$y' = x e^{x^2-y} + \frac{2x}{e^y} \qquad y(1) = 1 - \ln 2$

$y' = x e^{x^2} \cdot e^{-y} + 2x e^{-y} = e^{-y} \cdot (x e^{x^2} + 2x) = h(y) g(x)$ szeparábilis $\Rightarrow$

$\Rightarrow \int \frac{1}{h(y)} dy = \int g(x) dx$

$e^y = \int e^y dy = \int x e^{x^2} + 2x dx = \frac{1}{2} \int \underbrace{2x e^{x^2}}_{\text{" } f'(x) \cdot e^{f(x)} \text{ "}} dx + 2 \int x dx = \frac{1}{2} e^{x^2} + x^2 + C$

$\Rightarrow y(x) = \ln\left(\frac{1}{2} e^{x^2} + x^2 + C\right) \qquad \begin{pmatrix} y \in \mathbb{R} \\ \frac{1}{2} e^{x^2} + x^2 + C \in (0, +\infty) \end{pmatrix}$

Kezdeti értékből C meghatározható:

$y(1) = 1 - \ln 2 \iff \ln\left(\frac{1}{2} \cdot e^{1^2} + 1^2 + C\right) = 1 - \ln 2$

$\ln\left(\frac{1}{2} e + 1 + C\right) = 1 - \ln 2 = \ln e - \ln 2 = \ln \frac{e}{2}$

$\iff \boxed{C = -1}$

k.é.p. megoldása:

$\Rightarrow y(x) = \ln\left(\frac{1}{2} e^{x^2} + x^2 - 1\right) \qquad \begin{pmatrix} \text{pl. } x \in [1, +\infty) =: I \\ y \in \mathbb{R} =: J \end{pmatrix}$