19.1 Section 5.7, Exercise 359, 360

The given function $T : S \to R$, $T(u,v) = (x,y)$ on the region $S = \{(u,v) : 0 \le u \le 1, 0 \le v \le 1\}$ bounded by the unique square is given. $R \subset \mathbb{R}^2$ is the image of S under T.

(a) Justify that T is a C^1 transformation

(b) Find the images of the vertices of the unit square S through the function T

(c) Determine the image R of the unit square S and graph it

359. $= 2u - v$, $y = u + 2v$

360. $x = u^2$, $y = v^2$

$x = 2u - v$

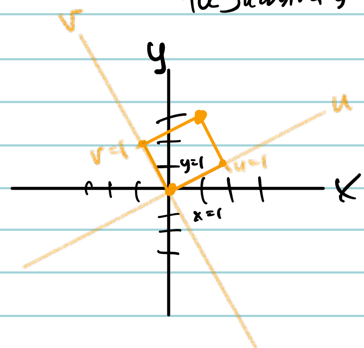
$y = u + 2v$

Jacobian matrix

$$\begin{array}{c c} & \begin{array}{cc} u & v \end{array} \\ \begin{array}{c} x \\ y \end{array} & \begin{bmatrix} x_u & x_v \\ y_u & y_v \end{bmatrix} \end{array} = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

this is connected to linear algebra, note this T is a linear transformation

the Jacobian is $\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4 + 1 = 5$



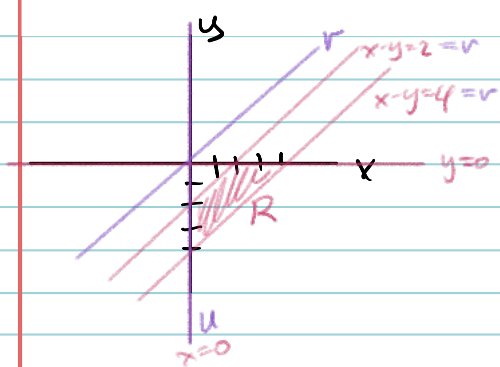
(u,v)	(x,y)
$(0,0)$	$(2\cdot 0 - 0,\ 0 + 2\cdot 0) = (0,0)$
$(0,1)$	$(2\cdot 0 - 1,\ 0 + 2\cdot 1) = (-1,2)$
$(1,0)$	$(2\cdot 1 - 0,\ 1 + 2\cdot 0) = (2,1)$
$(1,1)$	$(2\cdot 1 - 1,\ 1 + 2\cdot 1) = (1,3)$

19.2 Section 5.7, Checkpoint 5.46

Make appropriate changes of variables in the integral

$$\iint_R \frac{4}{(x-y)^2}\, dy\, dx$$

where R is the trapezoid bounded by the lines $x - y = 2$, $x - y = 4$, $x = 0$, and $y = 0$. Write the resulting integral.



let $u = x - y$, $v = x$ $\rightarrow$ $J = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1$

how? well, $x-y=2$ and $x-y=4$ suggest $x-y=u$ is a good sub.

then, we draw the sketch, and pick v s.t. we preserve right-handedness of $(x,y) \to (u,v)$.

v-axis: $u=0 = x-y \rightarrow y=x$

u-axis: $v=0 = x \rightarrow x=0$

$2 \le x-y \le 4$ ↳ $2 \le u \le 4$

$x=0$ ↳ $v=0$

$y=0 = x-(x-y)$ ↳ $0 = v-u$ ↳ $u=v$

So $$\iint_R \frac{4}{(x-y)^2}\, dy\, dx = \int_{u=2}^{4} \int_{v=0}^{u} \frac{4}{u^2}\, dv\, du = \int_{u=2}^{4} \left[\frac{4v}{u^2}\right]_{v=0}^{u} du$$

$$= \int_{u=2}^{4} \frac{4}{u}\, du = 4(\ln|4| - \ln|2|) = 4\ln\left(\frac{4}{2}\right) = 4\ln 2$$

I claimed Area(R) = 6, while

$$\iint_R 1\, dy\, dx = \int_{u=2}^{4} \int_{v=0}^{u} dv\, du = \int_{u=2}^{4} u\, du = \left[\frac{u^2}{2}\right]_2^4 = \frac{16-4}{2} = \frac{12}{2} = 6,$$ as expected.

19.3 Section 5.7, Checkpoint 5.47

Using the substitutions $x = v$, $y = \sqrt{u+v}$, evaluate the integral

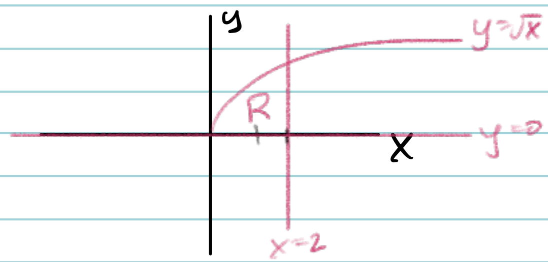
$$\iint_R y \sin\left(y^2 - x\right)\, dA$$

where R is the region bounded by the lines $\sqrt{x}$, $x = 2$, and $y = 0$.

$x = v$

$y = \sqrt{u+v}$

$$J = \begin{vmatrix} 0 & 1 \\ \frac{1}{2\sqrt{u+v}} & \frac{1}{2\sqrt{u+v}} \end{vmatrix} = \frac{-1}{2\sqrt{u+v}}$$



$$\iint_R y \sin(y^2 - x)\, dA = \iint_R \frac{-1}{2\sqrt{u+v}} \sqrt{u+v} \sin\left(\left(\sqrt{u+v}\right)^2 - v\right) dA$$

$$= \frac{-1}{2} \iint \sin(u)\, dA$$

$y = \sqrt{x} \rightarrow \sqrt{u+v} = \sqrt{v} \rightarrow u+v = v \rightarrow u = 0$

$x = 2 \rightarrow v = 2$ $(0 \le x \le 2 \rightarrow 0 \le v \le 2)$

$y = 0 \rightarrow \sqrt{u+v} = 0 \rightarrow u+v = 0 \rightarrow u = -v$

$$\int_{v=0}^{2} \int_{u=-v}^{0} \sin u \, du\, dv = \int_{v=0}^{2} \left[-\cos u\right]_{-v}^{0} dv = \int_{v=0}^{2} -1 + \cos(-v)\, dv$$

$$= \int_{v=0}^{2} -1 - \cos(v)\, dv = \left[-v - \sin v\right]_0^2 = -2 - \sin(2)$$