19 11-04-2025

PROBLEM-SOLVING STRATEGY

Change of Variables

1. Sketch the region given by the problem in the xy-plane and then write the equations of the curves that form the boundary.
2. Depending on the region or the integrand, choose the transformations $x = g(u, v)$ and $y = h(u, v)$.
3. Determine the new limits of integration in the uv-plane.
4. Find the Jacobian $J(u, v)$.
5. In the integrand, replace the variables to obtain the new integrand.
6. Replace $dy\,dx$ or $dx\,dy$, whichever occurs, by $J(u, v)\,du\,dv$.

19.1 Section 5.7, Exercise 359, 360

The given function $T : S \to R$, $T(u, v) = (x, y)$ on the region $S = \{(u, v) : 0 \le u \le 1, 0 \le v \le 1\}$ bounded by the unique square is given. $R \subset \mathbb{R}^2$ is the image of S under T.

(a) Justify that T is a C^1 transformation

(b) Find the images of the vertices of the unit square S through the function T

(c) Determine the image R of the unit square S and graph it

359. $= 2u - v$, $y = u + 2v$

360. $x = u^2$, $y = v^2$

19.2 Section 5.7, Checkpoint 5.46

Make appropriate changes of variables in the integral

$$\iint_R \frac{4}{(x-y)^2}\ dy\ dx$$

where R is the trapezoid bounded by the lines $x - y = 2$, $x - y = 4$, $x = 0$, and $y = 0$. Write the resulting integral.

19.3 Section 5.7, Checkpoint 5.47

Using the substitutions $x = v$, $y = \sqrt{u+v}$, evaluate the integral

$$\iint_R y \sin\left(y^2 - x\right)\, dA$$

where R is the region bounded by the lines $\sqrt{x}$, $x = 2$, and $y = 0$.

19.4 Section 5.7, Checkpoint 5.48

Let D be the region in xyz-space defined by $1 \le x \le 2$, $0 \le xy \le 2$, and $0 \le z \le 1$. Evaluate

$$\iiint_D (x^2 y + 3xyz)\, dx\, dy\, dz$$

by using the transformation $u = x$, $v = xy$, and $w = 3z$.