

NSG-304

# UNPUBLISHED PRELIMINARY DATA

## ATMOSPHERIC OSCILLATIONS

by A. J. Lineberger and H. D. Edwards

Georgia Tech Project A-652-001

GPO PRICE \$ \_\_\_\_\_

OTS PRICE(S) \$ \_\_\_\_\_

Contract No. AF19(628)-393

Hard copy (HC) 2.00

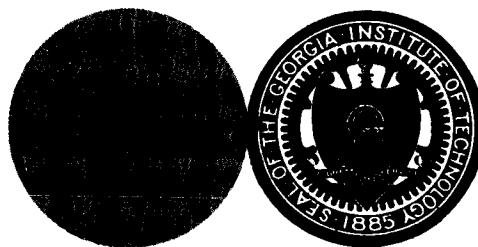
Microfiche (MF) .50

Prepared for  
National Aeronautics and Space Administration  
Washington 25, D. C.

N65-25009

FACILITY FORM 602

(ACCESSION NUMBER)	(THRU)
<u>33</u>	<u>1</u>
(PAGES)	(CODE)
<u>CR 62898</u>	<u>13</u>
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)



April 1965

Engineering Experiment Station  
GEORGIA INSTITUTE OF TECHNOLOGY  
Atlanta, Georgia

"Requests for additional copies by Agencies of the Department of Defense, their contractors, and other Government agencies should be directed to the:

DEFENSE DOCUMENTATION CENTER (DDC)  
CAMERON STATION  
ALEXANDRIA, VIRGINIA 22314

Department of Defense contractors must be established for DDC services or have their 'need-to-know' certified by the cognizant military agency of their project or contract."

"All other persons and organizations should apply to the:

U.S. DEPARTMENT OF COMMERCE  
OFFICE OF TECHNICAL SERVICES  
WASHINGTON 25, D. C."

**CASE FILE COPY**

\*  
ATMOSPHERIC OSCILLATIONS

by

A. J. Lineberger and H. D. Edwards

Georgia Tech Project A-652-001

Prepared for

National Aeronautics and Space Administration  
Washington 25, D. C.

Contract No. NSG 304-63

April 1965

\*The studies reported here were also supported by the  
Air Force Cambridge Research Laboratories under  
Contract AF19(628)-393.

## ATMOSPHERIC OSCILLATIONS

Aileen J. Lineberger and Howard D. Edwards  
Space Sciences Laboratory  
Georgia Institute of Technology  
Atlanta, Georgia

### ABSTRACT

25009

The development of present theories of atmospheric oscillations is outlined in the following report with special emphasis being given to points of interest to persons studying upper atmospheric motions. The general mathematical attack has been summarized and references to complete derivations have been included. Current research on atmospheric oscillations has been related to studies of atmospheric phenomena conducted in the Georgia Tech Space Sciences Laboratory. Particular attention has been given to the relation of postulated gravity waves to observed wind motion with reference to the following: a downward propagation of gravity wave phase velocity, a phase change in the region of a negative temperature gradient, and the energy flux from the lower atmosphere to the upper atmosphere.

Author

## INTRODUCTION

Evidence of periodic changes in the atmosphere was first obtained from barometric observations made in the 18th century. In 1882 Kelvin was able to demonstrate that the fluctuation of barometric pressure through the day was the sum of Fourier components with 24, 12, and 8 hour periods. He noted that in high latitudes the 12 hour component was larger than the 24 hour component. The reverse of this observation would be expected if the forcing function for the oscillation was solar diurnal heating. Kelvin attempted to explain this effect by a "resonance theory" in which he postulated a free period of the atmosphere close to 12 hours. Wilkes [1949] noted that the maximum of the pressure variation occurred at approximately 10 a.m. and 10 p.m.

The argument for a resonance of the atmosphere was based on the following. If the gravitational forces of the sun and moon dominated the pressure variation, then the lunar force, having almost twice the effective force on the earth, should be the primary cause of oscillation. Consequently one should observe changes in pressure to have a period of 12.5 hours. However, the period of semidiurnal oscillation was found to be much closer to 12 hours than 12.5 hours.\* Thus, it may be concluded that the influence of the sun must be stronger than that of the moon. For this to be the case, the temperature effect must be larger than the gravitational effect. The temperature variation is diurnal, however. Therefore, there must be a strong resonance of approximately 12 hours, such that the 12 hour component of temperature variation would be larger than the 24 hour component.

Prior to Kelvin's investigations, Laplace had worked out, under simplifying assumptions, the equations of oscillation of a homogeneous ocean of uniform depth. He was able to apply his results to tides of a uniform isothermal atmos-

---

\* Chapman [1941] quoted Hough as computing that the free period must be within 2 or 3 minutes of the 12 hours observed.

where, if he made the assumption that the scale height of the atmosphere was the equivalent depth for which the atmosphere would obey the ocean approximation. The scale height,  $H$ , equals  $c^2/\gamma g$ , where  $c$  is the speed of sound,  $\gamma$  is the ratio of the specific heats and  $g$  is the acceleration of gravity. Lamb [1932] later assumed that pressure changes in the atmosphere occurred adiabatically and came to the conclusion that the equivalent depth of the atmosphere was equal to the scale height, which substantiates Laplace's assumption. For the semidiurnal variation to be predominant, i.e. for a 12 hour period, Lamb [1932] computed the equivalent depth to be approximately 26,000 feet.

Later evidence showed a free oscillation period of 10.5 hours which seemed to contradict Lamb's work. The period of 10.5 hours was computed from the time that was required for the waves generated by a point pulse to travel around the earth. The point pulses which were large enough to be observed were the eruption of the volcano Krackatoa in 1883, the Great Siberian Meteor in 1908, and several Soviet megaton nuclear explosions in 1951 to 1962. These pulses were analyzed respectively by Pekeris [1939], Donn and Ewig [1962], and Press and Harkrider [1962].

The early theories of atmospheric oscillations were based on Lamb's work. In 1936 Taylor used the mathematical device of approximating the depth of the earth's atmosphere by its equivalent depth as an ocean. He approximated the temperature as a function of altitude, the velocity and pressure as functions of the altitude and latitude, and the variations of velocity and pressure as functions of  $e^{i(\sigma t + s\phi)}$  where  $2\pi/\sigma$  is the period of oscillation,  $\phi$  is the longitude,  $t$  is the time, and  $s$  is a constant. He then explained the 10.5 hour free period observed in terms of the free period of an ocean of equivalent depth.

The 10.5 and 12 hour periods were explained by Pekeris [1937] by assuming

a layered atmosphere with several equivalent depths. He approximated the temperature by a function of altitude illustrated in Figure 1. The changes in the temperature gradient from negative to zero (points A and D) in the temperature versus altitude curve gave two equivalent depths. Pekeris also found that oscillations traveling upward would experience phase shifts at points A and D. The oscillations would also be amplified due to the decreasing density and pressure by a factor of 100 at 100 km. In a paper in 1939 Pekeris examined the records of pressure fluctuations excited by the Krackatoa eruptions to ascertain if modes of the 12 hour component could be detected. He had computed the ratio of the 10.5 hour component to the 12 hour component to be 5:2. In the barographic records the fluctuations caused by the 12 hour component were too small to be positively identified, but there was no evidence to contradict the existence of a 12 hour component.

The next significant step was made when Weeks and Wilkes [1947] organized the theory developed up to that time and analyzed the energy trapped in a certain region of the atmosphere, between a temperature minimum and the earth. They used a differential analyzer to study the free oscillations for different given temperature distributions. They assumed that most of the energy supplied to the atmosphere enters the lower atmosphere in the more dense regions as gravitational energy. Later Wilkes [1951] extended the mathematical analysis to include solar thermal input.

The energy forcing function (thermal or gravitational) is understood to excite a series of modes of oscillation which depend on latitude and longitude. The energy for each mode is introduced at low altitude and spreads as a spherical wave front in the atmosphere. The motion of the air particle has components both parallel to and perpendicular to the direction of propagation. One will recall that a sound wave is considered to be a compression and rarefaction longitudinal to the direction of propagation.

According to Weeks and Wilkes [1947] the diurnal mode of oscillation is damped out by viscosity at 100 to 300 km. The semidiurnal oscillations, because of their periods, are reflected in the 50-100 km region by the temperature minimum and negative temperature gradient. Modes with periods of the order of those of the semidiurnal modes will be trapped and multiply reflected between the earth and the temperature minima at 30 and 80 km. The multiple reflection allowed pressure oscillations caused by Krackatoa and similar sources to propagate around the earth several times. With each reflection some fraction of the energy was transmitted and might then be observed in the upper atmosphere.

M. L. White [1955, 1956, 1960a] further developed the theory to cover oscillations caused by gravitational forces at low altitudes and thermal input at all altitudes. Recently White [1960b] combined thermally and gravitationally excited oscillations with the ionospheric dynamo effect for an electron and positive ion gas in an imposed static magnetic field.

Recently enough data from radio wave reflection techniques of E-region drift have been collected to imply that the oscillation phase in the altitude region 95-115 km is consistent with the phase observed at the ground. Studies from meteor trails show that a phase reversal exists at 85 km as would be expected in conjunction with the temperature minimum. The temperature variations would affect the phase angle and amplitude. The region of thermal input would alter the rate of change of amplitude and of phase with height.

Superposed on the periodic pattern of oscillations are seemingly random oscillations. The random oscillations may be grouped into acoustic and gravity waves according to their frequency. The acoustic and gravity waves are derived from dynamical equations and are governed by gravitational and compressional forces. These oscillations will be described later mathematically. Depending on frequency, these random modes may be reflected or transmitted at certain

altitudes under the same conditions as the periodic modes. Thus, random as well as periodic oscillations should be observed in the upper atmosphere.

Gossard [1962] observed gravity waves in the troposphere which persisted for 10 to 12 hours. He attempted to show that gravity waves generated in the troposphere might propagate into the upper atmosphere. Gossard [1962] listed three principal mechanisms for generating random internal gravity waves in the troposphere. First, internal gravity waves may be generated as standing waves in the lee of topographic features. Second, internal gravity waves may be produced by the motion of a boundary between two cells of air of different densities inverted with respect to density. In this second case a very regular, sinusoidal gravity wave train may be generated as the wake, if the velocity of the boundary is of the proper magnitude relative to the height and intensity of the inversion and to the slope of the boundary. Third, large tropospheric storms and large scale features associated with stable layers in the lower atmosphere will produce oscillations of long duration.

In some rare instances the gravity waves may be visible in the lower atmosphere as layering in cloud formations. Gossard has photographs of the waves on page 747 of his 1962 article. Hines also mentions that noctilucent clouds occasionally reveal the gravity wave pattern. The noctilucent clouds occasionally form in long parallel bands 9 km apart.

One, then, should observe continuous periodic motion in the upper atmosphere from the transmitted diurnal and semidiurnal modes as well as random oscillations. The wavelength of the random modes of oscillation should be roughly the same magnitude as the periodic modes of oscillation, since reflection by the thermal gradient and dissipation by viscous, eddy, and kinematic effects remove all but certain wavelengths at high altitudes.

Essentially all of the energy of the atmosphere comes from radiation or gravitational forces with the motion being caused by the conversion of this energy to kinetic energy. There is little generation of entropy. The net heating of the atmosphere is due to the difference between solar radiation absorbed and infrared radiation emitted by the atmosphere. The next section describes the governing equations for this motion which is found to be oscillatory in many considerations. The oscillatory motion is broken into internal gravity wave motion and acoustic wave motion by most authors.

## DYNAMICS OF THE ATMOSPHERE

Equations describing oscillations of the atmosphere were first obtained by adapting hydrodynamic equations of nonviscous, compressible fluids, i.e. gases. Laplace performed the first major work in this area by relating tides of an ocean to an atmosphere of an equivalent depth.

Lamb's book, Hydrodynamics [1932] is a classic in this field and is the basis for the theoretical work of Taylor, Pekeris, Wilkes, and others. Lamb related hydrodynamic equations to atmospheric tidal oscillations for a number of special cases. He made the justifiable approximation that, for changes in the atmosphere as a whole, viscosity and nonadiabatic losses may be neglected. Only in a highly turbulent region is this approximation poor. This approximation is used in all of the work considered unless stated otherwise.

The mathematical manipulations were carried out in either rectangular or spherical coordinates. For a viscous, compressible fluid undergoing changes adiabatically, one may obtain the equations governing the motion of the atmosphere from the following three equations. The equations are 1, the equation of motion; 2, the equation of continuity, and 3, the equation of adiabatic state.

$$\frac{\partial \underline{v}}{\partial t} = - \frac{1}{\rho} \nabla p - 2 \underline{\omega} \times \underline{v} + \frac{1}{\rho} \text{Fr} + g \quad (1)$$

$$\nabla (\rho \underline{v}) = \underline{v} \cdot \nabla \rho + \rho \nabla \cdot \underline{v} = - \frac{\partial \rho}{\partial t} \quad (2)$$

$$\frac{\partial p}{\partial t} + \underline{v} \cdot \nabla p = c^2 \left( \frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho \right) \quad (3)$$

In the above equations  $\underline{v} = u\underline{i} + v\underline{j} + w\underline{k}$ , is the velocity,  $\rho$  is the density,  $p$  is the pressure,  $c$  is the speed of sound,  $\omega$  is the angular velocity of the

earth, and  $F_r$  is the frictional force. Also,  $g$  is the "observed gravitational force" which is the sum of  $-\phi$  and  $\omega^2 R$ , the gravitational potential and the centrifugal force.

To solve the above set of equations one commonly uses a perturbation analysis and linearizes the resulting equations. One considers the stationary values,  $\rho_0$ ,  $p_0$ ,  $T_0$ , and  $\underline{v}_0$  for density, pressure, temperature, and velocity, and lets  $\rho'$ ,  $p'$ ,  $T'$ , and  $\underline{v}'$  be the perturbation of these quantities. The linear approximation is fairly good below 100 km where fluctuations in the density are less than 10 per cent. According to Hines [1960] the density may fluctuate as much as 20 per cent above 100 km. In the regions above 100 km the linear approximation is not as good.

If one replaces  $\rho$ ,  $p$ , and  $\underline{v}$ , in equations 1, 2, and 3, by  $\rho_0 + \rho'$ ,  $p_0 + p'$ , and  $\underline{v}_0 + \underline{v}'$  and simplifies one obtains the following:

$$\rho_0 \frac{\partial u}{\partial t} = - \frac{\partial p'}{\partial x} - 2 \omega [(\cos \alpha)w' - (\sin \alpha)v'] \quad (4a)$$

$$\rho_0 \frac{\partial v}{\partial t} = - \frac{\partial p'}{\partial y} - 2 \omega (\sin \alpha)u' \quad (4b)$$

$$\rho_0 \frac{\partial w}{\partial t} = - \frac{\partial p'}{\partial z} + g p' - 2 \omega (-\cos \alpha)u' \quad (4c)$$

$$\frac{\partial p'}{\partial t} = - \rho_0 \nabla \cdot \underline{v} - \underline{v}' \cdot \nabla \rho \quad (5)$$

$$\frac{\partial p'}{\partial t} + \underline{v} \cdot \nabla p_0 = c^2 \left[ \frac{\partial p'}{\partial t} + \underline{v} \cdot \nabla \rho_0 \right] \quad (6)$$

where  $\alpha$  is the latitude, and  $\underline{v}_0$  is set equal to zero.

One may now solve the equations as they are written above, as Wilkes [1949] outlines. An alternative is to further simplify the equations by making approxi-

mations on both the equations and the model of the atmosphere described. The simplified equations will be discussed first; then the more general approach will be described.

A model frequently used is that of a flat, nonrotating earth. The temperature is assumed either to be constant, to increase or decrease monotonically with altitude, or to be stratified. Gravity is usually considered to be constant. Density and pressure are usually considered to vary exponentially with altitude.

The most one can profitably simplify the problem is to consider an isothermal atmosphere, plane level surfaces, and a nonrotating earth. This case has been handled by Eckart [1960], Lamb [1932], and Hines [1960]. The simplification is not valid for small effects, but general, large effects may be described and discussed. Hines tried with apparent success to relate his results to effects observed experimentally. Eckart went over nearly the same derivation as Hines but included more detail. However, Hines used notation that is more physically meaningful. Both used linearized equations for small perturbations on a stationary system. Eckart used entropy concepts, while Hines used the approximation of an adiabatic state. Both found a high and low set of allowed frequencies separated by a region of forbidden frequencies. Waves with frequencies below the forbidden region were called gravity waves and waves with frequencies above the forbidden region were called acoustic waves.

In particular Hines assumed wave solutions for density, pressure, vertical, and horizontal velocity to be of the form  $C_j \exp i (\omega t - K_x X - K_z Z)$ . He substituted this into the equations 4, 5, and 6, neglected  $\omega \times y$  terms, and obtained a dispersion relation

$$\omega^4 - \omega_c^2 (K_x^2 + K_z^2) + (\gamma - 1) g^2 K_x^2 + i \gamma g \omega^2 K_z = 0$$

where  $\omega$  is the frequency of oscillation,  $\gamma$  is the ratio of specific heats, and

$K_x$  and  $K_z$  are wave numbers given by  $2\pi$  times inverse wave lengths. To interpret the dispersion relation Hines assumed that  $K_x$  is real  $= k_x$  and, therefore,  $K_z$  is purely imaginary or is  $= k_z + \frac{i\gamma g}{2c}$  where  $k_z$  is real. Hines chose the second alternative to allow for vertical phase propagation. As a result of this assumption Hines was able to interpret the phase change in the oscillations of the upper atmosphere as gravity waves. He noted that in the absence of gravity the dispersion relation becomes  $\omega^2 = (k_x^2 + K_z^2)c^2$  which is the familiar equation for sound propagation. Then for simple sound waves  $K_x$  and  $K_z$  would be real.

When Hines solved the dispersion relation under the condition  $K_z = k_z + k/2H$  he found that  $4/\omega^4$  has two positive roots, and is double valued for real wave number pairs  $(k_x, k_z)$ . He designated the two choices of  $\omega$  as corresponding to acoustic or gravity waves.\* The frequencies for acoustic waves are greater than  $\omega_a = \gamma g/2c$  and the frequencies for internal gravity waves are less than  $\omega_g = (\gamma - 1)^{1/2}g/c$ . Since  $\gamma < 2$  then  $\omega_a > \omega_g$ . There is a gap of forbidden frequencies  $\omega_i$  such that  $\omega_g < \omega_i < \omega_a$ . Recently Pitteway and Hines [1963] extended their model to include viscous damping of atmospheric gravity waves.

Eckart [1960] went through a second derivation in which the effect of the earth's rotation was included. The other conditions are the same as the first case discussed. He again found that certain frequencies are not allowed and the acoustic and gravity waves are similar to the ones already described. Figures 2 and 3 show these allowed frequencies versus wave number in the cases of a non-rotating and a rotating earth. The unshaded area represents an imaginary propagation surface.

\*

There exists some ambiguity in the use of the term gravity wave for various media, i.e. liquids or gases. A surface gravity wave must be distinguished from an internal gravity wave with which we are concerned. Also, different terms may appear in the equations of motion of gravity waves depending upon the assumptions made and the media described. It appears to be usual, however, to call the set of waves with lower frequency, of the two sets of allowed frequencies, gravity waves.

The gravity wave propagates energy upward in modes whose phase progression is downward, while acoustic wave energy propagates in nearly the same direction as the phase.

Acoustic and gravity waves are governed by compressional and gravitational forces; the rotational force modifies but does not change the type of wave which is found. Eckart described one important difference between acoustic and gravity waves. On page 120, Eckart [1960] discussed the idea that gravity waves with short wave lengths "have one outstanding characteristic which distinguishes them from sound waves.\* In the latter, the ratio of particle velocity to pressure amplitude is very small--on the order of magnitude of  $1/\rho c$ . In the gravity waves this ratio becomes much larger and approaches infinity for short wave lengths. This is also a characteristic of the fluctuations in wind velocity that occur without marked pressure fluctuations. One may therefore make a tentative identification of the gravity waves with the fluctuating component of the wind."

A more general approach uses spherical coordinates. Wilkes [1949] outlined the basic mathematical equations in his book, and ramifications were developed in papers by Wilkes [1951] and by White [1955, 1956, 1960a, 1960b]. The basic equations of motion are taken to be, in the linearized, perturbation form,

$$\frac{\partial u}{\partial t} - 2 \omega v \cos \theta = - \frac{1}{a} \frac{\partial}{\partial \theta} \left( \frac{p'}{\rho_0} + \Omega \right)$$

$$\frac{\partial v}{\partial t} + 2 \omega u \cos \theta = - \frac{1}{a \sin \theta} \frac{\partial}{\partial \phi} \left( \frac{p'}{\rho_0} + \Omega \right)$$

$$\frac{\partial p}{\partial z} = - g p' - \rho_0 \frac{\partial \Omega}{\partial z}$$

---

\* Eckart used the term sound wave in the sense in which this paper uses acoustic wave.

where  $a$  is the radius of the earth,  $\omega$  is the angular velocity of the earth,  $\theta$  is the latitude,  $\phi$  is the longitude,  $z$  is the height above the earth's surface,  $u$  is the southward component of air velocity at  $(z, \theta, \phi)$ ,  $v$  is the eastward component,  $w$  is the vertically upward component,  $c$  is the velocity of sound at height  $z$ , and  $\Omega$  is the tide producing potential, gravitational in origin. In the above the earth is considered to be spherical, and the variation of radius vector, gravity, and  $\frac{\partial \Omega}{\partial z}$  with height are neglected. Also, the vertical acceleration is considered to be negligible. Temperature, density, and pressure are functions of the altitude. The equation of continuity becomes

$$\frac{\partial \rho}{\partial t} + w \frac{\partial \rho_0}{\partial z} + \rho_0 \operatorname{div} \underline{V} = 0$$

The adiabatic gas law is

$$\frac{dp}{dt} - c^2 \frac{d\rho}{dt} = 0$$

if the thermal forcing function is ignored. If one considers a thermal forcing function,  $Q$ , one must use

$$\frac{dp}{dt} = + c^2 \frac{d\rho}{dt} + g \rho Q$$

where  $Q = Q(z, \theta, \phi)$ . Note that in this analysis the forcing functions are considered, while Hines simply looked for allowed motions under certain conditions. Wilkes considered only the gravitational forcing function; Sen and White [1955] considered thermal and gravitational forcing functions acting at ground level. White [1956] extended the theory to include a ground level gravitational forcing function and a thermal forcing function which varies with altitude.

Solutions to these differential equations are worked out in the papers referenced.

The NS wind velocity was found by White [1956] to be

$$u = \frac{i\sigma g}{4a\omega_e} \frac{pH}{p_o} \frac{\left( \frac{d}{d\theta} + \frac{s \cot \theta}{f} \right) \Theta_r^s(\theta)}{\left( f^2 - \cos^2 \theta \right) \Theta_r^s(\theta)}$$

When the ratio of NS to EW velocity is formed the dependence on altitude cancels.

The ratio of the NS to EW velocity components is

$$\frac{u}{v} = \frac{\left( \frac{d}{d\theta} + \frac{s \cot \theta}{f} \right) \Theta_r^s(\theta) e^{i\sigma t}}{i \left( \frac{\cos \theta}{f} \frac{d}{d\theta} + \frac{s}{\sin \theta} \right) \Theta_r^s(\theta) e^{i\sigma t}}$$

where  $\sigma = 2\pi / \text{period of oscillation}$ ,  $s$  is a constant,  $r$  is the component of oscillation considered,  $\omega_e$  = angular velocity of earth,  $f = \sigma/2 \omega_e$ , and  $\theta$  = co-latitude. Also,  $\Theta_2^2 = P_2^2(\theta) - B P_{1/4}^2(\theta)$  for the solar semidiurnal oscillation.  $P_1^m$  is the associated Legendre function and  $B$  is a constant determined empirically from experimental data. These equations may be used to make approximations to wind motions.

Pekeris solved the governing equations for the case of a purely gravitational forcing function. In his solutions he derived an expression for the pressure. For characteristic values of the period,  $\frac{2\pi}{\sigma}$ , he showed that the amplitude becomes infinite, and a free period, or resonance occurs.

Lower boundary conditions are usually set by specifying that the vertical velocity must be zero at the earth's surface. To set the upper boundary condition

it is usual to consider the rate of flow of energy in a column of air of constant cross section. One considers the horizontal energy flow to be constant and assumes the energy to decrease vertically, going to zero at infinity. Since energy is assumed to enter at the low altitudes it is apparent that at some high altitude the energy must be flowing outward only, which justifies the assumption that the energy will go to zero at infinity. Wilkes [1949] on page 49 of his book obtained the refractive index for atmospheric waves by making an analogy to electromagnetic waves. He found the refractive index  $\mu$  to be given by

$$\mu^2 = -\frac{1}{4} + \frac{1}{h} \left( \frac{dH}{dX} + \frac{\gamma - 1}{\gamma} H \right)$$

If  $\mu^2$  is negative at certain altitudes some of the energy will be transmitted and some will be reflected. Low temperatures and negative temperature gradients may cause  $\mu^2$  to become negative. For various values of  $h$ , which is a function of the mode of oscillation and arises as a separation constant in the differential equation, some waves will be reflected and some transmitted.

## CORRELATION WITH EXPERIMENTAL OBSERVATION

Several authors have conducted theoretical studies which can be correlated with experimental observations carried out in our laboratory. Motions characteristic of gravity waves were evident in our data.

Several analyses will be discussed, but the one described by Hines [1960] was the most successful in relating experimental observations to a model. Gossard [1954, 1962] related the energy flux from the troposphere into the upper atmosphere to gravity waves. White [1960b] expanded the theory to cover the dynamo effect and has graphically related the theory of semidiurnal tidal components to experimental observation.

Hines listed six observed properties which he correlated with a simplified model of the atmosphere. The points were (1) wide variations in the wind component with altitude, (2) persistence of a wind pattern for time intervals as large as 100 minutes, (3) a ratio of horizontal scale size to vertical scale size of 20 to 1, (4) dominant horizontal motions and negligible vertical wind accelerations, (5) increasing speed of dominant irregular winds with altitude, (6) smallest vertical structure size increasing with altitude. Hines obtained these properties from experimental observations made before 1959. These properties are consistent with our data, and it is then reasonable to assume that his model will hold for the winds observed by this laboratory.

Upon analyzing a dispersion relation, Hines finds that there exist two sets of allowed frequencies, gravity waves and acoustic waves. A characteristic of the gravity waves is that while energy is carried upward the phase propagates downward with time.

This laboratory has attempted to demonstrate the existence of gravity waves in the following manner. Our observations show that the wind vector viewed from above performs clockwise rotation with increasing altitude at a given time, and

performs clockwise rotation with time at a given altitude. More than 75% of the wind data show anticyclonic motion between 100 and 115 km, and over 90% of the wind data show anticyclonic motion between 110 and 112 km. One may relate the two observed rotations of the wind vector by assuming that gravity and tidal waves were propagating upward with an attendant downward propagation of phase in the region under observation.

Under the above assumptions the phase velocity was computed for two sets of sodium release data obtained from rocket flights over Eglin Air Force Base, Florida. For the first set released on 3 December 1962 at 17:20, 18:01, 21:45, and 22:45 CST, the rate of rotation of the wind vector at a given altitude as a function of time, and at a given time as a function of altitude was computed. Averaged between 98 and 113 km the wind vector was found to rotate  $15^{\circ}$  per km change in altitude and  $0.4^{\circ}$  per minute at a given altitude. Over this altitude range the wind vectors consistently moved clockwise with increasing altitude and with increasing time. Upon dividing one obtains a vertical phase velocity of 0.03 km/min or approximately 0.44 m/s. A similar analysis was performed on the four releases on 17 May 1963 at 19:06 and 22:19 CST and on 18 May 1963 at 02:56 and 04:06 CST. Averaged between 106 and 113 km the wind vector was found to rotate  $10^{\circ}$  per km and  $0.52^{\circ}$  per minute.

The phase velocity in this case was 0.8 m/s. One may assume that the wind pattern is descending at the above rates and compare the wind component curves for the two sets of four wind determinations. In Figures 4 and 5, each of the wind curves has been shifted up along the z axis a distance corresponding to its computed descent in the elapsed time between wind measurements. As one observes there is definitely a correlation in the two sets of four wind patterns. In Figure 4 the total descent of the wind pattern between the 19:06 wind determination and the 04:06 wind determination was 27 km. In Figure 5 the total descent of the

wind pattern between 17:20 and 22:45 was 8.5 km.

A downward shift of the wind pattern has been discussed in the paper by Rosenberg and Edwards [1964]. A study of time and spatial variations of winds was recently made by Rosenberg, Edwards, and Justus [in preparation]. The single sodium trail release on 17 May 1963 at 19:06 CST was observed to exhibit the same rotation previously discussed and to reveal a wind pattern with a downward motion of 1.3 m/s over an observed period of approximately 15 minutes. The downward velocity of this single release of the 17 May 1963 series is larger than the average phase velocity computed for all four releases. The phase velocities observed seem to vary over a fairly narrow range for the winds observed thus far. The variation might be explained as the changing superposition of a number of gravity waves.

Gossard [1962] observes that fluctuations of pressure due to random gravity waves in the lower atmosphere are seen to persist as long as 10 to 12 hours. Since random oscillations are superposed on diurnal and semidiurnal wind motion, it would seem that one should observe better correlation between wind patterns measured at closely spaced intervals than widely spaced intervals but there should still be observable correlation throughout the day. The determination of the rotation of the wind vector with time at a given altitude for the two sets of four rocket releases averages to approximately  $0.5^{\circ}/\text{min}$  or approximately two revolutions per day. Apparently this rotation is predominantly a semidiurnal effect.

The wind motion is considered to be the sum of a general drift, a periodic oscillation and a random component. No effort has been made yet to separate these motions in connection with the phase velocity computed here.

One may make a comparison between the energy which would be carried by the gravity waves from the troposphere to the ionosphere and the energy dissipated by turbulence in the ionosphere. Gossard [1962] considers a negligibly viscous

atmosphere and neglects energy reflected by thermal barriers. He notes that the larger waves become nonlinear above certain altitudes and deposit some of their energy in the turbulence spectrum. For several different observations he computes the total energy density of the gravity wave to range between  $0.73 \text{ ergs/cm}^3$  and  $3.2 \text{ ergs/cm}^3$ . On days of high gravity wave activity the maximum energy flux is on the order of several hundred  $\text{ergs/cm}^2 \text{ sec}$ . If one takes the energy density of the gravity wave to be approximately  $1 \text{ erg/cm}^3$  and the energy flux to be approximately  $100 \text{ ergs/cm}^2 \text{ sec}$ , then 100 ergs must be carried through a cubic centimeter in one second. This implies that the velocity of the energy being transported is  $100 \text{ cm/s}$ .

Justus and Edwards [NASA Technical Note in Press] have shown that at 100 km the energy dissipation is approximately  $0.1 \text{ j/kg sec}$ . From this value one may compute the energy dissipation per unit volume per unit time to be  $4.97 \times 10^{-7} \text{ ergs/cm}^3 \text{ sec}$ . The energy flux which Gossard shows may leave the troposphere is seen to be much larger than the dissipation due to turbulence in the ionosphere. Turbulent dissipation is low between the troposphere and the ionosphere. As Gossard mentions, energy will be lost due to reflection and turbulence. The amount of energy dissipated by turbulence decreases from the ionosphere to the upper troposphere according to the limited data available in the study by Justus and Edwards. Energy dissipation increases quite rapidly, however, in the region above 100 km. Reflection will probably be the primary mechanism which keeps energy from the troposphere from reaching the ionosphere.

Gossard [1962] also computed the amplification of the gravity waves which reach the ionosphere. The vertical wave lengths and wind velocity perturbations which Gossard computed are the same magnitude as the wave lengths which we observed.

Another method for determining phase velocities is suggested in a paper by Axford [1963] in which the Dungey process for the formation of sporadic E is de-

scribed. This process is simply that the component of the electrically neutral wind parallel to the magnetic field drives the free ions and electrons along field lines and the perpendicular component distorts the field slightly. Thus, if the wind profile is sinusoidal along the vertical axis, the free ions and electrons will be forced to the point where the velocity is zero until their partial pressure gradient balances the force exerted by the neutral particles. Then, if the phase velocity is downward, and the points of zero velocity move downward with time, the ionization will tend to move down with the zero points. Then, layers of sporadic E separated by half the wave length of the gravity wave should move down with a velocity equal to the phase velocity of the gravity wave. A layering of sporadic E has been observed. At the present no systematic review of the literature has been undertaken to clarify the motion of the layers.

Axford [1963] presents a table which includes the following "typical" values.

Altitude	km	145	120	100
Horizontal velocity	m/s	50	50	30
Phase velocity	m/s	1	1	0.5
Wave length	km	12	10	5

The magnitude of his "typical" phase velocity is very close to the phase velocity at the corresponding altitudes and horizontal velocities.

## CONCLUSIONS

The studies presented here indicate that winds observed in the upper atmosphere may be composed of diurnal and semidiurnal motions upon which are superposed random gravity waves. If the interpretation given in this paper relative to phase velocity is correct, then the propagation of the phase downward and energy upward might be related by means of the observed rotation of the wind vector and used to demonstrate the existence of gravity waves. Further study may relate the rate of change of the phase of the wind vector to temperature gradients. In addition, one might be able to relate the energy dissipated in turbulence in the ionosphere to the energy flux which is generated from the lower atmosphere and carried to the upper atmosphere. Characteristics of the motion of sporadic E may also be related to the gravity waves.

#### ACKNOWLEDGMENTS

We are indebted to Professor C. O. Hines of the University of Chicago for reviewing the draft and for offering many helpful suggestions. Much credit is due our colleague, C. G. Justus for discussions during the study.

Financial support for the work has been supplied by the National Aeronautics and Space Administration under Grant NsG-304-63 and by the Air Force Cambridge Research Laboratories under Contract AF 19(628)-393.

# REFERENCES

1. Anderson, D. L., "Surface Waves on a Spherical Earth," J. Geophys. Res., 68, 3483 (1963)
2. Axford, W. I., "Wind Shear and Sporadic E," J. Geophys. Res., 68, 769 (1963)
3. Birkhoff, G., Hydrodynamics, Princeton University Press, University of Cincinnati (1950)
4. Chapman, S., Compendium of Meteorology, Am. Met. Soc., p. 510 (1951)
5. Charney, J. G., Drazin, J. Geophys. Res., 66, 83 (1961)
6. Donn, W. L. and M. Ewig, J. Geophys. Res., 67, 1855 (1962)
7. Eckart, Carl, Hydrodynamics of Oceans and Atmospheres, Pergamon Press, New York (1960)
8. Gossard, E. E. and W. Munk, J. Meteorology, 11, 259 (1954)
9. Gossard, E. E., J. Geophys. Res., 67, 745 (1962)
10. Greenhow, J. S. and E. L. Neufeld, J. Geophys. Res., 64, 2129 (1959)
11. Hines, C. O., "Motions in the Ionosphere," Proc. I. R. E., 47, 176 (1959)
12. Hines, C. O., Can. J. Phys., 38, 1441 (1960)
13. Hines, C. O., Quart. J. Roy. Met. Soc., 89, 1 (1963)
14. Justus, C. G. and H. D. Edwards, "Turbulence in the Upper Atmosphere," NASA Technical Note, in Press.
15. Kampe, H. J., J. Geophys. Res., 67, 4243 (1962)
16. Kochanski, A., J. Geophys. Res., 68, 213 (1963)
17. Kuethe, A. M. and J. D. Jchetzter, Foundations of Aerodynamics, John Wiley and Sons, Inc., New York (1959)
18. Lamb, Sir Horace, Hydrodynamics, Cambridge of the University Press, 6th Edition (1932)
19. Mack, L. R., J. Geophys. Res., 67, 829 (1962)
20. Martyn, D. F., "Atmospheric Tides in the Ionosphere," Proc. Roy. Soc. London, 194, 429 (1948)
21. Martyn, D. F., Proc. Roy. Soc. London, 194, 445 (1948)
22. Martyn, D. F., Proc. Roy. Soc. London, 201, 216 (1950)

23. Milne-Thomson, L. M., Theoretical Hydrodynamics, MacMillan Company, New York (1955)
24. Nicholson, J. R., and W. R. Steigler, J. Geophys. Res., 68, 3577 (1963)
25. Pekeris, C. L., "Atmospheric Oscillations," Proc. Roy. Soc. London, 157, 650, (1937)
26. Pekeris, C. L., "The Propagation of a Pulse in the Atmosphere," Proc. Roy. Soc. London, 171, 534 (1939)
27. Pekeris, C. L., "The Propagation of a Pulse in the Atmosphere," Phys. Rev., 73, 145 (1948)
28. Petterssen, Sverre, Weather Analysis and Forecasting, Vol. I, McGraw-Hill Book Co., Inc., New York (1956)
29. Phillips, O. M., J. Geophys. Res., 66, 2889 (1961)
30. Pitteway, M. L. V., and C. O. Hines, Can. J. Phys., 41, 1935 (1963)
31. Press, F., and D. Harkrider, J. Geophys. Res., 67, 3889 (1962)
32. "Proceedings of the International Symposium on Fluid Mechanics in the Ionosphere," J. Geophys. Res., 64, (1959)
33. Ratcliffe, J. A., Physics of the Upper Atmosphere, Academic Press, New York (1960)
34. Rosenberg, N. W. and H. D. Edwards, J. Geophys. Res., 69, 2819 (1964)
35. Rosenberg, N. W. and H. D. Edwards and C. G. Justus, "Time and Spatial Variations," in preparation
36. Sen, H. K., M. L. White, Thermal and Gravitational Excitation of Atmospheric Oscillations," J. Geophys. Res., 60, 483 (1955)
37. Small, K. A. and S. T. Butler, J. Geophys. Res., 66, (1961)
38. Stephens, R. W. B. and A. E. Bate, Wave Motion and Sound, Edward Arnold and Co., London (1950)
39. Stoker, J. J., Water Waves, Interscience Pub., Inc., New York (1957)
40. Taylor, G. I., "The Oscillations of the Atmosphere," Proc. Roy. Soc. London, 156, 318 (1936)
41. Titridge, J. E., J. Geophys. Res., 68, 3399 (1963)
42. U. S. Standard Atmospheres, 1962, U. S. Printing Office, Washington, D. C.
43. Von Arx, William S., Introduction to Physical Oceanography, Addison Wesley Co., Inc., Reading, Mass. 1962

44. Weeks, K. and M. V. Wilkes, "Atmospheric Oscillations and Resonance Theory," Proc. Roy. Soc. London, 192, 80 (1947)
45. Weston, V. H., Can. J. Phys., 40, 446 (1962)
46. Wexler, H., J. Geophys. Res., 67, 3875 (1962)
47. White, M. L., J. Geophys. Res., 61, 489 (1956)
48. White, M. L., J. Atmos. and Terrest. Phys., 17, 220 (1960a)
49. White, M. L., J. Geophys. Res., 65, 153 (1960b)
50. Wilkes, M. V., Oscillations of the Earth's Atmosphere, Cambridge University Press (1949)
51. Wilkes, M. V., Proc. Roy. Soc. London, 207, 358 (1951)
52. Woodbridge, D. D., "Ionospheric Winds," J. Geophys. Res., 67, 4221 (1962)

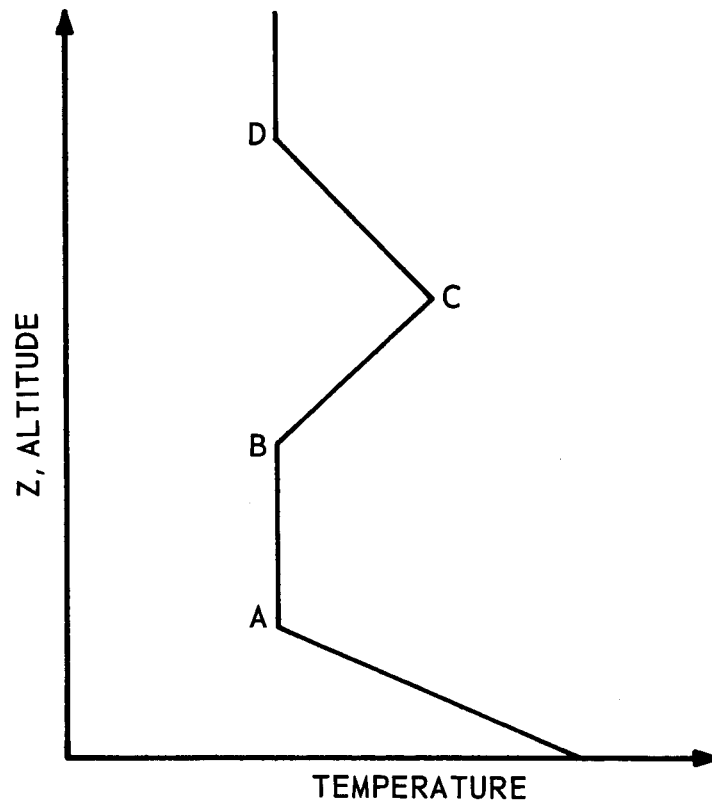


Figure 1. The Assumed Temperature Variation as a Function of Altitude.

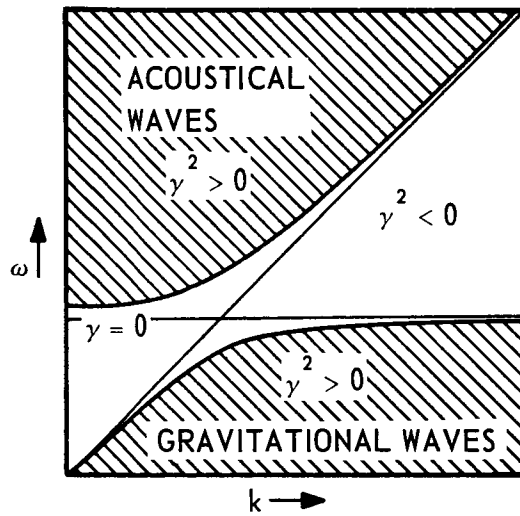


Figure 2. Wave Number,  $k$ , as a Function of Frequency,  $\omega$ , for an Isothermal Atmosphere.

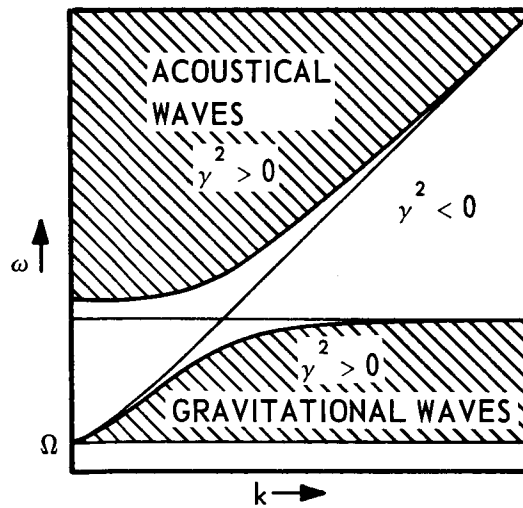


Figure 3. Wave Number,  $k$ , as a Function of Frequency,  $\omega$ , for an Isothermal Atmosphere Rotating About a Vertical Axis with Angular Velocity  $\Omega$ .

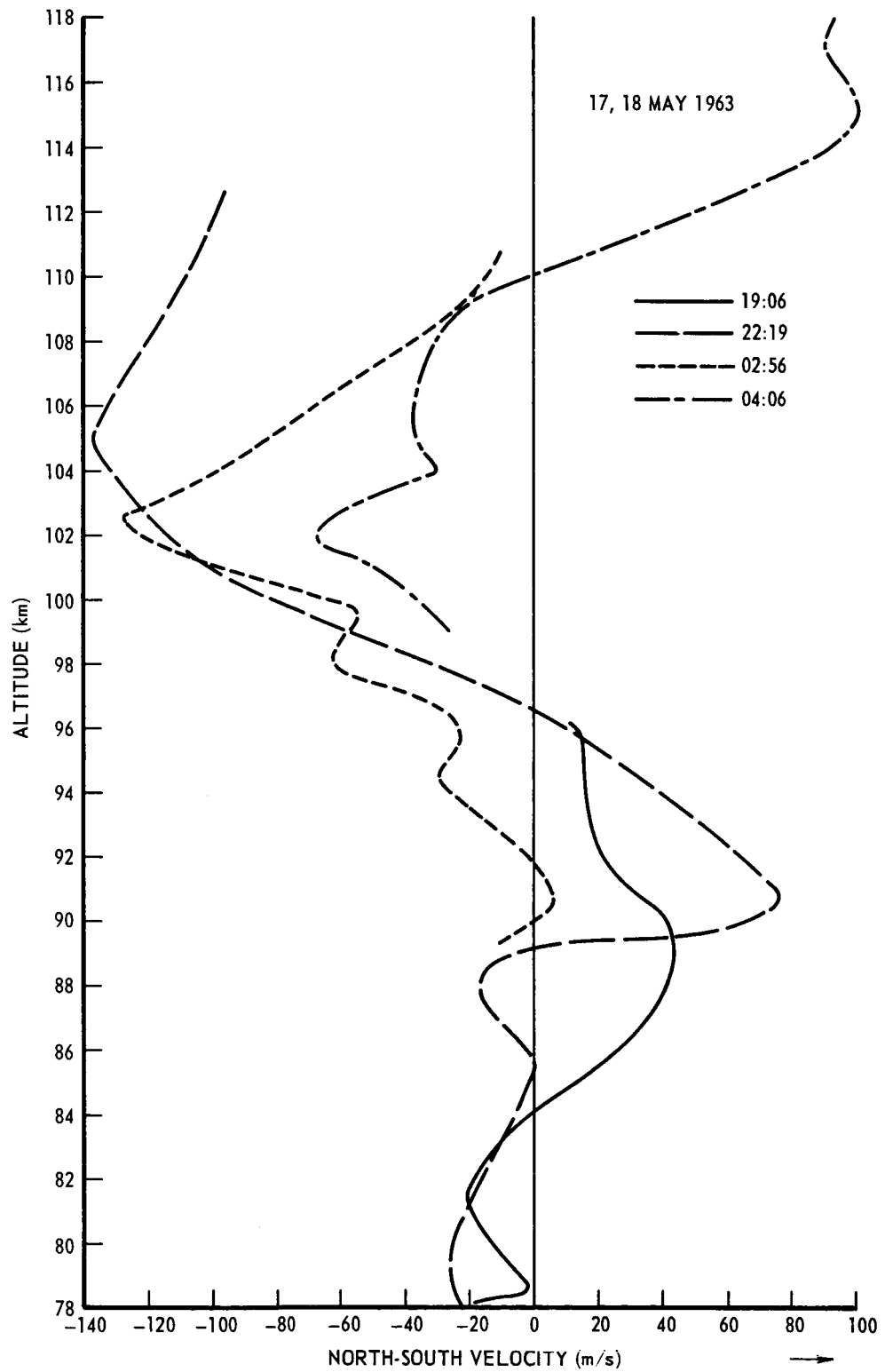


Figure 4a. The North-South Components of Wind Velocity Shifted Along the Ordinate Relative to the 04:06 Release. North is Taken as Positive.

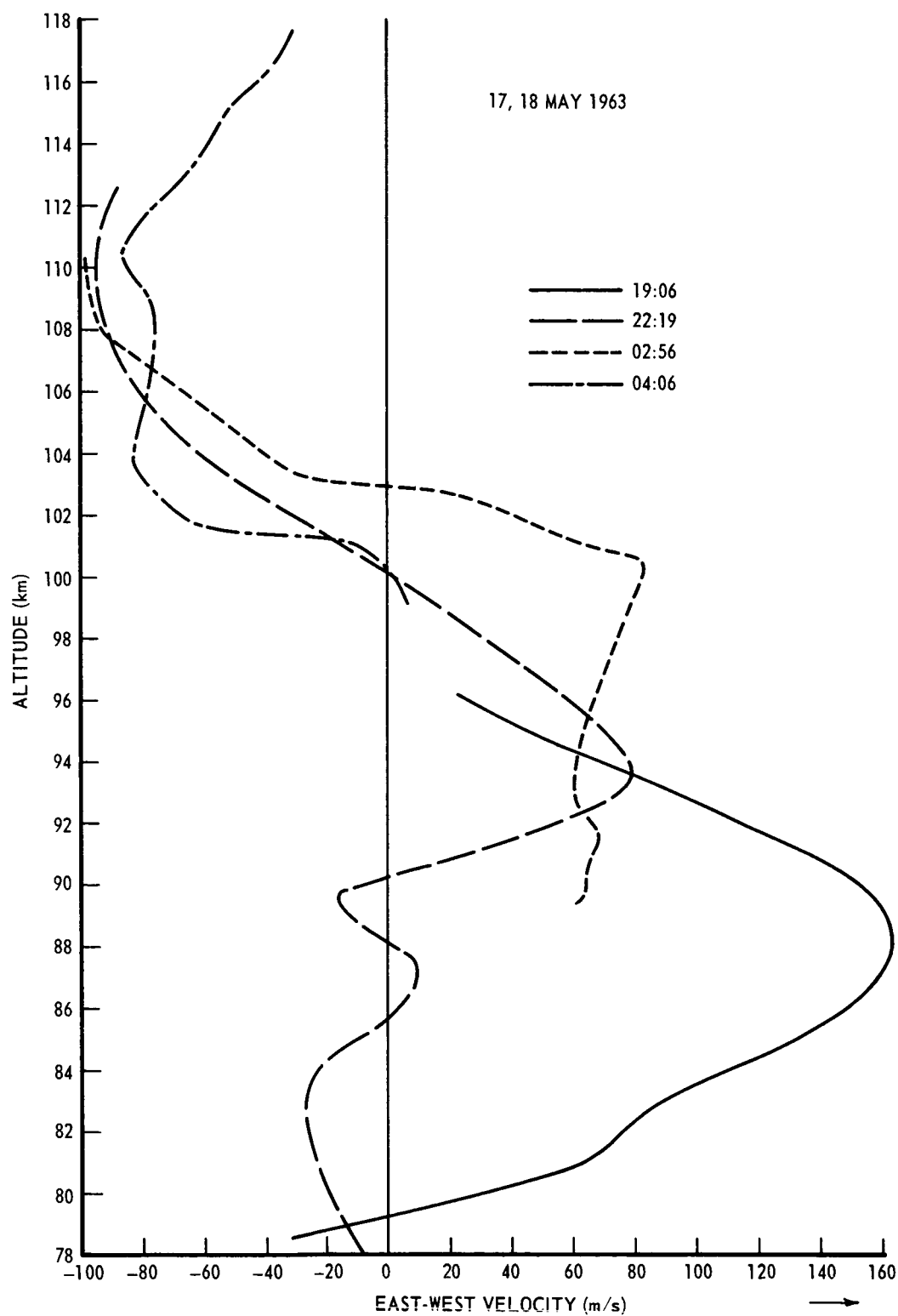


Figure 4b. The East-West Components of Wind Velocity Shifted Along the Ordinate Relative to the 04:06 Release. East is Taken as Positive.

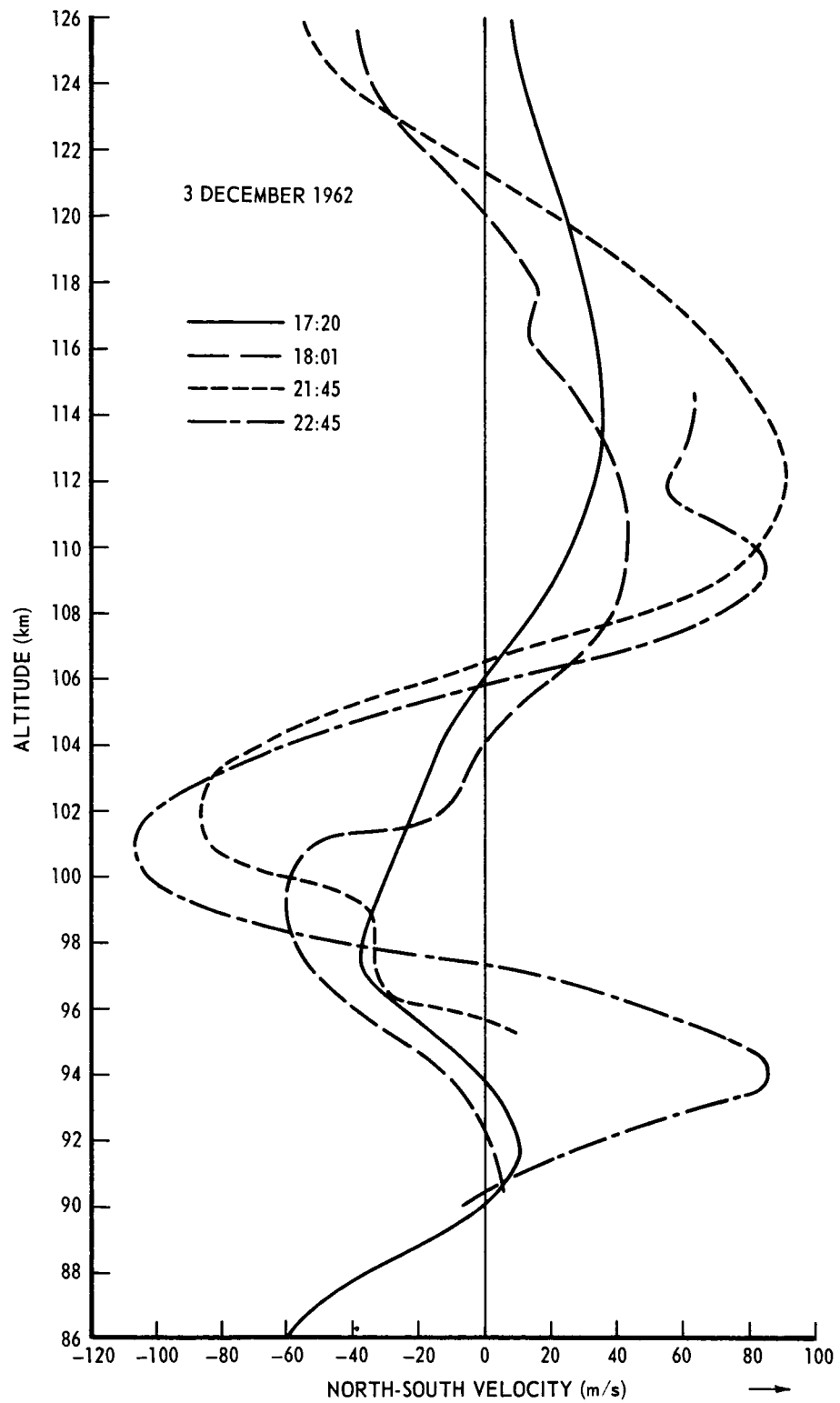


Figure 5a. The North-South Components of Wind Velocity Shifted Along the Ordinate Relative to the 22:45 Release. North is Taken as Positive.

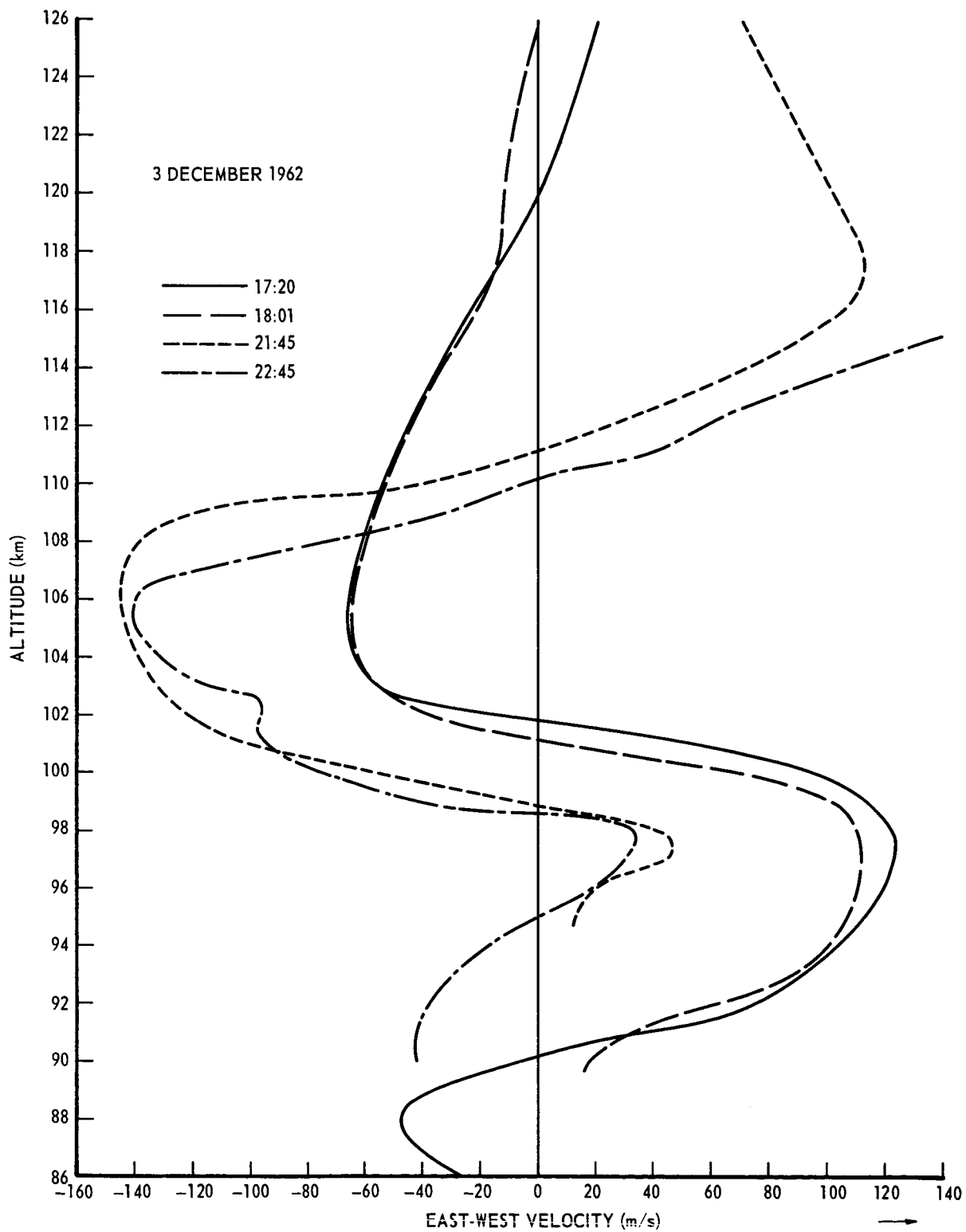


Figure 5b. The East-West Component of Wind Velocity Shifted Along the Ordinate Relative to the 22:45 Release. East is Taken as Positive.