



# Channel estimation strategies for underwater acoustic (UWA) communication: An overview

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## Abstract

Underwater wireless communication techniques have played the most important role in the exploration of oceans and other aquatic environments. Underwater acoustic (UWA) communication faces a lot of hurdles such as environmental characteristics, variety of noises, temperature, pressure, salinity, etc which makes the UWA channel unique. UWA channel modeling is the most demanding task due to its time-varying property and due to its double dispersion property resulting in severe multipath spread and time variation. Accurate estimation and tracking of channel state information (CSI) are required for receiver design and channel capacity analysis in an underwater environment. The goal of this study is to provide a comprehensive survey on channel estimation of the latest researches in the field of UWA communication. The previous works are summarized, reviewed and compared according to their years of publication. This paper provides an overview on the channel model, methods, and algorithms for channel estimation and equalization in an underwater environment. It also includes a journey of channel estimation from time-varying UWA multipath model towards the estimation of a multipath underwater channel in the Multiple-input Multiple-output Orthogonal Frequency Division Multiplexing (MIMO–OFDM) environment.

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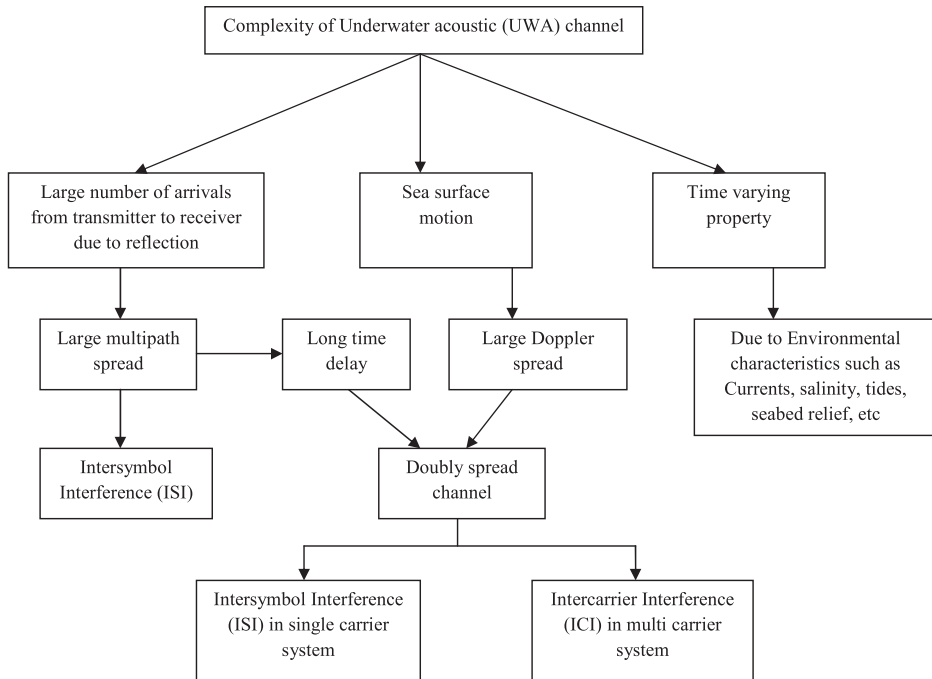


Fig. 1. Factors to be considered for designing UWA channel.

## 1. Introduction

Underwater acoustic (UWA) communications are the only approach for data transmission with high speed and high reliability in underwater [1]. With the rapid expansion in technology, underwater communications have become a fast-growing field, with extensive applications in commercial and military water based systems. The need for underwater wireless communications exists in applications such as remote control in the off-shore oil industry, pollution monitoring in environmental systems, collection of scientific data from ocean-bottom stations, disaster detection and early warning, national security, and in defense for intrusion detection, as well as new resource discovery. Thus, the research of new underwater wireless communication techniques has played the most important role in the exploration of oceans and other aquatic environments.

The complexity of underwater acoustic channels includes large Doppler spread due to sea surface motion, multipath delay due to a large number of arrivals and time-varying property due to environmental characteristics as shown in Fig. 1. The environmental characteristics include currents, tides, salinity, seabed relief, and movement of underwater acoustic systems [2, 3]. The UWA channel has non-stationary and time-varying channel impulse response (CIR) which severely limits data transmission rate due to multipath time-varying propagation [4, 5]. Gaining knowledge on the dispersing nature of UWA channel due to multipath effect, low speed of propagation, and strong multipath interference is highly necessary for data transmission in an underwater environment [6, 7]. Disperse from time domain due to large delay result in Intersymbol interference (ISI) and from frequency domain due to the motion

of sea surface and drift of transmitter and receiver result in the time-varying channel [7, 8]. Many times doubly selective channel result in Intersymbol interference (ISI) in a single carrier system and Intercarrier interference (ICI) in a multicarrier system [9]. The UWA channel is also considered as a sparse channel means a channel which has a large number of zero taps in time domain response [10].

Channel estimation is a challenging task in an underwater environment acoustic communication [11–20]. Different researchers with different system models have estimated the underwater channel. Researchers estimated the underwater channel using ray theory model [21], adaptive signal processing method based on least squares [22], and Channel estimation in MIMO system [23], Sparse recovery methods to shallow water [24–29].

In this work we review, summarize, and compare different channel estimation schemes for multipath channel model, SISO and SIMO system model, Pilot assisted OFDM system model, Cyclic prefix (CP) OFDM system model, Zero-padded (ZP) OFDM system model, MIMO system model, and finally MIMO–OFDM system model as per system capacity, efficiency and effectiveness of system. Since the estimation scheme completely depends on the system model, hence the system model of an underwater channel model is a necessary entity. Many researchers have focused not only on channel estimation but also on detection, coding, equalization, and transmission but here we have focused only on channel estimation and equalization. The paper further proceeds as follows: Section 2 represents a brief review on UWA systems. Section 3 represents a brief review of algorithms for channel estimation in UWA communication. Section 4 represents the conclusion and future scope. Section 5 represents the references used in this paper.

## 2. Underwater acoustic (UWA) systems review

This section represents a brief review of the research on channel estimation, symbol detection and error measurement as per system model. System models for UWA communication are categorized as Multipath channel model, SISO and SIMO system model, Pilot assisted OFDM system model, Cyclic prefix (CP) OFDM system model, Zero-padded (ZP) OFDM system model, MIMO system model, and MIMO–OFDM system model.

Continuous estimation of time-varying channel via linear and nonlinear equalization includes adaptive decision feedback techniques which are demanding in algorithm stability, computational complexity, and selection of channel parameters [30]. Existing time domain methods include time reversal [31], Passive phase conjugate [32], time-domain adaptive equalizer with PLL [33, 34], and multichannel decision feedback equalizer (DFE) uses error control coding schemes to reduce bit error rate (BER) in underwater communication but at the expense of bandwidth efficiency due to coding. Estimation of channel state information (CSI) based on blind methods which depend on statistical characteristics of the received signal result in poor CSI [12]. Pilot assisted methods though decrease bandwidth efficiency need a large number of pilots due to large multipath delay and Doppler spread in underwater acoustic communication (UWA) [11]. Many Doppler estimation techniques based on single carrier modulation [35–37] enjoy high spectral efficiency with time-domain equalization but with high complexity due to Doppler spread and multipath delay [38–43]. Hence multicarrier modulation is used to achieve high spectral efficiency with low complexity.

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation that will be applicable for a doubly selective channel with a reduction in complexity by exploiting

channel sparsity and channel correlation in frequency, time, and angle domain which makes underwater communication different from the cellular network [44, 45]. ISI is perfectly eliminated by the OFDM system using a cyclic prefix or zero padding (ZP) whose length should be larger or equal to channel impulse response (CIR) [46]. The Cyclic prefix OFDM system will result in a decrease in energy efficiency. Hence zero padded OFDM system for saving transmission power or energy is giving preference over cyclic prefix (CP) [47] in UWA communication. But in some cases, CP OFDM is giving preference to avoid signal to noise ratio (SNR) loss [48] and receiver complexity over ZP OFDM [49]. The biggest advantage of using OFDM system is that it converts a frequency selective channel into a set of parallel flat subchannels, thus greatly solving the equalization at the receiver in one tap. OFDM eliminates the need for complex time domain equalizer. But in an OFDM system, an error due to carrier frequency offset (CFO) exploits the orthogonality between subcarriers leading to Inter-carrier interference (ICI) in channel estimation.

Further OFDM introduces frequency and time synchronization to sustain orthogonality between subcarriers and resist CFO. Channel capacity increases using multiple input multiple output (MIMO) system compared to single input multiple output (SIMO) system [50, 51]. MIMO increases system capacity through multiple transmitting and receiving antenna. MIMO system is different from other spatial diversity systems as MIMO system transmits different coded data from different transmitting antenna through the parallel transmission. Finally, MIMO with OFDM named as MIMO-OFDM is used for high-speed underwater communication with low complexity over limited bandwidth frequency selective channel.

### 3. Algorithms for channel estimation for UWA communication

The underwater acoustic channel is sparse in nature in the sense that few propagation paths are carrying channel energy. Sparse recovery algorithms are implemented for sparse signal reconstruction. Sparse recovery algorithms are roughly divided into two categories. In one category belong convex optimization algorithms such as  $l_1$ -LS [52], YALLI [53], and SpaRSA [54] and the other category includes greedy algorithms such as Matching Pursuit (MP), Orthogonal matching pursuit (OMP), Compressive sampling matching pursuit (CoSaMP) [55], Subspace pursuit (SP) [56], and Homotopy [57]. Least square (LS) and RLS (Recursive Least Square) as an adaptive channel estimator giving promising bit error rate (BER) performance and average mean square error (MSE) performance better than Linear minimum mean square error (LMMSE) [58]. The problem with LS and LMMSE is that they don't consider sparseness of underwater channel which leads many pilots for severely double spread channel estimation. RLS adaptive algorithm gives faster convergence than LMS (Least Mean Square) based algorithm but at the expense of higher complexity compared to LMS [42]. Further LMS and RLS algorithm are unable to reconcile rapid channel variation due to algorithm limited tracking capability. CoSaMP and SP have the problem that they require explicit knowledge of sparsity. Further, to overcome all such problems compressive sensing (CS) techniques are considered.

CS includes Matching Pursuit (MP), Basic Pursuit (BP) and Orthogonal matching pursuit (OMP) at the sub nyquist rate for sparse channel estimation when ICI is considered to improve spectral efficiency [59]. MP iteratively estimate channel taps by maximizing the correlation of mixture matrix with a residual signal [60]. Further, some researchers have also focused on exponential smoothing based simultaneous OMP (ES-SOMP). OMP algorithm used to develop low complexity, angle domain, and sparse frequency selective CS algorithm for the underwa-

ter MIMO–OFDM system [61–63]. Homotopy complexity is in the same order as OMP and sparse recovery performance is as good obtained from convex optimization. Hence to solve compressing sensing sometimes in addition to BP, OMP another algorithm called Least Absolute Shrinkage and Selection Operator (LASSO) [64] is used. These algorithms are unable to exploit cluster sparsity structure of CIR for overall performance. To solve this problem authors many times used group LASSO [65], Block OMP [66], and block compressing sampling matching pursuit [67]. The problem with these methods is that they need prior information which is very difficult and almost impossible in a practical communication system. In such a case, sparse Bayesian Learning (SBL) plays its role for sparse signal reconstruction.

#### 4. Channel estimation in UWA systems

This section represents the research on channel estimation, symbol detection and error measurement as per system models. The summary of various methods with mathematical equations, data acquisition location, parameters for experiments or simulation along with simulated/experimental outcomes and advantages are represented in Table 1.

##### 4.1. Channel estimation in multipath channel model

To reduce computational complexity through significant channel estimate components and to improve accuracy, receiver structure matched to channel physical characteristics [68] is designed which rely on the decision-feedback equalizer to compute the parameter of channel estimator with optimal coefficient selection. This estimation cancels the post-cursor ISI term and channel response recorded with MSE. Multipath delayed/dominant arrivals can be further compressed through MP to improve the performance of Passive Phase Conjugation (PPC) at the receiver. The time reversal receiver structure PPC-DFE and PPC-McDFE are used for tap coefficients of DFE with fast convergence with RLS algorithm [69].

A non-convex sparse optimization approach for fast tracking of sparse coefficient and measurement of tracking accuracy is achieved through a sparse reconstruction of rapid non-stationary transitions due to motion and fluid. Prediction error and tracking computation time for NCMNS are least compared to L1-LS, GPSR, GPSS-BB and YALL1 [70]. Further measurement of channel tracking error as signal prediction error is done through channel components in signal subspace with the help of subspace tracker ASRAE-RLS to track variation in time based on CIR estimation [71].

A cluster-sparsity  $l_{2,0}$  norm shrinkage LMS algorithm ( $l_{2,0}$ -SH-LMS) is applied to identify a millimeter-wave frequency band channel ranging from 150 KHz to 1500 KHz to improve the performance of channel estimation efficiently. The proposed  $l_{2,0}$  norm shrinkage LMS algorithm in [72] results in faster convergence rate with lower steady state misalignment over Block Sparse Least Mean Square (BS-LMS) algorithm.

In [73], the authors focused on non-uniform sampling across the different frequency range with compressive sampling across Doppler frequency and full-rate sampling at lower Doppler frequency. Finally, the channel is estimated through a 2D frequency domain representation of shallow water acoustic channel.

In [74], the authors focused on minimization of logarithmic cost function of 1st order methods, LCLMA, LCLMS, and 2nd order methods, LCRLA, LCRLS using adaptive filtering resulting in trade-off between steady-state performance and convergence rate. Further, the authors have also focused on frequency and phase offsets. At last steady state performance

Table 1

Summary of review of channel estimation for various models in UWA communication system.

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ for Simulations/Experiments	Advantages	Outcomes
<b>4.1 Multipath channel/system model for UWA communication system</b>					
Stojanovic M.	2008 [68]	Channel estimator aided decision feedback equalization in conjunction with multichannel spatial diversity combining. The data symbol estimate is calculated as: $\hat{d}(n) = \sum_{p=1}^P \alpha_p [x_p \theta(n) - \sum_{i>0} \hat{f}_p(i) \hat{d}(n-i)]$ The phase corrected signals are represented as: $\hat{f}_p(n) = \lambda_{ch} \hat{f}_p[n-1] + (1 - \lambda_{ch}) x_{p\theta}(n) d^*(n),$ $P=1, \dots, P$ Computation of post-cursor ISI term is done as: $x_{pb}(n+1) = \downarrow x_{pb}(n) + \hat{f}_p(1, n) d(n),$ $P=1, \dots, P$	Continental Shelf region near the coast of New England. Water depth range=100 m to 200 m Carrier frequency ( $f_c$ )=25 KHz Modulation=QPSK Bit rate=10Kbits/s 8 hydrophone on vertical array spaced by 0.03m	Reduction in complexity due to selection of significant components.	Excellent result when applied to estimate data symbols where real data transmitted at 10Kbps over 3Km in shallow water. Experimental output: Channel response was recorded with mean square error (MSE), the phase estimate, and output scatter plot of estimated symbol for full complexity and reduced complexity was obtained for multichannel processing.
Zhang G et.al.	2012 [69]	Matching Pursuit (MP) algorithm for sparse channel estimation Passive-Phase Conjunction (PPC) Processing. The tap value is estimated as $\hat{H}_k^{lp} = \frac{(I)_{lp}^H r_{p-1}}{\ (I)_{lp}\ ^2}$ And $r_p$ is updated by $r_p = r_{p-1} - \frac{(I)_{lp}^H r_{p-1}}{\ (I)_{lp}\ ^2} (I)_{lp}$	Trondheim harbor in Norway. Sampling frequency at receiver ( $F_s$ )=96KHz carrier frequency ( $F_c$ )=12 KHz symbol rate (R)=1.2 Kbps No. of observation symbols (M)=100 Over sampling factor=4 No. feed-forward filter taps ( $N_g$ )=16 No. of feedback filter taps ( $N_b$ )=2 No. of training symbols ( $N_t$ )=72 $\Lambda$ =RLS forgetting factor=0.999 No. of channels (K)=8 Proportional tracking constant in PLL ( $K_1$ )=0.01 Integral tracking constant in PLL ( $K_2$ )=0.001	Sparse channel estimation using MP algorithm which improves the performance of PPC-McDFE receiver structure at 1 kbps and 2 kbps	Passive phase conjunction multichannel DFE (PPC-McDFE) achieved superior performance through MP (matching Pursuit) of 11.8 at 1 kb/s and 10.8 at 2 kb/s. Experimental output: MP processing improves performance through dominant arrivals.

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Sen Gupta A. et al.	2012 [70]	<p>Conventional <math>L_2</math> constrained <math>L_1</math> minimization (non-convex sparse optimization)</p> <p>Estimation of sparse channel through mixed norm formulation is expressed as:</p> $u_{opt} = \underset{u \in C^{KL}}{\operatorname{argmin}} ((1 - \lambda) \ u\ _1 + \lambda \ Cu - y\ _2^2)$ <p>where <math>\lambda</math> is a design parameter that chooses weighting between <math>L_1</math> and <math>L_2</math> of mixed norm cost. <math>u</math> is the vector of sparse coefficients to be estimated, <math>C</math> is the system matrix, and <math>y</math> is the received signal matrix.</p> <p>The geometric mixed norm for the above equation is represented as: <math>Z_{opt} = \underset{z \in C^{KL}}{\operatorname{argmin}} (1 - \lambda) \ z\ _2^2 + \lambda \ CZ^2 1_{KL \times 1} - y\ _2^2)</math></p> <p>Where <math>z</math> is a diagonal matrix, <math>1_{KL \times 1}</math> is KL-length vector</p> <p>Normalized prediction error is mathematically represented as:</p> $\epsilon_{pred}(i) = \ C(i + 1)\hat{u} - y(i + 1)\ _2^2$	<p>Shallow water acoustic channel of depth = 15 m</p> <p>Range = 200 m</p> <p>Wind conditions vary from moderate to high</p>	<p>Non-convex mixed norm solver (NCMNS) outperforms over <math>L_1</math> regularized least squares (L1-LS), Gradient projection-based sparse reconstruction (GPSR) techniques, Basic GPSR and its Barzilai-Borwein variant (GPSS-BB) and Alternating direction mixed norm Algorithm (YALL1) in shallow water acoustic channel of depth 15 m and range = 200 m with least tracking computation time and Prediction error.</p>	<p>Performance comparison of NCMNS, L1-LS, GPSR, GPSS-BB and YALL1 through prediction error and tracking computation time.</p> <p>Experimental output:</p> <p>Performance comparison of Normalized prediction error and tracking computation time for NCMNS, L1-LS, GPSR, GPSS-BB, and YALL1.</p>

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ for Simulations/Experiments	Advantages	Outcomes
Huang S.H. et al	2014 [71]	Adaptive subspace tracking reduced rank amplitude estimation using RLS method (ASRAE-RLS) algorithm for cross correlated taps. The signal prediction in terms of mean-squared is expressed as: $E \xi_r(n) ^2 = \sigma_d^2(\sum_{k=r+1}^N \chi_k + \sigma_\beta^2) + \sigma_v^2$ $\sigma_\beta^2$ is the variance of the estimation noise The channel impulse response (CIR) correlation matrix is estimated recursively as: $R_h(n) \cong \sum_{i=1}^n \beta^{n-1} h(i)h^H(i)$ Based on signal subspace projection the CIR is estimated as: $\hat{h}_r(n-D) = \hat{Q}_r(n-D)\hat{Q}_r^H(n-D)\hat{h}^{LMS}(n)$ The channel component are also estimated through mean squared signal prediction as: $z_r(n) = \arg \min_{z_r} \sum_{k=1}^n \lambda^{n-k} [ x(k) - z_r^H d_Q(k) ^2]$	2007 Autonomous Underwater Vehicle Festival (AUVFest'07) experiment Carrier Frequency = 17 KHz Bandwidth = 4 KHz Sampling rate = 80k samples/s Source to receiver range = 5 km (calm sea) Source to receiver range = 2.3 km (rough sea) ASIAEX01 Data Source receiver range = 31.28 m BPSK modulation 511 chips Carrier frequency ( $F_c$ ) = 400 Hz Bandwidth = 100 Hz	ASRAE-RLS improve signal prediction error by 12 dB over conventional RLS.	Accurate tracking of channel impulse response and reduction in signal prediction error by 12 dB Experimental output (AUVFest'07 experiment) : Signal Prediction MSE using ASRCR, ASRAE-RLS, and ASRMAE algorithms. Measurement for AUVFest'07 calm Sea data Experimental output (ASIAEX01 Data ): Measurement CIR of type I, type II and type III channel
Youwen Z. et al.	2016 [72]	Cluster-Sparsity $l_{2,0}$ -norm shrinkage LMS algorithm Noise free posterior error vector is obtained as: $e_f^+(n) = (1 - \mu \ x(n)\ ^2) e_f(n) - \mu \ x(n)\ ^2 v(n)$ The recursion is described as: $E[e_f(n)^2] = \alpha E[e_f(n-1)^2] + (1 - \alpha) \tilde{e}_f^2(n)$ Misalignment = $20 \log_{10} [\frac{\ h_0 - h(n)\ ^2}{\ h_0\ ^2}]$ Where $h_0$ = actual tap weight, $h(n)$ = estimated one	South China Sea Depth of water = 260 m Range between source and hydrophone = 1 Km Center frequency ( $f_c$ ) = 300 KHz Sample set = 100 KHz Launch angle = $[-45^\circ, 45^\circ]$	$l_{2,0}$ -norm shrinkage LMS algorithm outperform over Block Sparse Least Mean Square (BS-LMS) in terms of convergence rate and steady-state misalignment	Better performance and high channel estimation accuracy. Experimental output: Millimeter-wave communication using $l_{2,0}$ norm LMS algorithm achieves superior performance than block-sparse LMS (BS-LMS) algorithm

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Ansari N. et al.	2016 [73]	Non-uniform sampling across different frequency range with compressive sampling across Doppler frequency and full-rate sampling at lower Doppler frequency and NMSE for performance evaluation SNR of noisy channel is represented as: $\text{SNR} = 10 \log_{10} \left( \frac{\frac{1}{LK} \sum_{i=0}^{L-1} \sum_{k=0}^{K-1}  H(i,k) ^2}{\sigma_n^2} \right)$ Where $\sigma_n^2$ represents noise variance. For estimated channel, Performance is evaluated using Normalized mean squared error (NMSE) and is computed as: $\text{NMSE} = 10 \log_{10} \left( \frac{\sum_{i=0}^{L-1} \sum_{k=0}^{K-1}  H(i,k) - \hat{H}(i,k) ^2}{\sum_{i=0}^{L-1} \sum_{k=0}^{K-1}  H(i,k) ^2} \right)$ Using Compressed sensing(CS) channel is estimated as $\min_U \ U_{sub} - \mathfrak{R}_{T^c} U\ _2^2 \text{ subject to } \ U\ _1 \leq \tau$	2008 Surface Processes and Acoustic Communications Experiment (SPACE08) in Martha's Vineyard No. of transmitter=1 No. of receiver=4 Data collected at a depth of 15 m BPSK modulation with 4095 point Symbol rate=6.5 Kbps Carrier frequency=13 KHz	Channel estimation through 2D frequency domain representation of shallow water acoustic channel	NMSE performance using channel estimation for 1st five non zero Doppler frequency with SNR=10 dB and SNR=50 dB Experimental output: Performance of NMSE results on channel estimation using CS with SNR=10 dB, 5dB

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Kari D. et al.	2017 [74]	Adaptive filtering based on minimization of logarithm cost function. The logarithmic cost least mean squares (LCLMS): $\hat{h}_{m+1} = \hat{h}_m + \mu' \frac{e_m  e_m ^2}{1 + a  e_m ^2} s_m^*$ The logarithmic cost least mean absolutes (LCLMA): $\hat{h}_{m+1} = \hat{h}_m + \mu' \frac{a  e_m }{1 + a  e_m } [csgn(e_m)] s_m^*$ Where $csgn(e_m) = \frac{1}{\sqrt{2}}$ $(\text{sign}(\text{Re}(e_m)) + j \text{sign}(\text{Im}(e_m)))$ The second order updating algorithm is: $P_m = \lambda^{-1} (P_{m-1} - g_m s_m^H P_{m-1})$ $g_m = \frac{w(e_m) P_{m-1} s_m}{\lambda + w(e_m) s_m^H P_{m-1} s_m}$ $w(e_m) = \frac{ e_m ^2}{1 + a  e_m ^2}$ for the logarithmic cost recursive least squares (LCRLS) $w(e_m) = \frac{1}{1 + a  e_m }$ for the logarithmic cost recursive least absolutes (LCRLA)	Depth of Transmitter ( $T_x$ )=80 m Depth of Receiver ( $R_x$ )=50 m Distance between $T_x$ and $R_x$ =1 km Carrier frequency ( $f_c$ )=15 KHz Signal bandwidth=5 KHz Sample rate=16 KHz Frequency resolution (df)=16 Hz Time resolution (dt)=1/16 ms Symbol rate=4 KHz Maximum multipath delay ( $\tau_{max}$ )=35 ms Coherence time of the small scale variants (TSS)=0.4 s Total duration of simulated channel ( $T_{tot}$ )=16 s	LCLMS and LCLMA algorithms give superior performance in 1st Order environment and LCRLS algorithm in 2nd Order environment both in BER vs. SNR and MSE performance.	Accurately simulating underwater acoustic channels. Experimental output: LCLMS and LCLMA algorithms give superior performance in 1st Order environment and LCRLS algorithm in 2nd Order environment both in BER vs. SNR and MSE performance.

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
<b>4.2 SISO and SIMO system model for UWA communication system</b>					
Rosa Zheng Yahong	2007 [75]	A time-domain CS, equalization, and phase correction scheme in single input multiple output (SIMO) UWA Communication system. GroupWise Phase estimation and correction after equalization and multiple channels combining. Phase rotation of equalized data can be compensated using: $\tilde{x}_p(k) = \tilde{x}_p((p-1)N_s + k)e^{-j\psi_p},$ $k=1,2,3,\dots,N_s$ $p=1,2,3,\dots,N_g$	Saint Margaret's Bay, Canada Water depth=30 m Source-receiver range=3.06Km QPSK with bandwidth=2 KHz Carrier frequency ( $f_c$ )=17 KHz 8 hydrophones spaced interval of 3.06 Km	To overcome phase drift and rotation problem arises due to time-domain adaptive equalizer.	Obtained BER in the order of $10^{-4}$ with high stability and also obtained plot for equalized QPSK. Experimental output: Achieving a bit error rate of $10^{-4}$ over existing methods. (Symbol wise phase detector).
Rosa ZhangY. et al.	2010 [76]	Single carrier Frequency-domain channel estimation and equalization scheme with the phase correction method The channel transfer function is estimated using minimum mean square error criterion: $\lambda_m \tilde{H}_m(n) = \frac{Y_m(n)X^*(n)}{ X(n) ^2 + \sigma^2}$ $n=1,2,3,\dots,2N$ Equalized Frequency domain block data represented as: $\tilde{x} = \left( \sum_{m=1}^{N_g} \Delta_m \Phi_m \right) x + \hat{v}$ Compensation of phase rotation of equalized data can be done as: $\tilde{x}_p(k) = \tilde{x}_p((p-1)N_s + k)e^{-j\psi_p}$ $k=1,2,\dots,N_s$ $p=1,2,\dots,N_g$	AUVFest'07 experiment at panama city, Florida, USA Separation between two fixed ACD <sub>s</sub> in shallow water=5.06 km Unequally spaced over 1.86 m 8 hydrophones placed in vertical array Range between source and fixed receiver 1 to 3 km Carrier frequency ( $f_c$ )=17KHz Bandwidth=5KHz Offset frequency range=-2 to +2 KHz	Channel estimation and equalization of the SIMO UWA communication system through 2N-point FFT. To overcome combat phase rotation larger than $\pi/2$ through averaging the phase variation over a group of symbols against symbol wise PLL method, GroupWise phase correction with time- reversal method and Decision feedback equalization.	Achieved a symbol rate of 4Kbps and bit error rate of $1 \times 10^{-4}$ in a range of 5.06KM for fixed to a fixed channel. For moving delay spread of 5ms in a range of 1.3Km with an uncoded bit error rate of $1 \times 10^{-3}$ . Experimental output: Achieved uncoded bit rate of $1 \times 10^{-4}$ for fixed to fixed channels and for moving to fixed source achieving uncoded bit rate of $1 \times 10^{-3}$ .

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ for Simulations/Experiments	Advantages	Outcomes
Gong G. et al.	2012 [77]	Space alternating generalized expectation maximization (SAGE) technique for estimation detection and equalization Data update rule for SAGE algorithm is represented as: $\hat{\mu}_h^{(i+1)}(k) = \frac{Q(\frac{(\mu_h^{(i)})^{ISTROKE} \Phi_k^{ISTROKE} r - \sum_{q=1, q \neq k}^L S^{(i)}(q) \text{tr}(\Phi_q (\sum_H^{(i)} + \mu_h^{(i)} (\mu_h^{(i)})^{ISTROKE}) \Phi_k^{ISTROKE})}{\text{tr}(\Phi_k (\sum_H^{(i)} + \mu_h^{(i)} (\mu_h^{(i)})^{ISTROKE}) \Phi_k^{ISTROKE})})}{Q}$ Where Q is the quantization process The MMSE estimation of channel parameters is determined as: $\hat{h}_{MMSE} = (\Phi_p^{ISTROKE} \Phi_p + N_0 (\sum_h^{(o)})^{-1})^{-1} \Phi_p^{ISTROKE} r_p$ Where $N_0$ is the variance of Gaussian noise	Carrier frequency = 10KHz Symbol rate = 2000b/s Max. Doppler spread = 10 Hz Max. Delay spread = 6ms Block length (N) = 1500 Modulation = BPSK	Computational complexity is reduced through Discrete Fourier basic expansion channel model.	Scatter plot and BER vs. SNR performance for estimation, detection and equalization. Experimental output: Obtained a graph between BER vs. SNR for joint CS, equalization and symbol detection.
<b>4.3 Pilot assisted OFDM system model for UWA communication system</b>					
Yonggang W. et.al.	2011 [78]	Estimating frequency response of channel through different kinds of pilot pattern that are known to receiver. Sampled channel estimation by adopting LS algorithm $\hat{H}_p(k) = H_p(k) + n_{ICI} + n$ Where $H(k) = \sum_i \alpha_i e^{j \prod f_{Di} T} \frac{\sin(\prod f_{Di} T)}{\prod f_{Di} T} e^{-j \frac{2 \prod f_{Di} k}{N}}$ Where $f_{Di}$ = Doppler frequency offset	Tank of Harbin Engg. University Length, Width and Depth of Tank = 22.5 m, 21.5 m and 1.5 m respectively	Estimating the frequency response of channel through a pilot pattern that is known to the receiver.	BER performance to different types of pilot patterns (Block, Comb and Scatter) and obtained 2.667, 2.4 and 2 Kbit/s at 5KHz, 10KHz, and 20kHz respectively. Experimental output: Three pilot patterns (Block, comb, Scatter) are analyzed and finally Scatter pilot outperform both in time selective and frequency selective channel.

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Han J. et al.	2014 [79]	Channel estimation through data embedded pilot. From chosen pilot channel is estimated as: $\hat{H} = H \{1 + \sum_{k=0}^{N-1} \frac{D(k) \frac{p^*(k)}{ p(k) ^2}}{N}\} + w$	Coastal Sea of Bhusan Data symbol length 1000 symbols Data Embedded Pilot length 250 symbols Power( $\alpha$ ) 0.3 Training symbol length 50 symbols	Time variant channel estimation through data embedded pilot without overload and with increased spectral efficiency and data rate	Time variant channel estimation without adding overhead to increase the data rate. Experimental output: Data embedded pilot technique estimates the time-variant channel with increased data rate.
Kim H. et al.	2014 [80]	Parametric Model through linear phase progression Preamble and postamble reduced length can be obtained as: $K = \left\lceil \frac{w^2(N-2p)}{2(1-w^2)} \right\rceil$ W = Superimposed pilot sequence weight (N-2p) = data sequence	Donghae-si, Republic of Korea Depth of transmitter and receiver is 60m and 107.5m Carrier frequency ( $f_c$ ) = 6KHz Bandwidth = 4KHz QPSK modulation	Chanel estimation through preamble, postamble and superimposed pilot and error minimization through ML (Maximum Likelihood)approach	MSE Performance with varying weights. Experimental output: Phase changes over time and phase variation are calculated as 5.2 radians per second.

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Panayirchi E. et.al.	2016 [81]	MP with Maximum a posteriori Probability (MAP) based Space-alternating generalized expectation maximization technique (SAGE method) The variance estimator of the CGM components at $(i+1)^{\text{th}}$ iteration is given as: $(\sigma_l^2)^{(i+1)} = \frac{v_l^{-1(i)}  h_l^{(i)} ^2}{\sum_{m=1}^L v_m^{-1(i)}} \quad 1=1,2,\dots,L$ ML variance estimates $s$ are found through optimization problem: $\hat{s} = \arg \max_s \log p(z s)$ Subject to: $\sum_{l=1}^L s_l = 1, \text{ and } s_l \geq 0 \text{ for } l=1,2,3,\dots,L$	SPACE'08 experiment carrier frequency ( $f_c$ )=13 KHz channel bandwidth (BW)=2.442 KHz number of subcarriers (K) =256 OFDM symbol duration (T)=104.8 ms Subcarrier spacing ( $f_s=1/T$ )=9.54 Hz cyclic prefix duration (TCP)=12.4 ms number of paths on each link (L $\times$ )=3, 6, 9, 12 maximum Doppler rate ( $b_{\max}$ )=10–3, $5 \times 10$ –3, 10–2, $5 \times 10$ –2 Doppler spread resolution ( $\Delta v$ )=10–3 modulation formats=BPSK, QPSK, 16QAM number of CGM-MP-SAGE iterations=5 pilot spacing ( $\Delta_p$ )=1, 2, 4, 8 oversampling factor (Q)=1, 2, 4, 8, 16 35 discrete paths with 5 taps. 16QAM OFDM system. Number of subcarrier (N)=512 Pilot subcarriers ( $N_p$ )=128 Cyclic Prefix (CP) length=64 $\frac{1}{2}$ convolutional code Forgetting factor ( $\lambda$ )=0.98	Improving channel estimation through admissible hidden data using CGM-MP-SAGE algorithm for an OFDM system	Excellent symbol error rate (SER) and channel estimation performance with robust to the effect of Doppler mismatch. Experimental output: MSE vs. SNR and SER vs. SNR performance comparison of the CGM-MP-SAGE and MP algorithm for different constellation using different oversampling factor, Doppler shift, pilot spacing, and channel settings.
SHI X. et.al.	2016 [82]	Adaptive channel estimation (flat fading channel) based on LS and RLS algorithm for OFDM system. Channel impulse response(CIR) for LS estimate is represented as: $\hat{h}_{LS} = A^H A^{-1} A^H r$ The RLS estimator is expressed as: $\hat{h}(m, l) = \hat{w}(m-1, l) \hat{h}(m-1, l)$ Where $\hat{w}(m-1, l)$ is coefficient estimation vector and $\hat{h}(m-1, l)$ is estimated channel coefficient vector Performance is evaluated through MSE(mean square error) $\text{MSE(dB)} = 10 \log_{10} \left( \frac{1}{N_m} \sum_{n=1}^{N_m} \ \hat{h} - h\ ^2 \right)$ Where $N_m$ is the total number of iterations		Fast fading complexity reduced as significant path considered by LS-RLS method which outperforms over LS and LMMSE in BER and MSE vs. SNR.	Perfect tracking with MSE and BER performance by LS_RLS over LS and LMMSE. Experimental output: Proposed LS-RLS method outperforms over LS and LMMSE in MSE vs. SNR performance and BER performance.

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Tang L. et al.	2017 [83]	Pilot routing algorithm The mutual coherence is expressed as: $\mu(T) = \max_{1 \leq i, j \leq L, i \neq j} \frac{ \zeta_i^H \zeta_j }{\ \zeta_i\  \ \zeta_j\ }$ Where $\zeta_i, \zeta_j$ are the $i^{\text{th}}$ and $j^{\text{th}}$ column of measurement matrix. Adjusted mutual coherence is expressed as: $\gamma(T) = \sum_{1 \leq i, j \leq L, i \neq j} ( g_{ij}  \geq t) \cdot  g_{ij} ^3$ Where t is the threshold value which is 0.1	No. of subcarrier (N)=512 No. of pilot subcarrier ( $N_p$ )=36 No. of cyclic prefix=50 Doppler shift =100H <sub>z</sub> Length of the channel (L)=50 Sparsity (K)=6 Pilot placement samples (A)=60	Proposed algorithm increasing the accuracy of channel estimation and transmission efficiency through BER vs. SNR performance with $\gamma = 1:3453$ and MSE vs. SNR $\gamma = 1:3453$	Enhancing the accuracy of channel estimation and transmission efficiency Experimental output: BER vs. SNR, MSE Vs SNR is obtained for different values of mutual coherence.
Jing L. et al.	2017 [84]	Markov chain Monte Carlo method over sparse UAC MMSE parameter estimation is done using: $\hat{\theta}_{MMSE} = E\{\{v u\}\} = \frac{1}{I} \sum_{i=1}^I v^{(i)}$ The NMSE for each simulation can be calculated as: $NMSE = E\{\ h_l - \hat{h}_l\ ^2 / \ h_l\ ^2\}$	Extrinsic information transfer (EXIT) analysis for convergence	Though Markov Chain Monte Carlo (MCMC) outperform over Generalized approximate message passing method (GAMP) in BER performance but GAMP deliver best channel estimate	Improvement in BER Experimental output: EXIT charts are obtained at SNR 5 dB with BER vs. SNR and NMSE vs. SNR for 128 pilot subcarriers

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Chen P. et.al.	2017 [85]	Compressed sensing based algorithm The frequency domain representation of received signal is represented as: $r_f = D h_f + v_f + w_f$ where $v_f = F v$ and $w_f = F w$ are impulsive noise and other noise, $D$ =diagonal matrix The received signals in the pilot subcarriers are $r_p = D_p F_p h_{p,t} + F_p v_{p,t} + w_p$ $F_p$ is an $N_p \times N_p$ DFT matrix. $P$ =is an $N_p \times N_c$ matrix select $N_p$ pilot subcarriers out of $N_c$ subcarriers. Vector containing Samples of impulsive noise can be estimated as in Least-Squares(LS) approach: $\hat{v}_l = (F_l^H F_l)^{-1} F_l^H F_p \hat{v}_{p,t}$ Vector containing Samples of impulsive noise can be estimated as in DFT approach: $\hat{v}[i] = \begin{cases} \sqrt{k} e^{\frac{j2\pi(i-1)(mp-1)}{N_c}} \hat{v}_{p,t}[i/N_p], & i \in I_f \\ 0, & \text{otherwise} \end{cases}$ Estimated data symbol in Data Aided is given by $r_f = \hat{D} F h_f + F v + \tilde{w} f$	UA communication experiment in estuary of the Swan River, Western Australia No. of OFDM blocks ( $N_b$ )=5 Bandwidth ( $B$ )=4 kHz Carrier of frequency ( $f_c$ )=12 KHz Sampling rate ( $f_s$ )= 96 KHz No.of subcarriers ( $N_c$ )=512 Subcarrier Spacing ( $f_{sc}$ )=7.8 Hz Length of OFDM symbol ( $T$ )=128 ms Length of CP ( $T_{cp}$ )=25 ms	Improvement in System accuracy and BER performance through mitigation of impulsive noise with channel estimation using CS based Joint channel and Impulsive noise estimation	Reducing BER through a performance of BER vs. SNR at SIR=-10 dB and BER vs. SIR at SNR=7 dB and showing system accuracy through MSE vs. SNR performance Experimental output: Estimated SIR and SNR with a performance comparison of various algorithms for T83, T84 and T85 file using QPSK modulation
Chen H. et.al.	2017 [86]	Fast Bayesian Matching Pursuit (FBMP) Approximate estimation of sparsity $h$ is expressed as: $\hat{h}_{AMMSE} = \sum_{\Lambda^*} h_e P_e$ Where $p_e \triangleq \frac{e^{v(z)}}{\sum_{z' \in \Lambda^*} e^{v(z')}}$ and $h_e = \sigma_1^2 B_z^H y$	No. of channel taps=10 Length of CIR sequence ( $N$ )=100	FBMP outperform over OMP. FBMP save SNR of 4 dB compared to OMP	BER performance of FBMP and OMP Experimental output: Fast Bayesian matching pursuit outperform over OMP both in terms of BER and MSE

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Qian C. et al.	2017 [87]	<p>Filtered multitone (FMT) OMP for Sparse Channel</p> <p>The measurement model is expressed as</p> $y_{pi} = s_{pi}A \sum_{p=1}^{N_{pa}} \sum_{n_a=0}^{N_{amp}} \xi_p^{n_a} T_s'' + w_{pi}$ <p>Where <math>s_{pi}</math> is pilot vector, <math>y_{pi}</math> is measurement observation vector and <math>w_{pi}</math> is noise vector</p> <p>Doppler scale is calculated as</p> $\beta_p = \frac{(\hat{\tau}_n' - \hat{\tau}_n)}{t}, n = 1, 2, \dots, N_{pa}$	<p>BELLHOP beam tracing model</p> <p>Channel depth = 100 m</p> <p>TX and RX location depth = 90 m and 40 m respectively</p> <p>Delay spread = 35ms</p>	<p>The Complexity of the system is reduced by splitting bandwidth into sub-bands</p>	<p>MSE performance with different schemes.</p> <p>Experimental output: UWA channel is estimated at SNR=15dB</p>
<b>4.4 CP-OFDM system model for UWA communication system</b>					
Kang T. et al.	2008 [88]	<p>Iterative LDPC-coded OFDM receivers analysis using EXIT chart and MP algorithm for channel estimation</p> <p>carrier frequency offset (CFO) is estimated as:</p> $\hat{\epsilon} = \arg \max_{\epsilon} y^H E(\epsilon) \hat{T} (\hat{T}^H \hat{T})^{-1} \hat{T}^H E(\epsilon)^H y$ <p>Channel coefficient estimate at <math>q_k</math> position is done using:</p> $\hat{f}_{qk} = \frac{(v_k)_{qk}}{\ (A)_{qk}\ ^2}$	<p>Viapahu Lagoon, Moorea, French Polynesia</p> <p>Depth = 3 m</p> <p>Distance = 330 m</p> <p>OFDM data symbols = 25</p> <p>Delay spread = 1.5 ms</p> <p>Maximum Doppler spread = 0.5 Hz</p>	<p>Equalization, Synchronization and channel estimation symbol by symbol basis through iterative receiver with reduction in computational complexity against non- linear filtering algorithm subject to divergence</p>	<p>OMP outperform in channel tracking over MP and LS channel estimation through asymptotic convergence behavior of different realizations of the iterative receiver component</p> <p>Experimental output: OMP CE outperforms over conventional CE.</p> <p>Type-2 receiver perform better than Type-3 receiver for <math>E_p/E_b = 0.707</math></p> <p><math>E_p</math> = pilot subcarrier power</p> <p><math>E_b</math> = data subcarrier power</p>

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Liao J. et al.	2014 [89]	Piecewise linear approximation for channel estimation with differential coding to mitigate ICI Channel frequency response of pilots is estimated as per LS and MMSE criteria is: $\hat{H}_p = H_p + \frac{IC I_p + W_p}{S_p}$ MSE is expressed as: $MSE =   \frac{IC I_p + W_p}{S_p}  $	Modulation = 8PSK Sampling interval = 31.25 $\mu$ s No. of subcarriers = 1024 No. of pilots = 512 OFDM symbol duration = 256 ms	Channel estimator with ICI mitigation method outperform over LS detector	Performance graph between average BER vs. average received SNR for Doppler of 9%.Proposed method. Experimental output: Performance graph between average BER vs. average received SNR for Doppler of 9%. The Proposed method combination of time-varying channel estimator with interference auto cancellation outperform over LS detector, time- varying channel estimator, and differential coding interference auto cancellation.
Lin Na et.al.	2015 [90]	Channel estimation through OMP algorithm The time domain channel estimation calculated as: $\hat{h} [m,l] = \begin{cases} h[m,l] + \xi[m.l], & \text{significant taps} \\ 0 & \text{zero-valued taps} \end{cases}$ Where $\xi[m.l]$ = noise in time domain The performance of different predictors are compared in terms of NMSE as: $NMSE = \frac{E[\ h(m) - \hat{h}(m)\ ^2]}{E[\ h(m)\ ^2]}$ Where $h(m)$ and $\hat{h}(m)$ are predicted time domain channel coefficient vectors	FFT length = 1024 Guard interval = 85.333 ms Symbol duration = 255.848 ms Effective bandwidth = 6 KHz Center frequency = 11 KHz Effective speed = 8 Kbit/s Mapping = QPSK Pilot type = Comb-type	Sparse channel estimation through prediction (time- domain predictor) in adaptive underwater environment considering a small number of significant channel paths to reduce complexity.	Better performance with reliable channel state information (CSI) over LS and LMMSE predictor Experimental output: CS prediction outperforms over LS prediction and LMMSE-prediction method. MSE and BER performance of CS-Prediction, LS-Prediction and LMMSE Prediction

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Liu L. et al.	2016 [91]	PN sequence based method for underwater acoustic OFDM system (Sliding cross correlation method) Channel frequency response is expressed as: $\hat{H} = H + \frac{N}{P}$ Normalized carrier frequency offset (CFO) is expressed as: $\epsilon_c = \frac{\arg(\sum_{n=0}^{\frac{N}{2}-1} y^*(n)y(n+\frac{N}{2}))}{\pi}$	Sampling frequency=96 KHz FFT size = 1024 Frequency band=6-18 KHz CP length = 256 Carrier frequency = 12 KHz Symbol duration = 85.33 ms Subcarrier spacing = 11.72 Hz CP duration = 21.33 ms Mapping mode = QPSK Convolution Coding = [171,133] Code rate = 1/2 Data rate (code) = 9.6kbps	Sliding cross-correlation method outperforms LFM with an estimation of $10^{-5}$ against $10^{-4}$	Estimation performance analysis between sliding cross- correlation (15 m/s) and LFM cross correlation (15 m/s) with CFO constellation. Experimental output: Proposed Sliding Cross-correlation method outperforms over Linear Frequency Modulation (LFM) with an estimation error of $10^{-4}$ in estimation error vs. SNR performance.
Wang Z. et al.	2017 [92]	Joint Sparse Method (JSM) and exponential Smoothing (ES) as channel estimation method for OFDM system The received pilot vector becomes: $Y_p = X_p F_p h + W_p$ The $m^{\text{th}}$ pilot is represented as: $\hat{H}_p(m) = \sum_{n=0}^{N-1} F_p^{mn} (h(n) + w(n) X_p^{-1}(m))$ Where $h(n)$ = $n^{\text{th}}$ sample of UWA channel $w(n)$ = AWGN, M= no.of pilot $X_p^{-1}(m)$ = $m^{\text{th}}$ entry of diagonal matrix $X_p^{-1}$	Sampling rate = 48KHz Bandwidth = 4.8KHz Symbol duration = 116.58 ms FFT size = 4096 No. of active subcarriers = 512 Guard interval = 31.25 ms Frequency interval = 11.72 Hz Modulation mode = QPSK Frequency range = 12-18KHz	Sparse channel estimation using ES-SOMP (exponentially smoothing based simultaneous orthogonal matching pursuit) outperform over OMP and SOMP about 2 to 3 dB	Improving accuracy of Sparse Channel estimation with outperform against pilots inserted in one individual (OMP algorithm) in OFDM about 2~3 dB with MSE, phase estimate and scatter plot. Experimental output: Exponential smoothing based simultaneous orthogonal matching pursuit (ES-SOMP) as against Sparse model outperforms over OMP and SOMP about 2 to 3 dB.

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Altabbaa M. et.al.	2017 [93]	MP-MAP algorithm The maximum a posteriori(MAP) estimator for channel gain is represented as: $\hat{h}_{MAP} = (A_c^{ISTROKE} A_c + \frac{1}{\beta} \Gamma^{-1})^{-1} (A_c^{ISTROKE} Z_p + \frac{1}{\beta} \Gamma^{-1} \mu)$ Where $A_c^{ISTROKE} A_c$ = banded matrix ML estimation for variance $\sigma^2$ and mean $\mu$ vectors of channel gain is represented as: $\hat{\mu}_{ML} = (A_{c_{po}}^{ISTROKE} C_z^{-1} A_{c_{po}})^{-1} \Re\{A_{c_{po}}^{ISTROKE} C_z^{-1} Z_p\}$ $\hat{\sigma}_{p,ML}^2 = (\frac{ e_p ^2 \beta - \ a_{c_p}\ ^2}{2 \ a_{c_p}\ ^4 \beta}) +$	carrier frequency ( $f_c$ )=18 KHz channel bandwidth (BW)=7 KHz number of subcarriers (K)=512 OFDM symbol duration (T)=73:15 ms Subcarrier spacing=13:67 Hz guard interval duration (T <sub>g</sub> )=35 ms number of paths on the link (L <sub>X</sub> ) 3 maximum Doppler shift =10 <sup>-2</sup> , 5 × 10 <sup>-3</sup> 10 <sup>-3</sup> Doppler spread resolution =10 <sup>-3</sup> modulation formats=QPSK, 16QAM pilot spacing=4 oversampling factor (Q)=8	MA-MAP outperform over MP in MSE and Block error rate (BER)	MSE vs. SNR performance and SER vs. SNR performance for MP-MAP Experimental output: Channel estimation MP-MAP algorithm results in excellent MSE and symbol error rate (SER) compared to MP algorithm.
<b>4.5 OFDM (Zero-padded format/method) or ZP-OFDM system model for UWA communication system</b>					
R.Berger C. et.al.	2010 [94]	Root-Music and ESPRIT subspace algorithm, Compressed sensing in form of Orthogonal Matching Pursuit (OMP) and Basic Pursuit (BP) algorithm Based on pilot subcarriers, the complex path gains $\xi_p$ are estimated as: $\{\xi_p\}_{LS} = \arg \min_{\xi_p} \sum_{m \in S_p}  H_m - \sum_{p=1}^{N_p} \xi_{pe-j2\pi \frac{n_i}{T}} \tau_p ^2$ The channel response on data subcarriers is reconstructed as: $H_m = \sum_{p=1}^{N_p} \xi_{pe-j2\pi \frac{n_i}{T}} \tau_p m S_D$ The minimum mean square error (MMSE) receiver applied for data demodulation is: $S = (H^H H + N_o I)^{-1} H^H z$ Where $N_o$ = noise power	GLINT'08 experiment(for mild Doppler spread scenario) Carrier frequency ( $f_c$ )= 25KHz Bandwidth (B)=7.8125 KHz No. of carriers (K)=1024 Symbol duration (T)=131.072 ms Subcarrier spacing ( $\Delta f$ )=(1/T) =7.63 Hz Guard interval (T <sub>g</sub> )=25 ms SPACE'08 experiment Carrier frequency ( $f_c$ )=13KHz Bandwidth (B)=9.77 KHz No. of carriers (K)=1024 Symbol duration (T)=104.86 ms Subcarrier spacing ( $\Delta f$ )=(1/T)=9.54 Hz Guard interval (T <sub>g</sub> )=24.6 ms	Subspace method outperform over LS for low Doppler spread and compressed sensing outperform over LS for large Doppler spread in channel estimation	Mitigating the effect of ICI. Experimental output (GLINT'08 experiment): BLER vs. phones performance using 16-QAM and 64-QAM. Experimental output (SPACE'08 experiment) BLER vs. phones performance using 16-QAM and 64-QAM

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Huang J. et al.	2010 [95]	Sparse channel estimation through BP algorithm (l1_1 s, SparSA and YALL1) For ICI aware SIMO systems the observation vector by collecting all path gains is expressed as: $z = \sum_{q=1}^{N_d} S(a_q)W\xi_q + v$ For ICI-ignorant MIMO systems the received signal for $v^{\text{th}}$ receiver: $z_v = \sum_{\mu=1}^{N_t} S_{\mu}W\xi^{v,\mu} + v_v$	2008 Surface Processes and Acoustic Communications Experiment (SPACE08) in Martha's Vineyard No. of subcarriers = 1024 Pilot subcarriers = 352 Null subcarriers = 96 Data subcarriers = 576 Bandwidth = 9.77 KHz Symbol length = 104.86 ms Guard interval = 24.6 ms	For Sparse channel estimation among BP solvers, SparSA and YALL1 outperform over l1_1 s and reducing runtime	CPU time performance for l1_1 s, SparSA and YALL1 is obtained. SparSA, YALL1 outperforms in CPU time over l1_1 s both in simulation and experimental analysis Experimental output: SparSA and Yall1 reduce the runtime comparison to l1_1 s, and OMP.
Wang Z. et al.	2011 [96]	Sparse channel estimation for zero-padded OFDM using cluster of channel prediction Offset parameters is estimated based on pilot subcarriers is: $\{\hat{\gamma}_i, \Delta \hat{\tau}_i\}_{i=1}^{N_c} = \underset{\{\gamma_i, \Delta \tau_i\}}{\operatorname{argmin}} E_{m \in S_p} \ z(n) - \sum_{i=1}^{N_c} \hat{z}_i(n n-1)\ _m^2$ Sparse channel estimation is expressed as $\hat{h}(n) = \underset{\{\hat{z}_i(n), \hat{A}(n)\}}{\operatorname{argmin}} \left\{ \frac{\ z(n) - \hat{A}(n)h(n)\ ^2}{\hat{\sigma}_h^2(n)} + \sum_{i=1}^{N_c} \frac{\ \hat{z}_i(n) - \hat{A}(n)h(n)\ ^2}{\hat{\sigma}_i^2(n)} + \xi  h(n) _1 \right\}$ Symbol is estimated using linear minimum –mean-squared-error (LMMSE) equalizer: $\hat{s}(n) = \hat{H}(n)^H \hat{H}(n) + \hat{\sigma}_o^2(n) I_k^{-1} \hat{H}(n)^H z(n)$	SPACE08 in Martha's Vineyard Water depth = 15 m Receiver from which data collected = 60 m Carrier frequency = 13 KHz Bandwidth = 9.77 KHz Total subcarrier = 1024 Symbol duration = 104.86 ms Guard interval = 24.6 ms Pilot subcarrier = 256 Data subcarrier = 672	Sparse channel estimation through cluster prediction with exploiting path clustering behavior and outperform over the block by block channel estimation	Outstanding performance of sparse channel estimation against the block by block conventional channel estimation Experimental output: Block error rate Performance (BLER) measurement for ICI ignorant /ICI aware for 128 pilots, 96 pilots, 64 pilots for D=0 and D=3 using different channel estimation schemes using 16 QAM

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Qi C. et al.	2012 [97]	Complex Homotopy Algorithm, Gauss-Markov (GM) model and RLS with CS First order Gauss-Markov Process (GM) to model the channel is represented as: $h^{(m)} = kh^{(m-1)} + v^{(m)}$ with $k = J_0(2\pi f_d T_s)$ After applying complex Homotopy to RLS algorithm, the iterative procedure is expressed as: $z^{(m)} = \frac{\beta - \beta^m}{1 - \beta^m} z^{(m-1)} + \frac{1 - \beta}{1 - \beta^m} y^{(m)}$	No. of subcarriers ( $N_d$ )=512 No. of null Subcarriers ( $N_u$ )=110 No. of pilot Subcarriers ( $N_p$ )=20 Length of zero padding ( $N_G$ )=64 No. of multipath (S)=5 Length of CIR (L)=50 Carrier frequency ( $f_c$ )=24Khz Signal Bandwidth (B)=4Khz Doppler Spread at ( $f_c$ )=2.7hz FFT Size=1024 No. of active subcarriers=968 No. of null subcarriers=56 No. of pilots per OFDM block=128 Modulation BPSK Symbol duration=85.33 ms Guard interval duration=10 ms Frequency separation between subcarriers=11.72Hz Band width=12 KHz	CPU runtime of Homotopy algorithm(0.025) must smaller than OMP(0.5266) and YALL1(2.3136)	Faster and more accurately UWA channel estimation through CPU running time. Experimental output: Performance comparison of different CS algorithms with GM and RLS (Homotopy).
Priyaajali K. et al.	2014 [98]	Improved least square (LS) based technique for Sparse Channel for OFDM system Channel impulse response (CIR) estimation for improved LS channel estimation is expressed as: $\hat{h}_{i,l} = \frac{1}{N_p} (\sum_{k=0}^{N_p-1} H_{k(k'),l} e^{\frac{j2\pi k' l}{N_p}} + \sum_{k=0}^{N_p-1} \frac{n_{k(k'),l}}{p_{k(k'),l}} e^{\frac{j2\pi k' l}{N_p}})$	Improved LS (considering significant taps with predetermined threshold) outperform over conventional LS of OFDM system	MSE and BER significantly better than conventional LS. Improvement of approximately 4dB in SNR at BER $10^{-2}$ Experimental output: BER and MSE performance of Improved Least Square better over Conventional Least Square for Delay spread of 6ms and 10ms.	
Huang Yi et.al.	2014 [99]	Data driven sparsity Learning approach based on LMMSE equalizer for OFDM system To control sparsity, the sparse estimation by taking $L_q$ as regularization is expressed as: $\hat{x} = \underset{x}{\operatorname{argmin}} \frac{1}{2} \ z_p - Ax\ _2^2 + \lambda \ x\ _q^q$ $L_2$ algorithms-LSQR $\min_x \frac{1}{2} \ z_p - Ax\ _2^2 + \lambda \ x\ _2^2$ $L_0$ algorithms-OMP $\min_x \ x\ _{0s.t.} \ z_p - Ax\ _2^2 \leq \delta$ $L_1$ algorithms $\min_x \frac{1}{2} \ z_p - Ax\ _2^2 + \lambda \ x\ $ $L_{1/2}$ algorithm $\min_x \frac{1}{2} \ z_p - Ax\ _2^2 + \lambda \ x\ _{1/2}^{1/2}$	SPACE08 in Martha's Vineyard Water depth =15m Each receiver consisting elements spaced by 0.1m	SPaRSA, FISTA, Nesterov, and Twist Block error rate performance on channel estimation outperform over $L_0$ and $L_2$ algorithm	SPaRSA, FISTA, Nesterov, and Twist outperform over $L_0$ and $L_2$ algorithm. Experimental output: Block error rate) BLER performance of ICI-ignorant and ICI aware receiver for Space08-S1(60m) and Space08-S3(200m)

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Yu F. et al.	2014 [100]	Iterative OFDM receiver design through compressed channel estimation, soft in soft out equalization (SISO) and LDPC decoder To estimate channel response, the matrix expression is given as: $\mathbf{r} = \sum_{p=1}^{N_p} \xi_p \wedge_p \Gamma_p \mathbf{s} + \mathbf{n} = \mathbf{H} \mathbf{s} + \mathbf{n}$ where $\xi_p = \frac{A_p}{1+b_p} e^{-j2\pi f_c \tau_p}$ $[\wedge_p]_{m,k} = \begin{cases} e^{-2\pi j f_c \tau_p}, & m=k \\ 0, & m \neq k \end{cases}$ $[L_p]_{m,k} = \gamma_{m,k}^{(p)}$ $N_p$ = total number of multipath $\tau_p$ = delay of pth path $b_p$ = doppler scale of pth path Fast block-Fourier transform (FFT) based OMP algorithm In Block-FFT based OMP algorithm , FFT is used to calculate the inner product as: $\gamma(r, n) = \sum_{l_0=0}^{\frac{L}{d}-1} E(b_m, l_0 d + r) \exp(-j2\pi l_0 \frac{n_0 + \frac{l}{d}}{d})$	EXIT chart analysis (Convergence analysis) Time duration (T)=104.86 ms Zero padding length ( $T_g$ )=24.6 ms Number of subcarrier (K)=1024 Maximum Doppler shift = $3 \times 10^{-4}$ Number of pilot set $ S_p  = 256$ Carrier frequency ( $f_c$ )=13KHz	Iterative way of channel estimation through BER performance and EXIT chart analysis with limited number of iteration to reduce complexity for OFDM system	BER performance measurement through number of iteration and EXIT chart analysis Experimental output: BER performance increases with the number of iteration. BER performance also increases with Doppler scale but up to limit. Low ICI and low SNR lead to lower BER for EXIT chart analysis.
Yu F. et al.	2015 [101]	In Block-FFT based OMP algorithm , FFT is used to calculate the inner product as: $\gamma(r, n) = \sum_{l_0=0}^{\frac{L}{d}-1} E(b_m, l_0 d + r) \exp(-j2\pi l_0 \frac{n_0 + \frac{l}{d}}{d})$		Estimation of the compressed channel (reducing number of pilots) to reduce complexity through Fast block-Fourier transform (FFT) based OMP algorithm against original OMP algorithm	The computational complexity of the proposed algorithm reduced compared to original Orthogonal matching pursuit (OMP) Experimental output: Comparison of MSE performance between proposed and OMP algorithm.

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Li C. et.al.	2017 [102]	Algorithm carry orthogonal matching pursuit (OMP) based on compression sensing The complexity can be expressed as: $O_{\psi} I_{mf} = \frac{w}{\Delta w} O_2 + 2w + K + 2Kq - 2q^2$	SPACE08 number of subcarriers K=1024 symbol duration T (ms)=104.86 guard interval T <sub>g</sub> (ms)=24.6 pilot length (ω)=352 $\frac{T}{T_g K} = 0.0042$ Marthas Vineyard island in New England number of subcarriers K=128, 256, 512, 1024 symbol duration T (ms)=13, 26, 52, 105 guard interval T <sub>g</sub> (ms) =16 pilot length ω=352 only for K=1024	Reduction in complexity during sparse channel estimation using OMP where ICI channel matrix is used in the objective function	Better accuracy with the reduction in complexity in tracking Experimental output: Obtained through graph normalized computation complexity gain vs. varying oversampling factor for different k.

#### 4.6 MIMO system model for UWA communication system

Ling J. et al.	2009 [103]	Sparse learning via iterative minimization (SLIM) algorithm with symbol detection using (RELAX-BLAST) For channel estimation the channel parameter $h, p, \eta$ are estimated using maximum a posterior (MAP) criterion: $\min_{h,p,\eta} (d_y \log \eta + \frac{\ y - Xh\ ^2}{\eta} + \sum_{n=1}^N R \log p_n + \sum_{n=1}^N R \frac{\ h_n\ ^2}{p_n})$ The linear minimum mean squared error (LMMSE) is expressed as: $f_{n,A} = (\sum_{j=1, j \neq A}^N \hat{H}_j \hat{H}_j^H + \bar{\eta} I)^{-1} s_n \quad n = 1, 2, \dots, N$	SPACE08 in WHOI, South to the coast of Martha's Vineyard Water depth=15 m No. of transmit transducers=4 No. of hydrophones at receiver configuration=32 at 60 m (cross mounted), 24 at 200 m (Vertical line mounted), 12 at 1Km (vertical line mounted) Carrier frequency=13KHz Bandwidth=10KHz Training symbols=512 QPSK payload symbols=8000	Symbol detection through REL AX-BLAST approach outperform over vertical Bell labs layered space-time (V-BLAST) and enhanced channel estimation SLIM (at a slightly higher complexity)	BER performance is determined and achieved a coded BER of $1.47 \times 10^{-5}$ with the data rate of 31.25 kbps. Experimental output: Achieving a coded BER of $1.47 \times 10^{-5}$ with data rate of 31.25 Kbps using good, bad and ugly days.
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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Gwun B. et al.	2013 [104]	L1-norm minimization with LMS channel estimation To estimate $h_m$ using L1-norm minimization is expressed as: $h_m = \underset{h_m}{\operatorname{argmin}} \ S\ _1 \text{ subject to }  r_m - Sh_m  < \epsilon$ LMS channel estimation is expressed as: $\hat{h}_m(k+1) = \hat{h}_m(k) + \mu e_m S_m$	Range between source and receiver = 1Km Carrier frequency = 12KHz Sampling frequency = 192KHz BPSK modulation with 0.5 Kbps data rate	Channel tracking by LMS method for MIMO environment is faster on MSE over the conventional method.	Performance comparison of LMS channel estimation with conventional method. Experimental output: The Proposed method based on L1-norm minimization, channel regeneration and LMS channel estimation outperform on MSE over conventional method with reduces computational complexity.
Yang Z. et al.	2016 [105]	Improved proportionate normalized least mean squares (IPNLMS) algorithm Robustness in the sparsity of UWA channel is achieving using diagonal elements of Improved proportionate normalized least mean squares (IPNLMS) which is expressed as: $g_{m,n}(t, l) = \frac{1-\alpha}{2NL} + (1 + \alpha) \frac{ \hat{h}_{m,n}(t, l) }{2\ \hat{h}_m(t)\ _1 + \epsilon}$ Where $\alpha$ determines sparsity The LC-MMSE estimate of symbol is represented as: $\hat{s}_{n,k} = f_n^H (y^k - \hat{H} \hat{s}_n^k)$ $f_n = (\sigma_w^2 I_M + \bar{v}^{-1} \hat{H} \hat{H}^H)^{-1} \hat{h}_n$ Where $\hat{H}$ is the estimated channel matrix	SPACE08 in Martha's Vineyard Symbol period ( $T_s$ ) = 0.1024ms Modulation = QPSK, 8PSK and 16 QAM Carrier frequency ( $f_c$ ) = 13KHz Roll off factor = 0.2 Channel bandwidth = 11.7188KHz Inter transducer spacing = 50cm	IPNLMS algorithm outperform over Normalized LMS (NLMS) and minimum mean square error (MMSE) in sparseness measurement of the channel but computational complexity is more compared to NLMS	The CIR is estimated before the 1st iteration and after 5th iteration using IPNLMS and LC-MMSE equalizer. IPNLMS achieves minor sparseness gain over NLMS and MMSE. IPNLMS has slightly better BER over NLMS but computational complexity is more than NLMS. Experimental output: BER Performance of $2 \times 6, 3 \times 9$ and $4 \times 12$ MIMO transmission using LC-MMSE after 5 iterations over 200m, 1000m. Sparseness measurement for the channel is also done for MMSE, NLMS and IPNLMS.

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Zhang Y. et.al.	2018 [106]	(Homotopy RLS-DCD) Adaptive algorithm in posteriori soft decision driven sparse channel Priori error of adaptive channel estimator Is represented as: $e_m(k) = y_m(k) - x^{ISTROKE}(k)\hat{h}_m(k)$ $X(k)$ =training signal vector $y_m(k)$ = received signal Exponentially-weighted mean squared error(MSE) for conventional RLS algorithm: $\min_{\hat{H}(k)} \{\xi(k) \triangleq \sum_{l=1}^k \lambda^{k-l} \ y(l) - \hat{H}(k)x(l)\ _2^2\}$ The tap weight matrix $\hat{H}(k)$ minimizes the cost function $\xi'(k)$ : $\min_{\hat{H}(k)} \{\xi'(k) \triangleq \frac{1}{\sigma^2} \xi(k) + f_p[\hat{H}(k)]\}$ The output from posteriori soft decision equalizer is obtained as: $\hat{x}_n(k) = \hat{f}_n^H(k)(r_k - \hat{H}(k)\tilde{S}_n(k))$	Songhua Lake, China Lake depth = 48.6 m Two transducers below surface at a distance of 5 m and 6 m. 48 hydrophones with communication range of 2.1 km Carrier frequency ( $f_c$ ) = 3 KHz Symbol rate = 2k symbols/second Roll-off factor = 0.2 Bandwidth = 2.4 KHz Sampling rate = 25KHz	Sparse channel estimation accurately with EW-HRLS-DCD (exponential weighted Homotopy recursive least square) algorithm outperform over TEQ (Turbo Equalization) based EW-RLS algorithm.	Improvement in system performance through CIR at 1st transducer and last hydrophone with convergence performance for $2 \times 4$ , $2 \times 8$ , $2 \times 12$ MIMO system. Experimental output: Estimated CIR over 8000 symbols with QPSK modulation using exponential weighted (EW-RLS) algorithm. Turbo equalization (TEQ) based EW-HRLS-DCD algorithm outperforms the TEQ based EW-RLS algorithm.
Kaddouri S. et.al.	2018 [107]	MIMO P:iterative greedy orthogonal matching pursuit algorithm Channel impulse response (CIR) estimated is represented as: $\hat{h}_j[n] = [\hat{h}_j[n, 0] \ \hat{h}_j[n, 1] \ \dots \ \hat{h}_j[n, L_c - 1]]^T$ Root mean square error (RMSE) between modeled CIR and estimated CIR is calculated as: $RMSE = \sqrt{\frac{\sum_{l=0}^{L_c-1}  h(\tau_l) - \hat{h}(\tau_l) ^2}{\sum_{l=0}^{L_c-1}  h(\tau_l) ^2}}$	Florida Atlantic University (FAU) Sampling frequency ( $f_s$ ) = 75KHz Center frequency ( $f_c$ ) = 300 KHz Bandwidth (BW) = 75KHz Source level (SL) = 179 dB Absorption Coefficient ( $\alpha$ ) = 80 dB/Km Temperature (T) = 23 °C pH = 7	Time variation in the channel impulse response (CIR) is closely tracked by MIMO-OMP method with 1.9% over LS estimation of 4.9% and Trended-LS estimation of 1.8%.	Improvement in estimated channel impulse response through Root mean square error (RMSE) at SNR 30 dB, 80 dB, 10 dB MIMO-OMP technique capable of tracking the time-variations in the channel even if SNR decreased.

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
<b>4.7 MIMO- OFDM system model for UWA communication system</b>					
Ceballos Carrascosa P. et al	2010 [108]	<p>Low complexity adaptive channel estimation algorithm(LMS algorithm) with non uniform Doppler prediction and tracking</p> <p>The received signal after demodulation is given as:</p> $y_k^r(n) = \sum_{t=1}^{N_t} H_k^{tr}(n) d_k^t(n) e^{j\theta_k^t(n)} + z_k^r(n)$ <p>The LS estimate of data symbols is given as: <math>\hat{d}_k(n) = \theta_k^{-1}(n)[H_k'(n)H_k(n)]^{-1}H_k'(n)y_k(n)</math></p> <p>The LS estimate of the channel is given as:</p> $\hat{H}_k^r(n) = [\hat{D}_k'(n)\hat{D}_k(n)]^{-1}\hat{D}_k'(n)\hat{y}_k^r(n)$	<p>Expt 1: Panama city Beach, FL</p> <p><math>N_t</math> (Transmit elements)=2,4</p> <p><math>N_r</math> (Receive elements)=8</p> <p>Distance[m]=500,1000,1500</p> <p>Frequency band [KHz] =24-48</p> <p>No. of carriers (K) =128-2048</p> <p>Block Duration T [ms]=5.3-85.3</p> <p>Guard time <math>T_g</math> [ms] =25</p> <p>R/B (Symbol rate/Bandwidth) = 0.175-0.773</p> <p>Expt 2: Buzzards Bay, MA</p> <p><math>N_t</math> (Transmit elements)=2</p> <p><math>N_r</math> (Receive elements)=6</p> <p>Distance[m]=600</p> <p>Frequency band [KHz] =75-137.5</p> <p>No. of carriers K=1024-4096</p> <p>Block Duration T [ms]=16.4-65.5</p> <p>Guard time <math>T_g</math> [ms] =16</p> <p>R/B (Symbol rate/Bandwidth) = 0.506-0.804</p> <p>Expt 3: Narragansett Bay, RI</p> <p><math>N_t</math> (Transmit elements)=2</p> <p><math>N_r</math> (Receive elements)=6</p> <p>Distance[m]=400,1000</p> <p>Frequency band [KHz] =10-12.4</p> <p>No. of carriers K=128-256</p> <p>Block Duration T[ms]=52.4-104</p> <p>Guard time <math>T_g</math> [ms]=15</p> <p>R/B (Symbol rate/Bandwidth) = 0.777-0.874</p>	<p>Compensation of motion-induced non-uniform Doppler frequency offset across wide 24Khz signal Bandwidth and MIMO OFDM channel estimation through frequency and time correlation result in low computational complexity.</p>	<p>Error free performance with 24KHZ bandwidth, maximum data rate is 23Kb/s with 2048 carriers</p> <p>Experimental output (Expt. 1):</p> <p>Estimation of the channel with phase estimation for K=1024 carriers and 4 transmitters at 1500m. BER performance at 500, 1000, 1500 m.</p> <p>Experimental output(Expt. 2):</p> <p>Phase estimation for all carriers for K=1024 at a transmission distance of 600m along with BER performance.</p> <p>Experimental output(Expt. 3):</p> <p>Signal processing results at a transmission distance of 1Km with BER performance for various configuration.</p>

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Table 1 (continued)

Authors	Year/ Ref	MethodsMathematics for Estimation, Detection and Error measurement	Data Acquisition Location/ Parameters for Simulations/Experiments	Advantages	Outcomes
Kim Sunwoo	2012 [109]	Fully angle domain frequency selective sparse channel estimation algorithm Non zero tap index are selected as follows: $q_i = \arg \min_{l \neq q_1, q_2, \dots, q_{i-1}} \left\{ \min_{H_l} \ Y^i - H_l X_l\ _F^2 \right\}$ Non zero elements on $i^{\text{th}}$ iteration is: $\hat{w}^i = ((F^i)^H F^i)^{-1} (F^i)^H r$		Reduced computational complexity through non-zero tap position and non zero angle domain coefficient for MIMO–OFDM UAC	Reduced complexity using 2-stage for MIMO OFDM UAC system. Experimental output: Proposed 2 stage OMP algorithm outperforms both 2 stage MP and conventional 1-stage OMP algorithm.
Shao J. et al.	2014 [110]	Channel estimation for MIMO–OFDM through CS-MP, OMP and LS MSE performance is expressed as $\text{MSE} = \frac{E[\sum_k  H(k) - \hat{H}(k) ^2]}{E \sum_k  H(k) ^2}$	No. of subcarriers = 256 No. of pilot = 64 Channel length L = 60 No. of TX and RX = $2 \times 2$ Modulation type = QAM Carrier Frequency $f_c = 15\text{KHZ}$	Compressed sensing matching pursuit (CS-MP) outperforms over OMP and LS in UWA channel. Estimation through MSE and calculation time for MIMO–OFDM system	CS-MP perform best than LS and OMP to improve frequency spectrum efficiency Experimental output: Performance and Comparison of MSE and calculation time among LS, OMP and CS-MP algorithm. CS-MP outperform over LS and OMP

obtained both in impulsive free and impulse noise environment considering both frequency and phase offsets. The author has also compared the computational complexity of different algorithms; MSE and BER vs. SNR are also obtained for both 1st order and 2nd order methods.

#### 4.2. Channel estimation in SISO and SIMO system model

Group wise phase estimation and correction through time domain channel estimation, equalization, and phase correction scheme is achieved to overcome the problem of convergence and stability in the UWA SIMO system. The main focus is on computing phase estimation and correction after equalization [75].

Further frequency-domain channel estimation and equalization with phase correction scheme for single carrier underwater communication are also achieved. Here the channel is estimated through a pilot signal block in each transmitter packet then re-estimated by detected symbols using the MMSE criterion. The frequency-domain equalized signal is converted into a time domain. The Phase correction algorithm is applied to time domain equalized signal to overcome the phase rotation problem due to varying Doppler drift and achieving better robustness against interference and noise [76].

In [77], the authors represented SAGE technique for joint channel estimation, equalization and symbol detection for underwater communication. Computational complexity is reduced through the representation of discrete Fourier basic expansion channel model.

#### 4.3. Channel estimation in pilot assisted OFDM system model

In [78], the authors estimated channel frequency response by pilots already known to the receiver. Three different kinds of pilot patterns are analyzed here (Block, Comb, and Scatter) through simulation and experiment. Scatter pilot pattern outperforms both in time selective and frequency selective channel over Block and Comb pattern. However to increase the data rate channel is estimated through data embedded pilot without extra overload [79]. Channel is also estimated through preamble and post-amble at the beginning and end of the frame through LS and minimum mean squared error (MMSE) algorithm. Here a channel estimation strategy that does not require additional bandwidth is directly added to the data sequence. The proposed model also minimizes the phase error of the estimated channel response [80].

The combination of MP and maximum a posteriori probability (MAP) based on SAGE maximization technique is used to improve the channel estimation with low complexity. A matching pursuit (MP) approach is used to estimate the complex-valued taps from source to destination. The sparseness here is modeled by Continuous Gaussian Mixture (CGM) [81]. Fast fading adaptive Channel is estimated in underwater acoustic communication using LS and RLS algorithm [82]. In [83], the authors propose a pilot routing algorithm to minimize mutual coherence in order to improve the accuracy of channel estimation.

Channel estimation in UWA communication can also be improved by considering clustering and sparsity. In [84], the authors consider clustering and sparsity to improve channel estimation in UWA communication. Complexity is reduced through Markov chain Monte Carlo (MCMC) algorithm with turbo equalization. Finally, the author shows improved bit error rate performance with existing algorithms.

In [85], the authors have proposed a frame-based coded underwater acoustic (UA) in which coded sequence at the transmitter is obtained through encoding, interleaving and puncturing.

The coded sequence is mapped into data symbols that are taken from PSK and QAM constellation. In [86], the authors used Fast Bayesian matching pursuit (FBMP) to determine bit error rate (BER) performance against OMP and came out with the result that FBMP outperforms over OMP with a saving of 4 dB SNR. Filtered multitone (FMT) OMP for sparse channel estimation is used to reduce complexity through splitting sub-bands in [87].

#### 4.4. Channel estimation in cyclic prefix (CP) OFDM system model

A low-density parity-check (LDPC) coded the OFDM system with the assumption that channel state information (CSI) and carrier frequency offset (CFO) is unavailable to both transmitter and receiver. Asymptotic performance and convergence behavior of channel response are evaluated using Extrinsic information transfer (EXIT) chart method [88]. Further piecewise linear approximation for channel estimation with ICI mitigation is presented in [89].

A time-domain predictor for exploiting the sparse nature of the underwater acoustic channel, ultimately estimating the channel with low complexity as a small number only of channel paths are predicted and the predictor parameter is computed using RLS algorithm and giving a comparison of different channel estimation methods [90]. Finally, the author had obtained BER comparison of different channel predictor with RLS predictor that has better tracking ability and the author has presented NMSE comparison also.

A Joint Sparse model with exponential smoothing (ES) for OFDM systems is used to consider the correlation between channels. The author has simulated NMSE for exponential smoothing based simultaneous orthogonal matching pursuit (ES-SOMP) with different smoothing factor  $\lambda$ . The author is also derived the NMSE of different algorithms [92]. In [93], the authors implemented Matching pursuit with maximum a posteriori (MP-MAP) algorithm to determine MSE vs. SNR performance and finally came out with the result that MP-MAP outperforms over MP in MSE and Block error rate performance (BLER).

#### 4.5. Channel estimation in zero-padded (ZP) OFDM system model

A path-based model with the limited number of channels is proposed in [94]. Channel with limited Doppler spread is estimated using Root-Music and ESPRIT (Subspace algorithm) and channels with Doppler spread are estimated using a compressed sensing approach (OMP, BP algorithm). In [95], authors have given performance analysis of BP solvers  $l1\_ls$ , SparSA, and Yalli are obtained through simulated and experimental data. The performance of SparSA and Yalli is better than  $l1\_ls$  in terms of the complexity and CPU run time. In [96], the authors used a cluster of channel paths to exploit time coherence of UWA channels to estimate the underwater channel. In [97], the authors proposed a complex Homotopy algorithm for sparse channel estimation which consists of the Gauss Markov (GM) model in 1st stage and RLS with CS in 2nd stage to reduce computation time comparison to OMP and YALLI.

The authors in [98] considered the Improved LS algorithm through significant channel taps based on the predetermined threshold. The BER performance of improved LS algorithm is better than conventional LS algorithm for the delay spread of 6 ms and 10 ms. In [99], the authors proposed a data-driven learning sparsity method based on LMMSE equalizer to compare block error rate (BER) through different sparse algorithms under  $L_0$ ,  $L_{1/2}$ ,  $L_1$  and  $L_2$  constraints.

Bit error rate (BER) performance through an iterative OFDM receiver is considered to reduce performance degradation caused by ICI in a zero-padding OFDM system (ZP-OFDM) [100]. Here modulation symbol is consisting of both data symbol and pilot symbol. BER performance Vs SNR for different iteration number along with EXIT chart of iteration with different SNR is obtained. In [101], the authors achieved lower complexity through compressed channel estimation (CCE) based on the Fast block Fourier transform (FFT) Orthogonal matching pursuit (OMP) algorithm. In [102], the authors focused on channel estimation through Orthogonal matching Pursuit (OMP) compressing sensing technique with minimum computational complexity. Minimum computation complexity will be achieved by exploiting ICI interferences on the adjacent subcarrier. Finally, the authors obtained normalized computation complexity gain Vs varying oversampling factor for different  $k$ .

#### 4.6. Channel estimation in MIMO system model

The SLIM algorithm is proposed in MIMO UWA communication system with  $N$  number of transducers at transmitter and  $M$  number of hydrophones at receiver [103]. QPSK with a Gray code is used to extract symbols. Further RELAX-BLAST scheme is implemented for symbol detection. In [104], the authors used an  $N \times M$  MIMO underwater acoustic communication system with  $N$  number of transmits transducers and  $M$  number of receives hydrophones. At transmitter before transmission encoding, interleaving and modulation (QPSK, 8PSK, and 16QAM) are performed. At  $m$ th hydrophone the received signal is represented as  $y_m^{ce} \approx \sum_{n=1}^N X_n h_{m,n} + w_m = X h_m + w_m$  where  $h_m = (X^H X + \sigma_w^2 I_{N \times L})^{-1} X^H y_m^{ce}$  where  $\sigma_w^2$  is noise variance.

A bit-interleaved coding modulation (BICM) scheme based on  $N \times M$  MIMO single carrier UWA communication system is also proposed in which  $N$  number of the transducer at the transmitter and  $M$  number of hydrophones at the receiver is available. The received baseband signal at  $m$ th hydrophone is represented as  $y_m(k) = \sum_{p=0}^{P-1} \sum_{n=1}^N h_{m,n}^p(k) x_n(k-p) + \eta_m(k)$  where  $h_{m,n}^p(k)$  represents channel impulse response between  $m$ th hydrophone and  $n^{\text{th}}$  transducer at time instant  $k$ .  $\eta_m(k)$  represents additive noise. The authors extended Homotopy recursive least square dichotomous coordinate descent (Homotopy RLS-DCD) to estimate time-varying MIMO sparse channel adaptively. A posteriori soft-decision symbol or a priori soft decision symbol applied to MMSE equalizer resulted in improvement in turbo equalization with a reduction in pilot overhead [106].

In [107], the authors used a MIMO system in which the relationship between the received signal and the transmitted signal is represented as  $r_j[n] = S[n]h_j[n] + w_j[n]$  where  $h_j[n]$  represents coefficients of channel impulse response between transmitters and receivers.  $S[n]$  is formed from  $s_i[n]$  and is called the augmented matrix. Here the authors had given a comparison of proposed MIMO P-iterative greedy OMP algorithm with the least square estimation method using experimental and simulated data. A robust channel estimation technique closely tracks the time variation at high frequency. The author has obtained the MIMO-OMP algorithm which is robust to both LS and trended-LS technique and tracking capability of the MIMO-OMP technique is high even when SNR decreases.

#### 4.7. Channel estimation in MIMO-OFDM system model

The data rate can be increased through spatial multiplexing using independent data streams. In [108], the authors proposed a low complexity adaptive LMS algorithm to increase the data rate by compensating the Doppler frequency offset. A single matrix of size  $N_t \times N_t$  per carrier is used in a low-complexity approach and performed for  $K$  carrier. Magnitude truncation for channel sparseness is also performed while transforming the estimated frequency domain (transfer function) coefficients into time-domain (impulse response). Further, the authors have also performed Phase prediction to reduce pilot overhead. In [109], the author reduces the computational complexity through non zero angle domain coefficients and non zero tap positions to estimate the frequency-selective sparse channel for MIMO-OFDM UAC systems. The author called it as an application of the OMP algorithm for UWA channel estimation. Finally, the author has obtained the error in channel estimation over 200 Monte Carlo runs.

ISI effect is avoided by taking the condition that the cyclic-prefix (CP) exceeds the maximum multipath time delay in a MIMO-OFDM system with  $N_t$  transmitter and  $N_r$  receiver with  $N_s$  orthogonal sub-carriers. In [110] channel estimation is performed through MSE performance between OMP, LS and Compressed sensing matching pursuit (CS-MP) in a MIMO-OFDM system environment.

### 5. Conclusion and future scope

This paper gives a survey on channel estimation with different system models in the field of UWA communication. As an underwater acoustic channel suffers from frequency selective fading, Doppler spread, significant delay, double side spreading, it is not possible to design a single communication system with specific algorithms/methods. Hence in this paper, we have discussed different system models with different algorithms/methods. We have also represented a comparison between methods through complexity calculation in channel tracking, MSE, and NMSE performance with the number of iterations, and BER vs. SNR performance at the receiver.

Initially, to reduce computational complexity a decision feedback equalizer is focused which suffers from error propagation due to feedback. DFE in conjunction with spatial diversity combining is represented and complexity of channel response with MSE over DFE is reduced by selecting significant components. MP algorithm is implemented on PPC-McDFE to improve sparse channel estimation performance over PPC-DFE by compressing the dominant arrivals. The channel tracking accuracy is also improved through non-convex sparse optimization approach. Non-convex sparse optimization approach such as NCMNS reduces channel tracking computation time over other methods through fast-tracking of sparse coefficients. Minimization of logarithm cost function also plays a vital role in accurately simulating UWA channels. The convergence rate for LCLMS and LCLMA algorithms is faster compared to the LCRLS algorithm for 1st order methods whereas for 2nd order methods LCRLS outperforms over LCLMS and LCLMA. Subspace methods that give improvement on LS estimation also explained for sparse channel estimation, equalization and symbol detection for moderate Doppler spread through SLIM and SAGE algorithm. SAGE technique reduces computational complexity over other techniques through the representation of discrete fourier basic expansion channel model. Phase correction scheme to overcome phase drift and rotation problem due to time-varying Doppler drift in addition to channel estimation using the MMSE criterion and equalization are also discussed in this paper. Channel estimation using pilot patterns is



also discussed in this paper in which the Scatter pilot pattern outperforms in channel tracking over Block and Comb pilot pattern. Channel estimation using clustering and sparsity is also discussed in this paper in which MCMC shows the least complexity in channel tracking over all other algorithms when clustering and sparsity are taking into consideration. The correlation between channels is also discussed using a joint sparse model with exponential smoothing (ES). In such a case NMSE, MSE vs. SNR performance is considered and came out with a result that MP-MAP gives better channel tracking over MP in MSE and BER performance. Finally, we can say that the compressed sensing technique for sparse reconstruction is best used to improve the MSE, NMSE performance with lower complexity in channel tracking and with low hardware cost in different system model environments starting from pilot assisted OFDM system to MIMO-OFDM system.

As the demand for high-speed communication in underwater is increased day by day, so MIMO channels with OFDM is also focused here to increase spectral efficiency, system capacity for frequency selective channel.

MIMO-OFDM in UWA communication is a new research area. This system opens doors to implement various algorithms/methods with their benefits to exploit characteristics of the UWA channel. In this paper, the implementation of LS, OMP, and CS-MP algorithms along with their MSE performance in the MIMO-OFDM environment is also described. It is also shown that CS-MP outperforms over OMP and LS in UWA channel estimation through MSE and calculation time performance. Further researchers can implement IPNLMS algorithms, FFT based OMP algorithm, exponential smoothing as channel estimation, Fast Bayesian Matching Pursuit (FBMP), Filtered multi-tone (FMT) OMP in MIMO-OFDM environment with their benefits to exploit sparsely of UWA channel. Proper channel estimation in an underwater environment using MIMO-OFDM will fulfill the demand for high speed and reliable communication with multimedia applications.

## References

- [1] A.B. Baggeroer, An overview of acoustic communications from 2000-2012, *Underw. Commun.: Channel Model. Valid.* 5 (2012) 201–207.
- [2] J.A. Catipovic, et al., Performance limitations in underwater acoustic telemetry, *IEEE J. Ocean. Eng.* 15 (3) (Jul. 1990) 205–216.
- [3] M. Stojanovic, J. Catipovic, J.G. Proakis, Adaptive multichannel combining and equalization for underwater acoustic communications, *J. Acoust. Soc. Am.* 94 (3) (Sep. 1993) 1621–1631.
- [4] M. Stojanovic, Acoustic (Underwater) communications, *Encyclopedia of Telecommunications*, Wiley, New York, NY, USA, 2003.
- [5] J.C. Preisig, G.B. Deane, Surface wave focusing and acoustic communications in the surf zone, *J. Acoust. Soc. Am.* 116 (4) (2004) 2067–2080.
- [6] F. Qu, Z. Wang, L. Yang, Z. Wu, A journey toward modeling and resolving doppler in underwater acoustic communications, *IEEE Commun. Mag.* 54 (2) (2016) 49–55.
- [7] D. Kilfoyle, A. Baggeroer, The state of the art in underwater acoustic telemetry, *IEEE J. Ocean. Eng.* 25 (1) (Jan. 2000) 4–27.
- [8] T.H. Eggen, A.B. Baggeroer, J.C. Preisig, Communication over Doppler spread channels – Part I: channel and receiver presentation, *IEEE J. Ocean. Eng.* 25 (1) (Jan. 2000) 62–71.
- [9] M. Stojanovic, J.C. Preisig, Underwater acoustic communication channels: propagation models and statistical characterization, *IEEE Commun. Mag.* 47 (1) (2009) 84–89.
- [10] C.R. Berger, S. Zhou, J.C. Preisig, P. Willett, Sparse channel estimation for multicarrier underwater acoustic communication: from subspace methods to compressed sensing, *IEEE Trans. Signal Process.* 58 (3) (Mar. 2010) 1708–1721.
- [11] W. Li, J.C. Preisig, Estimation of rapidly time-varying sparse channels, *IEEE J. Ocean. Eng.* 32 (4) (Oct. 2007) 927–939.

- [12] C.R. Berger, Z. Wang, J. Huang, S. Zhou, Application of compressive sensing to sparse channel estimation, *IEEE Commun. Mag.* 48 (11) (Nov. 2010) 164–174.
- [13] T. Kang, R.A. Iltis, Iterative carrier frequency offset and channel estimation for underwater acoustic OFDM systems, *IEEE J. Sel. Areas Commun.* 26 (9) (Dec. 2008) 1650–1661.
- [14] M. Stojanovic, OFDM for underwater acoustic communications: adaptive synchronization and sparse channel estimation, in: *Proceedings of the IEEE International Acoustics, Speech, and Signal Processing (ICASSP)*, Mar./Apr. 2008, pp. 5288–5291.
- [15] C.R. Benson, M.R. Frater, High data rates in the high frequency acoustic channel, in: *Proceedings of the MTS/IEEE Washington OCEANS*, Oct. 2015, pp. 1–4.
- [16] S. Roy, T.M. Duman, V. McDonald, J.G. Proakis, High-rate communication for underwater acoustic channels using multiple transmitters and space-time coding: receiver structures and experimental results, *IEEE J. Ocean. Eng.* 32 (3) (Jul. 2007) 663–688.
- [17] P. Flandrin, An empirical model for electronic submissions to conferences, *Adv. Complex Syst.* 13 (3) (2010) 439–449.
- [18] L.M. Brekhovskikh, Y.P. Lysanov, *Fundamentals of Ocean Acoustics*, Springer, New York, NY, USA, 2003.
- [19] Z. Yang, Y.R. Zheng, Iterative channel estimation and turbo equalization for multiple-input multiple-output underwater acoustic communications, *IEEE J. Ocean. Eng.* 41 (1) (Jan. 2016) 232–242.
- [20] B. Paradis, C.L. Bachand, P.J. Gendron, D.A. Brown, Development of a high frequency underwater acoustic communication modem, in: *Proceedings of Meetings on Acoustics (ASA)*, 21, 2014.
- [21] W. Li, J.C. Preisig, Estimation of rapidly time-varying sparse channels, *IEEE J. Ocean. Eng.* 32 (4) (2007) 927–939.
- [22] B. Li, S. Zhou, M. Stojanovic, L. Freitag, P. Willett, Multicarrier communication over underwater acoustic channels with nonuniform doppler shifts, *IEEE J. Ocean. Eng.* 33 (2) (2008) 198–209.
- [23] P.C. Carrascosa, M. Stojanovic, Adaptive channel estimation and data detection for underwater acoustic MIMO-OFDM systems, *IEEE J. Ocean. Eng.* 35 (3) (2010) 635–646.
- [24] A.S. Gupta, J. Preisig, Tracking the time-varying sparsity of channel coefficients in shallow water acoustic communications, in: *Signals, Systems and Computers (ASILOMAR)*, 2010 Conference Record of the Forty-Fourth Asilomar Conference on, IEEE, 2010, pp. 1047–1049.
- [25] J.A. Tropp, A.C. Gilbert, Signal recovery from random measurements via orthogonal matching pursuit, *IEEE Trans. Inf. Theory* 53 (12) (2007) 4655–4666.
- [26] M.A. Figueiredo, R.D. Nowak, S.J. Wright, Gradient projection for sparse reconstruction: application to compressed sensing and other inverse problems, *IEEE J. Sel. Top. Signal Process.* 1 (4) (2007) 586–597.
- [27] A. Sen Gupta, Time-frequency localization issues in the context of sparse process modeling, in: *Proceedings of Meetings on Acoustics ICA 2013*, 19, ASA, 2013.
- [28] A.S. Gupta, N. Ansari, A. Gupta, Tracking the underwater acoustic channel using two-dimensional frequency sampling, in: *Underwater Technology (UT)*, 2015 IEEE, IEEE, 2015, pp. 1–5.
- [29] N. Ansari, A. Gupta, A.S. Gupta, Physics inspired cs based underwater acoustic channel estimation, in: *Signal and Information Processing (GlobalSIP)*, 2015 IEEE Global Conference on, IEEE, 2015, pp. 1106–1110.
- [30] M. Stojanovic, J. Capovic, J. Proakis, Reduced-complexity spatial and temporal processing of underwater acoustic communication signals, *J. Acoust. Soc. Am.* 98 (2) (1995) 961–972.
- [31] H.C. Song, W.S. Hodgkiss, W.A. Kuperman, M. Stevenson, T. Akal, Improvement of time-reversal communications using adaptive channel equalizers, *IEEE J. Ocean Eng.* 31 (Apr.) (2006) 487–496.
- [32] J.A. Flynn, J.A. Ritcey, D. Rouseff, W.L.J. Fox, Multichannel equalization by decision-directed passive phase conjugation: experimental results, *IEEE J. Ocean. Eng.* 29 (July) (2004) 824–836.
- [33] M. Stojanovic, J. Capovic, J. Proakis, Phase-coherent digital communications for underwater acoustic channels, *IEEE J. Ocean Eng.* 19 (Jan.) (1994) 100–111.
- [34] M. Stojanovic, Recent advances in high-speed underwater acoustic communications, *IEEE J. Ocean. Eng.* 21 (April) (1996) 125–136.
- [35] M. Johnson, L. Freitag, M. Stojanovic, Improved Doppler tracking and correction for underwater acoustic communications, *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process* 1 (1997) 575–578.
- [36] T.J. Riedl, A.C. Singer, Broadband Doppler compensation: principles and new results, in: *Proc. Conf. Rec. 45th Asilomar Conf. Signals, Syst. Comput.*, 2011, pp. 944–946.
- [37] K.A. Perrine, K.F. Nieman, T.L. Henderson, K.H. Lent, T.J. Brudner, B.L. Evans, Doppler estimation and correction for shallow underwater acoustic communications, in: *Proceedings of the IEEE Conference on Rec. 44th Asilomar Conference on Signals, Systems, and Computers*, 2010, pp. 746–750.

- [38] J. Li, Y.V. Zakharov, Multibranch autocorrelation method for doppler estimation in underwater acoustic channels, *IEEE J. Ocean. Eng.* (99) (Nov. 2017) 1–15 PP.
- [39] M. Stojanovic, J. Preisig, Underwater acoustic communication channels: propagation models and statistical characterization, *IEEE Commun. Mag.* 47 (1) (January 2009) 84–89.
- [40] T. Arikan, T. Riedl, A. Singer, J. Younce, Comparison of OFDM and single-carrier schemes for Doppler tolerant acoustic communications, in: *OCEANS 2015 - Genova, Genoa, Italy, May 2015*, pp. 1–7.
- [41] L. Cannelli, G. Leus, H. Dol, P. van Walree, Adaptive turbo equalization for underwater acoustic communication, in: *Proceedings of the MTS/IEEE OCEANS Conference, Bergen, Norway, Jun. 2013*, pp. 1–9.
- [42] J.W. Choi, T.J. Riedl, K. Kim, A.C. Singer, J.C. Preisig, Adaptive linear turbo equalization over doubly selective channels, *IEEE J. Ocean. Eng.* 36 (4) (Oct. 2011) 473–489.
- [43] C. Laot, N. Beuzeuglin, A. Bourre, Experimental results on MMSE turbo equalization in underwater acoustic communication using high order modulation, in: *Proceedings of the MTS/IEEE OCEANS Conference, Seattle, WA, USA, Sep. 2010*, pp. 1–6.
- [44] F. Wan, W.P. Zhu, M.N.S. Swamy, Semiblind sparse channel estimation for MIMO-OFDM systems, *IEEE Trans. Veh. Technol.* 60 (6) (2011) 2569–2582.
- [45] D. Hu, X. Wang, L. He, A new sparse channel estimation and tracking method for time-varying OFDM systems, *IEEE Trans. Veh. Technol.* 62 (9) (2013) 4648–4653.
- [46] B. Li, J. Huang, S. Zhou, K. Ball, M. Stojanovic, L. Freitag, P. Willett, MIMO-OFDM for high-rate underwater acoustic communications, *IEEE J. Ocean. Eng.* 34 (4) (2009) 634–644.
- [47] Christian R. Berger, Shengli Zhou, James C. Preisig, Peter Willett, Sparse channel estimation for multicarrier underwater acoustic communication: from subspace methods to compressed sensing, *IEEE Trans. Signal Process.* 58 (3) (March 2010) 1708–1721.
- [48] A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005, pp. 387–388.
- [49] B. Muquet, Z. Wang, G. Giannakis, M. de Courville, P. Duhamel, Cyclic prefixing or zero padding for wireless multicarrier transmissions? *IEEE Trans. Commun.* 50 (12) (2002) 2136–2148.
- [50] D. Tse, P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, New York, NY, 2005.
- [51] D. Kilfoyle, J.C. Preisig, A.B. Baggeroer, Spatial modulation experiments in the underwater acoustic channel, *IEEE J. Ocean. Eng.* 30 (April 2005) 406–415.
- [52] S.-J. Kim, K. Koh, M. Lustig, S. Boyd, D. Gorinevsky, An interior point method for large-scale  $\ell_1$ -regularized least squares, *IEEE Trans. Sig. Proc.* 1 (4) (Dec. 2007) 606–617.
- [53] J.Z. Huang, C.R. Berger, S. Zhou, J. Huang, Comparison of basis pursuit algorithms for sparse channel estimation in underwater acoustic OFDM, in: *Proceedings of the MTS/IEEE OCEANS, Sydney, Australia, May 2010*, pp. 1–6.
- [54] S.J. Wright, R.D. Nowak, M.A.T. Figueiredo, Sparse reconstruction by separable approximation, *IEEE Trans. Sig. Proc.* 57 (7) (July 2009) 2479–2493.
- [55] D. Needell, J.A. Tropp, CoSaMP: iterative signal recovery from incomplete and inaccurate samples, *Appl. Comput. Harmon. Anal.* 26 (3) (Apr. 2008) 301–321.
- [56] W. Dai, O. Milenkovic, Subspace pursuit for compressive sensing signal reconstruction, *IEEE Trans. Inf. Theory* 55 (5) (May 2009) 2230–2249.
- [57] D.L. Donoho, Y. Tsaig, Fast solution of  $\ell_1$ -norm minimization problems when the solution may be sparse, *IEEE Trans. Inf. Theory* 54 (11) (Nov. 2008) 4798–4812.
- [58] X. Shi, Y. Yang, Adaptive sparse channel estimation based on RLS for underwater acoustic OFDM systems, in: *Proceedings of the Sixth International Conference on Instrumentation and Measurement, Computer, Communication and Control, 2016*, pp. 266–269.
- [59] G. Taubock, F. Hlawatsch, D. Eiwen, H. Rauhut, Compressive estimation of doubly selective channels in multicarrier systems: leakage effects and sparsity-enhanced processing, *IEEE J. Sel. Top. Sig. Process.* 4 (2) (Apr 2010).
- [60] J. Xue, Z. Wenjun, C. En, A fast algorithm for sparse channel estimation via orthogonal matching pursuit, in: *Proceedings of the 73rd vehicular technology conference, 2011*, pp. 1–5. <http://dx.doi.org/10.1109/VETECS.2011.5956242>.
- [61] A. Zhang, S. Yang, G. Gui, Sparse channel estimation for MIMO OFDM two-way relay network with compressed sensing, *Int. J. Antennas Propag.* 2013 (Feb. 2013) Art. no. 914734.
- [62] Y. Peng, X. Yang, X. Zhang, W. Wang, B. Wu, Compressed MIMO OFDM channel estimation, in: *Proceedings of the 12th IEEE International Conference on Communication Technology (ICCT), Nov. 2010*, pp. 1291–1294.

- [63] G. Gui, Q. Wan, W. Peng, F. Adachi, Sparse Multipath Channel Estimation Using Compressive Sampling Matching Pursuit Algorithm, May 2010 [Online]. Available: <https://arxiv.org/abs/1005.2270>.
- [64] R. Tibshirani, Regression shrinkage and selection via the lasso, *J. R. Statist. Soc. B* 58 (1994) 267–288.
- [65] X. Lv, G. Bi, C. Wan, The group lasso for stable recovery of block-sparse signal representations, *IEEE Trans. Signal Process.* 59 (4) (2011) 1371–1382.
- [66] Y.C. Eldar, P. Kuppinger, H. Bolcskei, Block-sparse signals: uncertainty relations and efficient recovery, *IEEE Trans. Signal Process.* 58 (6) (2010) 3042–3054.
- [67] R.G. Baraniuk, V. Cevher, M.F. Duarte, et al., Model-based compressive sensing, *IEEE Trans. Inf. Theory* 56 (4) (2010) 1982–2001.
- [68] M. Stojanovic, Efficient processing of acoustic signals for high rate information transmission over sparse underwater channels, *Elsevier J. Physical Commun.* 1 (2) (2008) 146–161.
- [69] G.S. Zhang, B. Peng, H.F. Dong, Experimental assessment of sparse channel estimations for passive-phase conjugation communications, in: *IEEE International Conference on Acoustics Speech and Signal Processing*, 2012, pp. 2701–2704.
- [70] A.S. Gupta, J. Preisig, A geometric mixed norm approach to shallow water acoustic channel estimation and tracking, *Phys. Commun.* 5 (2) (2012) 119–128.
- [71] S.H. Huang, T.C. Yang, J. Tsao, Improving channel estimation for rapidly time-varying correlated underwater acoustic channels by tracking the signal subspace, *Ad Hoc Netw.* 34 (C) (2014) 17–30.
- [72] Y. Zhang, S. Xiao, L. Liu, D. Sun, Analysis and estimation of the underwater acoustic millimeter-wave communication channel, in: *IEEE IEEE/OES China Ocean Acoustics (COA)*, 2016, pp. 1–5.
- [73] N. Ansari, A. Gupta, A. Sen Gupta, Shallow water acoustic channel estimation using two-dimensional frequency characterization, *J. Acoust. Soc. Am. (JASA)* 140 (5) (2016) 3995–4009.
- [74] D. Kari, et al., Robust adaptive algorithms for underwater acoustic channel estimation and their performance analysis, *Digit. Signal Process.* 68 (2017) 57–68.
- [75] Y.R. Zheng, Channel estimation and phase-correction for robust underwater acoustic communications, in: *IEEE Intl. Military Conf.*, Orlando, FL, Oct., 2007.
- [76] Y.R. Zheng, C. Xiao, T.C. Yang, W.B. Yang, Frequency-domain channel estimation and equalization for shallow-water acoustic communications, *Else. J. Phys. Commun.* (3) (2010) 48–63.
- [77] G. Gong, W. Yao, Joint channel estimation, equalization, and symbol detection for underwater communication, in: *IEEE International Conference on Consumer Electronics, Communications and Networks (CECNet)*, 2012, pp. 1096–1099.
- [78] W. Yonggang, Underwater acoustic channel estimation for pilot based OFDM, in: *IEEE International Conference on Communications and Mobile Computing*, 2011, pp. 573–577.
- [79] J. Han, C. Hwang, H. Lee, Ki. Kim, M. Nordenvaad, J. Kim, Channel estimation using data embedded pilot in underwater acoustic communication, in: *IEEE OCEANS 2014 - TAIPEI*, 2014, pp. 1–4.
- [80] H. Kim, J. Chung, M. Nordenvaad, J. Kim, Superimposed pilots aided estimation of phase varying channels for underwater acoustic communication, in: *IEEE OCEANS 2014 - TAIPEI*, 2014, pp. 1–4.
- [81] E. Panayirci, H. Senol, M. Uysal, et al., Sparse channel estimation and equalization for OFDM-based underwater cooperative systems with amplify-and-forward relaying, *IEEE Trans. Signal Process.* 64 (1) (2016) 214–228.
- [82] X.L. Shi, Y.X. Yang, Adaptive sparse channel estimation based on RLS for underwater acoustic OFDM systems, in: *Sixth International Conference on Instrumentation & Measurement Computer Communication and Control (IMCCC)*, 2016, pp. 1–4. 2016.
- [83] L. Tang, H. Wu, R. Jiang, C. Lu, An improved pilot routing algorithm for compressed sensing-based channel estimation in underwater acoustic OFDM system, in: *IEEE International Conference on Advanced Infocomm Technology (ICAIT)*, 2017, pp. 90–94.
- [84] L.Y. Jing, C.B. He, J.G. Huang, Z. Ding, Joint channel estimation and detection using Markov chain Monte Carlo method over sparse underwater acoustic channels, *IET Commun.* 11 (2017) 1789–1796.
- [85] P. Chen, Y. Rong, S. Nordholm, Z. He, A. Duncan, Joint channel estimation and impulsive noise mitigation in underwater acoustic OFDM communication systems, *IEEE Trans. Wirel. Commun.* 16 (9) (Sep. 2017) 6165–6178.
- [86] H. Chen, C. Qi, Underwater acoustic channel estimation via fast Bayesian matching pursuit, in: *IEEE International Conference on Wireless Communications and Signal Processing (WCSP)*, 2017, pp. 1–6.
- [87] C. Qian, Z. Wang, X. Lu, Y. Wu, F. Qu, Sparse channel estimation for filtered multitone in underwater communications, in: *Proceedings of the MTS/IEEE Oceans*, 2017, pp. 1–7.

- [88] T. Kang, R.A. Iltis, Iterative carrier frequency offset and channel estimation for underwater acoustic OFDM systems, *IEEE J. Select. Areas Commun.* 26 (9) (Dec. 2008) 1650–1661.
- [89] J. Liao, et al., Time-varying channel estimation with inter-carrier interference mitigation in underwater OFDM systems, in: *Proceedings of the OCEANS TAIPEI*, Apr. 2014, pp. 1–6.
- [90] N. Lin, et al., Prediction based sparse channel estimation for underwater acoustic OFDM, *Appl. Acoust.* 96 (2015) 94–100.
- [91] L. Lanjun, PN sequence based Doppler and channel estimation for underwater acoustic OFDM communication, in: *2016 IEEE International Conference on Signal Processing Communications and Computing (ICSPCC)*, 2016, pp. 1–6.
- [92] Z. Wang, H. Wu, S. Liu, An improved sparse underwater acoustic OFDM channel estimation method based on joint sparse model and exponential smoothing, in: *IEEE International Conference on Signal Processing, Communications and Computing (ICSPCC)*, 2017, pp. 1–6.
- [93] M. Altabbaa, E. Panayirci, Channel estimation and equalization algorithm for OFDM-based underwater acoustic communications systems, in: *International Conference on Wireless Communications, Networking and Mobile Computing*, 2017, pp. 1–4.
- [94] C.R. Berger, S. Zhou, J.C. Preisig, P. Willett, Sparse channel estimation for multicarrier underwater acoustic communication: from subspace methods to compressed sensing, *IEEE Trans. Signal Process.* 58 (no.3) (2010) 1708–1721.
- [95] J.-Z. Huang, C.R. Berger, S. Zhou, J. Huang, Comparison of basis pursuit algorithms for sparse channel estimation in underwater acoustic OFDM, in: *Proc. of OCEANS, Sydney, Australia, May 2010* 24–27.
- [96] Z.-H. Wang, S. Zhou, J. Preisig, K. Pattipati, P. Willett, Per-cluster-prediction based sparse channel estimation for multicarrier underwater acoustic communications, in: *IEEE International Conference on Signal Processing, Communications and Computing*, 2011-Sep.
- [97] C. Qi, L. Wu, X. Wang, Underwater acoustic channel estimation via complex Homotopy, in: *2012 IEEE International Conference on Communications, ICC*, 2012, pp. 3821–3825.
- [98] K.S. Priyanjali, A.V. Babu, An improved least square channel estimation technique for OFDM systems in sparse underwater acoustic channel, in: *Proc. the 2014 International Conference on Advances in Computing Communications and Informatics ICACCI 2014*, 2014, pp. 2521–2525.
- [99] Y. Huang, L. Wan, S. Zhou, et al., Comparison of sparse recovery algorithms for channel estimation in underwater acoustic OFDM with datadriven sparsity learning, *Phys. Commun.* 13 (2014) 156–167.
- [100] F. Yu, Q. Guo, D. Li, Performance of iterative OFDM receiver in doubly spread underwater acoustic communication channel, in: *IEEE International Conference on Signal Processing (ICSP)*, 2014, pp. 1579–1583.
- [101] F. Yu, D. Li, Q. Guo, et al., Block-FFT based OMP for compressed channel estimation in underwater acoustic communications, *IEEE Commun. Lett.* 19 (11) (2015) 1937–1940.
- [102] Ch. Li, K. Song, L. Yang, Low computational complexity design over sparse channel estimator in underwater acoustic OFDM communication system, *IET Commun.* 11 (7) (2017) 1143–1151.
- [103] J. Ling, T. Yardibi, X. Su, H. He, J. Li, Enhanced channel estimation and symbol detection for high speed multi-input multi-output underwater acoustic communications, *J. Acoust. Soc. Am.* 125 (5) (May 2009) 3067–3078.
- [104] B. Gwun, J. Han, Ki. Kim, J. Jung, MIMO underwater communication with sparse channel estimation, in: *IEEE International Conference on Ubiquitous and Future Networks (ICUFN)*, 2013, pp. 32–36.
- [105] Z. Yang, Y.R. Zheng, Iterative channel estimation and turbo equalization for multiple-input multiple-output underwater acoustic communications, *IEEE J. Ocean. Eng.* 41 (1) (Jan. 2016) 232–242.
- [106] Y. Zhang, Y.V. Zakharov, J. Li, Soft-decision-driven sparse channel estimation and turbo equalization for MIMO underwater acoustic communications, *IEEE Access* 6 (2018) 4955–4973.
- [107] S. Kaddouri, P.-P.J. Beaujean, P.-J. Bouvet, High-frequency acoustic estimation of time-varying underwater sparse channels using multiple sources and receivers operated simultaneously, in: *IEEE Access*, 6, Feb. 2018, pp. 10569–10580.
- [108] P.C. Carrascosa, M. Stojanovic, Adaptive channel estimation and data detection for underwater acoustic MIMO-OFDM systems, in: *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, 35, July 2010, pp. 635–646.
- [109] S. Kim, Angle-domain frequency-selective sparse channel estimation for underwater MIMO-OFDM systems, *IEEE Commun. Lett.* 16 (5) (2012) 685–687.
- [110] J. Shao, X. Zhang, Y. Liu, Channel estimation based on compressed sensing for high-speed underwater acoustic communication, in: *IEEE International Congress on Image and Signal Processing*, 2014, pp. 1017–1021.