Linear Quadratic Regulator (LQR)

$$O = PA + A^{T}P + Q - PBR^{T}B^{T}P$$

$$U = -KX = -R^{-1}B^{T}P \times (\pm)$$

$$eig(\dot{x} = Ax + Bu = (A - BK)x$$

$$[K,P,p_c1] = 1qr(A,B,Q,R)$$

$$[P1,K1,p_c1] = icare(A,B,Q,R,[],[],[],[])$$

Pole Placement Design Using State Feedback

Poles
$$-\int \omega_{n} \pm \omega_{n} \int_{\mathbb{S}^{2}-1}^{\pi} \omega_{n} = \frac{\pi}{T_{p}\sqrt{1-\zeta^{2}}}$$

$$\Delta_{d}(s) = (s-\mu_{1})(s-\mu_{2})(s-\mu_{3})$$

$$\phi(A) = \phi(s=A)$$

$$= A^{3} + \mu_{A}^{2} + 6 \cdot A \qquad \phi(s) = \Delta_{d}(s)$$

$$+ 200 I_{2}$$

$$K = [0 \cdots 0 \quad 1]U^{-1}\Delta_{d}(A)$$

$$|sI - A + BK| \qquad \leftarrow \text{For SYMS}$$

$$\dot{z} = (A - BK)z \qquad u = -Kx$$

$$L = \Delta_d(s)|_{s=A} V^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|sI - (A - LC)| = 0 \leftarrow \text{For SYMS}$$

$$\begin{bmatrix} \dot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} r$$

Realizations of LTI Systems

$$\min_{x,\lambda} H(x,\lambda) \coloneqq L(x) + \lambda^T f(x)$$

Controllability in Canonical Forms

Observability in Canonical Forms

Obtaining the State Equations Without Derivatives of u(t)

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix} \quad \begin{bmatrix} b_0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \quad D = 0$$
With Derivatives
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & 1 & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -b_n a_0 + b_0 & -b_n a_1 + b_1 & \cdots & -b_n a_{n-1} + b_{n-1} \end{bmatrix}$$

$$D = b_n \quad n = \text{order}$$

$$\exists b_0 > b_1 = b_0$$

0 1 0 ··· : : 1 ·· 0 0 0 ···

State-Space to a Transfer Function

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

Controllable Kalman Decomposition

$$P = \begin{bmatrix} c_1 & c_2 & \cdots & c_{\mu} \\ \hline From U & Appended vectors \\ \hline A = \begin{bmatrix} A_c & A_{12} \\ \hline 0 & A_{c} \end{bmatrix}_{n-\mu}^{\mu} & A = P & P \\ \hline C = CP & A & C & P \end{bmatrix}$$

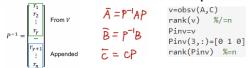
$$u = \operatorname{ctrb}(A, B)$$

$$rank(u)$$

$$p = u$$

$$p(:,3) = [0 \ 1 \ 0]$$

Observable Kalman Decomposition



Nonhomogeneous Solution

