3. $\left.\begin{array}{l} e^{x_1+x_2x_3}-x_2^2=0 \\ 2x_1x_3+x_2-x_3^2=0 \end{array}\right\}$ $f(x_1;x_2,x_3)=\begin{pmatrix} e^{x_1+x_2x_3}-x_2^2 \\ 2x_1x_3+x_2-x_3^2 \end{pmatrix}$

$(\Omega_1\subset\mathbb{R} \quad \Omega_2\subset\mathbb{R}^2 \quad f:\Omega_1\times\Omega_2\to\mathbb{R}^2)$

$a=1 \quad \varphi(a)=b=\begin{pmatrix}-1\\1\end{pmatrix}$, ekkor $f(a;b)=f(a;\varphi(a))=\begin{pmatrix} e^{1+(-1)\cdot 1}-(-1)^2 \\ 2\cdot 1\cdot 1+(-1)-(-1)^2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$\partial_2 f(x_1;x_2,x_3)=\begin{pmatrix} e^{x_1+x_2x_3}\cdot x_3-2x_2 & e^{x_1+x_2x_3}\cdot x_2 \\ 1 & 2x_1-x_3\end{pmatrix}$

" $\partial_2=\begin{pmatrix}\frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3}\end{pmatrix}$ " $\qquad \partial/\partial x_2 \qquad \partial/\partial x_3$

$\partial_2 f(a;b)=\partial_2 f(1;-1,1)=\begin{pmatrix} e^0\cdot 1-2(-1) & e^0\cdot(-1) \\ 1 & 2-2\end{pmatrix}=\begin{pmatrix}3 & -1\\ 1 & 0\end{pmatrix}:=A$

$\det \partial_2 f(a;b)=\det A=3\cdot 0-(-1)\cdot 1=1\neq 0$

Impl. fv. tétel

$\Rightarrow$ $\exists\varphi:\mathbb{R}\to\mathbb{R}^2$ implicitfüggvény, hogy $f(x_1;\varphi_1(x_1),\varphi_2(x_1))=0 \quad (\forall x_1\in K(a))$

és $\varphi\in C^1$ $\cancel{\quad}$ $(\partial_2 f(a;b))^{-1}(-\partial_1 f(a;b))=\varphi'(a)$

$\partial_1 f(x_1;x_2,x_3)=\begin{pmatrix} e^{x_1+x_2x_3} \\ 2x_3\end{pmatrix} \Rightarrow \partial f(a;b)=\partial_1 f(1;-1,1)=\begin{pmatrix}e^0\\2\cdot 1\end{pmatrix}=\begin{pmatrix}1\\2\end{pmatrix}=:B$

$\frac{\partial}{\partial x_1}$

$\Rightarrow \varphi'(a)=\varphi'(1)=-A^{-1}\cdot B=-\frac{1}{1}\cdot\begin{pmatrix}0 & 1\\ -1 & 3\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=-\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}-2\\-5\end{pmatrix}$

4. $\left.\begin{array}{l} f(x,y)=3x+2y \\ g(x,y)=\frac{x^2}{2}+y^2/4-1=0\end{array}\right\}$ $F(x,y)=3x+2y+\lambda(x^2/2+y^2/4-1)$

$\left.\begin{array}{l}\partial_1 F(x,y)=3+\lambda x=0 \\ \partial_2 F(x,y)=2+\frac{2}{2}y=0\end{array}\right\}$ ha $\lambda=0$ akkor az első egyenletből $\Rightarrow 3=0$ ↯

$\Rightarrow \lambda\neq 0 \Rightarrow \boxed{\begin{array}{l}x=-3/\lambda\\ y=-4/\lambda\end{array}}$

$x^2/2+y^2/4-1=0 \Rightarrow \frac{1}{2}\left(\frac{3}{\lambda}\right)^2+\frac{1}{4}\left(\frac{4}{\lambda}\right)^2-1=0 \Leftrightarrow \frac{9}{\lambda^2}+\frac{8}{\lambda^2}-2=0$

$\Leftrightarrow 9+8-2\lambda^2=0 \Leftrightarrow \lambda^2=\frac{17}{2} \Leftrightarrow \lambda=\pm\sqrt{\frac{17}{2}}$

ha $\lambda=+\sqrt{\frac{17}{2}} \Rightarrow f\left(-3\sqrt{\frac{2}{17}},-4\sqrt{\frac{2}{17}}\right)=-3\cdot 3\cdot\sqrt{\frac{2}{17}}+2\cdot(-4)\sqrt{\frac{2}{17}}=-17\cdot\sqrt{\frac{2}{17}}=-\sqrt{34}$ abszolút Min

ha $\lambda=-\sqrt{\frac{17}{2}} \Rightarrow f\left(3\sqrt{\frac{2}{17}},4\sqrt{\frac{2}{17}}\right)=3\cdot 3\cdot\sqrt{\frac{2}{17}}+2\cdot 4\sqrt{\frac{2}{17}}=17\sqrt{\frac{2}{17}}=\sqrt{34}$ abszolút Max