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Derivation and Definition of a Linear Aircraft Model

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SUMMARY

This report documents the derivation and definition of a linear aircraft model for a rigid aircraft of constant mass flying over a flat, nonrotating earth. The derivation makes no assumptions of reference trajectory or vehicle symmetry. The linear system equations are derived and evaluated along a general trajectory and include both aircraft dynamics and observation variables.

INTRODUCTION

The need for linear models of aircraft for the analysis of vehicle dynamics and control law design is well known. These models are widely used, not only for computer applications but also for quick approximations and desk calculations. Whereas the use of these models is well understood and well documented, their derivation is not. The lack of documentation and, occasionally, understanding of the derivation of linear models is a hindrance to communication, training, and application.

This report details the development of the linear model of a rigid aircraft of constant mass, flying over a flat, nonrotating earth. This model consists of a state equation and an observation (or measurement) equation. The system equations have been broadly formulated to accommodate a wide variety of applications. The linear state equation is derived from the nonlinear six-degree-of-freedom equations of motion. The linear observation equation is derived from a collection of nonlinear equations representing state variables, time derivatives of state variables, control inputs, and flightpath, air data, and other parameters. The linear model is developed about a nominal trajectory that is general.

Whereas it is common to assume symmetric aerodynamics and mass distribution, or a straight and level trajectory, or both (Clancy, 1975; Dommasch and others, 1967; Etkin, 1972; McRuer and others, 1973; Northrop Aircraft, 1952; Thelander, 1965), these assumptions limit the generality of the linear model. The principal contribution of this report is a solution of the general problem of deriving a linear model of a rigid aircraft without making these simplifying assumptions. By defining the initial conditions (of the nominal trajectory) for straight and level flight and setting the asymmetric aerodynamic and inertia terms to zero, one can easily obtain the more traditional linear models from the linear model derived in this report.

Another significant contribution of this report is the derivation and definition of a linear observation (measurement) model. The observation model is often entirely neglected in standard texts. A thorough treatment of common aircraft measurements is presented by Gainer and Hoffman (1972), and Gracey (1980) provides a detailed discussion of speed and altitude measurements. However, neither of these references present linear models of these measurements. This report relies heavily on these two references and uses their results as one of the bases for the nonlinear measurement equations from which the linear measurement model is derived. Also included in this report is a large number of other measurements or variables for observation that have been found to be useful in vehicle analysis and control law design.

Duke and others (1987) describe a FORTRAN program called LINEAR that derives a linear aircraft model by numerical differencing (Dieudonne, 1978). The program LINEAR produces a linear aircraft model (both state and observation matrices) that is equivalent to the linear models defined in this report.

This report is divided into two main sections that define the reference systems and nonlinear state and observation equations (section 1) and derive a linear model presented in the appendixes (section 2). The appendixes contain a definition of the linear aerodynamic model used in this report (app. A), a derivation of the wind axis translational acceleration parameters (app. B), generalized linear derivatives of the nonlinear state and observation equations (app. C), and the individual derivatives of the state and observation equations (app. D). The details of the principal results of this report are presented in appendix D.

SYMBOLS

A	total aerodynamic axial force, lb
a	speed of sound, ft/sec
a_{n}	normal accelerometer output, g
$a_{\mathrm{n},i}$	output of normal accelerometer not at vehicle center of gravity, g
a_x	output of accelerometer aligned with vehicle body x axis, g
$a_{x,i}$	output of accelerometer aligned with body x axis, not at vehicle center of gravity, g
$a_{x,\mathrm{k}}$	kinematic acceleration in vehicle body x axis, g
a_y	output of accelerometer aligned with vehicle body y axis, g
$a_{y,i}$	output of accelerometer aligned with body y axis, not at vehicle center of gravity, g
$a_{y,\mathrm{k}}$	kinematic acceleration in the vehicle body y axis, g
a_z	output of accelerometer aligned with vehicle body z axis, g
$a_{z,i}$	output of accelerometer aligned with body z axis, not at vehicle center of gravity, g
$a_{z,\mathrm{k}}$	kinematic acceleration in vehicle body z axis, g
b	reference span, ft
C_ξ	generalized force or moment coefficient
C_{ξ_x}	derivative of generalized force or moment coefficient with respect to arbitrary variable x
$\bar{c}$	reference aerodynamic chord, ft
D	total aerodynamic drag, lb
D_x	$I_z - I_y$
D_y	$I_x - I_z$
D_z	$I_y - I_x$
E_{s}	specific energy, ft
F	arbitrary force or moment
fpa	flightpath acceleration, g
g	acceleration due to gravity, ft/sec^2
g_0	acceleration due to gravity at sea level, ft/sec^2
h	altitude, ft
$h_{,i}$	altitude measurement not at vehicle center of gravity, ft
I	inertia tensor
I_x	moment of inertia about x body axis, slug-ft^2
I_{xy}	product of inertia in x–y body axis plane, slug-ft^2
I_{xz}	product of inertia in x–z body axis plane, slug-ft^2
I_y	moment of inertia about y body axis, slug-ft^2
I_{yz}	product of inertia in y–z body axis plane, slug-ft^2
I_z	moment of inertia about z body axis, slug-ft^2
I_1	$I_x I_z - I_{yz}^2$
I_2	$I_{xy} I_z + I_{yz} I_{xz}$
I_3	$I_{xy} I_{yz} + I_y I_{xz}$
I_4	$I_x I_z - I_{xz}^2$
I_5	$I_x I_{yz} + I_{xy} I_{xz}$
I_6	$I_x I_y - I_{xy}^2$
L	total moment about x body axis, ft-lb; or, total aerodynamic lift, lb
ℓ	unit length, ft
M	total moment about y body axis, ft-lb; or, Mach number

m	vehicle mass, slugs
N	total moment about z body axis, ft-lb; or, total aerodynamic normal force, lb
n	load factor
P_s	specific power, ft/sec
p	roll rate (about x body axis), rad/sec
p_a	static or free-stream pressure, lb/ft^2
p_s	stability axis roll rate, rad/sec
p_t	total pressure, lb/ft^2
q	pitch rate (about y body axis), rad/sec
$\bar{q}$	dynamic pressure, lb/ft^2
q_c	impact pressure, lb/ft^2
q_c/p_a	Mach meter calibration ratio
q_s	stability axis pitch rate, rad/sec
Re	Reynolds number
Re$'$	Reynolds number per unit length, ft^{-1}
r	yaw rate (about z body axis), rad/sec
r_s	stability axis yaw rate, rad/sec
S	surface area of wing, ft^2
T	total angular momentum; or, ambient or free-stream temperature, °R
T_t	total temperature, °R
t	time
u	velocity along x body axis, ft/sec
V	vehicle velocity, ft/sec
v	velocity along y body axis, ft/sec
w	velocity along z body axis, ft/sec
X_a	total aerodynamic force along x body axis, lb
X_g	total gravitational force along x body axis, lb
X_T	total thrust force along x body axis, lb
x	vehicle position along x earth axis, ft
Y	total aerodynamic sideforce, lb
Y_a	total aerodynamic force along y body axis, lb
Y_g	total gravitational force along y body axis, lb
Y_T	total thrust force along y body axis, lb
y	vehicle position along y earth axis, ft
Z_a	total aerodynamic force along z body axis, lb
Z_g	total gravitational force along z body axis, lb
Z_T	total thrust force along z body axis, lb
z	vehicle position along z earth axis, ft
α	angle of attack, rad
$\alpha_{,i}$	angle-of-attack measurement not at vehicle center of gravity, rad
β	angle of sideslip, rad
$\beta_{,i}$	angle-of-sideslip measurement not at vehicle center of gravity, rad
γ	flightpath angle, rad
δ_i	ith control surface deflection
θ	pitch angle, rad
μ	coefficient of viscosity, lb/ft-sec
ρ	density of air, lb/ft^3
Φ	arbitrary function

ϕ	bank angle, rad
ψ	heading angle, rad

Vectors

$\mathbf{a}$	body axis acceleration vector
$\mathbf{E}$	attitude vector of Euler angles
$\mathbf{F}$	total force vector
$\mathbf{f}$	state vector function
$\mathbf{g}$	observation vector function
$\mathbf{H}$	total angular momentum vector
$\mathbf{h}$	sum of higher order terms in Taylor series
$\mathbf{M}$	total moment vector
$\mathbf{R}$	position vector in earth axis system
$\mathbf{u}$	input or control vector
$\mathbf{V}$	vehicle velocity vector
$\mathbf{x}$	state vector
$\mathbf{y}$	observation vector
$\delta\mathbf{u}$	perturbation of control vector
$\delta\mathbf{x}$	perturbation of state vector
$\delta\dot{\mathbf{x}}$	perturbation of time derivative of state vector
Ω	rotational velocity vector

Matrices

A	state matrix of the generalized state equation, $C\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$
A'	state matrix of the state equation, $\dot{\mathbf{x}} = A'\mathbf{x} + B'\mathbf{u}$
B	control matrix of the generalized state equation, $C\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$
B'	control matrix of the state equation, $\dot{\mathbf{x}} = A'\mathbf{x} + B'\mathbf{u}$
C	system matrix of the generalized state equation, $C\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$
F	feedforward matrix of the generalized observation equation, $y = H\mathbf{x} + G\dot{\mathbf{x}} + F\mathbf{u}$
F'	feedforward matrix of the observation equation, $y = H'\mathbf{x} + F'\mathbf{u}$
G	derivative observation matrix of the generalized observation equation, $y = H\mathbf{x} + G\dot{\mathbf{x}} + F\mathbf{u}$
H	observation matrix of the generalized observation equation, $y = H\mathbf{x} + G\dot{\mathbf{x}} + F\mathbf{u}$
H'	observation matrix of the observation equation, $y = H'\mathbf{x} + F'\mathbf{u}$
I	intertia tensor
J'	scaling matrix for inertia tensor
L_{BV}	transformation matrix from earth to body axes
R	transformation matrix from earth to body axes
T	angular velocity matrix in the generalized state equation, $T\dot{\mathbf{x}} = \mathbf{f}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)]$
$0_{n\times m}$	$n \times m$ matrix of 0 values
$1_{n\times m}$	an $n \times m$ matrix with values of 1 on the diagonal

Subscripts

a	aerodynamic; or static or, free stream
b	body axis system
D	drag
g	gravitational
h	displacement of altitude instrument
$\dot{h}$	displacement of altitude rate instrument
$, i$	not at vehicle center of gravity
, k	kinematic
L	lift
ℓ	rolling moment
m	pitching moment
n	yawing moment
n	orthogonal
P	power plant induced
s	stability axis; or, specific
T	thrust
t	total
v	vehicle-carried vertical axis system
w	wind reference axis system
x	displacement in x body axis
xy	x–y body axis plane
xz	x–z body axis plane
Y	sideforce
y	displacement in y body axis
yz	y–z body axis plane
z	displacement in the z body axis
0	at sea level, standard day conditions; or, nominal conditions

Superscript

T	transpose

1 NONLINEAR SYSTEM EQUATIONS

The motion of an aircraft as a rigid body can be described by a set of six nonlinear simultaneous second-order differential equations. These equations, representing the translational and rotational motion of the vehicle, can be formulated in the notation of Kwakernaak and Sivan (1972) and Dieudonne (1978) as a time-invariant system expressed as

$$\dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t), \mathbf{u}(t)] \tag{1-1}$$

where $\mathbf{x}(t)$ is the 12-dimensional time-varying state vector (t being time), $\dot{\mathbf{x}}(t)$ is the derivative of $\mathbf{x}(t)$ with respect to time, $\mathbf{u}(t)$ is the k-dimensional time-varying input or control vector, and $\mathbf{f}$ is a 12-dimensional nonlinear function expressing the six-degree-of-freedom rigid body equations.

Measurements of the vehicle state can be represented by the observation equation

$$\mathbf{y}(t) = \mathbf{g}[\mathbf{x}(t), \mathbf{u}(t)] \tag{1-2}$$

where $\mathbf{y}(t)$ is an ℓ-dimensional time-varying observation vector and $\mathbf{g}$ is an ℓ-dimensional nonlinear function expressing the relationship of the true vehicle state and control vectors to the observed parameters. Typically, the function $\mathbf{g}$ characterizes the dynamics and location of the sensors.

For the aircraft analysis and design problem, both the nonlinear and linear system equations are formulated more broadly than just described (Edwards, 1976; Maine and Iliff, 1980, 1986). The nonlinear system equations include $\dot{\mathbf{x}}(t)$ terms in both the state and observation functions. In fact, in the most extended form the state equation is expressed in terms of transformed variables (discussed in section 1.2.1). These generalized equations form the basis of the analysis in this report. The generalized system equations are

$$T\dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)] \tag{1-3}$$

$$\mathbf{y}(t) = \mathbf{g}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)] \tag{1-4}$$

where T is a constant 12×12 angular velocity matrix.

1.1 Definition of Reference Systems

While numerous reference systems are used in aerospace applications, this report is limited to four reference systems: the body, the wind, the vehicle-carried vertical, and the topodetic reference systems. The stability axes are also defined even though this reference system is used only to define the stability axis rotational rates (section 1.3.8).

Within this report the translational equations are referenced to the wind axes, and the rotational equations are referenced to the body axes. Measurement equations are primarily referenced to the body axes when the use of a reference system is needed. The use of this mixed axis system definition in both the nonlinear and linear models is related to the measurability and meaningfulness of quantities. Because the aerodynamic forces act in the wind axes, this reference system is used for the translational equations. For instance, angle of attack, velocity, and angle of sideslip are either directly measurable or closely related to directly measurable quantities, while the body axis velocities (u, v, and w in the x, y, and z directions, respectively) are not. The body axis rotational rates are measured by sensors fixed in the body axes; wind axis rates can be derived only from these quantities through axis transformations.

The first reference system to be described is the topodetic reference system, also called the earth-fixed reference frame (Etkin, 1972), the earth axes (Thelander, 1965), and the Eulerian axes (Northrop Aircraft, 1952). The topodetic reference frame is considered fixed in space (and hence, inertial) with the orientation of the axes as shown in figure 1; the x axis is directed north, the y axis east, and the z axis down. The vehicle position (x and y) and altitude (h) are measured from the origin of this reference system.

The vehicle-carried vertical axis system (fig. 2; Etkin, 1972) has its origin at the center of gravity of the vehicle. The x_v axis is directed north, the y_v axis east, and the z_v axis down. This axis system is obtained by a translation of the topodetic axis system to the vehicle center of gravity. The attitude of the aircraft (heading, pitch, and bank angles ψ, θ, and ϕ, respectively) is described in terms of the orientation of the aircraft body axes with respect to the vehicle-carried vertical axes.

The origin of the body axis system (fig. 3) is the vehicle center of gravity. The x axis is directed toward the nose of the aircraft, the y axis toward the right wing, and the z axis toward the bottom of the aircraft. The specific orientation of the actual body axes relative to the vehicle body is somewhat arbitrary. For symmetrical aircraft, the x and z axes are in the plane of symmetry; for asymmetrical aircraft, these axes are located in a plane approximating what would be the plane of symmetry. The positive direction for the body axis rates (roll, pitch, and yaw rates, p, q, and r, respectively), the body axis velocities (u, v, and w), and the body axis moments (L, M, and N about the x, y, and z axes, respectively) are shown in figure 3.

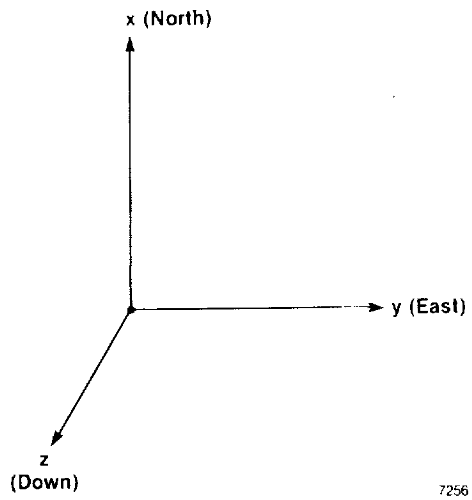


Figure 1. Topodetic axis system.

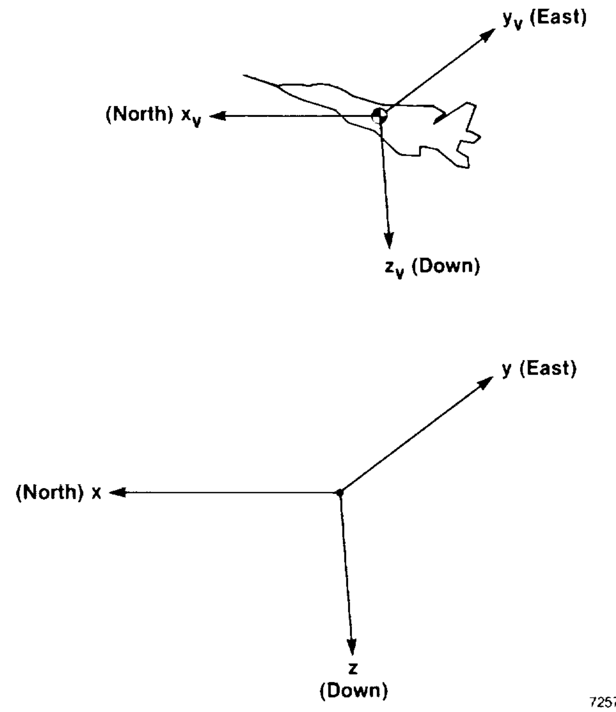


Figure 2. Relationship between topodetic and vehicle-carried vertical axis systems.

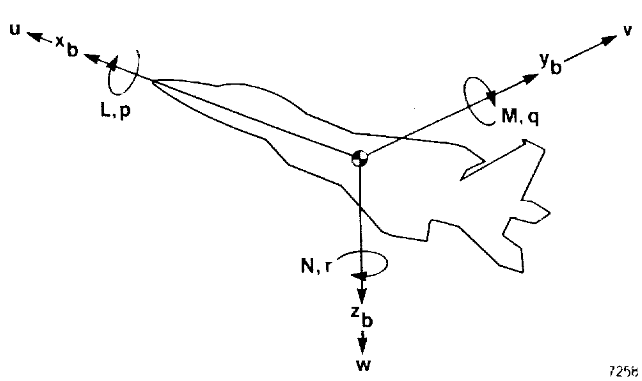


Figure 3. Body axis system.

The relationship between the vehicle-carried vertical and body axes is shown in figure 4. The Euler angles (ψ, θ, and ϕ) define the orientation of the body axes with respect to the vehicle-carried vertical axes. The rotations required to transform the vehicle-carried vertical axes to the body axes are shown in figure 5. The heading angle ψ is a rotation about the z vehicle-carried vertical axis into a new axis system (designated (x_1, y_1, z_1) in fig. 5); the pitch attitude θ is a rotation about the y_1 axis into the (x_2, y_2, z_2) axes system; the roll attitude ϕ is a rotation about the y_2 axis into the body axes.

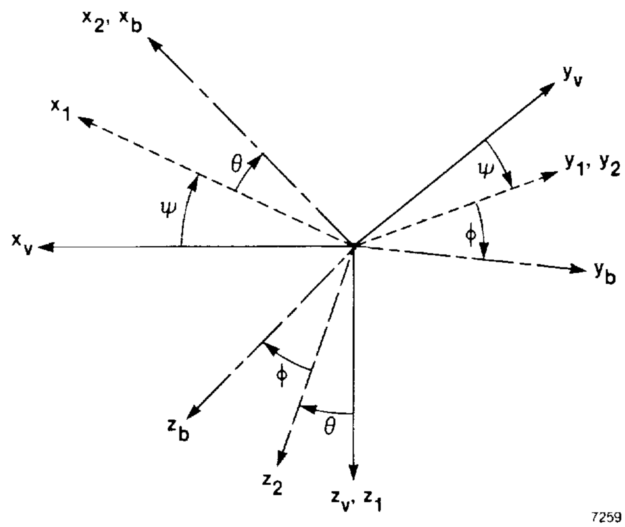


Figure 4. Relationship between vehicle-carried vertical and body axis systems.

These rotations are described by

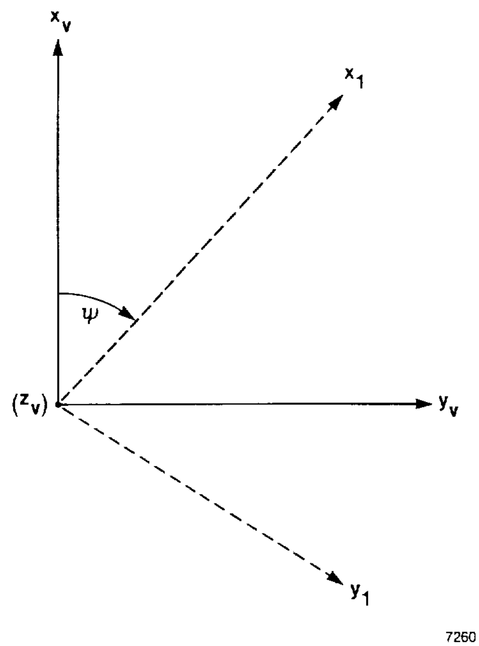
$$L_\psi = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1-5}$$

$$L_\theta = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{1-6}$$

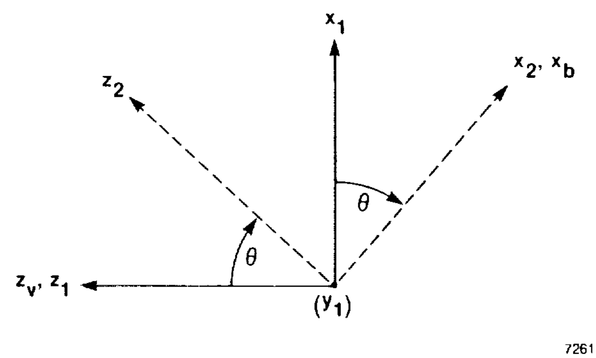
$$L_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \tag{1-7}$$

and the total rotation is described by

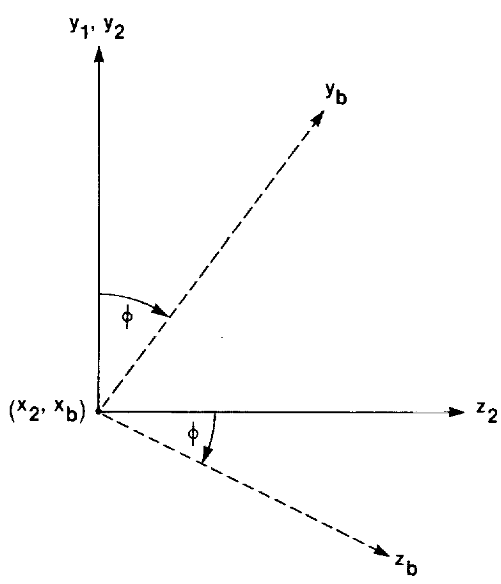
$$L_{\mathrm{BV}} = L_\psi L_\theta L_\phi = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi & \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ -\cos\phi\sin\psi & +\cos\phi\cos\psi & \\ \cos\phi\sin\theta\cos\psi & \cos\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ +\sin\phi\sin\psi & -\sin\phi\cos\psi & \end{bmatrix} \tag{1-8}$$



(a) Rotation through ψ about z_V axis.



(b) Rotation through θ about y_1 axis.



(c) Rotation through ϕ about x_b axis.

Figure 5. Rotation of axes through Euler angles.

Because L_{BV} is a unitary matrix, the transformation from the body axes to the vehicle-carried vertical axes is $L_{\mathrm{BV}}^{\mathrm{T}}$.

The relationships between the body, wind, and stability axes are shown in figure 6. All three axis systems have their origin at the center of gravity of the aircraft. The x axis in the wind reference system (x_{w}) is aligned with the velocity vector of the aircraft. The angle of sideslip β and angle of attack α define the orientation of the wind axes with respect to the body axes. (The stability axes are shown in figure 6 also. This reference system is displaced from the wind axis system by a rotation β and from the body axis system by a rotation $-\alpha$.)

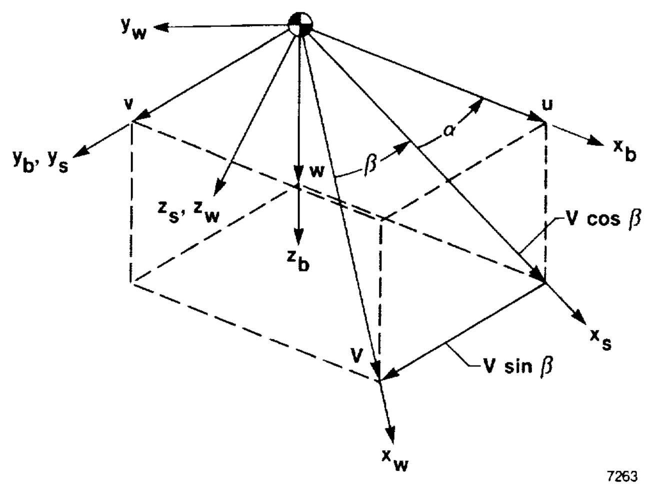


Figure 6. Relationship of body, stability, and wind axes.

Also shown in figure 6 are the components of the velocity vector $\mathbf{V}$ in the body axes (u, v, and w) and the definition of positive rotations for α and β. It should be noted that β is a positive rotation in a left-handed coordinate system, whereas the positive sense of all other rotations used in aircraft analysis are positive in a right-handed coordinate system.

The definitions of the body axis velocities (fig. 6) are

$$u = V\cos\alpha\cos\beta \tag{1-9}$$

$$v = V\sin\beta \tag{1-10}$$

$$w = V\sin\alpha\cos\beta \tag{1-11}$$

The total velocity V, angle of attack α, and angle of sideslip β can be expressed in terms of these body axis velocities as

$$V = |\mathbf{V}| = (u^2 + v^2 + w^2)^{1/2} \tag{1-12}$$

$$\alpha = \tan^{-1}\frac{w}{u} \tag{1-13}$$

$$\beta = \sin^{-1}\frac{v}{V} \tag{1-14}$$

1.2 Nonlinear State Equations

For the aircraft problem, the state vector $\mathbf{x}$ is 12×1 vector composed of four 3×1 subvectors representing the vehicle rotational velocity, the vehicle translational velocity, the vehicle attitude, and the vehicle location:

$$\mathbf{x} = [\mathbf{x}_1^{\mathrm{T}}\ \mathbf{x}_2^{\mathrm{T}}\ \mathbf{x}_3^{\mathrm{T}}\ \mathbf{x}_4^{\mathrm{T}}]^{\mathrm{T}} \tag{1-15}$$

where

$$\mathbf{x}_1 = [p\ \ q\ \ r]^{\mathrm{T}} \tag{1-16}$$

$$\mathbf{x}_2 = [V\ \ \alpha\ \ \beta]^{\mathrm{T}} \tag{1-17}$$

$$\mathbf{x}_3 = [\phi\ \ \theta\ \ \psi]^{\mathrm{T}} \tag{1-18}$$

$$\mathbf{x}_4 = [h\ \ x\ \ y]^{\mathrm{T}} \tag{1-19}$$

with $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$, and $\mathbf{x}_4$ being the rotational velocity, translational velocity, attitude, and position subvectors, respectively. The vehicle rotational and translational velocity are defined within the aircraft-fixed axis systems. In the formulation of the state used in this report, the vehicle rotations are body axis rates, whereas the vehicle velocity terms are stability axis parameters. The vehicle attitude and location parameters are earth relative.

The vector function $\mathbf{f}$, relating the state vector its time derivative, and the control vector to the time derivative of the state vector with respect to time, is a 12-dimensional vector function composed of four 3-dimensional vector subfunctions:

$$\mathbf{f}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)] = [\mathbf{f}_1^{\mathrm{T}}\ \mathbf{f}_2^{\mathrm{T}}\ \mathbf{f}_3^{\mathrm{T}}\ \mathbf{f}_4^{\mathrm{T}}]^{\mathrm{T}} \tag{1-20}$$

where $\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{f}_3$, and $\mathbf{f}_4$ are the vector functions that relate the $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$, and $\mathbf{u}(t)$ vectors to the rotational acceleration, translational acceleration, attitude rate, and earth-relative velocity subvectors of $\dot{\mathbf{x}}(t)$. In the following sections, each of these subfunctions will be developed separately. The details of the derivation of these subfunctions can be found in any of the standard references on aircraft dynamics (Etkin, 1972; McRuer and others, 1973; Thelander, 1965).

1.2.1 Rotational acceleration.—The subfunction $\mathbf{f}_1$ of $\mathbf{f}$ from which the rotational acceleration terms in the $\dot{\mathbf{x}}$ vector are derived is based on the moment equation

$$\mathbf{M} = \frac{d}{dt}\mathbf{H} \tag{1-21}$$

where $\mathbf{M}$ is the total moment on the vehicle and $\mathbf{H}$ is the total angular momentum of the vehicle. This expression can be expanded to

$$\mathbf{M} = \frac{\delta}{\delta t}(I\Omega) + \Omega \times (I\Omega) \tag{1-22}$$

where $\delta/\delta t$ is the time derivative operator in a moving reference frame (such as the vehicle body axis system) and the substitution

$$\mathbf{H} = I\Omega \tag{1-23}$$

has been used to replace the total angular momentum term with the product of the inertia tensor I and the rotational velocity vector Ω. (The inertia tensor is assumed to be constant with time.) The definition of the terms in equation (1-22) follow:

$$\mathbf{M} = \begin{bmatrix} \sum L \\ \sum M \\ \sum N \end{bmatrix} = \begin{bmatrix} L + L_{\mathrm{T}} \\ M + M_{\mathrm{T}} \\ N + N_{\mathrm{T}} \end{bmatrix} \tag{1-24}$$

with L, M, and N being the aerodynamic total moments about the x, y, and z body axes, respectively, and L_T, M_T, and N_T the sums of all power-plant-induced moments;

$$I = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{bmatrix} \tag{1-25}$$

where I_x, I_y, and I_z are the moments of inertia about the x, y, and z body axes, respectively, and I_{xy}, I_{xz}, and I_{yz} are the products of inertia in the x–y, x–z, and y–z body axis planes, respectively; and

$$\Omega = \mathbf{x}_1 = [p \;\; q \;\; r]^{\mathrm{T}} \tag{1-26}$$

where p, q, and r are the rotational rates about the x, y, and z body axes, respectively. Because it is assumed that the inertia tensor is a constant with respect to time, equation (1-22) can be rewritten as

$$\frac{\delta}{\delta t}\Omega = I^{-1}(\mathrm{M} - \Omega \times I\Omega) \tag{1-27}$$

This is the vector subfunction for the rotational acceleration. Designating this subfunction as $\mathbf{f}_1$, the following definition applies:

$$\mathbf{f}_1[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)] \equiv I^{-1}[\mathrm{M} - \Omega \times (I\Omega)] \tag{1-28}$$

where

$$\frac{\delta}{\delta t}\Omega = \mathbf{f}_1[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)] \tag{1-29}$$

$$\frac{\delta}{\delta t}\Omega = [\dot{p} \;\; \dot{q} \;\; \dot{r}]^{\mathrm{T}} \tag{1-30}$$

Since the inverse of the inertia tensor I^{-1} is given by

$$I^{-1} = \frac{1}{\det I}\begin{bmatrix} I_1 & I_2 & I_3 \\ I_2 & I_4 & I_5 \\ I_3 & I_5 & I_6 \end{bmatrix} \tag{1-31}$$

where

$$\det I = I_x I_y I_z - I_x I_{yz}^2 - I_z I_{xy}^2 - I_y I_{xz}^2 - 2 I_{yz} I_{xz} I_{xy} \tag{1-32}$$

$$I_1 = I_y I_z - I_{yz}^2 \tag{1-33}$$

$$I_2 = I_{xy} I_z + I_{yz} I_{xz} \tag{1-34}$$

$$I_3 = I_{xy} I_{yz} + I_y I_{xz} \tag{1-35}$$

$$I_4 = I_x I_z - I_{xz}^2 \tag{1-36}$$

$$I_5 = I_x I_{yz} + I_{xy} I_{xz} \tag{1-37}$$

$$I_6 = I_x I_y - I_{xy}^2 \tag{1-38}$$

the expression for the rotational accelerations can be expanded as a set of scalar equations:

$$\begin{aligned}\dot{p} = \frac{1}{\det I}[&\Sigma L I_1 + \Sigma M I_2 + \Sigma N I_3 - p^2(I_{xz}I_2 - I_{xy}I_3) + pq(I_{xz}I_1 - I_{yz}I_2 - D_z I_3) \\ &- pr(I_{xy}I_1 + D_y I_2 - I_{yz}I_3) + q^2(I_{yz}I_1 - I_{xy}I_3) - qr(D_x I_1 - I_{xy}I_2 + I_{xz}I_3) \\ &- r^2(I_{yz}I_1 - I_{xz}I_2)]\end{aligned} \tag{1-39}$$

$$\begin{aligned}\dot{q} = \frac{1}{\det I}[&\Sigma L I_2 + \Sigma M I_4 + \Sigma N I_5 - p^2(I_{xz}I_4 - I_{xy}I_5) + pq(I_{xz}I_2 - I_{yz}I_4 - D_z I_5) \\ &- pr(I_{xy}I_2 + D_y I_4 - I_{yz}I_5) + q^2(I_{yz}I_2 - I_{xy}I_5) - qr(D_x I_2 - I_{xy}I_4 + I_{xz}I_5) \\ &- r^2(I_{yz}I_2 - I_{xz}I_4)]\end{aligned} \tag{1-40}$$

$$\begin{aligned}\dot{r} = \frac{1}{\det I}[&\Sigma L I_3 + \Sigma M I_5 + \Sigma N I_6 - p^2(I_{xz}I_5 - I_{xy}I_6) + pq(I_{xz}I_3 - I_{yz}I_5 - D_z I_6) \\ &- pr(I_{xy}I_3 + D_y I_5 - I_{yz}I_6) + q^2(I_{yz}I_3 - I_{xy}I_6) - qr(D_x I_3 - I_{xy}I_5 + I_{xz}I_6) \\ &- r^2(I_{yz}I_3 - I_{xz}I_5)]\end{aligned} \tag{1-41}$$

where

$$D_x = I_z - I_y \tag{1-42}$$

$$D_y = I_x - I_z \tag{1-43}$$

$$D_z = I_y - I_x \tag{1-44}$$

Equation (1-3) defines the generalized nonlinear state equations as

$$T\dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)]$$

This equation, although more complicated than the nonlinear equations defined by equation (1-1), allows for a more tractable formulation of the state equation by using the matrix T to provide a means of addressing the rotational accelerations in a decoupled axis system.

The derivation of the rotational acceleration terms is based on the moment equation (1-22):

$$\mathbf{M} = \frac{\delta}{\delta t}(I\Omega) + \Omega \times I\Omega$$

Rearranging terms and assuming that the inertia tensor is constant with respect to time, the equation can be written as

$$I\frac{\delta}{\delta t}\Omega = \mathbf{M} - \Omega \times I\Omega \tag{1-45}$$

The rows of this vector equation are now scaled using the following scaling matrix:

$$J' = \begin{bmatrix} 1/I_x & 0 & 0 \\ 0 & 1/I_y & 0 \\ 0 & 0 & 1/I_z \end{bmatrix} \tag{1-46}$$

This matrix, when premultiplying equation (1-27), merely divides the first row by the roll inertia I_x, the second row by the pitch inertia I_y, and the third row by the yaw inertia I_z. Using the definition

$$J = J'I \tag{1-47}$$

the resulting equation is

$$J\frac{\delta}{\delta t}\Omega = J'M - J'(\Omega \times I\Omega) \tag{1-48}$$

and J can be written as

$$J = \begin{bmatrix} 1.0 & -I_{xy}/I_x & -I_{xz}/I_x \\ -I_{xy}/I_y & 1.0 & -I_{yz}/I_y \\ -I_{xz}/I_z & -I_{yz}/I_z & 1.0 \end{bmatrix} \tag{1-49}$$

Equation (1-48) can be expanded and expressed as

$$\begin{aligned} \begin{bmatrix} \dot{p}' \\ \dot{q}' \\ \dot{r}' \end{bmatrix} &\equiv \begin{bmatrix} 1.0 & -I_{xy}/I_x & -I_{xz}/I_x \\ I_{xy}/I_y & 1.0 & -I_{yz}/I_y \\ -I_{xz}/I_z & -I_{yz}/I_z & 1.0 \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} \\ &= \begin{bmatrix} \Sigma L/I_x - rpI_{xy}/I_x + pqI_{xz}/I_x + rqI_y/I_x + (q^2 - r^2)I_{yz}/I_x - qrI_z/I_x \\ \Sigma M/I_y - rpI_x/I_y + rqI_{xy}/I_y - pqI_{yz}/I_y + (r^2 - p^2)I_{xz}/I_y + prI_z/I_y \\ \Sigma N/I_z + qpI_x/I_z - qrI_{xz}/I_z + prI_{yz}/I_z + (p^2 - q^2)I_{xy}/I_z - pqI_y/I_z \end{bmatrix} \end{aligned} \tag{1-50}$$

where $\dot{p}'$, $\dot{q}'$, and $\dot{r}'$ are the decoupled rotational accelerations of the vehicle.

Using the definition of J in equation (1-49), the matrix transformation T can be defined as

$$T = \left[\begin{array}{cc:c} J & 0_{3\times3} & \\ 0_{3\times3} & 1_{3\times3} & 0_{6\times6} \\ \hdashline \multicolumn{2}{c:}{0_{6\times6}} & 1_{6\times6} \end{array}\right] \tag{1-51}$$

which would be an identity matrix except for the presence of the inertia terms in the upper left-hand corner. Thus, the vector subfunctions for the generalized state equation defining vehicle translational acceleration, vehicle attitude rates, and earth-relative velocities are the same as those defined for the standard nonlinear state equations in sections 1.2.2, 1.2.3, and 1.2.4, respectively.

1.2.2 Translational acceleration.—Derivation of the translational acceleration vector subfunction $\mathbf{f}_2$ is based on the force equation

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{V}) \tag{1-52}$$

where $\mathbf{F}$ is the total force acting on the vehicle and m is the vehicle mass. This expression can be expanded to

$$\mathbf{F} = m\left(\frac{\delta}{\delta t}\mathbf{V} + \mathbf{\Omega} \times \mathbf{V}\right) \tag{1-53}$$

with the assumption of constant mass with respect to time and the following definitions of $\mathbf{F}$ and $\mathbf{V}$:

$$\mathbf{F} = [\Sigma X \;\; \Sigma Y \;\; \Sigma Z]^{\mathrm{T}} \tag{1-54}$$

where ΣX, ΣY, and ΣZ are the sums of the aerodynamic, thrust, and gravitational forces in the x, y, and z body axes, respectively, and

$$\mathbf{V} = [u \;\; v \;\; w]^{\mathrm{T}} \tag{1-55}$$

Rearranging the terms of equation (1-52) gives an expression for the translational acceleration:

$$\frac{\delta}{\delta t}\mathbf{V} = \frac{1}{m}\mathbf{F} - \Omega \times \mathbf{V} \tag{1-56}$$

This equation expresses body axis accelerations in terms of body axis forces, angular rates, and velocities. However, the desired form of this relation requires the translational accelerations in the wind axis system; that is, in terms of the magnitude of the total vehicle velocity V, angle of attack α, and angle of sideslip β, which are expressed by equations (1-9) to (1-11)

$$\begin{aligned} u &= V \cos\alpha \cos\beta \\ v &= V \sin\beta \\ w &= V \sin\alpha \cos\beta \end{aligned}$$

and equations (1-12) to (1-14)

$$\begin{aligned} V &= |\mathbf{V}| = (u^2 + v^2 + w^2)^{1/2} \\ \alpha &= \tan^{-1}\left(\frac{w}{u}\right) \\ \beta &= \sin^{-1}\left(\frac{v}{V}\right) \end{aligned}$$

The wind axis translational acceleration terms (derived in app. B) are summarized as:

$$[\dot{V}\ \dot{\alpha}\ \dot{\beta}]^{\mathrm{T}} = \mathbf{f}_2[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)] \tag{1-57}$$

where

$$\begin{aligned} \dot{V} = \frac{1}{m}[&-D\cos\beta + Y\sin\beta + X_{\mathrm{T}}\cos\alpha\cos\beta + Y_{\mathrm{T}}\sin\beta + Z_{\mathrm{T}}\sin\alpha\cos\beta \\ &- mg(\cos\alpha\cos\beta\sin\theta - \sin\beta\sin\phi\cos\theta - \sin\alpha\cos\beta\cos\phi\cos\theta)] \end{aligned} \tag{1-58}$$

$$\begin{aligned} \dot{\alpha} = &\frac{1}{Vm\cos\beta}[-L + Z_{\mathrm{T}}\cos\alpha - X_{\mathrm{T}}\sin\alpha + mg(\cos\alpha\cos\phi\cos\theta + \sin\alpha\sin\theta)] \\ &+ q - \tan\beta\,(p\cos\alpha + r\sin\alpha) \end{aligned} \tag{1-59}$$

$$\begin{aligned} \dot{\beta} = \frac{1}{mV}&[D\sin\beta + Y\cos\beta - X_{\mathrm{T}}\cos\alpha\sin\beta + Y_{\mathrm{T}}\cos\beta - Z_{\mathrm{T}}\sin\alpha\sin\beta \\ &+ mg(\cos\alpha\sin\beta\sin\theta + \cos\beta\sin\phi\cos\theta - \sin\alpha\sin\beta\cos\phi\cos\theta)] + p\sin\alpha - r\cos\alpha \end{aligned} \tag{1-60}$$

with D being total aerodynamic drag; Y total aerodynamic sideforce; and X_{T}, Y_{T}, and Z_{T} total thrust force along the x, y, and z body axes, respectively.

1.2.3 Attitude rates.—The matrix R that transforms angular velocities in the earth-fixed axis system into body axis angular velocities is defined by

$$R = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \tag{1-61}$$

where R is derived by Maine and Iliff (1986) from the total angular velocity of the aircraft expressed in terms of the derivatives with respect to time of the Euler angles ($\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$):

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{1-62}$$

This transformation from earth-fixed to body axes can be expressed by the equation

$$\Omega = R\left(\frac{d}{dt}\mathbf{E}\right) \tag{1-63}$$

where $\mathbf{E}$ is an attitude vector whose components are the Euler angles:

$$\mathbf{E} = [\phi\ \theta\ \psi]^{\mathrm{T}} \tag{1-64}$$

Premultiplying both sides of equation (1-63) by R^{-1} and rearranging terms yields the equation for the attitude rates,

$$\frac{d}{dt}\mathbf{E} = R^{-1}\Omega \tag{1-65}$$

which can be expanded into the scalar equations

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \tag{1-66}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{1-67}$$

$$\dot{\psi} = q\sin\phi\sec\theta + r\cos\phi\sec\theta \tag{1-68}$$

1.2.4 Earth-relative velocity.—The matrix L_{BV} that transforms earth axis system vectors into the body axis system is defined by equation (1-8) as

$$L_{\mathrm{BV}} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & cos\phi\cos\theta \end{bmatrix}$$

The specific relationship between earth-relative velocities and body axis velocities is expressed by

$$\mathbf{V} = L_{\mathrm{BV}}\left(\frac{d}{dt}\mathbf{R}\right) \tag{1-69}$$

where $\mathbf{R}$ is the earth axis system vector defining the location of the vehicle:

$$\mathbf{R} = [x\ \ y\ \ z]^{\mathrm{T}} \tag{1-70}$$

with $z = -h$.

The equation for the earth-relative velocity can be formulated as

$$\frac{d}{dt}\mathbf{R} = L_{\mathrm{BV}}^{-1}\mathbf{V} \tag{1-71}$$

in which these velocities are expressed in terms of body axis velocities. Using equation (1-72) and the definitions of the body axis velocities in equations (1-12) to (1-14) allows the earth-relative velocities to be expressed in terms of V, α, and β:

$$\dot{h} = V(\cos\alpha\cos\beta\sin\theta - \sin\beta\sin\phi\cos\theta - \sin\alpha\cos\beta\cos\phi\cos\theta) \tag{1-72}$$

$$\begin{aligned}\dot{x} = V[&\cos\alpha\cos\beta\cos\theta\cos\psi + \sin\beta(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi)\\ &+ \sin\alpha\cos\beta(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)]\end{aligned} \tag{1-73}$$

$$\begin{aligned}\dot{y} = V[&\cos\alpha\cos\beta\cos\theta\sin\psi + \sin\beta(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi)\\ &+ \sin\alpha\cos\beta(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)]\end{aligned} \tag{1-74}$$

1.3 Nonlinear Observation Equations

No standard set of observation variables exists for the aircraft analysis and control design problem. However, for any guidance and control problem, the main observation variables generally will be a subset of the state variables. Other common observation variables are the vehicle body axis translational accelerations and air data parameters. Thus, the dimension of $\mathbf{g}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)]$ is not fixed and varies from application to application. The set of observation variables described in this section was selected to address a wide range of problems. The basic composition of the observation vector $\mathbf{y}$ as used in this report is given by

$$\mathbf{y} = [\mathbf{x}^{\mathrm{T}}\ \dot{\mathbf{x}}^{\mathrm{T}}\ \mathbf{u}^{\mathrm{T}}\ \mathbf{y}'^{\mathrm{T}}]^{\mathrm{T}} \tag{1-75}$$

where $\mathbf{x}$ and $\dot{\mathbf{x}}$ are the state vector and time derivative of the state vector described previously, $\mathbf{u}$ is the control vector, and $\mathbf{y}'$ is defined by

$$\mathbf{y}' = [\mathbf{y}_1'^{\mathrm{T}}\ \mathbf{y}_2'^{\mathrm{T}}\ \mathbf{y}_3'^{\mathrm{T}}\ \mathbf{y}_4'^{\mathrm{T}}\ \mathbf{y}_5'^{\mathrm{T}}\ \mathbf{y}_6'^{\mathrm{T}}\ \mathbf{y}_7'^{\mathrm{T}}\ \mathbf{y}_8'^{\mathrm{T}}]^{\mathrm{T}} \tag{1-76}$$

where

$$\mathbf{y}_1' = [a_{x,\mathrm{k}}\ a_{y,\mathrm{k}}\ a_{z,\mathrm{k}}\ a_x\ a_y\ a_z\ a_{\mathrm{n}}\ a_{x,i}\ a_{y,i}\ a_{z,i}\ a_{\mathrm{n},i}\ n]^{\mathrm{T}} \tag{1-77}$$

$$\mathbf{y}_2' = [a\ M\ Re\ Re'\ \bar{q}\ q_{\mathrm{c}}\ q_{\mathrm{c}}/p_{\mathrm{a}}\ p_{\mathrm{a}}\ p_{\mathrm{t}}\ T\ T_{\mathrm{t}}]^{\mathrm{T}} \tag{1-78}$$

$$\mathbf{y}_3' = [\gamma\ \mathrm{fpa}\ \ddot{\mathrm{h}}]^{\mathrm{T}} \tag{1-79}$$

$$\mathbf{y}_4' = [E_{\mathrm{s}}\ P_{\mathrm{s}}]^{\mathrm{T}} \tag{1-80}$$

$$\mathbf{y}_5' = [L\ D\ N\ A]^{\mathrm{T}} \tag{1-81}$$

$$\mathbf{y}_6' = [u\ v\ w\ \dot{u}\ \dot{v}\ \dot{w}]^{\mathrm{T}} \tag{1-82}$$

$$\mathbf{y}_7' = [\alpha_{,i}\ \beta_{,i}\ h_{,i}\ \dot{h}_{,i}]^{\mathrm{T}} \tag{1-83}$$

$$\mathbf{y}_8' = [T\ p_{\mathrm{s}}\ q_{\mathrm{s}}\ r_{\mathrm{s}}]^{\mathrm{T}} \tag{1-84}$$

with the elements of $\mathbf{y}'_1$ being terms related to the vehicle body axis acceleration, the elements of $\mathbf{y}'_2$ being air data terms, the elements of $\mathbf{y}'_3$ being flightpath-related terms, the elements of $\mathbf{y}'_4$ being terms related to vehicle energy, $\mathbf{y}'_5$ being a vehicle force vector, the elements of $\mathbf{y}'_6$ being body axis translational rates and the time derivatives of those terms, $\mathbf{y}'_7$ being a vector of variables representing measurements from instruments not located at the vehicle center of gravity, and the elements of $\mathbf{y}'_8$ being a collection of miscellaneous terms. Obviously, this grouping of terms is somewhat arbitrary and is done primarily to ease the definition of these terms in the following sections of this report. This grouping of observation variables parallels that used by Duke and others (1987).

The vector function $\mathbf{g}$ relating the state vector, the time derivative of the state vector, and the control vector to the observation vector is an ℓ-dimensional function composed of four subfunctions:

$$\mathbf{g}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)] = [\mathbf{x}^{\mathrm{T}} \ \dot{\mathbf{x}}^{\mathrm{T}} \ \mathbf{u}^{\mathrm{T}} \ \mathbf{g}'^{\mathrm{T}}] \tag{1-85}$$

where $\mathbf{x}$, $\dot{\mathbf{x}}$, and $\mathbf{u}$ are identity functions on the state vector, time derivative of the state vector, and control vector, respectively, and $\mathbf{g}'$ is composed of vector subfunctions defining the $\mathbf{y}'$ vector.

The state vector, time derivative of state vector, and control vector components of the observation vector are not discussed in detail in this section of the report. The equations for the elements of the time derivative of the state vector were developed in section 1.1. The observation equations for the state and control variables are simply identities. The equations for the remaining observation variables are obtained from a variety of sources. In addition to the previously cited sources, Clancy (1975), Dommasch and others (1967), Gainer and Hoffman (1972), and Gracey (1980) provide the background and derivation of the observation equations used in this report.

1.3.1 Accelerations.—The vehicle body axis accelerations and accelerometer outputs constitute the set of observation variables that, after the state variables themselves, are most important in the aircraft control analysis and design problem. These accelerations and accelerometer outputs are measured in units of g and are derived directly from the body axis forces defined in section 1.2.2. The body axis acceleration vector $\mathbf{a}$ can be expressed as

$$\mathbf{a} = \frac{d}{dt}V = \frac{\delta}{\delta t}V + \Omega \times V \tag{1-86}$$

It is important to note here that the $\dot{u}$, $\dot{v}$, and $\dot{w}$ body axis velocity rates, derived in appendix B and defined by equation (B-1), are not the body axis accelerations. The body axis accelerations contain not only the body axis velocity rates but also the rotational velocity and translational velocity cross-product terms. Thus, expanding equation (1-86) yields

$$\mathbf{a} = \begin{bmatrix} a_{x,\mathrm{k}} \\ a_{y,\mathrm{k}} \\ a_{z,\mathrm{k}} \end{bmatrix} = \begin{bmatrix} \dot{u} + qw - rv \\ \dot{v} + ru - pw \\ \dot{w} + pv - qu \end{bmatrix} \tag{1-87}$$

where $a_{x,\mathrm{k}}$, $a_{y,\mathrm{k}}$, and $a_{z,\mathrm{k}}$ are the kinematic accelerations in the vehicle body x, y, and z axes, respectively. Using

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} (1/m)(X_{\mathrm{T}} + X_{\mathrm{a}} + X_{\mathrm{g}}) + rv - qw \\ (1/m)(Y_{\mathrm{T}} + Y_{\mathrm{a}} + Y_{\mathrm{g}}) + pw - ru \\ (1/m)(Z_{\mathrm{T}} + Z_{\mathrm{a}} + Z_{\mathrm{g}}) + qu - pv \end{bmatrix} \tag{1-88}$$

(stated as eq. (B-1) in app. B), equation (1-87) can be rewritten as

$$\begin{bmatrix} a_{x,\mathrm{k}} \\ a_{y,\mathrm{k}} \\ a_{z,\mathrm{k}} \end{bmatrix} = \begin{bmatrix} (1/m)(X_{\mathrm{T}} + X_{\mathrm{a}} + X_{\mathrm{g}}) \\ (1/m)(Y_{\mathrm{T}} + Y_{\mathrm{a}} + Y_{\mathrm{g}}) \\ (1/m)(Z_{\mathrm{T}} + Z_{\mathrm{a}} + Z_{\mathrm{g}}) \end{bmatrix} \tag{1-89}$$

where X_a, Y_a, and Z_a are total aerodynamic forces and X_g, Y_g, and Z_g are total gravitational forces along the x, y, and z body axes, respectively. This can be expanded in terms of the gravitational and aerodynamic forces to give (in units of g)

$$\begin{bmatrix} a_{x,\mathrm{k}} \\ a_{y,\mathrm{k}} \\ a_{z,\mathrm{k}} \end{bmatrix} = \frac{1}{g_0 m} \begin{bmatrix} X_\mathrm{T} - D\cos\alpha + L\sin\alpha - gm\sin\theta \\ Y_\mathrm{T} + Y + gm\sin\phi\cos\theta \\ Z_\mathrm{T} - D\sin\alpha - L\cos\alpha + gm\cos\phi\cos\theta \end{bmatrix} \tag{1-90}$$

where g_0 is the acceleration due to gravity at sea level.

The outputs of body axis accelerometers at the vehicle center of gravity are simply the body axis accelerations due to the thrust and aerodynamic forces. The accelerometer output equations can be written directly from equation (1-90) as

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \frac{1}{g_0 m} \begin{bmatrix} X_\mathrm{T} - D\cos\alpha + L\sin\alpha \\ Y_\mathrm{T} + Y \\ Z_\mathrm{T} - D\sin\alpha - L\cos\alpha \end{bmatrix} \tag{1-91}$$

where a_x, a_y, and a_z are the outputs of accelerometers at the vehicle center of gravity and aligned with the vehicle body x, y, and z axes, respectively. Because the normal acceleration a_n is defined by

$$a_\mathrm{n} = -a_z \tag{1-92}$$

an expression for this variable can be extracted from equation (1-91):

$$a_\mathrm{n} = (-Z_\mathrm{T} + D\sin\alpha + L\cos\alpha)/g_0 m \tag{1-93}$$

The equations defining the output of accelerometers aligned with the vehicle body axes but displaced from the vehicle center of gravity are derived by Gainer and Hoffman (1972) using the definition of inertial acceleration given in equation (1-86)

$$\mathbf{a} = \frac{\delta}{\delta t}V + \Omega \times V$$

and the definition of inertial velocity

$$\mathbf{V} = \frac{\delta}{\delta t}\mathbf{r} + \Omega \times \mathbf{r} \tag{1-94}$$

The results from Gainer and Hoffman (1972) are reproduced here without rederivation:

$$\begin{bmatrix} a_{x,i} \\ a_{y,i} \\ a_{z,i} \end{bmatrix} = \begin{bmatrix} a_x - [(q^2 + r^2)x_x - (pq - \dot{r})y_x - (pr + \dot{q})z_x]/g_0 \\ a_y + [(pq + \dot{r})x_y - (p^2 + r^2)y_y - (qr - \dot{p})z_y]/g_0 \\ a_z + [(pr - \dot{q})x_z + (qr + \dot{p})y_z - (q^2 + p^2)z_z]/g_0 \end{bmatrix} \tag{1-95}$$

where $a_{x,i}$, $a_{y,i}$, and $a_{z,i}$ are outputs at accelerometers aligned with the x, y, and z body axes but not located at the vehicle center of gravity; the subscripts x, y, and z refer to the x, y, and z body axes, respectively; and the symbols x, y, and z refer to the x, y, and z body axis locations of the sensors relative to the vehicle center of gravity. Because the normal acceleration is the negative of the z body axis accelerometer, the output of a normal accelerometer not at the vehicle center of gravity but aligned with the z body axis, $a_{\mathrm{n},i}$, is given by

$$a_{\mathrm{n},i} = a_\mathrm{n} - [(pr - \dot{q})x_z + (qr + \dot{p})y_z - (q^2 + p^2)z_z]/g_0 \tag{1-96}$$

The final quantity included in the general category of accelerations is load factor n. This quantity is defined without inclusion of the z body axis force component as

$$n \doteq \frac{L}{mg} \tag{1-97}$$

1.3.2 Air data parameters.—The air data parameters having the greatest application to aircraft dynamics and control problems are the sensed parameters and the reference and scaling parameters. Chosen for inclusion as the sensed parameters are impact pressure q_c, static or free-stream pressure p_a, total pressure p_t, ambient or free-stream temperature T, and total temperature T_t. The selected reference and scaling parameters are Mach number M, dynamic pressure $\bar{q}$, speed of sound a, Reynolds number Re, Reynolds number per unit length Re′, and the Mach meter calibration ratio q_c/p_a. The derivation of these quantities is treated extensively by Gracey (1980).

The nonlinear equations defining these quantities are

$$a = \left[1.4 \frac{p_0}{\rho_0 T_0} T\right]^{1/2} \tag{1-98}$$

$$M = \frac{V}{a} \tag{1-99}$$

$$\mathrm{Re} = \frac{\rho V \ell}{\mu} \tag{1-100}$$

$$\mathrm{Re}' = \frac{\rho V}{\mu} \tag{1-101}$$

$$\bar{q} = \frac{1}{2}\rho V^2 \tag{1-102}$$

$$q_c = \begin{cases} [(1.0 + 0.2M^2)^{3.5} - 1.0]p_a & (M \le 1.0) \\ \{1.2M^2[5.76M^2/(5.6M^2 - 0.8)]^{2.5} - 1.0\}p_a & (M \ge 1.0) \end{cases} \tag{1-103}$$

$$\frac{q_c}{p_a} = \begin{cases} (1.0 + 0.2M^2)^{3.5} - 1.0 & (M \le 1.0) \\ 1.2M^2[5.76M^2/(5.6M^2 - 0.8)]^{2.5} - 1.0 & (M \ge 1.0) \end{cases} \tag{1-104}$$

$$T_t = T(1.0 + 0.2M^2) \tag{1-105}$$

where ρ is the density of the air, μ is the coefficient of viscosity, and the subscript 0 refers to sea level, standard day conditions. Free-stream pressure, free-stream temperature, and the coefficient of viscosity are properties of the atmosphere and are assumed to be functions of altitude alone.

1.3.3 Flightpath-related parameters.—Included in the observation variables are what might best be termed flightpath-related parameters for lack of better nomenclature. These terms include flightpath angle γ, flightpath acceleration fpa, and vertical acceleration $\ddot{h}$. The variables are defined by the following equations:

$$\gamma = \sin^{-1}\left(\frac{\dot{h}}{V}\right) \tag{1-106}$$

$$\mathrm{fpa} = \frac{\dot{V}}{g} \tag{1-107}$$

$$\ddot{h} = a_{x,\mathrm{k}} \sin\theta - a_{y,\mathrm{k}} \sin\phi\cos\theta - a_{z,\mathrm{k}} \cos\phi\cos\theta \tag{1-108}$$

1.3.4 Energy-related parameters.—Two energy-related parameters are included with the observation variables considered in this report: specific energy E_s, and specific power P_s, defined as

$$E_s = h + \frac{V^2}{2g} \tag{1-109}$$

$$P_s = \frac{dE_s}{dt} = \dot{h} + \frac{V\dot{V}}{g} \tag{1-110}$$

1.3.5 Force parameters.—The set of observation variables being considered also includes four force parameters. These quantities are total aerodynamic lift L, total aerodynamic drag D, total aerodynamic normal force N, and total aerodynamic axial force A, defined as

$$L = \bar{q}SC_{\mathrm{L}} \tag{1-111}$$

$$D = \bar{q}SC_{\mathrm{D}} \tag{1-112}$$

$$N = L\cos\alpha + D\sin\alpha \tag{1-113}$$

$$A = -L\sin\alpha + D\cos\alpha \tag{1-114}$$

where S is the surface area of the wing, C_{L} coefficient of lift, and C_{D} coefficient of drag.

1.3.6 Body axis rates and accelerations.—Because they are of interest in the control analysis and design problem, six body axis rates and accelerations are included as observation variables. These include the x body axis rate u, the y body axis rate v, and the z body axis rate w. Also included are the time derivatives of these quantities, $\dot{u}$, $\dot{v}$, and $\dot{w}$, respectively.

The definitions of the body axis rates are given in equations (1-9) to (1-11) as

$$\begin{aligned} u &= V\cos\alpha\cos\beta \\ v &= V\sin\beta \\ w &= V\sin\alpha\cos\beta \end{aligned}$$

The time derivatives of these terms can be defined using equation (B-1) and equations (B-8), (B-9), (B-10), and (1-56) as

$$\dot{u} = \frac{X_{\mathrm{T}} - gm\sin\theta - D\cos\alpha + L\sin\alpha}{m} + rV\sin\beta - qV\sin\alpha\cos\beta \tag{1-115}$$

$$\dot{v} = \frac{Y_{\mathrm{T}} + gm\sin\phi\cos\theta + Y}{m} + pV\sin\alpha\cos\beta - rV\cos\alpha\cos\beta \tag{1-116}$$

$$\dot{w} = \frac{Z_{\mathrm{T}} + gm\cos\phi\cos\theta - D\sin\alpha - L\cos\alpha}{m} + qV\cos\alpha\cos\beta - pV\sin\beta \tag{1-117}$$

1.3.7 Instruments displaced from the vehicle center of gravity.—The need to include measurements from instruments displaced from the vehicle center of gravity arises from the fact that not all aircraft instrumentation is located at the vehicle center of gravity. The most important of these quantities are undoubtedly the accelerometer outputs treated in section 1.3.1. In this section four additional parameters are presented: angle of attack ($\alpha_{,i}$), angle of sideslip ($\beta_{,i}$), altitude ($h_{,i}$), and altitude rate ($\dot{h}_{,i}$) measurements from instruments displaced from center of gravity by some x, y, and z body axis distances. The subscripts α, β, h, and $\dot{h}$ refer to the displacements of the angle-of-attack, angle-of-sideslip, altitude, and altitude rate instruments from the vehicle center of gravity. The equations used to compute these quantities are

$$\alpha_{,i} = \alpha + \frac{qx_\alpha - py_\alpha}{V} \tag{1-118}$$

$$\beta_{,i} = \beta + \frac{rx_\beta - pz_\beta}{V} \tag{1-119}$$

$$h_{,i} = h + x_h \sin\theta - y_h \sin\phi\cos\theta - z_h \cos\phi\cos\theta \tag{1-120}$$

$$\dot{h}_{,i} = \dot{h} + \dot{\theta}(x_{\dot{h}}\cos\theta + y_{\dot{h}}\sin\phi\sin\theta + z_{\dot{h}}\cos\phi\sin\theta) - \dot{\phi}(y_{\dot{h}}\cos\phi\cos\theta - z_{\dot{h}}\sin\phi\cos\theta) \tag{1-121}$$

1.3.8 Miscellaneous observation parameters.—The final set of observation parameters considered in this report is a miscellaneous collection of parameters of interest in analysis and design problems. These parameters are total angular momentum T, stability axis roll rate p_s, stability axis pitch rate q_s, and stability axis yaw rate r_s. The equations used to define these quantities are

$$T = \frac{1}{2}(I_x p^2 - 2I_{xy}pq - 2I_{xz}pr + I_y q^2 - 2I_{yz}qr + I_z r^2) \tag{1-122}$$

$$p_s = p\cos\alpha + r\sin\alpha \tag{1-123}$$

$$q_s = q \tag{1-124}$$

$$r_s = -p\sin\alpha + r\cos\alpha \tag{1-125}$$

2 LINEAR SYSTEM EQUATIONS

The standard state equation for a linear differential system has the form

$$\dot{\mathbf{x}}(t) = A'\mathbf{x}(t) + B'\mathbf{u}(t) \tag{2-1}$$

where, for a time-invariant system, A' is a constant $n \times n$ matrix and B' is a constant $n \times k$ matrix. The standard output equation has the form

$$\mathbf{y}(t) = H'\mathbf{x}(t) + F'\mathbf{u}(t) \tag{2-2}$$

where H' is a constant $\ell \times n$ matrix and F' is a constant $\ell \times k$ matrix. The generalized linear system equations used with an extended formulation compatible with the generalized nonlinear equations (1-3) and (1-4) can be characterized by

$$C\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \tag{2-3}$$

$$\mathbf{y}(t) = H\mathbf{x}(t) + G\dot{\mathbf{x}}(t) + F\mathbf{u}(t) \tag{2-4}$$

where C and A are constant $n \times n$ matrices, B is a constant $n \times k$ matrix, H and G are constant $\ell \times n$ matrices, and F is a constant $\ell \times k$ matrix. The nonlinear system equations developed in section 1 (eqs. (1-1) to (1-4)) can be linearized about a trajectory, and a linear model can be formulated that is similar to either the standard or the generalized linear system equations.

2.1 Linearization of the State Equation

If $\mathbf{u}_0(t)$ is given input to a system described by the state differential equation (1-3), and if $\mathbf{x}_0(t)$ is a known solution of the state differential equation, then approximations to the neighboring solutions can be found for small deviations in the initial state and in the input by using a linear state differential equation. The nonlinear state differential equation (1-3) can be linearized about a general trajectory, as by Kwakernaak and Sivan (1972) and Dieudonne (1978), so that $\mathbf{x}_0(t)$ satisfies

$$T\dot{\mathbf{x}}_0(t) = \mathbf{f}[\dot{\mathbf{x}}_0(t), \mathbf{x}_0(t), \mathbf{u}_0(t)]$$

Assuming that the system is operated at close to nominal conditions with $\mathbf{u}(t)$, $\mathbf{x}(t)$, and $\dot{\mathbf{x}}(t)$ deviating only slightly from $\mathbf{u}_0(t)$, $\mathbf{x}_0(t)$, and $\dot{\mathbf{x}}_0(t)$, the following expressions can be written:

$$\mathbf{u}(t) = \mathbf{u}_0(t) + \delta\mathbf{u}(t) \tag{2-5}$$

$$\mathbf{x}(t) = \mathbf{x}_0(t) + \delta\mathbf{x}(t) \tag{2-6}$$

$$\dot{\mathbf{x}}(t) = \dot{\mathbf{x}}_0(t) + \delta\dot{\mathbf{x}}(t) \tag{2-7}$$

where $\delta\mathbf{u}(t)$, $\delta\mathbf{x}(t)$, and $\delta\dot{\mathbf{x}}(t)$ are small perturbations to the control, state, and time derivative of the state vectors, respectively.

Substituting equations (2-5) to (2-7) into the nonlinear state differential equation (1-3), expanding in a Taylor series about $\dot{\mathbf{x}}_0(t)$, $\mathbf{x}_0(t)$, $\mathbf{u}_0(t)$, and assuming T constant with respect to $\dot{\mathbf{x}}(t)$ yields

$$T[\dot{\mathbf{x}}_0(t) + \delta\dot{\mathbf{x}}(t)] = \mathbf{f}[\mathbf{x}_0(t), \dot{\mathbf{x}}_0(t), \mathbf{u}(t)] + \frac{\partial\mathbf{f}}{\partial\mathbf{x}}\,\delta\mathbf{x} + \frac{\partial\mathbf{f}}{\partial\dot{\mathbf{x}}}\,\delta\dot{\mathbf{x}} + \frac{\partial\mathbf{f}}{\partial\mathbf{u}}\,\delta\mathbf{u} + \mathbf{h}(t) \tag{2-8}$$

where $\partial\mathbf{f}/\partial\mathbf{x}$, $\partial\mathbf{f}/\partial\dot{\mathbf{x}}$, and $\partial\mathbf{f}/\partial\mathbf{u}$ are defined in equations (2-9) to (2-11) and $\mathbf{h}(t)$ represents the sum of the higher order terms in the Taylor series, assumed to be small with respect to the perturbations. The matrices used in the Taylor series expansion are defined by the following relationships:

$$\frac{\partial\mathbf{f}}{\partial\mathbf{x}} \equiv \left.\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right|_{(\mathbf{x}_0,\dot{\mathbf{x}}_0,\mathbf{u}_0)} \tag{2-9}$$

$$\frac{\partial\mathbf{f}}{\partial\dot{\mathbf{x}}} \equiv \left.\frac{\partial\mathbf{f}}{\partial\dot{\mathbf{x}}}\right|_{(\mathbf{x}_0,\dot{\mathbf{x}}_0,\mathbf{u}_0)} \tag{2-10}$$

$$\frac{\partial\mathbf{f}}{\partial\mathbf{u}} \equiv \left.\frac{\partial\mathbf{f}}{\partial\mathbf{u}}\right|_{(\mathbf{x}_0,\dot{\mathbf{x}}_0,\mathbf{u}_0)} \tag{2-11}$$

the (i,j)th elements of which are defined as

$$\left(\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right)_{i,j} = \frac{\partial\mathbf{f}_i}{\partial\mathbf{x}_j} \tag{2-12}$$

$$\left(\frac{\partial\mathbf{f}}{\partial\dot{\mathbf{x}}}\right)_{i,j} = \frac{\partial\mathbf{f}_i}{\partial\dot{\mathbf{x}}_j} \tag{2-13}$$

$$\left(\frac{\partial\mathbf{f}}{\partial\mathbf{u}}\right)_{i,j} = \frac{\partial\mathbf{f}_i}{\partial\mathbf{u}_j} \tag{2-14}$$

respectively, where $\mathbf{f}_i$ is the ith simultaneous equation of the nonlinear state differential function in equation (1-3), $\mathbf{x}_j$ the jth element of the state vector, $\dot{\mathbf{x}}_j$ the jth element of the time derivative of the state vector, $\dot{u}_j$ the jth element of the control vector, and all derivatives are evaluated at the nominal condition $(\mathbf{x}_0(t), \dot{\mathbf{x}}_0(t), \mathbf{u}_0(t))$.

Subtracting equation (1-3) from (2-8), rearranging terms and neglecting the higher order terms yields a linearized state equation,

$$\left[T - \frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}}\right] \delta\dot{\mathbf{x}}(t) = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\, \delta\mathbf{x}(t) + \frac{\partial \mathbf{f}}{\partial \mathbf{u}}\, \delta\mathbf{u}(t) \tag{2-15}$$

where the arguments of the matrix functions have been dropped to simplify the notation and where it is understood that the matrices are to be evaluated along the nominal trajectory.

Letting

$$C = T - \frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}} \tag{2-16}$$

$$A = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \tag{2-17}$$

$$B = \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \tag{2-18}$$

equation (2-15) can be written as

$$C\, \delta\dot{\mathbf{x}}(t) = A\, \delta\mathbf{x}(t) + B\, \delta\mathbf{u}(t) \tag{2-19}$$

which is precisely the formulation of the generalized state equation desired.

Premultiplying both sides of equation (2-19) by C^{-1} results in the standard form of the linearized state differential equation,

$$\delta\dot{\mathbf{x}}(t) = C^{-1}A\, \delta\mathbf{x}(t) + C^{-1}B\, \delta\mathbf{u}(t) \tag{2-20}$$

Letting

$$A' = C^{-1}A \tag{2-21}$$

$$B' = C^{-1}B \tag{2-22}$$

equation (2-20) can be written in the more usual notation

$$\delta\dot{\mathbf{x}}(t) = A'\, \delta\mathbf{x}(t) + B'\, \delta\mathbf{u}(t) \tag{2-23}$$

2.2 Linearization of the Observation Equation

The technique used in section 2.1 to linearize the state equations can be applied to the nonlinear observation equation (1-4),

$$\mathbf{y}(t) = \mathbf{g}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)]$$

Performing a Taylor series expansion about the nominal trajectory $(\mathbf{x}_0(t), \dot{\mathbf{x}}_0(t), \mathbf{u}_0(t))$ yields

$$\mathbf{y}_0(t) + \delta\mathbf{y}(t) = \mathbf{g}[\mathbf{x}_0(t), \dot{\mathbf{x}}_0(t), \mathbf{u}_0(t)] + \frac{\partial \mathbf{g}}{\partial \mathbf{x}}\, \delta\mathbf{x} + \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}}\, \delta\dot{\mathbf{x}} + \frac{\partial \mathbf{g}}{\partial \mathbf{u}}\, \delta\mathbf{u} + \mathbf{h}(t) \tag{2-24}$$

where

$$\frac{\partial \mathbf{g}}{\partial \mathbf{x}} \equiv \left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{(\mathbf{x}_0, \dot{\mathbf{x}}_0, \mathbf{u}_0)} \tag{2-25}$$

$$\frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}} \equiv \left.\frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}}\right|_{(\mathbf{x}_0,\dot{\mathbf{x}}_0,\mathbf{u}_0)} \tag{2-26}$$

$$\frac{\partial \mathbf{g}}{\partial \mathbf{u}} \equiv \left.\frac{\partial \mathbf{g}}{\partial \mathbf{u}}\right|_{(\mathbf{x}_0,\dot{\mathbf{x}}_0,\mathbf{u}_0)} \tag{2-27}$$

the (i,j)th elements of which are defined by

$$\left(\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right)_{i,j} = \frac{\partial \mathbf{g}_i}{\partial \mathbf{x}_j} \tag{2-28}$$

$$\left(\frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}}\right)_{i,j} = \frac{\partial \mathbf{g}_i}{\partial \dot{\mathbf{x}}_j} \tag{2-29}$$

$$\left(\frac{\partial \mathbf{g}}{\partial \mathbf{u}}\right)_{i,j} = \frac{\partial \mathbf{g}_i}{\partial \mathbf{u}_j} \tag{2-30}$$

respectively, where $\mathbf{g}_i$ is the ith simultaneous equation of the nonlinear observation equation (1-4). Again, all derivatives are evaluated at the nominal condition $(\mathbf{x}_0(t),\ \dot{\mathbf{x}}_0(t),\ \mathbf{u}_0(t))$.

Subtracting equation (1-4) from equation (2-24), rearranging terms, and neglecting higher order terms results in a linear observation equation,

$$\delta \mathbf{y}(t) = \frac{\partial \mathbf{g}}{\partial \mathbf{x}}\,\delta \mathbf{x} + \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}}\,\delta \dot{\mathbf{x}} + \frac{\partial \mathbf{g}}{\partial \mathbf{u}}\,\delta \mathbf{u} \tag{2-31}$$

where the arguments of the matrix functions have been dropped to simplify notation. Letting

$$H = \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \tag{2-32}$$

$$G = \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}} \tag{2-33}$$

$$F = \frac{\partial \mathbf{g}}{\partial \mathbf{u}} \tag{2-34}$$

equation (2-31) can be rewritten as

$$\delta \mathbf{y}(t) = H\,\delta \mathbf{x}(t) + G\,\delta \dot{\mathbf{x}}(t) + F\,\delta \mathbf{u}(t) \tag{2-35}$$

which is the generalized linear observation equation desired.

The standard form of the observation equation can be derived by substituting for $\delta\dot{\mathbf{x}}$ from equation (2-23) into equation (2-33). This substitution results in

$$\delta \mathbf{y}(t) = H\,\delta \mathbf{x}(t) + G[A'\,\delta \mathbf{x}(t) + B'\,\delta \mathbf{u}(t)] + F\,\delta \mathbf{u}(t) \tag{2-36}$$

which can be written as

$$\delta \mathbf{y}(t) = [H + GA']\,\delta \mathbf{x}(t) + [F + GB']\,\delta \mathbf{u}(t) \tag{2-37}$$

By letting

$$H' = H + GA' \tag{2-38}$$

$$F' = F + GB' \tag{2-39}$$

equation (2-37) becomes

$$\delta\mathbf{y}(t) = H'\,\delta\mathbf{x}(t) + F'\,\delta\mathbf{u}(t) \tag{2-40}$$

2.3 Definition of Matrices in Linearized System Equations

The results of sections 2.1 and 2.2 can be used to define the matrices in the linearized system equations in terms of partial derivatives of the nonlinear state and observation functions taken with respect to the state, time derivative of state, and control vectors. All derivatives are understood to be evaluated along the nominal trajectory.

Using the nonlinear state equation (1-3),

$$T\mathbf{x}(t) = \mathbf{f}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)]$$

the terms in the generalized form of the linearized state equation (2-19),

$$C\,\delta\dot{\mathbf{x}}(t) = A\,\delta\mathbf{x}(t) + B\,\delta\mathbf{u}(t)$$

can be defined as

$$C = T - \frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}} \tag{2-41}$$

$$A = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \tag{2-42}$$

$$B = \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \tag{2-43}$$

The terms in the standard form of the linearized state equation (2-20),

$$\delta\dot{\mathbf{x}}(t) = A'\,\delta\mathbf{x}(t) + B'\,\delta\mathbf{u}(t)$$

can be defined as

$$A' = \left[T - \frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}}\right]^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \tag{2-44}$$

$$B' = \left[T - \frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}}\right]^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \tag{2-45}$$

In a similar manner, the nonlinear observation equation (1-4),

$$\mathbf{y}(t) = \mathbf{g}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)]$$

can be used to define the terms of the generalized linearized observation equation (2-35),

$$\delta\mathbf{y}(t) = H\,\delta\mathbf{x}(t) + G\,\delta\dot{\mathbf{x}}(t) + F\,\delta\mathbf{u}(t)$$

as

$$H = \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \tag{2-46}$$

$$G = \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}} \tag{2-47}$$

$$F = \frac{\partial \mathbf{g}}{\partial \mathbf{u}} \tag{2-48}$$

The terms in the standard form of the linearized observation equation (2-40),

$$\delta \mathbf{y}(t) = H' \, \delta \mathbf{x}(t) + F' \, \delta \mathbf{u}(t)$$

can be defined as

$$H' = \frac{\partial \mathbf{g}}{\partial \mathbf{x}} + \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}} \left[T - \frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}} \right]^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \tag{2-49}$$

$$F' = \frac{\partial \mathbf{g}}{\partial \mathbf{u}} + \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}} \left[T - \frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}} \right]^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \tag{2-50}$$

2.4 Elements of the Linearized System Matrices

The elements of the linearized system matrices derived in sections 2.1 and 2.2 are determined by applying the linearization method employed with the vector equations in those sections to the individual scalar equations constituting the vector equations that define the time derivatives of the state and observation variables. Thus, for a matrix, such as the state matrix A defined by equation (2-42),

$$A = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$$

the element occupying the ith row and jth column of A, $(A)_{i,j}$, can be represented as

$$(A)_{i,j} = \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \tag{2-51}$$

where $\mathbf{f}_i$ is the scalar function defining the time derivative of the ith state and $\mathbf{x}_j$ is the jth state. The individual terms used in the A, B, C, H, G, and F matrices are defined in appendix D based on the generalized derivatives derived in appendix C.

Using the state vector $\mathbf{x}$ defined in (1-7) as

$$\mathbf{x} = [p \; q \; r \; V \; \alpha \; \beta \; \phi \; \theta \; \psi \; h \; x \; y]^{\mathrm{T}}$$

the elements of the A matrix can be expressed as

$$A = \begin{bmatrix} \partial(\dot{p}')/\partial p & \partial(\dot{p}')/\partial q & \cdots & \partial(\dot{p}')/\partial y \\ \partial(\dot{q}')/\partial p & \partial(\dot{q}')/\partial q & \cdots & \partial(\dot{q}')/\partial y \\ \vdots & \vdots & & \vdots \\ \partial(\dot{x})/\partial p & \partial(\dot{x})/\partial q & \cdots & \partial(\dot{x})/\partial y \\ \partial(\dot{y})/\partial p & \partial(\dot{y})/\partial q & \cdots & \partial(\dot{y})/\partial y \end{bmatrix} \tag{2-52}$$

Substituting for these partial derivatives using the terms in appendix D gives

$$A = \begin{bmatrix} (1/I_x)[(\bar{q}Sb^2/2V_0)C_{\ell_p} + \partial L_T/\partial p & (1/I_x)[(\bar{q}Sb\bar{c}/2V_0)C_{\ell_q} + \partial L_T/\partial_q + I_{xz}p_0 & \cdots \\ \quad -I_{xy}r_0 + I_{xz}q_0] & \quad +2I_{yz}q_0 + r_0(I_y - I_z)] & \\ (1/I_y)[(\bar{q}Sb\bar{c}/2V_0)C_{m_p} + \partial M_T/\partial p & (1/I_y)(\bar{q}S\bar{c}^2/2V_0)C_{m_q} + \partial M_T/\partial q & \cdots \\ \quad -2I_{xz}p_0 - I_{yz}q_0 + r_0(I_z - I_x)] & \quad +I_{xy}r_0 - I_{yz}p_0] & \\ \vdots & \vdots & \end{bmatrix} \tag{2-53}$$

The elements of the B, C, H, G, and F matrices can be determined in a similar fashion, although some care must be taken in determining the elements of the matrices for the observation equation and the C matrix.

To determine the elements of the matrices for the observation equation, one must consider the definition of the nonlinear vector function $\mathbf{g}$ defining the observation variables (eq. (1-85)),

$$\mathbf{g}[\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{u}(t)] = [\mathbf{x}^{\mathrm{T}}\ \dot{\mathbf{x}}^{\mathrm{T}}\ \mathbf{u}^{\mathrm{T}}\ \mathbf{g}'^{\mathrm{T}}]$$

and the definitions of the matrices for the generalized linear observation equations (2-46) to (2-48) ,

$$H = \frac{\partial \mathbf{g}}{\partial \mathbf{x}}$$

$$G = \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{x}}}$$

$$F = \frac{\partial \mathbf{g}}{\partial \mathbf{u}}$$

These matrices may be expressed using a partitioning based on the vector subfunctions of $\mathbf{g}$ as

$$H = \begin{bmatrix} \frac{\partial \mathbf{x}}{\partial \mathbf{x}} \\ -- \\ \frac{\partial \dot{\mathbf{x}}}{\partial \mathbf{x}} \\ -- \\ \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \\ -- \\ \frac{\partial \mathbf{g}'}{\partial \mathbf{x}} \end{bmatrix} \tag{2-54}$$

$$G = \begin{bmatrix} \frac{\partial \mathbf{x}}{\partial \dot{\mathbf{x}}} \\ -- \\ \frac{\partial \dot{\mathbf{x}}}{\partial \dot{\mathbf{x}}} \\ -- \\ \frac{\partial \mathbf{u}}{\partial \dot{\mathbf{x}}} \\ -- \\ \frac{\partial \mathbf{g}'}{\partial \dot{\mathbf{x}}} \end{bmatrix} \tag{2-55}$$

$$F = \begin{bmatrix} \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \\ -- \\ \frac{\partial \dot{\mathbf{x}}}{\partial \mathbf{u}} \\ -- \\ \frac{\partial \mathbf{u}}{\partial \mathbf{u}} \\ -- \\ \frac{\partial \mathbf{g}'}{\partial \mathbf{u}} \end{bmatrix} \tag{2-56}$$

which become

$$H = \begin{bmatrix} 1_{12\times12} \\ ---- \\ 0_{12\times12} \\ ---- \\ 0_{k\times12} \\ ---- \\ \frac{\partial \mathbf{g}'}{\partial \mathbf{x}} \end{bmatrix} \tag{2-57}$$

$$G = \begin{bmatrix} 0_{12\times12} \\ ---- \\ 1_{12\times12} \\ ---- \\ 0_{k\times12} \\ ---- \\ \frac{\partial \mathbf{g}'}{\partial \dot{\mathbf{x}}} \end{bmatrix} \tag{2-58}$$

$$F = \begin{bmatrix} 0_{12\times k} \\ ---- \\ 0_{12\times k} \\ ---- \\ 1_{k\times k} \\ ---- \\ \frac{\partial \mathbf{g}'}{\partial \mathbf{u}} \end{bmatrix} \tag{2-59}$$

upon evaluating the partial derivatives of the identity functions $\mathbf{x}$, $\dot{\mathbf{x}}$, and $\mathbf{u}$.

The C matrix may be viewed as a partitioned matrix as

$$C = \left[\begin{array}{cc|c} C_{11} & C_{12} & \\ \hline 0_{3\times3} & C_{22} & 0_{6\times6} \\ \hline \multicolumn{2}{c|}{0_{6\times6}} & 1_{6\times6} \end{array}\right] \tag{2-60}$$

where, from equation (1-48),

$$C_{11} = J = \begin{bmatrix} 1.0 & -I_{xy}/I_x & -I_{xz}/I_x \\ -I_{xz}/I_y & 1.0 & -I_{xz}/I_y \\ -I_{xz}/I_z & -I_{yz}/I_z & 1.0 \end{bmatrix} \tag{2-61}$$

and

$$C_{12} = \begin{bmatrix} -\partial(\dot{p}')/\partial\dot{V} & -\partial(\dot{p}')/\partial\dot{\alpha} & -\partial(\dot{p}')/\partial\dot{\beta} \\ -\partial(\dot{q}')/\partial\dot{V} & -\partial(\dot{q}')/\partial\dot{\alpha} & -\partial(\dot{q}')/\partial\dot{\beta} \\ -\partial(\dot{r}')/\partial\dot{V} & -\partial(\dot{r}')/\partial\dot{\alpha} & -\partial(\dot{r}')/\partial\dot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & -(\bar{q}Sb\bar{c}/2V_0I_x)C_{\ell_{\dot{\alpha}}} & -(\bar{q}Sb^2/2V_0I_x)C_{\ell_{\dot{\beta}}} \\ 0 & -(\bar{q}S\bar{c}^2/2V_0I_y)C_{m_{\dot{\alpha}}} & -(\bar{q}Sb\bar{c}/2V_0I_y)C_{m_{\dot{\beta}}} \\ 0 & -(\bar{q}Sb\bar{c}/2V_0I_z)C_{n_{\dot{\alpha}}} & -(\bar{q}Sb^2/2V_0I_z)C_{n_{\dot{\beta}}} \end{bmatrix} \tag{2-62}$$

$$C_{22} = \begin{bmatrix} 1.0 - \partial(\dot{V})/\partial\dot{V} & -\partial(\dot{V})/\partial\dot{\alpha} & -\partial(\dot{V})/\partial\dot{\beta} \\ -\partial(\dot{\alpha})/\partial\dot{V} & 1.0 - \partial(\dot{\alpha})/\partial\dot{\alpha} & -\partial(\dot{\alpha})/\partial\dot{\beta} \\ -\partial(\dot{\beta})/\partial\dot{V} & -\partial(\dot{\beta})/\partial\dot{\alpha} & 1.0 - \partial(\dot{\beta})/\partial\dot{\beta} \end{bmatrix}$$

$$= \begin{bmatrix} 1.0 & (\bar{q}S\bar{c}/2V_0m)(\cos\beta_0\, C_{D_{\dot{\alpha}}} - \sin\beta_0\, C_{Y_{\dot{\alpha}}}) & (\bar{q}Sb/2V_0m)(\cos\beta_0\, C_{D_{\dot{\beta}}}) \\ 0 & 1.0 + (\bar{q}S\bar{c}/2V_0^2 m\cos\beta_0)C_{L_{\dot{\alpha}}} & (\bar{q}Sb/2V_0^2 m\cos\beta_0)C_{L_{\dot{\beta}}} \\ 0 & (\bar{q}S\bar{c}/2V_0^2m)(\sin\beta_0\, C_{D_{\dot{\alpha}}} + \cos\beta_0\, C_{Y_{\dot{\alpha}}}) & 1.0 - (\bar{q}Sb/2V_0^2m)(\sin\beta_0\, C_{D_{\dot{\beta}}} + \cos\beta_0\, C_{Y_{\dot{\beta}}}) \end{bmatrix} \tag{2-63}$$

The inverse of the C matrix, C^{-1}, can be expressed as a partitioned matrix in terms of the matrix subpartitions of the C matrix as

$$C^{-1} = \left[\begin{array}{cc|c} C_{11}^{-1} & -C_{11}^{-1}C_{12}C_{22}^{-1} & \\ \cline{1-2} 0_{3\times 3} & C_{22}^{-1} & 0_{6\times 6} \\ \hline \multicolumn{2}{c|}{0_{6\times 6}} & 1_{6\times 6} \end{array}\right] \tag{2-64}$$

The elements of the A', B', H', and F' matrices can be determined using the C^{-1} matrix defined in equation (2-64), the A, B, H, G, and F matrices, and the definitions for A', B', H', and F' given in equations (2-21), (2-22), (2-38), and (2-39).

3 CONCLUDING REMARKS

This report derives and defines a set of linearized system matrices for a rigid aircraft of constant mass, flying in a stationary atmosphere over a flat, nonrotating earth. Both generalized and standard linear system equations are derived from nonlinear six-degree-of-freedom equations of motion and a large collection of nonlinear observation (measurement) equations.

This derivation of a linear model is general and makes no assumptions on either the reference (nominal) trajectory about which the model is linearized or the symmetry of the vehicle mass and aerodynamic properties.

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APPENDIX A—AERODYNAMIC FORCES AND MOMENTS

The aerodynamic forces and moments acting on an aircraft are the result of multiple factors whose significance varies with flight condition as well as from vehicle to vehicle. In general, these forces and moments are nonlinear functions primarily of Mach number, angle of attack, angle of sideslip, altitude, rotational rates, and control-surface deflections. For the purposes of this report, the aerodynamic forces and moments are assumed to be functions having the following form:

$$F = \Phi(\alpha, \beta, V, h, p, q, r, \dot{\alpha}, \dot{\beta}, \delta_1, \ldots, \delta n) \tag{A-1}$$

where F is an arbitrary force or moment, Φ is an arbitrary function, and the δ_i are the n control surface deflections. These forces and moments are related to the nondimensional force and moment coefficients by the equations for the forces,

$$D = \bar{q}SC_{\mathrm{D}} \tag{A-2}$$

$$Y = \bar{q}SC_{\mathrm{Y}} \tag{A-3}$$

$$L = \bar{q}SC_{\mathrm{L}} \tag{A-4}$$

and the moments,

$$L = \bar{q}SbC_{\ell} \tag{A-5}$$

$$M = \bar{q}S\bar{c}C_m \tag{A-6}$$

$$N = \bar{q}SbC_n \tag{A-7}$$

where b is reference span and $\bar{c}$ is reference aerodynamic chord.

While the nondimensional aerodynamic force and moment coefficients are themselves nonlinear functions of the vehicle states, time derivatives of the vehicle states, and the control surface deflections, these coefficients are commonly expressed in linear form in terms of partial derivatives of these coefficients with respect to the functional variables. These linear equations for the aerodynamic force and moment coefficients are derived in the same way as the linearized system equations (section 2); therefore, this derivation will not be repeated here. These linear equations are

$$\begin{aligned} C_{\mathrm{L}} = {} & C_{\mathrm{L_0}} + C_{\mathrm{L_\alpha}}\alpha + C_{\mathrm{L_\beta}}\beta + C_{\mathrm{L_h}}h + C_{\mathrm{L_V}}V \\ & + \sum_{i=1}^{n} C_{\mathrm{L_{\delta_i}}}\delta_i + C_{\mathrm{L_p}}\hat{p} + C_{\mathrm{L_q}}\hat{q} + C_{\mathrm{L_r}}\hat{r} + C_{\mathrm{L_{\dot\alpha}}}\hat{\dot{\alpha}} + C_{\mathrm{L_{\dot\beta}}}\hat{\dot{\beta}} \end{aligned} \tag{A-8}$$

$$\begin{aligned} C_{\mathrm{D}} = {} & C_{\mathrm{D_0}} + C_{\mathrm{D_\alpha}}\alpha + C_{\mathrm{D_\beta}}\beta + C_{\mathrm{D_h}}h + C_{\mathrm{D_V}}V \\ & + \sum_{i=1}^{n} C_{\mathrm{D_{\delta_i}}}\delta_i + C_{\mathrm{D_p}}\hat{p} + C_{\mathrm{D_q}}\hat{q} + C_{\mathrm{D_r}}\hat{r} + C_{\mathrm{D_{\dot\alpha}}}\hat{\dot{\alpha}} + C_{\mathrm{D_{\dot\beta}}}\hat{\dot{\beta}} \end{aligned} \tag{A-9}$$

$$\begin{aligned} C_{\mathrm{Y}} = {} & C_{\mathrm{Y_0}} + C_{\mathrm{Y_\alpha}}\alpha + C_{\mathrm{Y_\beta}}\beta + C_{\mathrm{Y_h}}h + C_{\mathrm{Y_V}}V \\ & + \sum_{i=1}^{n} C_{\mathrm{Y_{\delta_i}}}\delta_i + C_{\mathrm{Y_p}}\hat{p} + C_{\mathrm{Y_q}}\hat{q} + C_{\mathrm{Y_r}}\hat{r} + C_{\mathrm{Y_{\dot\alpha}}}\hat{\dot{\alpha}} + C_{\mathrm{Y_{\dot\beta}}}\hat{\dot{\beta}} \end{aligned} \tag{A-10}$$

$$\begin{aligned} C_{\ell} = {} & C_{\ell_0} + C_{\ell_\alpha}\alpha + C_{\ell_\beta}\beta + C_{\ell_h}h + C_{\ell_V}V \\ & + \sum_{i=1}^{n} C_{\ell_{\delta_i}}\delta_i + C_{\ell_p}\hat{p} + C_{\ell_q}\hat{q} + C_{\ell_r}\hat{r} + C_{\ell_{\dot\alpha}}\hat{\dot{\alpha}} + C_{\ell_{\dot\beta}}\hat{\dot{\beta}} \end{aligned} \tag{A-11}$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_\beta}\beta + C_{m_h}h + C_{m_V}V$$
$$+ \sum_{i=1}^{n} C_{m_{\delta_i}}\delta_i + C_{m_p}\hat{p} + C_{m_q}\hat{q} + C_{m_r}\hat{r} + C_{m_{\dot\alpha}}\hat{\dot\alpha} + C_{m_{\dot\beta}}\hat{\dot\beta} \tag{A-12}$$

$$C_n = C_{n_0} + C_{n_\alpha}\alpha + C_{n_\beta}\beta + C_{n_h}h + C_{n_V}V$$
$$+ \sum_{i=1}^{n} C_{n_{\delta_i}}\delta_i + C_{n_p}\hat{p} + C_{n_q}\hat{q} + C_{n_r}\hat{r} + C_{n_{\dot\alpha}}\hat{\dot\alpha} + C_{n_{\dot\beta}}\hat{\dot\beta} \tag{A-13}$$

where C_{ξ_0} is the value of the coefficient along the nominal trajectory and the notation C_{ξ_x} is defined as

$$C_{\xi_x} \equiv \frac{\partial C_\xi}{\partial x} \tag{A-14}$$

with C_ξ being an arbitrary force or moment coefficient and x being an arbitrary state, time derivative of state, or control-related parameter that for the usual derivatives is nondimensional. However, the derivatives with respect to altitude and velocity are not taken with respect to a nondimensional quantity. The definitions of these nondimensional stability and control derivatives are given in terms of the coefficient C_ξ. The nondimensional stability derivatives are defined as

$$C_{\xi_\alpha} \equiv \frac{\partial C_\xi}{\partial \alpha} \tag{A-15}$$

$$C_{\xi_\beta} \equiv \frac{\partial C_\xi}{\partial \beta} \tag{A-16}$$

$$C_{\xi_p} \equiv \frac{\partial C_\xi}{\partial (bp/2V_0)} \tag{A-17}$$

$$C_{\xi_q} \equiv \frac{\partial C_\xi}{\partial (\bar{c}q/2V_0)} \tag{A-18}$$

$$C_{\xi_r} \equiv \frac{\partial C_\xi}{\partial (br/2V_0)} \tag{A-19}$$

$$C_{\xi_{\dot\alpha}} \equiv \frac{\partial C_\xi}{\partial (\bar{c}\dot\alpha/2V_0)} \tag{A-20}$$

$$C_{\xi_{\dot\beta}} \equiv \frac{\partial C_\xi}{\partial (b\dot\beta/2V_0)} \tag{A-21}$$

The two other stability derivatives are not nondimensional and are defined as

$$C_{\xi_V} \equiv \frac{\partial C_\xi}{\partial V} \tag{A-22}$$

$$C_{\xi_h} \equiv \frac{\partial C_\xi}{\partial h} \tag{A-23}$$

The control derivatives are defined as

$$C_{\xi_{\delta_i}} \equiv \frac{\partial C_\xi}{\partial \delta_i} \tag{A-24}$$

The rotational terms in equations (A-8) to (A-13) are nondimensional versions of the corresponding variable with

$$\hat{p} = \frac{bp}{2V_0} \tag{A-25}$$

$$\hat{q} = \frac{\bar{c}q}{2V_0} \tag{A-26}$$

$$\hat{r} = \frac{br}{2V_0} \tag{A-27}$$

$$\hat{\dot{\alpha}} = \frac{\bar{c}\dot{\alpha}}{2V_0} \tag{A-28}$$

$$\hat{\dot{\beta}} = \frac{b\dot{\beta}}{2V_0} \tag{A-29}$$

Because the C_{ξ_0} terms are included, the force and moment coefficients are total force and moment coefficients. The state, time derivative of state, and control parameters on the right-hand side of equations (A-8) to (A-13) are differentials.

APPENDIX B—DERIVATION OF THE WIND AXIS TRANSLATIONAL PARAMETERS $\dot{V}$, $\dot{\alpha}$, AND $\dot{\beta}$

The derivation of the wind axis translational acceleration parameters is based primarily on the definitions in equations (1-9) to (1-14), the body axis translational acceleration equations (1-56), and the expression of the force terms defined in equation (1-53). In the following sections, each of the wind axis translational acceleration terms is derived separately after stating some preliminary definitions applicable to all calculations.

B.1 Preliminary Definitions

Equation (1-56),

$$\frac{\delta}{\delta t}\mathbf{V} = \frac{1}{m}\mathbf{F} - \Omega \times \mathbf{V}$$

can be expanded, using equations (1-54), (1-55), and (1-26), to

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} (1/m)(X_{\mathrm{T}} + X_{\mathrm{a}} + X_{\mathrm{g}}) + rv - qw \\ (1/m)(Y_{\mathrm{T}} + Y_{\mathrm{a}} + Y_{\mathrm{g}}) + pw - ru \\ (1/m)(Z_{\mathrm{T}} + Z_{\mathrm{a}} + Z_{\mathrm{g}}) + qu - pv \end{bmatrix} \tag{B-1}$$

The body axis aerodynamic forces can be rewritten in terms of the stability axis forces lift L, drag D, and sideforce Y:

$$X_{\mathrm{a}} = -D\cos\alpha + L\sin\alpha \tag{B-2}$$

$$Y_{\mathrm{a}} = Y \tag{B-3}$$

$$Z_{\mathrm{a}} = -D\sin\alpha - L\cos\alpha \tag{B-4}$$

The gravitational forces can be resolved into body axis components such that

$$X_{\mathrm{g}} = -mg\sin\theta \tag{B-5}$$
$$Y_{\mathrm{g}} = mg\sin\phi\cos\theta \tag{B-6}$$
$$Z_{\mathrm{g}} = mg\cos\phi\cos\theta \tag{B-7}$$

These equations will be used in the derivations of the $\dot{V}$, $\dot{\alpha}$, and $\dot{\beta}$ equations. Thus, the total forces in the body axes can be defined and expanded as

$$\Sigma X = X_{\mathrm{T}} - D\cos\alpha + L\sin\alpha - gm\sin\theta \tag{B-8}$$

$$\Sigma Y = Y_{\mathrm{T}} + Y + gm\sin\phi\cos\theta \tag{B-9}$$

$$\Sigma Z = Z_{\mathrm{T}} - D\sin\alpha - L\cos\alpha + gm\cos\phi\cos\theta \tag{B-10}$$

B.2 Derivation of $\dot{V}$ Equation

Beginning with the definition of V in terms of u, v, and w in equation (1-12),

$$V = (u^2 + v^2 + w^2)^{1/2}$$

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the equation for $\dot{V}$ becomes

$$\dot{V} = \frac{d}{dt}V = \frac{d}{dt}(u^2 + v^2 + w^2)^{1/2} \tag{B-11}$$

which after expanding the derivative and cancelling terms, becomes

$$\dot{V} = \frac{1}{V}(u\dot{u} + v\dot{v} + w\dot{w}) \tag{B-12}$$

By substituting the definitions for u, v, and w from equations (1-9) to (1-11) and cancelling terms, equation (B-12) yields

$$\dot{V} = \dot{u}\cos\alpha\cos\beta + \dot{v}\sin\beta + \dot{w}\sin\alpha\cos\beta \tag{B-13}$$

The definitions for $\dot{u}$, $\dot{v}$, and $\dot{w}$ in equation (B-1) are now used with equation (B-13) to give

$$\begin{aligned}\dot{V} = {} & \frac{\cos\alpha\cos\beta}{m}(X_a + X_T + X_g) + \cos\alpha\cos\beta(rv - qw) \\ & + \frac{\sin\beta}{m}(Y_a + Y_T + Y_g) + \sin\beta(pw - ru) \\ & + \frac{\sin\alpha\cos\beta}{m}(Z_a + Z_T + Z_g) + \sin\alpha\cos\beta(qu - pv)\end{aligned} \tag{B-14}$$

Expanding (B-14) in terms of equations (B-2) through (B-7) and cancelling yields

$$\begin{aligned}\dot{V} = {} & \frac{1}{m}[-D\cos\beta + Y\sin\beta + X_T\cos\alpha\cos\beta + Y_T\sin\beta + Z_T\sin\alpha\cos\beta \\ & - mg(\cos\alpha\cos\beta\sin\theta - \sin\beta\sin\phi\cos\theta - \sin\alpha\cos\beta\cos\phi\cos\theta)] \\ & + rv\cos\alpha\cos\beta - qw\cos\alpha\cos\beta + pw\sin\beta - ru\sin\beta \\ & + qu\sin\alpha\cos\beta - pv\sin\alpha\cos\beta\end{aligned} \tag{B-15}$$

Equation (B-15) can be simplified by recognizing that the terms involving the vehicle rotational rates are identically zero, which becomes obvious after substituting for u, v, and w in these terms. Thus, the final equation becomes

$$\begin{aligned}\dot{V} = {} & \frac{1}{m}[-D\cos\beta + Y\sin\beta + X_T\cos\alpha\cos\beta + Y_T\sin\beta + Z_T\sin\alpha\cos\beta \\ & - mg(\cos\alpha\cos\beta\sin\theta - \sin\beta\sin\phi\cos\theta - \sin\alpha\cos\beta\cos\phi\cos\theta)]\end{aligned} \tag{B-16}$$

B.3 Derivation of $\dot{\alpha}$ Equation

The equation for $\dot{\alpha}$ can be derived from the definition of α in equation (1-13),

$$\alpha = \tan^{-1}\frac{w}{u}$$

Taking the derivative of α with respect to time,

$$\dot{\alpha} = \frac{d}{dt}\alpha = \frac{d}{dt}\tan^{-1}\frac{w}{u} \tag{B-17}$$

then expanding and cancelling terms, the equation becomes

$$\dot{\alpha} = \frac{1}{u^2 + w^2}(u\dot{w} - \dot{u}w) \tag{B-18}$$

Substituting the definitions of u and w from equations (1-9) and (1-11) into equation (B-18) gives

$$\dot{\alpha} = \frac{\dot{w}\cos\alpha - \dot{u}\sin\alpha}{V\cos\beta} \tag{B-19}$$

Using equation (B-1) to substitute for $\dot{u}$ and $\dot{w}$ and equations (B-8) to (B-10) to define the forces, equation (B-19) becomes, after rearranging terms,

$$\begin{aligned}\dot{\alpha} = {} & \frac{1}{Vm\cos\beta}[-L + Z_{\mathrm{T}}\cos\alpha - X_{\mathrm{T}}\sin\alpha + mg(\cos\alpha\cos\phi\cos\theta + \sin\alpha\sin\theta)] \\ & + \frac{1}{V\cos\beta}(qu\cos\alpha - pv\cos\alpha - rv\sin\alpha + qw\sin\alpha)\end{aligned} \tag{B-20}$$

which after substituting for u, v, and w from equations (1-9) to (1-11) and combining terms gives

$$\begin{aligned}\dot{\alpha} = {} & \frac{1}{Vm\cos\beta}[-L + Z_{\mathrm{T}}\cos\alpha - X_{\mathrm{T}}\sin\alpha + mg(\cos\alpha\cos\phi\cos\theta + \sin\alpha\sin\theta)] \\ & + q - \tan\beta\,(p\cos\alpha + r\sin\alpha)\end{aligned} \tag{B-21}$$

B.4 Derivation of $\dot{\beta}$ Equation

The equation for $\dot{\beta}$ is derived from the definition of β as given in equation (1-14),

$$\beta = \sin^{-1}\frac{v}{V}$$

Taking the derivative of β with respect to time yields

$$\dot{\beta} = \frac{d}{dt}\beta = \frac{d}{dt}\sin^{-1}\frac{v}{V} \tag{B-22}$$

which becomes, after expanding the derivative, substituting for V, and cancelling,

$$\dot{\beta} = \frac{1}{V}[-\dot{u}\cos\alpha\sin\beta + \dot{v}\cos\beta - \dot{w}\sin\alpha\sin\beta] \tag{B-23}$$

Using equation (B-1) to substitute for $\dot{u}$, $\dot{v}$, and $\dot{w}$ and equations (B-8) to (B-10) to define the forces,

$$\begin{aligned}\dot{\beta} = {} & \frac{1}{mV}[-\cos\alpha\sin\beta\,(-D\cos\alpha + L\sin\alpha + X_{\mathrm{T}} - mg\sin\theta) + \cos\beta\,(Y + Y_{\mathrm{T}} + mg\sin\phi\cos\theta) \\ & \quad - \sin\alpha\sin\beta\,(-D\sin\alpha - L\cos\alpha + Z_{\mathrm{T}} + mg\cos\phi\cos\theta)] \\ & + \frac{1}{V}[-\cos\alpha\sin\beta\,(rv - qw) + \cos\beta\,(pw - ru) - \sin\alpha\sin\beta\,(qu - pv)]\end{aligned} \tag{B-24}$$

Substituting into equation (B-24) for u, v, and w and rearranging terms yields the final equation

$$\begin{aligned}\dot{\beta} = {} & \frac{1}{mV}[D\sin\beta + Y\cos\beta - X_{\mathrm{T}}\cos\alpha\sin\beta + Y_{\mathrm{T}}\cos\beta - Z_{\mathrm{T}}\sin\alpha\sin\beta \\ & \quad + mg(\cos\alpha\sin\beta\sin\theta + \cos\beta\sin\phi\cos\theta - \sin\alpha\sin\beta\cos\phi\cos\theta)] \\ & + p\sin\alpha - r\cos\alpha\end{aligned} \tag{B-25}$$

APPENDIX C—GENERALIZED DERIVATIVES

The equations defining the time derivatives of the state variables (derived in sections 1.2.1 to 1.2.4) and those defining the observation variables (presented in sections 1.3.1 to 1.3.8) are used to determine the generalized partial derivatives of the quantities with respect to a dummy variable ξ. The purpose of these generalized derivatives is primarily to facilitate the derivation of the terms in the linearized equations presented in section 2.4; however, these equations have also proved to be useful for computer programs and were used to verify the results obtained using LINEAR (see Duke and others, 1987).

C.1 Generalized Derivatives of the Time Derivatives of State Variables

Equations (1-39) to (1-41) define the rotational accelerations of the vehicle. These equations are used to determine the generalized derivatives of these quantities.

$$\begin{aligned}\frac{\partial(\dot{p})}{\partial\xi} = \frac{1}{\det I}\bigg\{& I_1\frac{\partial L}{\partial\xi} + I_2\frac{\partial M}{\partial\xi} + I_3\frac{\partial N}{\partial\xi} + I_1\frac{\partial L_\mathrm{T}}{\partial\xi} + I_2\frac{\partial M_\mathrm{T}}{\partial\xi} + I_3\frac{\partial N_\mathrm{T}}{\partial\xi}\\ &- [2p(I_{xz}I_2 - I_{xy}I_3) - q(I_{xz}I_1 - I_{yz}I_2 - D_zI_3) + r(I_{xy}I_1 + D_yI_2 - I_{yz}I_3)]\frac{\partial p}{\partial\xi}\\ &+ [p(I_{xz}I_1 - I_{yz}I_2 - D_zI_3) + 2q(I_{yz}I_1 - I_{xy}I_3) - r(D_xI_1 - I_{xy}I_2 + I_{xy}I_3)]\frac{\partial q}{\partial\xi}\\ &- [p(I_{xy}I_1 + D_yI_2 - I_{yz}I_3) + q(DI_1 - I_{xy}I_2 + I_{xz}I_3) + 2r(I_{yz}I_1 - I_{xz}I_2)]\frac{\partial r}{\partial\xi}\bigg\}\end{aligned} \tag{C-1}$$

$$\begin{aligned}\frac{\partial(\dot{q})}{\partial\xi} = \frac{1}{\det \mathrm{I}}\bigg\{& I_2\frac{\partial L}{\partial\xi} + I_4\frac{\partial M}{\partial\xi} + I_5\frac{\partial N}{\partial\xi} + I_2\frac{\partial L_\mathrm{T}}{\partial\xi} + I_4\frac{\partial M_\mathrm{T}}{\partial\xi} + I_5\frac{\partial N_\mathrm{T}}{\partial\xi}\\ &- [2p(I_{xz}I_4 - I_{xy}I_5) - q(I_{xz}I_2 - I_{yz}I_4 - D_zI_5) + r(I_{xy}I_2 + D_yI_4 - I_{yz}I_5)]\frac{\partial p}{\partial\xi}\\ &+ [p(I_{xz}I_2 - I_{yz}I_4 - D_zI_5) + 2q(I_{yz}I_2 - I_{xy}I_5) - r(D_xI_2 - I_{xy}I_4 + I_{xy}I_5)]\frac{\partial q}{\partial\xi}\\ &- [p(I_{xy}I_2 + D_yI_4 - I_{yz}I_5) + q(D_xI_2 - I_{xy}I_4 + I_{xz}I_5) + 2r[I_{yz}I_2 - I_{xz}I_4]\frac{\partial r}{\partial\xi}\bigg\}\end{aligned} \tag{C-2}$$

$$\begin{aligned}\frac{\partial(\dot{r})}{\partial\xi} = \frac{1}{\det I}\bigg\{& I_3\frac{\partial L}{\partial\xi} + I_5\frac{\partial M}{\partial\xi} + I_6\frac{\partial N}{\partial\xi} + I_3\frac{\partial L_\mathrm{T}}{\partial\xi} + I_5\frac{\partial M_\mathrm{T}}{\partial\xi} + I_6\frac{\partial N_\mathrm{T}}{\partial\xi}\\ &- [2p(I_{xz}I_5 - I_{xy}I_6) - q(I_{xz}I_3 - I_{yz}I_5 - D_zI_6) + r(I_{xy}I_3 + D_yI_5 - I_{yz}I_6)]\frac{\partial p}{\partial\xi}\\ &+ [p(I_{xz}I_3 - I_{yz}I_5 - D_zI_6) + 2q(I_{yz}I_3 - I_{xy}I_6) - r(D_xI_3 - I_{xy}I_5 + I_{xz}I_6)]\frac{\partial q}{\partial\xi}\\ &- [p(I_{xy}I_3 + D_yI_5 - I_{yz}I_6) + q(D_zI_3 - I_{xy}I_5 + I_{xz}I_6) + 2r(I_{yz}I_3 - I_{xz}I_5)]\frac{\partial r}{\partial\xi}\bigg\}\end{aligned} \tag{C-3}$$

The quantities I_1, I_2, I_3, I_4, I_5, I_6, D_x, D_y, D_z, and $\det I$ are defined in equations (1-32) to (1-38) and (1-42) to (1-44).

Equation (1-50) defines the decoupled rotational accelerations of the vehicle ($\dot{p}'$, $\dot{q}'$, and $\dot{r}'$), which are used to determine the generalized derivatives of the decoupled quantities:

$$\begin{aligned}\frac{\partial(\dot{p}')}{\partial\xi} = \frac{1}{I_x}\bigg[&\frac{\partial L}{\partial\xi} + \frac{\partial L_T}{\partial\xi} - (rI_{xy} - qI_{xz})\frac{\partial p}{\partial\xi} + (pI_{xz} + rI_y + 2qI_{xz} - rI_z)\frac{\partial q}{\partial\xi}\\ &- (pI_{xy} - qI_y + 2rI_{xz} + qI_z)\frac{\partial r}{\partial\xi}\bigg]\end{aligned} \tag{C-4}$$

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$$\frac{\partial(\dot{q}')}{\partial\xi} = \frac{1}{I_y}\left[\frac{\partial M}{\partial\xi} + \frac{\partial M_\mathrm{T}}{\partial\xi} - (rI_x + qI_{yz} + 2pI_{xz} - rI_z)\,\frac{\partial p}{\partial\xi} + (rI_{xy} - pI_{yz})\,\frac{\partial q}{\partial\xi}\right.$$
$$\left. - (pI_x - qI_{xy} - 2rI_{xz} - pI_z)\,\frac{\partial r}{\partial\xi}\right] \tag{C-5}$$
$$\frac{\partial(\dot{r}')}{\partial\xi} = \frac{1}{I_z}\left[\frac{\partial N}{\partial\xi} + \frac{\partial N_\mathrm{T}}{\partial\xi} + (qI_x + rI_{yz} + 2pI_{xy} - qI_y)\,\frac{\partial p}{\partial\xi} + (pI_x - rI_{xz} - 2qI_{xy} - pI_y)\,\frac{\partial q}{\partial\xi}\right.$$
$$\left. - (qI_{xz} - pI_{yz})\,\frac{\partial r}{\partial\xi}\right] \tag{C-6}$$

Equations (1-58) to (1-60) define the translational accelerations of the vehicle. These equations are used to determine the generalized derivatives of these quantities:

$$\frac{\partial(\dot{V})}{\partial\xi} = \frac{1}{m}\left\{ -\cos\beta\,\frac{\partial D}{\partial\xi} + \cos\alpha\cos\beta\,\frac{\partial X_\mathrm{T}}{\partial\xi} + \sin\beta\,\frac{\partial Y}{\partial\xi} + \sin\alpha\cos\beta\,\frac{\partial Z_\mathrm{T}}{\partial\xi} + \sin\beta\,\frac{\partial Y_\mathrm{T}}{\partial\xi}\right.$$
$$+ [-X_\mathrm{T}\sin\alpha\cos\beta + Z_\mathrm{T}\cos\alpha\cos\beta + mg(\sin\theta\sin\alpha\cos\beta$$
$$+ \cos\theta\cos\phi\cos\alpha\cos\beta)]\,\frac{\partial\alpha}{\partial\xi}$$
$$+ [D\sin\beta + Y\cos\beta - X_\mathrm{T}\sin\beta\cos\alpha + Y_\mathrm{T}\cos\beta - Z_\mathrm{T}\sin\alpha\sin\beta$$
$$+ mg(\sin\theta\cos\alpha\sin\beta + \cos\theta\sin\phi\cos\beta - \cos\theta\cos\phi\sin\alpha\sin\beta)]\,\frac{\partial\beta}{\partial\xi}$$
$$- mg(-\cos\theta\cos\phi\sin\beta + \cos\theta\sin\phi\sin\alpha\cos\beta)\frac{\partial\phi}{\partial\xi}$$
$$\left. - mg(\cos\theta\cos\alpha\cos\beta + \sin\theta\sin\phi\sin\beta + \sin\theta\cos\phi\sin\alpha\cos\beta)\,\frac{\partial\theta}{\partial\xi}\right\} \tag{C-7}$$
$$\frac{\partial(\dot{\alpha})}{\partial\xi} = \frac{1}{mV\cos\beta}\left(-\frac{\partial L}{\partial\xi} + \cos\alpha\,\frac{\partial Z_\mathrm{T}}{\partial\xi} - \sin\alpha\,\frac{\partial X_\mathrm{T}}{\partial\xi}\right) - \tan\beta\cos\alpha\,\frac{\partial p}{\partial\xi} + \frac{\partial q}{\partial\xi} - \tan\beta\sin\alpha\,\frac{\partial r}{\partial\xi}$$
$$- \left\{\frac{1}{mV^2\cos\beta}[-L + Z_\mathrm{T}\cos\alpha - X_\mathrm{T}\sin\alpha + mg(\cos\theta\cos\phi\cos\alpha + \sin\theta\sin\alpha)]\right\}\,\frac{\partial V}{\partial\xi}$$
$$+ \left\{\frac{1}{V\cos\beta}[-Z_\mathrm{T}\sin\alpha - X_\mathrm{T}\cos\alpha - mg(\cos\theta\cos\phi\sin\alpha - \sin\theta\cos\alpha)]\right.$$
$$\left. + \tan\beta\,(p\sin\alpha - r\cos\alpha)\right\}\frac{\partial\alpha}{\partial\xi}$$
$$+ \left\{\frac{\tan\beta}{mV\cos\beta}[-L + Z_\mathrm{T}\cos\alpha - X_\mathrm{T}\sin\alpha\right.$$
$$\left. + mg(\cos\theta\cos\phi\cos\alpha + \sin\theta\sin\alpha)] - \frac{1}{\cos^2\beta}(p\cos\alpha + r\sin\alpha)\right\}\frac{\partial\beta}{\partial\xi}$$
$$- \left(\frac{g}{V\cos\beta}\cos\theta\sin\phi\cos\alpha\right)\frac{\partial\phi}{\partial\xi} - \left[\frac{g}{V\cos\beta}(\sin\theta\cos\phi\cos\alpha - \cos\theta\sin\alpha)\right]\frac{\partial\theta}{\partial\xi} \tag{C-8}$$

$$
\begin{aligned}
\frac{\partial(\dot{\beta})}{\partial \xi} = \frac{1}{mV}&\left[\sin\beta\,\frac{\partial D}{\partial \xi} + \cos\beta\,\frac{\partial Y}{\partial \xi} - \cos\alpha\sin\beta\,\frac{\partial X_{\mathrm{T}}}{\partial \xi} + \cos\beta\,\frac{\partial Y_{\mathrm{T}}}{\partial \xi} - \sin\alpha\sin\beta\,\frac{\partial Z_{\mathrm{T}}}{\partial \xi}\right] \\
&+ \sin\alpha\,\frac{\partial p}{\partial \xi} - \cos\alpha\,\frac{\partial r}{\partial \xi} \\
&- \frac{1}{mV^2}[D\sin\beta + Y\cos\beta - X_{\mathrm{T}}\cos\alpha\sin\beta + Y_{\mathrm{T}}\cos\beta - Z_{\mathrm{T}}\sin\alpha\sin\beta \\
&\qquad + mg(\sin\theta\cos\alpha\sin\beta + \cos\theta\sin\phi\cos\beta - \cos\theta\cos\phi\sin\alpha\sin\beta)]\,\frac{\partial V}{\partial \xi} \\
&+ \left\{\frac{1}{mV}[X_{\mathrm{T}}\sin\alpha\sin\beta - Z_{\mathrm{T}}\cos\alpha\sin\beta + mg(-\sin\theta\sin\alpha\sin\beta - \cos\theta\cos\phi\cos\alpha\sin\beta)]\right. \\
&\qquad \left. + p\cos\alpha + r\sin\alpha\right\}\frac{\partial \alpha}{\partial \xi} \\
&+ \frac{1}{mV}[D\cos\beta - Y\sin\beta - X_{\mathrm{T}}\cos\alpha\cos\beta - Y_{\mathrm{T}}\sin\beta - Z_{\mathrm{T}}\sin\alpha\cos\beta \\
&\qquad + mg(\sin\theta\cos\alpha\cos\beta - \cos\theta\sin\phi\sin\beta - \cos\theta\cos\phi\sin\alpha\cos\beta)]\,\frac{\partial \beta}{\partial \xi} \\
&+ \frac{g}{V}(\cos\theta\cos\phi\cos\beta + \cos\theta\sin\phi\sin\alpha\sin\beta)\,\frac{\partial \phi}{\partial \xi} \\
&+ \frac{g}{V}(\cos\theta\cos\alpha\sin\beta - \sin\theta\sin\phi\cos\beta + \sin\theta\cos\phi\sin\alpha\sin\beta)\,\frac{\partial \theta}{\partial \xi}
\end{aligned} \tag{C-9}
$$

Equations (1-66) to (1-68) define the vehicle attitude rates. These equations are used to determine the generalized derivatives of these quantities:

$$
\begin{aligned}
\frac{\partial(\dot{\phi})}{\partial \xi} &= \frac{\partial p}{\partial \xi} + \sin\phi\tan\theta\,\frac{\partial q}{\partial \xi} + \cos\phi\tan\theta\,\frac{\partial r}{\partial \xi} + (q\cos\phi\tan\theta - r\sin\phi\tan\theta)\,\frac{\partial \phi}{\partial \xi} \\
&\quad + (q\sin\phi\sec^2\theta + r\cos\phi\sec^2\theta)\,\frac{\partial \theta}{\partial \xi}
\end{aligned} \tag{C-10}
$$

$$
\frac{\partial(\dot{\theta})}{\partial \xi} = \cos\phi\,\frac{\partial q}{\partial \xi} - \sin\phi\,\frac{\partial r}{\partial \xi} - (q\sin\phi + r\cos\phi)\,\frac{\partial \phi}{\partial \xi} \tag{C-11}
$$

$$
\begin{aligned}
\frac{\partial(\dot{\psi})}{\partial \xi} &= \sin\phi\sec\theta\,\frac{\partial q}{\partial \xi} + \cos\phi\sec\theta\,\frac{\partial r}{\partial \xi} + (q\cos\phi\sec\theta - r\sin\phi\sec\theta)\,\frac{\partial \phi}{\partial \xi} \\
&\quad + (q\sin\phi\sec\theta\tan\theta + r\cos\phi\sec\theta\tan\theta)\,\frac{\partial \theta}{\partial \xi}
\end{aligned} \tag{C-12}
$$

Equations (1-72) to (1-74) define the earth-relative velocities of the vehicle. These equations are used to determine the generalized derivatives of these quantities:

$$\begin{aligned}
\frac{\partial(\dot{h})}{\partial\xi} = {} & [\cos\beta\cos\alpha\sin\theta - \sin\beta\sin\phi\cos\theta - \cos\beta\sin\alpha\cos\phi\cos\theta]\,\frac{\partial V}{\partial\xi} \\
& - V(\cos\beta\sin\alpha\sin\theta + \cos\beta\cos\alpha\cos\phi\cos\theta)\,\frac{\partial\alpha}{\partial\xi} \\
& - V(\sin\beta\cos\alpha\sin\theta + \cos\beta\sin\phi\cos\theta - \sin\beta\sin\alpha\cos\phi\cos\theta)\,\frac{\partial\beta}{\partial\xi} \\
& - V(\sin\beta\cos\phi\cos\theta - \cos\beta\sin\alpha\sin\phi\cos\theta)\,\frac{\partial\phi}{\partial\xi} \\
& + V(\cos\beta\cos\alpha\cos\theta + \sin\beta\sin\phi\sin\theta + \cos\beta\sin\alpha\cos\phi\sin\theta)\,\frac{\partial\theta}{\partial\xi}
\end{aligned} \tag{C-13}$$

$$\begin{aligned}
\frac{\partial(\dot{x})}{\partial\xi} = {} & [\cos\beta\cos\alpha\cos\theta\cos\psi + \sin\beta\,(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) \\
& + \cos\beta\sin\alpha\,(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)]\,\frac{\partial V}{\partial\xi} \\
& - V[\cos\beta\sin\alpha\cos\theta\cos\psi - \cos\beta\cos\alpha\,(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)]\,\frac{\partial\alpha}{\partial\xi} \\
& - V[\sin\beta\cos\alpha\cos\theta\cos\psi - \cos\beta\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi \\
& \quad + \sin\beta\sin\alpha\,(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)]\,\frac{\partial\beta}{\partial\xi} \\
& + V[\sin\beta\,(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) - \cos\beta\sin\alpha\,(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi)]\,\frac{\partial\phi}{\partial\xi} \\
& - V[\cos\beta\cos\alpha\sin\theta\cos\psi - \sin\beta\sin\phi\cos\theta\cos\psi - \cos\beta\sin\alpha\cos\phi\cos\theta\cos\psi]\,\frac{\partial\theta}{\partial\xi} \\
& - V[\cos\beta\cos\alpha\cos\theta\sin\psi + \sin\beta\,(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) \\
& \quad + \cos\beta\sin\alpha\,(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)]\,\frac{\partial\psi}{\partial\xi}
\end{aligned} \tag{C-14}$$

$$\begin{aligned}
\frac{\partial(\dot{y})}{\partial\xi} = {} & [\cos\beta\cos\alpha\cos\theta\sin\psi + \sin\beta\,(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi) \\
& + \cos\beta\sin\alpha\,(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)]\,\frac{\partial V}{\partial\xi} \\
& - V[\cos\beta\sin\alpha\cos\theta\sin\psi - \cos\beta\cos\alpha\,(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)]\,\frac{\partial\alpha}{\partial\xi} \\
& - V[\sin\beta\cos\alpha\cos\theta\sin\psi - \cos\beta\,(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi) \\
& \quad + \sin\beta\sin\alpha\,(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)]\,\frac{\partial\beta}{\partial\xi}
\end{aligned}$$

$$- V[\sin\beta\,(\sin\phi\cos\psi - \cos\phi\sin\theta\sin\psi) + \cos\beta\sin\alpha\,(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi)]\,\frac{\partial\phi}{\partial\xi}$$

$$- V(\cos\beta\cos\alpha\sin\theta\sin\psi - \sin\beta\sin\phi\cos\theta\sin\psi - \cos\beta\sin\alpha\cos\phi\cos\theta\sin\psi)\,\frac{\partial\theta}{\partial\xi}$$

$$+ V[\cos\beta\cos\alpha\cos\theta\cos\psi - \sin\beta\,(\cos\phi\sin\psi - \sin\phi\sin\theta\cos\psi)$$

$$+ \cos\beta\sin\alpha\,(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)]\,\frac{\partial\psi}{\partial\xi} \qquad \text{(C-15)}$$

C.2 Generalized Derivatives of the Observation Variables

The vector equation (1-90) defining the body axis kinematic accelerations is used to determine the generalized derivatives of the individual body axis accelerations:

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial\xi} = \frac{1}{g_0 m}\left[\frac{\partial X_\mathrm{T}}{\partial\xi} - \cos\alpha\,\frac{\partial D}{\partial\xi} + \sin\alpha\,\frac{\partial L}{\partial\xi} + (D\sin\alpha + L\cos\alpha)\,\frac{\partial\alpha}{\partial\xi} - gm\cos\theta\,\frac{\partial\theta}{\partial\xi}\right] \qquad \text{(C-16)}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial\xi} = \frac{1}{g_0 m}\left[\frac{\partial Y_\mathrm{T}}{\partial\xi} + \frac{\partial Y}{\partial\xi} + gm\cos\theta\cos\phi\,\frac{\partial\phi}{\partial\xi} - gm\sin\theta\sin\phi\,\frac{\partial\theta}{\partial\xi}\right] \qquad \text{(C-17)}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial\xi} = \frac{1}{g_0 m}\left[\frac{\partial Z_\mathrm{T}}{\partial\xi} - \sin\alpha\,\frac{\partial D}{\partial\xi} - \cos\alpha\,\frac{\partial L}{\partial\xi} - (D\cos\alpha - L\sin\alpha)\,\frac{\partial\alpha}{\partial\xi}\right.$$

$$\left. - gm\cos\theta\sin\phi\,\frac{\partial\phi}{\partial\xi} - gm\sin\theta\cos\phi\,\frac{\partial\theta}{\partial\xi}\right] \qquad \text{(C-18)}$$

Vector equation (1-91) defines the output of body axis accelerometers at the vehicle center of gravity and is used to determine the generalized derivatives of the individual body axis accelerometers:

$$\frac{\partial(a_x)}{\partial\xi} = \frac{1}{g_0 m}\left[\frac{\partial X_\mathrm{T}}{\partial\xi} - \cos\alpha\,\frac{\partial D}{\partial\xi} + \sin\alpha\,\frac{\partial L}{\partial\xi} + (D\sin\alpha + L\cos\alpha)\,\frac{\partial\alpha}{\partial\xi}\right] \qquad \text{(C-19)}$$

$$\frac{\partial(a_y)}{\partial\xi} = \frac{1}{g_0 m}\left(\frac{\partial Y_\mathrm{T}}{\partial\xi} + \frac{\partial Y}{\partial\xi}\right) \qquad \text{(C-20)}$$

$$\frac{\partial(a_z)}{\partial\xi} = \frac{1}{g_0 m}\left[\frac{\partial Z_\mathrm{T}}{\partial\xi} - \sin\alpha\,\frac{\partial D}{\partial\xi} - \cos\alpha\,\frac{\partial L}{\partial\xi} - (D\cos\alpha - L\sin\alpha)\,\frac{\partial\alpha}{\partial\xi}\right] \qquad \text{(C-21)}$$

Using equation (1-93), the generalized derivative of the output of a normal accelerometer at the vehicle center of gravity can be expressed as

$$\frac{\partial a_\mathrm{n}}{\partial\xi} = \frac{1}{g_0 m}\left[-\frac{\partial Z_\mathrm{T}}{\partial\xi} + \sin\alpha\,\frac{\partial D}{\partial\xi} + \cos\alpha\,\frac{\partial L}{\partial\xi} + (D\cos\alpha - L\sin\alpha)\,\frac{\partial\alpha}{\partial\xi}\right] \qquad \text{(C-22)}$$

The vector equation (1-95) defining the output of orthogonal accelerometers aligned with the body axes but displaced from the vehicle center of gravity is used to determine the generalized derivatives of these quantities:

$$\frac{\partial(a_{x,i})}{\partial\xi} = \frac{\partial a_x}{\partial\xi} + \frac{1}{g_0}\left[(qy_x + rz_x)\frac{\partial p}{\partial\xi} + (py_x - 2qx_x)\frac{\partial q}{\partial\xi} + (pz_x - 2rx_x)\frac{\partial r}{\partial\xi} + z_x\frac{\partial\dot{q}}{\partial\xi} - y_x\frac{\partial\dot{r}}{\partial\xi}\right] \quad \text{(C-23)}$$

$$\frac{\partial(a_{y,i})}{\partial\xi} = \frac{\partial a_y}{\partial\xi} - \frac{1}{g_0}\left[(2py_y - qx_y)\frac{\partial p}{\partial\xi} - (px_y + rz_y)\frac{\partial q}{\partial y} - (qz_y - 2ry_y)\frac{\partial r}{\partial\xi} + z_y\frac{\partial\dot{p}}{\partial\xi} - x_y\frac{\partial\dot{r}}{\partial\xi}\right] \quad \text{(C-24)}$$

$$\frac{\partial(a_{z,i})}{\partial\xi} = \frac{\partial a_z}{\partial\xi} - \frac{1}{g_0}\left[(2pz_z - rx_z)\frac{\partial p}{\partial\xi} + (2qz_z - ry_z)\frac{\partial q}{\partial\xi} - (px_z + qy_z)\frac{\partial r}{\partial\xi} - y_z\frac{\partial\dot{p}}{\partial\xi} + x_z\frac{\partial\dot{q}}{\partial\xi}\right] \quad \text{(C-25)}$$

Equation (1-96) defines the output of a normal accelerometer aligned with the z body axis but not located at vehicle center of gravity, $a_{n,i}$. This equation is used to determine the generalized derivative of $a_{n,i}$:

$$\frac{\partial(a_{n,i})}{\partial\xi} = \frac{\partial a_n}{\partial\xi} + \frac{1}{g_0}\left[(2pz_z - rx_z)\frac{\partial p}{\partial\xi} + (2qz_z - ry_z)\frac{\partial q}{\partial\xi} - (px_z + qy_z)\frac{\partial r}{\partial\xi} - y_z\frac{\partial\dot{p}}{\partial\xi} + x_z\frac{\partial\dot{q}}{\partial\xi}\right] \quad \text{(C-26)}$$

In equations (C-20) to (C-23), the partial derivatives of the vehicle rotational rates with respect to the dummy variable ξ are defined by equations (C-1) to (C-3). The partial derivatives of the outputs of the body axis accelerometers at the vehicle center of gravity are defined by equations (C-16) to (C-19). In these equations, as before, the subscripts x, y, and z refer to the x, y, and z body axes, respectively, and the symbols x, y, and z refer to x, y, and z body axis locations of the sensors relative to the vehicle center of gravity.

Using equation (1-97), the generalized derivative of the load factor can be defined as

$$\frac{\partial(n)}{\partial\xi} = \frac{1}{mg}\frac{\partial L}{\partial\xi} \quad \text{(C-27)}$$

Equations (1-98) to (1-105) define the air data parameters of interest for this report. These equations are used to determine the generalized derivatives of the air data parameters:

$$\frac{\partial(a)}{\partial\xi} = \frac{0.7p_0}{\rho_0 T_0[1.4(p_0/\rho_0 T_0)]^{1/2}}\frac{\partial T}{\partial\xi} \quad \text{(C-28)}$$

$$\frac{\partial(M)}{\partial\xi} = \frac{1}{a}\frac{\partial V}{\partial\xi} - \frac{V}{a^2}\frac{\partial a}{\partial\xi} \quad \text{(C-29)}$$

$$\frac{\partial(\mathrm{Re})}{\partial\xi} = \frac{\rho\ell}{\mu}\frac{\partial V}{\partial\xi} + \frac{V\ell}{\mu}\frac{\partial\rho}{\partial\xi} - \frac{\rho V\ell}{\mu^2}\frac{\partial\mu}{\partial\xi} \quad \text{(C-30)}$$

$$\frac{\partial(\mathrm{Re}')}{\partial\xi} = \frac{\rho}{\mu}\frac{\partial V}{\partial\xi} + \frac{V}{\mu}\frac{\partial\rho}{\partial\xi} - \frac{\rho V}{\mu^2}\frac{\partial\mu}{\partial\xi} \quad \text{(C-31)}$$

$$\frac{\partial(\bar{q})}{\partial\xi} = \rho V\frac{\partial V}{\partial\xi} + \frac{V^2}{2}\frac{\partial\rho}{\partial\xi} \quad \text{(C-32)}$$

$$\frac{\partial(q_c)}{\partial\xi} = \begin{cases} [(1.0+0.2M^2)^{3.5}-1.0]\,\dfrac{\partial p_a}{\partial\xi} \\ \quad + 1.4M(1.0+0.2M^2)^{2.5} p_a\,\dfrac{\partial M}{\partial\xi} & (M \le 1.0) \\ \left[1.2M^2\left(\dfrac{5.76M^2}{5.6M^2-0.8}\right)^{2.5} - 1.0\right]\dfrac{\partial p_a}{\partial\xi} \\ \quad + p_a\left\{2.4M\left(\dfrac{5.76M^2}{5.6M^2-0.8}\right)^{2.5} + 3.0M^2\left(\dfrac{5.76M^2}{5.6M^2-0.8}\right)^{1.5}\right. \\ \qquad \left.\left[\dfrac{9.216M}{(5.6M^2-0.8)^2}\right]\dfrac{\partial M}{\partial\xi}\right\} & (M \ge 1.0) \end{cases} \tag{C-33}$$

$$\frac{\partial(q_c/p_a)}{\partial\xi} = \begin{cases} 1.4M(1.0+0.2M^2)^{2.5}\,\dfrac{\partial M}{\partial\xi} & (M \le 1.0) \\ \left\{2.4M\left(\dfrac{5.76M^2}{5.6M^2-0.8}\right)^{2.5} + 3.0M^2\left(\dfrac{5.76M^2}{5.6M^2-0.8}\right)^{1.5}\right. \\ \quad \left.\left[\dfrac{9.216M}{(5.6M^2-0.8)^2}\right]\right\}\dfrac{\partial M}{\partial\xi} & (M \ge 1.0) \end{cases} \tag{C-34}$$

$$\frac{\partial(T_t)}{\partial\xi} = (1.0+0.2M^2)\,\frac{\partial T}{\partial\xi} + 0.4TM\,\frac{\partial M}{\partial\xi} \tag{C-35}$$

In the preceding equations, the generalized derivative of Mach number appears several times. This term can be expanded using equation (C-29).

The definitions of the flightpath-related parameters are presented in equations (1-106) to (1-108). These definitions are used to derive the generalized partial derivatives of the flightpath-related parameters:

$$\frac{\partial(\gamma)}{\partial\xi} = \frac{1}{(V^2-\dot{h}^2)^{1/2}}\left[-\frac{\dot{h}}{V}\,\frac{\partial V}{\partial\xi} + \frac{\partial\dot{h}}{\partial\xi}\right] \tag{C-36}$$

$$\frac{\partial(\text{fpa})}{\partial\xi} = \frac{1}{g}\,\frac{\partial\dot{V}}{\partial\xi} \tag{C-37}$$

$$\begin{aligned} \frac{\partial(\ddot{h})}{\partial\xi} &= [-a_{y,\text{k}}\cos\phi\cos\theta + a_{z,\text{k}}\sin\phi\cos\theta]\,\frac{\partial\phi}{\partial\xi} \\ &\quad + [a_{x,\text{k}}\cos\theta + a_{y,\text{k}}\sin\phi\sin\theta + a_{z,\text{k}}\cos\phi\sin\theta]\,\frac{\partial\theta}{\partial\xi} \\ &\quad + \sin\theta\,\frac{\partial a_{x,\text{k}}}{\partial\xi} - \sin\phi\cos\theta\,\frac{\partial a_{y,\text{k}}}{\partial\xi} - \cos\phi\cos\theta\,\frac{\partial a_{z,\text{k}}}{\partial\xi} \end{aligned} \tag{C-38}$$

The partial derivatives of altitude rate $\dot{h}$ and velocity rate $\dot{V}$ that appear on the right-hand side of these equations are defined in equations (C-13) and (C-7), respectively. The partial derivatives of the body axis accelerations appearing in equation (C-38) are defined in equations (C-16) to (C-18).

Using equations (1-109) and (1-110), the generalized derivatives of the energy-related parameters are defined. The partial derivatives of altitude rate and velocity rate appearing in equation (C-40) are defined in equations (C-13) and (C-10), respectively:

$$\frac{\partial(E_s)}{\partial\xi} = \frac{V}{g}\frac{\partial V}{\partial\xi} + \frac{\partial h}{\partial\xi} \tag{C-39}$$

$$\frac{\partial(P_s)}{\partial\xi} = \frac{\dot{V}}{g}\frac{\partial V}{\partial\xi} + \frac{V}{g}\frac{\partial\dot{V}}{\partial\xi} + \frac{\partial\dot{h}}{\partial\xi} \tag{C-40}$$

The derivatives of the force parameters, lift (eq. (1-111)) and drag (eq. (1-112)), are defined in section D.1. The generalized derivatives of the normal force (eq. (1-113)) and the axial force (eq. (1-114)) are presented in terms of the generalized derivatives of the lift and drag forces:

$$\frac{\partial(N)}{\partial\xi} = \cos\alpha\frac{\partial L}{\partial\xi} + \sin\alpha\frac{\partial D}{\partial\xi} - (L\sin\alpha - D\cos\alpha)\frac{\partial\alpha}{\partial\xi} \tag{C-41}$$

$$\frac{\partial(A)}{\partial\xi} = -\sin\alpha\frac{\partial L}{\partial\xi} + \cos\alpha\frac{\partial D}{\partial\xi} - (L\cos\alpha + D\sin\alpha)\frac{\partial\alpha}{\partial\xi} \tag{C-42}$$

The body axis rates are defined in equations (1-9) to (1-11). The time derivatives of these terms are defined in equations (1-115) to (1-117). These equations are used to derive the generalized derivatives of the body axis rates and accelerations:

$$\frac{\partial(u)}{\partial\xi} = \cos\alpha\cos\beta\frac{\partial V}{\partial\xi} - V\sin\alpha\cos\beta\frac{\partial\alpha}{\partial\xi} - V\cos\alpha\sin\beta\frac{\partial\beta}{\partial\xi} \tag{C-43}$$

$$\frac{\partial(v)}{\partial\xi} = \sin\beta\frac{\partial V}{\partial\xi} + V\cos\beta\frac{\partial\beta}{\partial\xi} \tag{C-44}$$

$$\frac{\partial(w)}{\partial\xi} = \sin\alpha\cos\beta\frac{\partial V}{\partial\xi} + V\cos\alpha\cos\beta\frac{\partial\alpha}{\partial\xi} - V\sin\alpha\sin\beta\frac{\partial\beta}{\partial\xi} \tag{C-45}$$

$$\begin{aligned}\frac{\partial(\dot{u})}{\partial\xi} = {} & \frac{1}{m}\left(\frac{\partial X_{\mathrm{T}}}{\partial\xi} - \cos\alpha\frac{\partial D}{\partial\xi} + \sin\alpha\frac{\partial L}{\partial\xi}\right) - V\sin\alpha\cos\beta\frac{\partial q}{\partial\xi} + V\sin\beta\frac{\partial r}{\partial\xi} \\ & + (r\sin\beta - q\sin\alpha\cos\beta)\frac{\partial V}{\partial\xi} + \left[\frac{1}{m}(D\sin\alpha + L\cos\alpha) - qV\cos\alpha\cos\beta\right]\frac{\partial\alpha}{\partial\xi} \\ & + (rV\cos\beta + qV\cos\alpha\sin\beta)\frac{\partial\beta}{\partial\xi} - g\cos\theta\frac{\partial\theta}{\partial\xi}\end{aligned} \tag{C-46}$$

$$\begin{aligned}\frac{\partial(\dot{v})}{\partial\xi} = {} & \frac{1}{m}\left(\frac{\partial Y_{\mathrm{T}}}{\partial\xi} + \frac{\partial Y}{\partial\xi}\right) + V\sin\alpha\cos\beta\frac{\partial p}{\partial\xi} - V\cos\alpha\cos\beta\frac{\partial r}{\partial\xi} \\ & + (p\sin\alpha\cos\beta - r\cos\alpha\cos\beta)\frac{\partial V}{\partial\xi} + (pV\cos\alpha\cos\beta + rV\sin\alpha\cos\beta)\frac{\partial\alpha}{\partial\xi} \\ & - (pV\sin\alpha\sin\beta - rV\cos\alpha\sin\beta)\frac{\partial\beta}{\partial\xi} + g\cos\theta\cos\phi\frac{\partial\phi}{\partial\xi} - g\sin\theta\sin\phi\frac{\partial\theta}{\partial\xi}\end{aligned} \tag{C-47}$$

$$\frac{\partial(\dot{w})}{\partial\xi} = \frac{1}{m}\left(\frac{\partial Z_{\mathrm{T}}}{\partial\xi} - \sin\alpha\,\frac{\partial D}{\partial\xi} - \cos\alpha\,\frac{\partial L}{\partial\xi}\right) - V\sin\beta\,\frac{\partial p}{\partial\xi} + V\cos\alpha\cos\beta\,\frac{\partial q}{\partial\xi}$$
$$+ (q\cos\alpha\cos\beta - p\sin\beta)\,\frac{\partial V}{\partial\xi} - \left[\frac{1}{m}(D\cos\alpha - L\sin\alpha) + qV\sin\alpha\cos\beta\right]\frac{\partial\alpha}{\partial\xi}$$
$$- (qV\cos\alpha\sin\beta + pV\cos\beta)\,\frac{\partial\beta}{\partial\xi} - g\cos\theta\sin\phi\,\frac{\partial\phi}{\partial\xi} - g\sin\theta\cos\phi\,\frac{\partial\theta}{\partial\xi} \tag{C-48}$$

The outputs of various instruments displaced from the vehicle center of gravity are defined in equations (1-118) to (1-121). These equations define angle of attack, angle of sideslip, altitude, and altitude rate instrument outputs. The generalized derivatives of the quantities are based on these equations:

$$\frac{\partial(\alpha_{,i})}{\partial\xi} = -\frac{y_\alpha}{V}\,\frac{\partial p}{\partial\xi} + \frac{x_\alpha}{V}\,\frac{\partial q}{\partial\xi} - \left(\frac{qx_\alpha - py_\alpha}{V^2}\right)\frac{\partial V}{\partial\xi} + \frac{\partial\alpha}{\partial\xi} \tag{C-49}$$

$$\frac{\partial(\beta_{,i})}{\partial\xi} = -\frac{z_\beta}{V}\,\frac{\partial p}{\partial\xi} + \frac{x_\beta}{V}\,\frac{\partial r}{\partial\xi} - \left(\frac{rx_\beta - pz_\beta}{V^2}\right)\frac{\partial V}{\partial\xi} + \frac{\partial\beta}{\partial\xi} \tag{C-50}$$

$$\frac{\partial(h_{,i})}{\partial\xi} = (-y_h\cos\phi\cos\theta + z_h\sin\phi\cos\theta)\,\frac{\partial\phi}{\partial\xi}$$
$$+(x_h\cos\theta + y_h\sin\phi\sin\theta + z_h\cos\phi\sin\theta)\,\frac{\partial\theta}{\partial\xi} + \frac{\partial h}{\partial\xi} \tag{C-51}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial\xi} = \left[\dot{\phi}(y_{\dot{h}}\sin\phi\cos\theta + z_{\dot{h}}\cos\phi\cos\theta) + \dot{\theta}(y_{\dot{h}}\cos\phi\sin\theta - z_{\dot{h}}\sin\phi\sin\theta)\right]\frac{\partial\phi}{\partial\xi}$$
$$+ \left[-\dot{\theta}(x_{\dot{h}}\sin\theta - y_{\dot{h}}\sin\phi\cos\theta - z_{\dot{h}}\cos\phi\cos\theta) + \dot{\phi}(y_{\dot{h}}\cos\phi\sin\theta - z_{\dot{h}}\sin\phi\sin\theta)\right]\frac{\partial\theta}{\partial\xi}$$
$$- (y_{\dot{h}}\cos\phi\cos\theta - z_{\dot{h}}\sin\phi\cos\theta)\,\frac{\partial\dot{\phi}}{\partial\xi} + (x_{\dot{h}}\cos\theta + y_{\dot{h}}\sin\phi\sin\theta + z_{\dot{h}}\cos\phi\sin\theta)\,\frac{\partial\dot{\theta}}{\partial\xi}$$
$$+\frac{\partial\dot{h}}{\partial\xi} \tag{C-52}$$

The generalized derivatives of bank angle rate, pitch attitude rate, and altitude rate with respect to the dummy variable ξ are defined in equations (C-10), (C-11), and (C-13), respectively.

The final set of observation variables is defined in equations (1-122) to (1-125). These equations, defining total angular momentum and the stability axis rotational rates, are used to determine the generalized derivatives of these quantities:

$$\frac{\partial(T)}{\partial\xi} = (I_xp - I_{xy}q - I_{xz}r)\,\frac{\partial p}{\partial\xi} + (I_yq - I_{xy}p - I_{yz}r)\,\frac{\partial q}{\partial\xi} + (I_zr - I_{xz}p - I_{yz}q)\,\frac{\partial r}{\partial\xi} \tag{C-53}$$

$$\frac{\partial(p_{\mathrm{s}})}{\partial\xi} = \cos\alpha\,\frac{\partial p}{\partial\xi} + \sin\alpha\,\frac{\partial r}{\partial\xi} - (p\sin\alpha - r\cos\alpha)\,\frac{\partial\alpha}{\partial\xi} \tag{C-54}$$

$$\frac{\partial(q_{\mathrm{s}})}{\partial\xi} = \frac{\partial q}{\partial\xi} \tag{C-55}$$

$$\frac{\partial(r_{\mathrm{s}})}{\partial\xi} = -\sin\alpha\,\frac{\partial p}{\partial\xi} + \cos\alpha\,\frac{\partial r}{\partial\xi} + (-p\cos\alpha - r\sin\alpha)\,\frac{\partial\alpha}{\partial\xi} \tag{C-56}$$

APPENDIX D—EVALUATION OF DERIVATIVES

The generalized partial derivatives presented in equations (C-1) to (C-56) contain partial derivatives of the state variables, thrust forces, and total aerodynamic forces and moments with respect to the dummy variable ξ. In this appendix, these partial derivatives are defined with respect to specific state, time derivatives of state, and control variables. The derivatives of atmospheric parameters are also discussed.

D.1 Preliminary Evaluation

First, the partial derivatives of the state variables with respect to the state, time derivatives of state, and control variables are considered. All partial derivatives of the state variables with respect to the state variables are either equal to zero or unity. Thus,

$$\frac{\partial p}{\partial p} = \frac{\partial q}{\partial q} = \frac{\partial r}{\partial r} = \frac{\partial V}{\partial V} = \frac{\partial \alpha}{\partial \alpha} = \frac{\partial \beta}{\partial \beta} = \frac{\partial \phi}{\partial \phi} = \frac{\partial \theta}{\partial \theta} = \frac{\partial \psi}{\partial \psi} = \frac{\partial h}{\partial h} = \frac{\partial x}{\partial x} = \frac{\partial y}{\partial y} = 1 \tag{D-1}$$

and all other derivatives of state variables with respect to state variables are equal to zero. The partial derivatives of the state variables with respect to the time derivatives of the state variables ($\dot{\alpha}$ and $\dot{\beta}$, in particular) are equal to zero. This is also true of the partial derivatives of the state variables with respect to the control variables.

Second, the partial derivatives of the aerodynamic forces and moments with respect to the state, time derivatives of state, and control variables are evaluated. Using the definitions of the force and moment coefficients presented in appendix A, the partial derivatives can be explicitly evaluated in terms of the stability and control derivatives.

D.1.1 Rolling moment derivatives.—

$$\frac{\partial L}{\partial p} = \frac{\bar{q}Sb^2}{2V} C_{\ell_p} \tag{D-2}$$

$$\frac{\partial L}{\partial q} = \frac{\bar{q}Sb\bar{c}}{2V} C_{\ell_q} \tag{D-3}$$

$$\frac{\partial L}{\partial r} = \frac{\bar{q}Sb^2}{2V} C_{\ell_r} \tag{D-4}$$

$$\frac{\partial L}{\partial V} = Sb\rho V C_\ell + \bar{q}SbC_{\ell_V} \tag{D-5}$$

$$\frac{\partial L}{\partial \alpha} = \bar{q}SbC_{\ell_\alpha} \tag{D-6}$$

$$\frac{\partial L}{\partial \beta} = \bar{q}SbC_{\ell_\beta} \tag{D-7}$$

$$\frac{\partial L}{\partial h} = \frac{1}{2}SbV^2 C_\ell \frac{\partial \rho}{\partial h} + \bar{q}SbC_{\ell_h} \tag{D-8}$$

$$\frac{\partial L}{\partial \dot{\alpha}} = \frac{\bar{q}Sb\bar{c}}{2V} C_{\ell_{\dot{\alpha}}} \tag{D-9}$$

$$\frac{\partial L}{\partial \dot{\beta}} = \frac{\bar{q}Sb^2}{2V} C_{\ell_{\dot{\beta}}} \tag{D-10}$$

$$\frac{\partial L}{\partial \delta_i} = \bar{q}SbC_{\ell_{\delta_i}} \tag{D-11}$$

D.1.2 Pitching moment derivatives.—

$$\frac{\partial M}{\partial p} = \frac{\bar{q}Sb\bar{c}}{2V}C_{m_p} \tag{D-12}$$

$$\frac{\partial M}{\partial q} = \frac{\bar{q}S\bar{c}^2}{2V}C_{m_q} \tag{D-13}$$

$$\frac{\partial M}{\partial r} = \frac{\bar{q}Sb\bar{c}}{2V}C_{m_r} \tag{D-14}$$

$$\frac{\partial M}{\partial V} = S\bar{c}\rho VC_m + \bar{q}S\bar{c}C_{m_V} \tag{D-15}$$

$$\frac{\partial M}{\partial \alpha} = \frac{\bar{q}S\bar{c}}{2V}C_{m_\alpha} \tag{D-16}$$

$$\frac{\partial M}{\partial \beta} = \bar{q}S\bar{c}C_{m_\beta} \tag{D-17}$$

$$\frac{\partial M}{\partial h} = \frac{1}{2}S\bar{c}V^2C_m\frac{\partial \rho}{\partial h} + \bar{q}S\bar{c}C_{m_h} \tag{D-18}$$

$$\frac{\partial M}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}^2}{2V}C_{m_{\dot{\alpha}}} \tag{D-19}$$

$$\frac{\partial M}{\partial \dot{\beta}} = \frac{\bar{q}Sb\bar{c}}{2V}C_{m_{\dot{\beta}}} \tag{D-20}$$

$$\frac{\partial M}{\partial_{\delta i}} = \bar{q}S\bar{c}C_{m_{\delta_i}} \tag{D-21}$$

D.1.3 Yawing moment derivatives.—

$$\frac{\partial N}{\partial p} = \frac{\bar{q}Sb^2}{2V}C_{n_p} \tag{D-22}$$

$$\frac{\partial N}{\partial q} = \frac{\bar{q}Sb\bar{c}}{2V}C_{n_q} \tag{D-23}$$

$$\frac{\partial N}{\partial r} = \frac{\bar{q}Sb^2}{2V}C_{n_r} \tag{D-24}$$

$$\frac{\partial N}{\partial V} = Sb\rho VC_n + \bar{q}SbC_{n_V} \tag{D-25}$$

$$\frac{\partial N}{\partial \alpha} = \bar{q}SbC_{n_\alpha} \tag{D-26}$$

$$\frac{\partial N}{\partial \beta} = \bar{q}SbC_{n_\beta} \tag{D-27}$$

$$\frac{\partial N}{\partial h} = \frac{1}{2}SbV^2C_n\frac{\partial \rho}{\partial h} + \bar{q}SbC_{n_h} \tag{D-28}$$

$$\frac{\partial N}{\partial \dot{\alpha}} = \frac{\bar{q}Sb\bar{c}}{2V}C_{n_{\dot{\alpha}}} \tag{D-29}$$

$$\frac{\partial N}{\partial \dot{\beta}} = \frac{\bar{q}Sb^2}{2V}C_{n_{\dot{\beta}}} \tag{D-30}$$

$$\frac{\partial N}{\partial_{\delta i}} = \bar{q}SbC_{n_{\delta_i}} \tag{D-31}$$

D.1.4 Drag force derivatives.—

$$\frac{\partial D}{\partial p} = \frac{\bar{q}Sb}{2V} C_{\mathrm{D}_p} \tag{D-32}$$

$$\frac{\partial D}{\partial q} = \frac{\bar{q}S\bar{c}}{2V} C_{\mathrm{D}_q} \tag{D-33}$$

$$\frac{\partial D}{\partial r} = \frac{\bar{q}Sb}{2V} C_{\mathrm{D}_r} \tag{D-34}$$

$$\frac{\partial D}{\partial V} = S\rho V C_{\mathrm{D}} + \bar{q}SC_{\mathrm{D}_V} \tag{D-35}$$

$$\frac{\partial D}{\partial \alpha} = \bar{q}SC_{\mathrm{D}_\alpha} \tag{D-36}$$

$$\frac{\partial D}{\partial \beta} = \bar{q}SC_{\mathrm{D}_\beta} \tag{D-37}$$

$$\frac{\partial D}{\partial h} = \frac{1}{2}SV^2 C_{\mathrm{D}} \frac{\partial \rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h} \tag{D-38}$$

$$\frac{\partial D}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V} C_{\mathrm{D}_{\dot{\alpha}}} \tag{D-39}$$

$$\frac{\partial D}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V} C_{\mathrm{D}_{\dot{\beta}}} \tag{D-40}$$

$$\frac{\partial D}{\partial_{\delta i}} = \bar{q}SC_{\mathrm{D}_{\delta_i}} \tag{D-41}$$

D.1.5 Sideforce derivatives.—

$$\frac{\partial Y}{\partial p} = \frac{\bar{q}Sb}{2V} C_{\mathrm{Y}_p} \tag{D-42}$$

$$\frac{\partial Y}{\partial q} = \frac{\bar{q}S\bar{c}}{2V} C_{\mathrm{Y}_q} \tag{D-43}$$

$$\frac{\partial Y}{\partial r} = \frac{\bar{q}Sb}{2V} C_{\mathrm{Y}_r} \tag{D-44}$$

$$\frac{\partial Y}{\partial V} = S\rho V C_{\mathrm{Y}} + \bar{q}SC_{\mathrm{Y}_V} \tag{D-45}$$

$$\frac{\partial Y}{\partial \alpha} = \bar{q}SC_{\mathrm{Y}_\alpha} \tag{D-46}$$

$$\frac{\partial Y}{\partial \beta} = \bar{q}SC_{\mathrm{Y}_\beta} \tag{D-47}$$

$$\frac{\partial Y}{\partial h} = \frac{1}{2}SV^2 C_{\mathrm{Y}} \frac{\partial \rho}{\partial h} + \bar{q}SC_{\mathrm{Y}_h} \tag{D-48}$$

$$\frac{\partial Y}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V} C_{\mathrm{Y}_{\dot{\alpha}}} \tag{D-49}$$

$$\frac{\partial Y}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V} C_{\mathrm{Y}_{\dot{\beta}}} \tag{D-50}$$

$$\frac{\partial Y}{\partial_{\delta i}} = \bar{q}SC_{\mathrm{Y}_{\delta_i}} \tag{D-51}$$

D.1.6 Lift force derivatives.—

$$\frac{\partial L}{\partial p} = \frac{\bar{q}Sb}{2V} C_{\mathrm{L}_p} \tag{D-52}$$

$$\frac{\partial L}{\partial q} = \frac{\bar{q}S\bar{c}}{2V} C_{\mathrm{L}_q} \tag{D-53}$$

$$\frac{\partial L}{\partial r} = \frac{\bar{q}Sb}{2V} C_{\mathrm{L}_r} \tag{D-54}$$

$$\frac{\partial L}{\partial V} = S\rho V C_{\mathrm{L}} + \bar{q}SC_{\mathrm{L}_V} \tag{D-55}$$

$$\frac{\partial L}{\partial \alpha} = \bar{q}SC_{\mathrm{L}_\alpha} \tag{D-56}$$

$$\frac{\partial L}{\partial \beta} = \bar{q}SC_{\mathrm{L}_\beta} \tag{D-57}$$

$$\frac{\partial L}{\partial h} = \frac{1}{2}SV^2 C_{\mathrm{L}} \frac{\partial \rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h} \tag{D-58}$$

$$\frac{\partial L}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V} C_{\mathrm{L}_{\dot{\alpha}}} \tag{D-59}$$

$$\frac{\partial L}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V} C_{\mathrm{L}_{\dot{\beta}}} \tag{D-60}$$

$$\frac{\partial L}{\partial_{\delta i}} = \bar{q}SC_{\mathrm{L}_{\delta_i}} \tag{D-61}$$

Next, the partial derivatives of the powerplant-induced forces and moments with respect to the state, time derivative of state, and control variables are considered. The partial derivatives of the powerplant-induced forces and moments are assumed to be zero except for moments taken with respect to the body axis rates (p, q, r), moments and forces taken with respect to the velocity and velocity orientation terms (V, α, β), and forces taken with respect to the control variables. These terms, assumed to be nonzero, are taken as primitives and not evaluated further. Thus, using F_{p} to represent a powerplant-induced force (X_{T}, Y_{T}, and Z_{T}) and M_{p} to represent a powerplant-induced moment (L_{T}, M_{T}, and N_{T}),

$$\frac{\partial F_{\mathrm{p}}}{\partial p} = \frac{\partial F_{\mathrm{p}}}{\partial q} = \frac{\partial F_{\mathrm{p}}}{\partial r} = \frac{\partial F_{\mathrm{p}}}{\partial \phi} = \frac{\partial F_{\mathrm{p}}}{\partial \theta} = \frac{\partial F_{\mathrm{p}}}{\partial \psi} = \frac{\partial F_{\mathrm{p}}}{\partial h} = \frac{\partial F_{\mathrm{p}}}{\partial x} = \frac{\partial F_p}{\partial y} = 0 \tag{D-62}$$

$$\frac{\partial M_{\mathrm{p}}}{\partial \phi} = \frac{\partial M_{\mathrm{p}}}{\partial \theta} = \frac{\partial M_{\mathrm{p}}}{\partial \psi} = \frac{\partial M_{\mathrm{p}}}{\partial h} = \frac{\partial M_{\mathrm{p}}}{\partial x} = \frac{\partial M_{\mathrm{p}}}{\partial y} = \frac{\partial M_{\mathrm{p}}}{\partial \delta_i} = 0 \tag{D-63}$$

and

$$\frac{\partial F_p}{\partial V}, \frac{\partial F_p}{\partial \alpha}, \frac{\partial F_p}{\partial \beta}, \frac{\partial F_p}{\partial \delta_i}, \frac{\partial M_p}{\partial p}, \frac{\partial M_p}{\partial q}, \frac{\partial M_p}{\partial r}, \frac{\partial M_p}{\partial V}, \frac{\partial M_p}{\partial \alpha}, \text{and} \frac{\partial M_p}{\partial \beta}$$

are taken as primitives and not evaluated further.

The final set of partial derivatives to be discussed are the derivatives of atmospheric parameters with respect to the state, time derivative of state, and control variables. In this report, all atmospheric parameters are assumed to be functions of altitude only. Thus, except for

$$\frac{\partial T}{\partial h}, \frac{\partial \rho}{\partial h}, \frac{\partial \mu}{\partial h}, \text{ and } \frac{\partial p_a}{\partial h},$$

all derivatives of ambient temperature, density, viscosity, and ambient pressure are assumed to be equal to zero. The nonzero quantities listed previously are dependent on an atmospheric model. Clancy (1975), Dommasch and others (1967), Etkin (1972), and Gracey (1980) present discussions of atmospheric models. In this report, the quantities will be taken as primitives and not evaluated further.

D.2 Evaluation of the Derivatives of the Time Derivatives of the State Variables

The generalized derivatives of the time derivatives of the state variables are defined in appendix C, equations (C-1) to (C-15). In this section, these generalized derivatives are evaluated in terms of the stability and control derivatives, primative terms, and the state, time derivative of state, and control variables. In this section, the notation $\partial(\dot{\mathbf{x}}_i)/\partial x_i$ is used to represent the more correct notation $\partial f_i/\partial x_j$ that is employed in the discussion at the beginning of section 3. This notation is used because there is no convenient notation available to express these quantities clearly—particularly not the usual notation employed in flight mechanics texts such as Etkin (1972) and McRuer and others (1973). The notation that defines quantities such as $L_p = \partial(\dot{p})/\partial p$ and $M_q = \partial(\dot{q})/\partial q$ is misleading in this context because the definitions of those terms (such as L_p, M_q) are based on assumptions of symmetric mass distributions, symmetric aerodynamics, and straight and level flight, and additionally do not include derivatives with respect to atmospheric quantities.

D.2.1 Roll acceleration derivatives.—

$$\frac{\partial(\dot{p})}{\partial p} = \frac{1}{\det I}\left[\frac{\bar{q}Sb}{2V_0}(I_1 bC_{\ell_p} + I_2\bar{c}C_{m_p} + I_3 bC_{n_p}) + \frac{\partial L_T}{\partial p} + \frac{\partial M_T}{\partial p} + \frac{\partial N_T}{\partial p} - 2p_0(I_{xz}I_2 - I_{xy}I_3) + q_0(I_{xz}I_1 - I_{yz}I_2 - D_z I_3) - r_0(I_{xy}I_1 + D_y I_2 - I_{yz}I_3)\right] \quad \text{(D-64)}$$

$$\frac{\partial(\dot{p})}{\partial q} = \frac{1}{\det I}\left[\frac{\bar{q}S\bar{c}}{2V_0}(I_1 bC_{\ell_q} + I_2\bar{c}C_{m_q} + I_3 bC_{n_q})\frac{\partial L_T}{\partial q} + \frac{\partial M_T}{\partial q} + \frac{\partial N_T}{\partial q} + p_0(I_{xz}I_1 - I_{yz}I_2 - D_z I_3) + 2q_0(I_{yz}I_1 - I_{xy}I_3) - r_0(D_x I_1 - I_{xy}I_2 + I_{xz}I_3)\right] \quad \text{(D-65)}$$

$$\frac{\partial(\dot{p})}{\partial r} = \frac{1}{\det I}\left[\frac{\bar{q}Sb}{2V_0}(I_1 bC_{\ell_r} + I_2\bar{c}C_{m_r} + I_3 bC_{n_r}) + \frac{\partial L_T}{\partial r} + \frac{\partial M_T}{\partial r} + \frac{\partial N_T}{\partial r} - p_0(I_{xy}I_1 - D_y I_2 - I_{yz}I_3) - q_0(D_x I_1 - I_{xy}I_2 + I_{xz}I_3) - 2r_0(I_{yz}I_1 - I_{xz}I_2)\right] \quad \text{(D-66)}$$

$$\frac{\partial(\dot{p})}{\partial V} = \frac{1}{\det I}\left[I_1 Sb(\rho V_0 C_\ell + \bar{q}C_{\ell_V}) + I_2 S\bar{c}(\rho V_0 C_m + \bar{q}C_{m_V}) + I_3 Sb(\rho V_0 C_n + \bar{q}C_{n_V}) + I_1\frac{\partial L_T}{\partial V} + I_2\frac{\partial M_T}{\partial V} + I_3\frac{\partial N_T}{\partial V}\right] \quad \text{(D-67)}$$

$$\frac{\partial(\dot{p})}{\partial \alpha} = \frac{1}{\det I}\left[\bar{q}S(I_1 bC_{\ell_\alpha} + I_2\bar{c}C_{m_\alpha} + I_3 bC_{n_\alpha}) + I_1\frac{\partial L_T}{\partial \alpha} + I_2\frac{\partial M_T}{\partial \alpha} + I_3\frac{\partial N_T}{\partial \alpha}\right] \quad \text{(D-68)}$$

$$\frac{\partial(\dot{p})}{\partial \beta} = \frac{1}{\det I}\left[\bar{q}S(I_1 bC_{\ell_\beta} + I_2\bar{c}C_{m_\beta} + I_3 bC_{n_\beta}) + I_1\frac{\partial L_T}{\partial \beta} + I_2\frac{\partial M_T}{\partial \beta} + I_3\frac{\partial N_T}{\partial \beta}\right] \quad \text{(D-69)}$$

$$\frac{\partial(\dot{p})}{\partial \phi} = 0 \quad \text{(D-70)}$$

$$\frac{\partial(\dot{p})}{\partial \theta} = 0 \quad \text{(D-71)}$$

$$\frac{\partial(\dot{p})}{\partial \psi} = 0 \quad \text{(D-72)}$$

$$\frac{\partial(\dot{p})}{\partial h} = \frac{S}{\det I}\left[I_1 b\left(\frac{1}{2}V_0^2 C_\ell\frac{\partial\rho}{\partial h} + \bar{q}C_{\ell_h}\right) + I_2\bar{c}\left(\frac{1}{2}V_0^2 C_m\frac{\partial\rho}{\partial h} + \bar{q}C_{m_h}\right) + I_3 b\left(\frac{1}{2}V_0^2 C_n\frac{\partial\rho}{\partial h} + \bar{q}C_{n_h}\right)\right] \quad \text{(D-73)}$$

$$\frac{\partial(\dot{p})}{\partial x} = 0 \tag{D-74}$$

$$\frac{\partial(\dot{p})}{\partial y} = 0 \tag{D-75}$$

$$\frac{\partial(\dot{p})}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 \det I}(I_1 b C_{\ell_{\dot{\alpha}}} + I_2 \bar{c} C_{m_{\dot{\alpha}}} + I_3 b C_{n_{\dot{\alpha}}}) \tag{D-76}$$

$$\frac{\partial(\dot{p})}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0 \det I}(I_1 b C_{\ell_{\dot{\beta}}} + I_2 \bar{c} C_{m_{\dot{\beta}}} + I_3 b C_{n_{\dot{\beta}}}) \tag{D-77}$$

$$\frac{\partial(\dot{p})}{\partial \delta_i} = \frac{\bar{q}S}{\det I}(I_1 b C_{\ell_{\delta_i}} + I_2 \bar{c} C_{m_{\delta_i}} + I_3 b C_{n_{\delta_i}}) \tag{D-78}$$

D.2.2 Pitch acceleration derivatives.—

$$\begin{aligned}\frac{\partial(\dot{q})}{\partial p} = \frac{1}{\det I}\Big[&\frac{\bar{q}Sb}{2V_0}(I_2 b C_{\ell_p} + I_4 \bar{c} C_{m_p} + I_5 b C_{n_p}) + I_2\,\frac{\partial L_{\mathrm{T}}}{\partial p} + I_4\,\frac{\partial M_{\mathrm{T}}}{\partial p} + I_5\,\frac{\partial N_{\mathrm{T}}}{\partial p} \\ &- 2p_0(I_{xz}I_4 - I_{xy}I_5) + q_0(I_{xz}I_2 - I_{yz}I_4 - D_z I_5) - r_0(I_{xy}I_2 + D_y I_4 - I_{yz}I_5)\Big]\end{aligned} \tag{D-79}$$

$$\begin{aligned}\frac{\partial(\dot{q})}{\partial q} = \frac{1}{\det I}\Big[&\frac{\bar{q}S\bar{c}}{2V_0}(I_2 b C_{\ell_q} + I_4 \bar{c} C_{m_q} + I_5 b C_{n_q}) + I_2\,\frac{\partial L_{\mathrm{T}}}{\partial q} + I_4\,\frac{\partial M_{\mathrm{T}}}{\partial q} + I_5\,\frac{\partial N_{\mathrm{T}}}{\partial q} \\ &+ p_0(I_{xz}I_2 - I_{yz}I_4 - D_z I_5) + 2q_0(I_{yz}I_2 - I_{xy}I_5) - r_0(D_x I_2 - I_{xy}I_4 + I_{xz}I_5)\Big]\end{aligned} \tag{D-80}$$

$$\begin{aligned}\frac{\partial(\dot{q})}{\partial r} = \frac{1}{\det I}\Big[&\frac{\bar{q}Sb}{2V_0}(I_2 b C_{\ell_r} + I_4 \bar{c} C_{m_r} + I_5 b C_{n_r}) + I_2\,\frac{\partial L_{\mathrm{T}}}{\partial r} + I_4\,\frac{\partial M_{\mathrm{T}}}{\partial r} + I_5\,\frac{\partial N_{\mathrm{T}}}{\partial r} \\ &- p_0(I_{xy}I_2 - D_y I_4 - I_{yz}I_5) - q_0(D_x I_2 - I_{xy}I_4 + I_{xz}I_5) - 2r_0(I_{yz}I_2 - I_{xz}I_4)\Big]\end{aligned} \tag{D-81}$$

$$\begin{aligned}\frac{\partial(\dot{q})}{\partial V} = \frac{1}{\det I}\Big[&I_2 Sb(\rho V_0 C_\ell + \bar{q}C_{\ell_V}) + I_4 S\bar{c}(\rho V_0 C_m + \bar{q}C_{m_V}) \\ &+ I_5 Sb(\rho V_0 C_n + \bar{q}C_{n_V}) + I_2\,\frac{\partial L_{\mathrm{T}}}{\partial V} + I_4\,\frac{\partial M_{\mathrm{T}}}{\partial V} + I_5\,\frac{\partial N_{\mathrm{T}}}{\partial V}\Big]\end{aligned} \tag{D-82}$$

$$\frac{\partial(\dot{q})}{\partial \alpha} = \frac{1}{\det I}\Big[\bar{q}S(I_2 b C_{\ell_\alpha} + I_4 \bar{c} C_{m_\alpha} + I_5 b C_{n_\alpha}) + I_2\,\frac{\partial L_{\mathrm{T}}}{\partial \alpha} + I_4\,\frac{\partial M_{\mathrm{T}}}{\partial \alpha} + I_5\,\frac{\partial N_{\mathrm{T}}}{\partial \alpha}\Big] \tag{D-83}$$

$$\frac{\partial(\dot{q})}{\partial \beta} = \frac{1}{\det I}\Big[\bar{q}S(I_2 b C_{\ell_\beta} + I_4 \bar{c} C_{m_\beta} + I_5 b C_{n_\beta}) + I_2\,\frac{\partial L_{\mathrm{T}}}{\partial \beta} + I_4\,\frac{\partial M_{\mathrm{T}}}{\partial \beta} + I_5\,\frac{\partial N_{\mathrm{T}}}{\partial \beta}\Big] \tag{D-84}$$

$$\frac{\partial(\dot{q})}{\partial \phi} = 0 \tag{D-85}$$

$$\frac{\partial(\dot{q})}{\partial \theta} = 0 \tag{D-86}$$

$$\frac{\partial(\dot{q})}{\partial \psi} = 0 \tag{D-87}$$

$$\begin{aligned}\frac{\partial(\dot{q})}{\partial h} = \frac{S}{\det I}\Big[&I_2 b\Big(\frac{1}{2}V_0^2 C_\ell\,\frac{\partial \rho}{\partial h} + \bar{q}C_{\ell_h}\Big) + I_4 \bar{c}\Big(\frac{1}{2}V_0^2 C_m\,\frac{\partial \rho}{\partial h} + \bar{q}C_{m_h}\Big) \\ &+ I_5 b\Big(\frac{1}{2}V_0^2 C_n\,\frac{\partial \rho}{\partial h} + \bar{q}C_{n_h}\Big)\Big]\end{aligned} \tag{D-88}$$

$$\frac{\partial(\dot{q})}{\partial x} = 0 \tag{D-89}$$

$$\frac{\partial(\dot{q})}{\partial y} = 0 \tag{D-90}$$

$$\frac{\partial(\dot{q})}{\partial\dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 \det I}(I_2 bC_{\ell_{\dot{\alpha}}} + I_4\bar{c}C_{m_{\dot{\alpha}}} + I_5 bC_{n_{\dot{\alpha}}}) \tag{D-91}$$

$$\frac{\partial(\dot{q})}{\partial\dot{\beta}} = \frac{\bar{q}Sb}{2V_0 \det I}(I_2 bC_{\ell_{\dot{\beta}}} + I_4\bar{c}C_{m_{\dot{\beta}}} + I_5 bC_{n_{\dot{\beta}}}) \tag{D-92}$$

$$\frac{\partial(\dot{q})}{\partial\delta_i} = \frac{\bar{q}S}{\det I}(I_2 bC_{\ell_{\delta_i}} + I_4\bar{c}C_{m_{\delta_i}} + I_5 bC_{n_{\delta_i}}) \tag{D-93}$$

D.2.3 Yaw acceleration derivatives.—

$$\begin{aligned}\frac{\partial(\dot{r})}{\partial p} = \frac{1}{\det I}\Big[&\frac{\bar{q}Sb}{2V_0}(I_3 bC_{\ell_p} + I_5\bar{c}C_{m_p} + I_6 bC_{n_p}) + I_3\,\frac{\partial L_{\mathrm{T}}}{\partial p} + I_5\,\frac{\partial M_{\mathrm{T}}}{\partial p} + I_6\,\frac{\partial N_{\mathrm{T}}}{\partial p}\\ &- 2p_0(I_{xz}I_5 - I_{xy}I_6) + q_0(I_{xz}I_3 - I_{yz}I_5 - D_z I_6) - r_0(I_{xy}I_3 + D_y I_5 - I_{yz}I_6)\Big]\end{aligned} \tag{D-94}$$

$$\begin{aligned}\frac{\partial(\dot{r})}{\partial q} = \frac{1}{\det I}\Big[&\frac{\bar{q}S\bar{c}}{2V_0}(I_3 bC_{\ell_q} + I_5\bar{c}C_{m_q} + I_6 bC_{n_q}) + I_3\,\frac{\partial L_{\mathrm{T}}}{\partial q} + I_5\,\frac{\partial M_{\mathrm{T}}}{\partial q} + I_6\,\frac{\partial N_{\mathrm{T}}}{\partial q}\\ &+ p_0(I_{xz}I_3 - I_{yz}I_5 - D_z I_6) + 2q_0(I_{yz}I_3 - I_{xz}I_6) - r_0(D_x I_3 - I_{xy}I_5 + I_{xz}I_6)\Big]\end{aligned} \tag{D-95}$$

$$\begin{aligned}\frac{\partial(\dot{r})}{\partial r} = \frac{1}{\det I}\Big[&\frac{\bar{q}Sb}{2V_0}(I_3 bC_{\ell_r} + I_5\bar{c}C_{m_r} + I_6 bC_{n_r}) + I_3\,\frac{\partial L_{\mathrm{T}}}{\partial r} + I_5\,\frac{\partial M_{\mathrm{T}}}{\partial r} + I_6\,\frac{\partial N_{\mathrm{T}}}{\partial r}\\ &- p_0(I_{xy}I_3 + D_y I_5 - I_{yz}I_6) - q_0(D_x I_3 - I_{xy}I_5 + I_{xz}I_6)\\ &- 2r_0(I_{yz}I_3 - I_{xz}I_5)\Big]\end{aligned} \tag{D-96}$$

$$\begin{aligned}\frac{\partial(\dot{r})}{\partial V} = \frac{1}{\det I}\Big[&I_3 Sb(\rho V_0 C_\ell + \bar{q}C_{\ell_V}) + I_5 S\bar{c}(\rho V_0 C_m + \bar{q}C_{m_V})\\ &+ I_6 Sb(\rho V_0 C_n + \bar{q}C_{n_V}) + I_3\,\frac{\partial L_{\mathrm{T}}}{\partial V} + I_5\,\frac{\partial M_{\mathrm{T}}}{\partial V} + I_6\,\frac{\partial N_{\mathrm{T}}}{\partial V}\Big]\end{aligned} \tag{D-97}$$

$$\frac{\partial(\dot{r})}{\partial\alpha} = \frac{1}{\det I}\left[\bar{q}S(I_3 bC_{\ell_\alpha} + I_5\bar{c}C_{m_\alpha} + I_6 bC_{n_\alpha}) + I_3\,\frac{\partial L_{\mathrm{T}}}{\partial\alpha} + I_5\,\frac{\partial M_{\mathrm{T}}}{\partial\alpha} + I_6\,\frac{\partial N_{\mathrm{T}}}{\partial\alpha}\right] \tag{D-98}$$

$$\frac{\partial(\dot{r})}{\partial\beta} = \frac{1}{\det I}\left[\bar{q}S(I_3 bC_{\ell_\beta} + I_5\bar{c}C_{m_\beta} + I_6 bC_{n_\beta}) + I_3\,\frac{\partial L_{\mathrm{T}}}{\partial\beta} + I_5\,\frac{\partial M_{\mathrm{T}}}{\partial\beta} + I_6\,\frac{\partial N_{\mathrm{T}}}{\partial\beta}\right] \tag{D-99}$$

$$\frac{\partial(\dot{r})}{\partial\phi} = 0 \tag{D-100}$$

$$\frac{\partial(\dot{r})}{\partial\theta} = 0 \tag{D-101}$$

$$\frac{\partial(\dot{r})}{\partial\psi} = 0 \tag{D-102}$$

$$\begin{aligned}\frac{\partial(\dot{r})}{\partial h} = \frac{S}{\det I}\Big[&I_3 b\left(\frac{1}{2}V_0^2 C_\ell\,\frac{\partial\rho}{\partial h} + \bar{q}C_{\ell_h}\right) + I_5\bar{c}\left(\frac{1}{2}V_0^2 C_m\,\frac{\partial\rho}{\partial h} + \bar{q}C_{m_h}\right)\\ &+ I_6 b\left(\frac{1}{2}V_0^2 C_n\,\frac{\partial\rho}{\partial h} + \bar{q}C_{n_h}\right)\Big]\end{aligned} \tag{D-103}$$

$$\frac{\partial(\dot{r})}{\partial x} = 0 \tag{D-104}$$

$$\frac{\partial(\dot{r})}{\partial y} = 0 \tag{D-105}$$

$$\frac{\partial(\dot{r})}{\partial\dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 \det I}(I_3 bC_{\ell_{\dot{\alpha}}} + I_5\bar{c}C_{m_{\dot{\alpha}}} + I_6 bC_{n_{\dot{\alpha}}}) \tag{D-106}$$

$$\frac{\partial(\dot{r})}{\partial\dot{\beta}} = \frac{\bar{q}Sb}{2V_0 \det I}(I_3 bC_{\ell_{\dot{\beta}}} + I_5\bar{c}C_{m_{\dot{\beta}}} + I_6 bC_{n_{\dot{\beta}}}) \tag{D-107}$$

$$\frac{\partial(\dot{r})}{\partial\delta_i} = \frac{\bar{q}S}{\det I}(I_3 bC_{\ell_{\delta_i}} + I_5\bar{c}C_{m_{\delta_i}} + I_6 bC_{n_{\delta_i}}) \tag{D-108}$$

D.2.4 Decoupled roll acceleration derivatives.—

$$\frac{\partial(\dot{p}')}{\partial p} = \frac{1}{I_x}\left[\frac{\bar{q}Sb^2}{2V_0}C_{\ell_p} + \frac{\partial L_{\mathrm{T}}}{\partial p} - I_{xy}r_0 + I_{xz}q_0\right] \tag{D-109}$$

$$\frac{\partial(\dot{p}')}{\partial q} = \frac{1}{I_x}\left[\frac{\bar{q}Sb\bar{c}}{2V_0}C_{\ell_q} + \frac{\partial L_{\mathrm{T}}}{\partial q} + I_{xz}p_0 + 2I_{yz}q_0 + r_0(I_y - I_z)\right] \tag{D-110}$$

$$\frac{\partial(\dot{p}')}{\partial r} = \frac{1}{I_x}\left[\frac{\bar{q}Sb^2}{2V_0}C_{\ell_r} + \frac{\partial L_{\mathrm{T}}}{\partial r} - I_{xy}p_0 + q_0(I_y - I_z) - 2I_{yz}r_0\right] \tag{D-111}$$

$$\frac{\partial(\dot{p}')}{\partial V} = \frac{1}{I_x}\left[Sb(\rho V_0 C_\ell + \bar{q}C_{\ell_V}) + \frac{\partial L_{\mathrm{T}}}{\partial V}\right] \tag{D-112}$$

$$\frac{\partial(\dot{p}')}{\partial\alpha} = \frac{1}{I_x}\left(\bar{q}SbC_{\ell_\alpha} + \frac{\partial L_{\mathrm{T}}}{\partial\alpha}\right) \tag{D-113}$$

$$\frac{\partial(\dot{p}')}{\partial\beta} = \frac{1}{I_x}\left(\bar{q}SbC_{\ell_\beta} + \frac{\partial L_{\mathrm{T}}}{\partial\beta}\right) \tag{D-114}$$

$$\frac{\partial(\dot{p}')}{\partial\phi} = 0 \tag{D-115}$$

$$\frac{\partial(\dot{p}')}{\partial\theta} = 0 \tag{D-116}$$

$$\frac{\partial(\dot{p}')}{\partial\psi} = 0 \tag{D-117}$$

$$\frac{\partial(\dot{p}')}{\partial h} = \frac{Sb}{I_x}\left(\frac{1}{2}V_0^2 C_\ell \frac{\partial\rho}{\partial h} + \bar{q}C_{\ell_h}\right) \tag{D-118}$$

$$\frac{\partial(\dot{p}')}{\partial x} = 0 \tag{D-119}$$

$$\frac{\partial(\dot{p}')}{\partial y} = 0 \tag{D-120}$$

$$\frac{\partial(\dot{p}')}{\partial\dot{\alpha}} = \frac{\bar{q}Sb\bar{c}}{2V_0 I_x}C_{\ell_{\dot{\alpha}}} \tag{D-121}$$

$$\frac{\partial(\dot{p}')}{\partial\dot{\beta}} = \frac{\bar{q}Sb^2}{2V_0 I_x}C_{\ell_{\dot{\beta}}} \tag{D-122}$$

$$\frac{\partial(\dot{p}')}{\partial\delta_i} = \frac{\bar{q}Sb}{I_x}C_{\ell_{\delta_i}} \tag{D-123}$$

D.2.5 Decoupled pitch acceleration derivatives.—

$$\frac{\partial(\dot{q}')}{\partial p} = \frac{1}{I_y}\left[\frac{\bar{q}Sb\bar{c}}{2V_0}C_{m_p} + \frac{\partial M_T}{\partial p} - 2I_{xz}p_0 - I_{yz}q_0 + r_0(I_z - I_x)\right] \tag{D-124}$$

$$\frac{\partial(\dot{q}')}{\partial q} = \frac{1}{I_y}\left(\frac{\bar{q}S\bar{c}^2}{2V_0}C_{m_q} + \frac{\partial M_T}{\partial q} + I_{xy}r_0 - I_{yz}p_0\right) \tag{D-125}$$

$$\frac{\partial(\dot{q}')}{\partial r} = \frac{1}{I_y}\left[\frac{\bar{q}Sb\bar{c}}{2V_0}C_{m_r} + \frac{\partial M_T}{\partial r} + p_0(I_z - I_x) + I_{xy}q_0 + 2I_{xz}r_0\right] \tag{D-126}$$

$$\frac{\partial(\dot{q}')}{\partial V} = \frac{1}{I_y}\left[S\bar{c}(\rho V_0 C_m + \bar{q}C_{m_V}) + \frac{\partial M_T}{\partial V}\right] \tag{D-127}$$

$$\frac{\partial(\dot{q}')}{\partial \alpha} = \frac{1}{I_y}\left(\bar{q}S\bar{c}C_{m_\alpha} + \frac{\partial M_T}{\partial \alpha}\right) \tag{D-128}$$

$$\frac{\partial(\dot{q}')}{\partial \beta} = \frac{1}{I_y}\left(\bar{q}S\bar{c}C_{m_\beta} + \frac{\partial M_T}{\partial \beta}\right) \tag{D-129}$$

$$\frac{\partial(\dot{q}')}{\partial \phi} = 0 \tag{D-130}$$

$$\frac{\partial(\dot{q}')}{\partial \theta} = 0 \tag{D-131}$$

$$\frac{\partial(\dot{q}')}{\partial \psi} = 0 \tag{D-132}$$

$$\frac{\partial(\dot{q}')}{\partial h} = \frac{S\bar{c}}{I_y}\left(\frac{1}{2}V_0^2 C_m \frac{\partial \rho}{\partial h} + \bar{q}C_{m_h}\right) \tag{D-133}$$

$$\frac{\partial(\dot{q}')}{\partial x} = 0 \tag{D-134}$$

$$\frac{\partial(\dot{q}')}{\partial y} = 0 \tag{D-135}$$

$$\frac{\partial(\dot{q}')}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}^2}{2V_0 I_y}C_{m_{\dot{\alpha}}} \tag{D-136}$$

$$\frac{\partial(\dot{q}')}{\partial \dot{\beta}} = \frac{\bar{q}Sb\bar{c}}{2V_0 I_y}C_{m_{\dot{\beta}}} \tag{D-137}$$

$$\frac{\partial(\dot{q}')}{\partial \delta_i} = \frac{\bar{q}S\bar{c}}{I_y}C_{m_{\delta_i}} \tag{D-138}$$

D.2.6 Decoupled yaw acceleration derivatives.—

$$\frac{\partial(\dot{r}')}{\partial p} = \frac{1}{I_z}\left[\frac{\bar{q}Sb^2}{2V_0}C_{n_p} + \frac{\partial N_T}{\partial p} + 2I_{xy}p_0 + q_0(I_x - I_y) + I_{yz}r_0\right] \tag{D-139}$$

$$\frac{\partial(\dot{r}')}{\partial q} = \frac{1}{I_z}\left[\frac{\bar{q}Sb\bar{c}}{2V_0}C_{n_q} + \frac{\partial N_T}{\partial q} + p_0(I_x - I_y) - 2I_{xy}q_0 - I_{xz}r_0\right] \tag{D-140}$$

$$\frac{\partial(\dot{r}')}{\partial r} = \frac{1}{I_z}\left(\frac{\bar{q}Sb^2}{2V_0}C_{n_r} + \frac{\partial N_T}{\partial r} - I_{xz}q_0 + I_{yz}p_0\right) \tag{D-141}$$

$$\frac{\partial(\dot{r}')}{\partial V} = \frac{1}{I_z}\left[Sb(\rho V_0 C_n + \bar{q}C_{n_V}) + \frac{\partial N_T}{\partial V}\right] \tag{D-142}$$

$$\frac{\partial(\dot{r}')}{\partial\alpha} = \frac{1}{I_z}\left(\bar{q}SbC_{n_\alpha} + \frac{\partial N_\mathrm{T}}{\partial\alpha}\right) \tag{D-143}$$

$$\frac{\partial(\dot{r}')}{\partial\beta} = \frac{1}{I_z}\left(\bar{q}SbC_{n_\beta} + \frac{\partial N_\mathrm{T}}{\partial\beta}\right) \tag{D-144}$$

$$\frac{\partial(\dot{r}')}{\partial\phi} = 0 \tag{D-145}$$

$$\frac{\partial(\dot{r}')}{\partial\theta} = 0 \tag{D-146}$$

$$\frac{\partial(\dot{r}')}{\partial\psi} = 0 \tag{D-147}$$

$$\frac{\partial(\dot{r}')}{\partial h} = \frac{Sb}{I_z}\left(\frac{1}{2}V_0^2C_n\,\frac{\partial\rho}{\partial h} + \bar{q}C_{n_h}\right) \tag{D-148}$$

$$\frac{\partial(\dot{r}')}{\partial x} = 0 \tag{D-149}$$

$$\frac{\partial(\dot{r}')}{\partial y} = 0 \tag{D-150}$$

$$\frac{\partial(\dot{r}')}{\partial\dot{\alpha}} = \frac{\bar{q}Sb\bar{c}}{2V_0I_z}C_{n_{\dot{\alpha}}} \tag{D-151}$$

$$\frac{\partial(\dot{r}')}{\partial\dot{\beta}} = \frac{qSb^2}{2V_0I_z}C_{n_{\dot{\beta}}} \tag{D-152}$$

$$\frac{\partial(\dot{r}')}{\partial\delta_i} = \frac{\bar{q}Sb}{I_z}C_{n_{\delta_i}} \tag{D-153}$$

D.2.7 Total vehicle acceleration derivatives.—

$$\frac{\partial(\dot{V})}{\partial p} = \frac{\bar{q}Sb}{2V_0m}(-\cos\beta_0\,C_{\mathrm{D}_p} + \sin\beta_0\,C_{\mathrm{Y}_p}) \tag{D-154}$$

$$\frac{\partial(\dot{V})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0m}(-\cos\beta_0\,C_{\mathrm{D}_q} + \sin\beta_0\,C_{\mathrm{Y}_q}) \tag{D-155}$$

$$\frac{\partial(\dot{V})}{\partial r} = \frac{\bar{q}Sb}{2V_0m}(-\cos\beta_0\,C_{\mathrm{D}_r} + \sin\beta_0\,C_{\mathrm{Y}_r}) \tag{D-156}$$

$$\begin{aligned}\frac{\partial(\dot{V})}{\partial V} = \frac{1}{m}\Big[&-S\cos\beta_0\,(\rho V_0C_\mathrm{D} + \bar{q}C_{\mathrm{D}_V}) + S\sin\beta_0\,(\rho V_0C_\mathrm{Y} + \bar{q}C_{\mathrm{Y}_r})\\ &+\cos\alpha_0\cos\beta_0\,\frac{\partial X_\mathrm{T}}{\partial V} + \sin\alpha_0\cos\beta_0\,\frac{\partial Z_\mathrm{T}}{\partial V} + \sin\beta_0\,\frac{\partial Y_\mathrm{T}}{\partial V}\Big]\end{aligned} \tag{D-157}$$

$$\begin{aligned}\frac{\partial(\dot{V})}{\partial\alpha} = \frac{1}{m}\Big[&-\bar{q}S\cos\beta_0\,C_{\mathrm{D}_\alpha} + \bar{q}S\sin\beta_0\,C_{\mathrm{Y}_\alpha} + \cos\alpha_0\cos\beta_0\,\frac{\partial X_\mathrm{T}}{\partial\alpha}\\ &+\sin\alpha_0\cos\beta_0\,\frac{\partial Z_\mathrm{T}}{\partial\alpha} + \sin\beta_0\,\frac{\partial Y_\mathrm{T}}{\partial\alpha} - X_\mathrm{T}\sin\alpha_0\cos\beta_0\\ &+Z_\mathrm{T}\cos\alpha_0\cos\beta_0 + mg(\sin\theta_0\sin\alpha_0\cos\beta_0 + \cos\theta_0\cos\phi_0\cos\alpha_0\cos\beta_0)\Big]\end{aligned} \tag{D-158}$$

$$\frac{\partial(\dot V)}{\partial\beta} = \frac{1}{m}\Big[\bar q S(-\cos\beta_0\, C_{\mathrm{D}_\beta} + \sin\beta_0\, C_{\mathrm{D}} + \sin\beta_0\, C_{\mathrm{Y}_\beta} + \cos\beta_0\, C_{\mathrm{Y}})$$
$$+ \cos\alpha_0\cos\beta_0\frac{\partial X_{\mathrm{T}}}{\partial\beta} + \sin\alpha_0\cos\beta_0\frac{\partial Z_{\mathrm{T}}}{\partial\beta} + \sin\beta_0\,\frac{\partial Y_{\mathrm{T}}}{\partial\beta}$$
$$- X_{\mathrm{T}}\sin\beta_0\cos\alpha_0 - Z_{\mathrm{T}}\sin\alpha_0\sin\beta_0 + Y_{\mathrm{T}}\cos\beta_0$$
$$+ mg(\sin\theta_0\cos\alpha_0\sin\beta_0 + \cos\theta_0\sin\phi_0\cos\beta_0 - \cos\theta_0\cos\phi_0\sin\alpha_0\sin\beta_0)\Big] \tag{D-159}$$

$$\frac{\partial(\dot V)}{\partial\phi} = g(\cos\theta_0\cos\phi_0\sin\beta_0 - \cos\theta_0\sin\phi_0\sin\alpha_0\cos\beta_0) \tag{D-160}$$

$$\frac{\partial(V)}{\partial\theta} = g(-\cos\theta_0\cos\alpha_0\cos\beta_0 - \sin\theta_0\sin\phi_0\sin\beta_0 - \sin\theta_0\cos\phi_0\sin\alpha_0\cos\beta_0) \tag{D-161}$$

$$\frac{\partial(\dot V)}{\partial\psi} = 0 \tag{D-162}$$

$$\frac{\partial(\dot V)}{\partial h} = \frac{S}{m}\left[-\cos\beta_0\left(\frac{1}{2}V_0^2 C_{\mathrm{D}}\,\frac{\partial\rho}{\partial h} + \bar q C_{\mathrm{D}_h}\right) + \sin\beta_0\left(\frac{1}{2}V_0^2 C_{\mathrm{Y}}\,\frac{\partial\rho}{\partial h} + \bar q C_{\mathrm{Y}_h}\right)\right] \tag{D-163}$$

$$\frac{\partial(\dot V)}{\partial x} = 0 \tag{D-164}$$

$$\frac{\partial(\dot V)}{\partial y} = 0 \tag{D-165}$$

$$\frac{\partial(\dot V)}{\partial\dot\alpha} = \frac{\bar q S\bar c}{2V_0 m}(-\cos\beta_0\, C_{\mathrm{D}_{\dot\alpha}} + \sin\beta_0\, C_{\mathrm{Y}_{\dot\alpha}}) \tag{D-166}$$

$$\frac{\partial(\dot V)}{\partial\dot\beta} = \frac{\bar q S b}{2V_0 m}(-\cos\beta_0\, C_{\mathrm{D}_{\dot\beta}} + \sin\beta_0\, C_{\mathrm{Y}_{\dot\beta}}) \tag{D-167}$$

$$\frac{\partial(\dot V)}{\partial\delta_i} = \frac{\bar q S}{m}(-\cos\beta_0\, C_{\mathrm{D}_{\delta_i}} + \sin\beta_0\, C_{\mathrm{Y}_{\delta_i}})$$
$$+ \frac{1}{m}\left(\cos\alpha_0\cos\beta_0\frac{\partial X_{\mathrm{T}}}{\partial\delta_i} + \sin\beta_0\,\frac{\partial Y_{\mathrm{T}}}{\partial\delta_i} + \sin\alpha_0\cos\beta_0\,\frac{\partial Z_{\mathrm{T}}}{\partial\delta_i}\right) \tag{D-168}$$

D.2.8 Angle-of-attack rate derivatives.—

$$\frac{\partial(\dot\alpha)}{\partial p} = -\frac{\bar q S b}{2V_0^2 m\cos\beta_0}C_{\mathrm{L}_p} - \tan\beta_0\cos\alpha_0 \tag{D-169}$$

$$\frac{\partial(\dot\alpha)}{\partial q} = -\frac{\bar q S\bar c}{2V_0^2 m\cos\beta_0}C_{\mathrm{L}_q} + 1.0 \tag{D-170}$$

$$\frac{\partial(\dot\alpha)}{\partial r} = -\frac{\bar q S b}{2V_0^2 m\cos\beta_0}C_{\mathrm{L}_r} - \tan\beta_0\sin\alpha_0 \tag{D-171}$$

$$\frac{\partial(\dot\alpha)}{\partial V} = -\frac{1}{mV_0\cos\beta_0}\Big\{s(V_0\rho C_{\mathrm{L}} + \bar q C_{\mathrm{L}_V}) - \cos\alpha_0\,\frac{\partial Z_{\mathrm{T}}}{\partial V} + \sin\alpha_0\,\frac{\partial X_{\mathrm{T}}}{\partial V}$$
$$+ \frac{1}{V_0}[-\bar q S C_{\mathrm{L}} + Z_{\mathrm{T}}\cos\alpha_0 - X_{\mathrm{T}}\sin\alpha_0$$
$$+ mg(\cos\theta_0\cos\phi_0\cos\alpha_0 + \sin\theta_0\sin\alpha_0)]\Big\} \tag{D-172}$$

$$\frac{\partial(\dot{\alpha})}{\partial\alpha} = -\frac{1}{mV_0\cos\beta_0}\Big[\bar{q}SC_{\mathrm{L}_\alpha} - \cos\alpha_0\,\frac{\partial Z_{\mathrm{T}}}{\partial\alpha} + \sin\alpha_0\,\frac{\partial X_{\mathrm{T}}}{\partial\alpha} + Z_{\mathrm{T}}\sin\alpha_0 + X_{\mathrm{T}}\cos\alpha_0$$
$$+ mg(\cos\theta_0\cos\phi_0\sin\alpha_0 - \sin\theta_0\cos\alpha_0)$$
$$+ \tan\beta_0\,(p_0\sin\alpha_0 - r_0\cos\alpha_0)\Big] \tag{D-173}$$

$$\frac{\partial(\dot{\alpha})}{\partial\beta} = -\frac{1}{mV_0\cos\beta_0}\Big\{\bar{q}SC_{\mathrm{L}_\beta} - \cos\alpha_0\,\frac{\partial Z_{\mathrm{T}}}{\partial\beta} + \sin\alpha_0\,\frac{\partial X_{\mathrm{T}}}{\partial\beta}$$
$$- \tan\beta_0\,[-\bar{q}SC_{\mathrm{L}} + Z_{\mathrm{T}}\cos\alpha_0 - X_{\mathrm{T}}\sin\alpha_0$$
$$+ mg(\cos\theta_0\cos\phi_0\cos\alpha_0 + \sin\theta_0\sin\alpha_0)]\Big\}$$
$$-\frac{1}{\cos^2\beta_0}(p_0\cos\alpha_0 + r_0\sin\alpha_0) \tag{D-174}$$

$$\frac{\partial(\dot{\alpha})}{\partial\phi} = -\frac{g}{V_0\cos\beta_0}\,\cos\theta_0\sin\phi_0\cos\alpha_0 \tag{D-175}$$

$$\frac{\partial(\dot{\alpha})}{\partial\theta} = -\frac{g}{V_0\cos\beta_0}(\sin\theta_0\cos\phi_0\cos\alpha_0 - \cos\theta_0\sin\alpha_0) \tag{D-176}$$

$$\frac{\partial(\dot{\alpha})}{\partial\psi} = 0 \tag{D-177}$$

$$\frac{\partial(\dot{\alpha})}{\partial h} = -\frac{S}{mV_0\cos\beta_0}\left(\frac{1}{2}V_0^2C_{\mathrm{L}}\,\frac{\partial\rho}{\partial h} + \bar{q}C_{\mathrm{L}_h}\right) \tag{D-178}$$

$$\frac{\partial(\dot{\alpha})}{\partial x} = 0 \tag{D-179}$$

$$\frac{\partial(\dot{\alpha})}{\partial y} = 0 \tag{D-180}$$

$$\frac{\partial(\dot{\alpha})}{\partial\dot{\alpha}} = -\frac{\bar{q}S\bar{c}}{V_0^2m\cos\beta_0}C_{\mathrm{L}_{\dot{\alpha}}} \tag{D-181}$$

$$\frac{\partial(\dot{\alpha})}{\partial\dot{\beta}} = -\frac{\bar{q}Sb}{2V_0^2m\cos\beta_0}C_{\mathrm{L}_{\dot{\beta}}} \tag{D-182}$$

$$\frac{\partial(\dot{\alpha})}{\partial\delta_i} = \frac{1}{mV_0\cos\beta_0}\left[-\bar{q}SC_{\mathrm{L}_{\delta_i}} + \cos\alpha_0\,\frac{\partial Z_{\mathrm{T}}}{\partial\delta_i} - \sin\alpha_0\,\frac{\partial X_{\mathrm{T}}}{\partial\delta_i}\right] \tag{D-183}$$

D.2.9 Angle-of-sideslip rate derivatives.—

$$\frac{\partial(\dot{\beta})}{\partial p} = \frac{\bar{q}Sb}{2V_0^2m}(\sin\beta_0\,C_{\mathrm{D}_p} + \cos\beta_0\,C_{\mathrm{Y}_p}) + \sin\alpha_0 \tag{D-184}$$

$$\frac{\partial(\dot{\beta})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0^2m}(\sin\beta_0\,C_{\mathrm{D}_q} + \cos\beta_0\,C_{\mathrm{Y}_q}) \tag{D-185}$$

$$\frac{\partial(\dot{\beta})}{\partial r} = \frac{\bar{q}Sb}{2V_0^2m}(\sin\beta_0\,C_{\mathrm{D}_r} + \cos\beta_0\,C_{\mathrm{Y}_r}) - \cos\alpha_0 \tag{D-186}$$

$$\frac{\partial(\dot\beta)}{\partial V} = \frac{1}{mV_0}\Big[S\sin\beta_0\,(\rho V_0 + \bar q C_{\mathrm{D}_V}) + S\cos\beta_0\,(\rho V_0 C_{\mathrm{Y}} + \bar q C_{\mathrm{Y}_V})$$
$$- \cos\alpha_0\sin\beta_0\,\frac{\partial X_{\mathrm{T}}}{\partial V} + \cos\beta_0\,\frac{\partial Y_{\mathrm{T}}}{\partial V} - \sin\alpha_0\sin\beta_0\,\frac{\partial Z_{\mathrm{T}}}{\partial V}\Big]$$
$$- \frac{1}{mV_0^2}[\bar q S(\sin\beta_0\,C_{\mathrm{D}} + \cos\beta_0\,C_{\mathrm{Y}}) - X_{\mathrm{T}}\cos\alpha_0\sin\beta_0$$
$$+ Y_{\mathrm{T}}\cos\beta_0 - Z_{\mathrm{T}}\sin\alpha_0\sin\beta_0] \tag{D-187}$$

$$\frac{\partial(\dot\beta)}{\partial\alpha} = \frac{1}{mV_0}\Big[\bar q S(\sin\beta_0\,C_{\mathrm{D}_\alpha} + \cos\beta_0\,C_{\mathrm{Y}_\alpha}) - \cos\alpha_0\sin\beta_0\,\frac{\partial X_{\mathrm{T}}}{\partial\alpha} + \cos\beta_0\,\frac{\partial Y_{\mathrm{T}}}{\partial\alpha}$$
$$- \sin\alpha_0\sin\beta_0\,\frac{\partial Z_{\mathrm{T}}}{\partial\alpha} + X_{\mathrm{T}}\sin\alpha_0\sin\beta_0 - Z_{\mathrm{T}}\,\cos\alpha_0\sin\beta_0$$
$$- mg(\sin\theta_0\sin\alpha_0\sin\beta_0 + \cos\theta_0\cos\phi_0\cos\alpha_0\sin\beta_0)\Big]$$
$$+ p_0\cos\alpha_0 + r_0\sin\alpha_0 \tag{D-188}$$

$$\frac{\partial(\dot\beta)}{\partial\beta} = \frac{1}{mV_0}\Big\{\bar q S[\sin\beta_0\,(C_{\mathrm{D}_\beta} - C_{\mathrm{Y}}) + \cos\beta_0\,(C_{\mathrm{D}} + C_{\mathrm{Y}_\beta})]$$
$$- \cos\alpha_0\sin\beta_0\,\frac{\partial X_{\mathrm{T}}}{\partial\beta} + \cos\beta_0\,\frac{\partial Y_{\mathrm{T}}}{\partial\beta} - \sin\alpha_0\sin\beta_0\,\frac{\partial Z_{\mathrm{T}}}{\partial\beta}$$
$$- X_{\mathrm{T}}\cos\alpha_0\cos\beta_0 - Y_{\mathrm{T}}\sin\beta_0 - Z_{\mathrm{T}}\sin\alpha_0\cos\beta_0$$
$$+ mg(\sin\theta_0\cos\alpha_0\cos\beta_0 - \cos\theta_0\sin\phi_0\sin\beta_0 - \cos\theta_0\cos\phi_0\sin\alpha_0\cos\beta_0)\Big\} \tag{D-189}$$

$$\frac{\partial(\dot\beta)}{\partial\phi} = \frac{g}{V_0}(\cos\theta_0\cos\phi_0\cos\beta_0 + \cos\theta_0\sin\phi_0\sin\alpha_0\sin\beta_0) \tag{D-190}$$

$$\frac{\partial(\dot\beta)}{\partial\theta} = \frac{g}{V_0}(\cos\theta_0\cos\alpha_0\sin\beta_0 - \sin\theta_0\sin\phi_0\cos\beta_0$$
$$+ \sin\theta_0\cos\phi_0\sin\alpha_0\sin\beta_0) \tag{D-191}$$

$$\frac{\partial(\dot\beta)}{\partial\psi} = 0 \tag{D-192}$$

$$\frac{\partial(\dot\beta)}{\partial h} = \frac{S}{mV_0}\left[\sin\beta_0\,\left(\frac{1}{2}V_0^2 C_{\mathrm{D}}\,\frac{\partial\rho}{\partial h} + \bar q C_{\mathrm{D}_h}\right) + \cos\beta_0\,\left(\frac{1}{2}V_0^2 C_{\mathrm{Y}}\,\frac{\partial\rho}{\partial h} + \bar q C_{\mathrm{Y}_h}\right)\right] \tag{D-193}$$

$$\frac{\partial(\dot\beta)}{\partial x} = 0 \tag{D-194}$$

$$\frac{\partial(\dot\beta)}{\partial y} = 0 \tag{D-195}$$

$$\frac{\partial(\dot\beta)}{\partial\dot\alpha} = \frac{\bar q S\bar c}{2V_0^2 m}(\sin\beta_0\,C_{\mathrm{D}_{\dot\alpha}} + \cos\beta_0\,C_{\mathrm{Y}_{\dot\alpha}}) \tag{D-196}$$

$$\frac{\partial(\dot\beta)}{\partial\dot\beta} = \frac{\bar q S b}{2V_0^2 m}(\sin\beta_0\,C_{\mathrm{D}_{\dot\beta}} + \cos\beta_0\,C_{\mathrm{Y}_{\dot\beta}}) \tag{D-197}$$

$$\frac{\partial(\dot\beta)}{\partial\delta_i} = \frac{1}{mV_0}\Big[\bar q S(\sin\beta_0\,C_{\mathrm{D}_{\delta_i}} + \cos\beta_0\,C_{\mathrm{Y}_{\delta_i}}) - \cos\alpha_0\sin\beta_0\,\frac{\partial X_{\mathrm{T}}}{\partial\delta_i} + \cos\beta_0\,\frac{\partial Y_{\mathrm{T}}}{\partial\delta_i}$$
$$- \sin\alpha_0\sin\beta_0\,\frac{\partial Z_{\mathrm{T}}}{\partial\delta_i}\Big] \tag{D-198}$$

D.2.10 Roll attitude rate derivatives.—

$$\frac{\partial(\dot\phi)}{\partial p} = 1 \tag{D-199}$$

$$\frac{\partial(\dot\phi)}{\partial q} = \sin\phi_0 \tan\theta_0 \tag{D-200}$$

$$\frac{\partial(\dot\phi)}{\partial r} = \cos\phi_0 \tan\theta_0 \tag{D-201}$$

$$\frac{\partial(\dot\phi)}{\partial V} = 0 \tag{D-202}$$

$$\frac{\partial(\dot\phi)}{\partial \alpha} = 0 \tag{D-203}$$

$$\frac{\partial(\dot\phi)}{\partial \beta} = 0 \tag{D-204}$$

$$\frac{\partial(\dot\phi)}{\partial \phi} = q_0 \cos\phi_0 \tan\theta_0 - r_0 \sin\phi_0 \tan\theta_0 \tag{D-205}$$

$$\frac{\partial(\dot\phi)}{\partial \theta} = q_0 \sin\phi_0 \sec^2\theta_0 + r_0 \cos\phi_0 \sec^2\theta_0 \tag{D-206}$$

$$\frac{\partial(\dot\phi)}{\partial \psi} = 0 \tag{D-207}$$

$$\frac{\partial(\dot\phi)}{\partial h} = 0 \tag{D-208}$$

$$\frac{\partial(\dot\phi)}{\partial x} = 0 \tag{D-209}$$

$$\frac{\partial(\dot\phi)}{\partial y} = 0 \tag{D-210}$$

$$\frac{\partial(\dot\phi)}{\partial \dot\alpha} = 0 \tag{D-211}$$

$$\frac{\partial(\dot\phi)}{\partial \dot\beta} = 0 \tag{D-212}$$

$$\frac{\partial(\dot\phi)}{\partial \delta_i} = 0 \tag{D-213}$$

D.2.11 Pitch attitude rate derivatives.—

$$\frac{\partial(\dot\theta)}{\partial p} = 0 \tag{D-214}$$

$$\frac{\partial(\dot\theta)}{\partial q} = \cos\phi_0 \tag{D-215}$$

$$\frac{\partial(\dot\theta)}{\partial r} = -\sin\phi_0 \tag{D-216}$$

$$\frac{\partial(\dot\theta)}{\partial V} = 0 \tag{D-217}$$

$$\frac{\partial(\dot{\theta})}{\partial \alpha} = 0 \tag{D-218}$$

$$\frac{\partial(\dot{\theta})}{\partial \beta} = 0 \tag{D-219}$$

$$\frac{\partial(\dot{\theta})}{\partial \phi} = -q_0 \sin\phi_0 - r_0 \cos\phi_0 \tag{D-220}$$

$$\frac{\partial(\dot{\theta})}{\partial \theta} = 0 \tag{D-221}$$

$$\frac{\partial(\dot{\theta})}{\partial \psi} = 0 \tag{D-222}$$

$$\frac{\partial(\dot{\theta})}{\partial h} = 0 \tag{D-223}$$

$$\frac{\partial(\dot{\theta})}{\partial x} = 0 \tag{D-224}$$

$$\frac{\partial(\dot{\theta})}{\partial y} = 0 \tag{D-225}$$

$$\frac{\partial(\dot{\theta})}{\partial \dot{\alpha}} = 0 \tag{D-226}$$

$$\frac{\partial(\dot{\theta})}{\partial \dot{\beta}} = 0 \tag{D-227}$$

$$\frac{\partial(\dot{\theta})}{\partial \delta_i} = 0 \tag{D-228}$$

D.2.12 Heading rate derivatives.—

$$\frac{\partial(\dot{\psi})}{\partial p} = 0 \tag{D-229}$$

$$\frac{\partial(\dot{\psi})}{\partial q} = \sin\phi_0 \sec\theta_0 \tag{D-230}$$

$$\frac{\partial(\dot{\psi})}{\partial r} = \cos\phi_0 \sec\theta_0 \tag{D-231}$$

$$\frac{\partial(\dot{\psi})}{\partial V} = 0 \tag{D-232}$$

$$\frac{\partial(\dot{\psi})}{\partial \alpha} = 0 \tag{D-233}$$

$$\frac{\partial(\dot{\psi})}{\partial \beta} = 0 \tag{D-234}$$

$$\frac{\partial(\dot{\psi})}{\partial \phi} = q_0 \cos\phi_0 \sec\theta_0 - r_0 \sin\phi_0 \sec\theta_0 \tag{D-235}$$

$$\frac{\partial(\dot{\psi})}{\partial \theta} = q_0 \sin\phi_0 \sec\theta_0 \tan\theta_0 + r_0 \cos\phi_0 \sec\theta_0 \tan\theta_0 \tag{D-236}$$

$$\frac{\partial(\dot{\psi})}{\partial h} = 0 \tag{D-237}$$

$$\frac{\partial(\dot{\psi})}{\partial x} = 0 \tag{D-238}$$
$$\frac{\partial(\dot{\psi})}{\partial y} = 0 \tag{D-239}$$
$$\frac{\partial(\dot{\psi})}{\partial \dot{\alpha}} = 0 \tag{D-240}$$
$$\frac{\partial(\dot{\psi})}{\partial \dot{\beta}} = 0 \tag{D-241}$$
$$\frac{\partial(\dot{\psi})}{\partial \delta_i} = 0 \tag{D-242}$$

D.2.13 Altitude rate derivatives.—

$$\frac{\partial(\dot{h})}{\partial p} = 0 \tag{D-243}$$
$$\frac{\partial(\dot{h})}{\partial q} = 0 \tag{D-244}$$
$$\frac{\partial(\dot{h})}{\partial r} = 0 \tag{D-245}$$
$$\frac{\partial(\dot{h})}{\partial V} = \sin\theta_0\cos\beta_0\cos\alpha_0 - \sin\phi_0\cos\theta_0\sin\beta_0 - \cos\phi_0\cos\theta_0\cos\beta_0\sin\alpha_0 \tag{D-246}$$
$$\frac{\partial(\dot{h})}{\partial \alpha} = -V_0(\cos\beta_0\sin\alpha_0\sin\theta_0 + \cos\beta_0\cos\alpha_0\cos\phi_0\cos\theta_0) \tag{D-247}$$
$$\frac{\partial(\dot{h})}{\partial \beta} = -V_0(\sin\beta_0\cos\alpha_0\sin\theta_0 + \cos\beta_0\sin\phi_0\cos\theta_0 - \sin\beta_0\sin\alpha_0\cos\phi_0\cos\theta_0) \tag{D-248}$$
$$\frac{\partial(\dot{h})}{\partial \phi} = -V_0(\sin\beta_0\cos\phi_0\cos\theta_0 - \cos\beta_0\sin\alpha_0\sin\phi_0\cos\theta_0) \tag{D-249}$$
$$\frac{\partial(\dot{h})}{\partial \theta} = V_0(\cos\beta_0\cos\alpha_0\cos\theta_0 + \sin\beta_0\sin\phi_0\sin\theta_0 + \cos\beta_0\sin\alpha_0\cos\phi_0\sin\theta_0) \tag{D-250}$$
$$\frac{\partial(\dot{h})}{\partial \psi} = 0 \tag{D-251}$$
$$\frac{\partial(\dot{h})}{\partial h} = 0 \tag{D-252}$$
$$\frac{\partial(\dot{h})}{\partial x} = 0 \tag{D-253}$$
$$\frac{\partial(\dot{h})}{\partial y} = 0 \tag{D-254}$$
$$\frac{\partial(\dot{h})}{\partial \dot{\alpha}} = 0 \tag{D-255}$$
$$\frac{\partial(\dot{h})}{\partial \dot{\beta}} = 0 \tag{D-256}$$
$$\frac{\partial(\dot{h})}{\partial \delta_i} = 0 \tag{D-257}$$

D.2.14 North acceleration derivatives.—

$$\frac{\partial(\dot{x})}{\partial p} = 0 \tag{D-258}$$

$$\frac{\partial(\dot{x})}{\partial q} = 0 \tag{D-259}$$

$$\frac{\partial(\dot{x})}{\partial r} = 0 \tag{D-260}$$

$$\begin{aligned}\frac{\partial(\dot{x})}{\partial V} = {} & \cos\beta_0\cos\alpha_0\cos\theta_0\cos\psi_0 + \sin\beta_0\,(\sin\phi_0\sin\theta_0\cos\psi_0 - \cos\phi_0\sin\psi_0) \\ & + \cos\beta_0\sin\alpha_0\,(\cos\phi_0\sin\theta_0\cos\psi_0 + \sin\phi_0\sin\psi_0)\end{aligned} \tag{D-261}$$

$$\frac{\partial(\dot{x})}{\partial \alpha} = V_0[\cos\beta_0\cos\alpha_0\,(\cos\phi_0\sin\theta_0\cos\psi_0 + \sin\phi_0\sin\psi_0) - \cos\beta_0\sin\alpha_0\cos\theta_0\cos\psi_0] \tag{D-262}$$

$$\begin{aligned}\frac{\partial(\dot{x})}{\partial \beta} = {} & V_0[\cos\beta_0\,(\sin\phi_0\sin\theta_0\cos\psi_0 - \cos\phi_0\sin\psi_0) - \sin\beta_0\cos\alpha_0\cos\theta_0\cos\psi_0 \\ & - \sin\beta_0\sin\alpha_0\,(\cos\phi_0\sin\theta_0\cos\psi_0 + \sin\phi_0\sin\psi_0)]\end{aligned} \tag{D-263}$$

$$\begin{aligned}\frac{\partial(\dot{x})}{\partial \phi} = {} & V_0[\sin\beta_0\,(\cos\phi_0\sin\theta_0\cos\psi_0 + \sin\phi_0\sin\psi_0) \\ & + \cos\beta_0\sin\alpha_0\,(\cos\phi_0\sin\psi_0 - \sin\phi_0\sin\theta_0\cos\psi_0)]\end{aligned} \tag{D-264}$$

$$\begin{aligned}\frac{\partial(\dot{x})}{\partial \theta} = {} & V_0(\sin\beta_0\sin\phi_0\cos\theta_0\cos\psi_0 - \cos\beta_0\cos\alpha_0\sin\theta_0\cos\psi_0 \\ & + \cos\beta_0\sin\alpha_0\cos\phi_0\cos\theta_0\cos\psi_0)\end{aligned} \tag{D-265}$$

$$\begin{aligned}\frac{\partial(\dot{x})}{\partial \psi} = {} & V_0[-\cos\beta_0\cos\alpha_0\cos\theta_0\sin\psi_0 - \sin\beta_0\,(\sin\phi_0\sin\theta_0\sin\psi_0 + \cos\phi_0\cos\psi_0) \\ & - \cos\beta_0\sin\alpha_0\,(\cos\phi_0\sin\theta_0\sin\psi_0 - \sin\phi_0\cos\psi_0)]\end{aligned} \tag{D-266}$$

$$\frac{\partial(\dot{x})}{\partial h} = 0 \tag{D-267}$$

$$\frac{\partial(\dot{x})}{\partial x} = 0 \tag{D-268}$$

$$\frac{\partial(\dot{x})}{\partial y} = 0 \tag{D-269}$$

$$\frac{\partial(\dot{x})}{\partial \alpha} = 0 \tag{D-270}$$

$$\frac{\partial(\dot{x})}{\partial \beta} = 0 \tag{D-271}$$

$$\frac{\partial(\dot{x})}{\partial \delta_i} = 0 \tag{D-272}$$

D.2.15 East acceleration derivatives.—

$$\frac{\partial(\dot{y})}{\partial p} = 0 \tag{D-273}$$

$$\frac{\partial(\dot{y})}{\partial q} = 0 \tag{D-274}$$

$$\frac{\partial(\dot{y})}{\partial r} = 0 \tag{D-275}$$

$$\frac{\partial(\dot{y})}{\partial V} = \cos\theta_0 \sin\psi_0 \cos\beta_0 \cos\alpha_0 + \sin\beta_0 (\cos\phi_0 \cos\psi_0 + \sin\phi_0 \sin\theta_0 \sin\psi_0) + \cos\beta_0 \sin\alpha_0 (\cos\phi_0 \sin\theta_0 \sin\psi_0 - \sin\phi_0 \cos\psi_0) \tag{D-276}$$

$$\frac{\partial(\dot{y})}{\partial \alpha} = V_0[\cos\beta_0 \cos\alpha_0 (\cos\phi_0 \sin\theta_0 \sin\psi_0 - \sin\phi_0 \cos\psi_0) - \cos\beta_0 \sin\alpha_0 \cos\theta_0 \sin\psi_0] \tag{D-277}$$

$$\frac{\partial(\dot{y})}{\partial \beta} = V_0[\cos\beta_0 (\cos\phi_0 \cos\psi_0 + \sin\phi_0 \sin\theta_0 \sin\psi_0) - \sin\beta_0 \cos\alpha_0 \cos\theta_0 \sin\psi_0 - \sin\beta_0 \sin\alpha_0 (\cos\phi_0 \sin\theta_0 \sin\psi_0 - \sin\phi_0 \cos\psi_0)] \tag{D-278}$$

$$\frac{\partial(\dot{y})}{\partial \phi} = V_0[\sin\beta_0 (\cos\phi_0 \sin\theta_0 \sin\psi_0 - \sin\phi_0 \cos\psi_0) - \cos\beta_0 \sin\alpha_0 (\sin\phi_0 \sin\theta_0 \sin\psi_0 + \cos\phi_0 \cos\psi_0)] \tag{D-279}$$

$$\frac{\partial(\dot{y})}{\partial \theta} = V_0(\sin\beta_0 \sin\phi_0 \cos\theta_0 \sin\psi_0 - \cos\beta_0 \cos\alpha_0 \sin\theta_0 \sin\psi_0 + \cos\beta_0 \sin\alpha_0 \cos\phi_0 \cos\theta_0 \sin\psi_0) \tag{D-280}$$

$$\frac{\partial(\dot{y})}{\partial \psi} = V_0[\cos\beta_0 \cos\alpha_0 \cos\theta_0 \cos\psi_0 - \sin\beta_0 (\cos\phi_0 \sin\psi_0 - \sin\phi_0 \sin\theta_0 \cos\psi_0) + \cos\beta_0 \sin\alpha_0 (\cos\phi_0 \sin\theta_0 \cos\psi_0 + \sin\phi_0 \sin\psi_0)] \tag{D-281}$$

$$\frac{\partial(\dot{y})}{\partial h} = 0 \tag{D-282}$$

$$\frac{\partial(\dot{y})}{\partial x} = 0 \tag{D-283}$$

$$\frac{\partial(\dot{y})}{\partial y} = 0 \tag{D-284}$$

$$\frac{\partial(\dot{y})}{\partial \dot{\alpha}} = 0 \tag{D-285}$$

$$\frac{\partial(\dot{y})}{\partial \dot{\beta}} = 0 \tag{D-286}$$

$$\frac{\partial(\dot{y})}{\partial \delta_i} = 0 \tag{D-287}$$

D.3 Evaluation of the Derivatives of the Observation Variables

The generalized derivatives of the observation variables are defined in appendix C, in equations (C-16) to (C-56). In this section, these generalized derivatives are evaluated in terms of the stability and control derivatives, primative terms, and the state, time derivative of state, and control variables.

D.3.1 Longitudinal kinematic acceleration derivatives.—

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial p} = \frac{\bar{q}Sb}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_p} + \sin\alpha_0\, C_{\mathrm{L}_p}) \tag{D-288}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_q} + \sin\alpha_0\, C_{\mathrm{L}_q}) \tag{D-289}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial r} = \frac{\bar{q}Sb}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_r} + \sin\alpha_0\, C_{\mathrm{L}_r}) \tag{D-290}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial V} = \frac{1}{g_0 m}\left[-S\cos\alpha_0\left(\rho V_0 C_\mathrm{D} + \bar{q}C_{\mathrm{D}_V}\right) + S\sin\alpha_0\left(\rho V_0 C_\mathrm{L} + \bar{q}C_{\mathrm{L}_V}\right) + \frac{\partial X_\mathrm{T}}{\partial V}\right] \tag{D-291}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial \alpha} = \frac{1}{g_0 m}\left\{\bar{q}S[-\cos\alpha_0\left(C_{\mathrm{D}_\alpha} - C_\mathrm{L}\right) + \sin\alpha_0\left(C_{\mathrm{L}_\alpha} + C_\mathrm{D}\right)] + \frac{\partial X_\mathrm{T}}{\partial \alpha}\right\} \tag{D-292}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial \beta} = \frac{1}{g_0 m}\left[\bar{q}S(-\cos\alpha_0\, C_{\mathrm{D}_\beta} + \sin\alpha_0\, C_{\mathrm{L}_\beta}) + \frac{\partial X_\mathrm{T}}{\partial \beta}\right] \tag{D-293}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial \phi} = 0 \tag{D-294}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial \theta} = -\frac{g}{g_0}\cos\theta \tag{D-295}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial \psi} = 0 \tag{D-296}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial h} = \frac{1}{g_0 m}\left[-\cos\alpha_0\left(\frac{1}{2}SV_0^2 C_\mathrm{D}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right) + \sin\alpha_0\left(\frac{1}{2}SV_0^2 C_\mathrm{L}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\right] \tag{D-297}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial x} = 0 \tag{D-298}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial y} = 0 \tag{D-299}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_{\dot{\alpha}}} + \sin\alpha_0\, C_{\mathrm{L}_{\dot{\alpha}}}) \tag{D-300}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_{\dot{\beta}}} + \sin\alpha_0\, C_{\mathrm{L}_{\dot{\beta}}}) \tag{D-301}$$

$$\frac{\partial(a_{x,\mathrm{k}})}{\partial \delta_i} = \frac{1}{g_0 m}\left[\bar{q}S(-\cos\alpha_0\, C_{\mathrm{D}_{\delta_i}} + \sin\alpha_0\, C_{\mathrm{L}_{\delta_i}}) + \frac{\partial X_\mathrm{T}}{\partial \delta_i}\right] \tag{D-302}$$

D.3.2 Lateral kinematic acceleration derivatives.—

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial p} = \frac{\bar{q}Sb}{2V_0 g_0 m}C_{\mathrm{Y}_p} \tag{D-303}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}C_{\mathrm{Y}_q} \tag{D-304}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial r} = \frac{\bar{q}Sb}{2V_0 g_0 m}C_{\mathrm{Y}_r} \tag{D-305}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial V} = \frac{1}{g_0 m}\left(S\rho V_0 C_\mathrm{Y} + \bar{q}SC_{\mathrm{Y}_V} + \frac{\partial Y_\mathrm{T}}{\partial V}\right) \tag{D-306}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial \alpha} = \frac{1}{g_0 m}\left(\bar{q}SC_{\mathrm{Y}_\alpha} + \frac{\partial Y_\mathrm{T}}{\partial \alpha}\right) \tag{D-307}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial \beta} = \frac{1}{g_0 m}\left(\bar{q}SC_{\mathrm{Y}_\beta} + \frac{\partial Y_\mathrm{T}}{\partial \beta}\right) \tag{D-308}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial \phi} = \frac{g}{g_0}\cos\theta_0\cos\phi_0 \tag{D-309}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial \theta} = -\frac{g}{g_0}\sin\theta_0\sin\phi_0 \tag{D-310}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial \psi} = 0 \tag{D-311}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial h} = \frac{1}{g_0 m}\left(\frac{1}{2}SV_0^2 C_{\mathrm{Y}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{Y}_h}\right) \tag{D-312}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial x} = 0 \tag{D-313}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial y} = 0 \tag{D-314}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}C_{\mathrm{Y}_{\dot{\alpha}}} \tag{D-315}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0 g_0 m}C_{\mathrm{Y}_{\dot{\beta}}} \tag{D-316}$$

$$\frac{\partial(a_{y,\mathrm{k}})}{\partial \delta_i} = \frac{1}{g_0 m}\left(\bar{q}SC_{\mathrm{Y}_{\delta_i}} + \frac{\partial Y_{\mathrm{T}}}{\partial \delta_i}\right) \tag{D-317}$$

D.3.3 Z-body axis kinematic acceleration derivatives.—

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial p} = -\frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_p} + \cos\alpha_0\, C_{\mathrm{L}_p}) \tag{D-318}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial q} = -\frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_q} + \cos\alpha_0\, C_{\mathrm{L}_q}) \tag{D-319}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial r} = -\frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_r} + \cos\alpha_0\, C_{\mathrm{L}_r}) \tag{D-320}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial V} = -\frac{1}{g_0 m}\left[S\sin\alpha_0\,(\rho V_0 C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V}) + S\cos\alpha_0\,(\rho V_0 C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) - \frac{\partial Z_{\mathrm{T}}}{\partial V}\right] \tag{D-321}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial \alpha} = -\frac{\bar{q}S}{g_0 m}[\sin\alpha_0\,(C_{\mathrm{D}_\alpha} - C_{\mathrm{L}}) + \cos\alpha_0\,(C_{\mathrm{L}_\alpha} + C_{\mathrm{D}})] + \frac{1}{g_0 m}\frac{\partial Z_{\mathrm{T}}}{\partial \alpha} \tag{D-322}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial \beta} = -\frac{\bar{q}S}{g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_\beta} + \cos\alpha_0\, C_{\mathrm{L}_\beta}) + \frac{1}{g_0 m}\frac{\partial Z_{\mathrm{T}}}{\partial \beta} \tag{D-323}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial \phi} = -\frac{g}{g_0}\cos\theta_0\sin\phi_0 \tag{D-324}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial \theta} = -\frac{g}{g_0}\sin\theta_0\cos\phi_0 \tag{D-325}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial \psi} = 0 \tag{D-326}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial h} = -\frac{1}{g_0 m}\left[\sin\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{D}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right) + \cos\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{L}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\right] \tag{D-327}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial x} = 0 \tag{D-328}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial y} = 0 \tag{D-329}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial \dot{\alpha}} = -\frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_{\dot{\alpha}}} + \cos\alpha_0\, C_{\mathrm{L}_{\dot{\alpha}}}) \tag{D-330}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial \dot{\beta}} = -\frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_{\dot{\beta}}} + \cos\alpha_0\, C_{\mathrm{L}_{\dot{\beta}}}) \tag{D-331}$$

$$\frac{\partial(a_{z,\mathrm{k}})}{\partial \delta_i} = -\frac{1}{g_0 m}\left[\bar{q}S(\sin\alpha_0\, C_{\mathrm{D}_{\delta_i}} + \cos\alpha_0\, C_{\mathrm{L}_{\delta_i}}) - \frac{\partial Z_{\mathrm{T}}}{\partial \delta_i}\right] \tag{D-332}$$

D.3.4 x body axis accelerometer output derivatives.—

$$\frac{\partial(a_x)}{\partial p} = \frac{\bar{q}Sb}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_p} + \sin\alpha_0\, C_{\mathrm{L}_p}) \tag{D-333}$$

$$\frac{\partial(a_x)}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_q} + \sin\alpha_0\, C_{\mathrm{L}_q}) \tag{D-334}$$

$$\frac{\partial(a_x)}{\partial r} = \frac{\bar{q}Sb}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_r} + \sin\alpha_0\, C_{\mathrm{L}_r}) \tag{D-335}$$

$$\frac{\partial(a_x)}{\partial V} = \frac{1}{g_0 m}\left[-S\cos\alpha_0\,(\rho V_0 C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V}) + S\sin\alpha_0\,(\rho V_0 C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) + \frac{\partial X_{\mathrm{T}}}{\partial V}\right] \tag{D-336}$$

$$\frac{\partial(a_x)}{\partial \alpha} = \frac{1}{g_0 m}\left\{\bar{q}S[-\cos\alpha_0\,(C_{\mathrm{D}_\alpha} - C_{\mathrm{L}}) + \sin\alpha_0\,(C_{\mathrm{L}_\alpha} + C_{\mathrm{D}})] + \frac{\partial X_{\mathrm{T}}}{\partial \alpha}\right\} \tag{D-337}$$

$$\frac{\partial(a_x)}{\partial \beta} = \frac{1}{g_0 m}\left[\bar{q}S(-\cos\alpha_0\, C_{\mathrm{D}_\beta} + \sin\alpha_0\, C_{\mathrm{L}_\beta}) + \frac{\partial X_{\mathrm{T}}}{\partial \beta}\right] \tag{D-338}$$

$$\frac{\partial(a_x)}{\partial \phi} = 0 \tag{D-339}$$

$$\frac{\partial(a_x)}{\partial \theta} = 0 \tag{D-340}$$

$$\frac{\partial(a_x)}{\partial \psi} = 0 \tag{D-341}$$

$$\frac{\partial(a_x)}{\partial h} = \frac{1}{g_0 m}\left[-\cos\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{D}}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right) + \sin\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{L}}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\right] \tag{D-342}$$

$$\frac{\partial(a_x)}{\partial x} = 0 \tag{D-343}$$

$$\frac{\partial(a_x)}{\partial y} = 0 \tag{D-344}$$

$$\frac{\partial(a_x)}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_{\dot{\alpha}}} + \sin\alpha_0\, C_{\mathrm{L}_{\dot{\alpha}}}) \tag{D-345}$$

$$\frac{\partial(a_x)}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_{\dot{\beta}}} + \sin\alpha_0\, C_{\mathrm{L}_{\dot{\beta}}}) \tag{D-346}$$

$$\frac{\partial(a_x)}{\partial \delta_i} = \frac{1}{g_0 m}\left[\bar{q}S(-\cos\alpha_0\, C_{\mathrm{D}_{\delta_i}} + \sin\alpha_0\, C_{\mathrm{L}_{\delta_i}}) + \frac{\partial X_{\mathrm{T}}}{\partial \delta_i}\right] \tag{D-347}$$

D.3.5 y body axis accelerometer output derivatives.—

$$\frac{\partial(a_y)}{\partial p} = \frac{\bar{q}Sb}{2V_0 g_0 m}C_{\mathrm{Y}_p} \tag{D-348}$$

$$\frac{\partial(a_y)}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}C_{\mathrm{Y}_q} \tag{D-349}$$

$$\frac{\partial(a_y)}{\partial r} = \frac{\bar{q}Sb}{2V_0 g_0 m}C_{\mathrm{Y}_r} \tag{D-350}$$

$$\frac{\partial(a_y)}{\partial V} = \frac{1}{g_0 m}\left(S\rho V_0 C_{\mathrm{Y}} + \bar{q}SC_{\mathrm{Y}_V} + \frac{\partial Y_{\mathrm{T}}}{\partial V}\right) \tag{D-351}$$

$$\frac{\partial(a_y)}{\partial\alpha} = \frac{1}{g_0 m}\left(\bar{q}SC_{\mathrm{Y}_\alpha} + \frac{\partial Y_{\mathrm{T}}}{\partial\alpha}\right) \tag{D-352}$$

$$\frac{\partial(a_y)}{\partial\beta} = \frac{1}{g_0 m}\left(\bar{q}SC_{\mathrm{Y}_\beta} + \frac{\partial Y_{\mathrm{T}}}{\partial\beta}\right) \tag{D-353}$$

$$\frac{\partial(a_y)}{\partial\phi} = 0 \tag{D-354}$$

$$\frac{\partial(a_y)}{\partial\theta} = 0 \tag{D-355}$$

$$\frac{\partial(a_y)}{\partial\psi} = 0 \tag{D-356}$$

$$\frac{\partial(a_y)}{\partial h} = \frac{1}{g_0 m}\left(\frac{1}{2}SV_0^2 C_{\mathrm{Y}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{Y}_h}\right) \tag{D-357}$$

$$\frac{\partial(a_y)}{\partial x} = 0 \tag{D-358}$$

$$\frac{\partial(a_y)}{\partial y} = 0 \tag{D-359}$$

$$\frac{\partial(a_y)}{\partial\dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}C_{\mathrm{Y}_{\dot{\alpha}}} \tag{D-360}$$

$$\frac{\partial(a_y)}{\partial\dot{\beta}} = \frac{\bar{q}Sb}{2V_0 g_0 m}C_{\mathrm{Y}_{\dot{\beta}}} \tag{D-361}$$

$$\frac{\partial(a_y)}{\partial\delta_i} = \frac{1}{g_0 m}\left(\bar{q}SC_{\mathrm{Y}_{\delta_i}} + \frac{\partial Y_{\mathrm{T}}}{\partial\delta_i}\right) \tag{D-362}$$

D.3.6 z body axis accelerometer output derivatives.—

$$\frac{\partial(a_z)}{\partial p} = -\frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_p} + \cos\alpha_0\, C_{\mathrm{L}_p}) \tag{D-363}$$

$$\frac{\partial(a_z)}{\partial q} = -\frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_q} + \cos\alpha_0\, C_{\mathrm{L}_q}) \tag{D-364}$$

$$\frac{\partial(a_z)}{\partial r} = -\frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_r} + \cos\alpha_0\, C_{\mathrm{L}_r}) \tag{D-365}$$

$$\frac{\partial(a_z)}{\partial V} = -\frac{1}{g_0 m}\left[S\sin\alpha_0\,(\rho V_0 C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V}) + S\cos\alpha_0\,(\rho V_0 C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) - \frac{\partial Z_{\mathrm{T}}}{\partial V}\right] \tag{D-366}$$

$$\frac{\partial(a_z)}{\partial\alpha} = -\frac{1}{g_0 m}\left\{\bar{q}S[\sin\alpha_0\,(C_{\mathrm{D}_\alpha} - C_{\mathrm{L}}) + \cos\alpha_0\,(C_{\mathrm{L}_\alpha} + C_{\mathrm{D}})] - \frac{\partial Z_{\mathrm{T}}}{\partial\alpha}\right\} \tag{D-367}$$

$$\frac{\partial(a_z)}{\partial\beta} = -\frac{1}{g_0 m}\left[\bar{q}S(\sin\alpha_0\, C_{\mathrm{D}_\beta} + \cos\alpha_0\, C_{\mathrm{L}_\beta}) - \frac{\partial Z_{\mathrm{T}}}{\partial\beta}\right] \tag{D-368}$$

$$\frac{\partial(a_z)}{\partial\phi} = 0 \tag{D-369}$$

$$\frac{\partial(a_z)}{\partial\theta} = 0 \tag{D-370}$$

$$\frac{\partial(a_z)}{\partial\psi} = 0 \tag{D-371}$$

$$\frac{\partial(a_z)}{\partial h} = -\frac{1}{g_0 m}\left[\sin\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{D}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right) + \cos\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{L}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\right] \tag{D-372}$$

$$\frac{\partial(a_z)}{\partial x} = 0 \tag{D-373}$$
$$\frac{\partial(a_z)}{\partial y} = 0 \tag{D-374}$$
$$\frac{\partial(a_z)}{\partial \dot{\alpha}} = -\frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_{\dot{\alpha}}} + \cos\alpha_0\, C_{\mathrm{L}_{\dot{\alpha}}}) \tag{D-375}$$
$$\frac{\partial(a_z)}{\partial \dot{\beta}} = -\frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_{\dot{\beta}}} + \cos\alpha_0\, C_{\mathrm{L}_{\dot{\beta}}}) \tag{D-376}$$
$$\frac{\partial(a_z)}{\partial \delta_i} = -\frac{1}{g_0 m}\left[\bar{q}S(\sin\alpha_0\, C_{\mathrm{D}_{\delta_i}} + \cos\alpha_0\, C_{\mathrm{L}_{\delta_i}}) - \frac{\partial Z_{\mathrm{T}}}{\partial \delta_i}\right] \tag{D-377}$$

D.3.7 Normal accelerometer output derivatives.—

$$\frac{\partial(a_{\mathrm{n}})}{\partial p} = \frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_p} + \cos\alpha_0\, C_{\mathrm{L}_p}) \tag{D-378}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_q} + \cos\alpha_0\, C_{\mathrm{L}_q}) \tag{D-379}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial r} = \frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_r} + \cos\alpha_0\, C_{\mathrm{L}_r}) \tag{D-380}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial V} = \frac{1}{g_0 m}\left[\sin\alpha_0\,(S\rho V_0 C_{\mathrm{D}} + \bar{q}SC_{\mathrm{D}_V}) + \cos\alpha_0\,(S\rho V_0 C_{\mathrm{L}} + \bar{q}SC_{\mathrm{L}_V}) - \frac{\partial Z_{\mathrm{T}}}{\partial V}\right] \tag{D-381}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial \alpha} = \frac{1}{g_0 m}\left\{\bar{q}S[\sin\alpha_0\,(C_{\mathrm{D}_\alpha} - C_{\mathrm{L}}) + \cos\alpha_0\,(C_{\mathrm{L}_\alpha} + C_{\mathrm{D}})] - \frac{\partial Z_{\mathrm{T}}}{\partial \alpha}\right\} \tag{D-382}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial \beta} = \frac{1}{g_0 m}\left[\bar{q}S(\sin\alpha_0\, C_{\mathrm{D}_\beta} + \cos\alpha_0\, C_{\mathrm{L}_\beta}) - \frac{\partial Z_{\mathrm{T}}}{\partial \beta}\right] \tag{D-383}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial \phi} = 0 \tag{D-384}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial \theta} = 0 \tag{D-385}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial \psi} = 0 \tag{D-386}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial h} = \frac{1}{g_0 m}\left[\sin\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{D}}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right) + \cos\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{L}}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\right] \tag{D-387}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial x} = 0 \tag{D-388}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial y} = 0 \tag{D-389}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_{\dot{\alpha}}} + \cos\alpha_0\, C_{\mathrm{L}_{\dot{\alpha}}}) \tag{D-390}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_{\dot{\beta}}} + \cos\alpha_0\, C_{\mathrm{L}_{\dot{\beta}}}) \tag{D-391}$$
$$\frac{\partial(a_{\mathrm{n}})}{\partial \delta_i} = \frac{1}{g_0 m}\left[\bar{q}S(\sin\alpha_0\, C_{\mathrm{D}_{\delta_i}} + \cos\alpha_0\, C_{\mathrm{L}_{\delta_i}}) - \frac{\partial Z_{\mathrm{T}}}{\partial \delta_i}\right] \tag{D-392}$$

D.3.8 Derivatives of x body axis accelerometer output not at the vehicle center of gravity.—

$$\frac{\partial(a_{x,i})}{\partial p} = \frac{\bar{q}Sb}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_p} + \sin\alpha_0\, C_{\mathrm{L}_p}) + \frac{1}{g_0}(q_0 y_x + r_0 z_x) \tag{D-393}$$

$$\frac{\partial(a_{x,i})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_q} + \sin\alpha_0\, C_{\mathrm{L}_q}) + \frac{1}{g_0}(p_0 y_x - 2q_0 z_x) \tag{D-394}$$

$$\frac{\partial(a_{x,i})}{\partial r} = \frac{\bar{q}Sb}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_r} + \sin\alpha_0\, C_{\mathrm{L}_r}) + \frac{1}{g_0}(p_0 z_x - 2r_0 x_x) \tag{D-395}$$

$$\frac{\partial(a_{x,i})}{\partial V} = \frac{1}{g_0 m}\left[-S\cos\alpha_0\,(\rho V_0 C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V}) + S\sin\alpha_0\,(\rho V_0 C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) + \frac{\partial X_{\mathrm{T}}}{\partial V}\right] \tag{D-396}$$

$$\frac{\partial(a_{x,i})}{\partial \alpha} = \frac{1}{g_0 m}\left\{\bar{q}S[-\cos\alpha_0\,(C_{\mathrm{D}_\alpha} - C_{\mathrm{L}}) + \sin\alpha_0\,(C_{\mathrm{L}_\alpha} + C_{\mathrm{D}})] + \frac{\partial X_{\mathrm{T}}}{\partial \alpha}\right\} \tag{D-397}$$

$$\frac{\partial(a_{x,i})}{\partial \beta} = \frac{1}{g_0 m}\left[\bar{q}S(-\cos\alpha_0\, C_{\mathrm{D}_\beta} + \sin\alpha_0\, C_{\mathrm{L}_\beta}) + \frac{\partial X_{\mathrm{T}}}{\partial \beta}\right] \tag{D-398}$$

$$\frac{\partial(a_{x,i})}{\partial \phi} = 0 \tag{D-399}$$

$$\frac{\partial(a_{x,i})}{\partial \theta} = 0 \tag{D-400}$$

$$\frac{\partial(a_{x,i})}{\partial \psi} = 0 \tag{D-401}$$

$$\frac{\partial(a_{x,i})}{\partial h} = \frac{1}{g_0 m}\left[-\cos\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{D}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right) + \sin\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{L}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\right] \tag{D-402}$$

$$\frac{\partial(a_{x,i})}{\partial x} = 0 \tag{D-403}$$

$$\frac{\partial(a_{x,i})}{\partial y} = 0 \tag{D-404}$$

$$\frac{\partial(a_{x,i})}{\partial \dot{p}} = 0 \tag{D-405}$$

$$\frac{\partial(a_{x,i})}{\partial \dot{q}} = \frac{z_x}{g_0} \tag{D-406}$$

$$\frac{\partial(a_{x,i})}{\partial \dot{r}} = -\frac{y_x}{g_0} \tag{D-407}$$

$$\frac{\partial(a_{x,i})}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_{\dot{\alpha}}} + \sin\alpha_0\, C_{\mathrm{L}_{\dot{\alpha}}}) \tag{D-408}$$

$$\frac{\partial(a_{x,i})}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0 g_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_{\dot{\beta}}} + \sin\alpha_0\, C_{\mathrm{L}_{\dot{\beta}}}) \tag{D-409}$$

$$\frac{\partial(a_{x,i})}{\partial \delta_i} = \frac{1}{g_0 m}\left[-\bar{q}S(-\cos\alpha_0\, C_{\mathrm{D}_{\delta_i}} + \sin\alpha_0\, C_{\mathrm{L}_{\delta_i}}) + \frac{\partial X_{\mathrm{T}}}{\partial \delta_i}\right] \tag{D-410}$$

D.3.9 Derivatives of y body axis accelerometer output not at vehicle center of gravity.—

$$\frac{\partial(a_{y,i})}{\partial p} = \frac{\bar{q}Sb}{2V_0 g_0 m}C_{\mathrm{Y}_p} - \frac{1}{g_0}(2p_0 y_y - q_0 x_y) \tag{D-411}$$

$$\frac{\partial(a_{y,i})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}C_{Y_q} + \frac{1}{g_0}(p_0 x_y + r_0 z_y) \quad \text{(D-412)}$$

$$\frac{\partial(a_{y,i})}{\partial r} = \frac{\bar{q}Sb}{2V_0 g_0 m}C_{Y_r} + \frac{1}{g_0}(q_0 z_y - 2r_0 y_y) \quad \text{(D-413)}$$

$$\frac{\partial(a_{y,i})}{\partial V} = \frac{1}{g_0 m}\left(S\rho V_0 C_Y + \bar{q}SC_{Y_V} + \frac{\partial Y_T}{\partial V}\right) \quad \text{(D-414)}$$

$$\frac{\partial(a_{y,i})}{\partial \alpha} = \frac{1}{g_0 m}\left(\bar{q}SC_{Y_\alpha} + \frac{\partial Y_T}{\partial \alpha}\right) \quad \text{(D-415)}$$

$$\frac{\partial(a_{y,i})}{\partial \beta} = \frac{1}{g_0 m}\left(\bar{q}SC_{Y_\beta} + \frac{\partial Y_T}{\partial \beta}\right) \quad \text{(D-416)}$$

$$\frac{\partial(a_{y,i})}{\partial \phi} = 0 \quad \text{(D-417)}$$

$$\frac{\partial(a_{y,i})}{\partial \theta} = 0 \quad \text{(D-418)}$$

$$\frac{\partial(a_{y,i})}{\partial \psi} = 0 \quad \text{(D-419)}$$

$$\frac{\partial(a_{y,i})}{\partial h} = \frac{1}{g_0 m}\left(\frac{1}{2}SV_0^2 C_Y \frac{\partial \rho}{\partial h} + \bar{q}SC_{Y_h}\right) \quad \text{(D-420)}$$

$$\frac{\partial(a_{y,i})}{\partial x} = 0 \quad \text{(D-421)}$$

$$\frac{\partial(a_{y,i})}{\partial y} = 0 \quad \text{(D-422)}$$

$$\frac{\partial(a_{y,i})}{\partial \dot{p}} = -\frac{z_y}{g_0} \quad \text{(D-423)}$$

$$\frac{\partial(a_{y,i})}{\partial \dot{q}} = 0 \quad \text{(D-424)}$$

$$\frac{\partial(a_{y,i})}{\partial \dot{r}} = -\frac{x_y}{g_0} \quad \text{(D-425)}$$

$$\frac{\partial(a_{y,i})}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}C_{Y_{\dot{\alpha}}} \quad \text{(D-426)}$$

$$\frac{\partial(a_{y,i})}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0 g_0 m}C_{Y_{\dot{\beta}}} \quad \text{(D-427)}$$

$$\frac{\partial(a_{y,i})}{\partial \delta_i} = \frac{1}{g_0 m}\left(\bar{q}SC_{Y_{\delta_i}} + \frac{\partial Y_T}{\partial \delta_i}\right) \quad \text{(D-428)}$$

D.3.10 Derivatives of z body axis accelerometer output not at vehicle center of gravity.—

$$\frac{\partial(a_{z,i})}{\partial p} = -\frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{D_p} + \cos\alpha_0\, C_{L_p}) - \frac{1}{g_0}(2p_0 z_z - r_0 x_z) \quad \text{(D-429)}$$

$$\frac{\partial(a_{z,i})}{\partial q} = -\frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\, C_{D_q} + \cos\alpha_0\, C_{L_q}) - \frac{1}{g_0}(2q_0 z_z - r_0 y_z) \quad \text{(D-430)}$$

$$\frac{\partial(a_{z,i})}{\partial r} = -\frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{D_r} + \cos\alpha_0\, C_{L_r}) + \frac{1}{g_0}(p_0 x_z + q_0 y_z) \quad \text{(D-431)}$$

$$\frac{\partial(a_{z,i})}{\partial V} = -\frac{1}{g_0 m}\left[S\sin\alpha_0\,(\rho V_0 C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V}) + S\cos\alpha_0\,(\rho V_0 C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) - \frac{\partial Z_{\mathrm{T}}}{\partial V}\right] \quad \text{(D-432)}$$

$$\frac{\partial(a_{z,i})}{\partial \alpha} = -\frac{1}{g_0 m}\left\{\bar{q}S[\sin\alpha_0\,(C_{\mathrm{D}_\alpha} - C_{\mathrm{L}}) + \cos\alpha_0\,(C_{\mathrm{L}_\alpha} + C_{\mathrm{D}})] - \frac{\partial Z_{\mathrm{T}}}{\partial \alpha}\right\} \quad \text{(D-433)}$$

$$\frac{\partial(a_{z,i})}{\partial \beta} = -\frac{1}{g_0 m}\left[\bar{q}S(\sin\alpha_0\,C_{\mathrm{D}_\beta} + \cos\alpha_0\,C_{\mathrm{L}_\beta}) - \frac{\partial Z_{\mathrm{T}}}{\partial \beta}\right] \quad \text{(D-434)}$$

$$\frac{\partial(a_{z,i})}{\partial \phi} = 0 \quad \text{(D-435)}$$

$$\frac{\partial(a_{z,i})}{\partial \theta} = 0 \quad \text{(D-436)}$$

$$\frac{\partial(a_{z,i})}{\partial \psi} = 0 \quad \text{(D-437)}$$

$$\frac{\partial(a_{z,i})}{\partial h} = -\frac{1}{g_0 m}\left[\sin\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{D}}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right) + \cos\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{L}}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\right] \quad \text{(D-438)}$$

$$\frac{\partial(a_{z,i})}{\partial x} = 0 \quad \text{(D-439)}$$

$$\frac{\partial(a_{z,i})}{\partial y} = 0 \quad \text{(D-440)}$$

$$\frac{\partial(a_{z,i})}{\partial \dot{p}} = \frac{y_z}{g_0} \quad \text{(D-441)}$$

$$\frac{\partial(a_{z,i})}{\partial \dot{q}} = -\frac{x_z}{g_0} \quad \text{(D-442)}$$

$$\frac{\partial(a_{z,i})}{\partial \dot{r}} = 0 \quad \text{(D-443)}$$

$$\frac{\partial(a_{z,i})}{\partial \dot{\alpha}} = -\frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\,C_{\mathrm{D}_{\dot\alpha}} + \cos\alpha_0\,C_{\mathrm{L}_{\dot\alpha}}) \quad \text{(D-444)}$$

$$\frac{\partial(a_{z,i})}{\partial \dot{\beta}} = -\frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\,C_{\mathrm{D}_{\dot\beta}} + \cos\alpha_0\,C_{\mathrm{L}_{\dot\beta}}) \quad \text{(D-445)}$$

$$\frac{\partial(a_{z,i})}{\partial \delta_i} = -\frac{1}{g_0 m}\left[\bar{q}S(\sin\alpha_0\,C_{\mathrm{D}_{\delta_i}} + \cos\alpha_0\,C_{\mathrm{L}_{\delta_i}}) - \frac{\partial Z_{\mathrm{T}}}{\partial \delta_i}\right] \quad \text{(D-446)}$$

D.3.11 Derivatives of normal accelerometer output not at vehicle center of gravity.—

$$\frac{\partial(a_{n,i})}{\partial p} = \frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\,C_{\mathrm{D}_p} + \cos\alpha_0\,C_{\mathrm{L}_p}) + \frac{1}{g_0}(2p_0 z_z - r_0 x_z) \quad \text{(D-447)}$$

$$\frac{\partial(a_{n,i})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\,C_{\mathrm{D}_q} + \cos\alpha_0\,C_{\mathrm{L}_q}) + \frac{1}{g_0}(2q_0 z_z - r_0 y_z) \quad \text{(D-448)}$$

$$\frac{\partial(a_{n,i})}{\partial r} = \frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\,C_{\mathrm{D}_r} + \cos\alpha_0\,C_{\mathrm{L}_r}) - \frac{1}{g_0}(p_0 x_z + q_0 y_z) \quad \text{(D-449)}$$

$$\frac{\partial(a_{n,i})}{\partial V} = \frac{1}{g_0 m}\left[S\sin\alpha_0\,(\rho V_0 C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V}) + S\cos\alpha_0\,(\rho V_0 C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) - \frac{\partial Z_{\mathrm{T}}}{\partial V}\right] \quad \text{(D-450)}$$

$$\frac{\partial(a_{n,i})}{\partial \alpha} = \frac{1}{g_0 m}\left\{\bar{q}S[\sin\alpha_0\,(C_{\mathrm{D}_\alpha} - C_{\mathrm{L}}) + \cos\alpha_0\,(C_{\mathrm{L}_\alpha} + C_{\mathrm{D}})] - \frac{\partial Z_{\mathrm{T}}}{\partial \alpha}\right\} \quad \text{(D-451)}$$

$$\frac{\partial(a_{n,i})}{\partial\beta} = \frac{1}{g_0 m}\left[\bar{q}S(\sin\alpha_0\, C_{\mathrm{D}_\beta} + \cos\alpha_0\, C_{\mathrm{L}_\beta}) - \frac{\partial Z_{\mathrm{T}}}{\partial\beta}\right] \tag{D-452}$$

$$\frac{\partial(a_{n,i})}{\partial\phi} = 0 \tag{D-453}$$

$$\frac{\partial(a_{n,i})}{\partial\theta} = 0 \tag{D-454}$$

$$\frac{\partial(a_{n,i})}{\partial\psi} = 0 \tag{D-455}$$

$$\frac{\partial(a_{n,i})}{\partial h} = \frac{1}{g_0 m}\left[\sin\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{D}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right) + \cos\alpha_0\left(\frac{1}{2}SV_0^2 C_{\mathrm{L}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\right] \tag{D-456}$$

$$\frac{\partial(a_{n,i})}{\partial x} = 0 \tag{D-457}$$

$$\frac{\partial(a_{n,i})}{\partial y} = 0 \tag{D-458}$$

$$\frac{\partial(a_{n,i})}{\partial\dot{p}} = -\frac{y_z}{g_0} \tag{D-459}$$

$$\frac{\partial(a_{n,i})}{\partial\dot{q}} = \frac{x_z}{g_0} \tag{D-460}$$

$$\frac{\partial(a_{n,i})}{\partial\dot{r}} = 0 \tag{D-461}$$

$$\frac{\partial(a_{n,i})}{\partial\dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_{\dot{\alpha}}} + \cos\alpha_0\, C_{\mathrm{L}_{\dot{\alpha}}}) \tag{D-462}$$

$$\frac{\partial(a_{n,i})}{\partial\dot{\beta}} = \frac{\bar{q}Sb}{2V_0 g_0 m}(\sin\alpha_0\, C_{\mathrm{D}_{\dot{\beta}}} + \cos\alpha_0\, C_{\mathrm{L}_{\dot{\beta}}}) \tag{D-463}$$

$$\frac{\partial(a_{n,i})}{\partial\delta_i} = \frac{1}{g_0 m}\left[\bar{q}S(\sin\alpha_0\, C_{\mathrm{D}_{\delta_i}} + \cos\alpha_0\, C_{\mathrm{L}_{\delta_i}}) - \frac{\partial Z_{\mathrm{T}}}{\partial\delta_i}\right] \tag{D-464}$$

D.3.12 Load factor derivatives.—

$$\frac{\partial(n)}{\partial p} = \frac{\bar{q}Sb}{2V_0 gm}C_{\mathrm{L}_p} \tag{D-465}$$

$$\frac{\partial(n)}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 gm}C_{\mathrm{L}_q} \tag{D-466}$$

$$\frac{\partial(n)}{\partial r} = \frac{\bar{q}Sb}{2V_0 gm}C_{\mathrm{L}_r} \tag{D-467}$$

$$\frac{\partial(n)}{\partial V} = \frac{1}{gm}(S\rho V_0 C_{\mathrm{L}} + \bar{q}SC_{\mathrm{L}_V}) \tag{D-468}$$

$$\frac{\partial(n)}{\partial\alpha} = \frac{\bar{q}S}{gm}C_{\mathrm{L}_\alpha} \tag{D-469}$$

$$\frac{\partial(n)}{\partial\beta} = \frac{\bar{q}S}{gm}C_{\mathrm{L}_\beta} \tag{D-470}$$

$$\frac{\partial(n)}{\partial\phi} = 0 \tag{D-471}$$

$$\frac{\partial(n)}{\partial\theta} = 0 \tag{D-472}$$

$$\frac{\partial(n)}{\partial\psi} = 0 \tag{D-473}$$
$$\frac{\partial(n)}{\partial h} = \frac{1}{gm}\left(\frac{1}{2}SV_0^2 C_{\mathrm{L}}\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right) \tag{D-474}$$
$$\frac{\partial(n)}{\partial x} = 0 \tag{D-475}$$
$$\frac{\partial(n)}{\partial y} = 0 \tag{D-476}$$
$$\frac{\partial(n)}{\partial\dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0gm}C_{\mathrm{L}_{\dot{\alpha}}} \tag{D-477}$$
$$\frac{\partial(n)}{\partial\dot{\beta}} = \frac{\bar{q}Sb}{2V_0gm}C_{\mathrm{L}_{\dot{\beta}}} \tag{D-478}$$
$$\frac{\partial(n)}{\partial\delta_i} = \frac{\bar{q}S}{gm}C_{\mathrm{L}_{\delta_i}} \tag{D-479}$$

D.3.13 Speed of sound derivatives.—

$$\frac{\partial(a)}{\partial p} = 0 \tag{D-480}$$
$$\frac{\partial(a)}{\partial q} = 0 \tag{D-481}$$
$$\frac{\partial(a)}{\partial r} = 0 \tag{D-482}$$
$$\frac{\partial(a)}{\partial V} = 0 \tag{D-483}$$
$$\frac{\partial(a)}{\partial\alpha} = 0 \tag{D-484}$$
$$\frac{\partial(a)}{\partial\beta} = 0 \tag{D-485}$$
$$\frac{\partial(a)}{\partial\phi} = 0 \tag{D-486}$$
$$\frac{\partial(a)}{\partial\theta} = 0 \tag{D-487}$$
$$\frac{\partial(a)}{\partial\psi} = 0 \tag{D-488}$$
$$\frac{\partial(a)}{\partial h} = \frac{0.7p_0}{\rho_0 T_0(1.4\, p_0/\rho_0 T_0)^{1/2}}\frac{\partial \mathrm{T}}{\partial h} \tag{D-489}$$
$$\frac{\partial(a)}{\partial x} = 0 \tag{D-490}$$
$$\frac{\partial(a)}{\partial y} = 0 \tag{D-491}$$
$$\frac{\partial(a)}{\partial\dot{\alpha}} = 0 \tag{D-492}$$
$$\frac{\partial(a)}{\partial\dot{\beta}} = 0 \tag{D-493}$$

$$\frac{\partial(a)}{\partial\delta_i} = 0 \tag{D-494}$$

D.3.14 Mach number derivatives.—

$$\frac{\partial(M)}{\partial p} = 0 \tag{D-495}$$

$$\frac{\partial(M)}{\partial q} = 0 \tag{D-496}$$

$$\frac{\partial(M)}{\partial r} = 0 \tag{D-497}$$

$$\frac{\partial(M)}{\partial V} = \frac{1}{a} \tag{D-498}$$

$$\frac{\partial(M)}{\partial\alpha} = 0 \tag{D-499}$$

$$\frac{\partial(M)}{\partial\beta} = 0 \tag{D-500}$$

$$\frac{\partial(M)}{\partial\phi} = 0 \tag{D-501}$$

$$\frac{\partial(M)}{\partial\theta} = 0 \tag{D-502}$$

$$\frac{\partial(M)}{\partial\psi} = 0 \tag{D-503}$$

$$\frac{\partial(M)}{\partial h} = -\frac{V_0}{a^2}\left[\frac{0.7p_0}{\rho_0 T_0(1.4\,p_0/\rho_0 T_0)^{1/2}}\right]\frac{\partial \mathrm{T}}{\partial h} \tag{D-504}$$

$$\frac{\partial(M)}{\partial x} = 0 \tag{D-505}$$

$$\frac{\partial(M)}{\partial y} = 0 \tag{D-506}$$

$$\frac{\partial(M)}{\partial\dot{\alpha}} = 0 \tag{D-507}$$

$$\frac{\partial(M)}{\partial\dot{\beta}} = 0 \tag{D-508}$$

$$\frac{\partial(M)}{\partial\delta_i} = 0 \tag{D-509}$$

D.3.15 Reynolds number derivatives.—

$$\frac{\partial(\mathrm{Re})}{\partial p} = 0 \tag{D-510}$$

$$\frac{\partial(\mathrm{Re})}{\partial q} = 0 \tag{D-511}$$

$$\frac{\partial(\mathrm{Re})}{\partial r} = 0 \tag{D-512}$$

$$\frac{\partial(\mathrm{Re})}{\partial V} = \frac{\rho \ell}{\mu} \tag{D-513}$$

$$\frac{\partial(\mathrm{Re})}{\partial \alpha} = 0 \tag{D-514}$$

$$\frac{\partial(\mathrm{Re})}{\partial \beta} = 0 \tag{D-515}$$

$$\frac{\partial(\mathrm{Re})}{\partial \phi} = 0 \tag{D-516}$$

$$\frac{\partial(\mathrm{Re})}{\partial \theta} = 0 \tag{D-517}$$

$$\frac{\partial(\mathrm{Re})}{\partial \psi} = 0 \tag{D-518}$$

$$\frac{\partial(\mathrm{Re})}{\partial h} = \frac{V_0 \ell}{\mu} \frac{\partial \rho}{\partial h} - \frac{\rho V_0 \ell}{\mu^2} \frac{\partial \mu}{\partial h} \tag{D-519}$$

$$\frac{\partial(\mathrm{Re})}{\partial x} = 0 \tag{D-520}$$

$$\frac{\partial(\mathrm{Re})}{\partial y} = 0 \tag{D-521}$$

$$\frac{\partial(\mathrm{Re})}{\partial \dot{\alpha}} = 0 \tag{D-522}$$

$$\frac{\partial(\mathrm{Re})}{\partial \dot{\beta}} = 0 \tag{D-523}$$

$$\frac{\partial(\mathrm{Re})}{\partial \delta_i} = 0 \tag{D-524}$$

D.3.16 Reynolds number per unit length derivatives.—

$$\frac{\partial(\mathrm{Re}')}{\partial p} = 0 \tag{D-525}$$

$$\frac{\partial(\mathrm{Re}')}{\partial q} = 0 \tag{D-526}$$

$$\frac{\partial(\mathrm{Re}')}{\partial r} = 0 \tag{D-527}$$

$$\frac{\partial(\mathrm{Re}')}{\partial V} = \frac{\rho}{\mu} \tag{D-528}$$

$$\frac{\partial(\mathrm{Re}')}{\partial \alpha} = 0 \tag{D-529}$$

$$\frac{\partial(\mathrm{Re}')}{\partial \beta} = 0 \tag{D-530}$$

$$\frac{\partial(\mathrm{Re}')}{\partial \phi} = 0 \tag{D-531}$$

$$\frac{\partial(\mathrm{Re}')}{\partial \theta} = 0 \tag{D-532}$$

$$\frac{\partial(\mathrm{Re}')}{\partial \psi} = 0 \tag{D-533}$$

$$\frac{\partial(\mathrm{Re}')}{\partial h} = \frac{V_0}{\mu}\frac{\partial \rho}{\partial h} - \frac{\rho V_0}{\mu^2}\frac{\partial \mu}{\partial h} \tag{D-534}$$

$$\frac{\partial(\mathrm{Re}')}{\partial x} = 0 \tag{D-535}$$

$$\frac{\partial(\mathrm{Re}')}{\partial y} = 0 \tag{D-536}$$

$$\frac{\partial(\mathrm{Re}')}{\partial \dot{\alpha}} = 0 \tag{D-537}$$

$$\frac{\partial(\mathrm{Re}')}{\partial \dot{\beta}} = 0 \tag{D-538}$$

$$\frac{\partial(\mathrm{Re}')}{\partial \delta_i} = 0 \tag{D-539}$$

D.3.17 Dynamic pressure derivatives.—

$$\frac{\partial(\bar{q})}{\partial p} = 0 \tag{D-540}$$

$$\frac{\partial(\bar{q})}{\partial q} = 0 \tag{D-541}$$

$$\frac{\partial(\bar{q})}{\partial r} = 0 \tag{D-542}$$

$$\frac{\partial(\bar{q})}{\partial V} = \rho V_0 \tag{D-543}$$

$$\frac{\partial(\bar{q})}{\partial \alpha} = 0 \tag{D-544}$$

$$\frac{\partial(\bar{q})}{\partial \beta} = 0 \tag{D-545}$$

$$\frac{\partial(\bar{q})}{\partial \phi} = 0 \tag{D-546}$$

$$\frac{\partial(\bar{q})}{\partial \theta} = 0 \tag{D-547}$$

$$\frac{\partial(\bar{q})}{\partial \psi} = 0 \tag{D-548}$$

$$\frac{\partial(\bar{q})}{\partial h} = \frac{V_0^2}{2}\frac{\partial \rho}{\partial h} \tag{D-549}$$

$$\frac{\partial(\bar{q})}{\partial x} = 0 \tag{D-550}$$

$$\frac{\partial(\bar{q})}{\partial y} = 0 \tag{D-551}$$

$$\frac{\partial(\bar{q})}{\partial \dot{\alpha}} = 0 \tag{D-552}$$

$$\frac{\partial(\bar{q})}{\partial \dot{\beta}} = 0 \tag{D-553}$$

$$\frac{\partial(\bar{q})}{\partial \delta_i} = 0 \tag{D-554}$$

D.3.18 Impact pressure derivatives.—

$$\frac{\partial(q_c)}{\partial p} = 0 \tag{D-555}$$

$$\frac{\partial(q_c)}{\partial q} = 0 \tag{D-556}$$

$$\frac{\partial(q_c)}{\partial r} = 0 \tag{D-557}$$

$$\frac{\partial(q_c)}{\partial V} = \begin{cases} \frac{1.4p_a}{a}M(1.0+0.2M^2)^{2.5} & (M \leq 1.0) \\ \frac{p_a}{a}\left[2.4M\left(\frac{5.76M^2}{5.6M^2-0.8}\right)^{2.5} \right. \\ \left. \quad + 3.0M^2\left(\frac{5.76M^2}{5.6M^2-0.8}\right)^{1.5}\frac{9.216M}{(5.6M^2-0.8)^2}\right] & (M \geq 1.0) \end{cases} \tag{D-558}$$

$$\frac{\partial(q_c)}{\partial \alpha} = 0 \tag{D-559}$$

$$\frac{\partial(q_c)}{\partial \beta} = 0 \tag{D-560}$$

$$\frac{\partial(q_c)}{\partial \phi} = 0 \tag{D-561}$$

$$\frac{\partial(q_c)}{\partial \theta} = 0 \tag{D-562}$$

$$\frac{\partial(q_c)}{\partial \psi} = 0 \tag{D-563}$$

$$\frac{\partial(q_c)}{\partial h} = \begin{cases} [(1.0+0.2M^2)^{3.5}-1.0]\frac{\partial p_a}{\partial h} - \frac{1.4V_0}{a^2}M(1.0+0.2M^2)^{2.5}p_a\frac{\partial a}{\partial h} & (M \leq 1.0) \\ \left[1.2M^2\left(\frac{5.76M^2}{5.6M^2-0.8}\right)^{2.5} - 1.0\right]\frac{\partial p_a}{\partial h} \\ \quad - \frac{p_a V_0}{a^2}\left[2.4M\left(\frac{5.76M^2}{5.6M^2-0.8}\right)^{2.5}\right. \\ \left. \quad\quad + 3.0M^2\left(\frac{5.76M^2}{5.6M^2-0.8}\right)^{1.5}\frac{9.216M}{(5.6M^2-0.8)^2}\right]\frac{\partial a}{\partial h} & (M \geq 1.0) \end{cases} \tag{D-564}$$

$$\frac{\partial(q_c)}{\partial x} = 0 \tag{D-565}$$

$$\frac{\partial(q_c)}{\partial y} = 0 \tag{D-566}$$

$$\frac{\partial(q_c)}{\partial \dot{\alpha}} = 0 \tag{D-567}$$

$$\frac{\partial(q_c)}{\partial \dot{\beta}} = 0 \tag{D-568}$$

$$\frac{\partial(q_c)}{\partial \delta_i} = 0 \tag{D-569}$$

D.3.19 Mach meter calibration ratio derivatives.—

$$\frac{\partial(q_c/p_a)}{\partial p} = 0 \tag{D-570}$$

$$\frac{\partial(q_c/p_a)}{\partial q} = 0 \tag{D-571}$$

$$\frac{\partial(q_c/p_a)}{\partial r} = 0 \tag{D-572}$$

$$\frac{\partial(q_c/p_a)}{\partial V} = \begin{cases} \frac{1.4}{a} M(1.0 + 0.2M^2)^{2.5} & (M \le 1.0) \\ \frac{1}{a}\left[2.4M\left(\frac{5.76M^2}{5.6M^2 - 0.8}\right)^{2.5} \right. & \\ \qquad \left. + 3.0M^2\left(\frac{5.76M^2}{5.6M^2 - 0.8}\right)^{1.5} \frac{9.216M}{(5.6M^2 - 0.8)^2}\right] & (M \ge 1.0) \end{cases} \tag{D-573}$$

$$\frac{\partial(q_c/p_a)}{\partial \alpha} = 0 \tag{D-574}$$

$$\frac{\partial(q_c/p_a)}{\partial \beta} = 0 \tag{D-575}$$

$$\frac{\partial(q_c/p_a)}{\partial \phi} = 0 \tag{D-576}$$

$$\frac{\partial(q_c/p_a)}{\partial \theta} = 0 \tag{D-577}$$

$$\frac{\partial(q_c/p_a)}{\partial \psi} = 0 \tag{D-578}$$

$$\frac{\partial(q_c/p_a)}{\partial h} = \begin{cases} -\frac{1.4V_0}{a^2} M(1.0 + 0.2M^2)^{2.5} \frac{\partial a}{\partial h} & (M \le 1.0) \\ -\frac{V_0}{a^2}\left[2.4M\left(\frac{5.76M^2}{5.6M^2 - 0.8}\right)^{2.5} \right. & \\ \qquad \left. + 3.0M^2\left(\frac{5.76M^2}{5.6M^2 - 0.8}\right)^{1.5} \frac{9.216M}{(5.6M^2 - 0.8)^2}\right] \frac{\partial a}{\partial h} & (M \ge 1.0) \end{cases} \tag{D-579}$$

$$\frac{\partial(q_c/p_a)}{\partial x} = 0 \tag{D-580}$$

$$\frac{\partial(q_c/p_a)}{\partial y} = 0 \tag{D-581}$$

$$\frac{\partial(q_c/p_a)}{\partial \dot{\alpha}} = 0 \tag{D-582}$$

$$\frac{\partial(q_c/p_a)}{\partial \dot{\beta}} = 0 \tag{D-583}$$

$$\frac{\partial(q_c/p_a)}{\partial \delta_i} = 0 \tag{D-584}$$

D.3.20 Total temperature derivatives.—

$$\frac{\partial(T_t)}{\partial p} = 0 \tag{D-585}$$

$$\frac{\partial(T_t)}{\partial q} = 0 \tag{D-586}$$

$$\frac{\partial(T_t)}{\partial r} = 0 \tag{D-587}$$

$$\frac{\partial(T_t)}{\partial V} = \frac{0.4\,TM}{a} \tag{D-588}$$

$$\frac{\partial(T_t)}{\partial \alpha} = 0 \tag{D-589}$$

$$\frac{\partial(T_t)}{\partial \beta} = 0 \tag{D-590}$$

$$\frac{\partial(T_t)}{\partial h} = \left\{1.0 + 0.2M^2 - \frac{0.4\,TMV_0}{a^2}\left[\frac{0.7p}{\rho_0 T_0(1.4p_0/\rho_0 T_0)^{1/2}}\right]\right\}\frac{\partial T}{\partial h} \tag{D-591}$$

$$\frac{\partial(T_t)}{\partial x} = 0 \tag{D-592}$$

$$\frac{\partial(T_t)}{\partial y} = 0 \tag{D-593}$$

$$\frac{\partial(T_t)}{\partial \dot{\alpha}} = 0 \tag{D-594}$$

$$\frac{\partial(T_t)}{\partial \dot{\beta}} = 0 \tag{D-595}$$

$$\frac{\partial(T_t)}{\partial \delta_i} = 0 \tag{D-596}$$

D.3.21 Flightpath angle derivatives.—

$$\frac{\partial(\gamma)}{\partial p} = 0 \tag{D-597}$$

$$\frac{\partial(\gamma)}{\partial q} = 0 \tag{D-598}$$

$$\frac{\partial(\gamma)}{\partial r} = 0 \tag{D-599}$$

$$\frac{\partial(\gamma)}{\partial V} = -\frac{\dot{h}_0}{V_0(V_0^2 - \dot{h}_0^2)^{1/2}} \tag{D-600}$$

$$\frac{\partial(\gamma)}{\partial \alpha} = 0 \tag{D-601}$$

$$\frac{\partial(\gamma)}{\partial \beta} = 0 \tag{D-602}$$

$$\frac{\partial(\gamma)}{\partial \phi} = 0 \tag{D-603}$$

$$\frac{\partial(\gamma)}{\partial \theta} = 0 \tag{D-604}$$

$$\frac{\partial(\gamma)}{\partial\psi} = 0 \tag{D-605}$$

$$\frac{\partial(\gamma)}{\partial h} = 0 \tag{D-606}$$

$$\frac{\partial(\gamma)}{\partial x} = 0 \tag{D-607}$$

$$\frac{\partial(\gamma)}{\partial y} = 0 \tag{D-608}$$

$$\frac{\partial(\gamma)}{\partial\dot{\alpha}} = 0 \tag{D-609}$$

$$\frac{\partial(\gamma)}{\partial\dot{\beta}} = 0 \tag{D-610}$$

$$\frac{\partial(\gamma)}{\partial\dot{h}} = \frac{1}{(V_0^2 - \dot{h}_0^2)^{1/2}} \tag{D-611}$$

$$\frac{\partial(\gamma)}{\partial\delta_i} = 0 \tag{D-612}$$

D.3.22 Flightpath acceleration derivatives.—

$$\frac{\partial(\mathrm{fpa})}{\partial p} = 0 \tag{D-613}$$

$$\frac{\partial(\mathrm{fpa})}{\partial q} = 0 \tag{D-614}$$

$$\frac{\partial(\mathrm{fpa})}{\partial r} = 0 \tag{D-615}$$

$$\frac{\partial(\mathrm{fpa})}{\partial V} = 0 \tag{D-616}$$

$$\frac{\partial(\mathrm{fpa})}{\partial\alpha} = 0 \tag{D-617}$$

$$\frac{\partial(\mathrm{fpa})}{\partial\beta} = 0 \tag{D-618}$$

$$\frac{\partial(\mathrm{fpa})}{\partial\phi} = 0 \tag{D-619}$$

$$\frac{\partial(\mathrm{fpa})}{\partial\theta} = 0 \tag{D-620}$$

$$\frac{\partial(\mathrm{fpa})}{\partial\psi} = 0 \tag{D-621}$$

$$\frac{\partial(\mathrm{fpa})}{\partial h} = 0 \tag{D-622}$$

$$\frac{\partial(\mathrm{fpa})}{\partial x} = 0 \tag{D-623}$$

$$\frac{\partial(\mathrm{fpa})}{\partial y} = 0 \tag{D-624}$$

$$\frac{\partial(\mathrm{fpa})}{\partial\dot{V}} = \frac{1}{g} \tag{D-625}$$

$$\frac{\partial(\text{fpa})}{\partial\dot{\alpha}} = 0 \tag{D-626}$$

$$\frac{\partial(\text{fpa})}{\partial\dot{\beta}} = 0 \tag{D-627}$$

$$\frac{\partial(\text{fpa})}{\partial\delta_i} = 0 \tag{D-628}$$

D.3.23 Vertical acceleration derivatives.—

$$\begin{aligned}\frac{\partial(\ddot{h})}{\partial p} = \frac{\bar{q}Sb}{2V_0g_0m}[&\sin\theta_0(-\cos\alpha_0\,C_{\mathrm{D}_p} + \sin\alpha_0\,C_{\mathrm{L}_p})\\ &- \sin\phi_0\ \cos\theta_0\,C_{\mathrm{Y}_p} + \cos\phi_0\ \cos\theta_0\,(\sin\alpha_0\,C_{\mathrm{D}_p} + \cos\alpha_0\,C_{\mathrm{L}_p})]\end{aligned} \tag{D-629}$$

$$\begin{aligned}\frac{\partial(\ddot{h})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0g_0m}[&\sin\theta_0\,(-\cos\alpha_0\,C_{\mathrm{D}_q} + \sin\alpha_0\,C_{\mathrm{L}_q}) - \sin\phi_0\cos\theta_0\,C_{\mathrm{Y}_q}\\ &+ \cos\phi_0\cos\theta_0\,(\sin\alpha_0\,C_{\mathrm{D}_q} + \cos\alpha_0\,C_{\mathrm{L}_q})]\end{aligned} \tag{D-630}$$

$$\begin{aligned}\frac{\partial(\ddot{h})}{\partial r} = \frac{\bar{q}Sb}{2V_0g_0m}[&\sin\theta_0\,(-\cos\alpha_0\,C_{\mathrm{D}_r} + \sin\alpha_0\,C_{\mathrm{L}_r}) - \sin\phi_0\cos\theta_0\,C_{\mathrm{Y}_r}\\ &+ \cos\phi_0\cos\theta_0\,(\sin\alpha_0\,C_{\mathrm{D}_r} + \cos\alpha_0\,C_{\mathrm{L}_r})]\end{aligned} \tag{D-631}$$

$$\begin{aligned}\frac{\partial(\ddot{h})}{\partial V} = \frac{1}{g_0m}\Bigg\{&\sin\theta_0\left[-S\cos\alpha_0\,(\rho V_0C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V}) + S\sin\alpha_0\,(\rho V_0C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) + \frac{\partial X_{\mathrm{T}}}{\partial V}\right]\\ &- \sin\phi_0\cos\theta_0\left(S\rho V_0C_{\mathrm{Y}} + \bar{q}SC_{\mathrm{Y}_V} + \frac{\partial Y_{\mathrm{T}}}{\partial V}\right)\\ &+ \cos\phi_0\cos\theta_0\bigg[S\sin\alpha_0\,(\rho V_0C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V})\\ &\qquad + S\cos\alpha_0\,(\rho V_0C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) - \frac{\partial Z_{\mathrm{T}}}{\partial V}\bigg]\Bigg\}\end{aligned} \tag{D-632}$$

$$\begin{aligned}\frac{\partial(\ddot{h})}{\partial \alpha} = \frac{1}{g_0m}\Bigg\{&\sin\theta_0\left[-\bar{q}S\cos\alpha_0\,(C_{\mathrm{D}_\alpha} - C_{\mathrm{L}}) + \bar{q}S\sin\alpha_0\,(C_{\mathrm{L}_\alpha} + C_{\mathrm{D}}) + \frac{\partial X_{\mathrm{T}}}{\partial \alpha}\right]\\ &- \sin\phi_0\cos\theta_0\left(\bar{q}SC_{\mathrm{Y}_\alpha} + \frac{\partial Y_{\mathrm{T}}}{\partial \alpha}\right)\\ &+ \cos\phi_0\cos\theta_0\left[\bar{q}S\sin\alpha_0\,(C_{\mathrm{D}_\alpha} - C_{\mathrm{L}}) + \bar{q}S\cos\alpha_0\,(C_{\mathrm{L}_\alpha} + C_{\mathrm{D}}) - \frac{\partial Z_{\mathrm{T}}}{\partial \alpha}\right]\Bigg\}\end{aligned} \tag{D-633}$$

$$\begin{aligned}\frac{\partial(\ddot{h})}{\partial \beta} = \frac{1}{g_0m}\Bigg[&\sin\theta_0\left(-\bar{q}S\cos\alpha_0\,C_{\mathrm{D}_\beta} + \bar{q}S\sin\alpha_0\,C_{\mathrm{L}_\beta} + \frac{\partial X_{\mathrm{T}}}{\partial \beta}\right) - \sin\phi_0\cos\theta_0\left(\bar{q}SC_{\mathrm{Y}_\beta} + \frac{\partial Y_{\mathrm{T}}}{\partial \beta}\right)\\ &+ \cos\phi_0\cos\theta_0\left(\bar{q}S\sin\alpha_0\,C_{\mathrm{D}_\beta} + \bar{q}S\cos\alpha_0\,C_{\mathrm{L}_\beta} + \frac{\partial Z_{\mathrm{T}}}{\partial \beta}\right)\Bigg]\end{aligned} \tag{D-634}$$

$$\frac{\partial(\ddot{h})}{\partial \phi} = -a_{y_0}\cos\phi_0\cos\theta_0 + a_{z_0}\sin\phi_0\cos\theta_0 \tag{D-635}$$

$$\frac{\partial(\ddot{h})}{\partial \theta} = a_{x_0}\cos\theta_0 + a_{y_0}\sin\theta_0\sin\phi_0 + a_{z_0}\cos\phi_0\sin\theta_0 \tag{D-636}$$

$$\frac{\partial(\ddot{h})}{\partial \psi} = 0 \tag{D-637}$$

$$\frac{\partial(\ddot{h})}{\partial h} = \frac{1}{g_0 m}\Bigg\{ \sin\theta_0\Bigg[-\cos\alpha_0\left(\frac{1}{2}SV_0^2C_\mathrm{D}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right) + \sin\alpha_0\left(\frac{1}{2}SV_0^2C_\mathrm{L}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\Bigg]$$
$$- \sin\phi_0\cos\theta_0\left(\frac{1}{2}SV_0^2C_\mathrm{Y}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{Y}_h}\right)$$
$$+ \cos\phi_0\cos\theta_0\Bigg[\sin\alpha_0\left(\frac{1}{2}SV_0^2C_\mathrm{D}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{D}_h}\right)$$
$$+ \cos\alpha_0\left(\frac{1}{2}SV_0^2C_\mathrm{L}\,\frac{\partial\rho}{\partial h} + \bar{q}SC_{\mathrm{L}_h}\right)\Bigg]\Bigg\} \tag{D-638}$$

$$\frac{\partial(\ddot{h})}{\partial x} = 0 \tag{D-639}$$

$$\frac{\partial(\ddot{h})}{\partial y} = 0 \tag{D-640}$$

$$\frac{\partial(\ddot{h})}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0g_0m}[\sin\theta_0\,(-\cos\alpha_0\,C_{\mathrm{D}_{\dot{\alpha}}} + \sin\alpha_0\,C_{\mathrm{L}_{\dot{\alpha}}}) - \sin\phi_0\cos\theta_0\,C_{\mathrm{Y}_{\dot{\alpha}}}$$
$$+ \cos\phi_0\cos\theta_0\,(\sin\alpha_0\,C_{\mathrm{D}_{\dot{\alpha}}} + \cos\alpha_0\,C_{\mathrm{L}_{\dot{\alpha}}})] \tag{D-641}$$

$$\frac{\partial(\ddot{h})}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0g_0m}[\sin\theta_0\,(-\cos\alpha_0\,C_{\mathrm{D}_{\dot{\beta}}} + \sin\alpha_0\,C_{\mathrm{L}_{\dot{\beta}}}) - \sin\phi_0\cos\theta_0\,C_{\mathrm{Y}_{\dot{\beta}}}$$
$$+ \cos\phi_0\cos\theta_0\,(\sin\alpha_0\,C_{\mathrm{D}_{\dot{\beta}}} + \cos\alpha_0\,C_{\mathrm{L}_{\dot{\beta}}})] \tag{D-642}$$

$$\frac{\partial(\ddot{h})}{\partial \delta_i} = \frac{1}{g_0 m}\Bigg\{\sin\theta_0\left[\bar{q}S(-\cos\alpha_0\,C_{\mathrm{D}_{\delta_i}} + \sin\alpha_0\,C_{\mathrm{L}_{\delta_i}}) + \frac{\partial X_\mathrm{T}}{\partial\delta_i}\right] - \sin\phi_0\cos\theta_0\left(\bar{q}SC_{\mathrm{Y}_{\delta_i}} + \frac{\partial Y_\mathrm{T}}{\partial\delta_i}\right)$$
$$+ \cos\phi_0\cos\theta_0\left[\bar{q}S(\sin\alpha_0\,C_{\mathrm{D}_{\delta_i}} + \cos\alpha_0\,C_{\mathrm{L}_{\delta_i}}) - \frac{\partial Z_\mathrm{T}}{\partial\delta_i}\right]\Bigg\} \tag{D-643}$$

D.3.24 Specific energy derivatives.—

$$\frac{\partial(E_\mathrm{s})}{\partial p} = 0 \tag{D-644}$$

$$\frac{\partial(E_\mathrm{s})}{\partial q} = 0 \tag{D-645}$$

$$\frac{\partial(E_\mathrm{s})}{\partial r} = 0 \tag{D-646}$$

$$\frac{\partial(E_\mathrm{s})}{\partial V} = \frac{V_0}{g} \tag{D-647}$$

$$\frac{\partial(E_\mathrm{s})}{\partial \alpha} = 0 \tag{D-648}$$

$$\frac{\partial(E_\mathrm{s})}{\partial \beta} = 0 \tag{D-649}$$

$$\frac{\partial(E_\mathrm{s})}{\partial \phi} = 0 \tag{D-650}$$

$$\frac{\partial(E_\mathrm{s})}{\partial \theta} = 0 \tag{D-651}$$

$$\frac{\partial(E_\mathrm{s})}{\partial \psi} = 0 \tag{D-652}$$

$$\frac{\partial(E_\mathrm{s})}{\partial h} = 1 \tag{D-653}$$

$$\frac{\partial(E_s)}{\partial x} = 0 \tag{D-654}$$
$$\frac{\partial(E_s)}{\partial y} = 0 \tag{D-655}$$
$$\frac{\partial(E_s)}{\partial \dot{\alpha}} = 0 \tag{D-656}$$
$$\frac{\partial(E_s)}{\partial \dot{\beta}} = 0 \tag{D-657}$$
$$\frac{\partial(E_s)}{\partial \delta_i} = 0 \tag{D-658}$$

D.3.25 Specific power derivatives.—

$$\frac{\partial(P_s)}{\partial p} = 0 \tag{D-659}$$
$$\frac{\partial(P_s)}{\partial q} = 0 \tag{D-660}$$
$$\frac{\partial(P_s)}{\partial r} = 0 \tag{D-661}$$
$$\frac{\partial(P_s)}{\partial V} = \frac{\dot{V}}{g} \tag{D-662}$$
$$\frac{\partial(P_s)}{\partial \alpha} = 0 \tag{D-663}$$
$$\frac{\partial(P_s)}{\partial \beta} = 0 \tag{D-664}$$
$$\frac{\partial(P_s)}{\partial \phi} = 0 \tag{D-665}$$
$$\frac{\partial(P_s)}{\partial \theta} = 0 \tag{D-666}$$
$$\frac{\partial(P_s)}{\partial \psi} = 0 \tag{D-667}$$
$$\frac{\partial(P_s)}{\partial h} = 0 \tag{D-668}$$
$$\frac{\partial(P_s)}{\partial x} = 0 \tag{D-669}$$
$$\frac{\partial(P_s)}{\partial y} = 0 \tag{D-670}$$
$$\frac{\partial(P_s)}{\partial \dot{V}} = \frac{V}{g} \tag{D-671}$$
$$\frac{\partial(P_s)}{\partial \dot{\alpha}} = 0 \tag{D-672}$$
$$\frac{\partial(P_s)}{\partial \dot{\beta}} = 0 \tag{D-673}$$
$$\frac{\partial(P_s)}{\partial \dot{h}} = 1 \tag{D-674}$$

$$\frac{\partial(P_s)}{\partial\delta_i} = 0 \tag{D-675}$$

D.3.26 Normal force derivatives.—

$$\frac{\partial(N)}{\partial p} = \frac{\bar{q}Sb}{2V_0}(\cos\alpha_0\, C_{L_p} + \sin\alpha_0\, C_{D_p}) \tag{D-676}$$

$$\frac{\partial(N)}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0}(\cos\alpha_0\, C_{L_q} + \sin\alpha_0\, C_{D_q}) \tag{D-677}$$

$$\frac{\partial(N)}{\partial r} = \frac{\bar{q}Sb}{2V_0}(\cos\alpha_0\, C_{L_r} + \sin\alpha_0\, C_{D_r}) \tag{D-678}$$

$$\frac{\partial(N)}{\partial V} = S[\cos\alpha_0\,(\rho V_0 C_L + \bar{q}C_{L_V}) + \sin\alpha_0\,(\rho V_0 C_D + \bar{q}C_{D_V})] \tag{D-679}$$

$$\frac{\partial(N)}{\partial\alpha} = \bar{q}S(\cos\alpha_0\, C_{L_\alpha} + \sin\alpha_0\, C_{D_\alpha} - \sin\alpha_0\, C_L + \cos\alpha_0\, C_D) \tag{D-680}$$

$$\frac{\partial(N)}{\partial\beta} = \bar{q}S(\cos\alpha_0\, C_{L_\beta} + \sin\alpha_0\, C_{D_\beta}) \tag{D-681}$$

$$\frac{\partial(N)}{\partial\phi} = 0 \tag{D-682}$$

$$\frac{\partial(N)}{\partial\theta} = 0 \tag{D-683}$$

$$\frac{\partial(N)}{\partial\psi} = 0 \tag{D-684}$$

$$\frac{\partial(N)}{\partial h} = S\left[\cos\alpha_0\left(\frac{1}{2}V_0^2 C_L\,\frac{\partial\rho}{\partial h} + \bar{q}C_{L_h}\right) + \sin\alpha_0\left(\frac{1}{2}V_0^2 C_D\,\frac{\partial\rho}{\partial h} + \bar{q}C_{D_h}\right)\right] \tag{D-685}$$

$$\frac{\partial(N)}{\partial x} = 0 \tag{D-686}$$

$$\frac{\partial(N)}{\partial y} = 0 \tag{D-687}$$

$$\frac{\partial(N)}{\partial\dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0}(\cos\alpha_0\, C_{L_{\dot{\alpha}}} + \sin\alpha_0\, C_{D_{\dot{\alpha}}}) \tag{D-688}$$

$$\frac{\partial(N)}{\partial\dot{\beta}} = \frac{\bar{q}Sb}{2V_0}(\cos\alpha_0\, C_{L_{\dot{\beta}}} + \sin\alpha_0\, C_{D_{\dot{\beta}}}) \tag{D-689}$$

$$\frac{\partial(N)}{\partial\delta_i} = \bar{q}S(\cos\alpha_0\, C_{L_{\delta_i}} + \sin\alpha_0\, C_{D_{\delta_i}}) \tag{D-690}$$

D.3.27 Axial force derivatives.—

$$\frac{\partial(A)}{\partial p} = \frac{\bar{q}Sb}{2V_0}(-\sin\alpha_0\, C_{L_p} + \cos\alpha_0\, C_{D_p}) \tag{D-691}$$

$$\frac{\partial(A)}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0}(-\sin\alpha_0\, C_{L_q} + \cos\alpha_0\, C_{D_q}) \tag{D-692}$$

$$\frac{\partial(A)}{\partial r} = \frac{\bar{q}Sb}{2V_0}(-\sin\alpha_0\, C_{L_r} + \cos\alpha_0\, C_{D_r}) \tag{D-693}$$

$$\frac{\partial(A)}{\partial V} = S[-\sin\alpha_0\,(\rho V_0 C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) + \cos\alpha_0\,(\rho V_0 C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V})] \tag{D-694}$$

$$\frac{\partial(A)}{\partial \alpha} = \bar{q}S(-\sin\alpha_0\,C_{\mathrm{L}_\alpha} + \cos\alpha_0\,C_{\mathrm{D}_\alpha} - \cos\alpha_0\,C_{\mathrm{L}} - \sin\alpha_0\,C_{\mathrm{D}}) \tag{D-695}$$

$$\frac{\partial(A)}{\partial \beta} = \bar{q}S(-\sin\alpha_0\,C_{\mathrm{L}_\beta} + \cos\alpha_0\,C_{\mathrm{D}_\beta}) \tag{D-696}$$

$$\frac{\partial(A)}{\partial \phi} = 0 \tag{D-697}$$

$$\frac{\partial(A)}{\partial \theta} = 0 \tag{D-698}$$

$$\frac{\partial(A)}{\partial \psi} = 0 \tag{D-699}$$

$$\frac{\partial(A)}{\partial h} = S\left[-\sin\alpha_0\left(\frac{1}{2}V_0^2 C_{\mathrm{L}}\,\frac{\partial\rho}{\partial h} + \bar{q}C_{\mathrm{L}_h}\right) + \cos\alpha_0\left(\frac{1}{2}V_0^2 C_{\mathrm{D}}\,\frac{\partial\rho}{\partial h} + \bar{q}C_{\mathrm{D}_h}\right)\right] \tag{D-700}$$

$$\frac{\partial(A)}{\partial x} = 0 \tag{D-701}$$

$$\frac{\partial(A)}{\partial y} = 0 \tag{D-702}$$

$$\frac{\partial(A)}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0}(-\sin\alpha_0\,C_{\mathrm{L}_{\dot{\alpha}}} + \cos\alpha_0\,C_{\mathrm{D}_{\dot{\alpha}}}) \tag{D-703}$$

$$\frac{\partial(A)}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0}(-\sin\alpha_0\,C_{\mathrm{L}_{\dot{\beta}}} + \cos\alpha_0\,C_{\mathrm{D}_{\dot{\beta}}}) \tag{D-704}$$

$$\frac{\partial(A)}{\partial \delta_i} = \bar{q}S(-\sin\alpha_0\,C_{\mathrm{L}_{\delta_i}} + \cos\alpha_0\,C_{\mathrm{D}_{\delta_i}}) \tag{D-705}$$

D.3.28 x body axis rate derivatives.—

$$\frac{\partial(u)}{\partial p} = 0 \tag{D-706}$$

$$\frac{\partial(u)}{\partial q} = 0 \tag{D-707}$$

$$\frac{\partial(u)}{\partial r} = 0 \tag{D-708}$$

$$\frac{\partial(u)}{\partial V} = \cos\alpha_0\cos\beta_0 \tag{D-709}$$

$$\frac{\partial(u)}{\partial \alpha} = -V_0\sin\alpha_0\cos\beta_0 \tag{D-710}$$

$$\frac{\partial(u)}{\partial \beta} = -V_0\cos\alpha_0\sin\beta_0 \tag{D-711}$$

$$\frac{\partial(u)}{\partial \phi} = 0 \tag{D-712}$$

$$\frac{\partial(u)}{\partial \theta} = 0 \tag{D-713}$$

$$\frac{\partial(u)}{\partial \psi} = 0 \tag{D-714}$$

$$\frac{\partial(u)}{\partial h} = 0 \quad \text{(D-715)}$$
$$\frac{\partial(u)}{\partial x} = 0 \quad \text{(D-716)}$$
$$\frac{\partial(u)}{\partial y} = 0 \quad \text{(D-717)}$$
$$\frac{\partial(u)}{\partial \dot{\alpha}} = 0 \quad \text{(D-718)}$$
$$\frac{\partial(u)}{\partial \dot{\beta}} = 0 \quad \text{(D-719)}$$
$$\frac{\partial(u)}{\partial \delta_i} = 0 \quad \text{(D-720)}$$

D.3.29 y body axis rate derivatives.—

$$\frac{\partial(v)}{\partial p} = 0 \quad \text{(D-721)}$$
$$\frac{\partial(v)}{\partial q} = 0 \quad \text{(D-722)}$$
$$\frac{\partial(v)}{\partial r} = 0 \quad \text{(D-723)}$$
$$\frac{\partial(v)}{\partial V} = \sin\beta_0 \quad \text{(D-724)}$$
$$\frac{\partial(v)}{\partial \alpha} = 0 \quad \text{(D-725)}$$
$$\frac{\partial(v)}{\partial \beta} = V_0 \cos\beta_0 \quad \text{(D-726)}$$
$$\frac{\partial(v)}{\partial \phi} = 0 \quad \text{(D-727)}$$
$$\frac{\partial(v)}{\partial \theta} = 0 \quad \text{(D-728)}$$
$$\frac{\partial(v)}{\partial \psi} = 0 \quad \text{(D-729)}$$
$$\frac{\partial(v)}{\partial h} = 0 \quad \text{(D-730)}$$
$$\frac{\partial(v)}{\partial x} = 0 \quad \text{(D-731)}$$
$$\frac{\partial(v)}{\partial y} = 0 \quad \text{(D-732)}$$
$$\frac{\partial(v)}{\partial \dot{\alpha}} = 0 \quad \text{(D-733)}$$
$$\frac{\partial(v)}{\partial \dot{\beta}} = 0 \quad \text{(D-734)}$$
$$\frac{\partial(v)}{\partial \delta_i} = 0 \quad \text{(D-735)}$$

D.3.30 z body axis rate derivatives.—

$$\frac{\partial(w)}{\partial p} = 0 \tag{D-736}$$

$$\frac{\partial(w)}{\partial q} = 0 \tag{D-737}$$

$$\frac{\partial(w)}{\partial r} = 0 \tag{D-738}$$

$$\frac{\partial(w)}{\partial V} = \sin\alpha_0 \cos\beta_0 \tag{D-739}$$

$$\frac{\partial(w)}{\partial \alpha} = V_0 \cos\alpha_0 \cos\beta_0 \tag{D-740}$$

$$\frac{\partial(w)}{\partial \beta} = -V_0 \sin\alpha_0 \sin\beta_0 \tag{D-741}$$

$$\frac{\partial(w)}{\partial \phi} = 0 \tag{D-742}$$

$$\frac{\partial(w)}{\partial \theta} = 0 \tag{D-743}$$

$$\frac{\partial(w)}{\partial \psi} = 0 \tag{D-744}$$

$$\frac{\partial(w)}{\partial h} = 0 \tag{D-745}$$

$$\frac{\partial(w)}{\partial x} = 0 \tag{D-746}$$

$$\frac{\partial(w)}{\partial y} = 0 \tag{D-747}$$

$$\frac{\partial(w)}{\partial \dot{\alpha}} = 0 \tag{D-748}$$

$$\frac{\partial(w)}{\partial \dot{\beta}} = 0 \tag{D-749}$$

$$\frac{\partial(w)}{\partial \delta_i} = 0 \tag{D-750}$$

D.3.31 x body axis acceleration derivatives.—

$$\frac{\partial(\dot{u})}{\partial p} = \frac{\bar{q}Sb}{2V_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_p} + \sin\alpha_0\, C_{\mathrm{L}_p}) \tag{D-751}$$

$$\frac{\partial(\dot{u})}{\partial q} = \frac{\bar{q}Sc}{2V_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_q} + \sin\alpha_0\, C_{\mathrm{L}_q}) - V_0 \sin\alpha_0 \cos\beta_0 \tag{D-752}$$

$$\frac{\partial(\dot{u})}{\partial r} = \frac{\bar{q}Sb}{2V_0 m}(-\cos\alpha_0\, C_{\mathrm{D}_r} + \sin\alpha_0\, C_{\mathrm{L}_r}) + V_0 \sin\beta_0 \tag{D-753}$$

$$\begin{aligned}\frac{\partial(\dot{u})}{\partial V} = {} & \frac{1}{m}\left\{S[-\cos\alpha_0\,(\rho V_0 C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V}) + \sin\alpha_0\,(\rho V_0 C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V})] + \frac{\partial X_{\mathrm{T}}}{\partial V}\right\} \\ & + r_0 \sin\beta_0 - q_0 \sin\alpha_0 \cos\beta_0\end{aligned} \tag{D-754}$$

$$\frac{\partial(\dot{u})}{\partial\alpha} = \frac{1}{m}\left[\bar{q}S(-\cos\alpha_0\,C_{\mathrm{D}_\alpha} + \sin\alpha_0\,C_{\mathrm{L}_\alpha} + \sin\alpha_0\,C_{\mathrm{D}} + \cos\alpha_0\,C_{\mathrm{L}}) + \frac{\partial X_{\mathrm{T}}}{\partial\alpha}\right] - q_0V_0\cos\alpha_0\cos\beta_0 \quad \text{(D-755)}$$

$$\frac{\partial(\dot{u})}{\partial\beta} = \frac{1}{m}\left[\bar{q}S(-\cos\alpha_0\,C_{\mathrm{D}_\beta} + \sin\alpha_0\,C_{\mathrm{L}_\beta}) + \frac{\partial X_{\mathrm{T}}}{\partial\beta}\right] + r_0V_0\cos\beta_0 + q_0V_0\cos\alpha_0\sin\beta_0 \quad \text{(D-756)}$$

$$\frac{\partial(\dot{u})}{\partial\phi} = 0 \quad \text{(D-757)}$$

$$\frac{\partial(\dot{u})}{\partial\theta} = -g\cos\theta_0 \quad \text{(D-758)}$$

$$\frac{\partial(\dot{u})}{\partial\psi} = 0 \quad \text{(D-759)}$$

$$\frac{\partial(\dot{u})}{\partial h} = \frac{S}{m}\left[-\cos\alpha_0\left(\frac{1}{2}V_0^2C_{\mathrm{D}}\frac{\partial\rho}{\partial h} + \bar{q}C_{\mathrm{D}_h}\right) + \sin\alpha_0\left(\frac{1}{2}V_0^2C_{\mathrm{L}}\frac{\partial\rho}{\partial h} + qC_{\mathrm{L}_h}\right)\right] \quad \text{(D-760)}$$

$$\frac{\partial(\dot{u})}{\partial x} = 0 \quad \text{(D-761)}$$

$$\frac{\partial(\dot{u})}{\partial y} = 0 \quad \text{(D-762)}$$

$$\frac{\partial(\dot{u})}{\partial\dot{\alpha}} = \frac{\bar{q}Sc}{2V_0m}(-\cos\alpha_0\,C_{\mathrm{D}_{\dot{\alpha}}} + \sin\alpha_0\,C_{\mathrm{L}_{\dot{\alpha}}}) \quad \text{(D-763)}$$

$$\frac{\partial(\dot{u})}{\partial\dot{\beta}} = \frac{\bar{q}Sb}{2V_0m}(-\cos\alpha_0\,C_{\mathrm{D}_{\dot{\beta}}} + \sin\alpha_0\,C_{\mathrm{L}_{\dot{\beta}}}) \quad \text{(D-764)}$$

$$\frac{\partial(\dot{u})}{\partial\delta_i} = \frac{1}{m}\left[\bar{q}S(-\cos\alpha_0\,C_{\mathrm{D}_{\delta_i}} + \sin\alpha_0\,C_{\mathrm{L}_{\delta_i}}) + \frac{\partial X_{\mathrm{T}}}{\partial\delta_i}\right] \quad \text{(D-765)}$$

D.3.32 y body axis acceleration derivatives.—

$$\frac{\partial(\dot{v})}{\partial p} = \frac{\bar{q}Sb}{2V_0m}C_{\mathrm{Y}_p} + V_0\sin\alpha_0\cos\beta_0 \quad \text{(D-766)}$$

$$\frac{\partial(\dot{v})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0m}C_{\mathrm{Y}_q} \quad \text{(D-767)}$$

$$\frac{\partial(\dot{v})}{\partial r} = \frac{\bar{q}Sb}{2V_0m}C_{\mathrm{Y}_r} - V_0\cos\alpha_0\cos\beta_0 \quad \text{(D-768)}$$

$$\frac{\partial(\dot{v})}{\partial V} = \frac{1}{m}\left[S(\rho V_0C_{\mathrm{Y}} + \bar{q}C_{\mathrm{Y}_V}) + \frac{\partial Y_{\mathrm{T}}}{\partial V}\right] + p_0\sin\alpha_0\cos\beta_0 - r_0\cos\alpha_0\cos\beta_0 \quad \text{(D-769)}$$

$$\frac{\partial(\dot{v})}{\partial\alpha} = \frac{1}{m}\left(\bar{q}SC_{\mathrm{Y}_\alpha} + \frac{\partial Y_{\mathrm{T}}}{\partial\alpha}\right) + p_0V_0\cos\alpha_0\cos\beta_0 + r_0V_0\sin\alpha_0\cos\beta_0 \quad \text{(D-770)}$$

$$\frac{\partial(\dot{v})}{\partial\beta} = \frac{1}{m}\left(\bar{q}SC_{\mathrm{Y}_\beta} + \frac{\partial Y_{\mathrm{T}}}{\partial\beta}\right) - p_0V_0\sin\alpha_0\sin\beta_0 - r_0V_0\cos\alpha_0\sin\beta_0 \quad \text{(D-771)}$$

$$\frac{\partial(\dot{v})}{\partial\phi} = g\cos\theta_0\cos\phi_0 \quad \text{(D-772)}$$

$$\frac{\partial(\dot{v})}{\partial\theta} = -g\sin\theta_0\sin\phi_0 \quad \text{(D-773)}$$

$$\frac{\partial(\dot{v})}{\partial\psi} = 0 \quad \text{(D-774)}$$

$$\frac{\partial(\dot{v})}{\partial h} = \frac{S}{m}\left(\frac{1}{2}V_0^2 C_{\mathrm{Y}}\frac{\partial\rho}{\partial h} + \bar{q}C_{\mathrm{Y}_h}\right) \tag{D-775}$$

$$\frac{\partial(\dot{v})}{\partial x} = 0 \tag{D-776}$$

$$\frac{\partial(\dot{v})}{\partial y} = 0 \tag{D-777}$$

$$\frac{\partial(\dot{v})}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 m}C_{\mathrm{Y}_{\dot{\alpha}}} \tag{D-778}$$

$$\frac{\partial(\dot{v})}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0 m}C_{\mathrm{Y}_{\dot{\beta}}} \tag{D-779}$$

$$\frac{\partial(\dot{v})}{\partial \delta_i} = \frac{1}{m}\left(\bar{q}SC_{\mathrm{Y}_{\delta_i}} + \frac{\partial Y_{\mathrm{T}}}{\partial \delta_i}\right) \tag{D-780}$$

D.3.33 z body axis acceleration derivatives.—

$$\frac{\partial(\dot{w})}{\partial p} = \frac{\bar{q}Sb}{2V_0 m}(-\sin\alpha_0\, C_{\mathrm{D}_p} - \cos\alpha_0\, C_{\mathrm{L}_p}) - V_0\sin\beta_0 \tag{D-781}$$

$$\frac{\partial(\dot{w})}{\partial q} = \frac{\bar{q}S\bar{c}}{2V_0 m}(-\sin\alpha_0\, C_{\mathrm{D}_q} - \cos\alpha_0\, C_{\mathrm{L}_q}) + V_0\cos\alpha_0\cos\beta_0 \tag{D-782}$$

$$\frac{\partial(\dot{w})}{\partial r} = \frac{\bar{q}Sb}{2V_0 m}(-\sin\alpha_0\, C_{\mathrm{D}_r} - \cos\alpha_0\, C_{\mathrm{L}_r}) \tag{D-783}$$

$$\begin{aligned}\frac{\partial(\dot{w})}{\partial V} = {} & \frac{1}{m}\left[-S\sin\alpha_0\,(\rho V_0 C_{\mathrm{D}} + \bar{q}C_{\mathrm{D}_V}) - S\cos\alpha_0\,(\rho V_0 C_{\mathrm{L}} + \bar{q}C_{\mathrm{L}_V}) + \frac{\partial Z_{\mathrm{T}}}{\partial V}\right] \\ & + q_0\cos\alpha_0\cos\beta_0 - p_0\sin\beta_0\end{aligned} \tag{D-784}$$

$$\begin{aligned}\frac{\partial(\dot{w})}{\partial \alpha} = {} & \frac{1}{m}\left[\bar{q}S(-\sin\alpha_0\, C_{\mathrm{D}_\alpha} - \cos\alpha_0\, C_{\mathrm{L}_\alpha} - \cos\alpha_0\, C_{\mathrm{D}} + \sin\alpha_0\, C_{\mathrm{L}}) + \frac{\partial Z_{\mathrm{T}}}{\partial \alpha}\right] \\ & - q_0 V_0\sin\alpha_0\cos\beta_0\end{aligned} \tag{D-785}$$

$$\frac{\partial(\dot{w})}{\partial \beta} = \frac{1}{m}\left[\bar{q}S(-\sin\alpha_0\, C_{\mathrm{D}_\beta} - \cos\alpha_0\, C_{\mathrm{L}_\beta}) + \frac{\partial Z_{\mathrm{T}}}{\partial \beta}\right] - q_0 V_0\cos\alpha_0\sin\beta_0 - p_0 V_0\cos\beta_0 \tag{D-786}$$

$$\frac{\partial(\dot{w})}{\partial \phi} = -g\cos\theta_0\sin\phi_0 \tag{D-787}$$

$$\frac{\partial(\dot{w})}{\partial \theta} = -g\sin\theta_0\cos\phi_0 \tag{D-788}$$

$$\frac{\partial(\dot{w})}{\partial \psi} = 0 \tag{D-789}$$

$$\frac{\partial(\dot{w})}{\partial h} = \frac{S}{m}\left[-\sin\alpha_0\left(\frac{1}{2}V_0^2 C_{\mathrm{D}}\frac{\partial\rho}{\partial h} + \bar{q}C_{\mathrm{D}_h}\right) - \cos\alpha_0\left(\frac{1}{2}V_0^2 C_{\mathrm{L}}\frac{\partial\rho}{\partial h} + \bar{q}C_{\mathrm{L}_h}\right)\right] \tag{D-790}$$

$$\frac{\partial(\dot{w})}{\partial x} = 0 \tag{D-791}$$

$$\frac{\partial(\dot{w})}{\partial y} = 0 \tag{D-792}$$

$$\frac{\partial(\dot{w})}{\partial \dot{\alpha}} = \frac{\bar{q}S\bar{c}}{2V_0 m}(-\sin\alpha_0\, C_{\mathrm{D}_{\dot{\alpha}}} - \cos\alpha_0\, C_{\mathrm{L}_{\dot{\alpha}}}) \tag{D-793}$$

$$\frac{\partial(\dot{w})}{\partial \dot{\beta}} = \frac{\bar{q}Sb}{2V_0 m}(-\sin\alpha_0\, C_{\mathrm{D}_{\dot{\beta}}} - \cos\alpha_0\, C_{\mathrm{L}_{\dot{\beta}}}) \tag{D-794}$$

$$\frac{\partial(\dot{w})}{\partial\delta_i} = \frac{1}{m}\left[\bar{q}S(-\sin\alpha_0\, C_{\mathrm{D}_{\delta_i}} - \cos\alpha_0\, C_{\mathrm{L}_{\delta_i}}) + \frac{\partial Z_{\mathrm{T}}}{\partial\delta_i}\right] \tag{D-795}$$

D.3.34 Angle-of-attack sensor output derivatives.—

$$\frac{\partial(\alpha_{,i})}{\partial p} = -\frac{y_\alpha}{V_0} \tag{D-796}$$

$$\frac{\partial(\alpha_{,i})}{\partial q} = \frac{x_\alpha}{V_0} \tag{D-797}$$

$$\frac{\partial(\alpha_{,i})}{\partial r} = 0 \tag{D-798}$$

$$\frac{\partial(\alpha_{,i})}{\partial V} = \frac{q_0 x_\alpha - p_0 y_\alpha}{V_0^2} \tag{D-799}$$

$$\frac{\partial(\alpha_{,i})}{\partial \alpha} = 1 \tag{D-800}$$

$$\frac{\partial(\alpha_{,i})}{\partial \beta} = 0 \tag{D-801}$$

$$\frac{\partial(\alpha_{,i})}{\partial \phi} = 0 \tag{D-802}$$

$$\frac{\partial(\alpha_{,i})}{\partial \theta} = 0 \tag{D-803}$$

$$\frac{\partial(\alpha_{,i})}{\partial \psi} = 0 \tag{D-804}$$

$$\frac{\partial(\alpha_{,i})}{\partial h} = 0 \tag{D-805}$$

$$\frac{\partial(\alpha_{,i})}{\partial x} = 0 \tag{D-806}$$

$$\frac{\partial(\alpha_{,i})}{\partial y} = 0 \tag{D-807}$$

$$\frac{\partial(\alpha_{,i})}{\partial \dot{\alpha}} = 0 \tag{D-808}$$

$$\frac{\partial(\alpha_{,i})}{\partial \dot{\beta}} = 0 \tag{D-809}$$

$$\frac{\partial(\alpha_{,i})}{\partial \delta_i} = 0 \tag{D-810}$$

D.3.35 Angle-of-sideslip sensor output derivatives.—

$$\frac{\partial(\beta_{,i})}{\partial p} = -\frac{z_\beta}{V_0} \tag{D-811}$$

$$\frac{\partial(\beta_{,i})}{\partial q} = 0 \tag{D-812}$$

$$\frac{\partial(\beta_{,i})}{\partial r} = \frac{x_\beta}{V_0} \tag{D-813}$$

$$\frac{\partial(\beta_{,i})}{\partial V} = -\frac{r_0 x_\beta - p_0 z_\beta}{V_0^2} \tag{D-814}$$

$$\frac{\partial(\beta_{,i})}{\partial \alpha} = 0 \tag{D-815}$$

$$\frac{\partial(\beta_{,i})}{\partial \beta} = 1 \tag{D-816}$$

$$\frac{\partial(\beta_{,i})}{\partial \phi} = 0 \tag{D-817}$$

$$\frac{\partial(\beta_{,i})}{\partial \theta} = 0 \tag{D-818}$$

$$\frac{\partial(\beta_{,i})}{\partial \psi} = 0 \tag{D-819}$$

$$\frac{\partial(\beta_{,i})}{\partial h} = 0 \tag{D-820}$$

$$\frac{\partial(\beta_{,i})}{\partial x} = 0 \tag{D-821}$$

$$\frac{\partial(\beta_{,i})}{\partial y} = 0 \tag{D-822}$$

$$\frac{\partial(\beta_{,i})}{\partial \dot{\alpha}} = 0 \tag{D-823}$$

$$\frac{\partial(\beta_{,i})}{\partial \dot{\beta}} = 0 \tag{D-824}$$

$$\frac{\partial(\beta_{,i})}{\partial \delta_i} = 0 \tag{D-825}$$

D.3.36 Altimeter output derivatives.—

$$\frac{\partial(h_{,i})}{\partial p} = 0 \tag{D-826}$$

$$\frac{\partial(h_{,i})}{\partial q} = 0 \tag{D-827}$$

$$\frac{\partial(h_{,i})}{\partial r} = 0 \tag{D-828}$$

$$\frac{\partial(h_{,i})}{\partial V} = 0 \tag{D-829}$$

$$\frac{\partial(h_{,i})}{\partial \alpha} = 0 \tag{D-830}$$

$$\frac{\partial(h_{,i})}{\partial \beta} = 0 \tag{D-831}$$

$$\frac{\partial(h_{,i})}{\partial \phi} = -y_h \cos\phi_0 \cos\theta_0 + z_h \sin\phi_0 \cos\theta_0 \tag{D-832}$$

$$\frac{\partial(h_{,i})}{\partial \theta} = x_h \cos\theta_0 + y_h \sin\phi_0 \sin\theta_0 + z_h \cos\phi_0 \sin\theta_0 \tag{D-833}$$

$$\frac{\partial(h_{,i})}{\partial \psi} = 0 \tag{D-834}$$

$$\frac{\partial(h_{,i})}{\partial h} = 1 \tag{D-835}$$

$$\frac{\partial(h_{,i})}{\partial x} = 0 \tag{D-836}$$

$$\frac{\partial(h_{,i})}{\partial y} = 0 \tag{D-837}$$

$$\frac{\partial(h_{,i})}{\partial \dot{\alpha}} = 0 \tag{D-838}$$

$$\frac{\partial(h_{,i})}{\partial \dot{\beta}} = 0 \tag{D-839}$$

$$\frac{\partial(h_{,i})}{\partial \delta_i} = 0 \tag{D-840}$$

D.3.37 Altitude rate sensor output derivatives.—

$$\frac{\partial(\dot{h}_{,i})}{\partial p} = 0 \tag{D-841}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial q} = 0 \tag{D-842}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial r} = 0 \tag{D-843}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial V} = 0 \tag{D-844}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial \alpha} = 0 \tag{D-845}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial \beta} = 0 \tag{D-846}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial \phi} = \dot{\phi}(y_{\dot{h}} \sin\phi_0 \cos\theta_0 + z_{\dot{h}} \cos\phi_0 \cos\theta_0) + \dot{\theta}(y_{\dot{h}} \cos\phi_0 \sin\theta_0 - z_{\dot{h}} \sin\phi_0 \sin\theta_0) \tag{D-847}$$

$$\begin{aligned}\frac{\partial(\dot{h}_{,i})}{\partial \theta} &= -\dot{\theta}(x_{\dot{h}} \sin\theta_0 - y_{\dot{h}} \sin\phi_0 \cos\theta_0 - z_{\dot{h}} \cos\phi_0 \cos\theta_0) \\ &\quad + \dot{\phi}(y_{\dot{h}} \cos\phi_0 \sin\theta_0 - z_{\dot{h}} \sin\phi_0 \sin\theta_0)\end{aligned} \tag{D-848}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial \psi} = 0 \tag{D-849}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial h} = 0 \tag{D-850}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial x} = 0 \tag{D-851}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial y} = 0 \tag{D-852}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial \dot{\alpha}} = 0 \tag{D-853}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial \dot{\beta}} = 0 \tag{D-854}$$

C-2

$$\frac{\partial(\dot{h}_{,i})}{\partial\dot{\phi}} = -y_{\dot{h}}\cos\phi_0\cos\theta_0 + z_{\dot{h}}\sin\phi_0\cos\theta_0 \tag{D-855}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial\dot{\theta}} = x_{\dot{h}}\cos\theta_0 + y_{\dot{h}}\sin\phi_0\sin\theta_0 + z_{\dot{h}}\cos\phi_0\sin\theta_0 \tag{D-856}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial\dot{h}} = 1 \tag{D-857}$$

$$\frac{\partial(\dot{h}_{,i})}{\partial\delta_i} = 0 \tag{D-858}$$

D.3.38 Total angular momentum derivatives.—

$$\frac{\partial(T)}{\partial p} = I_x p_0 - I_{xy} q_0 - I_{xz} r_0 \tag{D-859}$$

$$\frac{\partial(T)}{\partial q} = I_y q_0 - I_{xy} p_0 - I_{yz} r_0 \tag{D-860}$$

$$\frac{\partial(T)}{\partial r} = I_z r_0 - I_{xz} p_0 - I_{yz} q_0 \tag{D-861}$$

$$\frac{\partial(T)}{\partial V} = 0 \tag{D-862}$$

$$\frac{\partial(T)}{\partial\alpha} = 0 \tag{D-863}$$

$$\frac{\partial(T)}{\partial\beta} = 0 \tag{D-864}$$

$$\frac{\partial(T)}{\partial\phi} = 0 \tag{D-865}$$

$$\frac{\partial(T)}{\partial\theta} = 0 \tag{D-866}$$

$$\frac{\partial(T)}{\partial\psi} = 0 \tag{D-867}$$

$$\frac{\partial(T)}{\partial h} = 0 \tag{D-868}$$

$$\frac{\partial(T)}{\partial x} = 0 \tag{D-869}$$

$$\frac{\partial(T)}{\partial y} = 0 \tag{D-870}$$

$$\frac{\partial(T)}{\partial\dot{\alpha}} = 0 \tag{D-871}$$

$$\frac{\partial(T)}{\partial\dot{\beta}} = 0 \tag{D-872}$$

$$\frac{\partial(T)}{\partial\delta_i} = 0 \tag{D-873}$$

D.3.39 Stability axis roll rate derivatives.—

$$\frac{\partial(p_s)}{\partial p} = \cos\alpha_0 \tag{D-874}$$

$$\frac{\partial(p_s)}{\partial q} = 0 \tag{D-875}$$

$$\frac{\partial(p_s)}{\partial r} = \sin\alpha_0 \tag{D-876}$$

$$\frac{\partial(p_s)}{\partial V} = 0 \tag{D-877}$$

$$\frac{\partial(p_s)}{\partial \alpha} = -p_0 \sin\alpha_0 + r_0 \cos\alpha_0 \tag{D-878}$$

$$\frac{\partial(p_s)}{\partial \beta} = 0 \tag{D-879}$$

$$\frac{\partial(p_s)}{\partial \phi} = 0 \tag{D-880}$$

$$\frac{\partial(p_s)}{\partial \theta} = 0 \tag{D-881}$$

$$\frac{\partial(p_s)}{\partial \psi} = 0 \tag{D-882}$$

$$\frac{\partial(p_s)}{\partial h} = 0 \tag{D-883}$$

$$\frac{\partial(p_s)}{\partial x} = 0 \tag{D-884}$$

$$\frac{\partial(p_s)}{\partial y} = 0 \tag{D-885}$$

$$\frac{\partial(p_s)}{\partial \dot{\alpha}} = 0 \tag{D-886}$$

$$\frac{\partial(p_s)}{\partial \dot{\beta}} = 0 \tag{D-887}$$

$$\frac{\partial(p_s)}{\partial \delta_i} = 0 \tag{D-888}$$

D.3.40 Stability axis pitch rate derivatives.—

$$\frac{\partial(q_s)}{\partial p} = 0 \tag{D-889}$$

$$\frac{\partial(q_s)}{\partial q} = 1 \tag{D-890}$$

$$\frac{\partial(q_s)}{\partial r} = 0 \tag{D-891}$$

$$\frac{\partial(q_s)}{\partial V} = 0 \tag{D-892}$$

$$\frac{\partial(q_s)}{\partial \alpha} = 0 \tag{D-893}$$

$$\frac{\partial(q_s)}{\partial \beta} = 0 \tag{D-894}$$

$$\frac{\partial(q_s)}{\partial\phi} = 0 \tag{D-895}$$

$$\frac{\partial(q_s)}{\partial\theta} = 0 \tag{D-896}$$

$$\frac{\partial(q_s)}{\partial\psi} = 0 \tag{D-897}$$

$$\frac{\partial(q_s)}{\partial h} = 0 \tag{D-898}$$

$$\frac{\partial(q_s)}{\partial x} = 0 \tag{D-899}$$

$$\frac{\partial(q_s)}{\partial y} = 0 \tag{D-900}$$

$$\frac{\partial(q_s)}{\partial\dot{\alpha}} = 0 \tag{D-901}$$

$$\frac{\partial(q_s)}{\partial\dot{\beta}} = 0 \tag{D-902}$$

$$\frac{\partial(q_s)}{\partial\delta_i} = 0 \tag{D-903}$$

D.3.41 Stability axis yaw rate derivatives.—

$$\frac{\partial(r_s)}{\partial p} = -\sin\alpha_0 \tag{D-904}$$

$$\frac{\partial(r_s)}{\partial q} = 0 \tag{D-905}$$

$$\frac{\partial(r_s)}{\partial r} = \cos\alpha_0 \tag{D-906}$$

$$\frac{\partial(r_s)}{\partial V} = 0 \tag{D-907}$$

$$\frac{\partial(r_s)}{\partial\alpha} = -p_0\cos\alpha_0 - r_0\sin\alpha_0 \tag{D-908}$$

$$\frac{\partial(r_s)}{\partial\beta} = 0 \tag{D-909}$$

$$\frac{\partial(r_s)}{\partial\phi} = 0 \tag{D-910}$$

$$\frac{\partial(r_s)}{\partial\theta} = 0 \tag{D-911}$$

$$\frac{\partial(r_s)}{\partial\psi} = 0 \tag{D-912}$$

$$\frac{\partial(r_s)}{\partial h} = 0 \tag{D-913}$$

$$\frac{\partial(r_s)}{\partial x} = 0 \tag{D-914}$$

$$\frac{\partial(r_s)}{\partial y} = 0 \tag{D-915}$$

$$\frac{\partial(r_s)}{\partial\dot{\alpha}} = 0 \tag{D-916}$$

$$\frac{\partial(r_s)}{\partial\dot{\beta}} = 0 \tag{D-917}$$

$$\frac{\partial(r_s)}{\partial\delta_i} = 0 \tag{D-918}$$

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15. Supplementary Notes

16. Abstract

This report documents the derivation and definition of a linear aircraft model for a rigid aircraft of constant mass flying over a flat, nonrotating earth. The derivation makes no assumptions of reference trajectory or vehicle symmetry. The linear system equations are derived and evaluated along a general trajectory and include both aircraft dynamics and observation variables.

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