

PART – A

Answer any SEVEN of the following questions.

7x2=14

- a. Find the rank of the matrix A using the elementary row operation, where

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 1 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$$

- b. What are logical connectives? What are the symbols used to denote the logical connectives?

- c. State the Pigeonhole Principle and give an example.

- d. What is the necessary and sufficient condition for

i. Linear homogenous system to have a non – trivial solution.

ii. Linear non – homogenous system to have a non – trivial solution.

e. If $\begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$ find x, y, z.

- f. Write the differences between a relation and function.

- g. Find the truth set of the open sentence $p(x): 2x^2 - 7x + 6 = 0$, Where $x \in \mathbb{Z}$, the set of all integers.

h. Evaluate (i) $\frac{8!}{2! \times 4!}$ (ii) ${}^{10}C_4$

- i. Symbolize the following and negate: "Some students are lazy or all students are hard working."

- j. Determine the number of combinations of 5 cards out of a deck of 52 cards.

PART – B

Answer any SEVEN of the following questions.

7x5=35

- a. A bit is either 0 or 1, a byte is a sequence of 8 bits. Find

i. The number of bytes that can be formed

ii. The number of bytes that begin with 11 and end with 11.

iii. The number of bytes that begin with 11 and do not end with 11.

- b. If $p(x): x$ is a prime divisor of 210 and $q(x): x^2 < 40$ with the replacement set as set \mathbb{Z} of all integers. Find the solution set of $p(x) \vee q(x)$ and $p(x) \wedge q(x)$

- c. Find 'r' if $2 {}^7P_r = 5 {}^8P_{r-1}$

- d. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{x: x \text{ is an even number with } 2 \leq x < 10\}$, $B = \{1, 2, 3, 4, 5\}$.

Find $A \cup B$, $A \cap B$, A' , B' and $A \Delta B$

- e. Define rank of a matrix and reduce the following matrix to its row reduced echelon form

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & 4 & 7 \\ -1 & -3 & 2 & -1 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

- f. In a function each one shake hand with the other only once. If the total number of handshakes was 36, how many people were there in this function?
- g. Show that the equations $5x + 2y + 3z = a$; $3x + 5y + 2z = b$; $2x + 3y - 5z = c$ have a solution only if $a + b + c = 0$. Solve the equations when $a = 0$, $b = 1$ and $c = -1$.
- h. Find the real values of λ for which the following system has a non-zero solution
 $x + 2y + 3z = \lambda x$; $3x + y + 2z = \lambda y$; $2x + 3y + z = \lambda z$
- i. Explain function. With an example. Also explain one-one and onto function.
- j. Explain **Conjunction** and **Conditional** in mathematical logic using a truth table.

PART - C

Answer any THREE of the following questions.

- a. What is a consistent linear equation? Solve completely the following system of equations
 $x + 6y + 3z + 8w = 0$; $2x + 4y + 6z - w = 0$; $3x + 10y + 9z + 7w = 0$; $4x + 16y + 12z + 15w = 0$
- b. Alice wants to arrange 3 mathematics, 2 computer science and 4 language books on a shelf. If each of the book is of different publication. Find
- The number of possible arrangements
 - The number of possible arrangements if all the books of a subject are to be together
 - The number of possible arrangements if 2 computer science books must be next to each other.
- c. In a group of 100 persons, 62 can speak English and 48 can speak Kannada.
- How many can speak English only?
 - How many can speak Kannada only?
 - How many can speak both English and Kannada?
- d. What is a non - singular matrix? Find the inverse of the following matrix using elementary row transformations $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$.
- e. Prove the following result using (i) Direct Proof (ii) Indirect Proof and (iii) Proof by contradiction. "If n is an odd integer then $n + 11$ is even."

SECTION - A

Answer any FOUR of the following questions.

4×5=20

- a. Define rank of a matrix and find the rank of the matrix A using elementary row operation, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$.
- b. Examine the following system of equations for consistency and solve if it is consistent
 - i. $x - 7y + 15z = -14$
 - ii. $2x + 3y - 4z = 6$
 - iii. $3x - 4y + 11z = -8$
 - iv. $5x - y + 7z = -2$
- c. Check whether the following compound propositions are logically equal or not.
 $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$.
- d. In a class of 35 students, 17 have taken mathematics 10 have taken mathematics but not economics. Find the number of students who have taken economics but not Maths. Given that each student has taken either Maths or economics or both.
- e. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $A = \{1, 2, 3, 4\}$ $B = \{2, 4, 6, 8\}$ $C = \{3, 4, 5, 6\}$. Find
 - i. $A \cap B$
 - ii. $A \cup B$
 - iii. $B \cap C$
 - iv. $A \cap (B \cap C)$
 - v. $A \cap (B \cup C)$

SECTION - B

Answer any TWO of the following questions.

2×9=18

- a. Solve the following system of equation

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0. \end{aligned}$$
- b. For all the universe of all integers let
 $p(x): x > 0$, $q(x): x$ is even,
 $r(x): x$ is a perfect square, $s(x): x$ is divisible by 3, $t(x): x$ is divisible by 7.
 Write the following symbolic form in words and indicate the truth values
 - i. $\forall x, \sim r(x)$.
 - ii. $\exists x, [s(x) \wedge \sim q(x)]$.
 - iii. $\forall x, [r(x) \vee t(x)]$.

- c. How many arrangements are there for all letters in the word SOCIOLOGICAL? In how many of these arrangements
- A & G are adjacent?
 - All vowels are adjacent?

SECTION - C

3. Answer the following question.

- a. If $A = [1\ 5\ 6; 7\ 8\ 9; 10\ 11\ 12]$, $B = [4\ -2\ 3; 0\ -1\ 2; -3\ 4\ 5]$, $C = [2\ 3\ 1; 1\ 4\ 5; 7\ 8\ 3]$
Find $A+B$, $A-B$, $A+B+C$ and $A-B+C$ and Verify $A+(B+C)=(A+B)+C$ and $A+B=B+A$



SECTION - A

Answer any FOUR of the following questions.

4×5=20

- a. Define the rank of a matrix and find the rank of the matrix A using the elementary row operation, where $A = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$.
- b. Solve the system of equations,
- $$\begin{aligned} x + y - 2z &= 5, \\ x - 2y + z &= -2, \\ -2x + y - z &= 0. \end{aligned}$$
- c. Evaluate whether the following compound propositions are logically equally or not.
 $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$.
- d. Examine whether the function $f(x) = 4x+3$ from $R \rightarrow R$ is bijective. Find the inverse of the following: $f(1)$, $f(-1)$ and $f(3/7)$.
- e. Solve for n if $nP_3 : nP_2 = 3:1$.

SECTION - B

Answer any TWO of the following questions.

2×9=18

Evaluate the inverse of a matrix and find the inverse of the following matrix using

elementary row transformations $A = \begin{bmatrix} 8 & 4 & -3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

Examine if 6 boys and 4 girls got elected as a class representative in your college a five-member student Council has to be formed from the elected class representatives in how many ways this Council can be formed such that

- There are 3 boys and 2 girls
- At least two girls
- At most two boys

- c. Solve $g \cdot f$, $f \cdot g$, f^2 & g^2 for the function f and g defined by $f(x) = x^3$ and $g(x) = x^2$ and $vx \in \mathbb{R}$.

SECTION - C

3. Answer the following question.

1x12

Evaluate the rank and inverse of the following matrix using elementary row transformations.

where $M = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$.