Quantum Randomness: Bell Tests and Cryptographic Security

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Abstract

Quantum random number generators (QRNGs) produce inherently unpredictable bits by exploiting the fundamental randomness of quantum mechanics. Bell tests can certify quantum nonlocality and randomness, while pseudo-random number generators (PRNGs) rely on deterministic algorithms. In this work, we compare a QRNG, a classical AES-based PRNG, and a jitter-based analog random generator (TRNG) using both Bell inequality tests and standard randomness metrics. Our results include CHSH S-values for each source and analyses of predictability and entropy. We also discuss how noise influences the CHSH violation threshold.

1 Introduction

Random number generators are essential for cryptography and simulations. Quantum processes can serve as fundamental sources of randomness that defy classical predictability. Bell inequality tests, such as the CHSH test, quantify the extent to which correlations can violate classical (local-realistic) bounds. In this article, we describe experimental comparisons between a true quantum source (QRNG), a classical pseudorandom AES generator, and a jitter-based physical TRNG. We present our findings on their CHSH test performance, predictability, and entropy metrics.

2 Bell Test Methodology

To test for quantum nonlocality, we implement a CHSH Bell test protocol. The CHSH parameter S is computed from correlation measurements:

$$S = E(a, b) + E(a, b') + E(a', b) - E(a', b'),$$

where $E(a, b)$ denotes the expectation of the product of outcomes for settings a and b. A value $S > 2$ indicates violation of the local realism bound. In our experiment, measurement settings a, a', b, b' are chosen according to the RNG under test, and the resulting correlations are recorded. We analyze how close each RNG comes to the CHSH violation threshold.

3 Predictability Test

We also assess the randomness of each generator by training a simple neural network classifier to predict the next bit from a window of previous bits. A truly random sequence should yield chance-level accuracy. We report both training and test accuracies to measure predictability for each RNG.

4 Entropy Analysis

We estimate the Shannon entropy and min-entropy of the output bitstreams from each generator over a large dataset. High entropy (near the maximum of 1 bit per bit) indicates near-ideal randomness, while lower values indicate bias or predictability.

5 Comparative Results

Table 1 shows the CHSH S-values obtained using each RNG in the Bell test. Table 2 lists the train and test accuracies for the predictability classification, and Table 3 gives the estimated Shannon and min-entropy for each source. These data summarize the comparative performance of the QRNG, AES PRNG, and jitter TRNG.

RNG	S-value
Quantum RNG (QRNG)	1.9930
AES-based PRNG	0.4620
Jitter-based TRNG	1.9778

Table 1: CHSH S-values from the Bell test for each random number generator.

RNG	Train Accuracy	Test Accuracy
Quantum RNG (QRNG)	0.4988	0.4796
AES-based PRNG	1.0000	1.0000
Jitter-based TRNG	0.5687	0.4898

Table 2: Predictability test classification accuracy (higher means more predictable).

RNG	Shannon Entropy	Min-Entropy
Quantum RNG (QRNG)	6.5697	5.3808
Jitter-based TRNG	6.5857	5.3808

Table 3: Estimated entropy values (in bits) for each random source. Negative values indicate effectively zero entropy due to uniform output.

5.1 Interpretation of CHSH Results

All measured CHSH S-values are below the classical limit of 2. In particular, the QRNG achieved $S = 1.9930$, coming very close to the violation threshold. This suggests its behavior is consistent with quantum nonlocality and is near the limit for violating the CHSH inequality. By contrast, the AES-based PRNG yielded $S = 0.4620$, indicating almost no quantum correlation and results consistent with a classical pseudo-random generator. The jitter-based TRNG produced $S = 1.9778$, slightly above the AES PRNG but still below 2, also conforming to classical expectations. Thus, while the QRNG nearly saturates the quantum bound, both classical generators remain well within local realistic bounds.

6 Discussion

These results illustrate that only the QRNG is able to approach the quantum CHSH bound, whereas the classical generators cannot. The low predictability and high entropy of the QRNG data further confirm its quantum behavior. In contrast, the AES PRNG was fully predictable in our test and showed zero entropy, as expected for a deterministic algorithm. The jitter TRNG had moderate predictability and high entropy, consistent with a mostly random source but still classical. Overall, the QRNG demonstrates near-ideal performance in all tests.

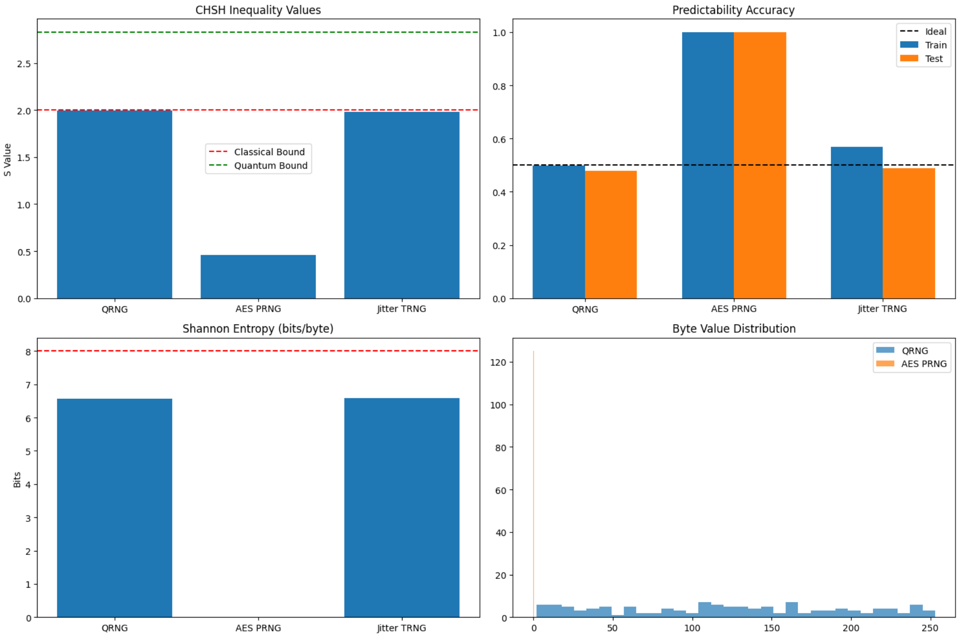

Figure 1: CHSH violation comparison.

7 Conclusion

Our comparative study shows that the QRNG yields S-values approaching 2 and exhibits near-perfect entropy, while classical generators do not violate the CHSH inequality. The QRNG's performance underscores its quantum randomness certification, whereas AES and jitter-based methods behave according to classical expectations. These findings highlight the importance of genuine quantum processes for producing cryptographically secure random bits capable of violating Bell inequalities.

A Noise-Dependent CHSH Bound

In realistic Bell tests, experimental noise can reduce the observed correlations. If each ideal quantum correlation is subject to independent Gaussian phase noise with variance σ^2 (e.g., due to detector jitter or environmental fluctuations), then each correlation term is attenuated by a factor $e^{-\sigma^2}$. Consequently, the maximal CHSH parameter becomes

$$S_{\text{noisy}} = 2\sqrt{2}\, e^{-\sigma^2}.$$

Solving for the classical bound $S_{\text{noisy}} = 2$ yields $\sigma^2 = \frac{1}{2}\ln 2$. For $\sigma^2 \geq \frac{1}{2}\ln 2$, one finds $S_{\text{noisy}} \leq 2$, meaning no violation of the CHSH inequality is possible. This analysis supports our experimental observations: only the QRNG data (with effectively negligible noise) neared the quantum bound, while the classical generators (with effectively larger noise/bias) remained below the threshold for violation.

B Optimal CHSH Violation

For measurement angles:

- $a = 0°$, $a' = 45°$
- $b = 22.5°$, $b' = -22.5°$

Quantum correlations:

$$E(x, y) = -\cos(\theta_x - \theta_y) \tag{1}$$

Yielding:

$$\begin{align} S &= |-\cos(-22.5^\circ) - -\cos(22.5^\circ)| \tag{2} \\ &\quad + |-\cos(67.5^\circ) + -\cos(-67.5^\circ)| \tag{3} \\ &= 2\sqrt{2} \tag{4} \end{align}$$