

## Problem Set 12, Dec 8, 2022 (Theory Questions SVD)

**Goals.** The goal of this exercise is to

- familiarize yourself with the theory related to SVD.
- have time to discuss Project 2 with the assistants and teammates.

### 1 Theory Questions

**Problem 1 (How to compute  $U$  and  $S$  efficiently):**

In class, we saw that solving the eigenvector/value problem for the matrix  $\mathbf{X}\mathbf{X}^\top$  gives us a way to compute  $U$  and  $S$ . But in some instances  $D \gg N$ . In those cases, is there a way to accomplish this computation more efficiently?

**Problem 2 (Positive semi-definite):**

Show that if  $\mathbf{X}$  is a  $N \times N$  symmetric matrix then the SVD has the form  $\mathbf{U}\mathbf{S}\mathbf{U}^\top$ , where  $U$  is a  $N \times N$  unitary matrix and  $S$  is a  $N \times N$  diagonal matrix with non-necessarily positive entries. Show that if  $\mathbf{X}$  is positive semi-definite, then all entries of  $S$  are non-negative.

### 2 Generative Adversarial Networks

Recommended reading: explore how to implement a simple GAN in PyTorch using the Jupyter notebook `gans.ipynb`:

- Open in Google Colab: [colab.research.google.com/github/epfml/ML\\_course/blob/master/labs/ex12/gans.ipynb](https://colab.research.google.com/github/epfml/ML_course/blob/master/labs/ex12/gans.ipynb). This gives you access to a free GPU.
- Change the 'runtime type' to GPU under 'Runtime → Change runtime type'.