1.)

x_i	$f[x_i]$					
-1	-1	0	1	$-\frac{1}{2}$	0	$\frac{1}{8}$
-1	-1	1	0	$-\frac{1}{2}$	$\frac{1}{4}$	
0	0	1	-1	0		
1	1	0	-1			
1	1	0				
1	1					

$$\boxed{H(x) = (-1) + (x+1)^2 - \frac{1}{2}(x+1)^2 x + \frac{1}{8}(x+1)^2 x (x-1)^2}$$

2.)

x_i	$f[x_i]$					
1	1	$\frac{1}{2}$	$-\frac{1}{8}$	$\frac{5}{216}$	$-\frac{1}{216}$	$\frac{3}{6400}$
1	1	$\frac{1}{2}$	$-\frac{1}{18}$	$\frac{1}{108}$	$-\frac{19}{21600}$	
1	1	$\frac{1}{3}$	$-\frac{1}{36}$	$\frac{1}{450}$		
4	2	$\frac{1}{4}$	$-\frac{1}{100}$			
4	2	$\frac{1}{5}$				
9	3					

$$\boxed{H(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{5}{216}(x-1)^3 - \frac{1}{216}(x-1)^3(x-4) + \frac{3}{6400}\cdot(x-1)^3(x-4)^2}$$

3.)

x_i	$f[x_i]$					
-2	4					
		-4				
-2	4		1			
		-4		0		
-2	4		1		0	
		-3		0		0
-1	1		1		0	
		0		0		
1	1		1			
		2				
1	1					

$$\boxed{H(x) = 4 - 4(x+2) + (x+2)^2 \; (= x^2)}$$

4.)

x_i	$f[x_i]$					
1	0					
		1				
1	0		-1			
		1		1		
1	0		-1		0	
		1		1		-3
1	0		0		-3	
		1		-2		
2	1		-2			
		-1				
2	1					

$$\boxed{H(x) = (x-1) - (x-1)^2 + (x-1)^3 - 3(x-1)^4(x-2)}$$

5.)

x_i	$f[x_i]$			
0	1			
		0		
0	1		-2	
		-2		4
1	-1		2	
		0		
1	-1			

$f(x) = \cos(\pi x)$

$f'(x) = -\pi \sin(\pi x)$

$f''(x) = -\pi^2 \cos(\pi x)$

$f'''(x) = \pi^3 \sin(\pi x)$

$f^{(4)}(x) = \pi^4 \cos(\pi x)$

$$\boxed{H(x) = 1 - 2x^2 + 4x^2(x-1)}$$

Merck Kft.
1113 Budapest, Bocskai út 134-146., Tel.: 1 463-8100, Fax: 1 463-8101
e-mail: kemia@merck.hu, honlap: www.merck.hu · www.merck-chemicals.hu

$$\left|f(x) - H(x)\right| \le \frac{M_4}{4!} \cdot \max_{x\in[0;1]} \left|\Omega(x)\right| = \frac{\pi^4}{24} \cdot \frac{1}{16} = \boxed{\frac{\pi^4}{384}}$$

$$M_4 = \max_{x\in[0;1]} \left|\pi^4 \cos(\pi x)\right| = \pi^4$$

$$\max_{x\in[0;1]} \left|\Omega(x)\right| = \max_{x\in[0;1]} \left[x(x-1)\right]^2 = \left[\frac{1}{2}\left(\frac{1}{2}-1\right)\right]^2 = \frac{1}{16}$$

6.)

$$f(x) = 3\sqrt{x}$$

$$f'(x) = \frac{3}{2} x^{-\frac{1}{2}}$$

x_i	$f[x_i]$			
1	3			
		$\frac{3}{2}$		
1	3		$-\frac{1}{6}$	
		1		$\frac{1}{36}$
4	6		$-\frac{1}{12}$	
		$\frac{3}{4}$		
4	6			

$$\boxed{H(x) = 3 + \frac{3}{2}(x-1) - \frac{1}{6}(x-1)^2 + \frac{1}{36}(x-1)^2(x-4)}$$

$$\left|f(x) - H(x)\right| \le \frac{M_4}{4!} \cdot \max_{x\in[1;4]} \left|\Omega(x)\right| = \frac{\frac{45}{16}}{24} \cdot \frac{81}{16} = \boxed{\frac{1215}{2048}}$$

$$f''(x) = -\frac{3}{4} x^{-\frac{3}{2}} \qquad f'''(x) = \frac{9}{8} x^{-\frac{5}{2}} \qquad f^{(4)}(x) = -\frac{45}{16} x^{-\frac{7}{2}}$$

$$M_4 = \max_{x\in[1;4]} \left|-\frac{45}{16} \cdot x^{-\frac{7}{2}}\right| = \frac{45}{16}$$

$$\max_{x\in[1;4]} \left|\Omega(x)\right| = \max_{x\in[1;4]} \left[(x-1)(x-4)\right]^2 = \left[\left(\frac{5}{2}-1\right)\left(\frac{5}{2}-4\right)\right]^2 = \frac{81}{16}$$

7.)

$$
\begin{array}{cccccc}
x_i & f[x_i] & & & \\
-1 & -1 & & & \\
 & & 3 & & \\
-1 & -1 & & -1 & \\
 & & 1 & & 1 \\
1 & 1 & & 1 & \\
 & & 3 & & \\
1 & 1 & & &
\end{array}
$$

$f(x) = x^3$

$f'(x) = 3x^2$

$f''(x) = 6x$

$f'''(x) = 6$

$f^{(4)}(x) = 0$

$$\boxed{H(x) = -1 + 3(x+1) - (x+1)^2 + (x+1)^2(x-1) \overset{!}{=} x^3}$$

$$|f(x) - H(x)| \leq \frac{M_4}{4!} \cdot \max_{x\in[1;1]} |\Omega(x)| = \boxed{0}$$

$$M_4 = \max_{x\in[1;1]} |0| = 0$$

8.)

$$
\begin{array}{cccccc}
x_i & f[x_i] & & & \\
-1 & 1 & & & \\
 & & -1 & & \\
-1 & 1 & & \frac{1}{2} & \\
 & & -\frac{1}{2} & & -\frac{1}{4} \\
0 & \frac{1}{2} & & \frac{1}{4} & \\
 & & -\frac{1}{4} & & \\
0 & \frac{1}{2} & & &
\end{array}
$$

$f(x) = (x+2)^{-1}$

$f'(x) = -(x+2)^{-2}$

$f''(x) = 2\cdot(x+2)^{-3}$

$f'''(x) = -6\cdot(x+2)^{-4}$

$f^{(4)}(x) = 24\cdot(x+2)^{-5}$

$$\boxed{H(x) = 1 - (x+1) + \frac{1}{2}(x+1)^2 - \frac{1}{4}(x+1)^2 x}$$

$$|f(x) - H(x)| \leq \frac{M_4}{4!} \cdot \max_{x\in[-1;0]} |\Omega(x)| = \frac{24}{24} \cdot \frac{1}{16} = \boxed{\frac{1}{16}}$$

$$M_4 = \max_{x\in[-1;0]} \left|24\cdot(x+2)^{-5}\right| = 24$$

$$\max_{x\in[1;0]} |\Omega(x)| = \max_{x\in[-1;0]} \left[(x+1)x\right]^2 = \left[\left(-\frac{1}{2}+1\right)\left(-\frac{1}{2}\right)\right]^2 = \frac{1}{16}$$

9.)

$$S(x) = \begin{cases} a_0 + a_1 x + a_2 x^2; & x \in [0; 1) \\ b_0 + b_1 x + b_2 x^2; & x \in [1; 2] \end{cases}$$

Interpoláció; $S \in C$

$S(0) = -1$: $a_0 = -1$

$S(1) = 1$: $a_0 + a_1 + a_2 = 1$

$b_0 + b_1 + b_2 = 1$

$S(2) = -1$: $b_0 + 2b_1 + 4b_2 = -1$

$S \in D$

$S'(1+0) = S'(1-0)$:

$a_1 + 2a_2 = b_1 + 2b_2$

Perem felt.

$S'(0) = 0$: $a_1 = 0$

Meo.:

$$\boxed{\begin{matrix} a_0 = -1 & a_1 = 0 & a_2 = 2 \\ b_0 = -9 & b_1 = 16 & b_2 = -6 \end{matrix} \qquad S(x) = \begin{cases} 2x^2 - 1; & x \in [0; 1) \\ -6x^2 + 16x - 9; & x \in [1; 2] \end{cases}}$$

10.)

$$S(x) = \begin{cases} a_0 + a_1 x + a_2 x^2; & x \in [-2; -1) \\ b_0 + b_1 x + b_2 x^2; & x \in [-1; 0] \end{cases}$$

Interpoláció; $S \in C$

$S(-2) = 2$: $a_0 - 2a_1 + 4a_2 = 2$

$S(-1) = 1$: $a_0 - a_1 + a_2 = 1$

$b_0 - b_1 + b_2 = 1$

$S(0) = -1$: $b_0 = -1$

$S \in D$

$S'(-1-0) = S'(-1+0)$:

$a_1 - 2a_2 = b_1 - 2b_2$

Perem felt.

$S'(0) = -2$: $b_1 = -2$

Meo.:

$$\boxed{\begin{matrix} a_0 = -2 & a_1 = -4 & a_2 = -1 \\ b_0 = -1 & b_1 = -2 & b_2 = 0 \end{matrix} \qquad S(x) = \begin{cases} -x^2 - 4x - 2; & x \in [-2; -1) \\ -2x - 1; & x \in [-1; 0] \end{cases}}$$

11.)
$$S(x)=\begin{cases} a_0+a_1x+a_2x^2; & x\in[-1;0) \\ b_0+b_1x+b_2x^2; & x\in[0;1] \end{cases}$$

Interpoláció; $S\in C$

$S(-1)=0$: $a_0-a_1+a_2=0$

$S(0)=1$: $a_0=1$

$b_0=1$

$S(1)=2$: $b_0+b_1+b_2=2$

$S\in D$

$S'(0-0)=S'(0+0)$:

$a_1=b_1$

Peremfelt.

$S'(-1)=0$: $a_1-2a_2=0$

Meo.:
$$\boxed{\begin{matrix} a_0=1 & a_1=2 & a_2=1 \\ b_0=1 & b_1=2 & b_2=-1 \end{matrix} \qquad S(x)=\begin{cases} x^2+2x+1; & x\in[-1;0) \\ -x^2+2x+1; & x\in[0;1] \end{cases}}$$

12.)
$$S(x)=\begin{cases} a_0+a_1x+a_2x^2; & x\in[-1;0) \\ b_0+b_1x+b_2x^2; & x\in[0;1] \end{cases}$$

Interpoláció; $S\in C$

$S(-1)=-1$: $a_0-a_1+a_2=-1$

$S(0)=1$: $a_0=1$

$b_0=1$

$S(1)=-1$: $b_0+b_1+b_2=-1$

$S\in D$

$S'(0-0)=S'(0+0)$:

$a_1=b_1$

Peremfelt.

$S'(-1)=0$: $a_1-2a_2=0$

Meo.:
$$\boxed{\begin{matrix} a_0=1 & a_1=4 & a_2=2 \\ b_0=1 & b_1=4 & b_2=-6 \end{matrix} \qquad S(x)=\begin{cases} 2x^2+4x+1; & x\in[-1;0) \\ -6x^2+4x+1; & x\in[0;1] \end{cases}}$$

13.) $S(x) = a_0 + a_1 x + a_2 x^2 + b(x-1)_+^2$

$S'(x) = a_1 + 2a_2 x + 2b(x-1)_+$

Interpoláció

$S(0) = -1: \quad a_0 = -1$

$S(1) = 1: \quad a_0 + a_1 + a_2 = 1$

$S(2) = -1: \quad a_0 + 2a_1 + 4a_2 + b = -1$

Peremfelt.

$S'(0) = 0: \quad a_1 = 0$

Meo.:

$$\boxed{\begin{array}{l} a_0 = -1 \quad a_1 = 0 \quad a_2 = 2 \quad b = -8 \\ S(x) = -1 + 2x^2 - 8(x-1)_+^2 \end{array}}$$

14.) $S(x) = a_0 + a_1 x + a_2 x^2 + b(x+1)_+^2$

$S'(x) = a_1 + 2a_2 x + 2b(x+1)_+$

Interpoláció

$S(-2) = 2: \quad a_0 - 2a_1 + 4a_2 = 2$

$S(-1) = 1: \quad a_0 - a_1 + a_2 = 1$

$S(0) = -1: \quad a_0 + b = -1$

Peremfelt.

$S'(0) = -2: \quad a_1 + 2b = -2$

Meo.:

$$\boxed{\begin{array}{l} a_0 = -2 \quad a_1 = -4 \quad a_2 = -1 \quad b = 1 \\ S(x) = -2 - 4x - x^2 + (x+1)_+^2 \end{array}}$$

15.) $S(x) = a_0 + a_1 x + a_2 x^2 + b(x-0)_+^2$

$S'(x) = a_1 + 2a_2 x + 2b(x-0)_+$

Interpoláció

$S(-1) = 0: \quad a_0 - a_1 + a_2 = 0$

$S(0) = 1: \quad a_0 = 1$

$S(1) = 2: \quad a_0 + a_1 + a_2 + b = 2$

Peremfelt.

$S'(-1) = 0: \quad a_1 - 2a_2 = 0$

Meo.:

$$\boxed{\begin{array}{l} a_0 = 1 \quad a_1 = 2 \quad a_2 = 1 \quad b = -2 \\ S(x) = 1 + 2x + x^2 - 2(x-0)_+^2 \end{array}}$$

16.) $S(x) = a_0 + a_1 x + a_2 x^2 + b(x-0)_+^2$

$S'(x) = a_1 + 2a_2 x + 2b(x-0)_+$

Interpoláció

$S(-1) = -1: \quad a_0 - a_1 + a_2 = -1$

$S(0) = 1: \quad a_0 = 1$

$S(1) = -1: \quad a_0 + a_1 + a_2 + b = -1$

Perem felt.

$S'(-1) = 0: \quad a_1 - 2a_2 = 0$

Meo.:

$a_0 = 1 \quad a_1 = 4 \quad a_2 = 2 \quad b = -8$

$S(x) = 1 + 4x + 2x^2 - 8(x-0)_+^2$

17.) $S(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + b(x-0)_+^3$

$S'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 3b(x-0)_+^2$

$S''(x) = 2a_2 + 6a_3 x + 6b(x-0)_+$

Interpoláció

$S(-\pi) = -1: \quad a_0 - \pi a_1 + \pi^2 a_2 - \pi^3 a_3 = -1$

$S(0) = 1: \quad a_0 = 1$

$S(\pi) = -1: \quad a_0 + \pi a_1 + \pi^2 a_2 + \pi^3 a_3 + \pi^3 b = -1$

Perem felt.

$S'(-\pi) = S'(\pi): \quad a_1 \cancel{+} - 2\pi a_2 + 3\pi^2 a_3 = a_1 + 2\pi a_2 + 3\pi^2 a_3 + 3\pi^2 b$

$S''(-\pi) = S''(\pi): \quad 2a_2 - 6\pi a_3 = 2a_2 + 6\pi a_3 + 6\pi b$

Meo.

$a_0 = 1 \quad a_1 = -\frac{12}{\pi} \quad a_2 = -\frac{6}{\pi^2} \quad a_3 = -\frac{4}{\pi^3} \quad b = \frac{8}{\pi^3}$

$S(x) = 1 - \frac{12}{\pi} x - \frac{6}{\pi^2} x^2 - \frac{4}{\pi^3} x^3 + \frac{8}{\pi^3}(x-0)_+^3$

18.) $S(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + b\left(x - \frac{\pi}{2}\right)_+^3$

$S'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 3b\left(x - \frac{\pi}{2}\right)_+^2$

$S''(x) = 2a_2 + 6a_3 x + 6b\left(x - \frac{\pi}{2}\right)_+$

Interpoláció

$S(0) = 0: \quad a_0 = 0$

$S\left(\frac{\pi}{2}\right) = 1: \quad a_0 + \frac{\pi}{2} a_1 + \frac{\pi^2}{4} a_2 + \frac{\pi^3}{8} a_3 = 1$

$S(\pi) = 0: \quad a_0 + \pi a_1 + \pi^2 a_2 + \pi^3 a_3 + \frac{\pi^3}{8} b = 0$

Perem feld.

$S''(0) = 0: \quad 2a_2 = 0$

$S''(\pi) = 0: \quad 2a_2 + 6\pi a_3 + 3\pi \cdot b = 0$

Meo.

$$\boxed{\begin{array}{l} a_0 = 0 \qquad a_1 = \frac{3}{\pi} \qquad a_2 = 0 \qquad a_3 = -\frac{4}{\pi^3} \qquad b = \frac{8}{\pi^3} \\ S(x) = \frac{3}{\pi} x - \frac{4}{\pi^3} x^3 + \frac{8}{\pi^3}\left(x - \frac{\pi}{2}\right)_+^3 \end{array}}$$

19.) $S(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + b(x-0)_+^3$ $\qquad f(x) = x^5$

$S'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 3b(x-0)_+^2$ $\qquad f'(x) = 5x^4$

Interpoláció

$S(-1) = -1: \quad a_0 - a_1 + a_2 - a_3 = -1$

$S(0) = 0: \quad a_0 = 0$

$S(1) = 1: \quad a_0 + a_1 + a_2 + a_3 + b = 1$

Perem felt.

$S'(-1) = f'(-1):$ $a_1 - 2a_2 + 3a_3 = 5$

$S'(1) = f'(1):$ $a_1 + 2a_2 + 3a_3 + 3b = 5$

Meo.

$$\boxed{\begin{array}{l} a_0 = 0 \qquad a_1 = -1 \qquad a_2 = 0 \qquad a_3 = 2 \qquad b = 0 \\ S(x) = -x + 2x^3 \end{array}}$$

20.) $S(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + b(x-0)_+^3$ $f(x) = -x^6$

$S'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 3b(x-0)_+^2$ $f'(x) = -6x^5$

Interpoláció

$S(-1) = -1:$ $a_0 - a_1 + a_2 - a_3 = -1$

$S(0) = 0:$ $a_0 = 0$

$S(1) = -1:$ $a_0 + a_1 + a_2 + a_3 + b = -1$

Perem felt.

$S'(-1) = f'(-1):$ $a_1 - 2a_2 + 3a_3 = 6$

$S'(1) = f'(1):$ $a_1 + 2a_2 + 3a_3 + 3b = -6$

Meo.

$$\boxed{\begin{array}{l} a_0 = 0 \qquad a_1 = 0 \qquad a_2 = 3 \qquad a_3 = 4 \qquad b = -8 \\ S(x) = 3x^2 + 4x^3 - 8(x-0)_+^3 \end{array}}$$