Chapter 6

Car-following

After this chapter, the student is able to:

- Predict the vehicle's trajectory given a CF model (know Newell's model by heart, others given in eqns)
- Interpret a model/FD at the other aggregation level (microscopic and macroscopic)

A car following model describes the longitudinal action of the vehicle as function of it's leader(s). That can be, it describes its position, speed or acceleration. There are many different forms, and all have their advantages and disadvantages. They all aim to describe driving behavior, and human behavior is inconsistent and hence difficult to capture in models.

This chapter does not give an overview of all models, nor a historical overview. That is given by Brackstone and McDonald (1999). Instead, in this chapter we discuss the simplest, Newell's, and discuss some characteristics which one might include.

6-1 Newell's car following model

The most easy car-following model is the model presented by Newell (2002a). It prescribes the position of the following x_{i+1} car as function of the position of the leader x_i. The model simply states that the position (and – by consequence – also speed, acceleration or jerk) of the follower is a distance s_j upstream of the position (or respectively speed, acceleration or jerk) of the leader of a time τ earlier

$$x_{i+1}(t) = x_i(t - \tau) - s_j \tag{6-1}$$

This means that the follower's trajectory is a copy of the leader's trajectory, translated over a vector $\{\tau, s_j\}$ in the xt-plane (see figure 6-1). This vector has also a direction, which can be computed by dividing its vertical component over the horizontal component.

$$w = \frac{s_j}{\tau} \tag{6-2}$$

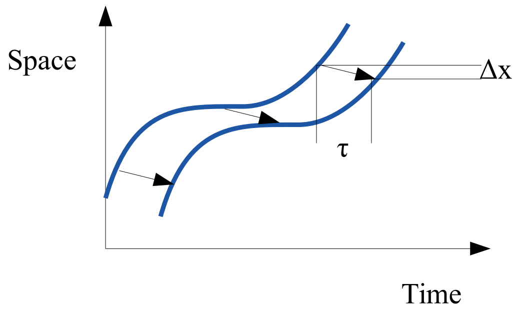

Figure 6-1: Newell's car-following model is translating a leader's trajectory over a vector $\{\tau, s_j\}$

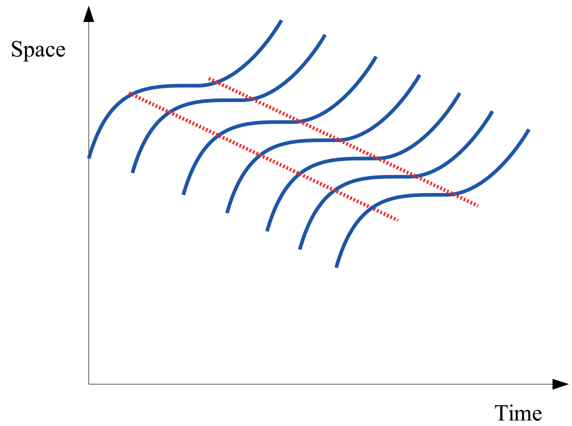

Figure 6-2: The average of the car-following behavior of drivers each with different parameters of the Newell model leads to a shock wave speed

This slope is the speed at which information travels backward in congested conditions, and hence is the slope of the congested branch of the fundamental diagram.

For each driver, different values for τ and s_j can be found. From empirical analysis (Chiabaut et al., 2010) it is shown that averaging the slopes over more than 12 drivers would yield a constant shock wave speed. This is illustrated in figure 6-2. Although the parameters of the car-following model are all different, the wave speed is an average translation of the disturbance, indicated by the red dotted line.

6-2 Characteristics

6-2-1 Dependencies

The car-following model describes the action of the following vehicle. That of course can depend on the movement of the leading vehicle. Some elements which are often included in a car-following model are:

- Acceleration of the leader: if the leader accelerates, the follower can get closer

- Speed: a higher speed would require a longer spacing
- Speed difference: if a follower is approaching his predecessor at a high speed, he needs to brake in time
- Spacing: if the spacing is large, he might (i) accelerate and/or (ii) be less influenced by its leader
- Desired speed: (i) the faster he wants to go, the more his desire to close a gap. But also (ii) even if the predecessor is far away, the follower will not exceed its desired speed

All these elements occur in car-following models. This list combines elements used for different type of models. For instance, some models might prescribe a distance, and hence use speed as input, whereas others might prescribe a speed, and use distance as input.

Surprisingly, many of the available car-following models are "incomplete", i.e. they lack one or more of the above elements and are therefore limited in their use. Using them for a dedicated task is of course allowable. A user should ensure that the model is suited for the task.

6-2-2 Reaction time

Human drivers have a reaction time. Mostly, the models are evaluated at time steps, which are chosen small, often in the order of 0.1 second. A good model allows to set the reaction time of the driver separately. Then, only information which is more than a reaction time earlier can influence a driver's acceleration.

Some models will use the model time step as reaction time. In that case the speed or acceleration of the model is evaluated every time step, which then is typically 1 second. Information of the previous time step is immediately used in the next time step, and in between there are no accelerations considered. The effect of reaction time on the traffic flow is described in Treiber et al. (2006). Generally speaking, a large reaction time can make the traffic flow unstable.

6-2-3 Multi leader car-following models

Human drivers can anticipate on the movement of their leader by looking ahead and considering more leaders. This has empirically been shown by Ossen (2008). Generally, one would adapt the car-following model allowing n times the spacing for the nth leader. Also, one might expect that a follower acts less sensitive on inputs from leaders further away – that might be in terms of lower acceleration (i.e., closer to zero) or later (i.e., a higher reaction time).

Two "mistakes" are common, leading to multi-leader car-following models which seem to make sense, but are not realistic.

1. In the car-following behavior, only vehicles in the same platoon should be considered, or vehicles which might influence the following vehicle. A car-following model which always takes three leaders regardless of the spacings might take a leader which is a long distance away, which in reality will not influence the driver.

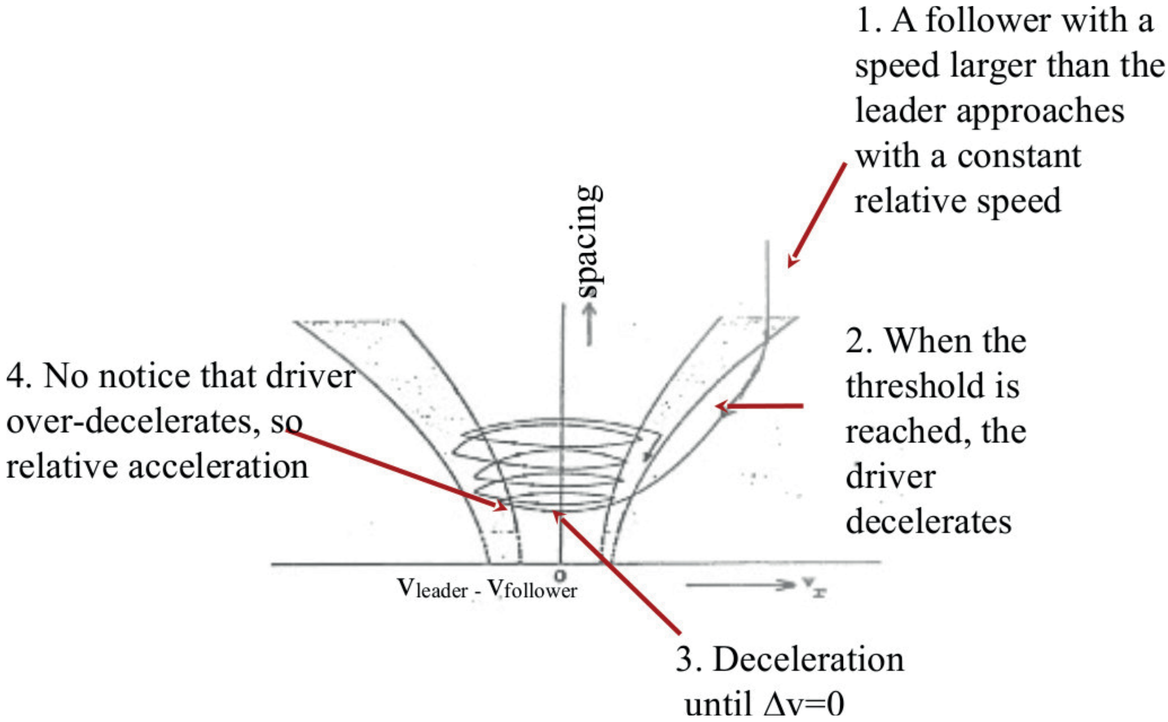

Figure 6-3: Schematic overview of the principles by Wiedemann

2. If the car following model predicts the *average* acceleration caused by each of the leaders, it might be that the follower crashes into its leader. Instead, a minimum operator is usually more suitable.

6-2-4 Insensitivity depending on distance

Most car-following models indicate drivers will adapt their speed based on the action of their leaders. Wiedemann (1974) states that drivers only react if the required action is above a certain threshold. Specifically, he indicates that within some bounds, drivers are unable to observe speed differences. One might also argue that within some boundaries, drivers are unwilling to adapt their speeds because the adaptation is too low. These bounds depend on the spacing between the leader and follower: the larger the spacing, the larger the speed difference needs to be before a driver is able to observe the speed difference. As an example, a 1 km/h speed difference might be unobservable (or unimportant to react upon) on a 200 meter spacing, but if the spacing is 10 meter, this is observable (or important).

It is relevant to draw a diagram relating the relative speed to the spacing, as is done in figure 6-3. The thresholds are indicated: within these bounds, the driver is insensitive and is considered to keep his current speed. Outside the bounds, the driver accelerates to match the traffic situation.

When the leader brakes, the follower closes in. That means that point is in the right half (the follower has a higher speed) and the line is going down (the spacing reduces because the follower is driving faster). At a certain moment, the follower comes closer to his leader and

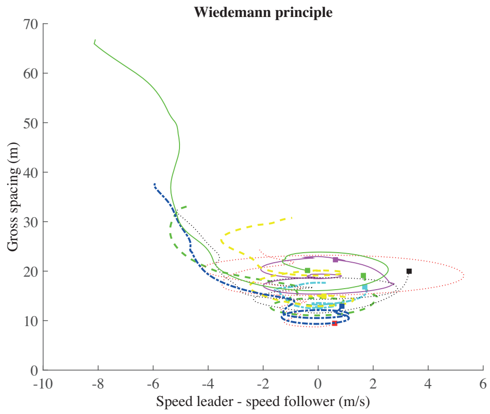

Figure 6-4: Empirical observations of leader-follower pairs in the relative speed - spacing plane

starts reacting to the speed difference, i.e. he will reduce speed. He can either not brake enough and have still a higher speed, or overreact. If the braking is not enough, the situation is the same as the starting situation, and the same situation will occur. Therefore, we will assume for the further reasoning that the follower overreacts.

The overreaction means that the follower will have a lower speed than the leader. That means that the speed difference will go through zero (the y-axis in the figure). At that moment, the speeds are equal, so the spacing remains momentarily the same. Therefore, when crossing the y-axis, the line is horizontal. After that, the leader drives at a higher speed than the follower, and the spacing will start increasing. For low speed differences, the speed difference is under the observation threshold and the follower does not adapt his acceleration. Only once the next bound is reached, he will notice the leader is shying away and start accelerating again.

If he will again overreact, the speed difference will become positive. Again, the speed difference will go through zero (the y-axis) where the line has a horizontal tangent line (a zero speed difference means that at that moment the spacing remains constant). One can continue constructing this figure, and it shows that in this phase plane (i.e., the plane relative speed - spacing) the leader-follower pair makes circles.

The wider the circles are, the wider the observation thresholds. The thresholds are considered to be higher for larger spacings. Empirical studies (e.g., Knoop et al. (2009); Hoogendoorn et al. (2011) indeed reveal these type of circles.

Caution is required in interpreting these figures. A Newell car-following model does not implement these type of observation thresholds. However, the relative speed - spacing figures will show circles due to the reaction time. Namely, once the leader brakes, the follower will not yet, and in the relative speed-spacing plane this will be shown as a part of a circle. A correct analysis tracks the car-following behaviour not at the same moments in time (vertical lines in the space time diagram), but at lines moving back with the shock wave speed. When doing so for the Newell model, one would not find any circles at all. A detailed analysis of this method is presented by Laval (2011)

Finally, some words on the principle in relation to car-following. Car-following models prescribe the position, speed or acceleration of the follower based on the position, speed, or acceleration of their leaders. The principle laid out in this section only mentions when the speed is *not* adapted, but does not specify the acceleration outside the thresholds. As such, it therefore cannot be considered a car-following model. Instead, it can be combined with another car-following model to get a complete description.

6-3 Examples

This section shows the some frequently used models, apart from the Newell car-following model described in 6-1.

6-3-1 Helly

The first model we describe here is the Helly model (Helly, 1959). The Helly model prescribes a desired spacing s^* as function of the speed v:

$$s^* = s_0 + Tv \tag{6-3}$$

Note that this could be considered as a spacing at standstill (jam spacing) plus a dynamic part where T is the net time headway (subtracting the jam spacing from desired spacing).

Now the acceleration is determined by a desire to drive at the same speed as the predecessor and a desire to drive at the desired headway. The model prescribes the following acceleration:

$$a(t) = \alpha\left(\Delta v(t-\tau)\right) + \gamma\left(s(t-\tau) - s^*\right) \tag{6-4}$$

In this equation, Δv is the speed difference, t is a moment in time and τ a reaction time.

6-3-2 Optimal Velocity Model

The optimal velocity model proposed by Bando et al. (1995) is a car-following model specifying the acceleration a as follows:

$$a = a_0(v^* - v) \tag{6-5}$$

In this equation, v is the speed of the vehicle, and a_0 a reference acceleration (tunable parameter, constant for a specific vehicle-driver combination). v^* is determined as follows:

$$v* = 16.8(\tanh(0.086(s - 25) + 0.913)) \tag{6-6}$$

In this equation, s is the spacing (in meters) between the vehicle and its leader, giving the speed in m/s.

6-3-3 Intelligent Driver model

Treiber et al. (2000) proposes the Intelligent Driver Model. This prescribes the following acceleration:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = a_0\left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right) \tag{6-7}$$

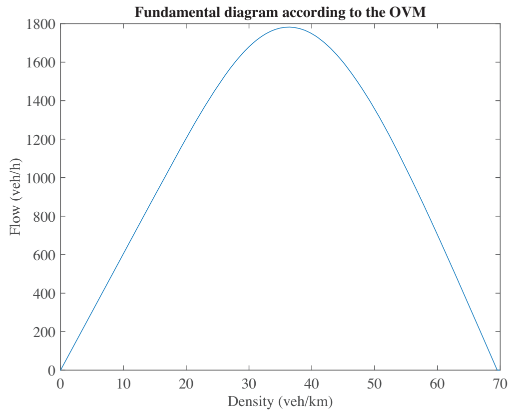

Figure 6-5: Fundamental diagram according to the OVM

with the desired spacing s^* as function of speed v and speed difference Δv:

$$s^*(v, \Delta v) = s_0 + vT + \frac{v \Delta v}{2\sqrt{a\,b}} \tag{6-8}$$

In this equation, Δv is the speed difference between the leader and the follower, a is acceleration and b a comfortable deceleration. a_0 is a reference acceleration (parameter).

6-4 Relation to fundamental diagram

The fundamental diagram gives in its usual form the relation between the density and the flow for homogeneous and stationary conditions. Also car-following models can describe homogeneous and stationary conditions. In that case, all vehicles should drive at the speed and should not change speed (stationary). Therefore, the acceleration of all vehicles must be zero.

Many car-following model prescribe the acceleration based on speed and spacing. The equilibrium conditions mean that the acceleration is zero (and one might argue the relative speeds as well). The relation between spacing and speed then gives an implicit equation for the spacing-speed diagram. Using that the density is the inverse of the (average) spacing, and the flow is (average) speed times density, one can reformulate the spacing-speed diagram from the car-following model into a fundamental diagram.

Take for example the OVM, equation 6-5 and equation 6-6. Equilibrium conditions prescribe that drivers do not accelerate, hence $a_0(v^* - v) = 0$. That means that either $a_0 = 0$, or

$$(v^* - v) = 0 \iff v^* = v \tag{6-9}$$

Since a_0 cannot be zero – in that case the vehicle would never accelerate. Therefore, in equilibrium conditions, equation 6-9 should hold. Using equation 6-6, we find:

$$v = 16.8(\tanh(0.086(s - 25) + 0.913)) \tag{6-10}$$

This gives a relation between speed and spacing. For a fundamental diagram in flow-density, that gives (using $q = kv$:

$$q = kv = k\left(16.8(\tanh(0.086(s-25)+0.913))\right) \tag{6-11}$$

The spacing can be changed into a density. Since all vehicles have the same spacing (equilibrium conditions), we may use

$$s = \langle s \rangle - 1/k \tag{6-12}$$

Substituting this in equation 6-11, we find an expression for the fundamental diagram.

$$q = k\left(16.8(\tanh(0.086(1/k-25)+0.913))\right) \tag{6-13}$$

Note that in this expression with the values as in equation 6-6, density should be in vehicles per meter and the resulting flow is given in veh/s. Converting these units, this results in the fundamental diagram as shown in figure 6-5.

Selected problems

For this chapter, consider problems: 11, A-2-4, A-3-2, A-4-2, 133, 154, 179, 181, 220, 222, A-10-4, 266, 277, 280, 281, 282, 283