Module-III: 12 Hours

Addition theorem (two and three events), conditional probability of two events, multiplication theorem, independence of events-pair wise and mutual, Bayes theorem.

Topic to be covered in this slide are as follows:

1. Addition theorem (two and three events).
2. Multiplication theorem.
3. Conditional probability of two events.
4. independence of events-pair wise and mutual.
5. Bayes theorem.

Theorem 149

For two events A and B $P(A \cap B) = P(A).P(B|A); P(A) > 0$

$= P(B).P(A|B); P(B) > 0$

Proof: $P(A) = \frac{n(A)}{n(S)}$ $P(B) = \frac{n(B)}{n(S)}$ and $P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

For the conditional event $A|B$ the favorable outcomes must be one of the sample points of B.;

$P(A|B) = \frac{n(A \cap B)}{n(B)}$

Rewriting ,

$P(A \cap B) = \frac{n(B)}{n(S)} \frac{n(A \cap B)}{n(B)}$

$P(A \cap B) = \frac{n(B)}{n(S)} P(A|B)$

Similarly we can prove that

$P(A \cap B) = P(A).P(B|A); P(A) > 0$ proved

. Remarks: Note:

1. for $P(B) > 0; P(A|B) \leq P(A)$
2. The conditional probability $P(A|B)$ is not defined if $P(B) = 0$

3. $P(B|B) = 1$

Theorem 150 (Multiplication Law of Probability for Independent Events)

If A and B are independent then

1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$

Proof: as we all know that for independent event
$P(A \cap B) = P(A) \times P(B)$
then put the value in the first theorem we get the final result. proved.

Theorem 151

For any three events A, Band C
$P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$

Proof: we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P[(A \cap C) \cup (B \cap C)] = P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$$

$$\frac{P[(A\cap C)\cup(B\cap C)]}{P(C)} = \frac{P(A\cap C)+P(B\cap C) - P(A\cap B\cap C)}{P(C)}$$

$$\frac{P[(A\cup B)\cap C)]}{P(C)} = \frac{P(A\cap C)}{P(C)} + \frac{P(B\cap C)}{P(C)} - \frac{P(A\cap B\cap C)}{P(C)}$$

$P(A\cup B|C) = P(A|C) + P(B|C) - P(A\cap B|C)$ proved

Theorem 152

For any three events A,B,C

$P(A\cap \bar{B}|C) + P(A\cap B|C) = P(A|C)$

Proof: $L.H.S$

$$P(A\cap \bar{B}|C)+P(A\cap B|C) = \frac{P(A\cap\bar{B}\cap C)}{P(C)} + \frac{P(A\cap B\cap C)}{P(C)}$$

$$P(A\cap \bar{B}|C)+P(A\cap B|C) = \frac{P(A\cap\bar{B}\cap C)+P(A\cap B\cap C)}{P(C)}$$

$$P(A\cap \bar{B}|C)+P(A\cap B|C) = \frac{P(A\cap C)}{P(C)}$$

$P(A\cap \bar{B}|C)+P(A\cap B|C) = P(A|C)$ Proved

Theorem 153

$P(S|B) = 1$

Proof: $P(S|B) = \frac{P(S \cap B)}{P(B)}$

$P(S|B) = \frac{P(B)}{P(B)}$

Therefore, $P(S|B) = 1$ proved

Theorem 154

If A and B are independent events then A and $\bar{B}$ are also independent events.

Proof: As we that for independent event

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(A \cap \bar{B}) = P(A) - P(A) \cdot P(B)$ [*because* independent event]

$P(A \cap \bar{B}) = P(A)[1 - P(B)]$

$P(A \cap \bar{B}) = P(A) \times P(\bar{B})$

Hence, it is clear that A and $\bar{B}$ are also independent events.

Proved

Theorem 155

If A and B are independent events then $\bar{A}$ and $\bar{B}$ are also independent events.

Proof: As we that for independent event

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) \; P(\bar{A}$$

$$\cap \bar{B}) = 1 - P(A \cup B)$$

$$P(\bar{A} \cap \bar{B}) = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A) - P(B) + P(A \cap B) \;\; P(\bar{A}$$

$$\cap \bar{B}) = 1 - P(A) - P(B) + P(A)P(B)$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A) - P(B)[1 - P(A)]$$

$$P(\bar{A} \cap \bar{B}) = 1\,[1 - P(A)] - P(B)[1 - P(A)]$$

$$P(\bar{A} \cap \bar{B}) = [1 - P(A)][1 - P(B)]$$

$$P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$$

Hence, it is clear that $\bar{A}$ and $\bar{B}$ are also independent events.
proved

Theorem 156

If A ,B,C are mutually independent event then $A \cup B$ and C are also independent

Proof: we need to prove

$P[(A \cup B) \cap C] = P(A \cup B)P(C)$

$L.H.S$

$P[(A \cup B) \cap C] = P(A \cap C) \cup P(B \cap C)$ [distributive law]

$P[(A \cup B) \cap C] = P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$ [addition law]

Since, A,B, C are independent events therefore, we get

$P[(A \cup B) \cap C] = P(A)P(C) + P(B)P(C) - P(A)P(B)P(C)$

$P[(A \cup B) \cap C] = P(C)[P(A) + P(B) - P(A)P(B)]$

$P[(A \cup B) \cap C] = P(C)[P(A) + P(B) - P(A \cap B)]$ P

$[(A \cup B) \cap C] = P(C)P(A \cup B)$

Example 157

A die is rolled, find the probability that the number obtained is greater than 4.

Solution: when a die is rolled then the the sample space is $n(S) = 6$
E= the event that obtained the number greater than 4 i.e., 5 and 6.
$n(E) = 2$
Therefore, the probability = $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Example 158

Two coins are tossed, find the probability that one head only is obtained.

Solution: When two coins are tossed together then the sample space is = $n(S) = 4$
The outcome(s) favorable to no head obtained is $\{TT\}$
The outcome(s) favorable to one head obtained is $\{HT, TH\}$
E= the event that obtain only one head i.e., $\{HT, TH\}$
$n(E) = 2$
Therefore, the probability = $P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Example 159

In a class, 40% of the students study math and science. 60% of the students study math. What is the probability of a student studying science given he/she is already studying math.

Solution: M= the students those who study mathematics.
S= the students those who study Science.
According to the question; The given information are as follows
40% of the students study math and science i.e.,
$P(M \cap S) = \frac{40}{100} = 0.40$
Additionally, 60% of the students study math i.e.,
$P(S) = \frac{60}{100} = 0.60$
Now our aim is to find, $P(S|M) = ?$
’ from the definition, we know that,
$P(S|M) = \frac{P(M \cap S)}{P(S)} = \frac{0.40}{0.60} = \frac{2}{3}$ ans

Example 160

What is the probability of the occurrence of a number that is odd and less than 5 when a fair die is rolled.

Solution: when a die is rolled then the the sample space is $n(S) = 6$

E= event that occurs a number that is odd and less than 5. $n(E) = \{1, 3\}$

$n(E) = 2$

Therefore, the probability = $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$ ans

Example 161

In a purse there are 30 coins, twenty one-rupee and remaining 50-paise coins. Eleven coins are picked simultaneously at random and are placed in a box. If a coin is now picked from the box, find the probability of it being a rupee coin

Solution: Total coins 30

In that,

1 rupee coins 20

50 paise coins 10

Probability of total 1 rupee coins = ${}^{20}C_{11}$
Probability that 11 coins are picked = ${}^{30}C_{11}$
Required probability of a coin now picked from the box is 1 rupee = $\frac{{}^{20}C_{11}}{{}^{30}C_{11}} = \frac{2}{3}$

Example 162

A bag contains 5 red smileys, 6 yellow smileys and 3 green smileys. If two smileys are picked at random, what is the probability that both are red or both are green in colour

Solution: Total number of smileys = 5+6+3=14
n(S)= 2 smileys need to pick from the pack of 14 smileys i.e., = ${}^{14}C_2$
n(R)= event that pick two smileys out of five red smileys i.e., = ${}^{5}C_2$
N(G)= event that pick two smileys out of three green smileys i.e., = ${}^{3}C_2$
Our aim is to find $n(R \cup G)$
from the question it is clear that, R, Y and G are independent events;

$$P(R \cup G) = P(R) + P(G)$$

$$P(R \cup G) = \frac{{}^{5}C_2}{{}^{14}C_2} + \frac{{}^{3}C_2}{{}^{14}C_2}$$

$$P(R \cup G) = \frac{5\times 4}{14\times 13} + \frac{3\times 2}{14\times 13}$$

$$P(R \cup G) = \frac{10}{91} + \frac{3}{91}$$

$P(R \cup G) = \frac{13}{91}$
$P(R \cup G) = \frac{1}{7}$

Example 163

Three newspapers A, B and C are published in a city and a survey of readers indicates the following: 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read with A and C, 4% read both B and C and 65% read none of the papers. A person is chosen at random. Find the probability that he read all the three papers.

Solution: According to question, the given information are as follows;
20% of the students read A i.e.,
$P(A) = \frac{20}{100} = 0.20$
16% of the students read B i.e.,
$P(B) = \frac{16}{100} = 0.16$
14% of the students read A i.e.,
$P(C) = \frac{14}{100} = 0.14$
8% of the students read both A and B i.e.,
$P(A \cap B) = \frac{8}{100} = 0.08$

5% of the students read both A and C i.e.,

$P(A \cap C) = \frac{5}{100} = 0.05$

4% of the students read both A and B i.e.,

$P(B \cap C) = \frac{4}{100} = 0.04$

65% of the students read none of above i.e.,

$P(A \cup B \cup C)^c = P(\overline{A \cup B \cup C}) = \frac{65}{100} = 0.65$

$P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C}) = 1 - 0.65 = 0.35$

our aim is to find the $P(A \cap B \cap C)$=?

by using the addition formula for three events, we get

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

now put all the above value, we get

$0.35 = 0.2 + 0.16 + 0.14 - 0.08 - 0.05 - 0.04 + P(A \cap B \cap C)$

$P(A \cap B \cap C) = 0.02$ ans

Example 164

The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting at least one contract is 4/5, what is the probability that he will not get either

Solution: Given data

P(Plumbing) = $\frac{2}{3}$

P(Elecrtric) = $\frac{5}{9}$

$P(Plumbing \cup Electric) = \frac{4}{5}$

By addition theorem of probability;

$P(Plumbing \cup Electric) = P(Plumb) + P(Elec) - P(Plumb \cap Elec)$

$\frac{4}{5} = \frac{2}{3} + \frac{5}{9} - P(Plumb \cap Elec)$

$P(Plumbing \cap Electric) = \frac{2}{3} + \frac{5}{9} - \frac{4}{5}$

$P(Plumbing \cap Electric) = \frac{30+25-36}{45}$

$P(Plumbing \cap Electric) = \frac{19}{45}$

Example 165

Three persons A, B, and C fire at a target simultaneously. The probabilities that A, B and C can hit the target is 1/3, 1/4 and 1/5 respectively. Find the probability that atleast two of them hit the target.

Solution: given information P(A)=1/3; then $P(\bar{A}) = 2/3$

P(B)=1/4; then $P(\bar{B}) = 3/4$

P(C)=1/5; then $P(\bar{C})$ =4/5
hints: at first find the exactly two person hits the target.
P(A & B hits & C misses) + P(A & C hits & B misses) + P(B & C hits & A misses)
here A,B,C all are independent events.
$P(A) \times P(B) \times P(\bar{C}) + P(A) \times P(C) \times P(\bar{B}) + P(B) \times P(C) \times P(\bar{A})$
our aim is to find atleast two of them hits the target i.e.,
So P(atleast 2 persons hit the target)= P(Exactly two hit) + P(all three hit)
P(atleast 2 persons hit the target)= P(Exactly two hit) + P(A)P(B)P(C)

Example 166

A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is (1/7) and the probability of wife's selection is (1/5). What is the probability that only one of them is selected.

Solution: Given data; Let E_1 = event that the husband is selected and
E_2 = event that the wife is selected.
Then,
$P(E_1) = \frac{1}{7}$ and $P(\bar{E}_1) = 1 - \frac{1}{7} = \frac{6}{7}$

$P(E_2) = \frac{1}{5}$ and $P(\bar{E}_2) = 1 - \frac{1}{5} = \frac{4}{5}$

Required probability = P [(A and not B) or (B and not A)]

$= P\,[(E_1 \cap \bar{E}_2) or (E_2 \cap \bar{E}_1)]$

$= P\,(E_1 \cap \bar{E}_2) + P\,(E_2 \cap \bar{E}_1)$

$= P\,(E_1)\,P\,(\bar{E}_2) + P\,(E_2)\,P\,(\bar{E}_1)$

$= \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{1}{5} \times \frac{6}{7}\right)$

$= \frac{10}{35} = \frac{2}{7}$ ans

Example 167

A basket contains 10 apples and 20 oranges out of which 3 apples and 5 oranges are defective. If we choose two fruits at random, what is the probability that either both are oranges or both are non defective.

Total number of fruits in the basket is 20+10=30

n(S)= pick two fruits at random i.e., ${}^{30}C_2$

total number of defective fruits are= 3+5=8

therefore, total number of non-defective fruits are 30-8=22

let us consider a event which contains two oranges i.e. $n(O) = {}^{20}C_2$

another event that are non-defective fruits= $n(ND) = {}^{22}C_2$

And $n(O \cap ND)$ be the event of getting two non-defective oranges = P(oranges and non defective) = ${}^{15}C_2$

now, using addition theorem,

$$P(O \cup ND) = P(O) + P(ND) - P(O \cap ND)$$

$$P(O \cup ND) = \frac{{}^{20}C_2}{{}^{30}C_2} + \frac{{}^{22}C_2}{{}^{30}C_2} - \frac{{}^{15}C_2}{{}^{30}C_2}$$

$$P(O \cup ND) = \frac{20 \times 19}{30 \times 29} + \frac{22 \times 21}{30 \times 29} - \frac{15 \times 14}{30 \times 29}$$

$$P(O \cup ND) = \frac{632}{870}$$

$$P(O \cup ND) = \frac{316}{435}$$

Example 168

A basket contains 10 apples and 20 oranges out of which 3 apples and 5 oranges are defective. If we choose two fruits at random, what is the probability that either both are oranges or both are non defective.

Total number of fruits in the basket is 20+10=30
n(S)= pick two fruits at random i.e., ${}^{30}C_2$
total number of defective fruits are= 3+5=8
therefore, total number of non-defective fruits are 30-8=22
let us consider a event which contains two oranges i.e. $n(O) = {}^{20}C_2$
another event that are non-defective fruits= $n(ND) = {}^{22}C_2$
And $n(O \cap ND)$ be the event of getting two non-defective oranges = P(oranges and non defective) = ${}^{15}C_2$
now, using addition theorem,

$$P(O \cup ND) = P(O) + P(ND) - P(O \cap ND)$$

$$P(O \cup ND) = \frac{{}^{20}C_2}{{}^{30}C_2} + \frac{{}^{22}C_2}{{}^{30}C_2} - \frac{{}^{15}C_2}{{}^{30}C_2}$$

$$P(O \cup ND) = \frac{20\times 19}{30\times 29} + \frac{22\times 21}{30\times 29} - \frac{15\times 14}{30\times 29}$$

$P(O \cup ND) = \frac{632}{870}$

$P(O \cup ND) = \frac{316}{435}$

Example 169

A box contains 10 bulbs, of which just three are defective. If a random sample of five bulbs is drawn, find the probability that the sample contains exactly one defective bulb.

Solution: Total number of elementary events = ${}^{10}C_5$

10C5 Number of ways of selecting exactly one defective bulb out of 3 and 4 non-defective out of 7 is = ${}^{3}C_1 \times {}^{7}C_4$

So,required probability = $\frac{{}^{3}C_1 \times {}^{7}C_4}{{}^{10}C_5} = \frac{5}{12}$

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Example 170

Two friend Harshita and Srikanta appeared for an exam. The probability of selection of Harshita is 1/7 and that of Srikanta is 2/9. Find the probability that both of them are selected.

Solution $P(H) = \frac{1}{7}$

$P(S) = \frac{2}{9}$

Our objective is to find the probability that both of them are selected i.e., $P(H \cap S)$

Since both are independent events i.e.,

$P(H \cap S) = P(H) \times P(S)$

$P(H \cap S) = \frac{1}{7} \times \frac{2}{9}$

$P(H \cap S) = \frac{2}{63}$

Example 171

In a race, the odd favour of cars P,Q,R,S are 1:7, 1:8, 1:9 and 1:10 respectively. Find the probability that one of them wins the race.

Solution: $P(P) = \frac{1}{7}$; $P(Q) = \frac{1}{8}$; $P(R) = \frac{1}{9}$; $P(S) = \frac{1}{10}$

All the events are mutually exclusive hence,

Required probability $= P(P) + P(Q) + P(R) + P(S)$

$= \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10}$

Example 172

USNs numbered 1 to 20 are mixed up and then a USN is drawn at random. What is the probability that the USN drawn has a number which is a multiple of 3 or 5.

Solution: Here, $S = \{1, 2, 3, 4, ..., 19, 20\}$.

$n(S) = {}^{20}C_1 = 20$

Let E = event of getting a multiple of 3 or 5 i.e.,

$(3 \cup 5) = \{3, 6, 9, 12, 15, 18, 5, 10, 20\}$.

$n(3 \cup 5) = {}^{9}C_1 = 9$

$P(3 \cup 5) = \frac{n(3 \cup 5)}{n(S)} = \frac{9}{20}$

Example 173

A problem is given to three persons P, Q, R whose respective chances of solving it are 2/7, 4/7, 4/9 respectively. What is the probability that the problem is solved

Solution:Let A, B, C be the respective events of solving the problem and $\bar{A}, \bar{B}, \bar{C}$ be the respective events of not solving the problem.

It is clear that, A, B, C are independent event.

From the theorem we know that, if A, B, C are independent event then $\bar{A}, \bar{B}, \bar{C}$ also are independent event.

now, P(A)= $\frac{2}{7}$ then $P(\bar{A}) = 1 - \frac{2}{7} = \frac{5}{7}$

P(B)= $\frac{4}{7}$ then $P(\bar{B}) = 1 - \frac{4}{7} = \frac{3}{7}$

P(C)= $\frac{4}{9}$ then $P(\bar{C}) = 1 - \frac{4}{9} = \frac{5}{9}$

P(none solves the problem) = P(not A) and (not B) and (not C)

$= P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A})P(\bar{B})P(\bar{C}) = \frac{5\ 3\ 5}{7\ 7\ 9}$

$= \frac{75}{441}$

Hence, P(the problem will be solved) = 1 - P(none solves the problem)

$= 1 - \frac{75}{441} = \frac{366}{441} = \frac{122}{147}$

Example 174

Two cards are drawn from the pack of 52 cards. Find the probability that both are diamonds or both are kings.

Solution: Let A = event of getting both diamond cards.

B = event of getting both kings.

n(S)= two cards drawn at a time = ${}^{52}C_2$ = 1326
n(A) = ${}^{13}C_2$ =78
n(B) = ${}^{4}C_2$ =6
$n(A \cap B)$ =1
now from the addition theorem, we get,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cup B) = \frac{78}{1326} + \frac{6}{1326} - \frac{1}{1326}$
$P(A \cup B) = \frac{83}{1326}$ ans

Example 175 (homework)

From a pack of cards, three cards are drawn at random. Find the probability that each card is from different suit.

Example 176

in answering a question on a multiple choice test a student either knows the answer or guess. Let 3/4 be the probability that he know the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with the probability 1/4. what is the probability that a student knows the answer given that he answered it correctly.

Solution: Consider the following events:

E_1 = student know the answer E_2

= student guesses the answer. A

= student answers correctly. we

have,

$P(E_1) = \frac{3}{4}$ and $P(E_2) = \frac{1}{4}$ also given that $P(A|E_2) = \frac{1}{4}$ and $P(A|E_1) = 1$

As per the bayes theorem;

The required probability = $P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$$= P(E_1|A) = \frac{\left(\frac{3}{4}\times 1\right)}{\left(\frac{3}{4}\times 1\right)+\left(\frac{1}{4}\times\frac{1}{4}\right)} = \frac{\frac{3}{4}}{\frac{13}{16}} = \frac{12}{13}$$

Example 177

A laboratory blood test is 99% effective in detecting a certain disease when it is, in fact, present, however, the test also yields a false positive result for 0.5% of the healthy person tested (i.e., if a healthy person is tested , then , with probability 0.005, the test will imply he has the disease). if 0.1% of the population actually has the disease, what is the probability thar a person has the disease given that his test result is positive

Solution: Consider the following events:

E_1 = person selected has the disease

E_2 = person selected does not have the disease

A = test result is positive.

we have,

$P(E_1) = \frac{1}{1000}$ and $P(E_2) = \frac{999}{1000}$ also given that $P(A|E_2) = \frac{5}{1000}$ and $P(A|E_1) = \frac{99}{100}$

As per the bayes theorem;

The required probability $= P(E_1|A) = \dfrac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$$= P(E_1|A) = \frac{\left(\frac{1}{1000}\times\frac{99}{100}\right)}{\left(\frac{1}{1000}\times\frac{99}{100}\right)+\left(\frac{999}{1000}\times\frac{5}{1000}\right)} = \frac{\frac{99}{100000}}{\frac{5985}{1000000}} = \frac{990}{5985}$$

Example 178

there are three coins. One is a two headed coin, another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. one of the three coin is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin.

Solution: Consider the following events:

E_1 = selecting two headed coin

E_2 = selecting biased coin. E_3

= selecting unbiased coin. A

= getting head on the coin.

we have,

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ and $P(A|E_3) = \frac{1}{2}$ also given that $P(A|E_2) = \frac{75}{100}$ and $P(A|E_1) = 1$

As per the bayes theorem;

The required probability $= P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)++P(E_3)P(A|E_3)}$

$$= P(E_1|A) = \frac{\frac{1}{3}}{\frac{9}{12}} = \frac{4}{9}$$

Example 179

Two groups are competing for the position on the board of directors of a corporation. the probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. further, if the first group wins , the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins, find the probability that the new product was introduced by the second group.

Solution: Consider the following events:
E_1 = that the first group win
E_2 = the event the second group win
A = event to introduce the new product.
we have,
$P(E_1) = 0.6$ and $P(E_2) = 0.4$ also given that $P(A|E_2) = 0.3$ and $P(A|E_1) = 0.7$
As per the bayes theorem;
The required probability = $P(E_1|A) = \dfrac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$= P(E_2|A) = \dfrac{(0.4\times0.3)}{(0.4\times0.3)+(0.6\times0.7)} = \dfrac{0.12}{0.54} = \dfrac{2}{9}$

Example 180

A bag contains 4 Red and 4 Black balls another bag contains 2 Red and 6 Black balls. one of the two bags is selected at random and a ball is drawn from the bag which is found to be Red. find the probability that the ball is drawn from the first bag.

Solution: Consider the following events:

E_1 = event of selecting the first bag

E_2 = event of selecting the second bag

A = event of selecting the red ball.

we have,

$P(E_1) = P(E_2) = \frac{1}{2}$

also given that $P(A|E_2) = \frac{2}{8}$ and $P(A|E_1) = \frac{4}{8}$

As per the Bayes theorem;

The required probability $= P(E_1|A) = \dfrac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$$= P(E_1|A) = \frac{\left(\frac{1}{2}\times\frac{1}{2}\right)}{\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right)} = \frac{\frac{1}{4}}{\frac{3}{8}} = \frac{2}{3}$$

Example 181

A company has two plants to manufacture TVs. The first plant manufacture 70% of the TV and rest are manufacture by the other plant. 80% of the TVs manufacture by the first plant are rated of standard quality, while that of the second plant only 70% are of standard quality. If a TV chosen at random is found to be of standard quality, find the probability that it was produced by the first plant.

Solution: Consider the following events:

E_1 = selecting TV manufacturing in the first plant.

E_2 = selecting TV manufacturing in the second plant.

A = selecting TV in standard quality.

we have,

$P(E_1) = 0.7, P(E_2) = \frac{30}{100}$ also given that $P(A|E_2) = \frac{70}{100}$ and $P(A|E_1) = \frac{80}{100}$

As per the Bayes theorem;

The required probability = $P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$$= P(E_1|A) = \frac{\left(\frac{70}{100}\times\frac{80}{100}\right)}{\left(\frac{70}{100}\times\frac{80}{100}\right)+\left(\frac{30}{100}\times\frac{70}{100}\right)} = \frac{\frac{56}{100}}{\frac{77}{100}} = \frac{8}{11}$$

Example 182

A factory has three machines A,B and C, which produce 100, 200 and 300 items of a particular type daily. The machines produced 2% , 3% and 5% defective items respectively. One day when the production over, and item was picked up randomly and it was found to be defective. find the probability that it was produced by machine A.

Solution: Consider the following events:

E_1 = selecting the production of machine A

E_2 = selecting the production of machine A

E_3 = selecting the production of machine A

A = selecting the product of defective items.

we have,

$P(E_1) = \frac{100}{600}$, $P(E_2) = \frac{200}{600}$, $P(E_3) = \frac{300}{600}$ and $P(A|E_3) = 0.05$ also given that $P(A|E_2) = 0.03$ and $P(A|E_1) = 0.02$

As per the bayes theorem;

The required probability = $P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$

$= P(E_1|A) = \frac{\left(\frac{1}{6}\times 0.02\right)}{\left(\frac{1}{6}\times 0.02\right)+\left(\frac{1}{3}\times 0.03\right)+\left(\frac{1}{2}\times 0.05\right)} = \frac{2}{23}$

Example 183

The chance that a doctor will diagnose a disease correctly is 60%. The chance that a patient will die after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. If a patient dies, what is the chance that his disease was not correctly diagnosed?

Solution: Consider the following events:

E_1 = disease X is diagnosed correctly by Dr. A

E_2 = disease X is diagnosed not correctly by Dr. A

E = A patient of Dr. A who had disease X dies. we have,

$P(E_1) = 0.6, P(E_2) = 1 - P(E_1) = \frac{40}{100}$

also given that $P(E|E_2) = \frac{70}{100}$ and $P(E|E_1) = \frac{40}{100}$

As per the Bayes theorem;

The required probability $= P(E_2|E) = \frac{P(E_2)P(E|E_2)}{P(E_1)P(E|E_1)+P(E_2)P(E|E_2)}$

$= P(E_2|E) = \frac{\left(\frac{70}{100}\times\frac{40}{100}\right)}{\left(\frac{60}{100}\times\frac{40}{100}\right)+\left(\frac{40}{100}\times\frac{70}{100}\right)} = \frac{0.28}{0.52} = \frac{7}{13}$

Example 184

A letter is known to have either come from TATANAGAR or CALCUTTA. on the envelop just two consecutive letters “TA” are visible. what is the probability that the letter came from CALCUTTA.

Solution: Consider the following events:

E_1 = event that the letter came from TATANAGAR

E_2 = event that the letter came from CALCUTTA

E = that two consecutive visible letters on the envelop are “TA”.

we have,

$P(E_1) = P(E_2) = \frac{1}{2}$,

also given that $P(E|E_2) = \frac{1}{7}$ and $P(E|E_1) = \frac{2}{8}$

As per the Bayes theorem;

The required probability = $P(E|E_2) = \dfrac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$$= P(E_2|R) = \frac{\left(\frac{1}{2}\times\frac{1}{7}\right)}{\left(\frac{1}{2}\times\frac{1}{7}\right)+\left(\frac{1}{2}\times\frac{1}{8}\right)} = \frac{4}{11}$$

THANK YOU