Contents

1 Basic

1.1 Default Code

```cpp
//Challenge: Accepted
#pragma GCC optimize("Ofast")
#include <bits/stdc++.h>
using namespace std;
#ifdef zisk
void debug(){cout << endl;}
template<class T, class ... U> void debug(T a, U ... b){
    cout << a << " ", debug(b...);}
template<class T> void pary(T l, T r) {
  while (l != r) cout << *l << " ", l++;
  cout << endl;
}
#else
#define debug(...) 0
#define pary(...) 0
#endif
#define ll long long
#define maxn 50005
#define pii pair<int, int>
#define ff first
#define ss second
#define io ios_base::sync_with_stdio(0);cin.tie(0);
#define iter(v) v.begin(),v.end()
#define SZ(v) (int)v.size()
#define pb emplace_back
int main() {
  io

}
```

1.2 .vimrc

```
sy on
se nu rnu bs=2 sw=4 ts=4 hls ls=2 si acd bo=all mouse=a
map <F9> :w<bar>!g++ "%" -o %:r -std=c++17 -Wall -Wshadow -
    Dzisk -g -fsanitize=undefined,address<CR>
map <F8> :!./%:r<CR>
map <C-a> <ESC>ggVG
```

2 Data Structure

2.1 Heavy-Light Decomposition

```cpp
struct Heavy_light_Decomposition { // 1-base
  int n, up[maxn], dep[maxn], to[maxn], siz[maxn], pa[maxn
      ];
  int C, ti[maxn], ord[maxn], wdown[maxn], edge[maxn], et =
       0;
  vector<pii> G[maxn];
  void init(int _n) {
    n = _n, C = 0, et = 1;
    for (int i = 1;i <= n;i++)
      G[i].clear(), to[i] = 0;
  }
  void add_edge(int a, int b, int w) {
    G[a].push_back(pii(b, et)), G[b].push_back(pii(a, et));
    edge[et++] = w;
  }
  void dfs(int u, int f, int d) {
    siz[u] = 1, pa[u] = f, dep[u] = d;
    for (auto &v: G[u])
      if (v.ff != f) {
        dfs(v.ff, u, d+1), siz[u] += siz[v];
        if (siz[to[u]] < siz[v]) to[u] = v;
      }
  }
```

```
  void cut(int u, int link) {
    ti[u] = C;
    ord[C++] = u, up[u] = link;
    if (!to[u]) return;
    cut(to[u], link);
    for (auto v:G[u]) {
      if (v.ff != pa[u] && v.ff != to[u]) cut(v.ff, v.ff);
    }
  }
  void build() { dfs(1, 1, 1), cut(1, 1); }
  int query(int a, int b) {
    int ta = up[a], tb = up[b], re = 0;
    while (ta != tb)
      if (dep[ta] < dep[tb])
        /*query*/, tb = up[b = pa[tb]];
      else /*query*/, ta = up[a = pa[ta]];
    if (a == b) return re;
    if (ti[a] > ti[b]) swap(a, b);
    /*query*/
    return re;
  }
};
```

2.2 Li-Chao Tree

```
struct LiChao_min {
  struct line {
    ll m, c;
    line(ll _m = 0, ll _c = 0) {
      m = _m;
      c = _c;
    }
    ll eval(ll x) { return m * x + c; }
  };
  struct node {
    node *l, *r;
    line f;
    node(line v) {
      f = v;
      l = r = NULL;
    }
  };
  typedef node *pnode;
  pnode root;
  int sz;
#define mid ((l + r) >> 1)
  void insert(line &v, int l, int r, pnode &nd) {
    if (!nd) {
      nd = new node(v);
      return;
    }
    ll trl = nd->f.eval(l), trr = nd->f.eval(r);
    ll vl = v.eval(l), vr = v.eval(r);
    if (trl <= vl && trr <= vr) return;
    if (trl > vl && trr > vr) {
      nd->f = v;
      return;
    }
    if (trl > vl) swap(nd->f, v);
    if (nd->f.eval(mid) < v.eval(mid))
      insert(v, mid + 1, r, nd->r);
    else swap(nd->f, v), insert(v, l, mid, nd->l);
  }
  ll query(int x, int l, int r, pnode &nd) {
    if (!nd) return inf;
    if (l == r) return nd->f.eval(x);
    if (mid >= x)
      return min(
          nd->f.eval(x), query(x, l, mid, nd->l));
    return min(
        nd->f.eval(x), query(x, mid + 1, r, nd->r));
  }
  /* -sz <= query_x <= sz */
  void init(int _sz) {
    sz = _sz + 1;
    root = NUll;
  }
  void add_line(ll m, ll c) {
    line v(m, c);
    insert(v, -sz, sz, root);
  }
  ll query(ll x) { return query(x, -sz, sz, root); }
};
```

2.3 Link Cut Tree

```
struct Splay { // xor-sum
  static Splay nil;
  Splay *ch[2], *f;
  int val, sum, rev, size;
  Splay(int _val = 0)
    : val(_val), sum(_val), rev(0), size(1) {
      f = ch[0] = ch[1] = &nil;
    }
  bool isr() {
    return f->ch[0] != this && f->ch[1] != this;
  }
  int dir() { return f->ch[0] == this ? 0 : 1; }
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void push() {
    if (!rev) return;
    swap(ch[0], ch[1]);
    if (ch[0] != &nil) ch[0]->rev ^= 1;
    if (ch[1] != &nil) ch[1]->rev ^= 1;
    rev = 0;
  }
  void pull() {
    // take care of the nil!
    size = ch[0]->size + ch[1]->size + 1;
    sum = ch[0]->sum ^ ch[1]->sum ^ val;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(), x->pull();
}
void splay(Splay *x) {
  vector<Splay *> splayVec;
  for (Splay *q = x;; q = q->f) {
    splayVec.pb(q);
    if (q->isr()) break;
  }
  reverse(ALL(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir() == x->f->dir())
      rotate(x->f), rotate(x);
    else rotate(x), rotate(x);
  }
}
Splay *access(Splay *x) {
  Splay *q = nil;
  for (; x != nil; x = x->f)
```

```
    splay(x), x->setCh(q, 1), q = x;
  return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
  root_path(x), x->rev ^= 1;
  x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
  chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
  root_path(x), chroot(y);
  x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
  split(x, y);
  if (y->size != 5) return;
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}
Splay *get_root(Splay *x) {
  for (root_path(x); x->ch[0] != nil; x = x->ch[0])
    x->push();
  splay(x);
  return x;
}
bool conn(Splay *x, Splay *y) {
  return get_root(x) == get_root(y);
}
Splay *lca(Splay *x, Splay *y) {
  access(x), root_path(y);
  if (y->f == nil) return y;
  return y->f;
}
void change(Splay *x, int val) {
  splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
  split(x, y);
  return y->sum;
}
```

2.4 Treap

```
struct node {
 int data, sz;
 node *l, *r;
 node(int k) : data(k), sz(1), l(0), r(0) {}
 void up() {
  sz = 1;
  if (l) sz += l->sz;
  if (r) sz += r->sz;
 }
 void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
 if (!a || !b) return a ? a : b;
 if (rand() % (sz(a) + sz(b)) < sz(a))
  return a->down(), a->r = merge(a->r, b), a->up(),
         a;
 return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
 if (!o) return a = b = 0, void();
 o->down();
 if (o->data <= k)
  a = o, split(o->r, a->r, b, k), a->up();
 else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
 if (sz(o) <= k) return a = o, b = 0, void();
 o->down();
 if (sz(o->l) + 1 <= k)
  a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
 else b = o, split2(o->l, a, b->l, k);
 o->up();
}
node *kth(node *o, int k) {
 if (k <= sz(o->l)) return kth(o->l, k);
 if (k == sz(o->l) + 1) return o;
 return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
 if (!o) return 0;
 if (o->data < key)
  return sz(o->l) + 1 + Rank(o->r, key);
 else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
 if (!o) return 0;
 if (o->data == k) {
  node *t = o;
  o->down(), o = merge(o->l, o->r);
  delete t;
  return 1;
 }
 node *&t = k < o->data ? o->l : o->r;
 return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
 node *a, *b;
 split(o, a, b, k),
  o = merge(a, merge(new node(k), b));
}
void interval(node *&o, int l, int r) {
 node *a, *b, *c;
 split2(o, a, b, l - 1), split2(b, b, c, r);
 // operate
 o = merge(a, merge(b, c));
}
```

3 Flow Matching

3.1 Bounded Flow

```
struct BoundedFlow { // 0-base
  struct edge {
    int to, cap, flow, rev;
  };
  vector<edge> G[N];
  int n, s, t, dis[N], cur[N], cnt[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n + 2; ++i)
      G[i].clear(), cnt[i] = 0;
  }
  void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
  }
  void add_edge(int u, int v, int cap) {
    G[u].pb(edge{v, cap, 0, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
  }
  int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
      edge &e = G[u][i];
      if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
        int df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
          e.flow += df, G[e.to][e.rev].flow -= df;
```

```
        return df;
      }
    }
  }
  dis[u] = -1;
  return 0;
}
bool bfs() {
  fill_n(dis, n + 3, -1);
  queue<int> q;
  q.push(s), dis[s] = 0;
  while (!q.empty()) {
    int u = q.front();
    q.pop();
    for (edge &e : G[u])
      if (!~dis[e.to] && e.flow != e.cap)
        q.push(e.to), dis[e.to] = dis[u] + 1;
  }
  return dis[t] != -1;
}
int maxflow(int _s, int _t) {
  s = _s, t = _t;
  int flow = 0, df;
  while (bfs()) {
    fill_n(cur, n + 3, 0);
    while ((df = dfs(s, INF))) flow += df;
  }
  return flow;
}
bool solve() {
  int sum = 0;
  for (int i = 0; i < n; ++i)
    if (cnt[i] > 0)
      add_edge(n + 1, i, cnt[i]), sum += cnt[i];
    else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
  if (sum != maxflow(n + 1, n + 2)) sum = -1;
  for (int i = 0; i < n; ++i)
    if (cnt[i] > 0)
      G[n + 1].pop_back(), G[i].pop_back();
    else if (cnt[i] < 0)
      G[i].pop_back(), G[n + 2].pop_back();
  return sum != -1;
}
int solve(int _s, int _t) {
  add_edge(_t, _s, INF);
  if (!solve()) return -1; // invalid flow
  int x = G[_t].back().flow;
  return G[_t].pop_back(), G[_s].pop_back(), x;
}
};
```

3.2 Dinic

```
struct MaxFlow { // 1-base
  struct edge {
    int to, cap, flow, rev;
  };
  vector<edge> g[maxn];
  int s, t, dis[maxn], ind[maxn], n;

  void init(int _n) {
    n = _n + 2;
    s = _n + 1, t = _n + 2;
    for (int i = 0; i <= n; ++i) g[i].clear();
  }
  void reset() {
    for (int i = 0; i <= n; ++i)
      for (auto &j : g[i]) j.flow = 0;
  }
  void add_edge(int u, int v, int cap) {
    g[u].pb(edge{v, cap, 0, (int)g[v].size()});
    g[v].pb(edge{u, 0, 0, (int)g[u].size() - 1});
    //change g[v] to cap for undirected graphs
  }
  bool bfs() {
    fill(dis, dis+n+1, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
      int cur = q.front(); q.pop();
      for (auto &e : g[cur]) {
        if (dis[e.to] == -1 && e.flow != e.cap) {
          q.push(e.to);
          dis[e.to] = dis[cur] + 1;
        }
      }
    }
    return dis[t] != -1;
  }
  int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = ind[u]; i < (int)g[u].size(); ++i) {
      edge &e = g[u][i];
      if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
        int df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
          e.flow += df;
          g[e.to][e.rev].flow -= df;
          return df;
        }
      }
    }
    dis[u] = -1;
    return 0;
  }
  int maxflow() {
    int flow = 0, df;
    while (bfs()) {
      fill(ind, ind+n+1, 0);
      while ((df = dfs(s, inf))) flow += df;
    }
    return flow;
  }
}flow;
```

3.3 Gomory Hu

```
MaxFlow Dinic;
int g[MAXN];
void GomoryHu(int n) { // 0-base
 fill_n(g, n, 0);
 for (int i = 1; i < n; ++i) {
  Dinic.reset();
  add_edge(i, g[i], Dinic.maxflow(i, g[i]));
  for (int j = i + 1; j <= n; ++j)
   if (g[j] == g[i] && ~Dinic.dis[j])
    g[j] = i;
 }
}
```

3.4 Hungarian Algorithm

```
int c[maxn][maxn]; //hungarian algorithm in O(n^3)
//1 base
int lx[maxn], ly[maxn], mx[maxn], my[maxn];
bool vx[maxn], vy[maxn];
int slack[maxn];
int tot;
bool dfs(int n, bool ch) {
  if (vx[n]) return false;
  vx[n] = 1;
  for (int v = 1;v <= tot;v++) {
    slack[v] = min(slack[v], lx[n] + ly[v] - c[n][v]);
    if (lx[n] + ly[v] - c[n][v] > 0) continue;
    vy[v] = 1;
```

```
    if (!my[v] || dfs(my[v], ch)) {
      if (ch) mx[n] = v, my[v] = n;
      return true;
    }
  }
  return false;
}
int main() {
  for (int i = 1;i <= n;i++) {
    for (int j = 1;j <= n;j++) vx[j] = vy[j] = 0;
    for (int j = 1;j <= n;j++) slack[j] = 1<<30;
    if (dfs(i, 1)) continue;
    bool aug = 0;
    while (!aug) {
      for (int j = 1;j <= n;j++) {
        if (!vy[j] && slack[j] == 0) {
          vy[j] = 1;
          if (dfs(my[j], 0)) {
            aug = 1;
            break;
          }
        }
      }
      if (aug) break;
      int delta = 1<<30;
      for (int j = 1;j <= n;j++) {
        if (!vy[j]) delta = min(delta, slack[j]);
      }
      for (int j = 1;j <= n;j++) {
        if (vx[j]) lx[j] -= delta;
        if (vy[j]) ly[j] += delta;
        else {
          slack[j] -= delta;
          if (slack[j] == 0 && !my[j]) aug = 1;
        }
      }
    }
    for (int j = 1;j <= n;j++) vx[j] = vy[j] = 0;
    dfs(i, 1);
  }
}
```

3.5 ISAP Algorithm

```
struct Maxflow { //to be modified
  static const int MAXV = 20010;
  static const int INF = 1000000;
  struct Edge {
    int v, c, r;
    Edge(int _v, int _c, int _r)
      : v(_v), c(_c), r(_r) {}
  };
  int s, t;
  vector<Edge> G[MAXV * 2];
  int iter[MAXV * 2], d[MAXV * 2], gap[MAXV * 2], tot;
  void init(int x) {
    tot = x + 2;
    s = x + 1, t = x + 2;
    for (int i = 0; i <= tot; i++) {
      G[i].clear();
      iter[i] = d[i] = gap[i] = 0;
    }
  }
  void addEdge(int u, int v, int c) {
    G[u].push_back(Edge(v, c, SZ(G[v])));
    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
  }
  int dfs(int p, int flow) {
    if (p == t) return flow;
    for (int &i = iter[p]; i < SZ(G[p]); i++) {
      Edge &e = G[p][i];
      if (e.c > 0 && d[p] == d[e.v] + 1) {
        int f = dfs(e.v, min(flow, e.c));
        if (f) {
          e.c -= f;
          G[e.v][e.r].c += f;
          return f;
        }
      }
    }
    if ((--gap[d[p]]) == 0) d[s] = tot;
    else {
      d[p]++;
      iter[p] = 0;
      ++gap[d[p]];
    }
    return 0;
  }
  int solve() {
    int res = 0;
    gap[0] = tot;
    for (res = 0; d[s] < tot; res += dfs(s, INF))
      ;
    return res;
  }
} flow;
```

3.6 KM Algorithm

```
int n, m; //1-base, max matching
int mx[maxn], my[maxn];
bool adj[maxn][maxn], vis[maxn];
bool dfs(int n) {
  if (vis[n]) return false;
  vis[n] = 1;
  for (int v = 1;v <= n;v++) {
    if (!adj[n][v]) continue;
    if (!my[v] || (my[v] && dfs(my[v]))) {
      mx[n] = v, my[v] = n;
      return true;
    }
  }
  return false;
}
//min vertex cover: take unmatched vertex in L and find
    alternating tree,
//ans is not reached in L + reached in R
```

3.7 Max Simple Graph Matching

```
struct GenMatch { // 1-base
  int V, pr[N];
  bool el[N][N], inq[N], inp[N], inb[N];
  int st, ed, nb, bk[N], djs[N], ans;
  void init(int _V) {
    V = _V;
    for (int i = 0; i <= V; ++i) {
      for (int j = 0; j <= V; ++j) el[i][j] = 0;
      pr[i] = bk[i] = djs[i] = 0;
      inq[i] = inp[i] = inb[i] = 0;
    }
  }
  void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
  }
  int lca(int u, int v) {
    fill_n(inp, V + 1, 0);
    while (1)
      if (u = djs[u], inp[u] = true, u == st) break;
      else u = bk[pr[u]];
    while (1)
      if (v = djs[v], inp[v]) return v;
      else v = bk[pr[v]];
    return v;
  }
```

```
  void upd(int u) {
    for (int v; djs[u] != nb;) {
      v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
      u = bk[v];
      if (djs[u] != nb) bk[u] = v;
    }
  }
  void blo(int u, int v, queue<int> &qe) {
    nb = lca(u, v), fill_n(inb, V + 1, 0);
    upd(u), upd(v);
    if (djs[u] != nb) bk[u] = v;
    if (djs[v] != nb) bk[v] = u;
    for (int tu = 1; tu <= V; ++tu)
      if (inb[djs[tu]])
        if (djs[tu] = nb, !inq[tu])
          qe.push(tu), inq[tu] = 1;
  }
  void flow() {
    fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
    iota(djs + 1, djs + V + 1, 1);
    queue<int> qe;
    qe.push(st), inq[st] = 1, ed = 0;
    while (!qe.empty()) {
      int u = qe.front();
      qe.pop();
      for (int v = 1; v <= V; ++v)
        if (el[u][v] && djs[u] != djs[v] &&
            pr[u] != v) {
          if ((v == st) ||
              (pr[v] > 0 && bk[pr[v]] > 0)) {
            blo(u, v, qe);
          } else if (!bk[v]) {
            if (bk[v] = u, pr[v] > 0) {
              if (!inq[pr[v]]) qe.push(pr[v]);
            } else {
              return ed = v, void();
            }
          }
        }
    }
  }
  void aug() {
    for (int u = ed, v, w; u > 0;)
      v = bk[u], w = pr[v], pr[v] = u, pr[u] = v,
       u = w;
  }
  int solve() {
    fill_n(pr, V + 1, 0), ans = 0;
    for (int u = 1; u <= V; ++u)
      if (!pr[u])
        if (st = u, flow(), ed > 0) aug(), ++ans;
    return ans;
  }
};
```

3.8 MCMF

```
struct MCMF { // 0-base
  struct edge {
    ll from, to, cap, flow, cost, rev;
  } * past[maxn];
  vector <edge> G[maxn];
  bitset <maxn> inq;
  ll dis[maxn], up[maxn], s, t, mx, n;
  bool BellmanFord(ll &flow, ll &cost) {
    fill(dis, dis + n, inf);
    queue<ll> q;
    q.push(s), inq.reset(), inq[s] = 1;
    up[s] = mx - flow, past[s] = 0, dis[s] = 0;
    while (!q.empty()) {
      ll u = q.front();
      q.pop(), inq[u] = 0;
      if (!up[u]) continue;
      for (auto &e : G[u])
        if (e.flow != e.cap &&
            dis[e.to] > dis[u] + e.cost) {
          dis[e.to] = dis[u] + e.cost, past[e.to] = &e;
          up[e.to] = min(up[u], e.cap - e.flow);
          if (!inq[e.to]) inq[e.to] = 1, q.push(e.to);
        }
    }
    if (dis[t] == inf) return 0;
    flow += up[t], cost += up[t] * dis[t];
    for (ll i = t; past[i]; i = past[i]->from) {
      auto &e = *past[i];
      e.flow += up[t], G[e.to][e.rev].flow -= up[t];
    }
    return 1;
  }
  ll MinCostMaxFlow(ll _s, ll _t, ll &cost) {
    s = _s, t = _t, cost = 0;
    ll flow = 0;
    while (BellmanFord(flow, cost));
    return flow;
  }
  void init(ll _n, ll _mx) {
    n = _n, mx = _mx;
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].pb(edge{a, b, cap, 0, cost, G[b].size()});
    G[b].pb(edge{b, a, 0, 0, -cost, G[a].size() - 1});
  }
};
```

3.9 Min Cost Circulation

```
//to be modified
struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
  memset(mark, false, sizeof(mark));
  memset(dist, 0, sizeof(dist));
  int upd = -1;
  for (int i = 0; i <= n; ++i) {
    for (int j = 0; j < n; ++j) {
      int idx = 0;
      for (auto &e : g[j]) {
        if (e.cap > 0 && dist[e.to] > dist[j] + e.cost) {
          dist[e.to] = dist[j] + e.cost;
          pv[e.to] = j, ed[e.to] = idx;
          if (i == n) {
            upd = j;
            while (!mark[upd]) mark[upd] = true, upd = pv[
                upd];
            return upd;
          }
        }
        idx++;
      }
    }
  }
  return -1;
}
int Solve(int n) {
  int rt = -1, ans = 0;
  while ((rt = NegativeCycle(n)) >= 0) {
    memset(mark, false, sizeof(mark));
    vector<pair<int, int>> cyc;
    while (!mark[rt]) {
      cyc.emplace_back(pv[rt], ed[rt]);
      mark[rt] = true;
      rt = pv[rt];
    }
```

```
    reverse(cyc.begin(), cyc.end());
    int cap = kInf;
    for (auto &i : cyc) {
      auto &e = g[i.first][i.second];
      cap = min(cap, e.cap);
    }
    for (auto &i : cyc) {
      auto &e = g[i.first][i.second];
      e.cap -= cap;
      g[e.to][e.rev].cap += cap;
      ans += e.cost * cap;
    }
  }
  return ans;
}
```

3.10 SW Mincut

```
// stoer wagner algorithm: global min cut
const int maxn = 505;
struct SW { // O(V^3) 0-based
  int n, vis[maxn], del[maxn];
  int edge[maxn][maxn], wei[maxn];
  void init(int _n) {
    n = _n;
    fill(del, del+n, 0);
    for (int i = 0;i < n;i++) fill(edge[i], edge[i] + n, 0)
        ;
  }
  void addEdge(int u, int v, int w) {
    edge[u][v] += w, edge[v][u] += w;
  }
  void search(int &s, int &t) {
    fill(vis, vis+n, 0);
    fill(wei, wei+n, 0);
    s = t = -1;
    while (1) {
      int ma = -1, cur = 0;
      for (int i = 0; i < n; ++i)
        if (!del[i] && !vis[i] && ma < wei[i])
          cur = i, ma = wei[i];
      if (mx == -1) break;
      vis[cur] = 1, s = t, t = cur;
      for (int i = 0; i < n; ++i)
        if (!vis[i] && !del[i]) wei[i] += edge[cur][i];
    }
  }
  int solve() {
    int ret = INF;
    for (int i = 0, x=0, y=0; i < n-1; ++i) {
      search(x, y), ret = min(res, wei[y]), del[y] = 1;
      for (int j = 0; j < n; ++j)
        edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return ret;
  }
};
```

4 Geometry

4.1 Geometry Template

```
using NumType = ll;
// using NumType = ld;
using Pt = pair<NumType, NumType>;
using Line = pair<Pt, Pt>;
#define X first
#define Y second
// ld eps = 1e-7;

Pt operator+(Pt a, Pt b)
  { return {a.X + b.X, a.Y + b.Y}; }
Pt operator-(Pt a, Pt b)
  { return {a.X - b.X, a.Y - b.Y}; }
Pt operator*(NumType i, Pt v)
  { return {i * v.X, i * v.Y}; }
Pt operator/(Pt v, NumType i)
  { return {v.X / i, v.Y / i}; }
NumType dot(Pt a, Pt b)
  { return a.X * b.X + a.Y * b.Y; }
NumType cross(Pt a, Pt b)
  { return a.X * b.Y - a.Y * b.X; }
NumType abs2(Pt v)
  { return v.X * v.X + v.Y * v.Y; };
int sgn(NumType v)
  { return v > 0 ? 1 : (v < 0 ? -1 : 0); }
// int sgn(NumType v){ return v > eps ? 1 : ( v < -eps ? -1
     : 0); }
int ori(Pt a, Pt b, Pt c)
  { return sgn(cross(b - a, c - a)); }
bool collinearity(Pt a, Pt b, Pt c)
  { return ori(a, b, c) == 0; }
bool btw(Pt p, Pt a, Pt b)
  { return collinearity(p, a, b) && sgn(dot(a - p, b - p))
    <= 0; }

bool intersect(Line a, Line b){
  Pt p1, p2, p3, p4;
  tie(p1, p2) = a; tie(p3, p4) = b;
  if(btw(p1, p3, p4) || btw(p2, p3, p4) || btw(p3, p1, p2)
    || btw(p4, p1, p2))
    return true;
  return ori(p1, p2, p3) * ori(p1, p2, p4) < 0 &&
      ori(p3, p4, p1) * ori(p3, p4, p2) < 0;
}
```

4.2 Convex Hull

```
vector<int> getConvexHull(vector<Pt>& pts){
  vector<int> id(SZ(pts));
  iota(iter(id), 0);
  sort(iter(id), [&](int x, int y){ return pts[x] < pts[y];
     });
  vector<int> hull;
  for(int tt = 0; tt < 2; tt++){
    int sz = SZ(hull);
    for(int j : id){
      Pt p = pts[j];
      while(SZ(hull) - sz >= 2 &&
          cross(pts[hull.back()] - pts[hull[SZ(hull) - 2]],
              p - pts[hull[SZ(hull) - 2]]) <= 0)
        hull.pop_back();
      hull.pb(j);
    }
    hull.pop_back();
    reverse(iter(id));
  }
  return hull;
}
```

4.3 Minimum Enclosing Circle

```
using NumType = ld;
pair<Pt, ld> MinimumEnclosingCircle(vector<Pt> &pts){
  random_shuffle(iter(pts));
  Pt c = pts[0];
  ld r = 0;
  for(int i = 1; i < SZ(pts); i++){
    if(abs(pts[i] - c) <= r) continue;
    c = pts[i]; r = 0;
    for(int j = 0; j < i; j++){
      if(abs(pts[j] - c) <= r) continue;
      c = (pts[i] + pts[j]) / 2;
```

```
      r = abs(pts[i] - c);
      for(int k = 0; k < j; k++){
        if(abs(pts[k] - c) > r)
          c = circumcenter(pts[i], pts[j], pts[k]);
      }
    }
  }
  return {c, r};
}
```

4.4 Minkowski Sum

```
void reorder_poly(vector<Pt>& pnts){
  int mn = 0;
  for(int i = 1; i < (int)pnts.size(); i++){
    if(pnts[i].Y < pnts[mn].Y || (pnts[i].Y == pnts[mn].Y
    && pnts[i].X < pnts[mn].X))
      mn = i;
  }
  rotate(pnts.begin(), pnts.begin() + mn, pnts.end());
}

vector<Pt> minkowski(vector<Pt> P, vector<Pt> Q){
  reorder_poly(P);
  reorder_poly(Q);
  int psz = P.size();
  int qsz = Q.size();
  P.eb(P[0]);
  P.eb(P[1]);
  Q.eb(Q[0]);
  Q.eb(Q[1]);
  vector<Pt> ans;
  int i = 0, j = 0;
  while(i < psz || j < qsz){
    ans.eb(P[i] + Q[j]);
    int t = sgn(cross(P[i + 1] - P[i], Q[j + 1] - Q[j]));
    if(t >= 0) i++;
    if(t <= 0) j++;
  }
  return ans;
}
```

4.5 Half Plane Intersection

```
// copy from 8BQube
bool isin( Line l0, Line l1, Line l2 ) {
 // Check inter(l1, l2) in l0
 pdd p = intersect(l1.X, l1.Y, l2.X, l2.Y);
 return sign(cross(l0.Y - l0.X,p - l0.X)) > 0;
}
/* Having solution, check intersect(ret[0], ret[1])
 * in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> lines) {
 vector<double> ata(SZ(lines)), ord(SZ(lines));
 for(int i = 0; i < SZ(lines); ++i) {
  ord[i] = i;
  pdd d = lines[i].Y - lines[i].X;
  ata[i] = atan2(d.Y, d.X);
 }
 sort(ALL(ord), [&](int i, int j) {
  if (fabs(ata[i] - ata[j]) >= eps)
   return ata[i] < ata[j];
  return ori(lines[i].X, lines[i].Y, lines[j].Y) < 0;
 });
 vector<Line> fin(1, lines[ord[0]]);
 for (int i = 1; i < SZ(lines); ++i)
  if (fabs(ata[ord[i]] - ata[ord[i - 1]]) > eps)
   fin.pb(lines[ord[i]]);
 deque<Line> dq;
 for (int i = 0; i < SZ(fin); ++i) {
  while (SZ(dq) >= 2 && !isin(fin[i], dq[SZ(dq) - 2], dq.
    back()))
   dq.pop_back();
  while (SZ(dq) >= 2 && !isin(fin[i], dq[0], dq[1]))
   dq.pop_front();
  dq.pb(fin[i]);
 }
 while (SZ(dq) >= 3 && !isin(dq[0], dq[SZ(dq) - 2], dq.back
    ()))
  dq.pop_back();
 while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
  dq.pop_front();
 return vector<Line>(ALL(dq));
```

5 Graph

5.1 Block Cut Tree

```
struct BlockCutTree{
  vector<vector<int>> tree; // 1-based
  vector<int> node;
  vector<int> type; // 0:square, 1:circle

  bool iscut(int v){
    return type[node[v]] == 1;
  }

  vector<int> getbcc(int v){
    if(!iscut(v)) return {node[v]};
    vector<int> ans;
    for(int i : tree[node[v]])
      ans.pb(i);
    return ans;
  }

  void build(int n, vector<vector<int>>& g){
    tree.resize(2 * n + 1);
    type.resize(2 * n + 1);
    node.resize(n + 1, -1);
    vector<int> in(n + 1);
    vector<int> low(n + 1);
    stack<int> st;

    int ts = 1;
    int bcc = 1;
    auto addv = [&](int id, int v){
      if(node[v] == -1){
        node[v] = id;
        return;
      }
      if(type[node[v]] == 0){
        int o = node[v];
        node[v] = bcc++;
        type[node[v]] = 1;
        tree[o].pb(node[v]);
        tree[node[v]].pb(o);
      }
      tree[id].pb(node[v]);
      tree[node[v]].pb(id);
    };
    function<void(int, int)> dfs = [&](int now, int p){
      in[now] = low[now] = ts++;
      st.push(now);
      int child = 0;
      for(int i : g[now]){
        if(i == p) continue;
        if(in[i]){
          low[now] = min(low[now], in[i]);
          continue;
        }
        child++;
        dfs(i, now);
```

```
        low[now] = min(low[now], low[i]);

        if(low[i] >= in[now]){
          int nowid = bcc++;
          while(true){
            int x = st.top();
            st.pop();
            addv(nowid, x);
            if(x == i) break;
          }
          addv(nowid, now);
        }
      }
      if(child == 0 && now == p) addv(bcc++, now);
    };
    dfs(1, 1);
  }
};
```

5.2 2-SAT

```
struct SAT{ // 1-based
  int n;
  vector<vector<int>> g, rg;
  bool ok = true;
  vector<bool> ans;

  void init(int _n){
    n = _n;
    g.resize(2 * n + 1);
    rg.resize(2 * n + 1);
    ans.resize(n + 1);
  }
  int neg(int v){
    return v <= n ? v + n : v - n;
  }
  void addEdge(int u, int v){
    g[u].eb(v);
    rg[v].eb(u);
  }
  void addClause(int a, int b){
    addEdge(a, b);
    addEdge(neg(b), neg(a));
  }
  void build(){
    vector<bool> vst(n + 1);
    vector<int> tmp, scc(n + 1, -1);
    int cnt = 1;
    function<void(int)> dfs = [&](int now){
      vst[now] = true;
      for(int i : rg[now]){
        if(vst[i]) continue;
        dfs(i);
      }
      tmp.pb(now);
    };
    for(int i = 1; i <= 2 * n; i++){
      if(!vst[i]) dfs(i);
    }
    reverse(iter(tmp));
    function<void(int, int)> dfs2 = [&](int now, int id){
      scc[now] = id;
      for(int i : g[now]){
        if(scc[i] != -1) continue;
        dfs2(i, id);
      }
    };
    for(int i : tmp){
      if(scc[i] == -1) dfs2(i, cnt++);
    }
    for(int i = 1; i <= n; i++){
      if(scc[i] == scc[neg(i)]){
        ok = false;
        return;
      }
      if(scc[i] < scc[neg(i)]) ans[i] = true;
      else ans[i] = false;
    }
  }

};
```

5.3 Dominator Tree

```
// copy from 8BQube
struct dominator_tree { // 1-base
 vector<int> G[N], rG[N];
 int n, pa[N], dfn[N], id[N], Time;
 int semi[N], idom[N], best[N];
 vector<int> tree[N]; // dominator_tree
 void init(int _n) {
  n = _n;
  for (int i = 1; i <= n; ++i)
   G[i].clear(), rG[i].clear();
 }
 void add_edge(int u, int v) {
  G[u].pb(v), rG[v].pb(u);
 }
 void dfs(int u) {
  id[dfn[u] = ++Time] = u;
  for (auto v : G[u])
   if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
 }
 int find(int y, int x) {
  if (y <= x) return y;
  int tmp = find(pa[y], x);
  if (semi[best[y]] > semi[best[pa[y]]])
   best[y] = best[pa[y]];
  return pa[y] = tmp;
 }
 void tarjan(int root) {
  Time = 0;
  for (int i = 1; i <= n; ++i) {
   dfn[i] = idom[i] = 0;
   tree[i].clear();
   best[i] = semi[i] = i;
  }
  dfs(root);
  for (int i = Time; i > 1; --i) {
   int u = id[i];
   for (auto v : rG[u])
    if (v = dfn[v]) {
     find(v, i);
     semi[i] = min(semi[i], semi[best[v]]);
    }
   tree[semi[i]].pb(i);
   for (auto v : tree[pa[i]]) {
    find(v, pa[i]);
    idom[v] =
     semi[best[v]] == pa[i] ? pa[i] : best[v];
   }
   tree[pa[i]].clear();
  }
  for (int i = 2; i <= Time; ++i) {
   if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
   tree[id[idom[i]]].pb(id[i]);
  }
 }
};
```

5.4 Virtual Tree

```
// copy from 8BQube
vector<int> vG[N];
int top, st[N];
```

```
void insert(int u) {
 if (top == -1) return st[++top] = u, void();
 int p = LCA(st[top], u);
 if (p == st[top]) return st[++top] = u, void();
 while (top >= 1 && dep[st[top - 1]] >= dep[p])
  vG[st[top - 1]].pb(st[top]), --top;
 if (st[top] != p)
  vG[p].pb(st[top]), --top, st[++top] = p;
 st[++top] = u;
}

void reset(int u) {
 for (int i : vG[u]) reset(i);
 vG[u].clear();
}

void solve(vector<int> &v) {
 top = -1;
 sort(ALL(v),
  [&](int a, int b) { return dfn[a] < dfn[b]; });
 for (int i : v) insert(i);
 while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
 // do something
 reset(v[0]);
}
```

6 Math

6.1 Extended Euclidean Algorithm

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void extgcd(ll x,ll y,ll &g,ll &a,ll &b) {
 if (y == 0) g=x,a=1,b=0;
 else extgcd(y,x%y,g,b,a),b-=(x/y)*a;
}
```

6.2 Floor & Ceil

```
int floor_div(int a,int b){
 return a/b-(a%b&&a<0^b<0);
}
int ceil_div(int a,int b){
 return a/b+(a%b&&a<0^b>0);
}
```

6.3 Legendre

```
// the Jacobi symbol is a generalization of the Legendre
    symbol,
// such that the bottom doesn't need to be prime.
// (n|p) -> same as legendre
// (n|ab) = (n|a)(n|b)
// work with long long
int Jacobi(int a, int m) {
 int s = 1;
 for (; m > 1; ) {
  a %= m;
  if (a == 0) return 0;
  const int r = __builtin_ctz(a);
  if ((r & 1) && ((m + 2) & 4)) s = -s;
  a >>= r;
  if (a & m & 2) s = -s;
  swap(a, m);
 }
 return s;
}

// 0: a == 0
// -1: a isn't a quad res of p
// else: return X with X^2 % p == a
// doesn't work with long long
int QuadraticResidue(int a, int p) {
 if (p == 2) return a & 1;
 const int jc = Jacobi(a, p);
 if (jc == 0) return 0;
 if (jc == -1) return -1;
 int b, d;
 for (; ; ) {
  b = rand() % p;
  d = (1LL * b * b + p - a) % p;
  if (Jacobi(d, p) == -1) break;
 }
 int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
 for (int e = (1LL + p) >> 1; e; e >>= 1) {
  if (e & 1) {
   tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) %
    p;
   g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
   g0 = tmp;
  }
  tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p
    ;
  f1 = (2LL * f0 * f1) % p;
  f0 = tmp;
 }
 return g0;
}
```

6.4 Simplex

```
#pragma once

typedef double T; // long double, Rational, double + mod<P
    >...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s]))
     s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[
          i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    }

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool simplex(int phase) {
    int x = m + phase - 1;
```

```
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
              < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }

  T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

7 Polynomial

7.1 FFT

```
template<int MAXN>
struct FFT {
  using val_t = complex<double>;
  const double PI = acos(-1);
  val_t w[MAXN];
  FFT() {
    for (int i = 0; i < MAXN; ++i) {
      double arg = 2 * PI * i / MAXN;
      w[i] = val_t(cos(arg), sin(arg));
    }
  }
  void bitrev(val_t *a, int n); // see NTT
  void trans(val_t *a, int n, bool inv = false); // see NTT
      ;
};
```

7.2 NTT

```
//to be modified
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, ll P, ll RT> //MAXN must be 2^k
struct NTT {
 ll w[MAXN];
 ll mpow(ll a, ll n);
 ll minv(ll a) { return mpow(a, P - 2); }
 NTT() {
  ll dw = mpow(RT, (P - 1) / MAXN);
  w[0] = 1;
  for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
 }
 void bitrev(ll *a, int n) {
  int i = 0;
  for (int j = 1; j < n - 1; ++j) {
   for (int k = n >> 1; (i ^= k) < k; k >>= 1);
   if (j < i) swap(a[i], a[j]);
  }
 }
 void operator()(ll *a, int n, bool inv = false) { //0 <= a
    [i] < P
  bitrev(a, n);
  for (int L = 2; L <= n; L <<= 1) {
   int dx = MAXN / L, dl = L >> 1;
   for (int i = 0; i < n; i += L) {
    for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
     ll tmp = a[j + dl] * w[x] % P;
     if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl] += P;
     if ((a[j] += tmp) >= P) a[j] -= P;
    }
   }
  }
  if (inv) {
   reverse(a + 1, a + n);
   ll invn = minv(n);
   for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
  }
 }
};
```

8 String

8.1 KMP Algorithm

```
void kmp(string s){
  int siz = s.size();
  vector<int> f(siz, 0);
  f[0] = 0;
  for (int i = 1;i < siz;i++) {
    f[i] = f[i-1];
    bool zero = 0;
    while (s[f[i]] != s[i]) {
      if (f[i] == 0) {
        zero = 1;
        break;
      }
      f[i] = f[f[i]-1];
    }
    if (!zero) f[i]++;
  }
}
```

8.2 Manacher Algorithm

```
vector<int> manacher(string s) {
  int n = s.size();
  vector<int> v(n);
  int pnt = -1, len = 1;
  for (int i = 0;i < n;i++) {
    int cor = 2 * pnt - i;
    if (cor >= 0) v[i] = min(v[cor], cor - pnt + len);
    while (i+v[i] < n && i-v[i] >= 0 && s[i+v[i]] == s[i-v[
        i]]) v[i]++;
    if (i + v[i] >= pnt + len) pnt = i, len = v[i];
  }
  for (int i = 0;i < n;i++) v[i] = 2 * v[i] - 1;
  return v;
}
```

8.3 MCP

```
string mcp(string s) { //Duval algorithm for Lyndon
    factorization
  s += s;
  int n = s.size(), i = 0, ans = 0;
  while (i < n/2) {
    ans = i;
    int j = i+1, k=i;
    while (j < n && s[k] <= s[j]) {
      if (s[k] < s[j]) k = i;
      else k++;
      j++;
    }
    while (i <= k) i += j - k;
  }
  return s.substr(ans, n/2);
}
```

8.4 Suffix Array

```
struct SuffixArray { //tested
  vector<int> sa, lcp, rank; //lcp[i] is lcp of sa[i] and
      sa[i-1]
  SuffixArray(string& s, int lim=256) { // or basic_string<
      int>
    int n = s.size() + 1, k = 0, a, b;
    vector<int> x(n, 0), y(n), ws(max(n, lim));
    rank.resize(n);
    for (int i = 0;i < n-1;i++) x[i] = (int)s[i];
    sa = lcp = y, iota(sa.begin(), sa.end(), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim =
        p) {
      p = j, iota(y.begin(), y.end(), n - j);
      for (int i = 0;i < n;i++) if (sa[i] >= j) y[p++] = sa
          [i] - j;
      for (int &i : ws) i = 0;
      for (int i = 0;i < n;i++) ws[x[i]]++;
      for (int i = 1;i < lim;i++) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      for(int i = 1;i < n;i++) a = sa[i - 1], b = sa[i], x[
          b] =
        (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p
            ++;
    }
    for (int i = 1;i < n;i++) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
      for (k && k--, j = sa[rank[i] - 1];
          s[i + k] == s[j + k]; k++);
  }
};
```

8.5 Suffix Array Automaton

```
//to be modified
const int MAXM = 1000010;
struct SAM {
 int tot, root, lst, mom[MAXM], mx[MAXM];
 int acc[MAXM], nxt[MAXM][33];
 int newNode() {
  int res = ++tot;
  fill(nxt[res], nxt[res] + 33, 0);
  mom[res] = mx[res] = acc[res] = 0;
  return res;
 }
 void init() {
  tot = 0;
  root = newNode();
  mom[root] = 0, mx[root] = 0;
  lst = root;
 }
 void push(int c) {
  int p = lst;
  int np = newNode();
  mx[np] = mx[p] + 1;
  for (; p && nxt[p][c] == 0; p = mom[p])
   nxt[p][c] = np;
  if (p == 0) mom[np] = root;
  else {
   int q = nxt[p][c];
   if (mx[p] + 1 == mx[q]) mom[np] = q;
   else {
    int nq = newNode();
    mx[nq] = mx[p] + 1;
    for (int i = 0; i < 33; i++)
     nxt[nq][i] = nxt[q][i];
    mom[nq] = mom[q];
    mom[q] = nq;
    mom[np] = nq;
    for (; p && nxt[p][c] == q; p = mom[p])
     nxt[p][c] = nq;
   }
  }
  lst = np;
 }
 void push(char *str) {
  for (int i = 0; str[i]; i++)
   push(str[i] - 'a' + 1);
 }
} sam;
```

8.6 Z-value Algorithm

```
int z[maxn];
void make_z(const string &s) {
  int l = 0, r = 0;
  for (int i = 1; i < SZ(s); ++i) {
    for (z[i] = max(0, min(r - i + 1, z[i - l]));
        i + z[i] < SZ(s) && s[i + z[i]] == s[z[i]];
        ++z[i])
      ;
    if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
  }
}
```

9 Formula

9.1 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k + c_1 x^{k-1} + \cdots + c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

9.2 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

9.3 Geometry

9.3.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left(1 - \left(\dfrac{a}{b+c}\right)^2\right)}$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\dfrac{\alpha+\beta}{2}}{\tan\dfrac{\alpha-\beta}{2}}$

Incenter:
$P_1 = (x_1, y_1), P_2 = (x_2, y_2), P_3 = (x_3, y_3)$
$s_1 = \overline{P_2P_3}, s_2 = \overline{P_1P_3}, s_3 = \overline{P_1P_2}$
$\dfrac{s_1P_1 + s_2P_2 + s_3P_3}{s_1 + s_2 + s_3}$

Circumcenter:
$P_0 = (0,0), P_1 = (x_1, y_1), P_2 = (x_2, y_2)$
$x_c = \frac{1}{2} \times \dfrac{y_2(x_1^2 + y_1^2) - y_1(x_2^2 + y_2^2)}{-x_2y_1 + x_1y_2}$
$y_c = \frac{1}{2} \times \dfrac{x_2(x_1^2 + y_1^2) - x_1(x_2^2 + y_2^2)}{-x_1y_2 + x_2y_1}$

Check if (x_0, y_0) is in the circumcircle:

$$\begin{vmatrix} x_1 - x_0 & y_1 - y_0 & (x_1^2 + y_1^2) - (x_0^2 + y_0^2) \\ x_2 - x_0 & y_2 - y_0 & (x_2^2 + y_2^2) - (x_0^2 + y_0^2) \\ x_3 - x_0 & y_3 - y_0 & (x_3^2 + y_3^2) - (x_0^2 + y_0^2) \end{vmatrix}$$

0: on edge, > 0: inside, < 0: outside

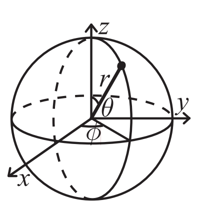
9.3.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

9.3.3 Spherical coordinates



$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \mathrm{acos}(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r\cos\theta & \phi &= \mathrm{atan2}(y, x) \end{aligned}$$

9.4 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\mathrm{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

9.5 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$\begin{aligned} 1 + 2 + 3 + \cdots + n &= \frac{n(n+1)}{2} \\ 1^2 + 2^2 + 3^2 + \cdots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \cdots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \cdots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \end{aligned}$$

9.6 Series

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty) \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1) \\ \sqrt{1+x} &= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1) \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty) \end{aligned}$$

9.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.
Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

9.7.1 Discrete distributions

Binomial distribution The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n, p)$, $n = 1, 2, \ldots, 0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n, p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

9.7.2 Continuous distributions

Uniform distribution If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

9.8 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n\mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k\in\mathbf{G}} a_{ik}p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k\in\mathbf{G}} p_{ki}t_k$.