1 **(a)** Explain what is meant by the *natural frequency of vibration* of a system. .[1]

(b) A block of metal is fixed to one end of a vertical spring. The other end of the spring is attached to an oscillator, as shown in Fig. 4.1. The amplitude of oscillation of the oscillator is constant.

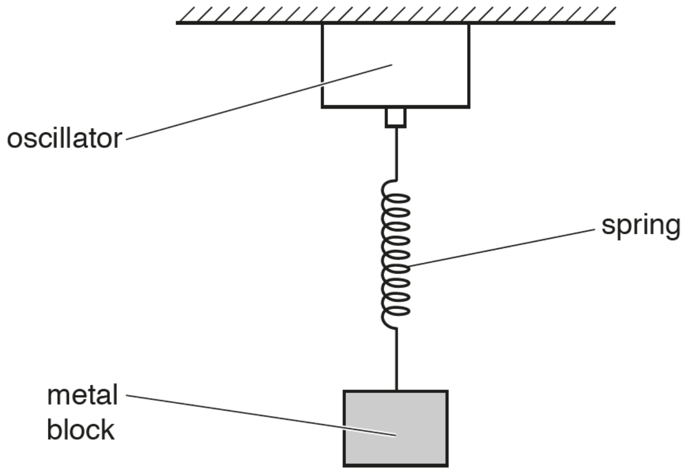

Fig. 4.1

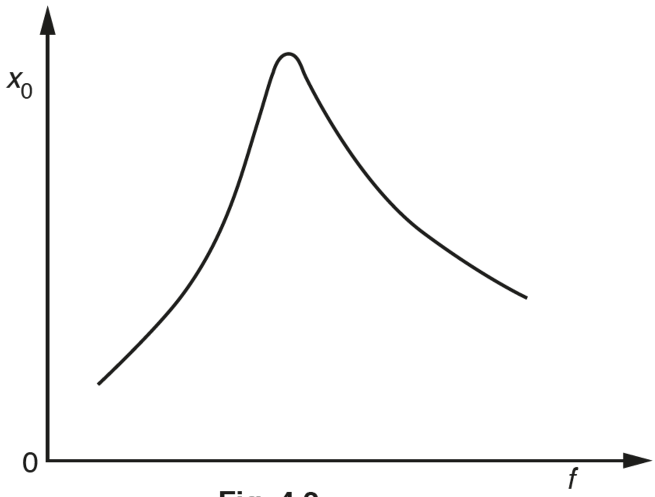

Fig. 4.2

The variation of the amplitude x_0 of the oscillations of the block with frequency f of the oscillations is shown in Fig. 4.2.

(i) Name the effect shown in Fig. 4.2. .[1]

(ii) State and explain whether the block is undergoing damped oscillations. .[2]

(c) State **one** example in which the effect shown in Fig. 4.2 is useful. [1]

2 A cylindrical tube, sealed at one end, has cross-sectional area A and contains some sand. The total mass of the tube and the sand is M.
The tube floats upright in a liquid of density ρ, as illustrated in Fig. 3.1.

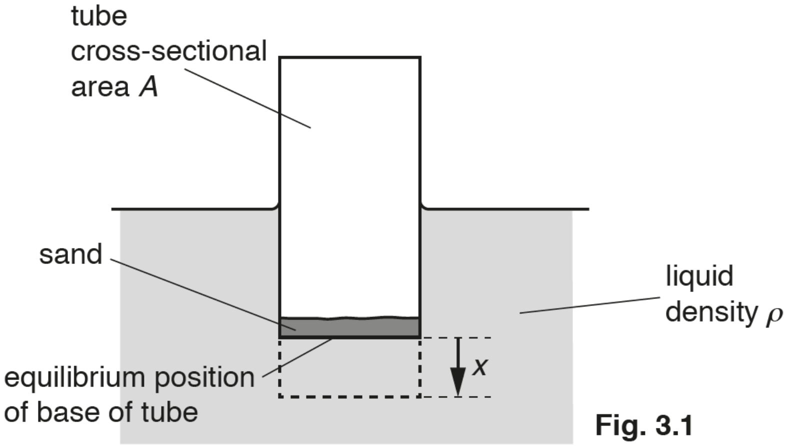

Fig. 3.1

The tube is pushed a short distance into the liquid and then released.

(a) **(i)** State the two forces that act on the tube immediately after its release. [1]

(ii) State and explain the direction of the resultant force acting on the tube immediately after its release. [2]

(b) The acceleration a of the tube is given by the expression $a = -\left(\frac{A\rho g}{M}\right)x$

where x is the vertical displacement of the tube from its equilibrium position.

Use the expression to explain why the tube undergoes simple harmonic oscillations in the liquid. [2]

(c) For a tube having cross-sectional area A of 4.5 cm^2 and a total mass M of 0.17 kg, the period of oscillation of the tube is 1.3 s.

(i) Determine the angular frequency ω of the oscillations. rad s^{-1} [2]

(ii) Use your answer in **(i)** and the expression in **(b)** to determine the density ρ of the liquid in which the tube is floating. kg m^{-3} [3]

3 A vertical spring supports a mass, as shown in Fig. 4.1.

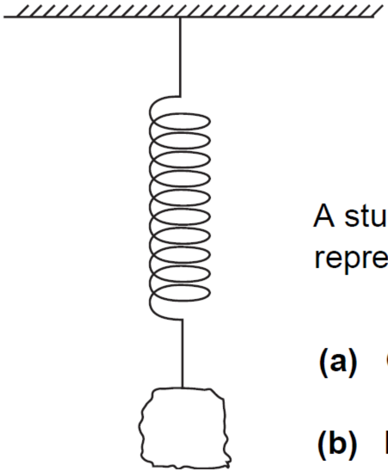

Fig. 4.1

The mass is displaced vertically then released. The variation with time t of the displacement y from its mean position is shown in Fig. 4.2.

A student claims that the motion of the mass may be represented by the equation $y = y_0 \sin\omega t$.

(a) Give two reasons why the use of this equation is inappropriate. [2]

(b) Determine the angular frequency ω of the oscillations. rad s^{-1} [2]

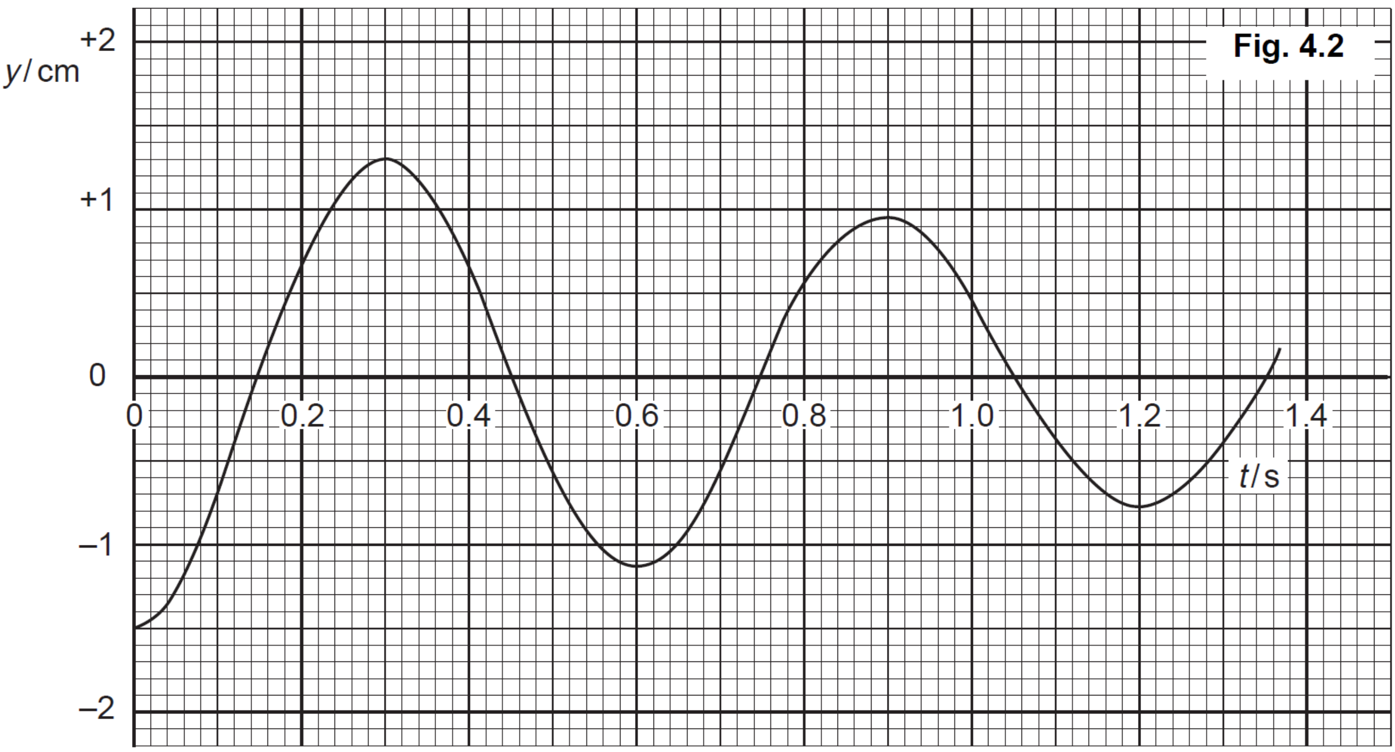

Fig. 4.2

(c) The mass is a lump of plasticine. The plasticine is now flattened so that its surface area is increased. The mass of the lump remains constant and the large surface area is horizontal.
The plasticine is displaced downwards by 1.5 cm and then released.
On Fig. 4.2, sketch a graph to show the subsequent oscillations of the plasticine. [3]

4 A spring is hung from a fixed point. A mass of 130 g is hung from the free end of the spring, as shown in Fig. 3.1. The mass is pulled downwards from its equilibrium position through a small distance *d* and is released.

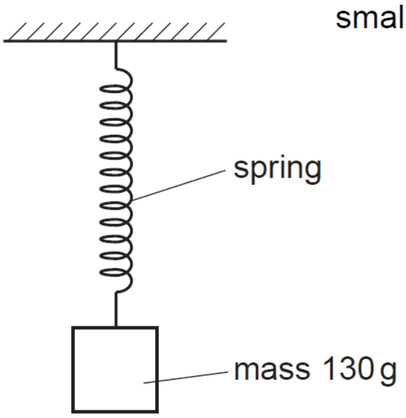

Fig. 3.1

The mass undergoes simple harmonic motion.

Fig. 3.2 shows the variation with displacement *x* from the equilibrium position of the kinetic energy of the mass.

(a) Use Fig. 3.2 to

(i) determine the distance *d* through which the mass was displaced initially, cm [1]

(ii) show that the frequency of oscillation of the mass is approximately 4.0 Hz. [6]

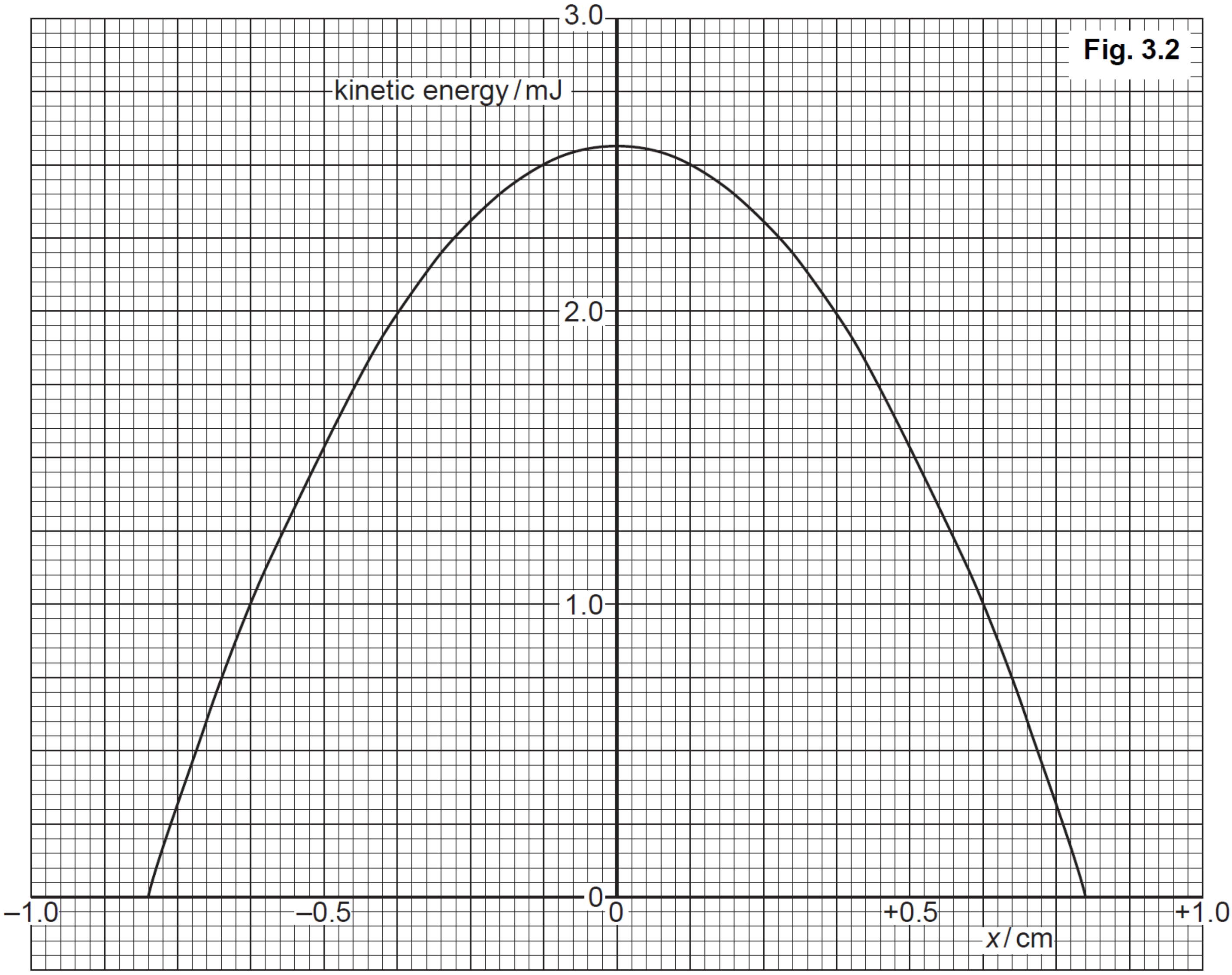

Fig. 3.2

(b) **(i)** On Fig. 3.2, draw a line to represent the total energy of the oscillating mass. [1]

(ii) After many oscillations, damping reduces the total energy of the mass to 1.0 mJ. For the oscillations with reduced energy,

1. state the frequency,

2. using the graph, or otherwise, state the amplitude. cm [2]

5 The needle of a sewing machine is made to oscillate vertically through a total distance of 22 mm, as shown in Fig. 3.1.

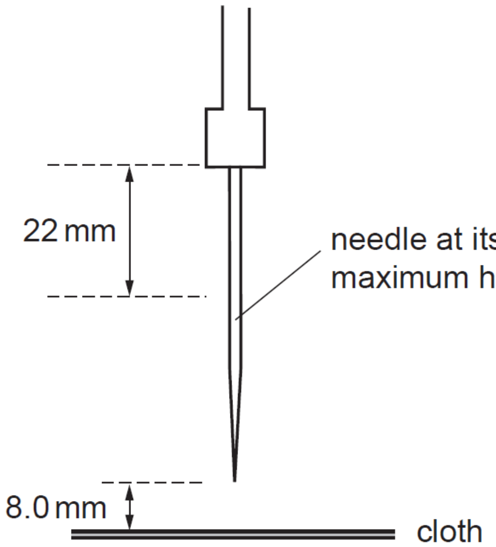

Fig. 3.1

The oscillations are simple harmonic with a frequency of 4.5 Hz.
The cloth that is being sewn is positioned 8.0 mm below the point of the needle when the needle is at its maximum height.

(a) State what is meant by *simple harmonic motion*. . [2]

(b) The displacement y of the point of the needle may be represented by the equation

$$y = a \cos \omega t.$$

(i) Suggest the position of the point of the needle at time $t = 0$. [1]

(ii) Determine the values of **1.** a, mm [1] **2.** ω. rad s^{-1} [2]

(c) Calculate, for the point of the needle,

(i) its maximum speed, . m s^{-1} [2]

(ii) its speed as it moves downwards through the cloth. . m s^{-1} [3]

6 A cylinder and piston, used in a car engine, are illustrated in Fig. 3.1.

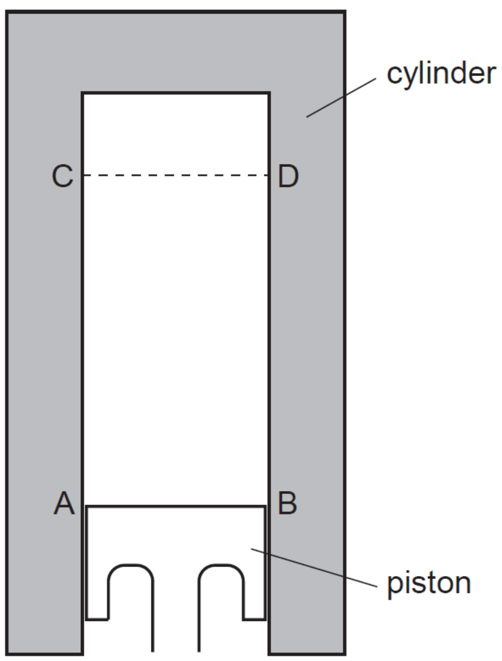

Fig. 3.1

The vertical motion of the piston in the cylinder is assumed to be simple harmonic.

The top surface of the piston is at AB when it is at its lowest position; it is at CD when at its highest position, as marked in Fig. 3.1.

(a) The displacement d of the piston may be represented by the equation $d = -4.0 \cos(220t)$ where d is measured in centimetres.

(i) State the distance between the lowest position AB and the highest position CD of the top surface of the piston. ..... cm [1]

(ii) Determine the number of oscillations made per second by the piston. [2]

(iii) On Fig. 3.1, draw a line to represent the top surface of the piston in the position where the speed of the piston is maximum. [1]

(iv) Calculate the maximum speed of the piston. cm s^{-1} [2]

(b) The engine of a car has several cylinders. Three of these cylinders are shown in Fig. 3.2.

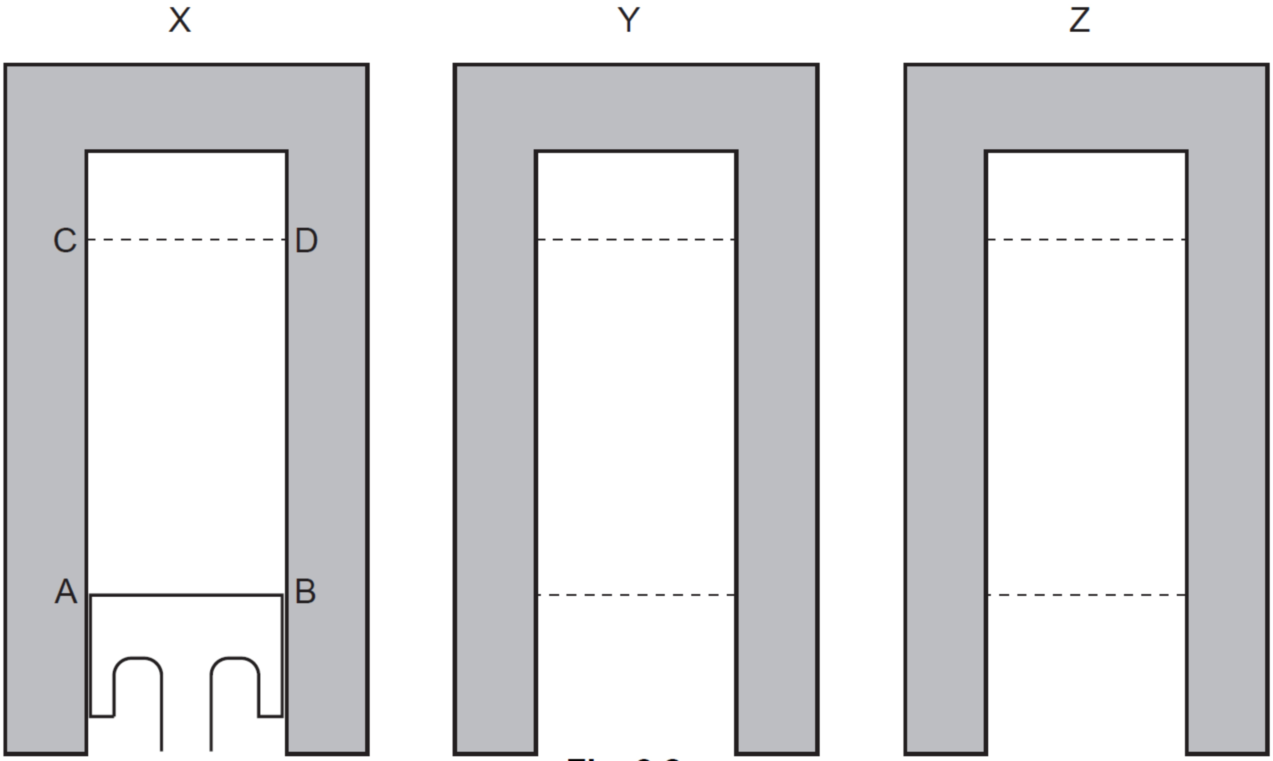

Fig. 3.2

X is the same cylinder and piston as in Fig. 3.1.
Y and Z are two further cylinders, with the lowest and the highest positions of the top surface of each piston indicated.
The pistons in the cylinders each have the same frequency of oscillation, but they are not in phase.
At a particular instant in time, the position of the top of the piston in cylinder X is as shown.

(i) In cylinder Y, the oscillations of the piston lead those of the piston in cylinder X by a phase angle of 120° ($\frac{2}{3}\pi$ rad).
Complete the diagram of cylinder Y, for this instant, by drawing

1. a line to show the top surface of the piston, [1]

2. an arrow to show the direction of movement of the piston. [1]

(ii) In cylinder Z, the oscillations of the piston lead those of the piston in cylinder X by a phase angle of 240° ($\frac{4}{3}\pi$ rad).
Complete the diagram of cylinder Z, for this instant, by drawing

1. a line to show the top surface of the piston, [1]

2. an arrow to show the direction of movement of the piston. [1]

(iii) For the piston in cylinder Y, calculate its speed for this instant. cm s^{-1} [2]