delta \mathbf{b}_1} \end{aligned}$$

$$\sigma_A^2 = H R H^T \quad (40)$$

$$R \equiv \begin{bmatrix} R_1 & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & R_2 & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & R_3 \end{bmatrix} \quad (41)$$

Polar Moment

$$\bar{H} = [\bar{\mathbf{h}}_1^T \quad \bar{\mathbf{h}}_2^T \quad \bar{\mathbf{h}}_3^T] \quad (42)$$

$$\begin{aligned} \bar{\mathbf{h}}_1^T &\equiv \frac{\delta J}{\delta a} \frac{\delta a}{\delta \mathbf{b}_1} + \frac{\delta J}{\delta c} \frac{\delta c}{\delta \mathbf{b}_1} + \frac{\delta J}{\delta A} \mathbf{h}_1^T \\ \bar{\mathbf{h}}_2^T &\equiv \frac{\delta J}{\delta a} \frac{\delta a}{\delta \mathbf{b}_2} + \frac{\delta J}{\delta b} \frac{\delta b}{\delta \mathbf{b}_2} + \frac{\delta J}{\delta A} \mathbf{h}_2^T \\ \bar{\mathbf{h}}_3^T &\equiv \frac{\delta J}{\delta b} \frac{\delta b}{\delta \mathbf{b}_3} + \frac{\delta J}{\delta c} \frac{\delta c}{\delta \mathbf{b}_3} + \frac{\delta J}{\delta A} \mathbf{h}_3^T \end{aligned}$$

$$\begin{aligned} \frac{\delta J}{\delta a} &= Aa/18, & \frac{\delta J}{\delta a} &= Ab/18, & \frac{\delta J}{\delta a} &= Ac/18 \\ & & \frac{\delta J}{\delta A} &= (a^2 + b^2 + c^2)/36 \end{aligned}$$

$$\sigma_J^2 = \bar{H} R \bar{H}^T \quad (45)$$

### 3.2.4 Pyramid

[26]

### 3.2.5 Rate Matching

[27]



### 3.2.6 Voting

[28]

### 3.2.7 Grid

[29]

## 3.3 Star-catalogue and searching for matching stars

### 3.3.1 Star Catalogue Generation

$$\mathbf{u} = \begin{bmatrix} \cos \alpha \cos \delta \\ \sin \alpha \cos \delta \\ \sin \delta \end{bmatrix} \quad (46)$$

$$m_i \leq m_{max} \quad (47)$$

$$m_j \leq m_{max} \quad (48)$$

$$\mathbf{u}_a^T \mathbf{u}_b \geq \cos \theta_{FOV} \quad (49)$$

### 3.3.2 Candidate Matching

### 3.3.3 Result Verification

### 3.3.4 k-vector

[30]

[31]

[32]

### 3.4 Attitude Determination

[1]

AIM (Attitude estimation using Image Matching)[3]

all [11] [33]

#### 3.4.1 The Predictive Attitude Determination Algorithm ?

[34]

#### 3.4.2 q-method

#### 3.4.3 QUEST

improvement to quest implementation [35]

kallman filtering [36]

#### 3.4.4 TRIAD

#### 3.4.5 The Fast Optimal Attitude Matrix

#### 3.4.6 DCM (direction cosine matrix) - (Singular Value Decomposition?)

[37] and

[15]

$$B = \sum_{i=1}^n b_i r_i^T \quad (50)$$

$$B = U S V^T \quad (51)$$

$$\mathbf{U}_+ = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det \mathbf{U} \end{bmatrix} \quad (52)$$

$$\mathbf{V}_+ = \mathbf{V} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det \mathbf{V} \end{bmatrix} \quad (53)$$

$$\mathbf{A} = \mathbf{U}_+ \mathbf{V}_+^T \quad (54)$$

## 4 Prototype

For now the following parts are finished in Python:

1. Centroiding
2. Planar Triangle Recognition with variations (nearly)
3. Pyramid alg
4. k-vector
5. QUEST (not started yet)

Testing

[38]

## 5 Complete program

## **6    Testing of star-tracker**

[39]

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## List of Figures

Robot Learning Darmstadt Problems with Euler Angles: Not Unique:  
Many angles result in the same rotation Hard to quantify differences between  
two Euler Angles Unit-Quaternion Solves the problems of singularities with  
the Euler Angles Easier to compute differences of orientations Important if  
we want to control the orientation of the end-effector See Siciliano or Spong  
Textbook!

Polar moment