Basic Math. [Class 15. [Basis Dimension 1. Consider the following Wechors in 124 V1= (3,0,-2,4) U, = (2/1/-1/3) V3= (-1141210) Vy= (-1,11,1,-1) W:= Span (U11/2/1/3/1/4 Select a basi's in W. What is climW=2

Solo a basis in W is a linearly independent generator system of W. So UniV21V31V9 is a generative cystem on W, we have to check if it's independent or not. so suppose that: X. V1+BV2+P. V3+8 V4=0 $\begin{array}{c} 2 \\ 2 \\ 4 \end{array} + 3 \\ 2 \\ 1 \\ -1 \\ 3 \end{array} + 7 \\ 2 \\ 4 \\ 5 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$

(=)
$$3x+2\beta-\gamma-\delta=0$$
 I.

 $\beta+4\gamma+\delta=0$ II

 $-2x-\beta+2\gamma+\delta=0$ II

 $4x+3\beta$ $-\delta=0$ IV

 $4x+3\beta$ we substitute

this into equations I_1I_1III
 $3x+2\beta-\gamma-(4x+3\beta)=0$ II

 $3x+2\beta-\gamma-(4x+3\beta)=0$ II

 $-2x-\beta+2\gamma+(4x+3\beta)=0$ II

 $-2x-\beta-\gamma=0=0$ $\gamma=-x-\beta$
 $4x+4\beta+4\gamma=0$ $1:4=0$
 $2x+2\beta+2\gamma=0$ $1:2=0$

of
$$+\beta + (-\lambda - \beta) = 0$$
 II.

II the some =)

 $0\lambda + 0\beta = 0$.

So differ are free variables

 $\Gamma = -\lambda - \beta$
 $\delta = 4\lambda + 3\beta$.

So for example:

 $\lambda = 1$
 $\beta = 0$
 $\delta = 1$
 $\delta = 0$
 $\delta = 1$
 $\delta = 0$
 $\delta = 0$

ave lin. dependent =) (reducing dependent rector system theorem) he an ouit one vector (with no 0 coefficient in a dependency equotion) 10 here we con ourit ex leave V11 V3 or V4 So W = Span (Un1Uz1U31U4) = = Span (Uni V21 V3) = = Span (U21 U31 U24) = Span (U1) UL 1U4)

we olso an see that a U1V21V31V4 is not a · let's omit V4 =) W= Span (V1/V2/V3). Now V11U21U3 is a gen. System i'n W, we have to check if they ove independent or dependent. Let LIBITEIR So flest & Un+ BUZ+PUz=0 $(=) \left(\frac{3}{0} \right) + 13 \left(\frac{2}{1} \right) + 3 \left(\frac{4}{2} \right) = \left(\frac{0}{0} \right)$

Find
$$d_1\beta_1\beta_1 = 0$$
 $3x + 2\beta - \beta = 0$
 $4x + 3\beta = 0$

$$\begin{array}{l}
\mathcal{L} = -\frac{3}{4}\beta \\
\beta \in \mathbb{R} \text{ free voriable} \\
\mathcal{D} = 3\mathcal{L} + 2\beta = 3(-\frac{3}{4}\beta) + 2\beta = \\
= -\frac{9}{4}\beta + 2\beta = -\frac{1}{4}\beta \\
\text{Sleakms} \mathcal{L} = -\frac{3}{4}\beta \cdot \beta \in \mathbb{R}, -\frac{1}{4}\beta \\
\text{For exp. if } \beta = 4 =)
\end{array}$$

$$\begin{array}{l}
\mathcal{L} = -3 \\
\beta = 4 =)
\end{array}$$

$$\begin{array}{l}
\mathcal{L} = -3 \\
\mathcal{L} = -1
\end{array}$$

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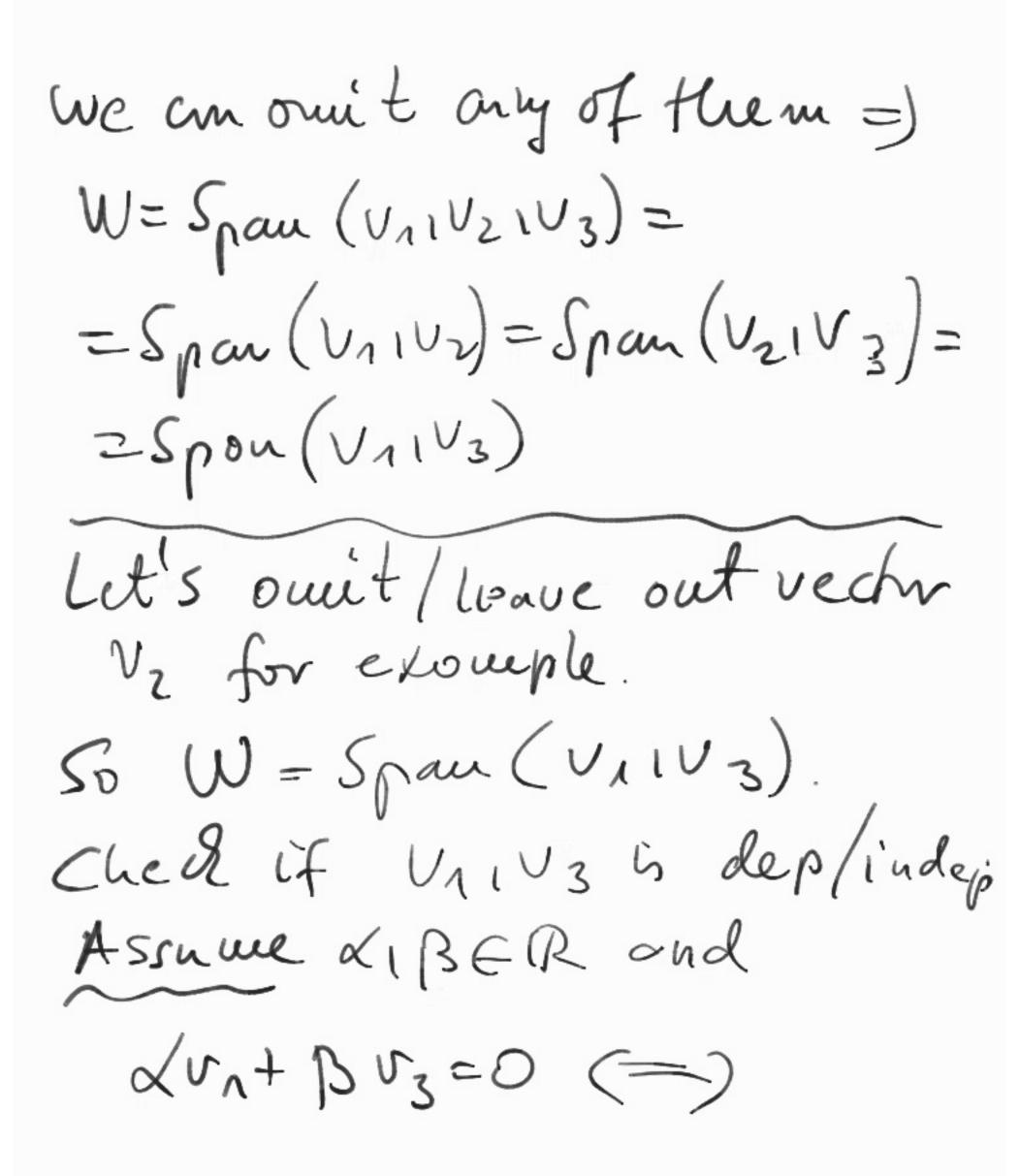
$$\begin{array}{l}
\mathcal{L} = -3 \\
\mathcal{L} = -1
\end{array}$$

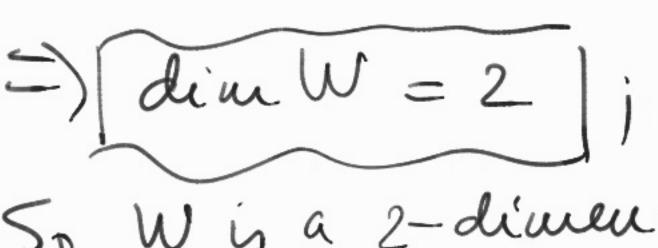
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So Wisa 2-démensional subspace in IR

- Determine whether the following vector systems form a basis in IR.
 - a) X11X2
 - b) X11×21×31 X41×5
 - C) X11X21X31X4 Where

Xn= (2131-217), Xz= (0/1/0/1)

X3=(1121-110)) X4=(-11-51210)

x5=(3,-1,1,2)

Sil: we know that vectors $e_1 = (1101010)$ are a liu. $e_2 = (0111010)$ independent $e_3 = (0101110)$ generator $e_4 = (0101011)$ System intr e1=(1101010)) So dim IR = 4, and overy basis in 112 must hove 4 element, Hum. So a) Xnixz connot be basis in M (2-elements ore not enough)

b)
$$x_1 x_2 x_3, x_4, x_5$$
 they are 5 different vectors 574 =) connut be basis in 1R4.

c) $x_1 x_2 x_3 x_4$ there is 9

of them so it can be basis.

Are they independent?

 $x_1 + y_2 + y_3 + x_5 + x_4 = 0 = 0$

-2×+13 =0 正 子×+13 =0 正

$$= \int [+ u] = \int [-1] = 0$$

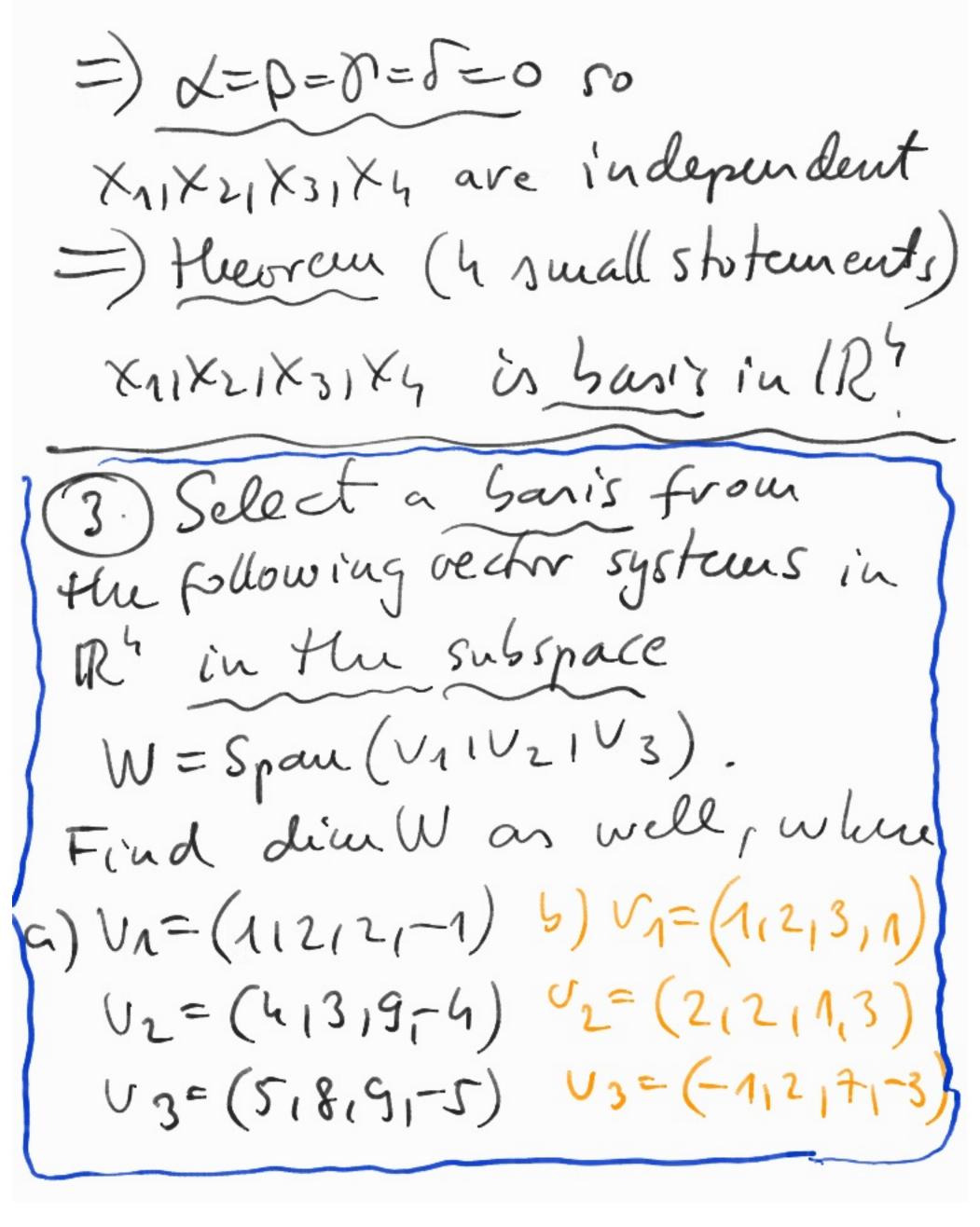
$$50 \int 2x + y = 0$$

$$3x + y + 2y = 0$$

$$-2x - y = 0$$

$$7x + x = 0$$

$$7x + x$$



$$5K.9$$
 $4\frac{1}{3}\frac{1}{9}+1$
 $4\frac{1}{3}\frac{1}{9}+1$

=)
$$3P=0$$
 =) $P=0$ =) $P=-P=0$

=) $A=-4B-5P=0$ $A=0$

=) $A=0$, $P=0$, $P=0$ is the only evention =) $V_1 V_2 V_3$ is liverly independent vector system, they also quente $W=0$ $V_1 V_2 V_3$ is basis in $W=0$ $V_1 V_2 V_3$ is basis in $W=0$

b)
$$V_{\lambda} = (\lambda_1 2_1 3_1 1)$$

 $V_2 = (2_1 2_1 \lambda_1 3)$
 $V_3 = (-\lambda_1 2_1 7_1 - 3)$

and now W==Span(v11/21/2)

We sow in closs 14/1/6

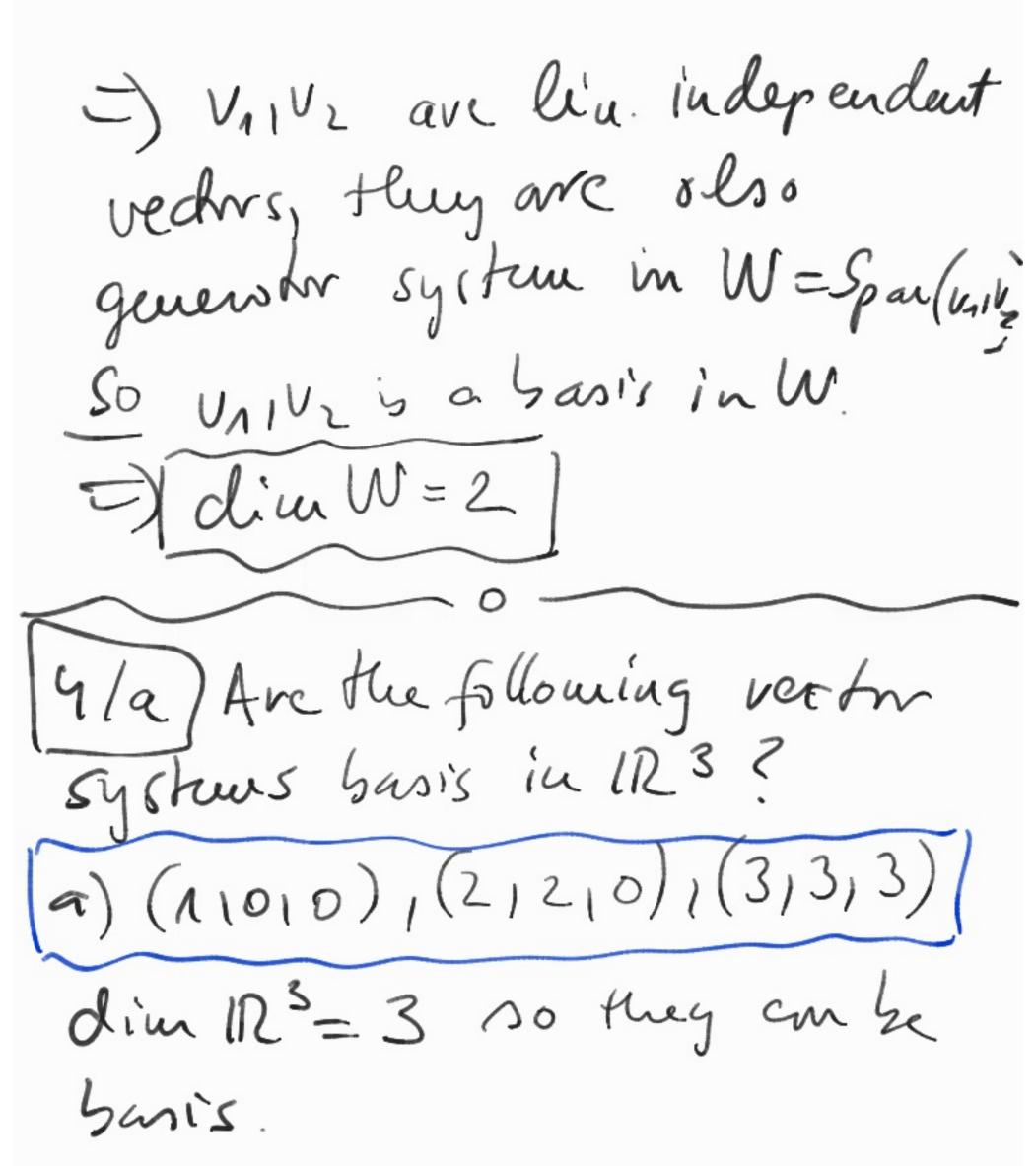
that v11/2/1/3 is liverly

dependent, we got for

exomple that:

-3 Vn+ 2 Vz + 1. V3 = 0. So the coefficients here are not 0 we am omit any of Vn1 V21 V3 no that their span does not change. We am omit for example N3 =) W= Span(V11V2).

We need a basis, dude if unive is indepldep. dv,+BV2=0 (=) $\left(\frac{1}{2}\right) + \beta \left(\frac{2}{2}\right) = \begin{pmatrix} 0\\0\\0\\0 \end{pmatrix} \left(\frac{2}{2}\right)$ X+2B=0 (C-1) 12x+2B=0 3 x + 13 = 0 / X+3B=0C 1 x=-2B=0) And oll often equotions are fullfilled so L=B=0 is the only soliting here.



$$\frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = \frac{1}{3} = \frac{1$$

$$2\left(\frac{3}{1}\right)+13\left(\frac{2}{5}\right)+12\left(\frac{1}{4}\right)=\left(\frac{3}{8}\right)$$

(F)=)[P=0]=)[B=0]=) a=-5B-47=0 (X=0) = the given 3 vectors one independent, ? they ore a banis in live. dim M=3 THE END