Házi feladatok III

Számítsa ki az alábbi mátrixok általánosított inverzét!

1.)

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^+ = (A^* A)^{-1} A^* = \frac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

$$B^+ = B^*(B B^*)^{-1} = \frac{1}{6}\begin{bmatrix} 0 & 2 \\ -3 & 5 \\ 3 & -1 \end{bmatrix}$$

2.)

$$A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A^+ = \frac{1}{6}\begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \end{bmatrix} \qquad B^+ = \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

3.) Igaz-e, hogy A^+ az A általánosított inverze?

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad A^+ = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4.)

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad A^+ = \frac{1}{5}\begin{bmatrix} 2 & 1 & -1 & 2 \\ -1 & 2 & 3 & -1 \end{bmatrix}$$

5.) (Példatár 271/8.):

x_i	-3	-2	0	1	4
y_i	-3	-2	1	2	5

a.) egyenes: $P(x) = \frac{7}{6}x + \frac{3}{5}$

b.) parabola: $P(x) = -\frac{6}{144}x^2 + \frac{29}{24}x + \frac{17}{20} =$

$= -\frac{1}{24}x^2 + \frac{29}{24}x + \frac{17}{20}$

6.)

X_i	-2	-1	0	1	2
y_i	1	1	0	2	2

a.) egyenes: $P(x) = \frac{3}{10}x + \frac{6}{5}$

b.) parabola: $P(x) = \frac{3}{14}x^2 + \frac{3}{10}x + \frac{27}{35}$

7.)

X_i	-1	0	1	2
y_i	0	2	2	4

a.) egyenes: $P(x) = \frac{6}{5}x + \frac{7}{5}$

b.) parabola:

$$\begin{array}{c|c} A^T A & A^T y \\ \begin{bmatrix} 4 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{bmatrix} & \begin{matrix} 8 \\ 10 \\ 18 \end{matrix} \end{array}$$

$P(x) = \frac{7}{5} + \frac{6}{5}x + 0 \cdot x^2$

8.)

X_i	-3	-1	0	1	3
Y_i	0	2	3	1	0

a.) egyenes: $P(x) = -\frac{1}{20}x + \frac{6}{5}$

b.) parabola: $P(x) = -\frac{1}{4}x^2 - \frac{1}{20}x + \frac{11}{5}$

9.)

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 1 & 2 \\ 4 & 1 & 1 & 4 \\ -8 & -1 & 1 & 8 \end{bmatrix}} \underset{\underline{c}}{\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}} = \underset{\underline{b}}{\begin{bmatrix} \int_{-2}^{2} 1\,dx = 4 \\ \int_{-2}^{2} x\,dx = 0 \\ \int_{-2}^{2} x^2\,dx = \frac{16}{3} \\ \int_{-2}^{2} x^3\,dx = 0 \end{bmatrix}}$$

$$A^{-1} = \frac{1}{12}\begin{bmatrix} -2 & 1 & 2 & -1 \\ 8 & -8 & -2 & 2 \\ 8 & 8 & -2 & -2 \\ -2 & -1 & 2 & 1 \end{bmatrix}$$

10.)

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 3 \\ 1 & 0 & 1 & 9 \\ -1 & 0 & 1 & 27 \end{bmatrix}} \underset{\underline{c}}{\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}} = \underset{\underline{b}}{\begin{bmatrix} \int_{-1}^{3} 1\,dx = 4 \\ \int_{-1}^{3} x\,dx = 4 \\ \int_{-1}^{3} x^2\,dx = \frac{28}{8} \\ \int_{-1}^{3} x^3\,dx = \frac{80}{4} = 20 \end{bmatrix}}$$

$$A^{-1} = \begin{bmatrix} 0 & -9 & 12 & -3 \\ 24 & -8 & -24 & 8 \\ 0 & 18 & 12 & -6 \\ 0 & -1 & 0 & 1 \end{bmatrix} \cdot \frac{1}{24}$$

11.)

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 4 & 9 \\ 0 & 1 & 8 & 27 \end{bmatrix}} \underset{\underline{c}}{\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}} = \underset{\underline{b}}{\begin{bmatrix} \int_{0}^{3} 1\,dx = 3 \\ \int_{0}^{3} x\,dx = \frac{9}{2} \\ \int_{0}^{3} x^2\,dx = 9 \\ \int_{0}^{3} x^3\,dx = \frac{81}{4} \end{bmatrix}}$$

$$A^{-1} = \begin{bmatrix} 6 & -11 & 6 & -1 \\ 0 & 18 & -15 & 3 \\ 0 & -9 & 12 & -3 \\ 0 & 2 & -3 & 1 \end{bmatrix} \cdot \frac{1}{6}$$

12.)

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ -\frac{3}{5} & -\frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ \frac{9}{25} & \frac{1}{25} & \frac{1}{25} & \frac{9}{25} \\ \frac{-81}{125} & \frac{-1}{125} & \frac{1}{125} & \frac{81}{125} \end{bmatrix}} \underset{\underline{c}}{\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}} = \underset{\underline{b}}{\begin{bmatrix} \int_{-1}^{1} 1\,dx = 2 \\ \int_{-1}^{1} x\,dx = 0 \\ \int_{-1}^{1} x^2\,dx = \frac{2}{3} \\ \int_{-1}^{1} x^3\,dx = 0 \end{bmatrix}}$$

$$A^{-1} = \begin{bmatrix} \frac{-1}{16} & \frac{5}{156} & \frac{25}{16} & \frac{-125}{156} \\ \frac{9}{16} & \frac{-135}{52} & \frac{-25}{16} & \frac{125}{52} \\ \frac{9}{16} & \frac{135}{52} & -\frac{25}{16} & -\frac{125}{52} \\ \frac{-1}{16} & \frac{-5}{156} & \frac{25}{16} & \frac{125}{156} \end{bmatrix}$$

(13) $$If = \int_{\frac{\pi}{2}}^{\pi} \cos(x)\cdot x \, dx \approx ?$$

$$= \left[\sin(x)\,x\right]_{\frac{\pi}{2}}^{\pi} - \int_{\frac{\pi}{2}}^{\pi} \sin(x)\,dx = -\frac{\pi}{2} - 1 \approx -2{,}5708$$

a.) érintő: $$I_0 f = f\left(\frac{\pi+\frac{\pi}{2}}{2}\right)\cdot\frac{\pi}{2} = f\left(\frac{3\pi}{4}\right)\cdot\frac{\pi}{2} = -\frac{1}{\sqrt{2}}\cdot\left(\frac{\pi}{2}\right)\cdot\frac{3\pi}{4} = -\frac{3\pi^2}{8\sqrt{2}} \approx -2{,}6171$$

Hiba: $$|I_0 f - If| \le \frac{1}{24}\cdot\|f''\|_{C[\frac{\pi}{2},\pi]}\cdot\left(\pi-\frac{\pi}{2}\right)^3$$

$f' = -\sin(x)\cdot x + \cos(x)$

$f'' = -\cos(x)\cdot x - \sin(x) - \sin x$
$= -\cos(x)\cdot x - 2\sin(x)$

$\|f''\| = |f''(\pi)| = \pi$

$$= \frac{1}{24}\cdot\pi\cdot\left(\frac{\pi}{2}\right)^3 = \frac{\pi^4}{3\cdot 2^6} \approx 0{,}5073 > |-2{,}5708+2{,}6171|$$

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b.) trapéz: $$I_1 f = \frac{\left(\pi-\frac{\pi}{2}\right)}{2}\cdot\left(f(\pi)+f\left(\frac{\pi}{2}\right)\right) = \frac{\pi}{4}(-\pi+0) = -\frac{\pi^2}{4} \approx -2{,}467$$

Hiba: $$|I_1 f - If| \le \frac{1}{12}\cdot\|f''\|_{C[\frac{\pi}{2},\pi]}\cdot\left(\pi-\frac{\pi}{2}\right)^3 = \frac{1}{12}\cdot\frac{\pi^4}{8}$$

Mj.: az érintő hibájának kétszerese!

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c.) Simpson: $$I_2 f = \left(\frac{\pi-\frac{\pi}{2}}{6}\right)\cdot\left(f\left(\frac{\pi}{2}\right) + 4\cdot f\left(\frac{3\pi}{4}\right) + f(\pi)\right) =$$

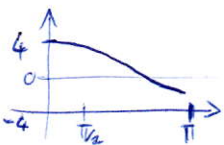
a.)-b.)-ben már kiszámoltuk az f-értékeit! →

$$= \frac{\pi}{12}\left(0 + \cancel{4}\cdot\left(-\frac{3\pi}{\cancel{4}}\cdot\frac{1}{\sqrt{2}}\right) - \pi\right) = \frac{\pi^2}{12}\left(-1-\frac{3}{\sqrt{2}}\right) \approx -2{,}567$$

$f''' = \sin(x)x - 3\cos(x)$

$f^{IV} = \cos(x)\cdot x + 4\sin(x)$

$\|f^{IV}\| = 4 = |f^{IV}(\frac{\pi}{2})|$



Hiba: $$|I_2 f - If| \le \frac{1}{2880}\cdot\|f^{IV}\|_{C[\frac{\pi}{2},\pi]}\cdot\left(\pi-\frac{\pi}{2}\right)^5 =$$

$$= \frac{4}{2880}\,\frac{\pi^5}{2^5} = \frac{\pi^5}{2880\cdot 8} \approx 0{,}0133$$

A Simpson-formula értéke

$$S(f) = \frac{1}{6} \cdot \left[0^2 + 4 \cdot \left(\frac{1}{2} \right)^2 + 1^2 \right] = \frac{1}{6} \cdot 2 = \frac{1}{3}.$$

Látjuk, hogy a Simpson-formula az x^2 polinom integráljára pontos értéket ad. Mivel a Simpson formulának 3 alappontja van, ezért minden legfeljebb másodfokú polinomra pontos integrálközelítést ad. Ellenőrizhetjük, hogy a Simpson-formula ennél többet tud, a harmadfokú polinomokra is pontos.

3.14 ~~**9.**~~ **a)** Az érintő-formula értéke

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$$E(f) = 1 \cdot \frac{1}{\frac{3}{2}} = \frac{2}{3}.$$

Az érintő-formula hibabecsléséhez szükségünk van az $M_2 = \|f''\|_\infty$ értékre.

$$f'(x) = -\frac{1}{x^2}$$
$$f''(x) = \frac{2}{x^3} \quad \to \quad |f''(x)| \le 2 = M_2, \ \ \forall \ x \in [1;2]$$

A érintő-formula hibabecslése

$$\left| \ln 2 - \frac{3}{2} \right| \le \frac{(2-1)^3}{24} M_2 = \frac{1}{12}.$$

b) A trapéz-formula értéke

$$T(f) = \frac{1}{2} \cdot \left[\frac{1}{1} + \frac{1}{2} \right] = \frac{3}{4}.$$

Az trapéz-formula hibabecslése

$$\left| \ln 2 - \frac{3}{4} \right| \le \frac{(2-1)^3}{12} M_2 = \frac{1}{6}.$$

c) A Simpson-formula értéke

$$S(f) = \frac{1}{6} \cdot \left[\frac{1}{1} + 4 \cdot \frac{1}{\frac{3}{2}} + \frac{1}{2} \right] = \frac{1}{6} \cdot \left[1 + \frac{8}{3} + \frac{1}{2} \right] = \frac{6 + 16 + 3}{36} = \frac{25}{36}.$$

A Simpson-formula hibabecsléséhez szükségünk van az $M_4 = \|f^{(4)}\|_\infty$ értékre.

$$f^{(3)}(x) = -\frac{6}{x^3}$$
$$f^{(4)}(x) = \frac{24}{x^4} \quad \to \quad |f^{(4)}(x)| \le 24 = 4! = M_4, \ \ \forall \ x \in [1;2]$$

A Simpson-formula hibabecslése

$$\left| \ln 2 - \frac{25}{36} \right| \le \frac{(2-1)^5}{4! \cdot 5!} M_4 = \frac{4!}{4! \cdot 5!} = \frac{1}{120}.$$

(14) Példatár 303/9 $\int_1^2 \frac{1}{x}\,dx = \ln 2 \approx ?$

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(15). $If = \int_0^2 x \cdot 2^x \,dx \approx ?$

$$= \left[\frac{2^x}{\ln 2}\cdot x\right]_0^2 - \int_0^2 \frac{2^x}{\ln 2}\,dx = \frac{8}{\ln 2} - \frac{3}{(\ln 2)^2} \approx \underline{\underline{5{,}2975}}$$

a.) érintő: $I_0 f = f\left(\frac{2+0}{2}\right)\cdot(2-0) = 2\cdot 2 = 4$

Hibabecslés: $|I_0 f - If| \leq \frac{1}{24}\cdot \|f''\|_{C[0,2]}\cdot(2-0)^3 \leq \frac{16}{\cancel{24}_3}\cdot \cancel{8}^1 = \frac{16}{3} \approx 5{,}33$

$f'(x) = 2^x + x\cdot 2^x\cdot \ln 2 = 2^x\cdot(1 + x\ln(2))$

$f''(x) = 2^x\left(\ln 2\cdot(1 + x\ln 2) + \ln 2\right) = 2^x\cdot \ln 2\left(2 + x\cdot\ln 2\right)$

$\|f''\|_{C[0,2]} = |f''(2)| = 4\underbrace{\ln 2}_{\wedge\, 1}(2 + 2\underbrace{\ln 2}_{\wedge\, 1}) \leq 4\cdot(2+2) = 16$

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b.) trapéz: $I_1 f = \left(\frac{2-0}{2}\right)\cdot\left(f(0) + f(2)\right) = 0 + 8 = \underline{\underline{8}}$

Hiba: $|I_1 f - If| \leq \frac{1}{12}\cdot\|f''\|_{C[0,2]}\cdot(2-0)^3 \leq \frac{32}{3} \approx 10{,}66$

érintő 2x-ese!

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d.) Simpson: $I_2 f = \frac{2-0}{6}\left(f(0) + 4\cdot f(1) + f(2)\right) = \frac{1}{3}(0 + 8 + 8) = \frac{16}{3} \approx 5{,}33$

Hiba: $|I_2 f - If| \leq \frac{1}{2880}\cdot\|f^{IV}\|_{C[0,2]}\cdot(2-0)^5 \leq \frac{15}{\cancel{2880}_{180}}\cdot\cancel{2^4} = \frac{15}{180}$

$\approx 0{,}0833$

$f'''(x) = 2^x\cdot(\ln 2)^2(2 + x\cdot\ln 2) + 2^x\cdot(\ln 2)^2 = 2^x \ln^2 x\,(3 + x\ln 2)$

$f^{IV}(x) = 2^x \ln^3 2\,(4 + x\ln 2)$

$\|f^{IV}\|_{C[0,2]} = |f^{IV}(2)| \leq \frac{15}{2}$

$\underset{7{,}17}{\approx}$

16.

$$If = \int_2^5 \frac{1}{\sqrt{x-1}}\,dx \approx ?$$

$$= \left[2\cdot\sqrt{x-1}\right]_2^5 = 2\sqrt{4} - 2 = 2(\sqrt{4}-1) = 2$$

a.) érintő: $I_0 f = (5-2)\cdot f\left(\frac{5+2}{2}\right) = \frac{3}{\sqrt{\frac{7}{2}-1}} = 3\cdot\sqrt{\frac{2}{5}} \approx \underline{\underline{1{,}897}}$

Hiba: $|If - I_0 f| \leq \frac{1}{24}\cdot\|f''\|_{C[2,5]}\cdot(5-2)^3 = \frac{3}{8}\cdot\frac{1}{4}\cdot 3^2 = \frac{27}{32} =$

$= 0{,}84375$

$f'(x) = -\frac{1}{2}\cdot(x-1)^{-3/2}$

$f''(x) = \frac{3}{4}\cdot(x-1)^{-5/2}$

$\|f''\|_{C[2,5]} = \frac{3}{4}$

b.) trapéz: $I_1 f = \frac{(5-2)}{2}\left(f(2)+f(5)\right) = \frac{3}{2}\left(1+\frac{1}{2}\right) = \frac{9}{4} = \underline{\underline{2{,}25}}$

Hiba: $|If - I_1 f| \leq \frac{1}{12}\cdot\|f''\|_{C[2,5]}\cdot(5-2)^3 = \frac{54}{32} = \frac{27}{16} = 1{,}6875$

c.) Simpson: $I_2 f = \frac{5-2}{6}\left(f(2) + 4\cdot f\left(\frac{7}{2}\right) + f(5)\right) = \frac{1}{2}\left(1 + 4\cdot\sqrt{\frac{2}{5}} + \frac{1}{2}\right) \approx$

$\approx \underline{\underline{2{,}015}}$

$f'''(x) = -\frac{15}{8}(x-1)^{-7/2}$

$f^{IV}(x) = \frac{105}{16}(x-1)^{-9/2}$

$\|f^{IV}\|_{C[2,5]} = |f^{IV}(2)| = \frac{105}{16}$

$|If - I_2 f| \leq \frac{\|f^{IV}\|_{C[2,5]}\cdot(5-2)^5}{2880} = \frac{105\cdot 3^5}{16\cdot 2880} \approx 0{,}55$

(17.) $\int_1^2 y^2 \sin\left(\frac{1}{y}\right) dy \approx ?$ Hány formula kell?

a.) érintő $|If - E_m f| \leq \frac{2-1}{24} \cdot \|f''\|_{C[1,2]} \cdot \left(\frac{2-1}{m}\right)^2 \leq *$

$f'(x) = 2y \sin\left(\frac{1}{y}\right) + y^2 \cos\left(\frac{1}{y}\right)\left(-\frac{1}{y^2}\right)$

$f''(x) = 2 \sin\left(\frac{1}{y}\right) - 2 \cdot \frac{1}{y} \cos\left(\frac{1}{y}\right) - \sin\left(\frac{1}{y}\right) \cdot \frac{1}{y^2}$

$x \in [1,2]$

$|f''(x)| \leq \left|2 + \frac{2}{y} + \frac{1}{y^2}\right| \leq 5 = \|f''\|_{C[1,2]}$

$* \leq \frac{5}{24} \cdot \left(\frac{1}{m}\right)^2 \overset{?}{\leq} 10^{-3}$

$\sqrt{\frac{5000}{24}} \leq m$

$\approx 14,4$

$\underline{\underline{m = 15}}$ már elég!

b.) trapéz: $|If - T_m f| \leq \frac{2-1}{12} \|f''\|_{C[1,2]} \cdot \left(\frac{2-1}{m}\right)^2 \leq \frac{5}{12} \cdot \frac{1}{m^2} \overset{?}{\leq} 10^{-3}$

$\sqrt{\frac{5000}{12}} \leq m^2 \Rightarrow \underline{\underline{m^* = 21}}$ már elég!

c.) Simpson: $|If - S_m f| \leq \frac{(2-1)}{2880} \cdot \|f^{IV}\|_{C[1,2]} \cdot \frac{1}{m^4} \leq (*)$

$f'''(y) = \cancel{-\frac{2}{y^2} \cos\left(\frac{1}{y}\right) + \frac{2}{y^2} \cdot \cos\left(\frac{1}{y}\right)} + \cancel{2 \frac{1}{y^3} \sin\left(\frac{1}{y}\right)} + \cancel{2 y^{-3} \sin\left(\frac{1}{y}\right)} + \cos\left(\frac{1}{y}\right) \cdot \frac{1}{y^4}$

$f^{IV}(y) = \sin\left(\frac{1}{y}\right) \cdot \frac{1}{y^6} - 4 \cdot \frac{1}{y^5} \cdot \cos\left(\frac{1}{y}\right)$

$|f^{IV}(y)| \leq \left|\frac{1}{y^6}\right| + \left|\frac{4}{y^5}\right| \overset{y \in [1,2]}{\leq} 5$

$\underline{\underline{m = 2}}$ már elég

$(*) \leq \frac{5}{2880} \cdot \frac{1}{m^4} \leq 10^{-3}$

$1,14 \approx \sqrt[4]{\frac{5000}{2880}} < m$

(18) $\int_1^2 \cos(\ln x)\,dx = ?$ Hány formula kell?

$f'(x) = -\sin(\ln x)\cdot\frac{1}{x}$

$f''(x) = \frac{\sin(\ln x)}{x^2} - \frac{\cos(\ln x)}{x^2}$

$f^{IV}(x) = -\frac{10\cdot\cos(\ln x)}{x^4}$

$f'''(x) = -\frac{2}{x^3}\left(\sin(\ln x) - \cos(\ln x)\right) + \frac{1}{x^3}\left(\cos(\ln x) + \sin(\ln x)\right)$

$= \frac{3\cos(\ln x)}{x^3} - \frac{\sin(\ln x)}{x^3}$

---

a.) $|If - E_m f| \le \frac{2-1}{24}\|f''\|_{C[1,2]}\cdot\left(\frac{2-1}{m}\right)^2 \le \frac{1}{\cancel{24}_{12}}\cdot\cancel{2}^1\cdot\frac{1}{m^2} \overset{?}{\le} 10^{-3}$

$x\in[1,2]$

$|f''(x)| \le 2$

$m > 9{,}12 \quad \Leftarrow \sqrt{\frac{1000}{12}} \le m$

Tehát $\underline{\underline{m=10}}$ már elég.

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b.) $|If - T_m f| \le \frac{2-1}{12}\|f''\|_{C[1,2]}\cdot\frac{1}{m^2} \le \frac{1}{6}\frac{1}{m^2} \overset{?}{\le} 10^{-3}$

$\sqrt{\frac{1000}{6}} \le m \Rightarrow 12{,}9 \le m \Rightarrow \underline{\underline{m=13}}$ már elég

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c.) $|If - S_m f| \le \frac{2-1}{2880}\|f^{IV}\|_{C[1,2]}\cdot\frac{1}{m^4} \le \frac{1}{288}\cdot\frac{1}{m^4} \overset{?}{\le} 10^{-3}$

$x\in[1,2]$

$|f^{IV}(x)| \le 10$

$1{,}3 \approx \sqrt[4]{\frac{1000}{288}} \le m \quad \underline{\underline{\Rightarrow m=2 \text{ már elég.}}}$

(19) $\int_1^3 \frac{\ln x}{x}\,dx \approx ?$ Hány formula kell?

$f'(x) = \frac{1}{x^2} - \frac{\ln x}{x^2}$

$f''(x) = \frac{2\ln x}{x^3} - \frac{3}{x^3}$

$f'''(x) = \frac{11}{x^4} - 6\frac{\ln x}{x^4}$

$f^{IV}(x) = 24\frac{\ln x}{x^5} - \frac{50}{x^5}$

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a.) Érintő: $|If - E_m f| \leq \frac{2}{24}\|f''\|_{C[1,3]} \cdot \left(\frac{2}{m}\right)^2 \leq \frac{\cancel{12}^1}{\cancel{24}_2} \cdot \cancel{4}^2 \cdot \frac{1}{m^2} = \frac{2}{m^2}$

$|f''(x)| \leq \frac{2\ln x}{x^3} + \frac{3}{x^3} \leq \frac{3}{x^3} + \frac{3}{x^3} \leq 6$

$\ln x < \frac{3}{2}$

$x \in [1,3]$

$\frac{2}{m^2} \leq 10^{-2} \Leftrightarrow$

$\sqrt{200} \leq m$

$m = 15$ már jó

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b.) Trapéz: $|If - T_m f| \leq \frac{2}{12} \cdot \|f''\|_{C[1,3]} \cdot \left(\frac{2}{m}\right)^2 \leq \frac{4}{m^2} \overset{?}{\leq} 10^{-2}$

$\Longleftrightarrow \sqrt{400} \leq m \Rightarrow m = 20$ már elég.

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c.) Simpson: $|If - S_m f| \leq \frac{2}{2880} \cdot \|f^{IV}\|_{C[1,3]} \cdot \left(\frac{2}{m}\right)^4 \overset{*}{\leq} \frac{86}{1440} \cdot \left(\frac{2}{m}\right)^4 \overset{?}{\leq} 10^{-2}$

$|f^{IV}(x)| \leq \frac{24\ln x + 50}{x^5} \leq \frac{86}{x^5} \leq 86$

$\ln(x) < \frac{3}{2}$ $(x \in [1,3])$

(*)

$\frac{43}{45} \cdot \frac{1}{m^4} \leq 10^{-2} \Longleftrightarrow m \geq 3{,}13 \Rightarrow m = 4$ már elég

20) $\int_2^5 x \ln x \, dx = ?$ Hány formula kell?

$f'(x) = \ln x + 1$ | $f'''(x) = -\frac{1}{x^2}$

$f''(x) = \frac{1}{x}$ | $f^{IV}(x) = \frac{2}{x^3}$

a.) érintő: $|If - E_m f| \leq \frac{5-2}{24} \cdot \|f''\|_{C[2,5]} \cdot \left(\frac{5-2}{m}\right)^2 = \frac{1}{16} \cdot \frac{9}{m^2} \overset{?}{\leq} 10^{-2}$

$\|f''(x)\|_{C[2,5]} = \frac{1}{2}$ | $\sqrt{\frac{900}{16}} \leq m$; ~~$m \geq 10{,}61 \Rightarrow m = 11$ már jó~~

$m \geq 7{,}5 \Rightarrow m = 8$ már jó

b.) Trapéz: $|If - T_m f| \leq \frac{5-2}{12} \|f''\|_{C[2,5]} \left(\frac{5-2}{m}\right)^2 = \frac{9}{8} \cdot \frac{1}{m^2} \overset{?}{\leq} 10^{-2}$

$\sqrt{\frac{900}{8}} \leq m \overset{m \geq 10,61}{\Rightarrow} m = 11$ már jó

c.) Simpson: $|If - S_m f| \leq \frac{5-2}{2880} \|f^{IV}\|_{C[2,5]} \cdot \left(\frac{5-2}{m}\right)^4 = \frac{27}{320} \cdot \frac{1}{4} \cdot \frac{1}{m^4}$

$\|f^{IV}(x)\|_{C[2,5]} = \frac{1}{4}$ | $\sqrt[4]{\frac{2700}{1280}} \leq m \Rightarrow m \geq 1{,}2 \Rightarrow$

$m = 2$ már elég