

Contents

1	Basic	1
1.1	Default Code	1
1.2	.vimrc	1
2	Data Structure	1
2.1	Heavy-Light Decomposition	1
2.2	Li-Chao Tree	1
2.3	Link Cut Tree	2
2.4	Treap	2
3	Flow Matching	3
3.1	Bounded Flow	3
3.2	Dinic	3
3.3	Gomory Hu	4
3.4	Hungarian Algorithm	4
3.5	ISAP Algorithm	4
3.6	KM Algorithm	5
3.7	Max Simple Graph Matching	5
3.8	MCMF	5
3.9	Min Cost Circulation	6
3.10	SW Mincut	6
4	Geometry	6
4.1	Geometry Template	6
4.2	Convex Hull	7
4.3	Minimum Enclosing Circle	7
4.4	Minkowski Sum	7
4.5	Half Plane Intersection	7
4.6	Dynamic Convex Hull	8
4.7	3D Point	8
4.8	ConvexHull3D	8
5	Graph	9
5.1	Block Cut Tree	9
5.2	2-SAT	10
5.3	Dominator Tree	10
5.4	Virtual Tree	11
6	Math	11
6.1	Extended Euclidean Algorithm	11
6.2	Floor & Ceil	11
6.3	Legendre	11
6.4	Simplex	11
6.5	Floor Sum	12
6.6	Miller Rabin & Pollard Rho	12
7	Misc	12
7.1	Fraction	12
7.2	Matroid	12
8	Polynomial	13
8.1	FFT	13
8.2	NTT	13
8.3	Polynomial Operation	13
8.4	Generating Function	14
8.4.1	Ordinary Generating Function	14
8.4.2	Exponential Generating Function	14
9	String	15
9.1	Rolling Hash	15
9.2	KMP Algorithm	15
9.3	Manacher Algorithm	15
9.4	MCP	15
9.5	Suffix Array	15
9.6	Suffix Array Automaton	16
9.7	Z-value Algorithm	16
9.8	Main Lorentz	16
9.9	AC Automaton	16

10	Formula	17
10.1	Recurrences	17
10.2	Trigonometry	17
10.3	Geometry	17
10.3.1	Triangles	17
10.3.2	Quadrilaterals	17
10.3.3	Spherical coordinates	17
10.3.4	Green's Theorem	18
10.4	Derivatives/Integrals	18
10.5	Sums	18
10.6	Series	18
10.7	Probability theory	18
10.7.1	Discrete distributions	18
10.7.2	Continuous distributions	18
10.8	Markov chains	18

1 Basic

1.1 Default Code

```
//Challenge: Accepted
//#pragma GCC optimize("Ofast")
#include <bits/stdc++.h>
using namespace std;

#define io ios_base::sync_with_stdio(0);cin.tie(0);cerr.tie(0)
#define iter(v) v.begin(),v.end()
#define SZ(v) (int)v.size()
#define pb emplace_back
#define mp make_pair
#define ff first
#define ss second

using ll = long long;
using pii = pair<int, int>;
using pll = pair<ll, ll>;

#ifdef zisk
void debug(){cerr << "\n";}
template<class T, class ... U>
void debug(T a, U ... b){cerr << a << " ", debug(b...);}
template<class T> void pary(T l, T r){
    while (l != r) cerr << *l << " ", l++;
    cerr << "\n";
}
#else
#define debug(...) void()
#define pary(...) void()
#endif
template<class A, class B>
ostream& operator<<(ostream& o, pair<A,B> p)
{ return o << '(' << p.ff << ',' << p.ss << ')'; }

int main(){
    io;
}
```

1.2 .vimrc

```
sy on
se nu rnu bs=2 sw=4 ts=4 hls ls=2 si acd bo=all mouse=a et
map <F9> :w<bar>!g++ "%" -o %:r -std=c++17 -Wall -Wextra -
    Wshadow -Dzisk -g -fsanitize=undefined,address<CR>
map <F8> :!./:%:r<CR>
map <C-a> <ESC>ggVG
inoremap {<CR> {<CR>}<ESC>ko
```

2 Data Structure

2.1 Heavy-Light Decomposition

```

struct Heavy_light_Decomposition { // 1-base
    int n, up[maxn], dep[maxn], to[maxn], siz[maxn], pa[maxn];
};
int C, ti[maxn], ord[maxn], wdown[maxn], edge[maxn], et = 0;
vector<pii> G[maxn];
void init(int _n) {
    n = _n, C = 0, et = 1;
    for (int i = 1; i <= n; i++)
        G[i].clear(), to[i] = 0;
}
void add_edge(int a, int b, int w) {
    G[a].push_back(pii(b, et)), G[b].push_back(pii(a, et));
    edge[et++] = w;
}
void dfs(int u, int f, int d) {
    siz[u] = 1, pa[u] = f, dep[u] = d;
    for (auto &v: G[u])
        if (v.ff != f) {
            dfs(v.ff, u, d+1), siz[u] += siz[v];
            if (siz[to[u]] < siz[v]) to[u] = v;
        }
}
void cut(int u, int link) {
    ti[u] = C;
    ord[C++] = u, up[u] = link;
    if (!to[u]) return;
    cut(to[u], link);
    for (auto v: G[u]) {
        if (v.ff != pa[u] && v.ff != to[u]) cut(v.ff, v.ff);
    }
}
void build() { dfs(1, 1, 1), cut(1, 1); }
int query(int a, int b) {
    int ta = up[a], tb = up[b], re = 0;
    while (ta != tb)
        if (dep[ta] < dep[tb])
            /*query*/, tb = up[b = pa[tb]];
        else /*query*/, ta = up[a = pa[ta]];
    if (a == b) return re;
    if (ti[a] > ti[b]) swap(a, b);
    /*query*/
    return re;
}
};

```

2.2 Li-Chao Tree

```

struct LiChao_min {
    struct line {
        ll m, c;
        line(ll _m = 0, ll _c = 0) {
            m = _m;
            c = _c;
        }
        ll eval(ll x) { return m * x + c; }
    };
    struct node {
        node *l, *r;
        line f;
        node(line v) {
            f = v;
            l = r = NULL;
        }
    };
    typedef node *pnode;
    pnode root;
    int sz;
    #define mid ((l + r) >> 1)
    void insert(line &v, int l, int r, pnode &nd) {
        if (!nd) {
            nd = new node(v);
            return;
        }
        ll trl = nd->f.eval(l), trr = nd->f.eval(r);
        ll vl = v.eval(l), vr = v.eval(r);

```

```

        if (trl <= vl && trr <= vr) return;
        if (trl > vl && trr > vr) {
            nd->f = v;
            return;
        }
        if (trl > vl) swap(nd->f, v);
        if (nd->f.eval(mid) < v.eval(mid))
            insert(v, mid + 1, r, nd->r);
        else swap(nd->f, v), insert(v, l, mid, nd->l);
    }
    ll query(int x, int l, int r, pnode &nd) {
        if (!nd) return inf;
        if (l == r) return nd->f.eval(x);
        if (mid >= x)
            return min(
                nd->f.eval(x), query(x, l, mid, nd->l));
        return min(
            nd->f.eval(x), query(x, mid + 1, r, nd->r));
    }
    /* -sz <= query_x <= sz */
    void init(int _sz) {
        sz = _sz + 1;
        root = NULL;
    }
    void add_line(ll m, ll c) {
        line v(m, c);
        insert(v, -sz, sz, root);
    }
    ll query(ll x) { return query(x, -sz, sz, root); }
};

```

2.3 Link Cut Tree

```

struct Splay { // subtree-sum, path-max
    static Splay nil;
    Splay *ch[2], *f;
    int val, rev, size, vir, id, type;
    pii ma;
    Splay(int _val = 0, int _id = 0)
        : val(_val), rev(0), size(0), vir(0), id(_id) {
        ma = make_pair(val, id);
        f = ch[0] = ch[1] = &nil;
        type = 0;
    }
    bool isr() { //is root
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push() {
        if (!rev) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev = 0;
    }
    void pull() {
        // take care of the nil!
        size = ch[0]->size + ch[1]->size + vir + type;
        ma = max(make_pair(val, id), max(ch[0]->ma, ch[1]->ma));
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
}

```

```

    x->setCh(p, !d);
    p->pull(), x->pull();
}
void splay(Splay *x) {
    vector<Splay *> splayVec;
    for (Splay *q = x;; q = q->f) {
        splayVec.pb(q);
        if (q->isr()) break;
    }
    reverse(iter(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir() == x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
Splay *access(Splay *x) {
    Splay *q = nil;
    for (; x != nil; x = x->f) {
        splay(x);
        x->vir -= q->size; x->vir += x->ch[1]->size;
        x->setCh(q, 1); x->pull();
        q = x;
    }
    return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
    root_path(x), x->rev ^= 1;
    x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
    chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
    chroot(x), root_path(y);
    x->f = y; y->vir += x->size;
}
void cut(Splay *x, Splay *y) {
    split(x, y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
    y->pull();
}
Splay *get_root(Splay *x) {
    for (root_path(x); x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    return get_root(x) == get_root(y);
}
Splay *lca(Splay *x, Splay *y) {
    access(x), root_path(y);
    if (y->f == nil) return y;
    return y->f;
}
void change(Splay *x, int val) {
    splay(x), x->val = val, x->pull();
}
pii query(Splay *x, Splay *y) {
    split(x, y);
    return y->ma;
}
}

```

2.4 Treap

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;

```

```

        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(),
            a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    split(o, a, b, k),
        o = merge(a, merge(new node(k), b));
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c;
    split2(o, a, b, l - 1), split2(b, b, c, r);
    // operate
    o = merge(a, merge(b, c));
}
}

```

3 Flow Matching

3.1 Bounded Flow

```

struct Dinic { // 1-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> g[maxN];
    int n, s, t, dis[maxN], ind[maxN], cnt[maxN];
    const int inf = 1e9;

    void init(int _n) {

```

```

n = _n + 2;
s = _n + 1, t = _n + 2;
for (int i = 0; i <= n; ++i) g[i].clear(), cnt[i] = 0;
}
void reset() {
    for (int i = 0; i <= n; ++i)
        for (auto &j : g[i]) j.flow = 0;
}
void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    g[u].pb(edge{v, rcap, lcap, (int)g[v].size()});
    g[v].pb(edge{u, 0, 0, (int)g[u].size() - 1});
}
void add_edge(int u, int v, int cap) {
    g[u].pb(edge{v, cap, 0, (int)g[v].size()});
    g[v].pb(edge{u, 0, 0, (int)g[u].size() - 1});
    //change g[v] to cap for undirected graphs
}
bool bfs() {
    fill(dis, dis+n+1, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int cur = q.front(); q.pop();
        for (auto &e : g[cur]) {
            if (dis[e.to] == -1 && e.flow != e.cap) {
                q.push(e.to);
                dis[e.to] = dis[cur] + 1;
            }
        }
    }
    return dis[t] != -1;
}
int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = ind[u]; i < (int)g[u].size(); ++i) {
        edge &e = g[u][i];
        if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
            int df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df;
                g[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill(ind, ind+n+1, 0);
        while ((df = dfs(s, inf))) flow += df;
    }
    return flow;
}
bool feasible() {
    int sum = 0;
    for (int i = 1; i <= n - 2; ++i)
        if (cnt[i] > 0)
            add_edge(n - 1, i, cnt[i], sum += cnt[i]);
        else if (cnt[i] < 0) add_edge(i, n, -cnt[i]);
    if (sum != maxflow(n - 1, n)) sum = -1;
    for (int i = 1; i <= n - 2; ++i)
        if (cnt[i] > 0)
            g[n - 1].pop_back(), g[i].pop_back();
        else if (cnt[i] < 0)
            g[i].pop_back(), g[n].pop_back();
    return sum != -1;
}
int boundedflow(int _s, int _t) {
    add_edge(_t, _s, inf);
    if (!feasible()) return -1; // infeasible flow
    int x = g[_t].back().flow;
    g[_t].pop_back(), g[_s].pop_back();
}

```

```

int y = maxflow(_t, _s);
return x-y;
}
};

```

3.2 Dinic

```

struct MaxFlow { // 1-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> g[maxn];
    int s, t, dis[maxn], ind[maxn], n;

    void init(int _n) {
        n = _n + 2;
        s = _n + 1, t = _n + 2;
        for (int i = 0; i <= n; ++i) g[i].clear();
    }
    void reset() {
        for (int i = 0; i <= n; ++i)
            for (auto &j : g[i]) j.flow = 0;
    }
    void add_edge(int u, int v, int cap) {
        g[u].pb(edge{v, cap, 0, (int)g[v].size()});
        g[v].pb(edge{u, 0, 0, (int)g[u].size() - 1});
        //change g[v] to cap for undirected graphs
    }
    bool bfs() {
        fill(dis, dis+n+1, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int cur = q.front(); q.pop();
            for (auto &e : g[cur]) {
                if (dis[e.to] == -1 && e.flow != e.cap) {
                    q.push(e.to);
                    dis[e.to] = dis[cur] + 1;
                }
            }
        }
        return dis[t] != -1;
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = ind[u]; i < (int)g[u].size(); ++i) {
            edge &e = g[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    g[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    int maxflow() {
        int flow = 0, df;
        while (bfs()) {
            fill(ind, ind+n+1, 0);
            while ((df = dfs(s, inf))) flow += df;
        }
        return flow;
    }
} flow;

```

3.3 Gomory Hu

```

MaxFlow Dinic;
int g[MAXN];
void GomoryHu(int n) { // 0-base

```

```

fill_n(g, n, 0);
for (int i = 1; i < n; ++i) {
    Dinic.reset();
    add_edge(i, g[i], Dinic.maxflow(i, g[i]));
    for (int j = i + 1; j <= n; ++j)
        if (g[j] == g[i] && ~Dinic.dis[j])
            g[j] = i;
}
}

```

3.4 Hungarian Algorithm

```

int c[maxn][maxn]; //hungarian algorithm in  $O(n^3)$ 
//1 base
int lx[maxn], ly[maxn], mx[maxn], my[maxn];
bool vx[maxn], vy[maxn];
int slack[maxn];
int tot;
bool dfs(int n, bool ch) {
    if (vx[n]) return false;
    vx[n] = 1;
    for (int v = 1; v <= tot; v++) {
        slack[v] = min(slack[v], lx[n] + ly[v] - c[n][v]);
        if (lx[n] + ly[v] - c[n][v] > 0) continue;
        vy[v] = 1;
        if (!my[v] || dfs(my[v], ch)) {
            if (ch) mx[n] = v, my[v] = n;
            return true;
        }
    }
    return false;
}
int main() {
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) vx[j] = vy[j] = 0;
        for (int j = 1; j <= n; j++) slack[j] = 1<<30;
        if (dfs(i, 1)) continue;
        bool aug = 0;
        while (!aug) {
            for (int j = 1; j <= n; j++) {
                if (!vy[j] && slack[j] == 0) {
                    vy[j] = 1;
                    if (dfs(my[j], 0)) {
                        aug = 1;
                        break;
                    }
                }
            }
            if (aug) break;
            int delta = 1<<30;
            for (int j = 1; j <= n; j++) {
                if (!vy[j]) delta = min(delta, slack[j]);
            }
            for (int j = 1; j <= n; j++) {
                if (vx[j]) lx[j] -= delta;
                if (vy[j]) ly[j] += delta;
                else {
                    slack[j] -= delta;
                    if (slack[j] == 0 && !my[j]) aug = 1;
                }
            }
        }
        for (int j = 1; j <= n; j++) vx[j] = vy[j] = 0;
        dfs(i, 1);
    }
}

```

3.5 ISAP Algorithm

```

struct Maxflow { //to be modified
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r)

```

```

        : v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV * 2];
    int iter[MAXV * 2], d[MAXV * 2], gap[MAXV * 2], tot;
    void init(int x) {
        tot = x + 2;
        s = x + 1, t = x + 2;
        for (int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
    int dfs(int p, int flow) {
        if (p == t) return flow;
        for (int &i = iter[p]; i < SZ(G[p]); i++) {
            Edge &e = G[p][i];
            if (e.c > 0 && d[p] == d[e.v] + 1) {
                int f = dfs(e.v, min(flow, e.c));
                if (f) {
                    e.c -= f;
                    G[e.v][e.r].c += f;
                    return f;
                }
            }
        }
        if ((--gap[d[p]]) == 0) d[s] = tot;
        else {
            d[p]++;
            iter[p] = 0;
            ++gap[d[p]];
        }
        return 0;
    }
    int solve() {
        int res = 0;
        gap[0] = tot;
        for (res = 0; d[s] < tot; res += dfs(s, INF))
            ;
        return res;
    }
} flow;

```

3.6 KM Algorithm

```

int n, m; //1-base, max matching
int mx[maxn], my[maxn];
bool adj[maxn][maxn], vis[maxn];
bool dfs(int n) {
    if (vis[n]) return false;
    vis[n] = 1;
    for (int v = 1; v <= n; v++) {
        if (!adj[n][v]) continue;
        if (!my[v] || (my[v] && dfs(my[v]))) {
            mx[n] = v, my[v] = n;
            return true;
        }
    }
    return false;
}
//min vertex cover: take unmatched vertex in L and find
//alternating tree,
//ans is not reached in L + reached in R

```

3.7 Max Simple Graph Matching

```

struct GenMatch { // 1-base
    int V, pr[N];
    bool el[N][N], inq[N], inp[N], inb[N];
    int st, ed, nb, bk[N], djs[N], ans;
    void init(int _V) {

```

```

V = _V;
for (int i = 0; i <= V; ++i) {
    for (int j = 0; j <= V; ++j) el[i][j] = 0;
    pr[i] = bk[i] = djs[i] = 0;
    inq[i] = inp[i] = inb[i] = 0;
}
}

void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
}

int lca(int u, int v) {
    fill_n(inp, V + 1, 0);
    while (1)
        if (u = djs[u], inp[u] = true, u == st) break;
        else u = bk[pr[u]];
    while (1)
        if (v = djs[v], inp[v]) return v;
        else v = bk[pr[v]];
    return v;
}

void upd(int u) {
    for (int v; djs[u] != nb;) {
        v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
        u = bk[v];
        if (djs[u] != nb) bk[u] = v;
    }
}

void blo(int u, int v, queue<int> &qe) {
    nb = lca(u, v), fill_n(inb, V + 1, 0);
    upd(u), upd(v);
    if (djs[u] != nb) bk[u] = v;
    if (djs[v] != nb) bk[v] = u;
    for (int tu = 1; tu <= V; ++tu)
        if (inb[djs[tu]])
            if (djs[tu] = nb, !inq[tu])
                qe.push(tu), inq[tu] = 1;
}

void flow() {
    fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
    iota(djs + 1, djs + V + 1, 1);
    queue<int> qe;
    qe.push(st), inq[st] = 1, ed = 0;
    while (!qe.empty()) {
        int u = qe.front();
        qe.pop();
        for (int v = 1; v <= V; ++v)
            if (el[u][v] && djs[u] != djs[v] &&
                pr[u] != v) {
                if ((v == st) ||
                    (pr[v] > 0 && bk[pr[v]] > 0)) {
                    blo(u, v, qe);
                } else if (!bk[v]) {
                    if (bk[v] = u, pr[v] > 0) {
                        if (!inq[pr[v]]) qe.push(pr[v]);
                    } else {
                        return ed = v, void();
                    }
                }
            }
    }
}

void aug() {
    for (int u = ed, v, w; u > 0;)
        v = bk[u], w = pr[v], pr[v] = u, pr[u] = v,
        u = w;
}

int solve() {
    fill_n(pr, V + 1, 0), ans = 0;
    for (int u = 1; u <= V; ++u)
        if (!pr[u])
            if (st = u, flow(), ed > 0) aug(), ++ans;
    return ans;
}
};

```

3.8 MCMF

```

struct MCMF { // 0-base
    struct edge {
        ll from, to, cap, flow, cost, rev;
    } * past[maxn];
    vector<edge> G[maxn];
    bitset<maxn> inq;
    ll dis[maxn], up[maxn], s, t, mx, n;
    bool BellmanFord(ll &flow, ll &cost) {
        fill(dis, dis + n, inf);
        queue<ll> q;
        q.push(s), inq.reset(), inq[s] = 1;
        up[s] = mx - flow, past[s] = 0, dis[s] = 0;
        while (!q.empty()) {
            ll u = q.front();
            q.pop(), inq[u] = 0;
            if (!up[u]) continue;
            for (auto &e : G[u])
                if (e.flow != e.cap &&
                    dis[e.to] > dis[u] + e.cost) {
                    dis[e.to] = dis[u] + e.cost, past[e.to] = &e;
                    up[e.to] = min(up[u], e.cap - e.flow);
                    if (!inq[e.to]) inq[e.to] = 1, q.push(e.to);
                }
        }
        if (dis[t] == inf) return 0;
        flow += up[t], cost += up[t] * dis[t];
        for (ll i = t; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            e.flow += up[t], G[e.to][e.rev].flow -= up[t];
        }
        return 1;
    }
    ll MinCostMaxFlow(ll _s, ll _t, ll &cost) {
        s = _s, t = _t, cost = 0;
        ll flow = 0;
        while (BellmanFord(flow, cost));
        return flow;
    }
    void init(ll _n, ll _mx) {
        n = _n, mx = _mx;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(ll a, ll b, ll cap, ll cost) {
        G[a].pb(edge{a, b, cap, 0, cost, G[b].size()});
        G[b].pb(edge{b, a, 0, 0, -cost, G[a].size() - 1});
    }
};

```

3.9 Min Cost Circulation

```

//to be modified
struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
    memset(mark, false, sizeof(mark));
    memset(dist, 0, sizeof(dist));
    int upd = -1;
    for (int i = 0; i <= n; ++i) {
        for (int j = 0; j < n; ++j) {
            int idx = 0;
            for (auto &e : g[j]) {
                if (e.cap > 0 && dist[e.to] > dist[j] + e.cost) {
                    dist[e.to] = dist[j] + e.cost;
                    pv[e.to] = j, ed[e.to] = idx;
                    if (i == n) {
                        upd = j;
                        while (!mark[upd]) mark[upd] = true, upd = pv[upd];
                        return upd;
                    }
                }
            }
            idx++;
        }
    }
}

```

```

    }
    return -1;
}
int Solve(int n) {
    int rt = -1, ans = 0;
    while ((rt = NegativeCycle(n)) >= 0) {
        memset(mark, false, sizeof(mark));
        vector<pair<int, int>> cyc;
        while (!mark[rt]) {
            cyc.emplace_back(pv[rt], ed[rt]);
            mark[rt] = true;
            rt = pv[rt];
        }
        reverse(cyc.begin(), cyc.end());
        int cap = kInf;
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            cap = min(cap, e.cap);
        }
        for (auto &i : cyc) {
            auto &e = g[i.first][i.second];
            e.cap -= cap;
            g[e.to][e.rev].cap += cap;
            ans += e.cost * cap;
        }
    }
    return ans;
}

```

3.10 SW Mincut

```

// stoer wagner algorithm: global min cut
const int maxn = 505;
struct SW { // O(V^3)  $\theta$ -based
    int n, vis[maxn], del[maxn];
    int edge[maxn][maxn], wei[maxn];
    void init(int _n) {
        n = _n;
        fill(del, del+n, 0);
        for (int i = 0; i < n; i++) fill(edge[i], edge[i] + n, 0);
    }
    void addEdge(int u, int v, int w) {
        edge[u][v] += w, edge[v][u] += w;
    }
    void search(int &s, int &t) {
        fill(vis, vis+n, 0);
        fill(wei, wei+n, 0);
        s = t = -1;
        while (1) {
            int ma = -1, cur = 0;
            for (int i = 0; i < n; ++i)
                if (!del[i] && !vis[i] && ma < wei[i])
                    cur = i, ma = wei[i];
            if (mx == -1) break;
            vis[cur] = 1, s = t, t = cur;
            for (int i = 0; i < n; ++i)
                if (!vis[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve() {
        int ret = INF;
        for (int i = 0, x=0, y=0; i < n-1; ++i) {
            search(x, y), ret = min(res, wei[y]), del[y] = 1;
            for (int j = 0; j < n; ++j)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return ret;
    }
};

```

4 Geometry

4.1 Geometry Template

```

using ld = ll;
using pdd = pair<ld, ld>;
using Line = pair<pdd, pdd>;
#define X first
#define Y second
// ld eps = 1e-7;

pdd operator+(pdd a, pdd b)
{ return {a.X + b.X, a.Y + b.Y}; }
pdd operator-(pdd a, pdd b)
{ return {a.X - b.X, a.Y - b.Y}; }
pdd operator*(ld i, pdd v)
{ return {i * v.X, i * v.Y}; }
pdd operator*(pdd v, ld i)
{ return {i * v.X, i * v.Y}; }
pdd operator/(pdd v, ld i)
{ return {v.X / i, v.Y / i}; }
ld dot(pdd a, pdd b)
{ return a.X * b.X + a.Y * b.Y; }
ld cross(pdd a, pdd b)
{ return a.X * b.Y - a.Y * b.X; }
ld abs2(pdd v)
{ return v.X * v.X + v.Y * v.Y; }
ld abs(pdd v)
{ return sqrt(abs2(v)); }
int sgn(ld v)
{ return v > 0 ? 1 : (v < 0 ? -1 : 0); }
// int sgn(NumType v){ return v > eps ? 1 : ( v < -eps ? -1 : 0); }

int ori(pdd a, pdd b, pdd c)
{ return sgn(cross(b - a, c - a)); }
bool collinearity(pdd a, pdd b, pdd c)
{ return ori(a, b, c) == 0; }
bool btw(pdd p, pdd a, pdd b)
{ return collinearity(p, a, b) && sgn(dot(a - p, b - p)) <= 0; }

bool seg_intersect(Line a, Line b){
    pdd p1, p2, p3, p4;
    tie(p1, p2) = a; tie(p3, p4) = b;
    if(btw(p1, p3, p4) || btw(p2, p3, p4) || btw(p3, p1, p2)
        || btw(p4, p1, p2))
        return true;
    return ori(p1, p2, p3) * ori(p1, p2, p4) < 0 &&
        ori(p3, p4, p1) * ori(p3, p4, p2) < 0;
}

pdd intersect(Line a, Line b){
    pdd p1, p2, p3, p4;
    tie(p1, p2) = a; tie(p3, p4) = b;
    ld a123 = cross(p2 - p1, p3 - p1);
    ld a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}

vector<int> getConvexHull(vector<pdd>& pts){
    vector<int> id(SZ(pts));
    iota(iter(id), 0);
    sort(iter(id), [&](int x, int y){ return pts[x] < pts[y];
    });
    vector<int> hull;
    for(int tt = 0; tt < 2; tt++){
        int sz = SZ(hull);
        for(int j : id){
            pdd p = pts[j];
            while(SZ(hull) - sz >= 2 &&
                cross(pts[hull.back()] - pts[hull[SZ(hull) - 2]],
                    p - pts[hull[SZ(hull) - 2]]) <= 0)
                hull.pop_back();
            hull.pb(j);
        }
        hull.pop_back();
        reverse(iter(id));
    }
    return hull;
}

```

4.2 Convex Hull

```
}

```

4.3 Minimum Enclosing Circle

```
using ld = long double;
pair<pdd, ld> circumcenter(pdd a, pdd b, pdd c);
pair<pdd, ld> MinimumEnclosingCircle(vector<pdd> &pts){
    random_shuffle(iter(pts));
    pdd c = pts[0];
    ld r = 0;
    for(int i = 1; i < SZ(pts); i++){
        if(abs(pts[i] - c) <= r) continue;
        c = pts[i]; r = 0;
        for(int j = 0; j < i; j++){
            if(abs(pts[j] - c) <= r) continue;
            c = (pts[i] + pts[j]) / 2;
            r = abs(pts[i] - c);
            for(int k = 0; k < j; k++){
                if(abs(pts[k] - c) > r)
                    tie(c, r) = circumcenter(pts[i], pts[j], pts[k]);
            }
        }
    }
    return {c, r};
}
```

4.4 Minkowski Sum

```
void reorder_poly(vector<pdd>& pts){
    int mn = 0;
    for(int i = 1; i < (int)pts.size(); i++){
        if(pts[i].Y < pts[mn].Y || (pts[i].Y == pts[mn].Y
            && pts[i].X < pts[mn].X))
            mn = i;
    }
    rotate(pts.begin(), pts.begin() + mn, pts.end());
}

vector<pdd> minkowski(vector<pdd> P, vector<pdd> Q){
    reorder_poly(P);
    reorder_poly(Q);
    int psz = P.size();
    int qsz = Q.size();
    P.eb(P[0]);
    P.eb(P[1]);
    Q.eb(Q[0]);
    Q.eb(Q[1]);
    vector<pdd> ans;
    int i = 0, j = 0;
    while(i < psz || j < qsz){
        ans.eb(P[i] + Q[j]);
        int t = sgn(cross(P[i + 1] - P[i], Q[j + 1] - Q[j]));
        if(t >= 0) i++;
        if(t <= 0) j++;
    }
    return ans;
}
```

4.5 Half Plane Intersection

```
// copy from 8BQube
bool isin( Line l0, Line l1, Line l2 ) {
    // Check inter(l1, l2) in l0
    pdd p = intersect(l1.X, l1.Y, l2.X, l2.Y);
    return sign(cross(l0.Y - l0.X, p - l0.X)) > 0;
}
/* Having solution, check intersect(ret[0], ret[1])
* in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
*/
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> lines) {
    vector<double> ata(SZ(lines)), ord(SZ(lines));
    for(int i = 0; i < SZ(lines); ++i) {
```

```
        ord[i] = i;
        pdd d = lines[i].Y - lines[i].X;
        ata[i] = atan2(d.Y, d.X);
    }
    sort(ALL(ord), [&](int i, int j) {
        if (fabs(ata[i] - ata[j]) >= eps)
            return ata[i] < ata[j];
        return ori(lines[i].X, lines[i].Y, lines[j].Y) < 0;
    });
    vector<Line> fin(1, lines[ord[0]]);
    for (int i = 1; i < SZ(lines); ++i)
        if (fabs(ata[ord[i]] - ata[ord[i - 1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for (int i = 0; i < SZ(fin); ++i) {
        while (SZ(dq) >= 2 && !isin(fin[i], dq[SZ(dq) - 2], dq.
            back()))
            dq.pop_back();
        while (SZ(dq) >= 2 && !isin(fin[i], dq[0], dq[1]))
            dq.pop_front();
        dq.pb(fin[i]);
    }
    while (SZ(dq) >= 3 && !isin(dq[0], dq[SZ(dq) - 2], dq.
        back()))
        dq.pop_back();
    while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    return vector<Line>(ALL(dq));
}
```

4.6 Dynamic Convex Hull

```
struct Line{
    ll a, b, l = MIN, r = MAX;
    Line(ll a, ll b): a(a), b(b) {}
    ll operator()(ll x) const{
        return a * x + b;
    }
    bool operator<(Line b) const{
        return a < b.a;
    }
    bool operator<(ll b) const{
        return r < b;
    }
};

ll iceil(ll a, ll b){
    if(b < 0) a *= -1, b *= -1;
    if(a > 0) return (a + b - 1) / b;
    else return a / b;
}

ll intersect(Line a, Line b){
    return iceil(a.b - b.b, b.a - a.a);
}

struct DynamicConvexHull{
    multiset<Line, less<>> ch;

    void add(Line ln){
        auto it = ch.lower_bound(ln);
        while(it != ch.end()){
            Line tl = *it;
            if(tl(tl.r) <= ln(tl.r)){
                it = ch.erase(it);
            }
            else break;
        }
        auto it2 = ch.lower_bound(ln);
        while(it2 != ch.begin()){
            Line tl = *prev(it2);
            if(tl(tl.l) <= ln(tl.l)){
                it2 = ch.erase(prev(it2));
            }
            else break;
        }
    }
}
```



```

it = ch.lower_bound(ln);
if(it != ch.end()){
    Line tl = *it;
    if(tl(tl.l) >= ln(tl.l)) ln.r = tl.l - 1;
    else{
        ll pos = intersect(ln, tl);
        tl.l = pos;
        ln.r = pos - 1;
        ch.erase(it);
        ch.insert(tl);
    }
}
it2 = ch.lower_bound(ln);
if(it2 != ch.begin()){
    Line tl = *prev(it2);
    if(tl(tl.r) >= ln(tl.r)) ln.l = tl.r + 1;
    else{
        ll pos = intersect(tl, ln);
        tl.r = pos - 1;
        ln.l = pos;
        ch.erase(prev(it2));
        ch.insert(tl);
    }
}
if(ln.l <= ln.r) ch.insert(ln);
}

ll query(ll pos){
    auto it = ch.lower_bound(pos);
    if(it == ch.end()) return 0;
    return (*it)(pos);
}
};

```

4.7 3D Point

```

// Copy from 8BQube
struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0): x(_x),
        y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};

Point operator-(const Point &p1, const Point &p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z); }
Point cross(const Point &p1, const Point &p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x - p1.
    x * p2.z, p1.x * p2.y - p1.y * p2.x); }
double dot(const Point &p1, const Point &p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z; }
double abs(const Point &a)
{ return sqrt(dot(a, a)); }
Point cross3(const Point &a, const Point &b, const Point &c
    )
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a); }
pdd proj(Point a, Point b, Point c, Point u) {
    // proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}

```

4.8 ConvexHull3D

```

// Copy from 8BQube
struct CH3D {
    struct face{int a, b, c; bool ok;} F[8 * N];

```

```

double dblcmp(Point &p, face &f)
{return dot(cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a]);}
int g[N][N], num, n;
Point P[N];
void deal(int p, int a, int b) {
    int f = g[a][b];
    face add;
    if (F[f].ok) {
        if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
        else
            add.a = b, add.b = a, add.c = p, add.ok = 1, g[p][b] =
                g[a][p] = g[b][a] = num, F[num++] = add;
    }
}
void dfs(int p, int now) {
    F[now].ok = 0;
    deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[now].b),
        deal(p, F[now].a, F[now].c);
}
bool same(int s, int t){
    Point &a = P[F[s].a];
    Point &b = P[F[s].b];
    Point &c = P[F[s].c];
    return fabs(volume(a, b, c, P[F[t].a])) < eps && fabs(
        volume(a, b, c, P[F[t].b])) < eps && fabs(volume(a, b,
            c, P[F[t].c])) < eps;
}
void init(int _n){n = _n, num = 0;}
void solve() {
    face add;
    num = 0;
    if(n < 4) return;
    if([&](){
        for (int i = 1; i < n; ++i)
            if (abs(P[0] - P[i]) > eps)
                return swap(P[1], P[i]), 0;
        return 1;
    }) || [&](){
        for (int i = 2; i < n; ++i)
            if (abs(cross3(P[i], P[0], P[1])) > eps)
                return swap(P[2], P[i]), 0;
        return 1;
    }) || [&](){
        for (int i = 3; i < n; ++i)
            if (fabs(dot(cross(P[0] - P[1], P[1] - P[2]), P[0] - P[
                i])) > eps)
                return swap(P[3], P[i]), 0;
        return 1;
    })return;
    for (int i = 0; i < 4; ++i) {
        add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c = (i +
            3) % 4, add.ok = true;
        if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
        g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.a] =
            num;
        F[num++] = add;
    }
    for (int i = 4; i < n; ++i)
        for (int j = 0; j < num; ++j)
            if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
                dfs(i, j);
                break;
            }
    for (int tmp = num, i = (num = 0); i < tmp; ++i)
        if (F[i].ok) F[num++] = F[i];
}
double get_area() {
    double res = 0.0;
    if (n == 3)
        return abs(cross3(P[0], P[1], P[2])) / 2.0;
    for (int i = 0; i < num; ++i)
        res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
    return res / 2.0;
}
double get_volume() {
    double res = 0.0;
    for (int i = 0; i < num; ++i)

```

```

    res += volume(Point(0, 0, 0), P[F[i].a], P[F[i].b], P[F[i].c]);
    return fabs(res / 6.0);
}
int triangle() {return num;}
int polygon() {
    int res = 0;
    for (int i = 0, flag = 1; i < num; ++i, res += flag, flag = 1)
        for (int j = 0; j < i && flag; ++j)
            flag &= !same(i, j);
    return res;
}
Point getcent(){
    Point ans(0, 0, 0), temp = P[F[0].a];
    double v = 0.0, t2;
    for (int i = 0; i < num; ++i)
        if (F[i].ok == true) {
            Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].c];
            t2 = volume(temp, p1, p2, p3) / 6.0;
            if (t2 > 0)
                ans.x += (p1.x + p2.x + p3.x + temp.x) * t2, ans.y +=
                    (p1.y + p2.y + p3.y + temp.y) * t2, ans.z += (p1.z + p2.z + p3.z + temp.z) * t2, v += t2;
        }
    ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v);
    return ans;
}
double pointmindis(Point p) {
    double rt = 99999999;
    for(int i = 0; i < num; ++i)
        if(F[i].ok == true) {
            Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].c];
            double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.z - p1.z) * (p3.y - p1.y);
            double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.x - p1.x) * (p3.z - p1.z);
            double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x - p1.x);
            double d = 0 - (a * p1.x + b * p1.y + c * p1.z);
            double temp = fabs(a * p.x + b * p.y + c * p.z + d) /
                sqrt(a * a + b * b + c * c);
            rt = min(rt, temp);
        }
    return rt;
}
};

```

5 Graph

5.1 Block Cut Tree

```

struct BlockCutTree{
    vector<vector<int>> tree; // 1-based
    vector<int> node;
    vector<int> type; // 0:square, 1:circle

    bool iscut(int v){
        return type[node[v]] == 1;
    }

    vector<int> getbcc(int v){
        if(!iscut(v)) return {node[v]};
        vector<int> ans;
        for(int i : tree[node[v]])
            ans.pb(i);
        return ans;
    }

    void build(int n, vector<vector<int>>& g){
        tree.resize(2 * n + 1);
        type.resize(2 * n + 1);
        node.resize(n + 1, -1);
        vector<int> in(n + 1);
        vector<int> low(n + 1);
    }
}

```

```

stack<int> st;

int ts = 1;
int bcc = 1;
auto addv = [&](int id, int v){
    if(node[v] == -1){
        node[v] = id;
        return;
    }
    if(type[node[v]] == 0){
        int o = node[v];
        node[v] = bcc++;
        type[node[v]] = 1;
        tree[o].pb(node[v]);
        tree[node[v]].pb(o);
    }
    tree[id].pb(node[v]);
    tree[node[v]].pb(id);
};

function<void(int, int)> dfs = [&](int now, int p){
    in[now] = low[now] = ts++;
    st.push(now);
    int child = 0;
    for(int i : g[now]){
        if(i == p) continue;
        if(in[i]){
            low[now] = min(low[now], in[i]);
            continue;
        }
        child++;
        dfs(i, now);
        low[now] = min(low[now], low[i]);

        if(low[i] >= in[now]){
            int nowid = bcc++;
            while(true){
                int x = st.top();
                st.pop();
                addv(nowid, x);
                if(x == i) break;
            }
            addv(nowid, now);
        }
    }
    if(child == 0 && now == p) addv(bcc++, now);
};
dfs(1, 1);
};

```

5.2 2-SAT

```

struct SAT{ // 1-based
    int n;
    vector<vector<int>> g, rg;
    bool ok = true;
    vector<bool> ans;

    void init(int _n){
        n = _n;
        g.resize(2 * n + 1);
        rg.resize(2 * n + 1);
        ans.resize(n + 1);
    }

    int neg(int v){
        return v <= n ? v + n : v - n;
    }

    void addEdge(int u, int v){
        g[u].eb(v);
        rg[v].eb(u);
    }

    void addClause(int a, int b){
        addEdge(a, b);
        addEdge(neg(b), neg(a));
    }

    void build(){
    }
}

```

```

vector<bool> vst(n + 1);
vector<int> tmp, scc(n + 1, -1);
int cnt = 1;
function<void(int)> dfs = [&](int now){
    vst[now] = true;
    for(int i : rg[now]){
        if(vst[i]) continue;
        dfs(i);
    }
    tmp.pb(now);
};
for(int i = 1; i <= 2 * n; i++){
    if(!vst[i]) dfs(i);
}
reverse(iter(tmp));
function<void(int, int)> dfs2 = [&](int now, int id){
    scc[now] = id;
    for(int i : g[now]){
        if(scc[i] != -1) continue;
        dfs2(i, id);
    }
};
for(int i : tmp){
    if(scc[i] == -1) dfs2(i, cnt++);
}
for(int i = 1; i <= n; i++){
    if(scc[i] == scc[neg(i)]){
        ok = false;
        return;
    }
    if(scc[i] < scc[neg(i)]) ans[i] = true;
    else ans[i] = false;
}
};

```

5.3 Dominator Tree

```

// copy from 8BQube
struct dominator_tree { // 1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; // dominator_tree
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].pb(v), rG[v].pb(u);
    }
    void dfs(int u) {
        id[dfn[u]] = ++Time; u;
        for (auto v : G[u])
            if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if (y <= x) return y;
        int tmp = find(pa[y], x);
        if (semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for (int i = 1; i <= n; ++i) {
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for (int i = Time; i > 1; --i) {
            int u = id[i];
            for (auto v : rG[u])

```

```

                if (v = dfn[v]) {
                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v]]);
                }
                tree[semi[i]].pb(i);
                for (auto v : tree[pa[i]]) {
                    find(v, pa[i]);
                    idom[v] =
                        semi[best[v]] == pa[i] ? pa[i] : best[v];
                }
                tree[pa[i]].clear();
            }
            for (int i = 2; i <= Time; ++i) {
                if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
                tree[id[idom[i]]].pb(id[i]);
            }
        }
    };

```

5.4 Virtual Tree

```

// copy from 8BQube
vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(v[0]);
}

```

6 Math

6.1 Extended Euclidean Algorithm

```

// ax+ny = 1, ax+ny == ax == 1 (mod n)
void extgcd(ll x, ll y, ll &g, ll &a, ll &b) {
    if (y == 0) g = x, a = 1, b = 0;
    else extgcd(y, x % y, g, b, a), b -= (x / y) * a;
}

```

6.2 Floor & Ceil

```

int floor_div(int a, int b) {
    return a / b - (a % b && a < 0 ^ b < 0);
}
int ceil_div(int a, int b) {
    return a / b + (a % b && a < 0 ^ b > 0);
}

```

6.3 Legendre

```
// the Jacobi symbol is a generalization of the Legendre
// symbol,
// such that the bottom doesn't need to be prime.
// (n/p) -> same as legendre
// (n|ab) = (n|a)(n|b)
// work with long long
int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

// 0: a == 0
// -1: a isn't a quad res of p
// else: return X with X^2 % p == a
// doesn't work with long long
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p))
                % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) %
            p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}
```

6.4 Simplex

```
#pragma once

typedef double T; // long double, Rational, double + mod<P>
// ...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s]))
    s=j

struct LPSolver {
    int m, n;
    vi N, B;
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
    }
};
```

```
rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
N[n] = -1; D[m+1][n] = 1;
}

void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
        T *b = D[i].data(), inv2 = b[s] * inv;
        rep(j,0,n+2) b[j] -= a[j] * inv2;
        b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
}

bool simplex(int phase) {
    int x = m + phase - 1;
    for (;) {
        int s = -1;
        rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
        if (D[x][s] >= -eps) return true;
        int r = -1;
        rep(i,0,m) {
            if (D[i][s] <= eps) continue;
            if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                < MP(D[r][n+1] / D[r][s], B[r])) r = i;
        }
        if (r == -1) return false;
        pivot(r, s);
    }
}

T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
        pivot(r, n);
        if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
        rep(i,0,m) if (B[i] == -1) {
            int s = 0;
            rep(j,1,n+1) ltj(D[i]);
            pivot(i, s);
        }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
}
};
```

6.5 Floor Sum

```
// from 8BQube
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m)
        ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m)
        ans += n * (b / m), b %= m;
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
} // sum^{n-1}_0 floor((a * i + b) / m) in log(n + m + a + b)
```

6.6 Miller Rabin & Pollard Rho

```
// n < 4,759,123,141 3 : 2, 7, 61
// n < 1,122,004,669,633 4 : 2, 13, 23, 1662803
```

```
// n < 3,474,749,660,383 6 : pirmes <= 13
// n < 2^64 7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
ll mul(ll a, ll b, ll n){
    return (__int128)a * b % n;
}
bool Miller_Rabin(ll a, ll n) {
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}
bool prime(ll n){
    vector<ll> tmp = {2, 325, 9375, 28178, 450775, 9780504,
        1795265022};
    for(ll i : tmp)
        if(!Miller_Rabin(i, n)) return false;
    return true;
}
map<ll, int> cnt;
void PollardRho(ll n) {
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2 == 0) return PollardRho(n / 2), ++cnt[2], void();
    ll x = 2, y = 2, d = 1, p = 1;
#define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p), y = f(f(y, n, p), n, p);
        d = gcd(abs(x - y), n);
    }
}
```

7 Misc

7.1 Fraction

```
struct Frac{
    ll p, q; // p / q
    Frac(ll _p, ll _q): p(_p), q(_q) { if(q < 0) p = -p, q = -q; }
};
Frac operator-(Frac a) { return Frac(-a.p, a.q); }
Frac operator+(Frac a, Frac b){
    ll q = a.q * b.q;
    ll p = a.p * b.q + b.p * a.q;
    return Frac(p, q);
}
Frac inv(Frac a){ return Frac(a.q, a.p); }
Frac operator-(Frac a, Frac b) { return a + (-b); }
Frac operator*(Frac a, Frac b) { return Frac({a.p * b.p, a.q * b.q}); }
Frac operator/(Frac a, Frac b) { return a * inv(b); }
ostream& operator<<(ostream& o, Frac a) { return o << a.p << '/' << a.q; }
```

7.2 Matroid

我們稱一個二元組 $M = (E, \mathcal{I})$ 為一個擬陣，其中 $\mathcal{I} \subseteq 2^E$ 為 E 的子集所形成的非空集合，若：

- 若 $S \in \mathcal{I}$ 以及 $S' \subsetneq S$ ，則 $S' \in \mathcal{I}$
- 對於 $S_1, S_2 \in \mathcal{I}$ 滿足 $|S_1| < |S_2|$ ，存在 $e \in S_2 \setminus S_1$ 使得 $S_1 \cup \{e\} \in \mathcal{I}$

除此之外，我們有以下的定義：

- 位於 \mathcal{I} 中的集合我們稱之為獨立集 (independent set)，反之不在 \mathcal{I} 中的我們稱為相依集 (dependent set)
- 極大的獨立集為基底 (base)、極小的相依集為迴路 (circuit)
- 一個集合 Y 的秩 (rank) $r(Y)$ 為該集中最大的獨立子集，也就是 $r(Y) = \max\{|X| \mid X \subseteq Y \text{ 且 } X \in \mathcal{I}\}$

性質：

1. $X \subseteq Y \wedge Y \in \mathcal{I} \implies X \in \mathcal{I}$
2. $X \subseteq Y \wedge X \notin \mathcal{I} \implies Y \notin \mathcal{I}$
3. 若 B 與 B' 皆是基底且 $B \subseteq B'$ ，則 $B = B'$
若 C 與 C' 皆是迴路且 $C \subseteq C'$ ，則 $C = C'$
4. $e \in E \wedge X \subseteq E \implies r(X) \leq r(X \cup \{e\}) \leq r(X) + 1$ i.e. 加入一個元素後秩不會降底，最多增加 1
5. $\forall Y \subseteq E, \exists X \subseteq Y, r(X) = |X| = r(Y)$

一些等價的性質：

1. 對於所有 $X \subseteq E$ ， X 的極大獨立子集都有相同的大小
2. 對於 $B_1, B_2 \in \mathcal{B} \wedge B_1 \neq B_2$ ，對於所有 $e_1 \in B_1 \setminus B_2$ ，存在 $e_2 \in B_2 \setminus B_1$ 使得 $(B_1 \setminus \{e_1\}) \cup \{e_2\} \in \mathcal{B}$
3. 對於 $X, Y \in \mathcal{I}$ 且 $|X| < |Y|$ ，存在 $e \in Y \setminus X$ 使得 $X \cup \{e\} \in \mathcal{B}$
4. 如果 $r(X \cup \{e_1\}) = r(X \cup \{e_2\}) = r(X)$ ，則 $r(X \cup \{e_1, e_2\}) = r(X)$ 。
如果 $r(X \cup \{e\}) = r(X)$ 對於所有 $e \in E'$ 都成立，則 $r(X \cup E') = r(X)$ 。

擬陣交

Data: 兩個擬陣 $M_1 = (E, \mathcal{I}_1)$ 以及 $M_2 = (E, \mathcal{I}_2)$

Result: I 為最大的位於 $\mathcal{I}_1 \cap \mathcal{I}_2$ 中的獨立集

$I \leftarrow \emptyset$

$X_1 \leftarrow \{e \in E \setminus I \mid I \cup \{e\} \in \mathcal{I}_1\}$

$X_2 \leftarrow \{e \in E \setminus I \mid I \cup \{e\} \in \mathcal{I}_2\}$

while $X_1 \neq \emptyset$ 且 $X_2 \neq \emptyset$ **do**

if $e \in X_1 \cap X_2$ **then**

$I \leftarrow I \cup \{e\}$

else

 構造交換圖 $\mathcal{D}_{M_1, M_2}(I)$

 在交換圖上找到一條 X_1 到 X_2 且沒有捷徑的路徑 P

$I \leftarrow I \Delta P$

end if

$X_1 \leftarrow \{e \in E \setminus I \mid I \cup \{e\} \in \mathcal{I}_1\}$

$X_2 \leftarrow \{e \in E \setminus I \mid I \cup \{e\} \in \mathcal{I}_2\}$

end while

8 Polynomial

8.1 FFT

```
using val_t = complex<double>;
template<int MAXN>
struct FFT {
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(vector<val_t> &a, int n) //same as NTT
    void trans(vector<val_t> &a, int n, bool inv = false) {
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    val_t tmp = a[j + dl] * (inv ? conj(w[x]) : w[x]);
                    ;
                    a[j + dl] = a[j] - tmp;
                    a[j] += tmp;
                }
            }
        }
    }
    if (inv) {
        for (int i = 0; i < n; ++i) a[i] /= n;
    }
}
```

```

    }
}
//multiplying two polynomials A * B:
//fft.trans(A, siz, 0), fft.trans(B, siz, 0):
//A[i] *= B[i], fft.trans(A, siz, 1);
};

```

8.2 NTT

```

//to be modified
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, ll P, ll RT> //MAXN must be 2^k
struct NTT {
    ll w[MAXN];
    ll mpow(ll a, ll n);
    ll minv(ll a) { return mpow(a, P - 2); }
    NTT() {
        ll dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
    }
    void bitrev(vector<ll> &a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(vector<ll> &a, int n, bool inv = false) {
        //0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    ll tmp = a[j + dl] * w[x] % P;
                    if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a.begin() + 1, a.begin() + n);
            ll invn = minv(n);
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
        }
    }
};

```

8.3 Polynomial Operation

```

// Copy from 8BQube
#define fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
template<int MAXN, ll P, ll RT> // MAXN = 2^k
struct Poly : vector<ll> { // coefficients in [0, P)
    using vector<ll>::vector;
    static NTT<MAXN, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int m) : vector<ll>(m) {
        copy_n(p.data(), min(p.n(), m), data());
    }
    Poly& irev() { return reverse(data(), data() + n()), *this; }
    Poly& isz(int m) { return resize(m), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if (((*this)[i] += rhs[i]) >= P) (*this)[i] -= P;
        return *this;
    }
    Poly& imul(ll k) {
        fi(0, n()) (*this)[i] = (*this)[i] * k % P;
    }
};

```

```

    return *this;
}
Poly Mul(const Poly &rhs) const {
    int m = 1;
    while (m < n() + rhs.n() - 1) m <= 1;
    Poly X(*this, m), Y(rhs, m);
    ntt(X.data(), m), ntt(Y.data(), m);
    fi(0, m) X[i] = X[i] * Y[i] % P;
    ntt(X.data(), m, true);
    return X.isz(n() + rhs.n() - 1);
}
Poly Inv() const { // (*this)[0] != 0, 1e5/95ms
    if (n() == 1) return {ntt.minv((*this)[0])};
    int m = 1;
    while (m < n() * 2) m <= 1;
    Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(m);
    Poly Y(*this, m);
    ntt(Xi.data(), m), ntt(Y.data(), m);
    fi(0, m) {
        Xi[i] *= (2 - Xi[i] * Y[i]) % P;
        if ((Xi[i] %= P) < 0) Xi[i] += P;
    }
    ntt(Xi.data(), m, true);
    return Xi.isz(n());
}
Poly Sqrt() const { // Jacobi((*this)[0], P) = 1, 1e5/235ms
    if (n() == 1) return {QuadraticResidue((*this)[0], P)};
    Poly X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
    return X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 + 1);
}
pair<Poly, Poly> DivMod(const Poly &rhs) const { // (rhs.back() != 0)
    if (n() < rhs.n()) return {{0}, *this};
    const int m = n() - rhs.n() + 1;
    Poly X(rhs); X.irev().isz(m);
    Poly Y(*this); Y.irev().isz(m);
    Poly Q = Y.Mul(X.Inv()).isz(m).irev();
    X = rhs.Mul(Q), Y = *this;
    fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
    return {Q, Y.isz(max(1, rhs.n() - 1))};
}
Poly Dx() const {
    Poly ret(n() - 1);
    fi(0, ret.n()) ret[i] = (i + 1) * (*this)[i + 1] % P;
    return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n()) ret[i + 1] = ntt.minv(i + 1) * (*this)[i] % P;
    return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<ll> _eval(const vector<ll> &x, const vector<Poly> &up) const {
    const int m = (int)x.size();
    if (!m) return {};
    vector<Poly> down(m * 2);
    // down[1] = DivMod(up[1]).second;
    // fi(2, m * 2) down[i] = down[i / 2].DivMod(up[i]).second;
    down[1] = Poly(up[1]).irev().isz(n()).Inv().irev()._tmul(m, *this);
    fi(2, m * 2) down[i] = up[i ^ 1]._tmul(up[i].n() - 1, down[i / 2]);
    vector<ll> y(m);
    fi(0, m) y[i] = down[m + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<ll> &x) {
    const int m = (int)x.size();
    vector<Poly> up(m * 2);
    fi(0, m) up[m + i] = {(x[i] ? P - x[i] : 0), 1};
};

```

```

    for (int i = m - 1; i > 0; --i) up[i] = up[i * 2].Mul(up[
        i * 2 + 1]);
    return up;
}
vector<ll> Eval(const vector<ll> &x) const { // 1e5, 1s
    auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector<ll> &x, const vector<
    ll> &y) { // 1e5, 1.4s
    const int m = (int)x.size();
    vector<Poly> up = _tree1(x), down(m * 2);
    vector<ll> z = up[1].Dx()._eval(x, up);
    fi(0, m) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, m) down[m + i] = {z[i]};
    for (int i = m - 1; i > 0; --i) down[i] = down[i * 2].Mul
        (up[i * 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
Poly Ln() const { // (*this)[0] == 1, 1e5/170ms
    return Dx().Mul(Inv()).Sz().isz(n());
}
Poly Exp() const { // (*this)[0] == 0, 1e5/360ms
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n()) if ((Y[i] == *this[i] - Y[i]) < 0) Y[i] += P;
    return X.Mul(Y).isz(n());
}
// M := P(P - 1). If k >= M, k := k % M + M.
Poly Pow(ll k) const {
    int nz = 0;
    while (nz < n() && !(*this)[nz]) ++nz;
    if (nz * min(k, (ll)n()) >= n()) return Poly(n());
    if (!k) return Poly(Poly {1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * k);
    const ll c = ntt.mpow(X[0], k % (P - 1));
    return X.Ln().imul(k % P).Exp().imul(c).irev().isz(n()).
        irev();
}
static ll LinearRecursion(const vector<ll> &a, const
    vector<ll> &coef, ll n) { // a_n = \sum c_j a_{n-j}
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly {1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    ll ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
}
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

8.4 Generating Function

8.4.1 Ordinary Generating Function

- $C(x) = A(rx)$: $c_n = r^n a_n$ 的一般生成函數。
- $C(x) = A(x) + B(x)$: $c_n = a_n + b_n$ 的一般生成函數。
- $C(x) = A(x)B(x)$: $c_n = \sum_{i=0}^n a_i b_{n-i}$ 的一般生成函數。
- $C(x) = A(x)^k$: $c_n = \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$ 的一般生成函數。
- $C(x) = xA(x)'$: $c_n = na_n$ 的一般生成函數。
- $C(x) = \frac{A(x)}{1-x}$: $c_n = \sum_{i=0}^n a_i$ 的一般生成函數。

常用展開式

- $\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots$

- $(1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n$, $\binom{a}{n} = \frac{a(a-1)(a-2)\dots(a-n+1)}{n!}$.

8.4.2 Exponential Generating Function

a_0, a_1, \dots 的指數生成函數：

$$\hat{A}(x) = \sum_{i=0}^{\infty} \frac{a_i}{i!} = a_0 + a_1 x + \frac{a_2}{2!} x^2 + \frac{a_3}{3!} x^3 + \dots$$

- $\hat{C}(x) = \hat{A}(x) + \hat{B}(x)$: $c_n = a_n + b_n$ 的指數生成函數
- $\hat{C}(x) = \hat{A}^{(k)}(x)$: $c_n = a_{n+k}$ 的指數生成函數
- $\hat{C}(x) = x\hat{A}(x)$: $c_n = na_n$ 的指數生成函數
- $\hat{C}(x) = \hat{A}(x)\hat{B}(x)$: $c_n = \sum_{k=0}^n \binom{n}{k} a_k b_{n-k}$ 的指數生成函數
- $\hat{C}(x) = \hat{A}(x)^k$: $\sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$ 的指數生成函數

9 String

9.1 Rolling Hash

```

int n;
string s;
vector<ll> h, rh;
vector<ll> kp;
const ll K = 26, MOD = 1000000007;

void topos(ll &a){
    a = (a % MOD + MOD) % MOD;
}

int ord(char c){
    return c - 'a';
}

pll geth(int l, int r){
    if (l > r) return mp(0, 0);
    ll ans = h[r] - h[l - 1] * kp[r - l + 1];
    topos(ans);
    return mp(ans, r - l + 1);
}

pll getrh(int l, int r){
    if (l > r) return mp(0, 0);
    l = n - l + 1;
    r = n - r + 1;
    swap(l, r);
    ll ans = rh[r] - rh[l - 1] * kp[r - l + 1];
    topos(ans);
    return mp(ans, r - l + 1);
}

pll concat(pll a, pll b){
    ll ans = a.F * kp[b.S] + b.F;
    ans %= MOD;
    return mp(ans, a.S + b.S);
}

void build(){
    n = s.size();
    s = " " + s;

    h.resize(n + 1);
    rh.resize(n + 1);
    kp.resize(n + 1);
    kp[0] = 1;
    for(int i = 1; i <= n; i++){
        kp[i] = kp[i - 1] * K % MOD;
    }
    for(int i = 1; i <= n; i++){
        h[i] = h[i - 1] * K % MOD + ord(s[i]);
        h[i] %= MOD;
        rh[i] = rh[i - 1] * K % MOD + ord(s[n - i + 1]);
        rh[i] %= MOD;
    }
}

```

}

9.2 KMP Algorithm

```
void kmp(string s){
    int siz = s.size();
    vector<int> f(siz, 0);
    f[0] = 0;
    for (int i = 1; i < siz; i++) {
        f[i] = f[i-1];
        bool zero = 0;
        while (s[f[i]] != s[i]) {
            if (f[i] == 0) {
                zero = 1;
                break;
            }
            f[i] = f[f[i]-1];
        }
        if (!zero) f[i]++;
    }
}
```

9.3 Manacher Algorithm

```
vector<int> manacher(string s) {
    int n = s.size();
    vector<int> v(n);
    int pnt = -1, len = 1;
    for (int i = 0; i < n; i++) {
        int cor = 2 * pnt - i;
        if (cor >= 0) v[i] = min(v[cor], cor - pnt + len);
        while (i+v[i] < n && i-v[i] >= 0 && s[i+v[i]] == s[i-v[i]]) v[i]++;
        if (i + v[i] >= pnt + len) pnt = i, len = v[i];
    }
    for (int i = 0; i < n; i++) v[i] = 2 * v[i] - 1;
    return v;
}
```

9.4 MCP

```
string mcp(string s) { //Duval algorithm for Lyndon
    factorization
    s += s;
    int n = s.size(), i = 0, ans = 0;
    while (i < n/2) {
        ans = i;
        int j = i+1, k=i;
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j]) k = i;
            else k++;
            j++;
        }
        while (i <= k) i += j - k;
    }
    return s.substr(ans, n/2);
}
```

9.5 Suffix Array

```
struct SuffixArray { //tested
    vector<int> sa, lcp, rank; //lcp[i] is lcp of sa[i] and
    sa[i-1]
    SuffixArray(string& s, int lim=256) { // or basic_string<
        int>
        int n = s.size() + 1, k = 0, a, b;
        vector<int> x(n, 0), y(n), ws(max(n, lim));
        rank.resize(n);
        for (int i = 0; i < n-1; i++) x[i] = (int)s[i];
        sa = lcp = y, iota(sa.begin(), sa.end(), 0);
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim =
            p) {
```

```
        p = j, iota(y.begin(), y.end(), n - j);
        for (int i = 0; i < n; i++) if (sa[i] >= j) y[p++] = sa
            [i] - j;
        for (int &i : ws) i = 0;
        for (int i = 0; i < n; i++) ws[x[i]]++;
        for (int i = 1; i < lim; i++) ws[i] += ws[i - 1];
        for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
        swap(x, y), p = 1, x[sa[0]] = 0;
        for (int i = 1; i < n; i++) a = sa[i - 1], b = sa[i], x[
            b] =
            (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p
            ++;
    }
    for (int i = 1; i < n; i++) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
        for (k && k--, j = sa[rank[i] - 1];
            s[i + k] == s[j + k]; k++);
    }
};
```

9.6 Suffix Array Automaton

```
//to be modified
const int MAXM = 1000010;
struct SAM {
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode() {
        int res = ++tot;
        fill(nxt[res], nxt[res] + 33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init() {
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c) {
        int p = lst;
        int np = newNode();
        mx[np] = mx[p] + 1;
        for (; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if (p == 0) mom[np] = root;
        else {
            int q = nxt[p][c];
            if (mx[p] + 1 == mx[q]) mom[np] = q;
            else {
                int nq = newNode();
                mx[nq] = mx[p] + 1;
                for (int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for (; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void push(char *str) {
        for (int i = 0; str[i]; i++)
            push(str[i] - 'a' + 1);
    }
} sam;
```

9.7 Z-value Algorithm

```
vector<int> z_function(string const& s) {
    int n = s.size();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; i++) {
```



```

    if (i <= r) z[i] = min(r-i+1, z[i-l]);
    while (i + z[i] < n && s[z[i]] == s[i+z[i]])
        z[i]++;
    if (i + z[i] - 1 > r)
        l = i, r = i + z[i] - 1;
}
return z;
}

```

9.8 Main Lorentz

```

vector<int> z_function(string const& s);
int get_z(vector<int> const& z, int i) {
    return (0 <= i && i < SZ(z)) ? z[i] : 0;
}
vector<pair<int, int>> rep;

void convert_to_rep(int shift, bool left, int cntr, int l,
    int k1, int k2) {
    for (int l1 = max(1, l - k2); l1 <= min(l, k1); l1++) {
        if (left && l1 == l) break;
        int l2 = l - l1;
        int pos = shift + (left ? cntr - l1 : cntr - l - l1 + 1);
        rep.emplace_back(pos, pos + 2*l - 1);
    }
}

void find_rep(string s, int shift = 0) {
    int n = s.size();
    if (n == 1) return;

    int nu = n / 2;
    int nv = n - nu;
    string u = s.substr(0, nu);
    string v = s.substr(nu);
    string ru(u.rbegin(), u.rend());
    string rv(v.rbegin(), v.rend());

    find_rep(u, shift);
    find_rep(v, shift + nu);

    vector<int> z1 = z_function(ru);
    vector<int> z2 = z_function(v + '#' + u);
    vector<int> z3 = z_function(ru + '#' + rv);
    vector<int> z4 = z_function(v);

    for (int cntr = 0; cntr < n; cntr++) {
        int l, k1, k2;
        if (cntr < nu) {
            l = nu - cntr;
            k1 = get_z(z1, nu - cntr);
            k2 = get_z(z2, nv + 1 + cntr);
        } else {
            l = cntr - nu + 1;
            k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
            k2 = get_z(z4, (cntr - nu) + 1);
        }
        if (k1 + k2 >= l)
            convert_to_rep(shift, cntr < nu, cntr, l, k1, k2);
    }
}

```

9.9 AC Automaton

```

// copy from nontoi
struct AhoCorasick {
    enum { P = 26, st = 'a'};
    struct node { // zero-based
        array<int, P> ch = {0};
        int fail = 0, cnt = 0, dep = 0;
    };
    int cnt;
    vector<node> v;
    vector<int> ans;
}

```

```

void init_(int mx) {
    v.clear();
    cnt = 1, v.resize(mx);
    v[0].fail = 0;
}

void insert(string s) {
    int p = 0, dep = 1;
    for(auto i : s) {
        int c = i - st;
        if(!v[p].ch[c]) {
            v[cnt].dep = dep;
            v[p].ch[c] = cnt++;
        }
        p = v[p].ch[c], dep++;
    }
    v[p].cnt++;
}

void build(vector<string> s) {
    for(auto i : s) insert(i);
    queue<int> q;
    for(int i = 0; i < P; i++) {
        if(v[0].ch[i]) q.push(v[0].ch[i]);
    }
    while(q.size()) {
        int p = q.front();
        q.pop();
        for(int i = 0; i < P; i++) if(v[p].ch[i]) {
            int to = v[p].ch[i], cur = v[p].fail;
            while(cur && !v[cur].ch[i]) cur = v[cur].fail;
            if(v[cur].ch[i]) cur = v[cur].ch[i];
            v[to].fail = cur;
            v[to].cnt += v[cur].cnt;
            q.push(to);
        }
    }
}

void traverse(string s) {
    int p = 0;
    ans.assign(cnt, 0);
    for(auto i : s) {
        int c = i - st;
        while(p && !v[p].ch[c]) p = v[p].fail;
        if(v[p].ch[c]) {
            p = v[p].ch[c];
            ans[p]++, v[p].cnt;
        }
    }
    vector<int> ord(cnt, 0);
    iota(all(ord), 0);
    sort(all(ord), [&](int a, int b) { return v[a].dep > v[b].dep; });
    for(auto i : ord) ans[v[i].fail] += ans[i];
    return;
}

int go(string s) {
    int p = 0;
    for(auto i : s) {
        int c = i - st;
        assert(v[p].ch[c]);
        p = v[p].ch[c];
    }
    return ans[p];
}
};

```

10 Formula

10.1 Recurrences

If $a_n = c_1 a_{n-1} + \dots + c_k a_{n-k}$, and r_1, \dots, r_k are distinct roots of $x^k + c_1 x^{k-1} + \dots + c_k$, there are d_1, \dots, d_k s.t.

$$a_n = d_1 r_1^n + \dots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

10.2 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$

$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$

$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W) \tan(v-w)/2 = (V-W) \tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w .

$$a \cos x + b \sin x = r \cos(x - \phi)$$

$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}$, $\phi = \text{atan2}(b, a)$.

10.3 Geometry

10.3.1 Triangles

Side lengths: a, b, c

$$\text{Semiperimeter: } p = \frac{a+b+c}{2}$$

$$\text{Area: } A = \sqrt{p(p-a)(p-b)(p-c)}$$

$$\text{Circumradius: } R = \frac{abc}{4A}$$

$$\text{Inradius: } r = \frac{A}{p}$$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc \left(1 - \left(\frac{a}{b+c}\right)^2\right)}$

$$\text{Law of sines: } \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$$

$$\text{Law of cosines: } a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$\text{Law of tangents: } \frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$$

Incenter:

$$P_1 = (x_1, y_1), P_2 = (x_2, y_2), P_3 = (x_3, y_3)$$

$$s_1 = P_2P_3, s_2 = P_1P_3, s_3 = P_1P_2$$

$$s_1P_1 + s_2P_2 + s_3P_3$$

$$\frac{s_1 + s_2 + s_3}{3}$$

Circumcenter:

$$P_0 = (0, 0), P_1 = (x_1, y_1), P_2 = (x_2, y_2)$$

$$x_c = \frac{1}{2} \times \frac{y_2(x_1^2 + y_1^2) - y_1(x_2^2 + y_2^2)}{-x_2y_1 + x_1y_2}$$

$$y_c = \frac{1}{2} \times \frac{x_2(x_1^2 + y_1^2) - x_1(x_2^2 + y_2^2)}{-x_1y_2 + x_2y_1}$$

Check if (x_0, y_0) is in the circumcircle:

$$\begin{vmatrix} x_1 - x_0 & y_1 - y_0 & (x_1^2 + y_1^2) - (x_0^2 + y_0^2) \\ x_2 - x_0 & y_2 - y_0 & (x_2^2 + y_2^2) - (x_0^2 + y_0^2) \\ x_3 - x_0 & y_3 - y_0 & (x_3^2 + y_3^2) - (x_0^2 + y_0^2) \end{vmatrix}$$

0: on edge, > 0 : inside, < 0 : outside

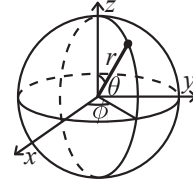
10.3.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

10.3.3 Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \text{atan2}(y, x) \end{aligned}$$

10.3.4 Green's Theorem

$$\text{Area} = \frac{1}{2} \int_C x \, dy - y \, dx$$

Circular sector:

$$x = x_0 + r \cos \theta$$

$$y = y_0 + r \sin \theta$$

$$\begin{aligned} A &= r \int_{\alpha}^{\beta} (y_0 + \sin \theta) \cos \theta + (x_0 + \cos \theta) \sin \theta \, d\theta \\ &= r(r\theta + x_0 \sin \theta - y_0 \cos \theta)|_{\alpha}^{\beta} \end{aligned}$$

10.4 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan x = 1 + \tan^2 x$$

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln |\cos ax|}{a}$$

$$\int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \text{erf}(x)$$

$$\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

10.5 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

10.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

10.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x . It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y ,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

10.7.1 Discrete distributions

Binomial distribution The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n, p)$, $n = 1, 2, \dots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$ is approximately $\text{Po}(np)$ for small p .

First success distribution The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\text{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

10.7.2 Continuous distributions

Uniform distribution If the probability density function is constant between a and b and 0 elsewhere it is $\text{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

10.8 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let X_1, X_2, \dots be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i . π_j/π_i is the expected number of visits in state j between two visits in state i .

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i 's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k \rightarrow \infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an *A-chain* if the states can be partitioned into two sets \mathbf{A} and \mathbf{G} , such that all states in \mathbf{A} are absorbing ($p_{ii} = 1$), and all states in \mathbf{G} leads to an absorbing state in \mathbf{A} . The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j , is $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is i , is $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ki} t_k$.

