Lesson 1

Topic: Gale Shapley Algorithm Quiz

QUIZ (10mins)

Question 1.

Which of the following best describes the Gale-Shapley algorithm?
a) A greedy algorithm that finds the maximum matching between two sets of elements.
b) A dynamic programming algorithm that finds the shortest path between two nodes in a graph.
c) A divide and conquer algorithm that sorts an array in O(n log n) time.
d) A randomized algorithm that finds the median of a set of numbers.

Answer: a) A greedy algorithm that finds the maximum matching between two sets of elements.

Question 2.

In the following preference table for 4 men and 4 women, what would be the result of the Gale-Shapley algorithm?

Men	Women	Preference Order
M1	W1	W3, W1, W2, W4
M2	W2	W1, W2, W4, W3
M3	W3	W2, W3, W1, W4
M4	W4	W3, W1, W4, W2

Women	Men	Preference Order
W1	M4	M1, M4, M3, M2
W2	M1	M2, M4, M1, M3
W3	M3	M4, M2, M1, M3
W4	M2	M3, M1, M2, M4

a) M1 and W1 would end up together
b) M2 and W2 would end up together
c) M3 and W3 would end up together
d) M4 and W4 would end up together

Answer: d) M4 and W4 would end up together

Question 3.

In a matching between two sets of elements using the Gale-Shapley algorithm, what is a stable matching?

a) A matching where every element is matched to their first choice.
b) A matching where no element prefers someone outside of their current match.
c) A matching where every element has made at least one proposal.
d) A matching where every element has been rejected at least once.

Answer: b) A matching where no element prefers someone outside of their current match

Question 4.

Consider the following preferences for four men and four women:

Men Preferences:

M1: W3 > W2 > W1 > W4

M2: W2 > W3 > W1 > W4

M3: W1 > W2 > W3 > W4

M4: W1 > W3 > W2 > W4

Women Preferences:

W1: M3 > M2 > M1 > M4

W2: M2 > M1 > M3 > M4

W3: M1 > M2 > M3 > M4

W4: M1 > M2 > M3 > M4

If the Gale-Shapley algorithm is run on these preferences, which of the following is a possible stable matching?

A) M1-W3, M2-W2, M3-W1, M4-W4

B) M1-W3, M2-W2, M3-W4, M4-W1

C) M1-W4, M2-W3, M3-W2, M4-W1

D) None of the above.

Answer: D) None of the above.

Explanation: If we run the Gale-Shapley algorithm on these preferences, the algorithm will converge to a stable matching, but none of the given matchings are stable. One way to see this is to note that M1 prefers W3 over his current partner W1, and W3 prefers M1 over her current partner M3. If M1 and W3 were to form a pair, they would both prefer each other over their current partners, leading to an unstable situation. Similarly, M2 prefers W2 over his current partner W3, and W2 prefers M2 over her current partner M1, so switching to M2-W2 would also be unstable. Finally, M4 prefers W3 over his current partner W4, and W3 prefers M1 over her current partner M2, so switching to M4-W3 would also be unstable. Therefore, none of the given matchings are stable, and the answer is D) None of the above.