Date ______ Module-3..

Basic Probability Definition,

① prob of Event A :

$$P(A) = \frac{\text{No of favourable outcomes}}{\text{Total no of outcomes}}$$

Favourable outcome = outcomes where even A occurs.
Total outcomes = total Possible outcomes in the Sample Space.

② Prob of complementary Event.
If A is an event, The prob of that A doesnot occur is

$$P(A') = 1 - P(A)$$

③ Addition theorem of Prob.
For 2 events A & B.

- If A & B are mutually exclusive (cannot happen together).

$$P(A \cup B) = P(A) + P(B)$$

- If A & B are not mutually exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

④ multiplication theorem of prob.

- If A & B are independent events:

$$P(A \cap B) = P(A) \times P(B)$$

- If A & B are dependent events:

$$P(A \cap B) = P(A) \times P(B/A).$$

$P(B/A)$ = Prob of B given that A has already occured.

Date

Prob of an Event (P(E))

Sample Space & Event

Sample Space denoted as S, is the set of all possible outcomes of an exp.

Ex: when rolling a six-sided die, the S.S is

$S = \{1, 2, 3, 4, 5, 6\}$.

Event: is the Subset of the S.S.

Exe: rolling even number

$E = \{2, 4, 6\}$.

Ex: From a bag containing 10 Black & 20 white balls, a ball is drawn at random, what is the probability that it is black?

Black = 10 white = 20 Total = 10 + 20 = 30.

P(S) = 30 P(B) = 20

$\therefore P(B) = \frac{10}{30} = \frac{1}{3} = 0.33$

① Random Experiment: A R.E is an action or Process that leads to one or more outcomes, but you can't Predict which one will happen in advance.

Ex: Tossing a coin or throwing dice.

② Outcomes: the results of Random Experiment are called outcomes also called as events

③ Mutually exclusive event: 2 or more event are M.E if they cannot happen at the same time.

Ex: in one coin Toss: A = Heads, B = Tails.

A & B are mutually exclusive bcoz you can't get both in 1 toss.

Date

(4) independent & dependent events: 2 or more events are said to be independent when the outcome of 1 does not affect by the other.

(5) Equally likely events: Events are E.L. if they have the same chance of happening.
Ex: in a fair die, each no (1 to 6) has an equal chance
$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$.

*. 5m Questions.

Probability of occurrence of an event.

$$P(E) = \frac{n(E)}{n(S)}$$

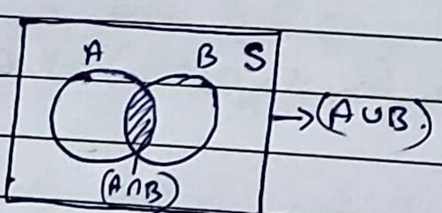
$n(E)$ = no of ways Event E can occur.
$n(S)$ = Total No of Possible outcomes / S.S.

(1) (5m) **Addition theorm.**

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \rightarrow$ not M.E
$P(A \cup B) = P(A) + P(B) \rightarrow$ M.E.

Stm: If A & B are any 2 events, then we have to show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof:



Given A & B are any 2 events.
from set theory→
$n(A \cup B) = n(A) + n(B) - n(A \cap B) \rightarrow (1)$

Date

Now divide both sides by total no of outcomes $n(S)$:

$$\therefore \frac{n(A\cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A\cap B)}{n(S)} \rightarrow (2)$$

From definition of Prob. we know that

$$P(E) = \frac{n(E)}{n(S)} \rightarrow (3)$$

apply (3) to (2).

$$P(A\cup B) = P(A) + P(B) - P(A\cap B).$$

Proved.

(5M)

(2) Theorem 2.

Prob of impossible event is zero: $(P(\phi) = 0)$.

Proof: Given S is a S.S. wKT impossible event contains no elements in it.

$(\phi = 0.)$

$\therefore S\cup\phi = S \rightarrow (1)$

apply Prob func to eq (1).

$$P(S\cup\phi) = P(S). \rightarrow (2)$$

from addition theorm wKT,

$$P(A\cup B) = P(A) + P(B) - P(A\cap B) \rightarrow (3)$$

apply (3) to (2).

$$\therefore P(S) + P(\phi) - P(S\cap\phi) = P(S)$$

$P(S\cap\phi) = 0 \rightarrow$ bcoz there is no common element in S & ϕ set.

$\therefore P(S) + P(\phi) = P(S).$

$\Rightarrow P(\phi) = P(S) - P(S)$

$\Rightarrow P(\phi) = 0$ Proved.

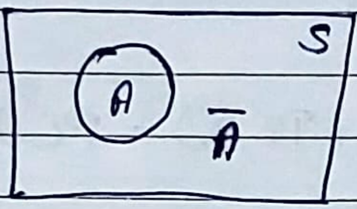
Date

③ Theorm 3

Prob of the complimentory Event $\bar{A}$ of A is given by

$$P(\bar{A}) = 1 - P(A).$$

Proof: let S be S.S.
& A be an event in S.



from venn dig, consider.

$$S = A \cup \bar{A} \rightarrow ①$$

apply prob func to eq ①

$$P(S) = P(A \cup \bar{A}) \rightarrow ②$$

from Addition theorm WKT

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \rightarrow ③$$

So apply ③ in ②

$$P(S) = P(A) + P(\bar{A}) - P(A \cap \bar{A})$$

$P(A \cap \bar{A}) = 0 \Rightarrow$ bcoz no common elements.

$$P(S) = P(A) + P(\bar{A})$$

$$P(S) = \frac{n(S)}{n(S)} = 1$$

$$\therefore \frac{n(S)}{n(S)} = P(A) + P(\bar{A})$$

$$1 = P(A) + P(\bar{A})$$

$$\therefore P(\bar{A}) = 1 - P(A) \quad \text{Proved.}$$

Date

Problems

1. what is the Prob of not drawing a face card (Jack, Q, K) from a standard deck of 52 cards?

⇒ $P(S) = 52$

$J = 4,\ K = 4,\ Q = 4 \quad \therefore 4+4+4 = 12$

$52 - 12 = 40$

$\therefore$ not picking Q, K, J $= \frac{40}{52} = 0.76\%$

Combination (^nC_r): Counts how many ways something can happen (just counting, no chance yet).

Ex: How many ways can I choose 2 red balls from 6?

⇒ $^6C_2 = \underline{15}$

Probability: Measures the chance of something happening.

Ex: whats the chance I pick 2 red balls & 1 blue ball?

⇒ $\frac{\text{favorable no.}}{\text{Total}}$

Q: what is the prob of getting 3 balls in a draw from a box containing 5 white & 4 black balls?

⇒ $P(S) = 5 + 4 = 9$

$P(E) = 3$ balls

$\therefore \frac{\cancel{3}}{\cancel{9}_3} = \frac{1}{3} = \underline{0.33}$