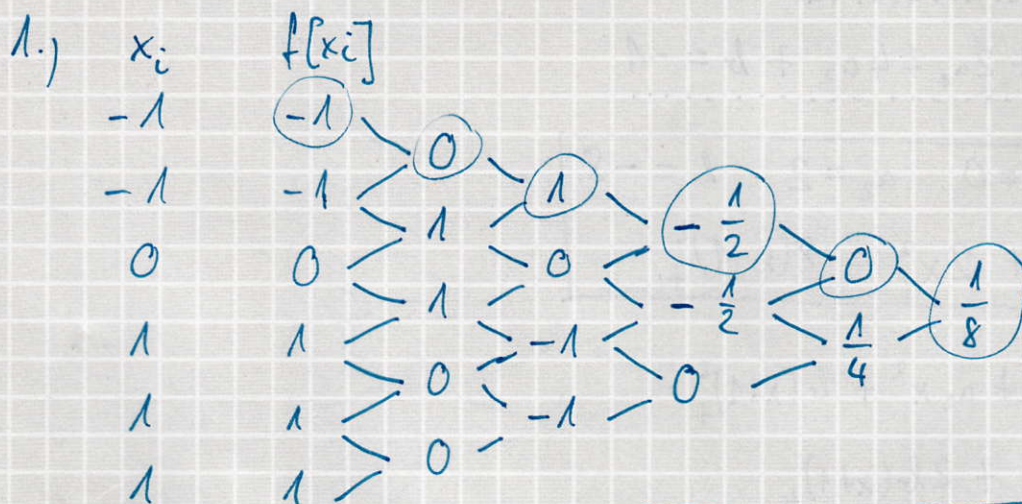
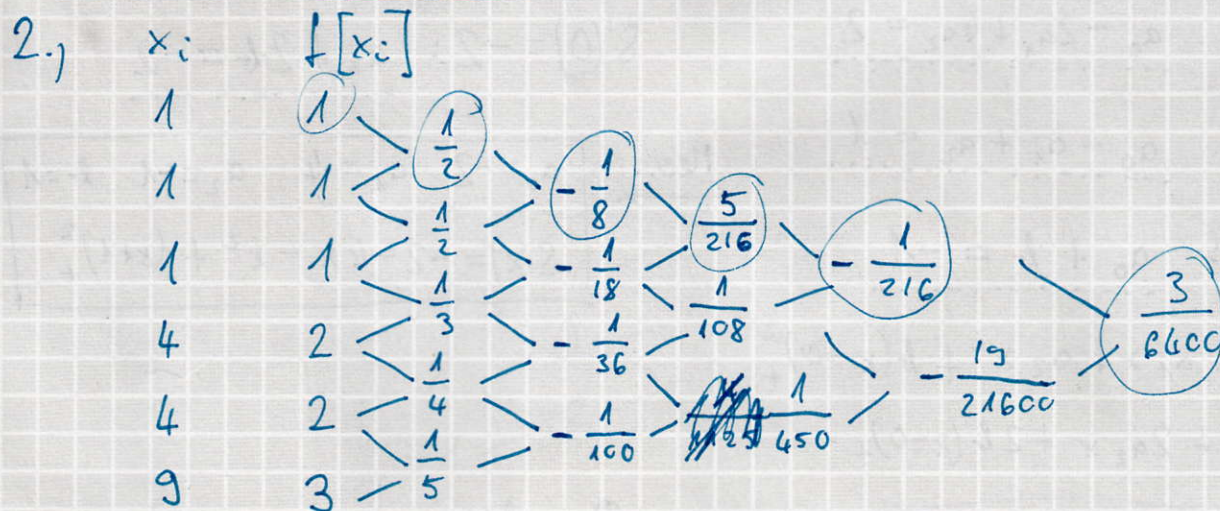


# jegyzetek notes Notizen



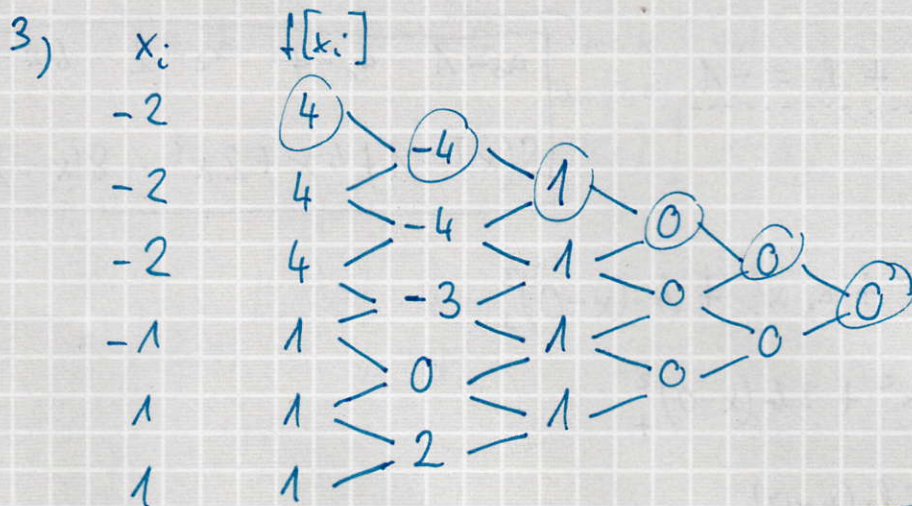
$$H(x) = (-1) + (x+1)^2 - \frac{1}{2} (x+1)^2 x + \frac{1}{8} (x+1)^2 x (x-1)^2$$



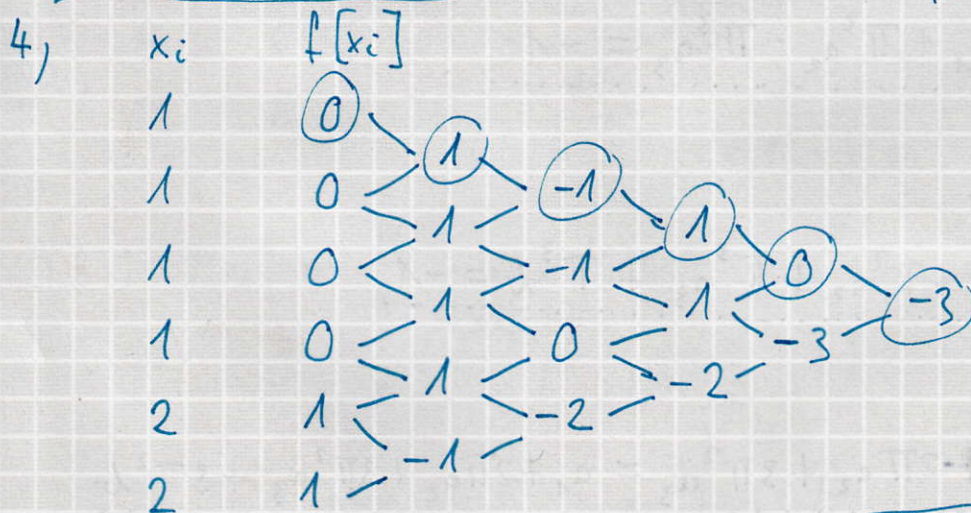
$$H(x) = 1 + \frac{1}{2} (x-1) - \frac{1}{8} (x-1)^2 + \frac{5}{216} (x-1)^3 - \frac{1}{216} (x-1)^3 (x-4) + \frac{3}{6400} \cdot (x-1)^3 (x-4)^2$$



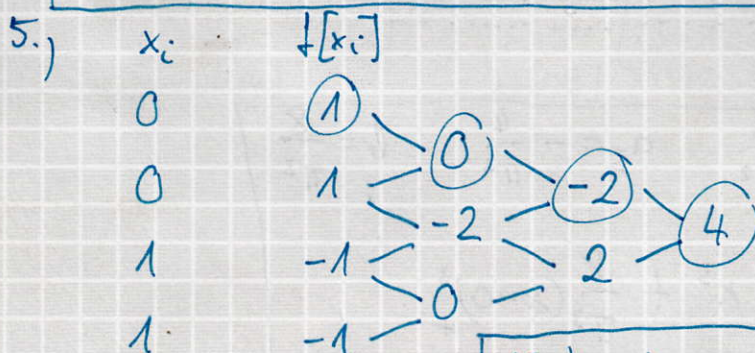
# jegyzetek notesNotizen



$$H(x) = 4 - 4(x+2) + (x+2)^2 (= x^2)$$



$$H(x) = (x-1) - (x-1)^2 + (x-1)^3 - 3(x-1)^4(x-2)$$



$$H(x) = 1 - 2x^2 + 4x^2(x-1)$$

$$f(x) = \cos(\pi x)$$

$$f'(x) = -\pi \sin(\pi x)$$

$$f''(x) = -\pi^2 \cos(\pi x)$$

$$f'''(x) = \pi^3 \sin(\pi x)$$

$$f^{(4)}(x) = \pi^4 \cos(\pi x)$$



# jegyzetek notes Notizen

$$|f(x) - H(x)| \leq \frac{M_4}{4!} \cdot \max_{x \in [0;1]} |\Omega(x)| = \frac{\pi^4}{24} \cdot \frac{1}{16} = \boxed{\frac{\pi^4}{384}}$$

$$M_4 = \max_{x \in [0;1]} |\pi^4 \cos(\pi x)| = \pi^4$$

$$\max_{x \in [0;1]} |\Omega(x)| = \max_{x \in [0;1]} [x(x-1)]^2 = \left[ \frac{1}{2} \left( \frac{1}{2} - 1 \right) \right]^2 = \frac{1}{16}$$

6.)

$x_i$	$f[x_i]$
1	3
1	3
4	6
4	6

Diagram showing the construction of the Hermite polynomial  $H(x)$  using the values of  $f$  and its derivatives at the nodes  $x_i$ . The diagram shows the following steps:

- From  $f[1] = 3$  and  $f'[1] = 3$ , we get  $\frac{3}{2}$ .
- From  $f[4] = 6$  and  $f'[4] = 6$ , we get  $\frac{1}{3}$ .
- From  $\frac{3}{2}$  and  $\frac{1}{3}$ , we get  $-\frac{1}{6}$ .
- From  $-\frac{1}{6}$  and  $\frac{1}{3}$ , we get  $\frac{1}{36}$ .

$$f(x) = 3\sqrt{x}$$

$$f'(x) = \frac{3}{2} x^{-\frac{1}{2}}$$

$$H(x) = 3 + \frac{3}{2}(x-1) - \frac{1}{6}(x-1)^2 + \frac{1}{36}(x-1)^2(x-4)$$

$$|f(x) - H(x)| \leq \frac{M_4}{4!} \cdot \max_{x \in [1;4]} |\Omega(x)| = \frac{\frac{45}{16}}{24} \cdot \frac{81}{16} = \boxed{\frac{1215}{2048}}$$

$$f''(x) = -\frac{3}{4} x^{-\frac{3}{2}} \quad f'''(x) = \frac{9}{8} x^{-\frac{5}{2}} \quad f^{(4)}(x) = -\frac{45}{16} x^{-\frac{7}{2}}$$

$$M_4 = \max_{x \in [1;4]} \left| -\frac{45}{16} \cdot x^{-\frac{7}{2}} \right| = \frac{45}{16}$$

$$\max_{x \in [1;4]} |\Omega(x)| = \max_{x \in [1;4]} [(x-1)(x-4)]^2 = \left[ \left( \frac{5}{2} - 1 \right) \left( \frac{5}{2} - 4 \right) \right]^2 = \frac{81}{16}$$



# jegyzetek notes Notizen

7.)

$x_i$	$f[x_i]$
-1	-1
-1	-1
1	1
1	1

$$H(x) = -1 + 3(x+1) - (x+1)^2 + (x+1)^2(x-1) \stackrel{!}{=} x^3$$

$$|f(x) - H(x)| \leq \frac{M_4}{4!} \cdot \max_{x \in [1;1]} |g_4(x)| = 0$$

$$M_4 = \max_{x \in [1;1]} |0| = 0$$

$$\begin{aligned} f(x) &= x^3 \\ f'(x) &= 3x^2 \\ f''(x) &= 6x \\ f'''(x) &= 6 \\ f^{(4)}(x) &= 0 \end{aligned}$$

8.)

$x_i$	$f[x_i]$
-1	1
-1	1
0	0
0	0

$$H(x) = 1 - (x+1) + \frac{1}{2}(x+1)^2 - \frac{1}{4}(x+1)^2 x$$

$$|f(x) - H(x)| \leq \frac{M_4}{4!} \cdot \max_{x \in [-1;0]} |g_4(x)| = \frac{24}{24} \cdot \frac{1}{16} = \frac{1}{16}$$

$$M_4 = \max_{x \in [-1;0]} |24 \cdot (x+2)^{-5}| = 24$$

$$\max_{x \in [-1;0]} |g_4(x)| = \max_{x \in [-1;0]} [(x+1)x]^2 = \left[ \left( -\frac{1}{2} + 1 \right) \left( -\frac{1}{2} \right) \right]^2 = \frac{1}{16}$$

$$\begin{aligned} f(x) &= (x+2)^{-1} \\ f'(x) &= -(x+2)^{-2} \\ f''(x) &= 2 \cdot (x+2)^{-3} \\ f'''(x) &= -6 \cdot (x+2)^{-4} \\ f^{(4)}(x) &= 24 \cdot (x+2)^{-5} \end{aligned}$$



# jegyzetek notes Notizen

9.) 
$$S(x) = \begin{cases} a_0 + a_1 x + a_2 x^2; & x \in [0; 1) \\ b_0 + b_1 x + b_2 x^2; & x \in [1; 2] \end{cases}$$

Interpoláció;  $S \in C$

$S(0) = -1: \quad a_0 = -1$

$S(1) = 1: \quad \begin{aligned} a_0 + a_1 + a_2 &= 1 \\ b_0 + b_1 + b_2 &= 1 \end{aligned}$

$S(2) = -1: \quad b_0 + 2b_1 + 4b_2 = -1$

$S \in D$

$S'(1+0) = S'(1-0):$

$a_1 + 2a_2 = b_1 + 2b_2$

Perem felt.

$S'(0) = 0: \quad a_1 = 0$

Meo.:

$\begin{aligned} a_0 &= -1 & a_1 &= 0 & a_2 &= 2 \\ b_0 &= -9 & b_1 &= 16 & b_2 &= -6 \end{aligned}$

$$S(x) = \begin{cases} 2x^2 - 1; & x \in [0; 1) \\ -6x^2 + 16x - 9; & x \in [1; 2] \end{cases}$$

10.) 
$$S(x) = \begin{cases} a_0 + a_1 x + a_2 x^2; & x \in [-2; -1) \\ b_0 + b_1 x + b_2 x^2; & x \in [-1; 0] \end{cases}$$

Interpoláció;  $S \in C$

$S(-2) = 2: \quad a_0 - 2a_1 + 4a_2 = 2$

$S(-1) = 1: \quad \begin{aligned} a_0 - a_1 + a_2 &= 1 \\ b_0 - b_1 + b_2 &= 1 \end{aligned}$

$S(0) = -1: \quad b_0 = -1$

$S \in D$

$S'(-1-0) = S'(-1+0):$

$a_1 - 2a_2 = b_1 - 2b_2$

Perem felt.

$S'(0) = -2: \quad b_1 = -2$

Meo.:

$\begin{aligned} a_0 &= -2 & a_1 &= -4 & a_2 &= -1 \\ b_0 &= -1 & b_1 &= -2 & b_2 &= 0 \end{aligned}$

$$S(x) = \begin{cases} -x^2 - 4x - 2; & x \in [-2; -1) \\ -2x - 1; & x \in [-1; 0] \end{cases}$$



# jegyzetek notes Notizen

$$11.) \quad S(x) = \begin{cases} a_0 + a_1 x + a_2 x^2; & x \in [-1; 0) \\ b_0 + b_1 x + b_2 x^2; & x \in [0; 1] \end{cases}$$

Interpoláció;  $S \in C$

$$S(-1) = 0: \quad a_0 - a_1 + a_2 = 0$$

$$S(0) = 1: \quad a_0 = 1$$

$$b_0 = 1$$

$$S(1) = 2: \quad b_0 + b_1 + b_2 = 2$$

$S \in D$

$$S'(0-0) = S'(0+0):$$

$$a_1 = b_1$$

Peremfelt.

$$S'(-1) = 0: \quad a_1 - 2a_2 = 0$$

Meo.:

$$\begin{array}{ccc} a_0 = 1 & a_1 = 2 & a_2 = 1 \\ b_0 = 1 & b_1 = 2 & b_2 = -1 \end{array} \quad S(x) = \begin{cases} x^2 + 2x + 1; & x \in [-1; 0) \\ -x^2 + 2x + 1; & x \in [0; 1] \end{cases}$$

$$12.) \quad S(x) = \begin{cases} a_0 + a_1 x + a_2 x^2; & x \in [-1; 0) \\ b_0 + b_1 x + b_2 x^2; & x \in [0; 1] \end{cases}$$

Interpoláció;  $S \in C$

$$S(-1) = -1: \quad a_0 - a_1 + a_2 = -1$$

$$S(0) = 1: \quad a_0 = 1$$

$$b_0 = 1$$

$$S(1) = -1: \quad b_0 + b_1 + b_2 = -1$$

$S \in D$

$$S'(0-0) = S'(0+0):$$

$$a_1 = b_1$$

Peremfelt.

$$S'(-1) = 0: \quad a_1 - 2a_2 = 0$$

Meo.:

$$\begin{array}{ccc} a_0 = 1 & a_1 = 4 & a_2 = 2 \\ b_0 = 1 & b_1 = 4 & b_2 = -6 \end{array} \quad S(x) = \begin{cases} 2x^2 + 4x + 1; & x \in [-1; 0) \\ -6x^2 + 4x + 1; & x \in [0; 1] \end{cases}$$

$$13.) \quad S(x) = a_0 + a_1 x + a_2 x^2 + b(x-1)_+^2$$

$$S'(x) = a_1 + 2a_2 x + 2b(x-1)_+$$

Interpolació

$$S(0) = -1: \quad a_0 = -1$$

$$S(1) = 1: \quad a_0 + a_1 + a_2 = 1$$

$$S(2) = -1: \quad a_0 + 2a_1 + 4a_2 + b = -1$$

Mec.:

$$\boxed{a_0 = -1 \quad a_1 = 0 \quad a_2 = 2 \quad b = -8}$$

$$S(x) = -1 + 2x^2 - 8(x-1)_+^2$$

Param.felt.

$$S'(0) = 0: \quad a_1 = 0$$

$$14.) \quad S(x) = a_0 + a_1 x + a_2 x^2 + b(x+1)_+^2$$

$$S'(x) = a_1 + 2a_2 x + 2b(x+1)_+$$

Interpolació

$$S(-2) = 2: \quad a_0 - 2a_1 + 4a_2 = 2$$

$$S(-1) = 1: \quad a_0 - a_1 + a_2 = 1$$

$$S(0) = -1: \quad a_0 + b = -1$$

Param.felt.

$$S'(0) = -2: \quad a_1 + 2b = -2$$

Mec.:

$$\boxed{a_0 = -2 \quad a_1 = -4 \quad a_2 = -1 \quad b = 1}$$

$$S(x) = -2 - 4x - x^2 + (x+1)_+^2$$

$$15.) \quad S(x) = a_0 + a_1 x + a_2 x^2 + b(x-0)_+^2$$

$$S'(x) = a_1 + 2a_2 x + 2b(x-0)_+$$

Interpolació

$$S(-1) = 0: \quad a_0 - a_1 + a_2 = 0$$

$$S(0) = 1: \quad a_0 = 1$$

$$S(1) = 2: \quad a_0 + a_1 + a_2 + b = 2$$

Param.felt.

$$S'(-1) = 0: \quad a_1 - 2a_2 = 0$$

Mec.:

$$\boxed{a_0 = 1 \quad a_1 = 2 \quad a_2 = 1 \quad b = -2}$$

$$S(x) = 1 + 2x + x^2 - 2(x-0)_+^2$$



$$16.) \quad S(x) = a_0 + a_1 x + a_2 x^2 + b(x-0)_+^2$$

$$S'(x) = a_1 + 2a_2 x + 2b(x-0)_+$$

Interpolació

$$S(-1) = -1: \quad a_0 - a_1 + a_2 = -1$$

$$S(0) = 1: \quad a_0 = 1$$

$$S(1) = -1: \quad a_0 + a_1 + a_2 + b = -1$$

Penam. felt.

$$S'(-1) = 0: \quad a_1 - 2a_2 = 0$$

Meo.:

$$\boxed{a_0 = 1 \quad a_1 = 4 \quad a_2 = 2 \quad b = -8}$$

$$S(x) = 1 + 4x + 2x^2 - 8(x-0)_+^2$$

$$17.) \quad S(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + b(x-0)_+^3$$

$$S'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 3b(x-0)_+^2$$

$$S''(x) = 2a_2 + 6a_3 x + 6b(x-0)_+$$

Interpolació

$$S(-\pi) = -1: \quad a_0 - \pi a_1 + \pi^2 a_2 - \pi^3 a_3 = -1$$

$$S(0) = 1: \quad a_0 = 1$$

$$S(\pi) = -1: \quad a_0 + \pi a_1 + \pi^2 a_2 + \pi^3 a_3 + \pi^3 b = -1$$

Penam. felt.

$$S'(-\pi) = S'(\pi): \quad a_1 - 2\pi a_2 + 3\pi^2 a_3 = a_1 + 2\pi a_2 + 3\pi^2 a_3 + 3\pi^2 b$$

$$S''(-\pi) = S''(\pi): \quad 2a_2 - 6\pi a_3 = 2a_2 + 6\pi a_3 + 6\pi b$$

Meo.

$$\boxed{a_0 = 1 \quad a_1 = -\frac{12}{\pi} \quad a_2 = -\frac{6}{\pi^2} \quad a_3 = -\frac{4}{\pi^3} \quad b = \frac{8}{\pi^3}}$$

$$S(x) = 1 - \frac{12}{\pi} x - \frac{6}{\pi^2} x^2 - \frac{4}{\pi^3} x^3 + \frac{8}{\pi^3} (x-0)_+^3$$



$$18.) S(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + b \left(x - \frac{\pi}{2}\right)_+^3$$

$$S'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 3b \left(x - \frac{\pi}{2}\right)_+^2$$

$$S''(x) = 2a_2 + 6a_3 x + 6b \left(x - \frac{\pi}{2}\right)_+$$

Interpolació

$$S(0) = 0: \quad \underline{a_0 = 0}$$

$$S\left(\frac{\pi}{2}\right) = 1: \quad \underline{a_0 + \frac{\pi}{2} a_1 + \frac{\pi^2}{4} a_2 + \frac{\pi^3}{8} a_3 = 1}$$

$$S(\pi) = 0: \quad \underline{a_0 + \pi a_1 + \pi^2 a_2 + \pi^3 a_3 + \frac{\pi^3}{8} b = 0}$$

Perem kld.

$$S''(0) = 0: \quad \underline{2a_2 = 0}$$

$$S''(\pi) = 0: \quad \underline{2a_2 + 6\pi a_3 + 3\pi \cdot b = 0}$$

Meo.

$a_0 = 0$	$a_1 = \frac{3}{\pi}$	$a_2 = 0$	$a_3 = -\frac{4}{\pi^3}$	$b = \frac{8}{\pi^3}$
$S(x) = \frac{3}{\pi} x - \frac{4}{\pi^3} x^3 + \frac{8}{\pi^3} \left(x - \frac{\pi}{2}\right)_+^3$				

$$19.) S(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + b(x-0)_+^3$$

$$f(x) = x^5$$

$$S'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 3b(x-0)_+^2$$

$$f'(x) = 5x^4$$

Interpolació

$$S(-1) = -1: \quad \underline{a_0 - a_1 + a_2 - a_3 = -1}$$

$$S(0) = 0: \quad \underline{a_0 = 0}$$

$$S(1) = 1: \quad \underline{a_0 + a_1 + a_2 + a_3 + b = 1}$$



Perem felt.

$$S'(-1) = f'(-1): \quad \underline{a_1 - 2a_2 + 3a_3 = 5}$$

$$S'(1) = f'(1): \quad \underline{a_1 + 2a_2 + 3a_3 + 3b = 5}$$

Mec.

$$a_0 = 0 \quad a_1 = -1 \quad a_2 = 0 \quad a_3 = 2 \quad b = 0$$

$$S(x) = -x + 2x^3$$

$$20.) \quad S(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + b(x-0)^3_+$$

$$f(x) = -x^6$$

$$S'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 3b(x-0)^2_+$$

$$f'(x) = -6x^5$$

Interpoláció

$$S(-1) = -1: \quad \underline{a_0 - a_1 + a_2 - a_3 = -1}$$

$$S(0) = 0: \quad \underline{a_0 = 0}$$

$$S(1) = -1: \quad \underline{a_0 + a_1 + a_2 + a_3 + b = -1}$$

Perem felt.

$$S'(-1) = f'(-1): \quad \underline{a_1 - 2a_2 + 3a_3 = 6}$$

$$S'(1) = f'(1): \quad \underline{a_1 + 2a_2 + 3a_3 + 3b = -6}$$

Mec.

$$a_0 = 0 \quad a_1 = 0 \quad a_2 = 3 \quad a_3 = 4 \quad b = -8$$

$$S(x) = 3x^2 + 4x^3 - 8(x-0)^3_+$$