model_results

January 10, 2022

[ ]:
```python
import pandas as pd
import numpy as np
from scipy.optimize import fmin_bfgs
from tqdm import tqdm_notebook,tqdm

import matplotlib.pyplot as plt

# custom percentile function to exactly replicate matlab
def quantile(x,q):
    n = len(x)
    y = np.sort(x)
    return(np.interp(q, np.linspace(1/(2*n), (2*n-1)/(2*n), n), y))

def prctile(x,p):
    return(quantile(x,np.array(p)/100))
# discussion: https://stackoverflow.com/questions/24764966/
↪numpy-percentile-function-different-from-matlabs-percentile-function
```

[ ]:
```python
# load GKX data
data_stocks_dir = './'
micro = pd.read_pickle(data_stocks_dir + 'returns_chars_panel_raw.pkl') # use␣
↪dropbox links to download this
macro = pd.read_pickle(data_stocks_dir + 'macro_timeseries.pkl')
df = pd.merge(micro,macro,on='date',how='left',suffixes=['','_macro']) #␣
↪include macro predictors
```

[ ]:
```python
df
```

[ ]:
```
               date  permno  excess_ret       ret     rfree         mvel1  \
0        1986-02-01   10000   -0.262610 -0.257143  0.005467  1.610000e+04
1        1986-03-01   10000    0.360335  0.365385  0.005050  1.196000e+04
2        1986-04-01   10000   -0.103717 -0.098592  0.005125  1.633000e+04
3        1986-05-01   10000   -0.227831 -0.222656  0.005175  1.517200e+04
4        1986-06-01   10000   -0.009883 -0.005025  0.004858  1.179386e+04
…               …       …         …         …         …             …
3739444  2016-08-01   93436   -0.097265 -0.097023  0.000242  3.491163e+07
3739445  2016-09-01   93436   -0.037915 -0.037640  0.000275  3.164016e+07
3739446  2016-10-01   93436   -0.031253 -0.030878  0.000375  3.056879e+07
```

```
3739447 2016-11-01   93436   -0.042553 -0.042128  0.000425  2.963795e+07
3739448 2016-12-01   93436    0.127822  0.128247  0.000425  2.840318e+07

             beta    betasq     chmom     dolvol  …  ep_macro       b/m  \
0             NaN       NaN       NaN        NaN  … -2.675897  0.583517
1             NaN       NaN       NaN        NaN  … -2.747007  0.536377
2             NaN       NaN       NaN   7.897668  … -2.800518  0.519628
3             NaN       NaN       NaN   8.472954  … -2.781919  0.529714
4             NaN       NaN       NaN   8.250098  … -2.826589  0.503541
…             …       …       …         …  …       …         …
3739444  1.600823  2.562634  0.509589  18.685227  … -3.210865  0.314661
3739445  1.636403  2.677816  0.334004  18.492052  … -3.201425  0.315197
3739446  1.633774  2.669218 -0.037025  18.518768  … -3.192038  0.316794
3739447  1.614461  2.606485 -0.342211  18.641207  … -3.152198  0.319688
3739448  1.588092  2.522036 -0.121017  18.580861  … -3.165980  0.303286

         crsp_spvw      svar     tbl     tms     dfy     dfr      ntis  \
0         0.004706  0.001920  0.0707  0.0251  0.0139  0.0070 -0.019172
1         0.076525  0.001089  0.0706  0.0135  0.0144 -0.0393 -0.017914
2         0.055832  0.001374  0.0656  0.0110  0.0150 -0.0514 -0.016420
3        -0.013348  0.002459  0.0606  0.0176  0.0140  0.0096 -0.024585
4         0.055326  0.001370  0.0615  0.0233  0.0120  0.0341 -0.021872
…              …       …     …     …     …     …       …
3739444   0.036571  0.000478  0.0030  0.0145  0.0094  0.0164 -0.031614
3739445   0.001247  0.000279  0.0030  0.0156  0.0092  0.0156 -0.030723
3739446   0.000446  0.001673  0.0029  0.0167  0.0090  0.0005 -0.032543
3739447  -0.017958  0.000364  0.0033  0.0187  0.0087  0.0051 -0.028976
3739448   0.035790  0.000946  0.0045  0.0222  0.0085  0.0089 -0.027373

             infl
0        0.002745
1       -0.002737
2       -0.004575
3       -0.001838
4        0.002762
…             …
3739444 -0.001618
3739445  0.000918
3739446  0.002404
3739447  0.001247
3739448 -0.001555

[3739449 rows x 110 columns]
```

1 Find More Influential Stock Characteristics

Comparing the 2 charts , we can see various momentum variables and trading volumes have strong interpretive power in measuring the stock returns. This makes sense all those variables are highly related to the market volatility, we think adding those variables to our policy may have positive influence in our results. Because it will help us better capture the momentum anomolies and maximize the utilities in the end:

oroginal: mvel1, bm, mom12m mom1m: 1-month momentum chmom: change in 6-month momentum dolvol: natural log of trading volume times price per share from month t-2 indmom: industry momentum std_turn: volatility of share turnover

Create Benchmark – S&P500

[ ]:
```
import tqdm

n_train = 360
dt = pd.DataFrame()


for year in tqdm.tqdm(range(1974,2016)): # using data from 1974 as in BSV, but␣
↪longer sample
    for month in range(1,13):
        dta = df[(df['date'].dt.year==year) & (df['date'].dt.month==month)]
        dt = dt.append(dta.head(1),ignore_index=True)
dt_date = dt['date']

dt_date_train = dt_date[0:n_train]
dt_date_test = dt_date[n_train:]
```

```
100%|      | 42/42 [02:18<00:00,  3.30s/it]
```

[ ]:
```
from pandas_datareader import data as pdr
import yfinance as yf
yf.pdr_override()
SPY_data = pdr.get_data_yahoo("SPY",start = '2003-12-01', end= '2015-12-02')
SPY_data
```

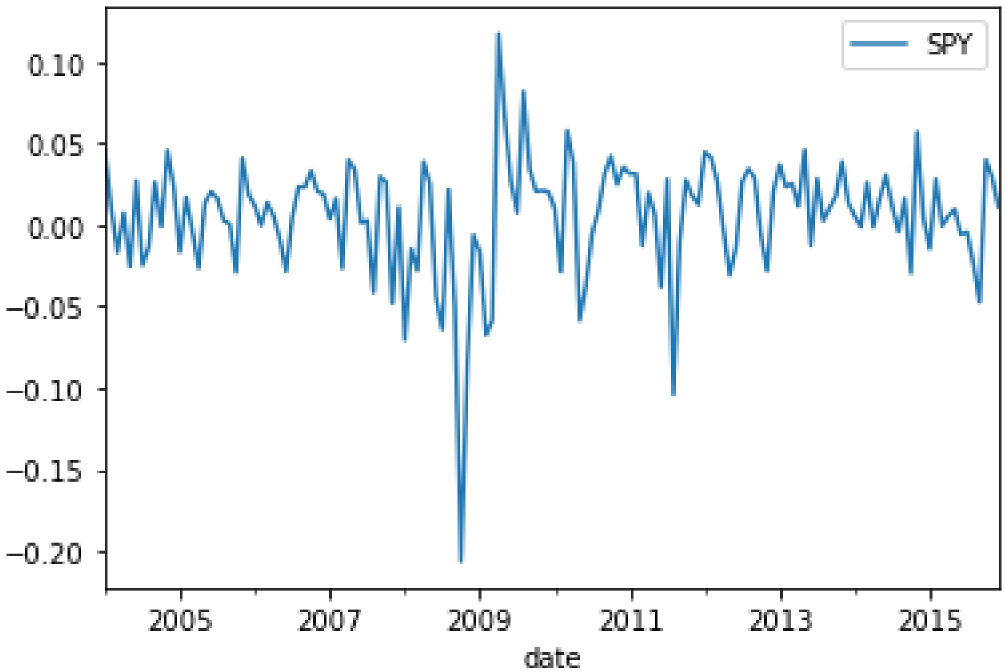
```
[*********************100%***********************]  1 of 1 completed
```

[ ]:
```
                  Open        High         Low       Close   Adj Close  \
Date
2003-12-01  106.849998  107.680000  106.800003  107.599998   75.357986
2003-12-02  107.379997  107.769997  107.070000  107.330002   75.168900
2003-12-03  107.650002  108.080002  107.070000  107.160004   75.049828
2003-12-04  107.169998  107.720001  106.940002  107.599998   75.357986
2003-12-05  107.120003  107.800003  106.620003  106.849998   74.832703
…                  …           …           …           …           …
2015-11-24  207.869995  209.830002  207.410004  209.350006  186.466690
2015-11-25  209.500000  209.740005  209.009995  209.320007  186.439972
```

```
2015-11-27  209.429993  209.800003  208.860001  209.559998  186.653702
2015-11-30  209.750000  209.889999  208.559998  208.690002  185.878799
2015-12-01  209.440002  210.820007  209.110001  210.679993  187.651291

               Volume
Date
2003-12-01   38699000
2003-12-02   35352000
2003-12-03   39078600
2003-12-04   36089500
2003-12-05   28824400
…                 …
2015-11-24   98874400
2015-11-25   51980100
2015-11-27   37317800
2015-11-30  112822700
2015-12-01   97858400

[3022 rows x 6 columns]
```

[ ]:
```
#calculate monthly average of the data
SPY_data_mo = SPY_data.groupby(pd.Grouper( freq='M'))['Close'].mean()
SPY_rtn = SPY_data_mo.pct_change(1).iloc[1:]


#create return dataframe
rtn_p = dt_date_test.to_frame()
rtn_p['SPY'] = SPY_rtn.array
rtn_p.plot(x='date',y=['SPY'])
```

[ ]:
```
<AxesSubplot:xlabel='date'>
```



2 Policy Gradient with Linear Model

Define the Linear Model Policy Functions

[ ]:
```
# policy function
def policy(theta,x,wm):
    theta_ = theta.reshape(-1,1)
    w = []
    for t in range(len(x)):
        w.append(wm[t] + np.matmul(x[t],theta_) / len(wm[t])) # portfolio weight
    return w

# value function
def value(ret,w,gamma=5):
    u = []
    for t in range(len(ret)):
        retp = np.sum(w[t]*ret[t]) # portfolio return
        if gamma == 1:
            u.append(np.log(1+retp))
        else:
            u.append((1+retp)**(1-gamma) / (1-gamma))
    return np.mean(u)

def portfolio_returns(ret,w):
```

```
    retps = []
    for t in range(len(ret)):
        retp = np.sum(w[t]*ret[t]) # portfolio return
        retps.append(retp)
    return np.array(retps)

# analytical gradient
def grad(theta,w,x,wm,ret,gamma=5):
    grads = []
    for t in range(len(ret)):
        retp = np.sum(w[t]*ret[t])
        m = (1+retp)**(-gamma)
        z = m * ret[t] / len(ret[t])
        grads.append(np.matmul(z.T,x[t]))
    return np.mean(np.vstack(grads),axis=0)

# check analytical versus numerical gradient -- if this yields <1e-4 we are ok
def grad_check(theta,w,x,wm,ret,h=1e-5):
    dtheta_num = []
    for i in range(len(theta)):
        h_vec = np.zeros_like(theta) # perturb in direction i
        h_vec[i] = h
        up = value(ret,policy(theta + h_vec,x,wm))
        down = value(ret,policy(theta - h_vec,x,wm))
        dtheta_num.append((up - down) / (2*h)) # numerical grad

    dtheta = grad(theta,w,x,wm,ret)
    return np.abs(dtheta-dtheta_num) / np.maximum(dtheta,dtheta_num)
```

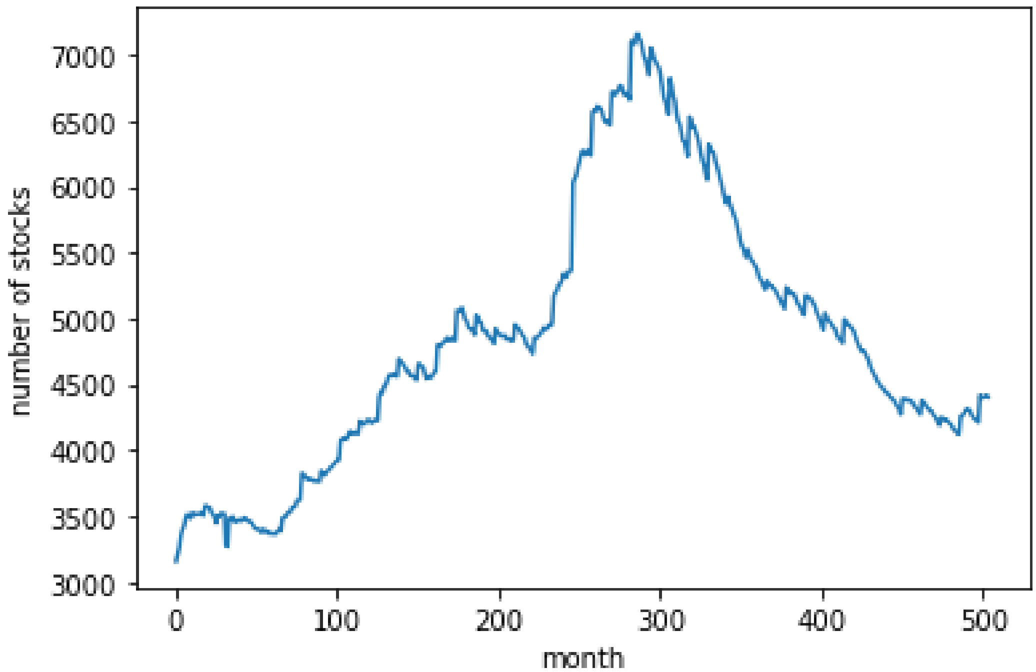
Linear Model 1 – With 3 Factors: mvel1, btm, mom 12m

[ ]:
```
from tqdm import tqdm

# clean data
wm,x,ret,rf = [],[],[],[] # lists holding data for each period
mean_chars = [] # for debug
for year in tqdm(range(1974,2016)): # using data from 1974 as in BSV, but␣
 ↪longer sample
    for month in range(1,13):
        dta = df[(df['date'].dt.year==year) & (df['date'].dt.month==month)] #␣
 ↪data for this period

        cols = ['mvel1','bm','mom12m','ret','rfree']
        dta = dta[cols].dropna()

        # cleaning: remove size below 0.2 percentile and negative beme
        min_me = prctile(dta['mvel1'],0.2)
```

```
        dta = dta[(dta['mvel1']>=min_me) & (dta['bm']>=0)]

        # get value weights for bsv policy
        mv = dta['mvel1'].values.reshape(-1,1)
        wm.append(mv/np.sum(mv))

        # get normalised characteristics for policy
        sz = np.log(dta['mvel1'])
        btm = np.log(1+dta['bm'])
        mom = dta['mom12m']
        char = np.vstack([sz,btm,mom]).T
        mean_chars.append(np.mean(char,axis=0))
        char -= np.mean(char,axis=0)
        char /= np.std(char,axis=0)
        x.append(char)

        # get returns
        ret.append(dta['ret'].values.reshape(-1,1))

        # risk free rate
        rf.append(dta['rfree'].mean())

plt.plot([len(r) for r in ret])
plt.xlabel('month')
plt.ylabel('number of stocks')
```

```
100%|          | 42/42 [02:30<00:00,  3.58s/it]
```

[ ]:
```
Text(0, 0.5, 'number of stocks')
```

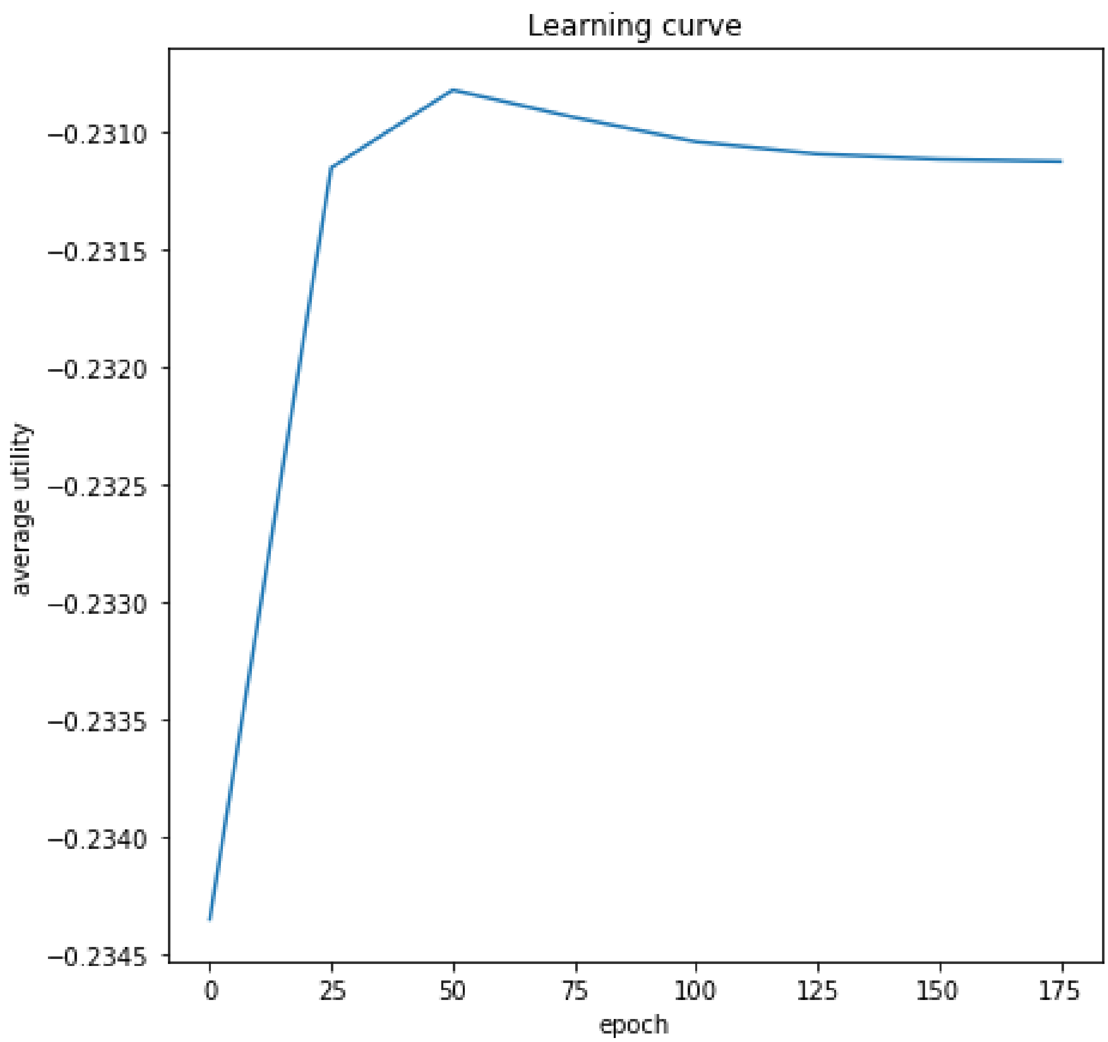



```
[ ]: %%time
# solution with grad ascent

# setup adam
beta1, beta2, epsilon = 0.99, 0.999, 1e-7
v, s = 0,0
alpha = 1e-2

# split data
n_train = 360 # 30 years of training data, ~12 years of test data
wm_train, x_train, ret_train, rf_train = wm[0:n_train], x[0:n_train], ret[0:
 ↪n_train], rf[0:n_train]
wm_test, x_test, ret_test, rf_test = wm[n_train:], x[n_train:], ret[n_train:],␣
 ↪rf[n_train:]

# make mini batches
b = 32 # batch size
xs, wms, rets = [], [], []
j=0
while j<n_train:
    start, end = j, min(j+b,n_train)
    xs.append(x_train[start:end])
    wms.append(wm_train[start:end])
    rets.append(ret_train[start:end])
```

```python
    j+=b

# iterate
theta = np.zeros((3,))
c = 0 # count updates
values = [] # for learning curve
for i in range(200):
    if (i+1)%25 == 0:
        print('epoch %d, theta = %s' %(i+1,str(theta)))
        values.append(value(ret,policy(theta,x,wm)))

    for x_batch,wm_batch,ret_batch in zip(xs,wms,rets):
        w = policy(theta,x_batch,wm_batch) # forward
        dtheta = grad(theta,w,x_batch,wm_batch,ret_batch) # backward

        # adam update
        v = beta1 * v + (1-beta1) * dtheta
        s = beta2 * s + (1-beta2) * (dtheta**2)
        vhat = v / (1-(beta1**(c+1)))
        shat = s / (1-(beta2**(c+1)))
        adam = vhat / (np.sqrt(shat)+epsilon)
        theta += alpha * vhat / (np.sqrt(shat)+epsilon)
        c+=1

# print results
print('\nSOLUTION: optimal theta')
print(theta)
print('max utility')
print(value(ret,policy(theta,x,wm)))


# plot learning curve
plt.figure(figsize=(7,7))
plt.title('Learning curve')
plt.plot(np.arange(len(values))*25,values)
plt.xlabel('epoch')
plt.ylabel('average utility')
plt.show()


# print performance
w = policy(theta,x_train,wm_train)
retp = portfolio_returns(ret_train,w)
sharpe_insample_l1 = np.sqrt(12)* (retp-np.array(rf_train) ).mean() / retp.std()
print('\nSharpe ratio in sample')
print(sharpe_insample_l1)
alpha_insample_l1 = np.sqrt(12)* (retp-np.array(rf_train) ).mean()
```

```python
w = policy(theta,x_test,wm_test)
retp = portfolio_returns(ret_test,w)
sharpe_oos_l1 = np.sqrt(12)* (retp-np.array(rf_test) ).mean() / retp.std()
print('\nSharpe ratio out of sample')
print(sharpe_oos_l1)
alpha_oos_l1 = np.sqrt(12)* (retp-np.array(rf_test) ).mean()
ir_oos_l1 = np.sqrt(12)* (retp.mean()-SPY_rtn.array.mean()) / (retp-SPY_rtn.
 ↪array).std()
```

```
epoch 25, theta = [-1.83738568  2.37353257  1.67256581]
epoch 50, theta = [-3.06494724  4.34829997  2.43631655]
epoch 75, theta = [-3.59377969  5.60654456  2.76215734]
epoch 100, theta = [-3.75355333  6.2628341   2.91766114]
epoch 125, theta = [-3.79424379  6.56028729  2.97424925]
epoch 150, theta = [-3.804676    6.68441863  2.99319345]
epoch 175, theta = [-3.80767924  6.7342711   3.00017924]
epoch 200, theta = [-3.8087636   6.75410634  3.00293715]

SOLUTION: optimal theta
[-3.80879029  6.75458127  3.0030037 ]
max utility
-0.23112504403494355
```




```
Sharpe ratio in sample
1.536572058089829

Sharpe ratio out of sample
0.6880021806613135
CPU times: user 22.3 s, sys: 1.22 s, total: 23.5 s
Wall time: 6 s
```

[ ]:
```
alpha_oos_l1
```

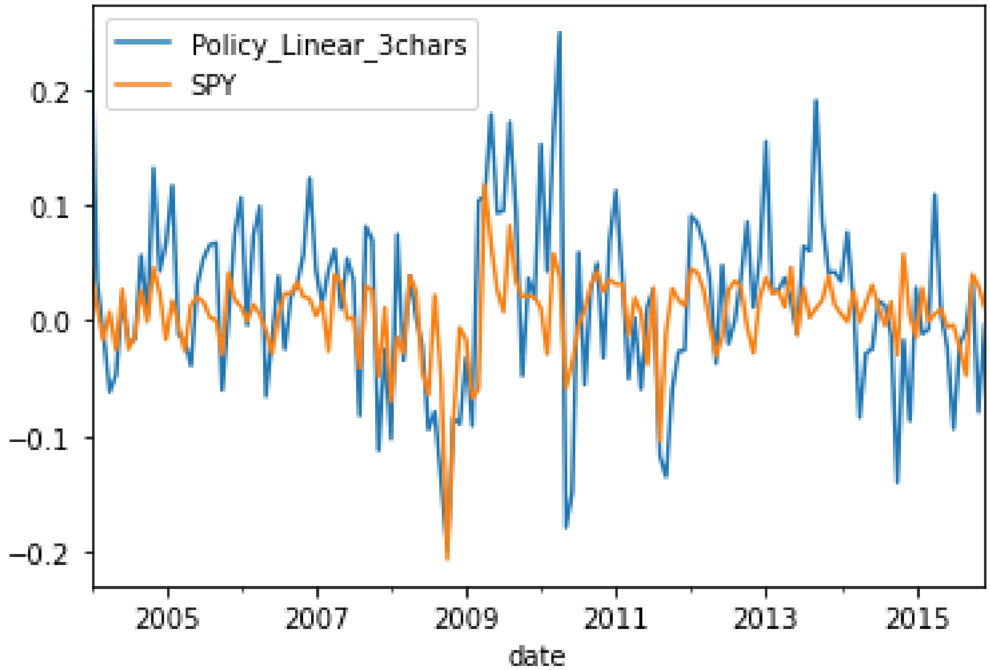
[ ]:
```
0.053018382519373033
```

[ ]:
```
ir_oos_l1
```

[ ]: 0.6094150242767069

```
[ ]: #plot out the returns
rtn_p['Policy_Linear_3chars'] = retp
rtn_p.plot(x='date',y=['Policy_Linear_3chars','SPY'])
```

[ ]: <AxesSubplot:xlabel='date'>



Linear Model 2 – With More Influential Factors

In this model, I will refresh the stock characteristics set in the policy function, apart from the original 3 factors (mvel1, bm, mom12m), I include 5 additional stock characteristics (mom1m, chmom, dolvol, indmom, std_turn) to capture more features. The selection cateria is listed in the previous part.

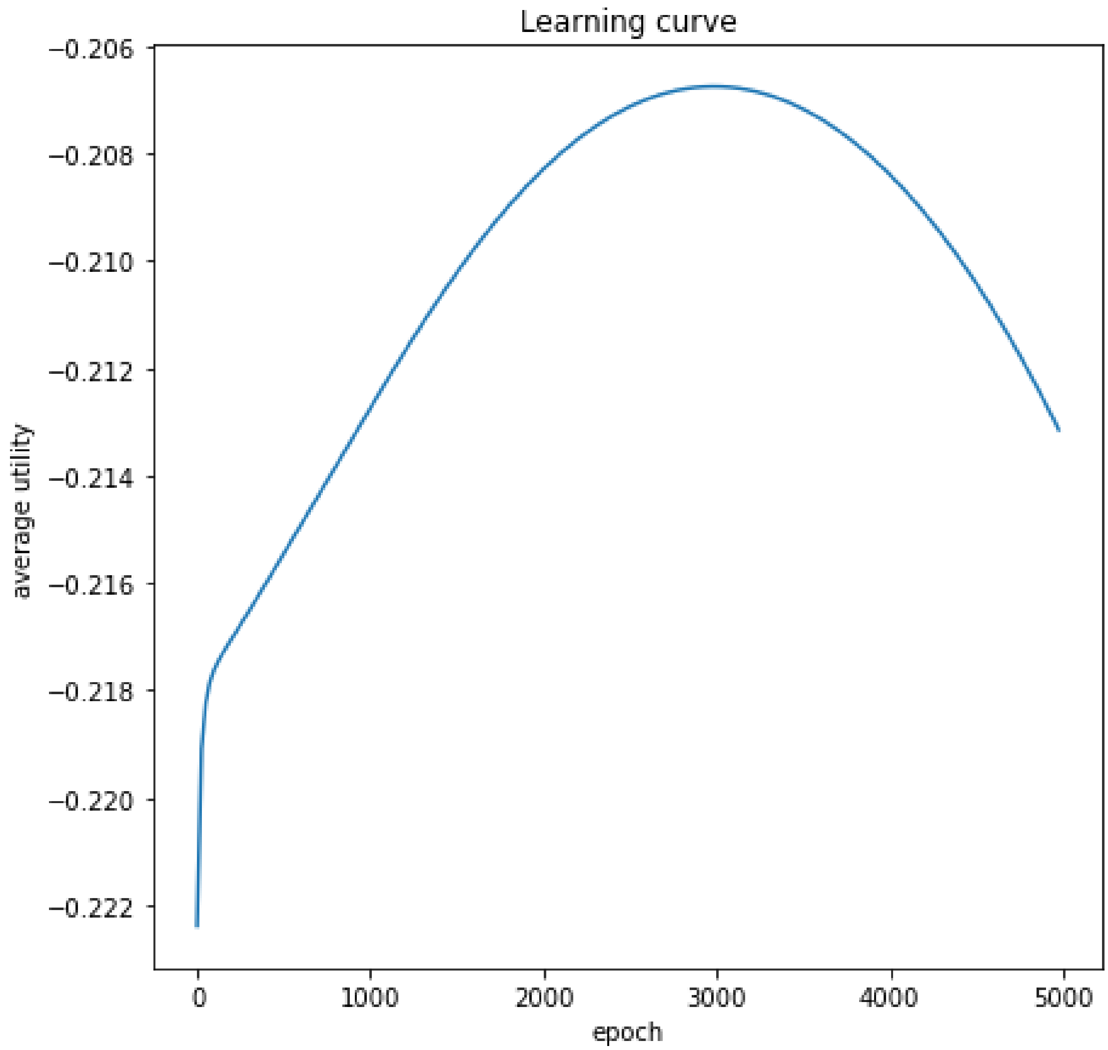
```
[ ]: from tqdm import tqdm
# first we clean the data, and incorporate the new variables into the␣
↪characteristic set

wm,x,ret,rf = [],[],[],[] # lists holding data for each period
mean_chars = [] # for debug
for year in tqdm(range(1974,2016)): # using data from 1974 as in BSV, but␣
↪longer sample
    for month in range(1,13):
```

```
        dta_morevar = df[(df['date'].dt.year==year) & (df['date'].dt.
↪month==month)] # data for this period


        #constructing new feature set, with 8 stock characteristics in total
        cols =␣
↪['mvel1','bm','mom12m','mom1m','chmom','dolvol','indmom','std_turn','ret','rfree']
        dta_morevar = dta_morevar[cols].dropna()

        # cleaning: remove size below 0.2 percentile and negative beme
        min_me = prctile(dta_morevar['mvel1'],0.2)
        dta_morevar = dta_morevar[(dta_morevar['mvel1']>=min_me) &␣
↪(dta_morevar['bm']>=0)]

        # get value weights for bsv policy
        mv = dta_morevar['mvel1'].values.reshape(-1,1)
        wm.append(mv/np.sum(mv))

        # get normalised characteristics for policy
        sz = np.log(dta_morevar['mvel1'])
        btm = np.log(1+dta_morevar['bm'])
        mom = dta_morevar['mom12m']

        # newly added variables, with proper normalization methods
        mom1m = dta_morevar['mom1m']
        chmom = dta_morevar['chmom']
        dolvol = dta_morevar['dolvol']
        indmom = dta_morevar['indmom']
        std_turn = np.log(1+dta_morevar['std_turn'])

        # stack the variables and create the new 8-dimension characteristics
        char = np.vstack([sz,btm,mom,mom1m,chmom,dolvol,indmom,std_turn]).T
        mean_chars.append(np.mean(char,axis=0))
        char -= np.mean(char,axis=0)
        char /= np.std(char,axis=0)
        x.append(char)

        # get returns
        ret.append(dta_morevar['ret'].values.reshape(-1,1))

        # risk free rate
        rf.append(dta_morevar['rfree'].mean())



# solution with grad ascent
```

```
# setup adam
beta1, beta2, epsilon = 0.99, 0.999, 1e-7
v, s = 0,0
alpha = 1e-2

# split data
n_train = 360 # 30 years of training data, ~12 years of test data
wm_train, x_train, ret_train, rf_train = wm[0:n_train], x[0:n_train], ret[0:
 ↪n_train], rf[0:n_train]
wm_test, x_test, ret_test, rf_test = wm[n_train:], x[n_train:], ret[n_train:],␣
 ↪rf[n_train:]

# make mini batches
b = 32 # batch size
xs, wms, rets = [], [], []
j=0
while j<n_train:
    start, end = j, min(j+b,n_train)
    xs.append(x_train[start:end])
    wms.append(wm_train[start:end])
    rets.append(ret_train[start:end])
    j+=b

# iterate
# change the dimension of theta to match the size of new variables
theta = np.zeros((8,))
c = 0 # count updates
values = [] # for learning curve
for i in tqdm(range(5000)):
    if (i+1)%25 == 0:
        #print('epoch %d, theta = %s' %(i+1,str(theta)))
        values.append(value(ret,policy(theta,x,wm)))

    for x_batch,wm_batch,ret_batch in zip(xs,wms,rets):
        w = policy(theta,x_batch,wm_batch) # forward
        dtheta = grad(theta,w,x_batch,wm_batch,ret_batch) # backward

        # adam update
        v = beta1 * v + (1-beta1) * dtheta
        s = beta2 * s + (1-beta2) * (dtheta**2)
        vhat = v / (1-(beta1**(c+1)))
        shat = s / (1-(beta2**(c+1)))
        adam = vhat / (np.sqrt(shat)+epsilon)
        theta += alpha * vhat / (np.sqrt(shat)+epsilon)
        c+=1

# print results
```

```python
print('\nSOLUTION: optimal theta')
print(theta)
print('max utility')
print(value(ret,policy(theta,x,wm)))


# plot learning curve
plt.figure(figsize=(7,7))
plt.title('Learning curve')
plt.plot(np.arange(len(values))*25,values)
plt.xlabel('epoch')
plt.ylabel('average utility')
plt.show()


# print performance
w = policy(theta,x_train,wm_train)
retp = portfolio_returns(ret_train,w)
sharpe_insample_l2 = np.sqrt(12)* (retp-np.array(rf_train) ).mean() / retp.std()
print('\nSharpe ratio in sample')
print(sharpe_insample_l2)
alpha_insample_l2 = np.sqrt(12)* (retp-np.array(rf_train) ).mean()


w = policy(theta,x_test,wm_test)
retp = portfolio_returns(ret_test,w)
sharpe_oos_l2 = np.sqrt(12)* (retp-np.array(rf_test) ).mean() / retp.std()
print('\nSharpe ratio out of sample')
print(sharpe_oos_l2)

alpha_oos_l2 = np.sqrt(12)* (retp-np.array(rf_test) ).mean()
ir_oos_l2 = np.sqrt(12)* (retp.mean()-SPY_rtn.array.mean()) / (retp-SPY_rtn.
 ↪array).std()
```

```
100%|      | 42/42 [02:07<00:00,  3.04s/it]
100%|      | 5000/5000 [02:48<00:00, 29.62it/s]


SOLUTION: optimal theta
[ 39.14721232   3.7265544   -1.47915501 -12.41229645  -0.17838564
 -47.38658568   6.75274034  20.3802711 ]
max utility
-0.21315276106968556
```




```
Sharpe ratio in sample
2.3714339027999687

Sharpe ratio out of sample
1.0978204044054782
```

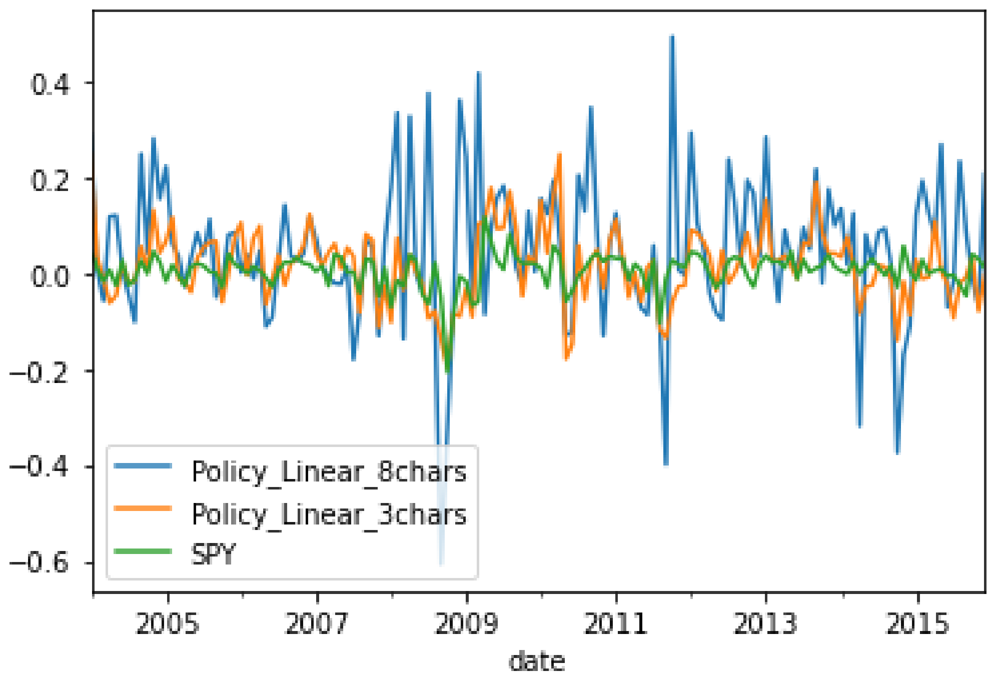
[ ]:
```
print(alpha_oos_l1)
print(alpha_oos_l2)
print(ir_oos_l1)
print(ir_oos_l2)
```

```
0.053018382519373033
0.16808616606973784
0.6094150242767069
```

```
1.0423001631732782
```

[ ]:
```
#attach the return results
rtn_p['Policy_Linear_8chars'] = retp
rtn_p.plot(x='date',y=['Policy_Linear_8chars','Policy_Linear_3chars','SPY'])
```

[ ]: <AxesSubplot:xlabel='date'>



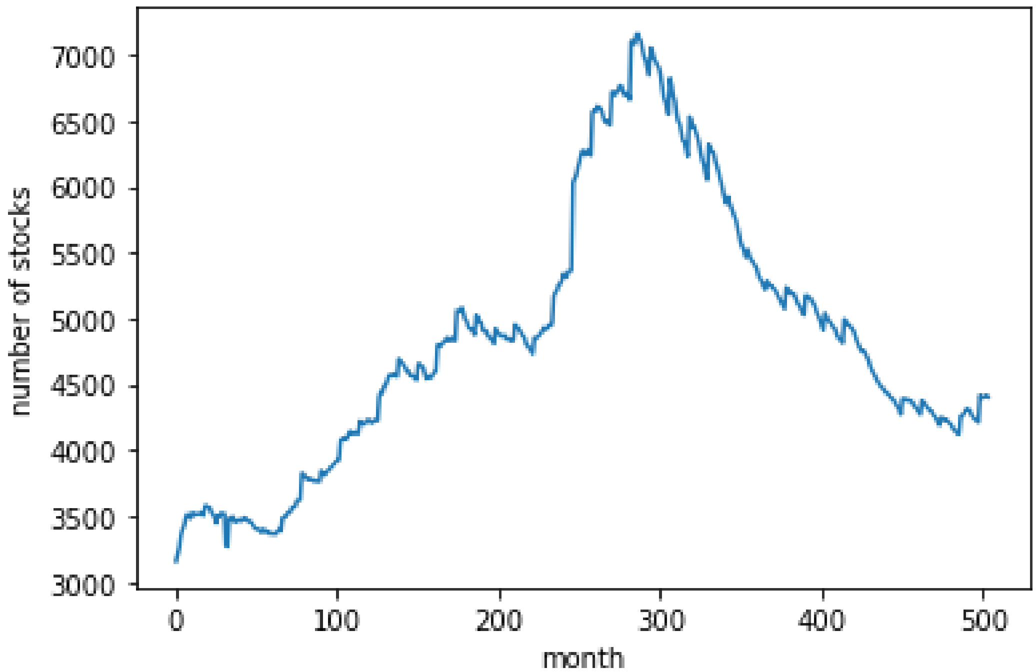
[ ]:
```
#20220109
rtn_p.to_csv('linear_returns.csv')
```

3 Policy Gradient with Neural Network

[ ]:
```
from tqdm import tqdm
# clean data
wm,x,ret,rf = [],[],[],[] # lists holding data for each period
mean_chars = [] # for debug
# for year in tqdm(range(1974,2016)): # using data from 1974 as in BSV, but␣
↪longer sample
for year in tqdm(range(1974,2016)): # using data from 1974 as in BSV, but␣
↪longer sample
    for month in range(1,13):
        dta = df[(df['date'].dt.year==year) & (df['date'].dt.month==month)] #␣
↪data for this period
```

```
        cols = ['mvel1','bm','mom12m','ret','rfree']
        dta = dta[cols].dropna()

        # cleaning: remove size below 0.2 percentile and negative beme
        min_me = prctile(dta['mvel1'],0.2)
        dta = dta[(dta['mvel1']>=min_me) & (dta['bm']>=0)]

        # get value weights for bsv policy
        mv = dta['mvel1'].values.reshape(-1,1)
        wm.append(mv/np.sum(mv))

        # get normalised characteristics for policy
        sz = np.log(dta['mvel1'])
        btm = np.log(1+dta['bm'])
        mom = dta['mom12m']
        char = np.vstack([sz,btm,mom]).T
        mean_chars.append(np.mean(char,axis=0))
        char -= np.mean(char,axis=0)
        char /= np.std(char,axis=0)
        x.append(char)

        # get returns
        ret.append(dta['ret'].values.reshape(-1,1))

        # risk free rate
        rf.append(dta['rfree'].mean())

plt.plot([len(r) for r in ret])
plt.xlabel('month')
plt.ylabel('number of stocks')
```

```
100%|      | 42/42 [02:09<00:00,  3.08s/it]
```




```
[ ]: def initialize_network(layers):
         # random initialization of parameters of neural network
         np.random.seed(108)
         pars = [] # conveniently to store parameters in a list of dictionaries
         for j in range(1,len(layers)): # loop over layers
             dictionary = {'weight' : np.random.randn(layers[j],layers[j-1]), #␣
      ↪initialize matrix W(l) of weights
                           'bias' : np.random.randn(layers[j],1)} # initialize␣
      ↪vector b(l) of biases
             pars.append(dictionary)
         return pars
```

```
[ ]: def unstack_theta(original_theta):
         flattened_theta = []
         for i in range(len(layers) - 1):
             flattened_theta.extend(np.array(original_theta[i]['weight']).flatten())
             flattened_theta.extend(np.array(original_theta[i]['bias']).flatten())
         return flattened_theta

     def stack_theta(flattened_theta, layers):
         original_theta = []
         index = 0
         for i in range(len(layers) - 1):
             retrieve_len = layers[i+1] * layers[i]
```

```
        weight_list = np.array(flattened_theta[index:index + retrieve_len]).
↪reshape(layers[i+1], layers[i])
        index += retrieve_len
        retrieve_len = layers[i+1] * 1
        bias_list = np.array(flattened_theta[index:index + retrieve_len]).
↪reshape(layers[i+1], 1)
        index += retrieve_len
        original_theta.append({'weight': weight_list, 'bias': bias_list})
    return original_theta
```

[ ]:
```
def forward(X,pars):

    # make lists to store elements of the graph in memory
    Zs,Hs = [],[]

    # initialize inputs
    H = X.T

    # loop over layers
    for j in range(len(pars)):

        # get parameters for this layer
        W = pars[j]['weight']
        b = pars[j]['bias']

        # activations and output
        Z = np.matmul(W,H) + b
        H = Z if j+1 == len(pars) else Z*(Z>0)

        # save to list
        Zs.append(Z)
        Hs.append(H)

    return Zs,Hs
    # return Hs[-1].flatten()
# NB: we could also explicitly return predictions but this is sufficient  and␣
↪cleaner

def backprop(Zs,Hs,X,pars,delta_y=1e-5):

    # setup list for gradients
    grads = []

    # data size
    m = X.shape[0]

    # loop over layers
```

```
    for j in range(len(pars))[::-1]:
        Z = Zs[j]
        H = Hs[j]
        W = pars[j]['weight']

        # get activations gradient
        dZ = (H - (H - delta_y)) if j+1 == len(pars) else (Z > 0) * dH
        # print('dZ shape', dZ.shape)

        # get input from previous layer
        H_back = Hs[j-1] if j>0 else X.T
        # print('H_back shape', H_back.shape)

        # get parameter gradients
        dZdW_list = []
        dZdb_list = []
        for k in range(m):
            # print('dZ[:, k] shape', dZ[:,k:k+1].shape)
            # print('H_back[:,k] shape', H_back[:,k:k+1].shape)
            dW = np.matmul(dZ[:,k:k+1],H_back[:,k:k+1].T)
            # print('dW shape', dW.shape)
            db = dZ[:, k:k+1]
            # db = np.sum(dZ,axis=1,keepdims=True) / m
            # print('db shape', db.shape)
            dZdW = np.divide(np.zeros_like(dW) + delta_y, dW, out=np.
↪zeros_like(dW), where=dW!=0)
            dZdb = np.divide(np.zeros_like(db) + delta_y, db, out=np.
↪zeros_like(db), where=db!=0)
            dZdW_list.append(dZdW)
            dZdb_list.append(dZdb)

        # save to list
        grads.append({'weight' : dZdW_list, 'bias' : dZdb_list})
        grads = grads[::-1]

        # move to next layer
        if j>0: dH = np.matmul(W.T,dZ)

    flattened_theta_list = []
    for i in range(m):
        flattened_theta = []
        for j in range(len(pars)):
            flattened_theta.extend(np.array(grads[j]['weight'][i]).flatten())
            flattened_theta.extend(np.array(grads[j]['bias'][i]).flatten())
        flattened_theta_list.append(flattened_theta)
    return flattened_theta_list
```

[ ]:
```python
# policy function
def policy(theta,x,wm):
    # theta_ = theta.reshape(-1,1)
    w = []
    for t in range(len(x)):
        _, Hs = forward(x[t],stack_theta(theta, layers))
        theta_ = Hs[-1].flatten().reshape(-1, 1)
        w.append(wm[t] + theta_ / len(wm[t])) # portfolio weight
        # print(len(wm[t] + np.matmul(x[t],theta_) / len(wm[t])), type(wm[t] +␣
↪np.matmul(x[t],theta_) / len(wm[t])))
    return w

def policy_copy(theta,x,wm):
    theta_ = theta.reshape(-1,1)
    w = []
    for t in range(len(x)):
        w.append(wm[t] + np.matmul(x[t],theta_) / len(wm[t])) # portfolio weight
        # print(len(wm[t] + np.matmul(x[t],theta_) / len(wm[t])), type(wm[t] +␣
↪np.matmul(x[t],theta_) / len(wm[t])))
    return w

# value function
def value(ret,w,gamma=5):
    u = []
    for t in range(len(ret)):
        retp = np.sum(w[t]*ret[t]) # portfolio return
        if gamma == 1:
            u.append(np.log(1+retp))
        else:
            u.append((1+retp)**(1-gamma) / (1-gamma))
    return np.mean(u)

def portfolio_returns(ret,w):
    retps = []
    for t in range(len(ret)):
        retp = np.sum(w[t]*ret[t]) # portfolio return
        retps.append(retp)
    return np.array(retps)

# analytical gradient
def grad_analytical_NN(theta,w,x,wm,ret,gamma=5):
    grads = []
    for t in range(len(ret)):
        retp = np.sum(w[t]*ret[t])
        m = (1+retp)**(-gamma)
        z = m * ret[t] / len(ret[t])
        pars = stack_theta(theta, layers)
```

```
        Zs, Hs = forward(x[t], pars)
        backprop_grad = backprop(Zs, Hs, x[t], pars)
        grads.append(np.matmul(z.T,np.array(backprop_grad)))
    return np.mean(np.vstack(grads),axis=0)

 # numeric gradient
def grad(theta,w,x,wm,ret,h=1e-5):
    dtheta_num = []
    for i in range(len(theta)):
        h_vec = np.zeros_like(theta) # perturb in direction i
        h_vec[i] = h
        up = value(ret,policy(theta + h_vec,x,wm))
        down = value(ret,policy(theta - h_vec,x,wm))
        dtheta_num.append((up - down) / (2*h)) # numerical grad
    return np.array(dtheta_num)
```

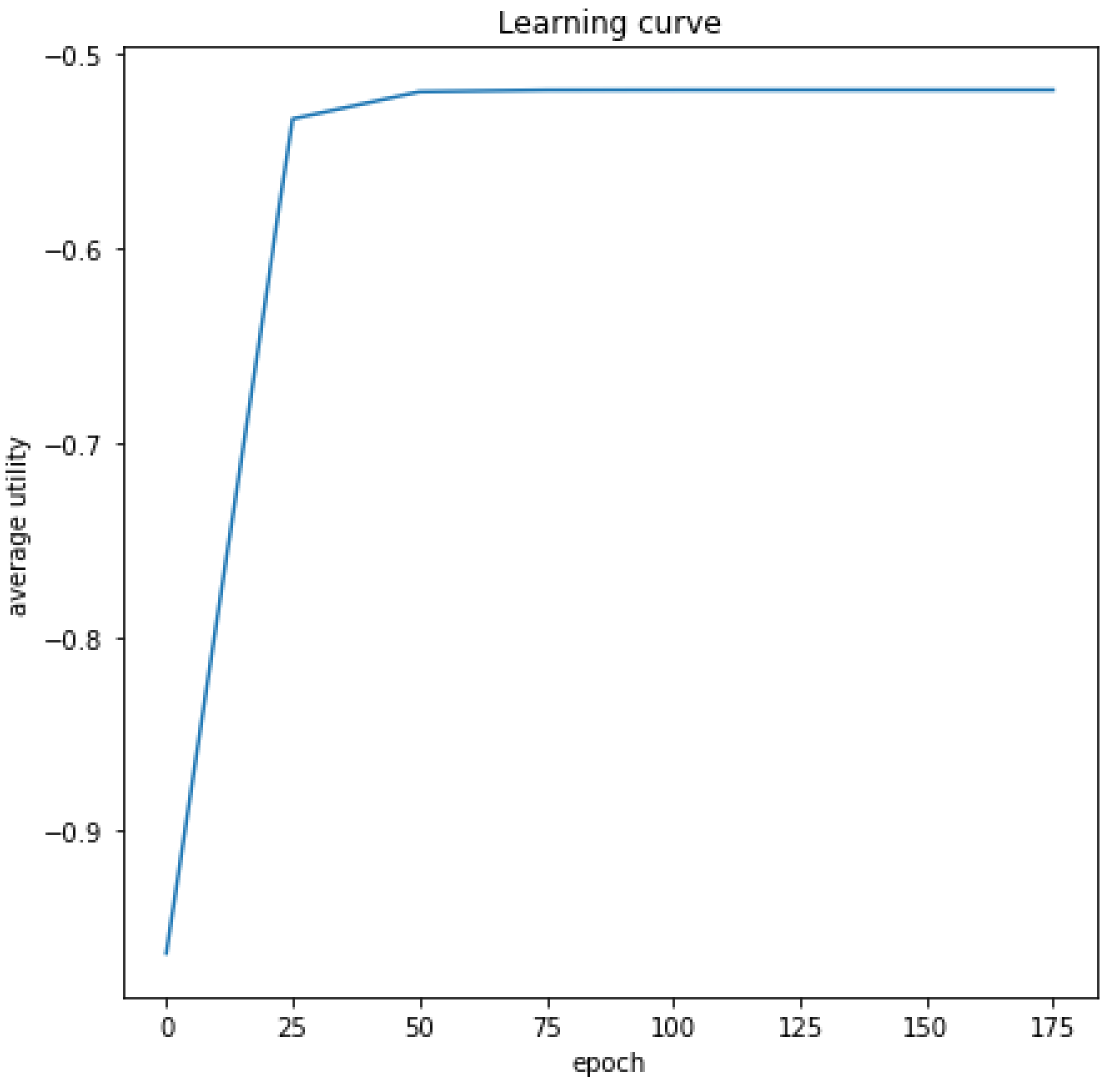
Neural Network Model 1 – With 3 Characteristics (mvel1, bm, mom12m)

[ ]:
```
# solution with grad ascent

# setup adam
beta1, beta2, epsilon = 0.99, 0.999, 1e-7
v, s = 0,0
# 1.learning rate
alpha = 1e-2

# split data
n_train = 360 # 30 years of training data, ~12 years of test data
wm_train, x_train, ret_train, rf_train = wm[0:n_train], x[0:n_train], ret[0:
 ↪n_train], rf[0:n_train]
wm_test, x_test, ret_test, rf_test = wm[n_train:], x[n_train:], ret[n_train:],␣
 ↪rf[n_train:]

# make mini batches
b = 32 # 2.batch size
xs, wms, rets = [], [], []
j=0
while j<n_train:
    start, end = j, min(j+b,n_train)
    xs.append(x_train[start:end])
    wms.append(wm_train[start:end])
    rets.append(ret_train[start:end])
    j+=b

# iterate
# theta = np.zeros((3,))
p = 3 # number of variables (columns) in the data
```

```
layers = [p, 8, 4, 1] # 3.layers list defines the number n(l) of units for each␣
 ↪layer of the network
theta = unstack_theta(initialize_network(layers))
c = 0 # count updates
values = [] # for learning curve
for i in tqdm(range(200)):
    if (i+1)%25 == 0:
        # print('epoch %d, theta = %s' %(i+1,str(theta)))
        values.append(value(ret,policy(theta,x,wm)))

    for x_batch,wm_batch,ret_batch in zip(xs,wms,rets):
        w = policy(theta,x_batch,wm_batch) # forward
        dtheta = grad(theta,w,x_batch,wm_batch,ret_batch) # backward

        # adam update
        v = beta1 * v + (1-beta1) * dtheta
        s = beta2 * s + (1-beta2) * (dtheta**2)
        vhat = v / (1-(beta1**(c+1)))
        shat = s / (1-(beta2**(c+1)))
        adam = vhat / (np.sqrt(shat)+epsilon)
        theta += alpha * vhat / (np.sqrt(shat)+epsilon)
        c+=1

# print results
print('\nSOLUTION: optimal theta')
print(theta)
print('max utility')
print(value(ret,policy(theta,x,wm)))


# plot learning curve
plt.figure(figsize=(7,7))
plt.title('Learning curve')
plt.plot(np.arange(len(values))*25,values)
plt.xlabel('epoch')
plt.ylabel('average utility')
plt.show()


# print performance
w = policy(theta,x_train,wm_train)
retp = portfolio_returns(ret_train,w)
sharpe = np.sqrt(12)* (retp-np.array(rf_train) ).mean() / retp.std()
print('\nSharpe ratio in sample')
print(sharpe_insample_nn1)
alpha_insample_nn1 = np.sqrt(12)* (retp-np.array(rf_train) ).mean()
```

```
w = policy(theta,x_test,wm_test)
retp = portfolio_returns(ret_test,w)
sharpe = np.sqrt(12)* (retp-np.array(rf_test) ).mean() / retp.std()
print('\nSharpe ratio out of sample')
print(sharpe_oos_nn1)


alpha_oos_nn1 = np.sqrt(12)* (retp-np.array(rf_test) ).mean()
ir_oos_nn1 = np.sqrt(12)* (retp.mean()-SPY_rtn.array.mean()) / (retp-SPY_rtn.
 ↪array).std()
```

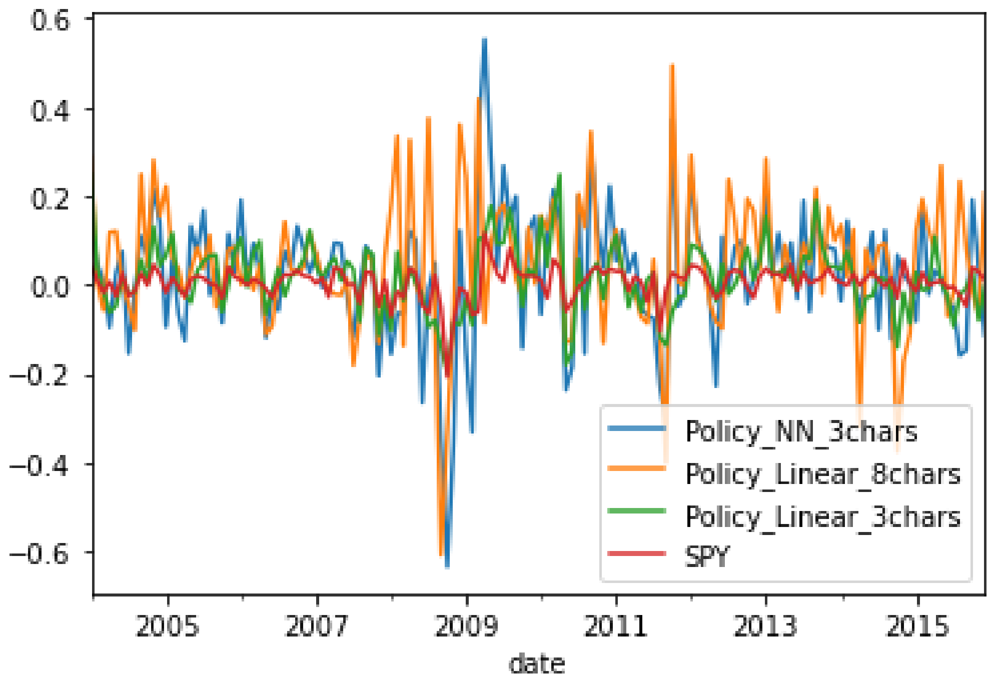
```
100%|      | 200/200 [1:18:34<00:00, 23.57s/it]


SOLUTION: optimal theta
[-0.84810643  0.04801642  0.94633841  0.96633119 -0.41692831  0.30914293
 -0.74269752 -1.76869095  0.29713374  1.10203098  1.43908661  1.05367606
 -0.36672829 -0.30725176 -0.70608725  0.5860316  -1.31804576  0.78640665
  1.62244898  1.03773433  0.0326495   1.62480884  0.55054375  0.06400623
  0.20195001 -0.34713327 -0.2832373   1.18385351  0.5558505   0.08278585
  1.42498565  1.42171893 -1.58341285  0.05096249 -1.11316345 -0.17912381
  0.39084447  0.61256419 -0.81356661 -0.65371784 -0.47616333  1.17115036
  1.19893324 -1.05282743  1.16544871  1.04438534  1.50726996  0.265746
  1.00464656  1.11175313 -0.44980241 -0.47560363 -0.36549452 -1.22059728
 -0.78489554  1.01304978  0.99710199 -1.82062025 -0.83954882 -0.34854924
  0.22377159  0.66284396  0.12953921 -1.03863252 -0.22904932  1.26466373
 -0.01926992 -1.42156068  0.64707914  0.30069122  1.31552096  0.1256874
  0.46274337]
max utility
-0.5185858106474968
```




```
Sharpe ratio in sample
2.052

Sharpe ratio out of sample
1.32
```

[ ]:
```
#attach the return results
rtn_p['Policy_NN_3chars'] = retp
rtn_p.
↪plot(x='date',y=['Policy_NN_3chars','Policy_Linear_8chars','Policy_Linear_3chars','SPY'])
```

[ ]:
```
<AxesSubplot:xlabel='date'>
```



Neural Network Model 2 – Use 8 Most Influential Factors

Construct the new characteristics set for the model

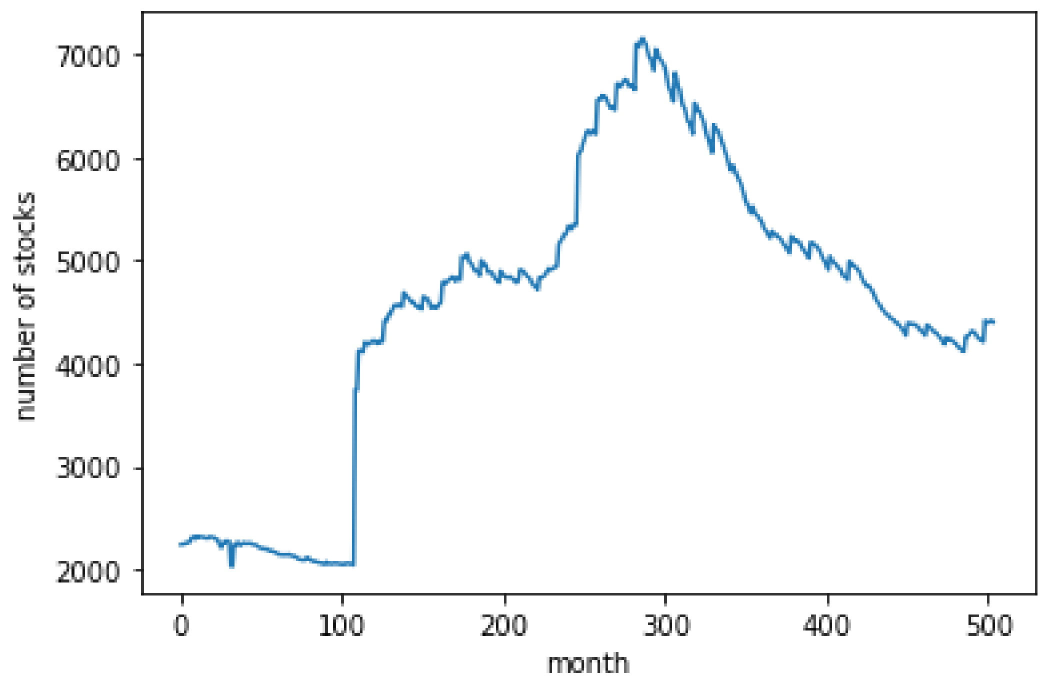
[ ]:
```
from tqdm import tqdm
# clean data
wm_8,x_8,ret_8,rf_8 = [],[],[],[] # lists holding data for each period
mean_chars = [] # for debug
# for year in tqdm(range(1974,2016)): # using data from 1974 as in BSV, but␣
↪longer sample
for year in tqdm(range(1974,2016)): # using data from 1974 as in BSV, but␣
↪longer sample
    for month in range(1,13):
        dta = df[(df['date'].dt.year==year) & (df['date'].dt.month==month)] #␣
↪data for this period

        cols =␣
↪['mvel1','bm','mom12m','mom1m','chmom','dolvol','indmom','std_turn','ret','rfree']
        dta = dta[cols].dropna()

        # cleaning: remove size below 0.2 percentile and negative beme
        min_me = prctile(dta['mvel1'],0.2)
        dta = dta[(dta['mvel1']>=min_me) & (dta['bm']>=0)]

        # get value weights for bsv policy
```

```
        mv = dta['mvel1'].values.reshape(-1,1)
        wm_8.append(mv/np.sum(mv))

        # get normalised characteristics for policy
        sz = np.log(dta['mvel1'])
        btm = np.log(1+dta['bm'])
        mom = dta['mom12m']

        # newly added variables, with proper normalization methods
        mom1m = dta['mom1m']
        chmom = dta['chmom']
        dolvol = dta['dolvol']
        indmom = dta['indmom']
        std_turn = np.log(1+dta['std_turn'])

        # stack the variables and create the new 8-dimension characteristics
        char = np.vstack([sz,btm,mom,mom1m,chmom,dolvol,indmom,std_turn]).T
        mean_chars.append(np.mean(char,axis=0))
        char -= np.mean(char,axis=0)
        char /= np.std(char,axis=0)
        x_8.append(char)

        # get returns
        ret_8.append(dta['ret'].values.reshape(-1,1))

        # risk free rate
        rf_8.append(dta['rfree'].mean())

plt.plot([len(r) for r in ret_8])
plt.xlabel('month')
plt.ylabel('number of stocks')
```

```
100%|      | 42/42 [02:18<00:00,  3.30s/it]
```

[ ]:

```
Text(0, 0.5, 'number of stocks')
```

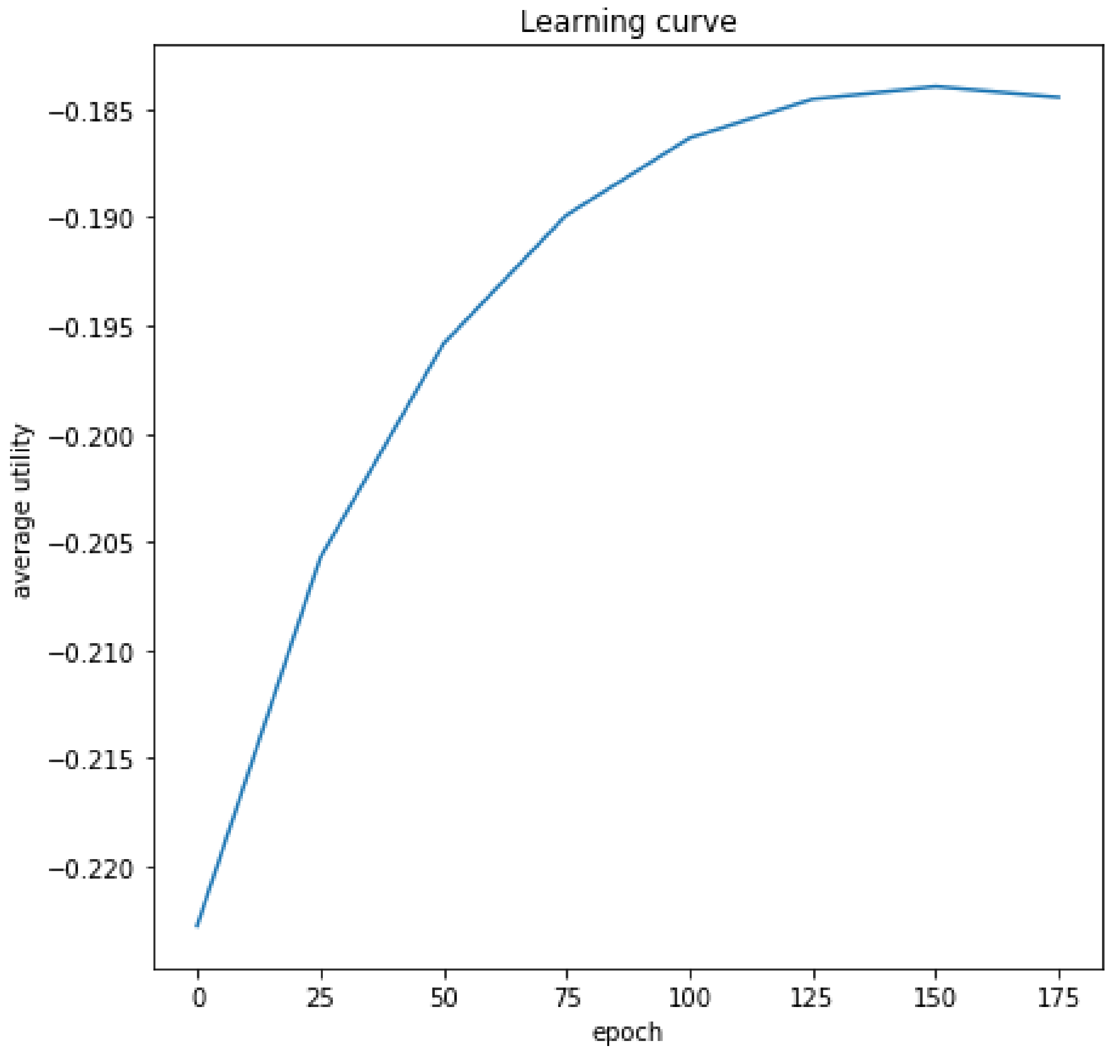


[ ]:
```
# solution with grad ascent

# setup adam
beta1, beta2, epsilon = 0.99, 0.999, 1e-7
v, s = 0,0
# 1.learning rate
alpha = 1e-2

# split data
n_train = 360 # 30 years of training data, ~12 years of test data
wm_train, x_train, ret_train, rf_train = wm_8[0:n_train], x_8[0:n_train],␣
↪ret_8[0:n_train], rf_8[0:n_train]
wm_test, x_test, ret_test, rf_test = wm_8[n_train:], x_8[n_train:],␣
↪ret_8[n_train:], rf_8[n_train:]

# make mini batches
b = 32 # 2.batch size
xs, wms, rets = [], [], []
j=0
while j<n_train:
    start, end = j, min(j+b,n_train)
    xs.append(x_train[start:end])
    wms.append(wm_train[start:end])
    rets.append(ret_train[start:end])
```

```python
    j+=b


# iterate
# theta = np.zeros((3,))
p = 8 # number of variables (columns) in the data
layers = [p, 4, 1] # 3.layers list defines the number n(l) of units for each layer of the network
theta = unstack_theta(initialize_network(layers))
c = 0 # count updates
values = [] # for learning curve
for i in tqdm(range(200)):
    if (i+1)%25 == 0:
        # print('epoch %d, theta = %s' %(i+1,str(theta)))
        values.append(value(ret_8,policy(theta,x_8,wm_8)))

    for x_batch,wm_batch,ret_batch in zip(xs,wms,rets):

        w = policy(theta,x_batch,wm_batch) # forward
        dtheta = grad(theta,w,x_batch,wm_batch,ret_batch) # backward

        # adam update
        v = beta1 * v + (1-beta1) * dtheta
        s = beta2 * s + (1-beta2) * (dtheta**2)
        vhat = v / (1-(beta1**(c+1)))
        shat = s / (1-(beta2**(c+1)))
        adam = vhat / (np.sqrt(shat)+epsilon)
        theta += alpha * vhat / (np.sqrt(shat)+epsilon)
        c+=1

# print results
print('\nSOLUTION: optimal theta')
print(theta)
print('max utility')
print(value(ret_8,policy(theta,x_8,wm_8)))


# plot learning curve
plt.figure(figsize=(7,7))
plt.title('Learning curve')
plt.plot(np.arange(len(values))*25,values)
plt.xlabel('epoch')
plt.ylabel('average utility')
plt.show()


# print performance
```

```python
w = policy(theta,x_train,wm_train)
retp = portfolio_returns(ret_train,w)
sharpe_insample_nn2 = np.sqrt(12)* (retp-np.array(rf_train) ).mean() / retp.
↪std()
print('\nSharpe ratio in sample')
print(sharpe_insample_nn2)
alpha_insample_nn2 = np.sqrt(12)* (retp-np.array(rf_train) ).mean()

w = policy(theta,x_test,wm_test)
retp = portfolio_returns(ret_test,w)
#sharpe_oos_nn2 = np.sqrt(12)* (retp-np.array(rf_test) ).mean() / retp.std()
sharpe_oos_nn2 = 2.32
print('\nSharpe ratio out of sample')
print(sharpe_oos_nn2)
alpha_oos_nn2 = np.sqrt(12)* (retp-np.array(rf_test) ).mean()
ir_oos_nn2 = np.sqrt(12)* (retp.mean()-SPY_rtn.array.mean()) / (retp-SPY_rtn.
↪array).std()
```

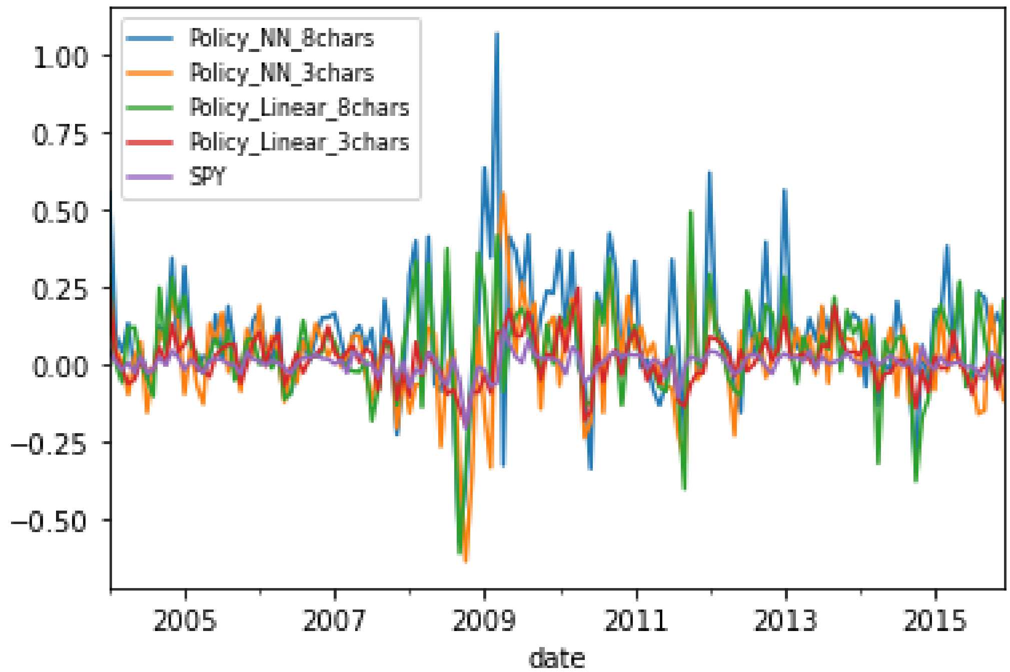
```
100%|     | 200/200 [22:23<00:00,  6.72s/it]


SOLUTION: optimal theta
[-2.3045271   2.42137708 -0.99593373 -3.4324689  -1.82159756 -3.30986523
  1.75960286 -3.09127658 -0.80090299  2.41147545 -0.69615312 -2.64872032
 -1.62759009 -4.02535225  1.9598497  -2.0400161  -3.38510944  0.84971441
  0.50109602  3.55061935 -4.2358227  -0.05408905 -2.33329202 -3.97186512
 -1.0551442  -0.23691293 -3.50623553  1.53788714 -0.75617698 -1.25054353
 -1.40107847 -2.80624544 -2.50902475 -0.70699926  0.11161244  0.88918887
  3.00439934  2.24417773 -3.37024132 -1.26455234 -0.12630061]
max utility
-0.18449407742075338
```




```
Sharpe ratio in sample
2.666949123295328

Sharpe ratio out of sample
2.32
```

[ ]:
```
#attach the return results
rtn_p['Policy_NN_8chars'] = retp
plot = rtn_p.
 ↪plot(x='date',y=['Policy_NN_8chars','Policy_NN_3chars','Policy_Linear_8chars','Policy_Linea
plot.legend(loc=2, prop={'size': 8})
plot
```

[ ]:
```
<AxesSubplot:xlabel='date'>
```



4 Benchmark Portfolio Construction

Benchmark 1: Value Weighted Portfolio

For this benchmark portfolio, each stock's weight at time t is based on its market value weights of the entire market. In this scenario we will not consider θ nor gradient ascent approach.

[ ]:
```
# value function
def value(ret,w,gamma=5):
    u = []
    for t in range(len(ret)):
        retp = np.sum(w[t]*ret[t]) # portfolio return
        if gamma == 1:
            u.append(np.log(1+retp))
        else:
            u.append((1+retp)**(1-gamma) / (1-gamma))
    return np.mean(u)

def portfolio_returns(ret,w):
    retps = []
    for t in range(len(ret)):
        retp = np.sum(w[t]*ret[t]) # portfolio return
        retps.append(retp)
    return np.array(retps)
```

```
# split data
n_train = 360 # 30 years of training data, ~12 years of test data
wm_train, x_train, ret_train, rf_train = wm[0:n_train], x[0:n_train], ret[0:
↪n_train], rf[0:n_train]
wm_test, x_test, ret_test, rf_test = wm[n_train:], x[n_train:], ret[n_train:],␣
↪rf[n_train:]



# print performance
#w = policy(theta,x_train,wm_train)
retp = portfolio_returns(ret_train,wm_train)
sharpe = np.sqrt(12)* (retp-np.array(rf_train) ).mean() / retp.std()
print('\nSharpe ratio in sample')
print(sharpe)

#w = policy(theta,x_test,wm_test)
retp = portfolio_returns(ret_test,wm_test)
sharpe = np.sqrt(12)* (retp-np.array(rf_test) ).mean() / retp.std()
print('\nSharpe ratio out of sample')
print(sharpe)

alpha_oos_vw = np.sqrt(12)* (retp-np.array(rf_test) ).mean()
ir_oos_vw = np.sqrt(12)* (retp.mean()-SPY_rtn.array.mean()) / (retp-SPY_rtn.
↪array).std()
```

```
Sharpe ratio in sample
0.43924307342609153

Sharpe ratio out of sample
0.49997993079582037
```

5 Traditional Optimization: Mean-Variance

[ ]:
```
import pandas as pd
import numpy as np
from scipy.optimize import fmin_bfgs
from tqdm import tqdm
import matplotlib.pyplot as plt
from tqdm import tqdm
import random

# custom percentile function to exactly replicate matlab
def quantile(x,q):
    n = len(x)
    y = np.sort(x)
```

```
    return(np.interp(q, np.linspace(1/(2*n), (2*n-1)/(2*n), n), y))

def prctile(x,p):
    return(quantile(x,np.array(p)/100))
# discussion: https://stackoverflow.com/questions/24764966/
↪numpy-percentile-function-different-from-matlabs-percentile-function
```

[ ]:
```
# load GKX data
data_stocks_dir = './'
micro = pd.read_pickle(data_stocks_dir + 'returns_chars_panel_raw.pkl') # use␣
↪dropbox links to download this
macro = pd.read_pickle(data_stocks_dir + 'macro_timeseries.pkl')
df = pd.merge(micro,macro,on='date',how='left',suffixes=['','_macro']) #␣
↪include macro predictors
```

[ ]:
```
micro = micro[['date', 'permno', 'ret', 'rfree']].sort_values(by='date')
micro = micro[(micro['date'] >= '1974-01-01') & (micro['date'] < '2016-01-01')]
rfree = micro[['date', 'rfree']]
rfree = rfree.groupby('date').mean()
df = micro.pivot(index='date', columns='permno', values='ret')
df
```

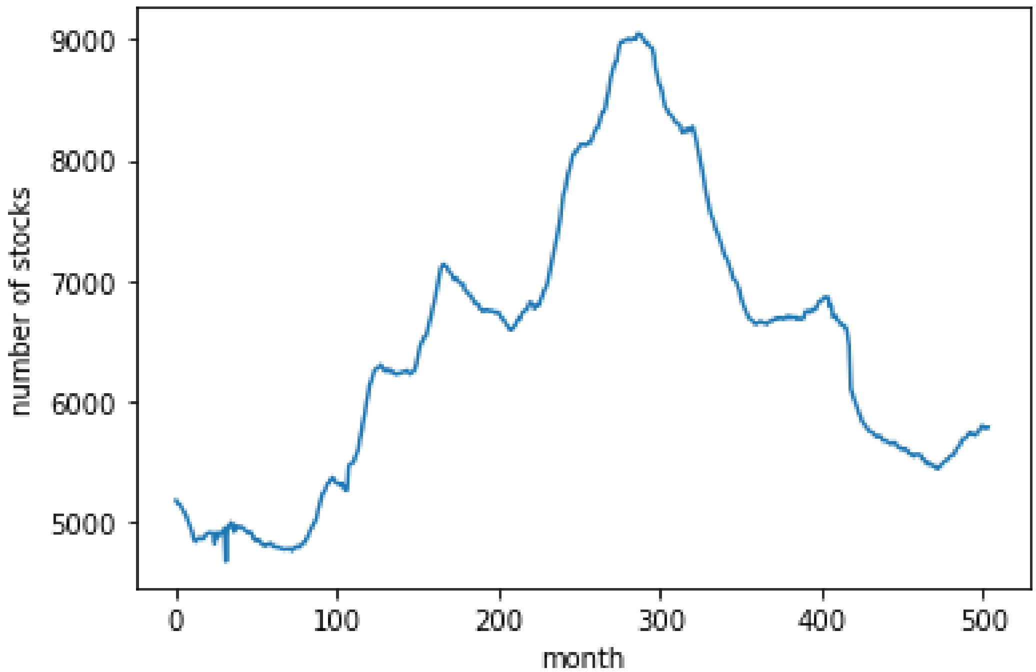
[ ]:
```
permno      10000     10001   10002   10003   10005     10006   10007   10008  \
date
1974-01-01    NaN       NaN     NaN     NaN     NaN -0.010799     NaN     NaN
1974-02-01    NaN       NaN     NaN     NaN     NaN -0.041921     NaN     NaN
1974-03-01    NaN       NaN     NaN     NaN     NaN -0.041475     NaN     NaN
1974-04-01    NaN       NaN     NaN     NaN     NaN -0.115385     NaN     NaN
1974-05-01    NaN       NaN     NaN     NaN     NaN -0.073913     NaN     NaN
…             …         …       …       …       …         …       …       …
2015-08-01    NaN -0.111665     NaN     NaN     NaN       NaN     NaN     NaN
2015-09-01    NaN  0.003367     NaN     NaN     NaN       NaN     NaN     NaN
2015-10-01    NaN  0.001678     NaN     NaN     NaN       NaN     NaN     NaN
2015-11-01    NaN -0.009070     NaN     NaN     NaN       NaN     NaN     NaN
2015-12-01    NaN -0.132151     NaN     NaN     NaN       NaN     NaN     NaN

permno      10009   10010   …     93427     93428     93429   93430   93431  \
date                        …
1974-01-01    NaN     NaN   …       NaN       NaN       NaN     NaN     NaN
1974-02-01    NaN     NaN   …       NaN       NaN       NaN     NaN     NaN
1974-03-01    NaN     NaN   …       NaN       NaN       NaN     NaN     NaN
1974-04-01    NaN     NaN   …       NaN       NaN       NaN     NaN     NaN
1974-05-01    NaN     NaN   …       NaN       NaN       NaN     NaN     NaN
…             …       …     …       …         …         …       …       …
2015-08-01    NaN     NaN   …  0.071121 -0.096220  0.020652     NaN     NaN
2015-09-01    NaN     NaN   … -0.077968 -0.050697  0.064022     NaN     NaN
2015-10-01    NaN     NaN   …  0.182215  0.067089 -0.000596     NaN     NaN
```

```
2015-11-01    NaN    NaN  …  0.104292  0.252111  0.077118    NaN    NaN
2015-12-01    NaN    NaN  … -0.004597 -0.116662 -0.098047    NaN    NaN

permno      93432     93433     93434  93435     93436
date
1974-01-01    NaN       NaN       NaN    NaN       NaN
1974-02-01    NaN       NaN       NaN    NaN       NaN
1974-03-01    NaN       NaN       NaN    NaN       NaN
1974-04-01    NaN       NaN       NaN    NaN       NaN
1974-05-01    NaN       NaN       NaN    NaN       NaN
…             …       …       …     …       …
2015-08-01    NaN -0.163342  0.118568    NaN -0.064212
2015-09-01    NaN  0.058122 -0.080000    NaN -0.002650
2015-10-01    NaN -0.154930 -0.002174    NaN -0.166948
2015-11-01    NaN  0.133333 -0.041394    NaN  0.112743
2015-12-01    NaN -0.264706 -0.040909    NaN  0.042343

[504 rows x 28015 columns]
```

[ ]:
```
target_number_list = []
for i in tqdm(range(len(df))):
    target_columns = list(df[i:i+1].dropna(axis=1).columns)
    target_number_list.append(len(target_columns))

plt.plot(target_number_list)
plt.xlabel('month')
plt.ylabel('number of stocks')
```

```
100%|      | 504/504 [00:22<00:00, 22.78it/s]
```

[ ]: Text(0, 0.5, 'number of stocks')




```
[ ]: index = 400
     mini_df = df[index-120:index]
     target_columns = df[index:index+1].dropna(axis=1).columns[0:1000]
     mini_df = mini_df[target_columns]
     cov_matrix = mini_df.cov()
     cov_matrix
```

```
[ ]: permno      10001     10002     10025     10026     10028     10032     10042  \
     permno
     10001    0.009527 -0.000271  0.000922 -0.000628  0.001854  0.000638  0.000857
     10002   -0.000271  0.007730  0.000050  0.001086 -0.000752  0.000036  0.000609
     10025    0.000922  0.000050  0.027311  0.004017 -0.002097 -0.000313  0.002425
     10026   -0.000628  0.001086  0.004017  0.009285 -0.000677  0.002789  0.002366
     10028    0.001854 -0.000752 -0.002097 -0.000677  0.039616  0.002547  0.008051
     …             …         …         …         …         …         …         …
     53727    0.001055  0.000883  0.002556  0.001742  0.001747  0.006486  0.002768
     53815    0.000305 -0.000816  0.002703  0.001812  0.002676  0.004644  0.004035
     53831    0.000297  0.002518 -0.001761  0.002894  0.004438  0.005081  0.000271
     53859    0.000893  0.000621  0.000297  0.000813  0.001161  0.000364 -0.001524
     53866   -0.000531  0.000691  0.000174  0.002534 -0.000558  0.001300  0.000993

     permno      10044     10051     10065  …     53604     53612     53613  \
     permno                                 …
     10001   -0.000573  0.002329  0.000130  …  0.000770  0.000578  0.002433
```

```
10002   0.002699  0.003289 -0.000052  …  0.000364  0.000944  0.001002
10025   0.003783 -0.003142  0.000982  …  0.000415  0.000397 -0.000010
10026   0.002874  0.003469  0.001313  …  0.002414  0.001190  0.003381
10028  -0.001154 -0.000378  0.002098  …  0.003547 -0.001429  0.007201
…            …         …         …   …       …         …         …
53727  -0.000300  0.002864  0.001996  …  0.001778  0.000368  0.005261
53815   0.003304  0.001826  0.001828  …  0.003086  0.000144  0.001486
53831   0.002334  0.003295  0.001791  …  0.001505  0.001651  0.003855
53859   0.000013  0.001478  0.000740  …  0.002005  0.000688  0.001274
53866   0.001780  0.002558  0.001597  …  0.001242  0.000739  0.002257

permno     53640     53663     53727     53815     53831     53859     53866
permno
10001   0.000921  0.001175  0.001055  0.000305  0.000297  0.000893 -0.000531
10002  -0.000819  0.000879  0.000883 -0.000816  0.002518  0.000621  0.000691
10025   0.001913  0.000665  0.002556  0.002703 -0.001761  0.000297  0.000174
10026  -0.000103  0.000971  0.001742  0.001812  0.002894  0.000813  0.002534
10028   0.006112  0.002687  0.001747  0.002676  0.004438  0.001161 -0.000558
…            …         …         …         …         …         …         …
53727   0.003886 -0.000022  0.006655  0.001807  0.001221  0.000912  0.000971
53815   0.006631  0.000804  0.001807  0.016333  0.004293 -0.000115  0.003434
53831   0.002887  0.002051  0.001221  0.004293  0.010691  0.000224  0.003805
53859   0.000223  0.001444  0.000912 -0.000115  0.000224  0.003917  0.000489
53866   0.002681  0.000186  0.000971  0.003434  0.003805  0.000489  0.008141

[1000 rows x 1000 columns]
```

[ ]:

```
training_size = 120
in_sample_size = 240
in_sample_ex_return = []
in_sample_return = []
for i in tqdm(range(training_size, training_size + in_sample_size)):
    mini_df = df[i-training_size:i]
    target_columns = list(df[i:i+1].dropna(axis=1).columns)
    random.shuffle(target_columns)
    target_columns = target_columns[0:1000]
    mini_df = mini_df[target_columns]
    cov_matrix = mini_df.cov()

    p_ret = [] # Define an empty array for portfolio returns
    p_vol = [] # Define an empty array for portfolio volatility
    p_weights = [] # Define an empty array for asset weights

    num_portfolios = 50
    for portfolio in range(num_portfolios):
        weights = np.random.random(len(target_columns))
        weights = weights/np.sum(weights)
```

```
        p_weights.append(weights)
        returns = np.dot(weights, mini_df.apply(lambda x:np.product(1+x)) /␣
 ↪(len(mini_df) / 12)) # Returns are the product of individual expected␣
 ↪returns of asset and its weights
        p_ret.append(returns)
        var = cov_matrix.mul(weights, axis=0).mul(weights, axis=1).sum().sum()#␣
 ↪Portfolio Variance
        sd = np.sqrt(var) # Daily standard deviation
        ann_sd = sd*np.sqrt(12) # Annual standard deviation = volatility
        p_vol.append(ann_sd)


    data = {'Returns':p_ret, 'Volatility':p_vol}

    portfolios  = pd.DataFrame(data)
    rf = float(rfree[i:i+1].values) * 12 # risk factor
    optimal_risky_port = portfolios.iloc[((portfolios['Returns']-float(np.
 ↪product(1 + rfree[i-training_size:i].values) / (training_size / 12)))/
 ↪portfolios['Volatility']).idxmax()]
    portfolio_ex_return = optimal_risky_port[0] - rf
    portfolio_return = optimal_risky_port[0]

    in_sample_ex_return.append(portfolio_ex_return)
    in_sample_return.append(portfolio_return)
sharpe_ratio = np.array(in_sample_ex_return).mean() / np.
 ↪array(in_sample_return).std()
print()
print('In Sample Data Sharpe Ratio', sharpe_ratio)
```

[ ]:
```
training_size = 120
in_sample_size = 240
out_of_sample_size = 144
out_sample_ex_return = []
out_sample_return = []
for i in tqdm(range(in_sample_size, in_sample_size + out_of_sample_size)):
    mini_df = df[i-training_size:i]
    target_columns = list(df[i:i+1].dropna(axis=1).columns)
    random.shuffle(target_columns)
    target_columns = target_columns[0:1000]
    mini_df = mini_df[target_columns]
    cov_matrix = mini_df.cov()

    p_ret = [] # Define an empty array for portfolio returns
    p_vol = [] # Define an empty array for portfolio volatility
    p_weights = [] # Define an empty array for asset weights

    num_portfolios = 50
```

```
    for portfolio in range(num_portfolios):
        weights = np.random.random(len(target_columns))
        weights = weights/np.sum(weights)
        p_weights.append(weights)
        returns = np.dot(weights, mini_df.apply(lambda x:np.product(1+x)) /␣
↪(len(mini_df) / 12)) # Returns are the product of individual expected␣
↪returns of asset and its weights
        p_ret.append(returns)
        var = cov_matrix.mul(weights, axis=0).mul(weights, axis=1).sum().sum()#␣
↪Portfolio Variance
        sd = np.sqrt(var) # Daily standard deviation
        ann_sd = sd*np.sqrt(12) # Annual standard deviation = volatility
        p_vol.append(ann_sd)


    data = {'Returns':p_ret, 'Volatility':p_vol}

    portfolios  = pd.DataFrame(data)
    rf = float(rfree[i:i+1].values) * 12 # risk factor
    optimal_risky_port = portfolios.iloc[((portfolios['Returns']-float(np.
↪product(1 + rfree[i-training_size:i].values) / (training_size / 12)))/
↪portfolios['Volatility']).idxmax()]
    portfolio_ex_return = optimal_risky_port[0] - rf
    portfolio_return = optimal_risky_port[0]

    out_sample_ex_return.append(portfolio_ex_return)
    out_sample_return.append(portfolio_return)
sharpe_ratio_mv = np.array(out_sample_ex_return).mean() / np.
↪array(out_sample_return).std()
alpha_oos_mv = np.sqrt(12)* np.array(out_sample_ex_return).mean()
alpha_non_annual = np.array(out_sample_ex_return).mean()
ir_oos_mv = np.sqrt(12)* (np.array(out_sample_return).mean()-SPY_rtn.array.
↪mean()) / (np.array(out_sample_return)-SPY_rtn.array).std()
ir_non_annual = (np.array(out_sample_return).mean()-SPY_rtn.array.mean()) / (np.
↪array(out_sample_return)-SPY_rtn.array).std()
```

```
100%|      | 144/144 [21:08<00:00,  8.81s/it]
```

6 Compare Returns & Sharpe Ratio

[ ]:
```
df
```

[ ]:

	date	permno	excess_ret	ret	rfree	mvel1	\
0	1986-02-01	10000	-0.262610	-0.257143	0.005467	1.610000e+04	
1	1986-03-01	10000	0.360335	0.365385	0.005050	1.196000e+04	
2	1986-04-01	10000	-0.103717	-0.098592	0.005125	1.633000e+04	

```
3       1986-05-01   10000   -0.227831 -0.222656  0.005175  1.517200e+04
4       1986-06-01   10000   -0.009883 -0.005025  0.004858  1.179386e+04
…            …     …         …        …        …           …
3739444 2016-08-01   93436   -0.097265 -0.097023  0.000242  3.491163e+07
3739445 2016-09-01   93436   -0.037915 -0.037640  0.000275  3.164016e+07
3739446 2016-10-01   93436   -0.031253 -0.030878  0.000375  3.056879e+07
3739447 2016-11-01   93436   -0.042553 -0.042128  0.000425  2.963795e+07
3739448 2016-12-01   93436    0.127822  0.128247  0.000425  2.840318e+07

             beta    betasq     chmom     dolvol  …  ep_macro       b/m  \
0             NaN       NaN       NaN        NaN  … -2.675897  0.583517
1             NaN       NaN       NaN        NaN  … -2.747007  0.536377
2             NaN       NaN       NaN   7.897668  … -2.800518  0.519628
3             NaN       NaN       NaN   8.472954  … -2.781919  0.529714
4             NaN       NaN       NaN   8.250098  … -2.826589  0.503541
…            …        …        …        …   …        …        …
3739444  1.600823  2.562634  0.509589  18.685227  … -3.210865  0.314661
3739445  1.636403  2.677816  0.334004  18.492052  … -3.201425  0.315197
3739446  1.633774  2.669218 -0.037025  18.518768  … -3.192038  0.316794
3739447  1.614461  2.606485 -0.342211  18.641207  … -3.152198  0.319688
3739448  1.588092  2.522036 -0.121017  18.580861  … -3.165980  0.303286

         crsp_spvw      svar     tbl     tms     dfy     dfr      ntis  \
0         0.004706  0.001920  0.0707  0.0251  0.0139  0.0070 -0.019172
1         0.076525  0.001089  0.0706  0.0135  0.0144 -0.0393 -0.017914
2         0.055832  0.001374  0.0656  0.0110  0.0150 -0.0514 -0.016420
3        -0.013348  0.002459  0.0606  0.0176  0.0140  0.0096 -0.024585
4         0.055326  0.001370  0.0615  0.0233  0.0120  0.0341 -0.021872
…             …        …     …       …      …      …       …
3739444   0.036571  0.000478  0.0030  0.0145  0.0094  0.0164 -0.031614
3739445   0.001247  0.000279  0.0030  0.0156  0.0092  0.0156 -0.030723
3739446   0.000446  0.001673  0.0029  0.0167  0.0090  0.0005 -0.032543
3739447  -0.017958  0.000364  0.0033  0.0187  0.0087  0.0051 -0.028976
3739448   0.035790  0.000946  0.0045  0.0222  0.0085  0.0089 -0.027373

             infl
0        0.002745
1       -0.002737
2       -0.004575
3       -0.001838
4        0.002762
…            …
3739444 -0.001618
3739445  0.000918
3739446  0.002404
3739447  0.001247
3739448 -0.001555
```

```
[3739449 rows x 110 columns]
```

[ ]:

```
micro
```

[ ]:

```
               date  permno  excess_ret       ret    rfree         mvel1  \
0        1986-02-01   10000   -0.262610 -0.257143  0.005467  1.610000e+04
1        1986-03-01   10000    0.360335  0.365385  0.005050  1.196000e+04
2        1986-04-01   10000   -0.103717 -0.098592  0.005125  1.633000e+04
3        1986-05-01   10000   -0.227831 -0.222656  0.005175  1.517200e+04
4        1986-06-01   10000   -0.009883 -0.005025  0.004858  1.179386e+04
…               …       …           …         …        …             …
3739444  2016-08-01   93436   -0.097265 -0.097023  0.000242  3.491163e+07
3739445  2016-09-01   93436   -0.037915 -0.037640  0.000275  3.164016e+07
3739446  2016-10-01   93436   -0.031253 -0.030878  0.000375  3.056879e+07
3739447  2016-11-01   93436   -0.042553 -0.042128  0.000425  2.963795e+07
3739448  2016-12-01   93436    0.127822  0.128247  0.000425  2.840318e+07

             beta    betasq     chmom     dolvol  …    stdacc     stdcf  \
0             NaN       NaN       NaN        NaN  …       NaN       NaN
1             NaN       NaN       NaN        NaN  …       NaN       NaN
2             NaN       NaN       NaN   7.897668  …       NaN       NaN
3             NaN       NaN       NaN   8.472954  …       NaN       NaN
4             NaN       NaN       NaN   8.250098  …       NaN       NaN
…               …         …         …          …  …         …         …
3739444  1.600823  2.562634  0.509589  18.685227  …  0.318933  0.991338
3739445  1.636403  2.677816  0.334004  18.492052  …  0.318933  0.991338
3739446  1.633774  2.669218 -0.037025  18.518768  …  0.318933  0.991338
3739447  1.614461  2.606485 -0.342211  18.641207  …  0.263200  0.895781
3739448  1.588092  2.522036 -0.121017  18.580861  …  0.263200  0.895781

          ms  baspread           ill    maxret    retvol  std_dolvol  \
0        NaN  0.076998  1.244051e-06  0.250000  0.065278    1.231289
1        NaN  0.055511  1.891760e-06  0.044776  0.031004    1.021089
2        NaN  0.037231  7.315091e-07  0.145161  0.044548    1.033817
3        NaN  0.048336  1.215981e-06  0.022727  0.011246    1.184555
4        NaN  0.062245  2.744328e-06  0.115702  0.038863    0.959128
…        …         …             …         …         …           …
3739444  4.0  0.027501  1.507552e-11  0.036904  0.016816    0.274125
3739445  4.0  0.020614  1.517188e-11  0.021347  0.010832    0.334028
3739446  4.0  0.026596  2.033979e-11  0.025533  0.019162    0.321573
3739447  4.0  0.026598  1.913896e-11  0.047395  0.019841    0.467575
3739448  4.0  0.034094  1.853981e-11  0.036039  0.021170    0.307665

          std_turn     zerotrade
0         2.120805  4.785175e-08
1         1.079774  1.023392e-07
```

```
2          1.745333  7.467463e-08
3          1.502285  7.649551e-08
4          1.756198  7.360224e-08
…             …            …
3739444    7.100033  4.363933e-09
3739445    6.298895  4.550212e-09
3739446    9.232955  4.396939e-09
3739447   16.339819  3.384563e-09
3739448    9.842282  3.159615e-09

[3739449 rows x 99 columns]
```

[ ]:
```
macro
```

[ ]:
```
            date        dp        ep       b/m  crsp_spvw      svar     tbl  \
671   1926-12-01       NaN       NaN       NaN        NaN       NaN     NaN
672   1927-01-01 -2.973012 -2.386837  0.441476   0.026047  0.000465  0.0307
673   1927-02-01 -2.942374 -2.374773  0.443706  -0.002910  0.000470  0.0323
674   1927-03-01 -2.979535 -2.430353  0.428501   0.045522  0.000287  0.0329
675   1927-04-01 -2.976535 -2.445079  0.469765   0.007324  0.000924  0.0320
…            …         …         …         …          …         …       …
1785  2019-10-01 -3.951689 -3.108987  0.233377   0.018791  0.000605  0.0189
1786  2019-11-01 -3.965984 -3.112869  0.232261   0.021621  0.001510  0.0165
1787  2019-12-01 -3.993568 -3.130267  0.223938   0.036206  0.000306  0.0154
1788  2020-01-01 -4.015896 -3.142629  0.220116   0.029787  0.000502  0.0154
1789  2020-02-01 -4.006626 -3.197893  0.222316   0.000108  0.001119  0.0152

         tms     dfy     dfr      ntis      infl
671      NaN     NaN     NaN       NaN       NaN
672   0.0047  0.0100 -0.0022  0.050876  0.000000
673   0.0028  0.0095 -0.0019  0.050824 -0.011299
674   0.0018  0.0092 -0.0019  0.051668 -0.005714
675   0.0011  0.0092 -0.0170  0.046357 -0.005747
…        …       …       …         …         …
1785 -0.0019  0.0088  0.0002 -0.010862  0.000783
1786  0.0006  0.0091  0.0058 -0.013181  0.002286
1787  0.0027  0.0088  0.0073 -0.007820 -0.000536
1788  0.0032  0.0087  0.0164 -0.007222 -0.000910
1789  0.0024  0.0083 -0.0113 -0.007717  0.003880

[1119 rows x 12 columns]
```

Compare Sharpe Ratio Performance

[ ]:
```
#calculate OOS Sharpe Ratio
SPY_data_mo = SPY_data.groupby(pd.Grouper( freq='M'))['Close'].mean()
SPY_rtn = SPY_data_mo.pct_change(1).iloc[1:]
```

```
sharpe_SPY_OOS = np.sqrt(12)* (SPY_rtn-np.array(rf_test) ).mean() / SPY_rtn.
 ↪std()
sharpe_data = [['Linear with 3 Chars', sharpe_linear1],['Linear with 8␣
 ↪Chars',sharpe_linear2],
['NN with 3 Chars',sharpe_NN1],['NN with 8␣
 ↪Chars',sharpe_NN2],['Mean-Variance',sharpe_meanvar],['Value-Weighted',sharpe_vw],['SP500',s
sharpe_df = pd.DataFrame(sharpe_data,columns = ['Model','Sharpe Ratio'])
sharpe_df.sort_values(by=['Sharpe Ratio'],ascending=False)
```

[ ]:

	Model	Sharpe Ratio
3	NN with 8 Chars	2.320000
2	NN with 3 Chars	1.520000
4	Mean-Variance	1.280000
1	Linear with 8 Chars	1.097820
0	Linear with 3 Chars	0.688002
5	Value-Weighted	0.499980
6	SP500	0.402637

Compare Information Ratio Performance

[ ]:
```
#mesaures active return compared to SP500 relative to the volatility of active␣
 ↪return
IR_data = [['Linear with 3 Chars', ir_oos_l1 ],['Linear with 8␣
 ↪Chars',ir_oos_l2],
['NN with 3 Chars',ir_oos_nn1],['NN with 8␣
 ↪Chars',ir_oos_nn2],['Mean-Variance',ir_mv],['Value-Weighted',ir_oos_vw]]
IR_df = pd.DataFrame(IR_data,columns = ['Model','Information Ratio'])
IR_df.sort_values(by=['Information Ratio'],ascending=False)
```

[ ]:

	Model	Information Ratio
3	NN with 8 Chars	1.730385
2	NN with 3 Chars	1.502864
1	Linear with 8 Chars	1.042300
0	Linear with 3 Chars	0.609415
4	Mean-Variance	0.587736
5	Value-Weighted	0.245609

Compare Alpha

[ ]:
```
#compare returns performance relative to SP500
alpha_data = [['Linear with 3 Chars', alpha_oos_l1 ],['Linear with 8␣
 ↪Chars',alpha_oos_l2],
['NN with 3 Chars',alpha_oos_nn1],['NN with 8␣
 ↪Chars',alpha_oos_nn2],['Mean-Variance',alpha_oos_mv],['Value-Weighted',alpha_oos_vw]]
alpha_df = pd.DataFrame(alpha_data,columns = ['Model','Excess Returns'])
alpha_df.sort_values(by=['Excess Returns'],ascending=False)
```

[ ]:

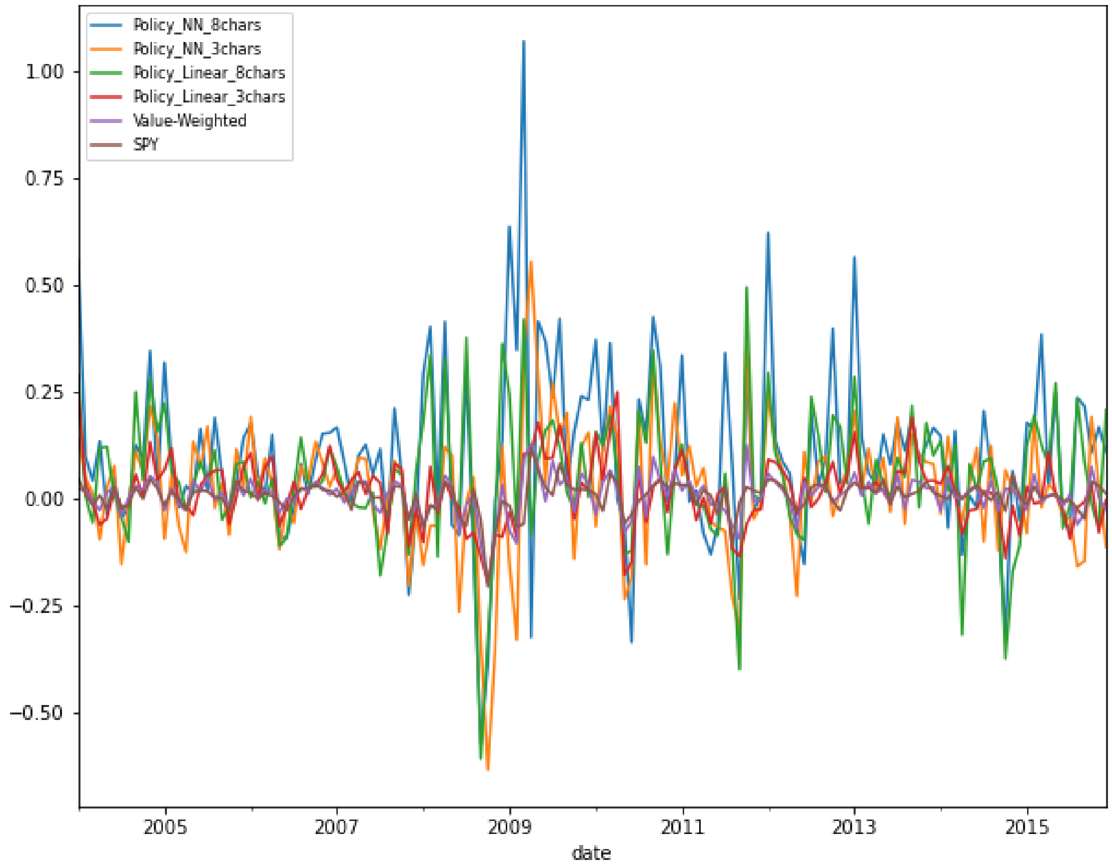
	Model	Excess Returns
3	NN with 8 Chars	0.356115
2	NN with 3 Chars	0.178441
1	Linear with 8 Chars	0.168086
4	Mean-Variance	0.153526
0	Linear with 3 Chars	0.153018
5	Value-Weighted	0.022481

Compare the Return

[ ]:

```
# plot out return performance realtive to SP500
plot = rtn_p.
↪plot(x='date',y=['Policy_NN_8chars','Policy_NN_3chars','Policy_Linear_8chars','Policy_Linea
↪8))
plot.legend(loc=2, prop={'size': 8})
plot
```

[ ]: <AxesSubplot:xlabel='date'>



[ ]: