Minta zh

ⁱ itt nem lehet x, y

2. $$\iint_H x^2+y^2 = \int_0^{1-2x}\underbrace{\int_0^{1/2} x^2+y^2 \, dx\, dy}$$

$H: \; 0 \le x \le \frac{1}{2}$

$\quad\; 0 \le y \le 1-2x$

$$= \int_0^{1/2}\int_0^{1-2x} x^2+y^2 \; dy\, dx = \int_0^{1/2} \underbrace{\left[x^2 y + \frac{y^3}{3}\right]_0^{1-2x}} dx$$

$$x^2(1-2x) + \frac{(1-2x)^3}{3} - 0$$

$$x^2 - 2x^3 + \frac{(1-2x)^3}{3}$$

$$\int_0^{1/2} x^2 - 2x^3 + \frac{(1-2x)^3}{3}\, dx = \left[\frac{x^3}{3} - 2\frac{x^4}{4} - \frac{1}{24}(1-2x)^4\right]_0^{1/2}$$

$$\left(\int \frac{(1-2x)^3}{3}\, dx = -\frac{1}{2}\cdot\frac{1}{3}\int \underbrace{(1-2x)^3}_{f}\cdot \underbrace{-2}_{f'} = -\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{(1-2x)^4}{4} \right.$$

kell még f'

SZABÁLY: $\int f'\cdot f^\alpha \, dx = \frac{f^{\alpha+1}}{\alpha+1}$

$$\left. = \underline{\underline{-\frac{1}{24}(1-2x)^4}} \right)$$

$$= \frac{\left(\frac{1}{2}\right)^3}{3} - \frac{\left(\frac{1}{2}\right)^4}{2} - \frac{1}{24}\underbrace{\left(1-\cancel{2}\cdot\frac{1}{\cancel{2}}\right)^4}_{0} - \left(-\frac{1}{24}\right)$$

$$\frac{\frac{1}{8}}{3} - \frac{\frac{1}{16}}{2} - \left(-\frac{1}{24}\right):$$

$$\frac{1}{8} \cdot \frac{1}{3} - \frac{1}{16} \cdot \frac{1}{2} - \left(-\frac{1}{24}\right)$$

$$\frac{1}{24} - \frac{1}{32} - \left(-\frac{1}{24}\right)$$

$$\frac{1}{96} - \left(-\frac{1}{24}\right) = \frac{1+4}{96} = \frac{5}{96}$$

4. $x^y = y^x$

$F(x,y) = x^y - y^x$

IFT:

- $F: \mathbb{R}^2 \to \mathbb{R}$, C^1 ✓
- $F(x_0, y_0) = 0$

$$x_0 = \frac{9}{4}, \quad y_0 = \frac{27}{8}$$

$$F\left(\frac{9}{4}, \frac{27}{8}\right) = 0 \quad ✓$$

- $\partial_2 F(x_0, y_0) \neq 0$ (regularitás)

$\partial_1 (x^y) = y \cdot x^{y-1}$

$\partial_2 (x^y) = x^y \cdot \ln x$

$\partial_2 (y^x) = x \cdot y^{x-1}$

$\partial_1 (y^x) = y^x \cdot \ln x$

$$\partial_1 F = y \cdot x^{y-1} - y^x \ln y \qquad x = \frac{9}{4}$$

$$\partial_2 F = x^y \cdot \ln x - x \cdot y^{x-1} \qquad y = \frac{27}{8}$$

$$\left(\frac{9}{4}\right)^{\frac{27}{8}} \cdot \ln \frac{9}{4} - \frac{9}{4} \cdot \left(\frac{27}{8}\right)^{\left(\frac{5}{4}\right)} \neq 0$$

$\Rightarrow \exists \varepsilon > 0$

$\Rightarrow \exists! \ \varphi : \left(\frac{9}{4} - \varepsilon, \ \frac{9}{4} + \varepsilon\right) \longrightarrow \mathbb{R}, \quad \varphi \in C^1$

úgy hogy

$$\varphi(x_0) = y_0$$

$$\varphi'(x_0) = - \frac{\partial_1 F(x_0, y_0)}{\partial_2 F(x_0, y_0)}$$

$$\varphi'\left(\frac{9}{4}\right) = - \frac{\frac{27}{8} \cdot \left(\frac{9}{4}\right)^{\frac{19}{8}} - \left(\frac{27}{8}\right)^{\frac{9}{4}} \cdot \ln\left(\frac{27}{8}\right)}{\left(\frac{9}{4}\right)^{\frac{27}{8}} \ln\left(\frac{9}{4}\right) - \frac{9}{4}\left(\frac{27}{8}\right)^{\frac{5}{4}}}$$

mon. csökken

$(1+\lambda)$ ... ∇ ... ált.

5. $f(x,y) = x^4 + \frac{1}{16}y^4$

$g(x,y) = 0$ mellékfeltétel

$g(x,y) = x + \frac{y}{2} + 1$

$g: \mathbb{R}^2 \to \mathbb{R}^1$

$f|_{\{g=0\}}$ ext.

OH nem működik a Lagrange-módszer akkor a rang felt. sérül:

$\iff g'(x,y) = (1 \quad 1/2)$ rangja max

$\iff$ rang $= 1$

att sérülne a rang-feltétel akkor g' rangja 0 lenne (0 mátrix)

$\Rightarrow$ ez most nem lehet

$\Rightarrow$ nincsenek extra helyek

$L(x,y,\lambda) = f(x,y) + \lambda \cdot g(x,y)$

$L(x,y,\lambda) = x^4 + \frac{1}{16}y^4 + \lambda \cdot \left(x + \frac{y}{2} + 1\right)$

1. rendű szükséges feltétel

$\nabla L = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\nabla L = \begin{pmatrix} 4x^3 + \lambda = 0 \\ \frac{1}{16} 4 \cdot y^3 + \frac{1}{2}\lambda = 0 \\ x + \frac{y}{2} + 1 = 0 \end{pmatrix}$ $\quad \frac{y^3}{4} + \frac{1}{2}\lambda$

I. $4x^3 + \lambda = 0 \quad \Rightarrow \quad \lambda = -4x^3$

II. $\frac{y^3}{4} + \frac{\lambda}{2} = 0 \qquad \frac{y^3}{4} - 2x^3 = 0$

III. $x + \frac{y}{2} + 1 = 0$

$$\frac{y^3}{4} = 2x^3 \quad | \cdot 4$$

$$y^3 = 8x^3$$

$$y = 2x$$

$$\frac{8x^3}{4} - \frac{4x^3}{2} = 2x^3 - 2x^3$$

$$y = -1$$

$$x + \frac{2x}{2} + 1$$

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2} \qquad \lambda = \frac{1}{2}$$

$$-\frac{1}{8}$$

$\exists$ sec: $x = -\frac{1}{2}, \quad y = -1, \quad \lambda = \frac{1}{2}$

$$L''(x, y, \lambda) = \begin{pmatrix} 12x^2 & 0 & 1 \\ 0 & \frac{3}{4}y^2 & \frac{1}{2} \\ 1 & \frac{1}{2} & 0 \end{pmatrix}$$

$3\frac{y^2}{4}$

$$L''\left(-\frac{1}{2}, -1, \frac{1}{2}\right) = \begin{pmatrix} 12 \cdot \frac{1}{4} & 0 \\ 0 & \frac{3}{4} \end{pmatrix} = \begin{pmatrix} \boxed{3} & 0 \\ 0 & \frac{3}{4} \end{pmatrix}$$

$(+, +, \dots +)$

poz. definit $\Rightarrow$

$\Rightarrow \exists$ lok. min hely

$x = -\frac{1}{2} \quad y = -1$ - ben!

$$\iiint_V \sin\left(\sqrt{x^2+y^2+z^2}\right)$$

$$x^2+y^2+z^2 \le 1 \quad , \quad x \ge 0 , \; y \ge 0 , \; z \ge 0$$

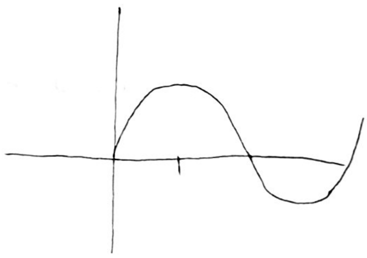
$$\iiint_V \sin\left(\sqrt{r^2\cos^2\varphi\cdot\cos^2\theta + r^2\sin^2\varphi\cos^2\theta + r^2\sin^2\theta}\right) \cdot r^2\cdot\cos \; dr \; d\varphi \; d$$

$$r^2\left(\cos^2\varphi\,\underline{\cos^2\theta} + \sin^2\varphi\,\underline{\cos^2\theta} + \sin^2\theta\right)$$

$$\underbrace{\cos^2\theta\,\underbrace{\left(\cos^2\varphi + \sin^2\varphi\right)}_{1} + \sin^2\theta}_{1}$$

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^{1} \underbrace{\sin\left(\sqrt{r^2\cdot 1}\right)\cdot r^2\cos\theta}_{\sin r \,\cdot\, r^2\cos\theta} \; dr \; d\varphi \; d\theta$$

$$\int_0^{\frac{\pi}{2}}\int_0^{1}\int_0^{\frac{\pi}{2}} \sin r\cdot r^2\cdot\cos\theta \; d\varphi \; dr \; d\theta$$



$$\int_0^{\frac{\pi}{2}}\int_0^{1} \sin r\cdot r^2\cdot\cos\theta\left(\frac{\pi}{2}-0\right) dr \; d\theta \qquad \overset{1}{\sin\frac{\pi}{2}} - \overset{0}{\sin 0}$$

$$\int_0^{1}\int_0^{\frac{\pi}{2}} \underbrace{\frac{\pi}{2}\sin r\cdot r^2}_{*}\cdot\cos\theta \; d\theta \; dr = \int_0^{1} * \left[\sin\theta\right]_0^{\frac{\pi}{2}}$$

$$\int_0^1 \frac{\pi}{2} \sin x \cdot x^2 \, dx$$

$$\frac{\pi}{2} \int_0^1 \underset{g'}{\sin x} \cdot \underset{f}{x^2} \, dx =$$

$g = -\cos x \quad f' = 2x$

$$\int f g' = f g - \int f' g$$

$$-x^2 \cos x + 2 \int \underset{f}{x} \, \underset{g'}{\cos x}$$

$f' = 1 \quad g = \sin x$

$$\underbrace{-x^2 \cos x + 2 \Big(x \cdot \sin x - \underbrace{\int 1 \cdot \sin x}_{-\cos x} \Big)}_{x \sin x + \cos x}$$

$$\Big[-x^2 \cos x + 2 x \sin x + 2 \cos x \Big]_0^1$$

$$\frac{\pi}{2}\left(-\cancel{1}^2\cdot\cos 1+2\sin 1+2\cos 1-\left(2\cos 0\right)\right)$$

$$-\cancel{\cos} 1+2\sin 1+2\cos 1-2$$

$$2\sin 1+\cos 1-2$$

$$\underbrace{\pi\cdot\sin 1+\frac{\pi}{2}\cos 1-\pi}$$