5 09-09-2025

5.1 Section 3.1, Exercise 8

Find the limit of the following vector valued function at the indicated value of t.

$$\lim_{t\to 4}\left\langle \sqrt{t-3}, \frac{\sqrt{t}-2}{t-4}, \tan\left(\frac{\pi}{t}\right)\right\rangle$$

5.2 Section 3.1, Exercise 22

Eliminate the parameter t, write the equation in Cartesian coordinates, then sketch the graphs of the vector-valued functions. *Hint: solve first equation for x in terms of t and substitute this result into the second equation.*

$$\mathbf{r}(t) = 2t\hat{\imath} + t^2\hat{\jmath} \qquad \text{let } x = 2t, y = t^2$$

THEOREM 3.3

Properties of the Derivative of Vector-Valued Functions

Let **r** and **u** be differentiable vector-valued functions of t, let f be a differentiable real-valued function of t, and let c be a scalar.

i.	$\frac{d}{dt}[c\mathbf{r}(t)]$	$=$	$c\mathbf{r}'(t)$	Scalar multiple
ii.	$\frac{d}{dt}[\mathbf{r}(t) \pm \mathbf{u}(t)]$	$=$	$\mathbf{r}'(t) \pm \mathbf{u}'(t)$	Sum and difference
iii.	$\frac{d}{dt}[f(t)\,\mathbf{u}(t)]$	$=$	$f'(t)\,\mathbf{u}(t) + f(t)\,\mathbf{u}'(t)$	Scalar product
iv.	$\frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{u}(t)]$	$=$	$\mathbf{r}'(t) \cdot \mathbf{u}(t) + \mathbf{r}(t) \cdot \mathbf{u}'(t)$	Dot product
v.	$\frac{d}{dt}[\mathbf{r}(t) \times \mathbf{u}(t)]$	$=$	$\mathbf{r}'(t) \times \mathbf{u}(t) + \mathbf{r}(t) \times \mathbf{u}'(t)$	Cross product
vi.	$\frac{d}{dt}[\mathbf{r}(f(t))]$	$=$	$\mathbf{r}'(f(t)) \cdot f'(t)$	Chain rule
vii.	If $\mathbf{r}(t) \cdot \mathbf{r}(t)$	$=$	c, then $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$.	

5.3 Section 3.2, Checkpoint 3.5

Calculate the derivative of the function

$$\mathbf{r}(t) = (t \ln t)\hat{\imath} + (5e^{t})\hat{\jmath} + (\cos t - \sin t)\hat{k}$$

5.4 Section 3.2, Checkpoint 3.7

Find the unit tangent vector for the vector-valued functions

$$\mathbf{r}(t) = (t^2 - 3)\hat{\imath} + (2t + 1)\hat{\jmath} + (t - 2)\hat{k}$$

5.5 Section 3.2, Checkpoint 3.8

Calculate the following integral:

$$\int_1^3 \left[(2t + 4)\hat{\imath} + (3t^2 - 4t)\hat{\jmath}\right]\; dt$$