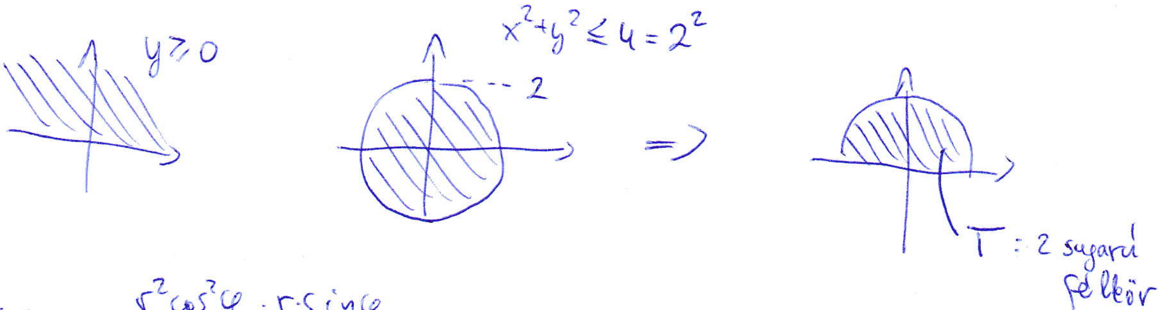
1. $f(x,y)=\dfrac{x^2y}{x^2+y^2}$



$$f(x,y)=f(r\cos\varphi, r\sin\varphi)=\frac{r^2\cos^2\varphi\cdot r\sin\varphi}{r^2}=r\cos^2\varphi\sin\varphi$$

$$\iint_T f(x,y)\,dx\,dy=\int_0^{\pi}\int_0^2 r\cos^2\varphi\sin\varphi\cdot \underset{\substack{\uparrow\\ (\text{helyettesítés!})}}{r}\,dr\,d\varphi=\int_0^{\pi}\int_0^2 r^2\cos^2\varphi\sin\varphi\,dr\,d\varphi=$$

$$=\int_0^{\pi}\underbrace{\cos^2\varphi}_{f^2}\underbrace{\sin\varphi}_{-f'}\,d\varphi\int_0^2 r^2\,dr=\left[-\frac{\cos^3\varphi}{3}\right]_0^{\pi}\cdot\left[\frac{r^3}{3}\right]_0^2=-\frac{1}{3}(-1-1)\cdot\left(\frac{8}{3}-0\right)=\frac{16}{9}$$

2. $f(x,y)=\left(\dfrac{e^{x+y}}{x^2},\ y\sin\left(xy+\frac{\pi}{2}\right)\right)\qquad a=(1,0)$

$$f(a)=f(1,0)=\left(\frac{e^{1+0}}{1^2},\ 0\cdot\sin\left(1\cdot 0+\frac{\pi}{2}\right)\right)=(e,0)$$

$\partial_1 f_1(x,y)=e^{x+y}\cdot\frac{1}{x^2}+e^{x+y}\cdot\left(-\frac{2}{x^3}\right)\ \Rightarrow\ \partial_1 f_1(a)=e-2e=-e$

$\partial_1 f_2(x,y)=y\cos\left(xy+\frac{\pi}{2}\right)\cdot y\ \Rightarrow\ \partial_1 f_2(a)=0$

$\partial_2 f_1(x,y)=e^{x+y}\cdot\frac{1}{x^2}\ \Rightarrow\ \partial_2 f_1(a)=e$

$\partial_2 f_2(x,y)=\sin\left(xy+\frac{\pi}{2}\right)+y\cos\left(xy+\frac{\pi}{2}\right)\cdot x\ \Rightarrow\ \partial_2 f_2(a)=\sin\frac{\pi}{2}+0=1$

$$\Bigg\}\ f'(a)=\begin{pmatrix}-e & e\\ 0 & 1\end{pmatrix}$$

$\det f'(a)=-e\cdot 1-e\cdot 0=-e\neq 0\ \xRightarrow{\text{Inverz fv. tétel}}$ $\exists\, U\subset\mathbb{R}$ $\exists\, V\subset\mathbb{R}^2$ nyíltak $a\in A$ $f(a)\in V$
$f_{|U}: U\to V$ bijekció $f^{-1}\in C^1$ és
$(f^{-1})'(f(a))=(f'(a))^{-1}$ $\quad(a\in U,\ f(a)\in V)$

$$\Rightarrow (f^{-1})'(f(a))=(f^{-1})'(e,0)=(f'(a))^{-1}=\begin{pmatrix}-e & e\\ 0 & 1\end{pmatrix}^{-1}=\underset{\substack{\uparrow\\ \text{determináns}}}{\frac{1}{-e}}\cdot\begin{pmatrix}1 & -e\\ 0 & -e\end{pmatrix}=\begin{pmatrix}-1/e & 1\\ 0 & 1\end{pmatrix}$$