Question 1

1 pts

Let $f \in \mathbb{R}^n \to \mathbb{R}^m$, $f \in D(a)$. Then

$A)\quad f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$B)\quad \lim_{h\to 0} \left(\frac{f(a+h)-f(a)}{||h||} - f'(a) \right) = 0$

$\boxed{C)\quad \lim_{h\to 0} \frac{f(a+h)-f(a)-f'(a)h}{||h||} = 0}$

$D)\quad \lim_{h\to 0} \frac{f(a+h)-f(a)+f'(a)h}{||h||} = 0$

- ○ B)
- ○ D)
- ● A)
- ○ C)

Question 4

1 pts

Let $I \subseteq \mathbb{R}$ be an open interval, $f, g : I \to \mathbb{R}$, $f, g \in D$, $f', g' \in C$. Then:

A) $\int g' \cdot f = f \cdot g - \int f' \cdot g$

B) $\forall\ a, b \in I,\ a < b :\quad \int g' \cdot f = [f \cdot g]_a^b - \int f' \cdot g$

C) $\int g' \cdot f = f \cdot g + \int f' \cdot g$

D) $\int g' \cdot f' = f \cdot g - \int f \cdot g$

- ○ D)
- ○ B)
- ● A)
- ○ C)

Question 3

1 pts

Let $g : \mathbb{R}^n \to \mathbb{R}^m$, $f : \mathbb{R}^m \to \mathbb{R}^p$, $g \in D(a)$, $f \in D(g(a))$. Then the size of the derivative matrix $(f \circ g)'(a)$ is $p \times n$.

- (selected) True
- False

Question 5

1 pts

6. Using the usual notations, the polar transformation formula of double integral is

A) $\iint\limits_R f(x,y)\,dR = \iint\limits_T f(\cos\varphi, \sin\varphi) \cdot r\,dT$

B) $\iint\limits_R f(x,y)\,dR = \iint\limits_T f(r\cos\varphi, r\sin\varphi) \cdot r\,dT$

C) $\iint\limits_R f(x,y)\,dR = \iint\limits_T f(\cos\varphi, \sin\varphi)\,dT$

D) $\iint\limits_R f(x,y)\,dR = \iint\limits_T f(r\cos\varphi, r\sin\varphi) \cdot r^2 \sin\varphi\,dT$

- A)
- D)
- C)
- (selected) B)

Question 6

1 pts

Give an example for function $f : \mathbb{R} \to \mathbb{R}$, for which $f \in D(a)$, $f'(a) = 0$ hold, however f has no local extreme value at a .

- $f(x) = x^2$ at the point $a = 0$.
- $\boxed{f(x) = x^3 \text{ at the point } a = 0 .}$
- $f(x) = |x|$ at the point $a = 0$.
- (selected) $f(x) = \text{sign } x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$

  at the point $a = 0$

Question 7

1 pts

The derivative of *arcsin x* is: $-\dfrac{1}{\sqrt{1-x^2}}$

- ○ True
- ● False

Question 8

1 pts

The polar transformation in double integral transforms the normal region into a rectangular region.

- ○ True
- ● False

Question 9

1 pts

5. Let $f \in \mathbb{R} \to \mathbb{R}$ be continuous, D_f is an open interval, $F : D_f \to \mathbb{R}$, $F' = f$. Then for any $[a, b] \subseteq D_f$ holds:

$A)\ \int\limits_a^b f(x)\,dx = \int\limits_a^b F(x)\,dx$ $\qquad B)\ \int\limits_a^b f(x)\,dx = f(b) - f(a)$

$C)\ \int\limits_a^b f(x)\,dx = F(b) + F(a)$ $\qquad D)\ \int\limits_a^b f(x)\,dx = F(b) - F(a)$

- ○ B)
- ○ C)
- ● D)
- ○ A)

Question 10

1 pts

If you compute a double integral over a normal region, then the boundaries of the integration are fixed.

- (●) True
- ( ) False

Question 9

1 pts

5. Let $f \in \mathbb{R} \to \mathbb{R}$ be continuous, D_f is an open interval, $F : D_f \to \mathbb{R}$, $F' = f$. Then for any $[a, b] \subseteq D_f$ holds:

$A)\ \int\limits_a^b f(x)\,dx = \int\limits_a^b F(x)\,dx$ $\qquad B)\ \int\limits_a^b f(x)\,dx = f(b) - f(a)$

$C)\ \int\limits_a^b f(x)\,dx = F(b) + F(a)$ $\qquad D)\ \int\limits_a^b f(x)\,dx = F(b) - F(a)$

- ( ) B)
- ( ) C)
- (●) D)
- ( ) A)

Question 14

1 pts

What is given by the integral $\int_0^1 \pi^3 x^2\,dx$?

- ( ) The arc length of $y = \pi^3 x^2 \quad (x \in [0;\ 1])$
- (●) The volume of the solid by revolution $y = \pi x \quad (x \in [0;\ 1])$ about the x-axis
- ( ) The area of the surface by revolution $y = \pi^2 x^2 \quad (x \in [0;\ 1])$ about the x-axis
- ( ) The volume of the solid by revolution $y = \pi^2 x^2 \quad (x \in [0;\ 1])$ about the x-axis

Question 15

1 pts

The n-th Taylor-polynomial of a function $\mathbb{R} \to \mathbb{R}$ may be $(n+1)$-th degree in certain cases.

○ True

◉ False

Question 2

1 pts

Choose the correct theorem about the differentiability and the derivative of the composition (Chain Rule).

○ Let $f, g : \mathbb{R} \to \mathbb{R}$, $g \in D(a)$, $f \in D(g(a))$. Then $f \circ g \in D(a)$ and $(f \circ g)'(a) = f'(g(a)) \cdot g'(a)$

○ Let $f, g : \mathbb{R} \to \mathbb{R}$, $f, g \in D(a)$. Then $f \circ g \in D(a)$ and $(f \circ g)'(a) = f'(a)\, g(a) + f(a) \cdot g'(a)$

○ Let $f, g : \mathbb{R} \to \mathbb{R}$, $f, g \in D(a)$. Then $f \circ g \in D(a)$ and $(f \circ g)'(a) = f'(a) \cdot g'(a)$

○ Let $f, g : \mathbb{R} \to \mathbb{R}$, $g \in D(a)$, furthermore $g(a) \in D_f$. Then $f \circ g \in D(a)$ and $(f \circ g)'(a) = f(g(a)) \cdot g'(a)$.

Question 1

1 pts

Give an example for a function $f : \mathbb{R} \to \mathbb{R}$ which is continuous at $a = 0$ but it is not differentiable at this point.

○ $$f(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ x & \text{if} \quad x \geq 0 \end{cases}$$

○ $$f(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ x^2 & \text{if} \quad x \geq 0 \end{cases}$$

○ There is no such an example.

○ $$f(x) = \text{sign } x = \begin{cases} -1 & \text{if} \quad x < 0 \\ 0 & \text{if} \quad x = 0 \\ 1 & \text{if} \quad x > 0 \end{cases}$$

Question 3

1 pts

If $f \in \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = \dfrac{xy^2}{\ln x}$, then

$A) \quad \partial_2 f(x, y) = \dfrac{2xy}{\frac{1}{x}}$

$B) \quad \partial_2 f(x, y) = \dfrac{2xy}{\ln x}$

$C) \quad \partial_2 f(x, y) = \dfrac{2xy \ln x - xy^2 \cdot \frac{1}{x}}{\ln^2 x}$

$D) \quad \partial_2 f(x, y) = \dfrac{y^2}{\frac{1}{x}}$

○ C)

○ A)

○ D)

○ B)

6. Using the usual notations, the polar transformation formula of double integral is

A) $\iint_R f(x,y)\ dR = \iint_T f(\cos\varphi, \sin\varphi)\cdot r\ dT$

B) $\iint_R f(x,y)\ dR = \iint_T f(r\cos\varphi, r\sin\varphi)\cdot r\ dT$

C) $\iint_R f(x,y)\ dR = \iint_T f(\cos\varphi, \sin\varphi)\ dT$

D) $\iint_R f(x,y)\ dR = \iint_T f(r\cos\varphi, r\sin\varphi)\cdot r^2\sin\varphi\ dT$

- ○ A)
- ○ B)
- ○ D)
- ○ C)

Question 5

3. Let $f:(\alpha,\beta)\to\mathbb{R},\ f\in C,\ f\in D^2, a\in(\alpha,\beta)$. Then

A) $\forall x\in(a,\beta)\ \exists\xi\in(a,x):\quad f(x)=f(a)+f'(a)(x-a)+\frac{f''(\xi)}{2}(x-a)^2$

B) $\forall x\in(a,\beta)\ \exists\xi\in(a,x):\quad f(x)=f(a)+f'(a)(x-a)+f''(\xi)(x-a)^2$

C) $\forall x\in(a,\beta)\ \exists\xi\in(a,x):\quad f(x)=f(a)-f'(a)(x-a)-\frac{f''(\xi)}{2}(x-a)^2$

D) $\forall x\in(a,\beta)\ \exists\xi\in(a,x):\quad f(x)=f(a)-f'(a)(x-a)-f''(\xi)(x-a)^2$

- ○ B)
- ○ D)
- ○ A)
- ○ C)

Using the usual notations:

$$\int f\left(g\left(x\right)\right)\ dx = F\left(g\left(x\right)\right),\ where\ F=\int f$$

- ○ True
- ○ False

Define the antiderivative.

○ Let $I \subseteq \mathbb{R}$ be an open interval, $f, F : I \to \mathbb{R}$.

We say that F is an antiderivative of f if

$F \in D$, and $\forall\, x \in I : \quad F'(x) = f(x)$.

○ Let $I \subseteq \mathbb{R}$ be an open interval, $f, F : I \to \mathbb{R}$.

We say that F is an antiderivative of f if $\forall\, x \in I : \quad F(x) = f(x) + C$, where $C \in \mathbb{R}$ is a constant.

○ Let $f, F \in \mathbb{R} \to \mathbb{R}$.

We say that F is an antiderivative of f, if $D_F = D_f$, $F \in D$, and $\forall\, x \in D_F \; : \quad F'(x) = f(x)$.

○ Let $f, F : [a, b] \to \mathbb{R}$. We say that F is an antiderivative of f if

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

If $f(x, y) = g(x) \cdot h(y)$, then the double integral of f over the unit circle equals the product of the integrals of f and g.

○ True

○ False

A continuous real function defined on an open interval has infinitely many antiderivatives.

○ True

○ False

If you compute a double integral over a normal region, then the boundaries of the integration are fixed.

○ False

○ True

The n-th Taylor-polynomial of a function $\mathbb{R} \to \mathbb{R}$ may be $(n+1)$-th degree in certain cases.

○ True

○ False

Let $f \in \mathbb{R}^n \to \mathbb{R}^m$, $f \in D(a)$. Then the j-th entry in the i-th row of $f'(a)$ equals $\partial_i f_j(a)$

○ True

○ False

If $f, g \in D(a)$, then

$A)\quad (fg)'(a) = f'(a)g'(a)$

$B)\quad (fg)'(a) = f'(a)g(a) + f(a)g'(a)$

$C)\quad (fg)'(a) = f'(a)g(a) - f(a)g'(a)$

$D)\quad (fg)'(a) = f'(a)g'(a) + f(a)g(a)$

○ B)

○ C)

○ D)

○ A)

Let $g : \mathbb{R}^n \to \mathbb{R}^m$, $f : \mathbb{R}^m \to \mathbb{R}^p$, $g \in D(a)$, $f \in D(g(a))$. Then the size of the derivative matrix $(f \circ g)'(a)$ is $p \times n$.

○ True

○ False

Question 8

1 pts

The polar transformation in double integral transforms the normal region into a rectangular region.

- ○ True
- ◉ False

Let $n \in \mathbb{N}_0$, $f \in D^n(a)$. The n-th Taylor-polynomial of f centered at the

A) $\sum_{k=0}^{n} \frac{f^{(k)}(x)}{k!} \cdot (x-a)^k$

B) $\sum_{k=0}^{n} \frac{f^{(k)}(x)}{(k+1)!} \cdot (x-a)^k$

C) $\boxed{\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} \cdot (x-a)^k}$

D) $\sum_{k=0}^{n} \frac{f^{(k)}(a)}{(k+1)!} \cdot (x-a)^k$