4(c)

What are the key ideas about why you use the Jacobian method for IK?

The FK of systems is generally non-linear and using Jacobian the function that maps joint angles to the end effector can be linearized in short proximity of a small change in joint angles.

Also, for higher DOFs and in general IK, calculating the function that maps the end effector to joint space is extremely difficult and due to its form factor and singularity issues may be impossible. Using the Jacobian and mainly Pseudo Inverse of Jacobian helps in performing IK and eliminating the singularity and non-inversability of the Jacobian.

4(d)

Explain how the Jacobian method for IK works and what are the assumptions it makes.

The Jacobian is essentially a linear approximation of the function that maps the joint angles to the end effector position. As this function is non-linear, a Jacobian can be generated using partial derivatives of the system. That is each column of the Jacobian is a partial derivative of the EE position with respect to a single joint angle to linearize and estimate the change in EE based on the small or instantaneous change in that specific angle.

Changing every angle separately by a very small delta a new position can be calculated using the FK of the system that is known. The ratio of the change in EE position with respect to the small delta in each angle generates the Jacobian matrix in each column. The Jacobian assumes that the function will be linear in that neighboring region.

This Jacobian will be the linear estimation of a function that takes dθ to de and using its inverse (pseudo inverse or even transpose) the mapping of de to dθ and IK will be possible.

4(e)

Write the Jacobian IK formula (in symbolic form) and point out, and label and explain each important part (i.e. the how in part 4.d)

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}, e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} \rightarrow \begin{cases} e = f(\theta) & is\ the\ FK \\ \theta = f^{-1}(e) & is\ the\ IK \end{cases}$$

$$J = \frac{de}{d\theta} = \begin{bmatrix} {de_1}/{d\theta_1} & {de_1}/{d\theta_2} & \cdots & {de_1}/{d\theta_n} \\ {de_2}/{d\theta_1} & {de_2}/{d\theta_2} & \cdots & {de_2}/{d\theta_n} \\ \vdots & \vdots & \cdots & \vdots \\ {de_m}/{d\theta_1} & {de_m}/{d\theta_2} & \cdots & {de_m}/{d\theta_n} \end{bmatrix} \approx \begin{bmatrix} {\Delta e_1}/{\Delta\theta_1} & {\Delta e_1}/{\Delta\theta_2} & \cdots & {\Delta e_1}/{\Delta\theta_n} \\ {\Delta e_2}/{\Delta\theta_1} & {\Delta e_2}/{\Delta\theta_2} & \cdots & {\Delta e_2}/{\Delta\theta_n} \\ \vdots & \vdots & \cdots & \vdots \\ {\Delta e_m}/{\Delta\theta_1} & {\Delta e_m}/{\Delta\theta_2} & \cdots & {\Delta e_m}/{\Delta\theta_n} \end{bmatrix}$$

$$each\ column\ of\ the\ Jacobian\ is\ the\ change\ in\ EE\ position\ based\ on\ a\ small\ change\ in\ \theta_i$$