NUMMÓD 1 ZH

Gépi számok:

$$M(t, k^-, k^+) = \left\{ \pm \left(\sum_{i=1}^{t} m_i 2^{-i} \right) 2^k \;\middle|\; \begin{array}{l} m_i \in \{0,1\} \\ m_1 = 1 \\ k^- \leq k < k^+ \end{array} \right\}$$

$$\varepsilon_0 = [10 \dots 0 | k^-] = \frac{1}{2} \cdot 2^{k^-} = 2^{k^- - 1}$$

$$\varepsilon_1 = \underbrace{[10 \dots 01|1]}_{\text{1-es rákövetkezője}} - \underbrace{[10 \dots 00|1]}_{\text{dec 1-es}} = \underbrace{[00 \dots 01|1]}_{\text{nem gépi szám}}$$

$$= [\underbrace{00 \dots}_{(t-1)\text{db}} \underbrace{1}_{1-(t-1)}010] = [10 \dots 0 | -t+2] = \frac{1}{2} \cdot 2^{2-t} = 2^{1-t}$$

$$|M(t, k^-, k^+)| = 2^t (k^+ - k^- + 1) + 1$$

$$M^\infty = (1 - 2^{-t}) 2^{k^+}$$

$M(6, -4, 4)$ $fl(4.21)$

4.21 → túlcsordulás

:2	4	maradék
	2	0
	1	0
	0	1

0	21	·2
0	42	
0	84	
1	68	
$\boxed{1}$	36	
0	72	
1	44	
0	88	

100.001$\boxed{1}$ → felfele kerekítünk

$\boxed{0}$ → $\emptyset$

100.001 ≈ 100.010

Felfele kerekítés esetén vizsgáljuk, hogy ehhez a számhoz vagy az ~~szám~~ alsó szomszédjához van e közelebb az eredeti szám.
Ha nem kerekítünk felfele akkor a felső szomszédot vizsgáljuk

$$[1000101|3] = \left(\frac{1}{2} + \frac{1}{32}\right) \cdot 2^3$$

pl 0.17 esetén az átváltásnál az elején levő nullákat elhagyjuk

$\Delta fl(x)$ $\quad M(t, k^-, k^+)$ $\quad \Delta fl(x) = 2^{-t} \cdot 2^{k} \cdot \frac{1}{2}$ (karakterisztika) $\quad$ ($)

$fl(\frac{1}{3}) - fl(\frac{1}{6})$ → közös karakterisztika - mindig a nagyobb

$$fl\left(\frac{1}{6}\right) = [10101011|-2] \hat{=} [010101011|-1] = [01010110|-1]$$

kivonás → összeadás $\quad a - b = a + (b \text{ 2-es komplemense}) +$ ~~túlcsordulás~~

$$01010110 \xrightarrow{\text{1-es}} 10101001 \xrightarrow{\text{2-es}} 10101010$$

Gauss-elimináció

$$Ax=b \rightarrow \text{ha } \det(A)\neq 0 \Rightarrow \exists A^{-1} \Rightarrow \underbrace{A^{-1}A}_{I}\,x = A^{-1}b$$

$$x = A^{-1}b$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 3 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 6 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & 3 \\ -1 & 3 & 1 & 6 \end{array}\right) \xrightarrow[\substack{2.\,sor - 1.\,sor\cdot 2 \\ 3\,sor + 1.\,sor\cdot 1}]{} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -5 & 5 & -5 \\ 0 & 5 & 0 & 10 \end{array}\right) \xrightarrow[3.\,sor + 2.\,sor\cdot 1]{} \overset{A'}{\left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -5 & 5 & -5 \\ 0 & 0 & 5 & 5 \end{array}\right)}$$

$$\det(A) = \det(A') = 1\cdot(-5)\cdot 5 = -25 \neq 0 \Rightarrow \exists!\, x \quad Ax=b$$

megoldás visszahelyettesítéssel:

$$\begin{aligned} 1x_1 + 2x_2 - 1x_3 &= 4 \\ 0x_1 + (-5x_2) + 5x_3 &= -5 \\ 0x_1 + 0x_2 + 5x_3 &= 5 \end{aligned} \qquad \text{mo: } x = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

megoldás sorműveletekkel:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -5 & 5 & -5 \\ 0 & 0 & 5 & 5 \end{array}\right) \xrightarrow[3.\,sor\cdot\frac{1}{5}]{} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -5 & 5 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow[\substack{2.\,sor - 3.\,sor\cdot 5 \\ 1.\,sor + 3.\,sor\cdot 1}]{} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & -5 & 0 & -10 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow[2.\,sor\cdot\frac{1}{5}]{} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow[1.\,sor - (2.\,sor)\cdot 2]{} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow x = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

LU felbontás - Cholesky felbontás:

$$\underset{A}{\begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix}} = \underset{L}{\begin{pmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{pmatrix}} \underset{U}{\begin{pmatrix} u_1 & u_2 & u_3 \\ 0 & u_4 & u_5 \\ 0 & 0 & u_6 \end{pmatrix}}$$

parkettaszerűen:

$$\begin{array}{cc} & \begin{pmatrix} u_1 & u_2 & u_3 \\ 0 & u_4 & u_5 \\ 0 & 0 & u_6 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{pmatrix} & \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix} \end{array}$$

$a_1 = u_1$ $\qquad a_5 = l_1 u_2 + u_4$

$a_2 = u_2$ $\qquad a_6 = l_1 u_3 + u_5$

$a_3 = u_3$

$a_4 = l_1 u_1$ $\qquad a_8 = l_2 u_2 + l_3 u_4$

$a_7 = l_2 u_1$ $\qquad a_9 = l_2 u_3 + u_5 l_3 + u_6$

$$A = LU = LDD^{-1}U = LDL^T \qquad D = \begin{pmatrix} u_1 & & 0 \\ & u_4 & \\ 0 & & u_6 \end{pmatrix}$$

$L^T = D^{-1}U$ $\qquad D^{-1}$ = főátlóban lévő elemek reciproka $\qquad D^{-1} = \begin{pmatrix} \frac{1}{u_1} & & 0 \\ & \frac{1}{u_4} & \\ 0 & & \frac{1}{u_6} \end{pmatrix}$

Cholesky:

$$A = LU = L\sqrt{D}\sqrt{D}L^T = \tilde{L}\tilde{L}^T$$

$$\tilde{L} = L \cdot \sqrt{D}$$

QR felbontás:

$$\underset{A}{\begin{array}{ccc} x_1 & x_2 & x_3 \\ \end{array}}$$

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix} = \underset{Q}{[q_1\ q_2\ q_3]} \cdot \underset{R}{\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{bmatrix}}$$

$s_1 = a_1$ $\qquad s_2 = x_2 - r_{12}q_1$ $\qquad s_3 = x_3 - r_{13}q_1 - r_{23}q_2$

$r_{11} = \|s_1\|_2 = \|x_1\|_2 = \sqrt{a_1^2 + a_4^2 + a_7^2}$ $\qquad r_{12} = \langle x_2, q_1 \rangle = \left\langle \begin{pmatrix} a_2 \\ a_5 \\ a_8 \end{pmatrix}, \begin{pmatrix} a_1 \\ a_4 \\ a_7 \end{pmatrix} \frac{1}{r_{11}} \right\rangle$

$q_1 = x_1 \cdot \frac{1}{r_{11}}$ $\qquad = \frac{1}{r_{11}} \begin{pmatrix} a_2 + a_1 \\ a_5 + a_4 \\ a_8 + a_7 \end{pmatrix}$

$q_2 = \overbrace{(x_2 - r_{12}q_1)}^{s_2} \frac{1}{r_{22}} = s_2 \frac{1}{r_{22}}$

$q_3 = \underbrace{(x_3 - r_{13}q_1 - r_{23}q_2)}_{s_3} \frac{1}{r_{33}} = s_3 \frac{1}{r_{33}}$

$r_{22} = \|s_2\|_2$ $\qquad r_{13} = \langle a_3, q_1 \rangle$

$r_{33} = \|s_3\|_2$ $\qquad r_{23} = \langle a_3, q_2 \rangle$

Householder-transzformáció:

első oszlopra:

a_1 vektort $k \cdot e_1$ alakra:

$$A = \begin{array}{ccc} a_1 & a_2 & a_3 \end{array}$$

$$\sigma_1 = -\mathrm{sgn}(a_{11}) \cdot \|a_1\|_2$$

$$v_1 = \frac{a - \sigma_1 \cdot e_1}{\|a - \sigma_1 \cdot e_1\|_2}$$

$$H(v_1) \cdot a_1 = \begin{bmatrix} \sigma \\ 0 \\ 0 \end{bmatrix}$$

$$H(v_1) \cdot a_2 = (I - 2 \cdot v_1 v_1^T) \cdot a_2 = a_2 - 2 \cdot v_1 (v_1^T \cdot a_2)$$

$$H(v_1) \cdot a_3 = (I - 2 \cdot v_1 v_1^T) \cdot a_3 = a_3 - 2 \cdot v_1 \cdot (v_1^T \cdot a_3)$$

$$R = H \cdot A = \begin{bmatrix} H(v_1) \cdot a_1 & H(v_1) \cdot a_2 & H(v_1) \cdot a_3 \end{bmatrix}$$

pl:

$$A = \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \end{array}$$

$$\sigma = -\mathrm{sgn}(a_{11}) \cdot \|a_1\|_2 = -\mathrm{sgn}(2) \cdot \sqrt{2^2 + (-1)^2} = -\sqrt{5}$$

$$a_1 - \sigma_1 \cdot e_1 = \begin{bmatrix} 2+\sqrt{5} \\ -1 \\ 0 \end{bmatrix} \qquad \sigma e_1 = \begin{bmatrix} \sigma \\ 0 \\ 0 \end{bmatrix}$$

$$\|a_1 - \sigma_1 \cdot e_1\|_2 = \sqrt{(2+\sqrt{5})^2 + 1^2} = \sqrt{10 + 4\sqrt{5}}$$

$$v = \frac{a_1 - \sigma_1 \cdot e_1}{\|a_1 - \sigma_1 \cdot e_1\|_2} = \frac{1}{\sqrt{10 + 4\sqrt{5}}} \cdot \begin{bmatrix} 2+\sqrt{5} \\ -1 \\ 0 \end{bmatrix}$$

$H(v_1) \cdot a_1$ a konstrukcióból tudjuk hogy: $\begin{bmatrix} -\sqrt{5} \\ 0 \\ 0 \end{bmatrix}$

$$H(v_1) \cdot a_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - 2 \cdot \frac{1}{\sqrt{10+4\sqrt{5}}} \cdot \frac{1}{\sqrt{10+4\sqrt{5}}} \cdot \begin{bmatrix} 2+\sqrt{5} \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2+\sqrt{5} & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \frac{2}{10+4\sqrt{5}} \cdot \begin{bmatrix} 2+\sqrt{5} \\ -1 \\ 0 \end{bmatrix} \cdot (2 + \sqrt{5} - 2) = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \frac{\sqrt{5}}{5+2\sqrt{5}} \begin{bmatrix} 2+\sqrt{5} \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - (\sqrt{5} - 2) \cdot \begin{bmatrix} 2+\sqrt{5} \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{5} \\ 0 \end{bmatrix}$$

$$H(v_1) \cdot a_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} - 2 \cdot \frac{1}{\sqrt{10+4\sqrt{5}}} \cdot \frac{1}{\sqrt{10+4\sqrt{5}}} \cdot \begin{bmatrix} 2+\sqrt{5} \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2+\sqrt{5} & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} - \frac{2}{10+4\sqrt{5}} \begin{bmatrix} 2+\sqrt{5} \\ -1 \\ 0 \end{bmatrix} \cdot 0 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$R = H \cdot A = \begin{bmatrix} -\sqrt{5} & 0 & 0 \\ 0 & \sqrt{5} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$