9 09-23-2025

9.1 Section 4.1, Checkpoint 4.4

Find the domain and range of the function $f(x, y) = \sqrt{36 - 9x^2 - 9y^2}$

9.2 Section 4.1, Checkpoint 4.5

Find the equation of the level surface of the function

$$g(x, y, z) = x^2 + y^2 + z^2 - 2x + 4y - 6z$$

corresponding to $c = 2$, and describe the surface, if possible.

9.3 Section 4.2, Exercise 60

Evaluate $\lim_{(x,y)\to(1,2)} x$ or explain why the limit does not exist.

9.4 Section 4.2, Exercise 77

Evaluate $\lim\limits_{(x,y)\to(0,0)} \ln\left(x^2 + y^2\right)$ or explain why the limit does not exist.

*Does the limit exist on **any** path?*

9.5 Section 4.2, Exercise 81

Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - 4y^4}{x + 2y^2}$ or explain why the limit does not exist.

Do you see any algebraic manipulations?

9.6 Section 4.2, Exercise 86

Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{xy+y^3}{x^2+y^2}$ or explain why the limit does not exist.

(a) Along the x-axis ($y=0$)

(b) Along the y-axis $x=0$

(c) Along the path $y=2x$

9.7 Section 4.2, Exercise 88

Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$ or explain why the limit does not exist.

(a) Along the x-axis ($y=0$)

(b) Along the y-axis $x=0$

(c) Along the path $y=x^2$

9.8 Section 4.2, Exercise 95

Determine the region in which the function is continuous. Explain your answer

$$f(x,y)=\begin{cases}\dfrac{x^2y}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$

Hint: maybe this coordinate system isn't helpful. Also remember the Squeeze Theorem? We do NOT expect you to use Squeeze Theorem with completely general functions