FSO Simulation Using MATLAB

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FSO System Block Diagram

The system block diagram of the simulation procedure is illustrated in Figure 1.

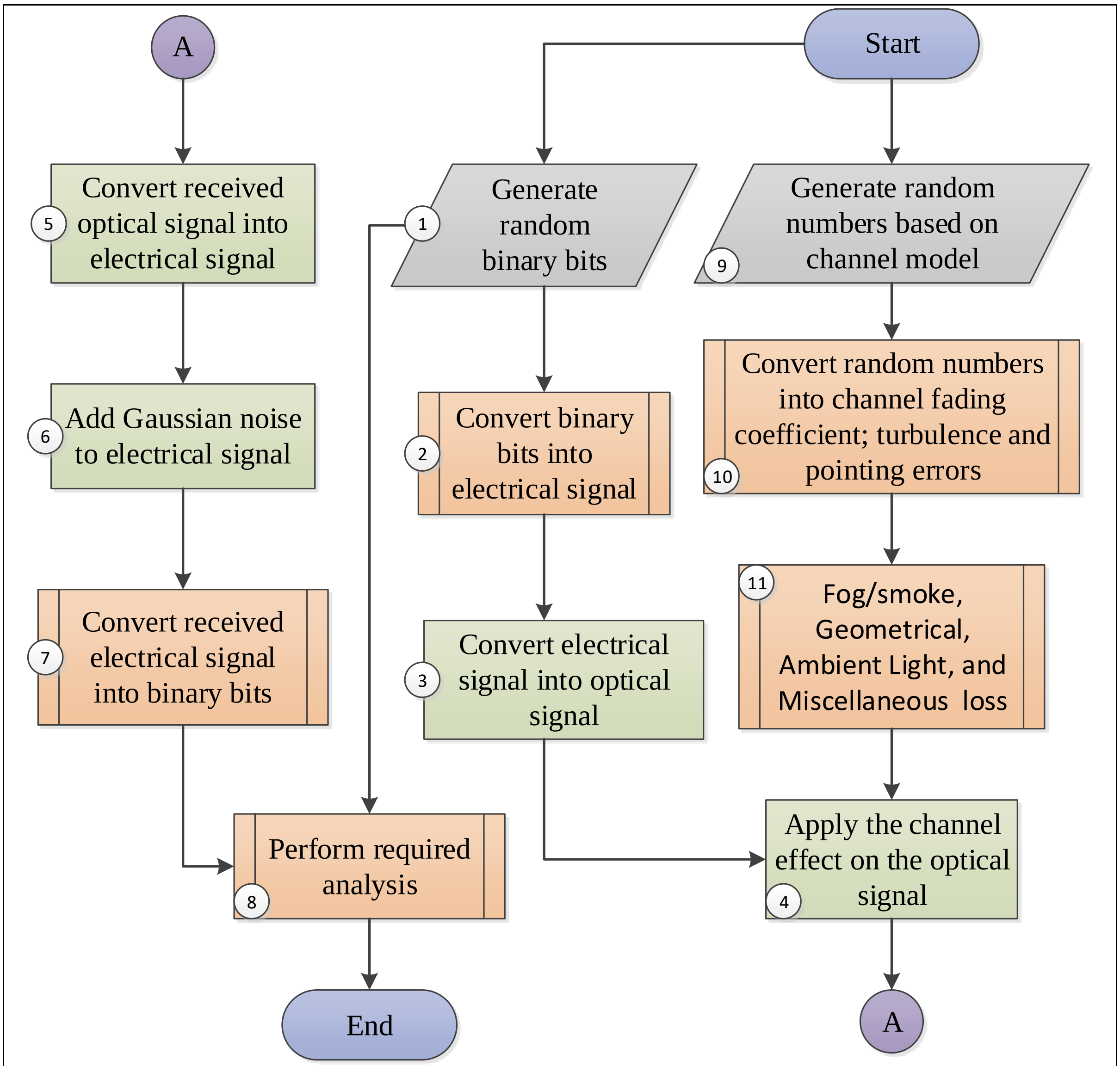

Figure 1: System block diagram of the simulation procedure

In the remaining text, I explain each block in more details.

1 – Generating random binary bits

The random binary bits or pseudorandom binary sequence (PRBS) represent any random input data for the FSO system. These bits are generated using a uniform random number generator engine. MATLAB function: *randi*.

2 – Bit to electrical signal conversion

The generated bits $PRBS$ are resampled so that each bit is represented by *NoS* sample. MATLAB function: *rectpulse*. The value of *NoS* depends on the available memory and the time window over which the train is assumed stationary. Obviously for high data rates or slow trains, larger number of samples could be generated. If the train moves with the speed of v over the distance of Δl then it takes $T_{\text{travel}} = \Delta l / v$ to finish the travelling. If the baud rate is $\text{BR} = 1/T_s$, where T_s is the symbol duration, then

$$NOS \leq \frac{T_{\text{travel}}}{T_s} = \frac{\Delta l \times \text{BR}}{v}$$

3 – Electrical to optical signal conversion

Knowing the average output optical power P_{avg} and extinction ratio ε, The following equations are used to calculate the high and low powers corresponding to bits 1 and 0 when implementing OOK modulation:

$$P_1 = \frac{2\, P_{avg}}{1 + \frac{1}{\varepsilon}}$$

$$P_0 = \frac{2\, P_{avg}}{1 + \varepsilon}$$

$$\Delta P = P_1 - P_0$$

By multiplying the generated electrical signal S_{elec} from step 2 by ΔP and adding the required optical power offset to accommodate the optical power average value, the optical signal at the transmitter side is generated $\Delta P \times S_{elec} + P_{avg}$.

4 – Applying the channel effects

Once calculating the channel coefficients h, the optical power generated from step 3 is multiplied by the coefficients. The outcome is the optical received power at the receiver side. Depending on the simulation purpose, the coefficient h may be applied to varying optical signal P_{sig} or average optical power P_{avg} part. If calculating BER is the target of the simulation, the received signal is $h \times \Delta P \times S_{elec}$. However, if simulating a real system is desired the received signal would be $h \times \left(\Delta P \times S_{elec} + P_{avg}\right)$.

5 – Optical to electrical signal conversion

To convert the received optical power to electrical signal, the responsivity of the photodiode as well as the TIA gain are used.

6 – Adding white Gaussian noise

If the SNR is given, by measuring the power of transmit signal in step 3, the required noise power P_n is calculated. To generate the additive noise, random numbers are generated based on random normal distribution $\mathcal{N}\left(0, \sqrt{P_n}\right)$. MATLAB function: *randn*.

7 – Bit extraction

A threshold level is set based on the average value of the received electrical signal. By comparing the midpoint of each received bit with the threshold, the received bit is determined to be 0 or 1. To perform adaptive thresholding, the length of averaging is changed from the whole received signal to smaller sections.

8 – Analysis

To perform the analysis, original transmit bits are compared to received bits, which leads to *BER* value. MATLAB function: *biterr*.

Another parameter extracted from the received signal is the Q-factor. Having the electrical signal level for bits 0 and 1, I can calculate Q-factor as

$$\mathrm{Q-factor} = \frac{|V_1 - V_0|}{\sigma_0 + \sigma_1}$$

where V_1 and V_0 are the mean values of received electrical signal corresponding to bits 1 and 0, respectively. While σ_1 and σ_0 are the standard deviation values of received electrical signal corresponding to bits 1 and 0, respectively.

The electrical SNR value is also calculated based on the signal power and existing noise power. From Section 3, the optical signal power is ΔP and at the receiver it results in voltage V_{sig} defined as

$$V_{\mathrm{sig}} = G \times \eta \times \Delta P$$

where G, and η are transimpendace (TIA) gain, and responsivity, respectively. Knowing the load impedance R_{Load} the electrical signal power will be

$$P_{\mathrm{sig}} = \frac{(G \times \eta \times \Delta P)^2}{R_{\mathrm{Load}}}$$

If noise equivalent power (NEP) of the receiver is given and the signal bandwidth BW is known, the detector noise power is obtained as

$$P_{\mathrm{det}} = \mathrm{NEP}\sqrt{\mathrm{BW}}$$

To take into account the background light and shot noise I have

$$i_n^2 = 2 \times q \times I \times \mathrm{BW}$$

where q is the electron charge and I is the induced current due to the noise. For background noise $I_{\mathrm{bg}} = \eta P_{\mathrm{bg}}$, where P_{bg} is the background illumination power. In case of shot noise, if the received average optical power is P_{r} then $I_{\mathrm{sn}} = \eta P_{\mathrm{r}}$. Finally the noise power at the output of TIA will be

$$P_n = \frac{2q\eta G^2 \mathrm{BW}}{R_{\mathrm{load}}}\left(P_{\mathrm{bg}} + P_{\mathrm{r}}\right)$$

And the total noise power will be

$$P_{\text{noise}} = \text{NEP}\sqrt{\text{BW}} + \frac{2q\eta G^2 \text{BW}}{R_{\text{load}}}\left(P_{\text{bg}} + P_{\text{r}}\right)$$

9 – Generating random number for channel fading

Based on the fading type, different kinds of random numbers are generated. In our simulation, turbulence and pointing errors are random phenomena. Different models are used to generate channel coefficient for each. I will briefly explain each process below:

A- Turbulence, Log-Normal model:

$$X = \mathcal{N}\left(\mu_{x,\text{turb}}, \sigma_{x,\text{turb}}\right)$$

$$h_{t-LN} = \exp(2X)$$

MATLAB function: *randn*.

B- Turbulence, Gamma-Gamma model:

$$X = \Gamma(\alpha, 1)$$

$$Y = \Gamma(\beta, 1)$$

$$h_{t-GG} = \frac{1}{\alpha\beta}XY$$

MATLAB function: *gamma*.

C- Pointing error, Log-Normal-Rician model:

$$X = \mathcal{N}\left(\mu_{x,PE}, \sigma_{j,PE}\right)$$

$$Y = \mathcal{N}\left(\mu_{y,PE}, \sigma_{j,PE}\right)$$

$$r = \sqrt{X^2 + Y^2}$$

$$h_{PE} = A_0 \exp\left(-\frac{2r^2}{w_{eq}^2}\right)$$

10 – Generating channel coefficient for turbulence and pointing errors

Once the values are generated, they are resampled to simulate the proper channel effect. If the bit rate is DR after resampling the bits in step two, the sampling frequency will be $F_s = DR \times NoS$. Typical channel temporal coherence for turbulence and pointing error is 1 – 1 msec. I pick $F_{fading} = 500$ Hz for our work. These values are used to resample fading effects to fit the whole signal. Two possible options exist to do resampling, either doing staircase resampling or resampling by using polyphaser anti-aliasing filter. MATLAB functions: *rectpulse*, and *resample*.

To confirm each model, the simulation results are compared with the available theory.

BER in clear channel:

$$\text{BER} = Q\left(\frac{\eta I_0}{\sqrt{2N_0}}\right)$$

where $Q(x) = \frac{1}{2\pi}\int_x^{\infty} \exp\left(-\frac{u^2}{2}\right) du$ is Q function. [1]

BER in pointing errors channel:

$$\text{BER} = \exp\left(-\frac{s^2}{2\sigma_s^2}\right) \times \sum_{i=1}^{k} \omega_i Q\left[\frac{\eta I_0 A_0}{\sqrt{2N_0}} \exp\left(-\frac{4\sigma_s^2}{w_{eq}^2} x_i\right)\right] I_0\left(s\sqrt{\frac{2x_i}{\sigma_s^2}}\right)$$

It is based on Gauss-Lagurerre quadrature formula (refer to Appendix A from Doc-F02-D2). $I_0(\cdot)$ is zero-th order modified Bessel function of first kind. [2] is used for PDF.

BER in turbulence channel – Log-Normal model:

$$\text{BER} = \frac{1}{\pi}\sum_{i=1}^{k} \omega_i Q\left[\frac{\eta I_0}{\sqrt{2N_0}} \exp\left(-2\sigma_x^2 + x_i\sqrt{9\sigma_x^2}\right)\right]$$

It is based on Gauss-Hermite quadrature formula [1].

BER in turbulence channel – Gamma-Gamma model:

$$\text{BER} = \frac{2^{\alpha+\beta-3}}{\sqrt{\pi^3}\Gamma(\alpha)\Gamma(\beta)} G_{5,2}^{2,4}\left[\left(\frac{2}{\alpha\beta}\right)^2 \times 2 \times \frac{\eta I_0}{\sqrt{2N_0}} \left| \begin{matrix} \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-\beta}{2}, \frac{2-\beta}{2}, 1 \\ 0, \frac{1}{2} \end{matrix} \right.\right]$$

where $G_{5,2}^{2,4}[\cdot]$ is Meijer's G function [3].

BER in pointing errors and turbulence channel – Log-Normal model:

$$\text{BER} = \frac{2^{\gamma^2-1}\Gamma\left(\frac{\gamma^2}{2}+\frac{1}{2}\right)\exp\left(\frac{s^2}{\sigma_s^2} + 2\sigma_x^2\gamma^2 + 2\sigma_x^2\gamma^4\right)}{\sqrt{\pi}(A_0)^{\gamma^2}} \times \left(\frac{\eta I_0}{\sqrt{2N_0}}\right)^{-\frac{\gamma^2}{2}}$$

This BER formula is an asymptotic .approximation for large SNR values [2].

BER in pointing errors and turbulence channel – GammaGamma model:

$$\text{BER} = \frac{2^{\beta-1}\Gamma\left(\frac{\beta}{2}+\frac{1}{2}\right)\exp\left(-\frac{s^2}{2\sigma_s^2} + \frac{-s^2\frac{\gamma^2}{\sigma_s^2}}{2\beta-2\gamma^2}\right)\left(\frac{\alpha\beta}{A_0}\right)^{\beta}}{\Gamma(\alpha)\Gamma(\beta)\sin[(\alpha-\beta)\pi]\,\Gamma(-(\alpha-\beta)+1)|\gamma^2-\beta|\beta} \times \Gamma\left(\frac{\beta+1}{2}\right)\sqrt{\pi}\gamma^2\left(\frac{\eta I_0}{\sqrt{2N_0}}\right)^{-\frac{\beta}{2}}$$

This BER formula is an asymptotic .approximation for large SNR values [2].

In case, the receiver has an aperture with diameter of d_s then for Gamma-Gamma model the parameters will be:

$$k = \frac{2\pi}{\lambda}$$

$$d = \left(k{d_s}^2/4L\right)^{0.5}$$

$$\sigma_{\ln X}^2 = \frac{0.49\sigma_R^2}{\left(1 + 0.65d^2 + 1.11\sigma_R^{12/5}\right)^{7/6}}$$

$$\sigma_{\ln Y}^2 = \frac{0.51\sigma_R^2\left(1 + 0.69\sigma_R^{12/5}\right)^{-5/6}}{1 + 0.90d^2 + 0.62d^2\sigma_R^{12/5}}$$

For Log-Normal model, it is given as:

$$\frac{\sigma_I^2(d_s)}{\sigma_I^2(0)} \approx [1 + 1.062d^2]^{-7/6}$$

11 – Fog/smoke loss

Based on the visibility Vis and by using Kim model, q parameter is defined [4]:

$$q = \begin{cases} 1.6, \mathrm{Vis} > 50 \\ 1.3, \quad 6 < \mathrm{Vis} < 50 \\ 1.6 \times \mathrm{Vis} + 0.34, \quad 1 < \mathrm{Vis} < 6 \\ \mathrm{Vis} - 0.5, \quad 0.5 < \mathrm{Vis} < 0.1 \\ 0, \quad \mathrm{Vis} < 0.1 \end{cases}$$

Having the laser wavelength λ (nm), the parameter β_λ in $1/km$ is defined as:

$$\beta_\lambda = -\frac{ln\, 0.02}{\mathrm{Vis}}\left(\frac{\lambda}{550}\right)^{-q}$$

And finally by using Beer's lambert law, the fog/smoke loss will be:

$$h_{\mathrm{FS}} = \exp\left(-\beta_\lambda L\right)$$

where L is the link distance in km.

11 – Geometrical loss

Geometrical or propagation loss for the elliptical beam is estimated based on the followings:

- receiver aperture to source L distance $\gg \lambda$
- receiver aperture area $A_{\mathrm{Rx-apr}} \ll$ beam size at receiver aperture plane $W(Z)$

The geometry of the beam and receiver is considered the same as Figure 2.

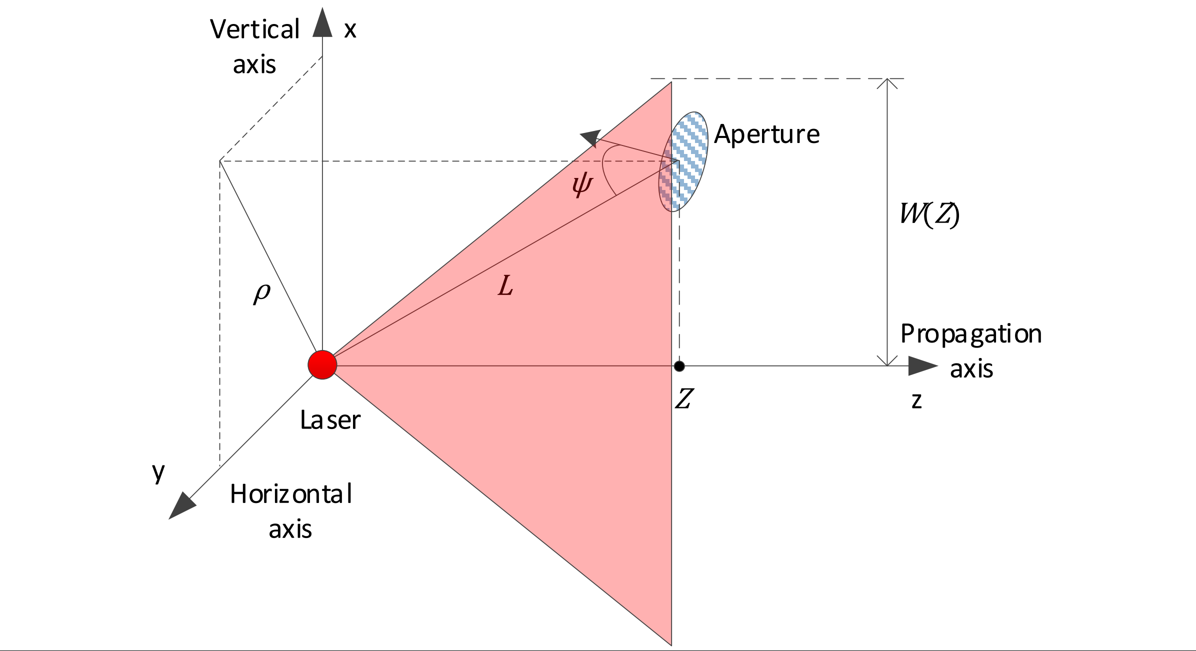

Figure 2: Laser beam and receiver aperture geometry

The geometrical loss of the aperture while the normalised intensity $I_n(\rho; Z)$ is given will be:

$$h_{\mathrm{GL}} = I_n(\rho; Z) \times T_{\mathrm{r}} \times A_{\mathrm{Rx-apr}} \times \cos\psi\,, \qquad \psi \leq \frac{1}{2}\mathrm{AFOV}$$

where AFOV, amd T_{r} are full-angle angular field-of-view, and transmittance of the receiver aperture. The angle ψ is defined as the angle between the vector connecting laser to the aperture and the vector normal to the aperture. I consider two radiation mechanism for the source, uniform and Gaussian. If the laser is a multimode-propagation source, it can be approximated with a uniform pattern. Otherwise a Gaussian propagation is considered.

A- Uniform radiation: when the intensity of the laser beam is uniform across the wavefront.

$$I_n(\rho; Z) = \frac{1}{\pi \times w_h(Z) \times w_v(Z)}$$

where P_{Tx} is the total power of the beam; $w_h(Z)$ and $w_v(Z)$ are beam radius along horizontal and vertical directions, respectively.

$$w_h(Z) = w_{0h} + Z\theta_{0h}$$
$$w_v(Z) = w_{0v} + Z\theta_{0v}$$

w_{0h} and w_{0v} are beam radii at transmitter side along horizontal and vertical directions, respectively. θ_{0h} and θ_{0v} are beam divergence ($1/e$ criterion) at transmitter side along horizontal and vertical directions, respectively.

B- Gaussian radiation: the intensity profile is Gaussian.

$$I_n(\rho; Z) = \frac{2}{\pi \times w_h(Z) \times w_v(Z)} \exp\left(-\frac{2x^2}{w_v(Z)^2} - \frac{2y^2}{w_h(Z)^2}\right)$$

$$w_h(Z) = w_{0h}\sqrt{1 + \varepsilon_h\left(\frac{\lambda Z}{\pi {w_{0h}}^2}\right)^2}$$

$$w_v(Z) = w_{0v}\sqrt{1 + \varepsilon_v\left(\frac{\lambda Z}{\pi {w_{0v}}^2}\right)^2}$$

$$\varepsilon_h = 1 + 2\frac{{w_{0h}}^2}{\rho_0(Z)^2}$$

$$\varepsilon_v = 1 + 2\frac{{w_{0v}}^2}{\rho_0(Z)^2}$$

$$\rho_0(Z) = (0.55C_n^2k^2Z)^{-\frac{3}{5}}$$

12 – References

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4- M. Ijaz, Z. Ghassemlooy, J. Pesek, O. Fiser, H. Le Minh and E. Bentley, "Modeling of Fog and Smoke Attenuation in Free Space Optical Communications Link Under Controlled Laboratory Conditions," in *Journal of Lightwave Technology*, vol. 31, no. 11, pp. 1720-1726, June1, 2013. doi: 10.1109/JLT.2013.2257683.