Date Module 4 — 5M + 9m = 14m.

Probability.

1. Addition Rule for 2 Events.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

2. Independence of Event.

$$P(A \cap B) = P(A) \times P(B).$$

⇒ from conditional Probability

wKT. $P(A/B) = \dfrac{P(A \cap B)}{P(B)} \Rightarrow P(A \text{ and } B) = P(A/B) \cdot P(B)$ → ①

$P(B/A) = \dfrac{P(A \cap B)}{P(A)} \Rightarrow P(A \text{ and } B) = \dfrac{P(B/A) \cdot P(A)}{\cancel{P(A)}}$ → ②

for independent events, from ① & ②

$P(A \text{ and } B) = P(A/B) \cdot P(B)$

$P(A \text{ and } B) = P(B/A) \cdot P(A).$

wKT $P(A \text{ and } B) = P(A \text{ and } B)$

$\therefore P(A \text{ and }) = P(A) \times P(B)$

for dependent events

$$P(A \text{ and } B) = P(A/B) \cdot P(B)$$

OR

$$P(A \text{ and } B) = P(B/A) \cdot P(A).$$

Date

(3) Conditional Probability.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

(4). Multiplication Rule

$$\left.\begin{aligned} P(A \cap B) &= P(B/A) \cdot P(A) \\ P(A \cap B) &= P(A/B) \cdot P(B) \end{aligned}\right\} \rightarrow \text{~~Ind~~ dependent}$$

$P(A \cap B) = P(A) \cdot P(B) \rightarrow$ independent.

(5) Baye's theorem.

$$P(A/B) = \frac{P(B/A) \times P(A)}{P(B)}.$$

Multiplication theorem (dependent)

$$\boxed{\begin{aligned} P(A \cap B) &= P(A) \times P(B/A). \\ P(A \cap B) &= P(B) \times P(A/B). \end{aligned}} \rightarrow \text{Prove this.}$$

From Prob definition

$$P(A) = \frac{n(A)}{n(S)} \qquad P(B) = \frac{n(B)}{n(S)}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

Date

consider $P(A\cap B) = \frac{n(A\cap B)}{n(S)} \rightarrow (1)$

multiple & divide numerator & denominator by $n(A)$

$$P(A\cap B) = \frac{n(A)}{n(A)} \times \frac{n(A\cap B)}{n(S)}$$

$$= \frac{n(A)}{n(S)} \times \frac{n(A\cap B)}{n(A)}$$

$\therefore P(A\cap B) = P(A) \times \frac{n(A\cap B)}{n(A)}$ $\boxed{\text{WKT } \frac{n(A)}{n(S)} = P(A)}$

divid $n(S)$ by both numerator & denominator

$$\therefore P(A\cap B) = P(A) \times \frac{\frac{n(A\cap B)}{n(S)}}{\frac{n(A)}{n(S)}}$$

WKT $\frac{n(A\cap B)}{n(S)} = P(A\cap B)$

$\frac{n(A)}{n(S)} = P(A)$

$\Rightarrow \cancel{\frac{n(A\cap B)}{n(S)} \times \frac{n(S)}{n}}$

$\therefore P(A\cap B) = P(A) \times \frac{P(A\cap B)}{P(A)} \rightarrow (2)$

from ~~dor~~ conditional Prob

$P(B/A) = \frac{P(A\cap B)}{P(A)} \rightarrow (3)$

(3) in (2)

Date ______

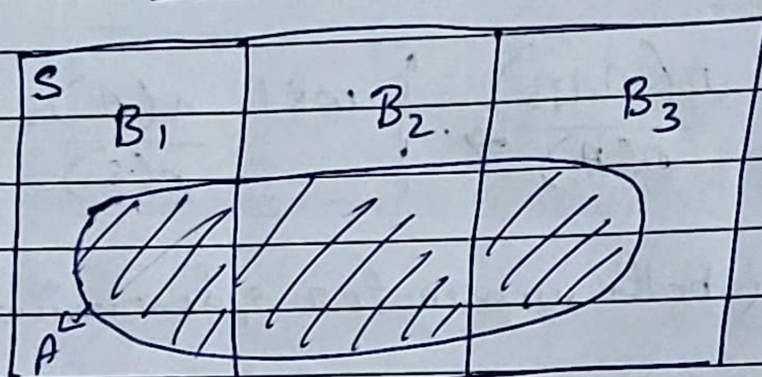
$$P(A\cap B) = P(B/A) \times P(A) \rightarrow (3)$$

(3) in (1)

$$P(A\cap B) = \cancel{P(A)} \times \frac{P(B/A) \times P(A)}{\cancel{P(A)}}$$

$$\boxed{P(A\cap B) = P(B/A) \times P(A)}$$ → proved.

- Total Probability theorem



$$A = (A\cap B_1) + (A\cap B_2) + (A\cap B_3)$$

It can be given as

$$P(A) = P(A\cap B_1) + P(A\cap B_2) + P(A\cap B_3) \rightarrow (1)$$

from conditional prob

w.k.T $P(A\cap B) = P(A/B) \cdot P(B) \rightarrow (2)$

(2) in (1)

$$\boxed{P(A) = P(A/B_1) * P(B_1) + P(A/B_2) * P(B_2) + P(A/B_3) * P}$$

Date

Bayes Theorem

$$\boxed{P(A/B) = \frac{P(B/A)\cdot P(A)}{P(B)}}$$

from conditional prob

$$P(A/B) = \frac{P(A\cap B)}{P(B)} \Rightarrow P(A\cap B) = P(A/B)\cdot P(B)$$

$$P(B/A) = \frac{P(A\cap B)}{P(A)} \Rightarrow P(A\cap B) = P(B/A)\cdot P(A)$$

$$P(A\cap B) = P(A\cap B)$$

$$\therefore P(A/B)\cdot P(B) = P(B/A)\cdot P(A)$$

$$\boxed{P(A/B) = \frac{P(B/A)\cdot P(A)}{P(B)}}$$

if $E_1, E_2, E_3 \ldots E_n$ are portion of S.S

$$\boxed{P(E_i/A) = \frac{P(A/E_i)\cdot P(E_i)}{\sum_{K=1}^{n} P(E_K)\cdot P(A/E_K)}} \rightarrow \text{Prove this}$$

from conditional prob. $\Rightarrow \boxed{P(A/B) = \frac{P(A\cap B)}{P(B)}}$

$$P(E_i/A) = \frac{P(E_i\cap A)}{P(A)} \rightarrow (1)$$

Multiplication theorem = $P(A\cap B) = P(A)\cdot P(B/A) \rightarrow (2)$

Total Prob Theorem =

$$P(A) = \sum_{K=1}^{n} P(E_K)\cdot P(A/E_K) \rightarrow (3)$$

Date ___ ___ ___

Substitute (2) & (3) in (1)

(1) $\Rightarrow$ $$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$$

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{k=1}^{n} P(E_k) \cdot P(A/E_k)}$$

$\therefore$ $$\boxed{P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{k=1}^{n} P(E_k) \cdot P(A/E_k)}}$$

Probability (both 3 and 4).

① Prob of Event A = $P(A) = \frac{\text{No of favorable outcomes}}{\text{Total No of outcomes}}$

$$\boxed{P(A) = \frac{n(A)}{n(S)}}$$

② complimentary event = $P(A') = 1 - P(A)$

③ Addition theorem/rule:

$P(A \cup B) = P(A) + P(B) \rightarrow$ (M.E)

$P(A \cup B = P(A) + P(B) - P(A \cap B) \rightarrow$ (Not M.E)

④ multiplication theorem

independent event = $P(A \cap B) = P(A) \times P(B)$

dependent event = $P(A \cap B) = P(A) \cdot P(B/A)$

$P(A \cap B) = P(B) \cdot P(A/B)$

⑤ impossible event = $P(\phi) = 0$

⑥ conditional Prob = $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

⑦ Total Prob theorem:

$P(A) = P(A/B_1) * P(B_1) + P(A/B_2) * P(B_2)$

$$\boxed{\Rightarrow P(A) = P(A/B_n) \times P(B_n)}$$

⑧ Bayes Theorem

$$P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$$