(BCA) ELEMENTS OF PROBABILITY & STATISTICS (SEM IV)

Assistant Professor
Department of MCA
Jain University, Jayanagar
Karnataka, India

Module-II: 12 Hours

Random experiment, Sample space, event, classical definition of probability,statistical regularity, field, sigma field, axiomatic definition of probability and simple properties.

Topic to be covered in this slide are as follows:

1. Random experiment
2. Sample space and event
3. classical definition of probability

Definition (Sample Space)

is a list of all possible outcomes of the experiment. The outcomes must be mutually exclusive and exhaustive. Mutually exclusive means they are distinct and non-overlapping. Exhaustive means complete.

for example : Sample space of all possible outcomes when two dice are tossed.

(1,1), (1,2), (1,3), (1,4), (1,5) (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Definition (Event)

is a subset of the sample space. An event can be classified as a simple event or compound event.

Definition 115 (Probability of an event)

The theoretical probability of an event is defined as the number of ways the event can occur divided by the number of events of the sample space. Using mathematical notation, we have

$P(E) = \frac{n(E)}{n(S)}$

$n(E)$ is the number of ways the event can occur and $n(S)$ represents the total number of events in the sample space.

Example 116

Two dice are tossed. The probability that the total score is a prime number is:

Solution: Sample space of all possible outcomes when two dice are tossed.

(1,1), (1,2), (1,3), (1,4), (1,5) (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Therefore total number of sample = $n(S) = 36$
event occur when the sum is prime i.e., 2
Since out of the above 36 possibilities, only one favourable to this event i.e., (1,1)
Therefore $P(E_1) = \frac{1}{36}$
Similarly, event occur when the sum is prime i.e., 3
Since out of the above 36 possibilities, only two favourable to this event i.e., (1,2),(2,1)
Therefore $P(E_2) = \frac{2}{36}$
Similarly, event occur when the sum is prime i.e., 5
Since out of the above 36 possibilities, only four favourable to this event i.e., (1,4),(2,3),(3,
Therefore $P(E_3) = \frac{4}{36}$
Similarly, event occur when the sum is prime i.e., 7
Since out of the above 36 possibilities, only six favourable to this event i.e., (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)
Therefore $P(E_4) = \frac{6}{36}$
Similarly, event occur when the sum is prime i.e., 11
Since out of the above 36 possibilities, only two favourable to this event i.e., (5,6),(6,5)
Therefore $P(E_5) = \frac{2}{36}$

Hence the final probability to getting sum is prime is $P(E_1)+P(E_2)+P(E_3)+P(E_4)+P(E_5) = \frac{1}{36}+\frac{2}{36}+\frac{4}{36}+\frac{6}{36}+\frac{2}{36}=\frac{15}{36}$

Example 117

The blood groups of 200 people is distributed as follows: 50 have type A blood, 65 have B blood type, 70 have O blood type and 15 have type AB blood. If a person from this group is selected at random, what is the probability that this person has O blood type?

Solution: Number of people having O blood type = 70 = $n(E)$
Total number of people = $n(S)$ =200
As We know that,
$P(E) = \frac{n(E)}{n(S)} = \frac{70}{200} = \frac{7}{20}$
So probability of people having O blood is $\frac{7}{20}$

Example 118

A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that this marble is white?

Solution: Total number of marbles in a bag is $3 + 7 + 10 = 20 = n(S)$
Total number of white marbles in a bag is $= n(E) = 10$
As We know that,
$P(E) = \frac{n(E)}{n(S)} = \frac{10}{20} = \frac{1}{2}$
So probability of that this marble is white is $\frac{1}{2}$

Example 119

In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green

Solution: Total number of marbles in a bag is $8 + 7 + 6 = 21 = n(S)$
Let E = event that the ball drawn is neither red nor green = event that the ball drawn is red.
$n(E) = 7$
As We know that,
$P(E) = \frac{n(E)}{n(S)} = \frac{7}{21} = \frac{1}{3}$
So probability of that the ball drawn is neither blue nor green is $\frac{1}{3}$

Example 120 (Homework)

In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither blue nor green

Example 121 (Homework)

In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor blue

Example 122

From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings

Let S be the sample space. pack of 52 cards two cards are drawn

Hence, $n(S) = {}^{52}C_2 = \frac{52\times51}{1\times2} = 1326$

Let E = event of getting 2 kings out of 4.

$n(E) = {}^{4}C_2 = \frac{4\times3}{1\times2} = 6$

$P(E) = \frac{n(E)}{n(S)} = \frac{6}{1326} = \frac{1}{221}$

So probability of getting two kings are: $\frac{1}{221}$

Example 123

Two dice are thrown together. What is the probability that the number obtained on one of the dice is multiple of number obtained on the other dice?

Solution: Sample space of all possible outcomes when two dice are tossed.
(1,1), (1,2), (1,3), (1,4), (1,5) (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)
Therefore total number of sample = $n(S) = 36$
event occur when the the number is 1 in one dice i.e.,
Since out of the above 36 possibilities, only six favourable to this event i.e., (1,1),(1,2),(1,3
(1,5), (1,6)
Therefore $P(E_1) = \frac{6}{36}$

Formulation with NumericalVIII

Similarly, event occur when the the number is 2 in one dice i.e.,
Since out of the above 36 possibilities, only four favourable to this event i.e., (2,1), (2,2), (2,4), (2,6)
Therefore $P(E_2) = \frac{4}{36}$
Similarly, event occur when the the number is 3 in one dice i.e.,
Since out of the above 36 possibilities, only three favourable to this event i.e., (3,1), (3,3), (3,6)
Therefore $P(E_3) = \frac{3}{36}$
Similarly, event occur when the the number is 4 in one dice i.e.,
Since out of the above 36 possibilities, only three favourable to this event i.e., (4,1), (4,2), (4,4)
Therefore $P(E_4) = \frac{3}{36}$
Similarly, event occur when the the number is 5 in one dice i.e.,
Since out of the above 36 possibilities, only two favourable to this event i.e., (5,1),(5,5)
Therefore $P(E_5) = \frac{2}{36}$
Similarly, event occur when the the number is 6 in one dice i.e.,
Since out of the above 36 possibilities, only four favourable to this event i.e., (6,1), (6,2), (6,3), (6,6)

Therefore $P(E_5) = \frac{4}{36}$

Hence the final probability to getting sum is prime is $P(E_1)+P(E_2)+P(E_3)+P(E_4)+P(E_5) = \frac{6}{36}+\frac{4}{36}+\frac{3}{36}+\frac{3}{36}+\frac{2}{36}+\frac{4}{36} = \frac{22}{36}$

Example 124 (Homework)

When two dice are thrown simultaneously, what is the probability that the sum of the two numbers that turn up is less than 12?

Example 125

Find the probability that a leap year has 53 Mondays.

In a leap year (which consists of 366 days) there are 52 complete weeks ($52 \times 7 = 364$) and 2 days over. The following are the Possible combinations for these two 'over' days:

- Sunday and Monday
- Monday and Tuesday
- Tuesday and Wednesday
- Wednesday and Thursday
- Thursday and Friday
- Friday and Saturday
- Saturday and Sunday

In order that it leap year selected at random should contain 53 Sundays, one of the two ’over" days must be Sunday.
Since out of the above 7 possibilities, 2 are favourable to this event,
Required probability = $\frac{2}{7}$

Example 126 (Homework)

Find the probability that a non-leap year has 53 Mondays.

Example 127

Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are queen?

Solution: Let S be the sample space. pack of 52 cards two cards are drawn
Hence, $n(S) = {}^{52}C_2 = \frac{52 \times 51}{1 \times 2} = 1326$
Let E = =event of getting 2 card is black or 2 card is queens
Total number of black cards = 26
Total number of queen cards = 4
Total number of cards which are both black and queen = 2

Hence, n(E)= number of ways of drawing 2 black from 26 or 2 queen from 4

$n(E) = n(B) + n(Q) - N(B \wedge Q)$

$n(E) = {}^{26}C_2 + {}^{4}C_2 - {}^{2}C_2 = 325 + 6 - 1 = 330$

$P(E) = \frac{n(E)}{n(S)} = \frac{330}{1326} = \frac{55}{221}$

So probability of getting two kings are: $\frac{55}{221}$

Example 128

If two letters are taken at random from the word HOME, what is the probability that none of the letters would be vowels?

Solution: Total number of letters in the word HOME is 4 and 2 letters are choosing.
Hence, total number of outcomes is given by

Hence, $n(S) = {}^{4}C_2 = \frac{4\times3}{1\times2} = 6$

O and E are vowels and H and M are consonants.
Therefore, there are two letters which are not vowel.
So, favorable outcomes is given by

$n(E) = {}^{2}C_2 = \frac{2\times1}{1\times2} = 1$

$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$

So the probability that none of the letters would be vowels: $\frac{1}{6}$

Example 129

A word consists of 9 letters; 5 consonants and 4 vowels. Three letters are chosen at random. What is the probability that more than one vowel will be selected ?

Solution: 3 letters can be choosen out of 9 letters
Hence, total number of outcomes is given by
Hence, $n(S) = {}^{9}C_3 = \frac{9\times8\times7}{1\times2\times3} = 84$
E= more than one vowels need to choose.
E_1 = the event when two vowels and 1 consonants choose
$n(E_1) = {}^{4}C_2 \times {}^{5}C_1 = 6 \times 5 = 30$
E_2 = the event when three vowels choose.
$n(E_2) = {}^{4}C_3 = 4$
the probability that more than one vowel will be selected
$P(E) = \frac{n(E_1)+n(E_2)}{n(S)} = \frac{30+4}{84}$
So the probability that none of the letters would be vowels: $\frac{17}{42}$

Example 130

A letter is taken out at random from "ASSISTANT" and another is taken out from "STATISTICS". The probability that they are the same letter is

Solution : Solution: 1 letters can be choosen out from "Assistant" and one from "Statistics"
Hence, total number of outcomes is given by
Hence, $n(S) = {}^{9}C_1 \times {}^{10}C_1 = 9 \times 10 = 90$
ASSISTANT → AA SSS I TT N = total letters 9
STATISTICS → SSS TTT A II C = total letters 10
Here N and C are not common and same letters can be A, I, S, T. Therefore
Probability of choosing A
Total no. of "A" in Assistant is 2 and in "Statistics" is 1 choose one at a time
$n(A) = {}^{2}C_1 \times {}^{1}C_1 = 2$
$P(A) = \frac{n(A)}{n(S)} = \frac{2}{90}$
similarly, Total no. of "I" in Assistant is 1 and in "Statistics is 2 choose one at a time
$n(I) = {}^{1}C_1 \times {}^{2}C_1 = 2$
$P(I) = \frac{n(I)}{n(S)} = \frac{2}{90}$

similarly, Total no. of “S" in Assistant is 3 and in “Statistics is 3 choose one at a time

$n(S) = {}^3C_1 \times {}^3C_1 = 9$

$P(S) = \frac{n(S)}{n(S)} = \frac{9}{90}$

similarly, Total no. of “T" in Assistant is 2 and in “Statistics is 3 choose one at a time

$n(T) = {}^2C_1 \times {}^3C_1 = 6$

$P(T) = \frac{n(A)}{n(S)} = \frac{6}{90}$

Hence, The probability that they are the same letter is

$P(A) + P(I) + P(S) + P(T) = \frac{2}{90} + \frac{2}{90} + \frac{9}{90} + \frac{6}{90} = \frac{19}{90}$

Example 131

In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that 1 girl and 2 boys are selected, is:

Solution: Let S be the sample space
Given Total numbers of Boys=15
Total number of girls =10
total number of student in a class is =10+15=25
Then, n(S) = Number ways of selecting 3 students out of 25
Hence, $n(S) = {}^{25}C_3 = \frac{25\times 24\times 23}{1\times 2\times 3} = 2300$
E be the event of selecting 1 girl and 2 boys.
$n(E) = {}^{10}C_1 \times {}^{15}C_2 = 10 \times \frac{15\times 14}{1\times 2} = 1050$
$P(E) = \frac{n(E)}{n(S)} = \frac{1050}{2300}$
So the probability that 1 girl and 2 boys are selected $\frac{21}{46}$

Example 132 (Homework)

A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, not replaced and then another pen is drawn. What is the probability of drawing 2 blue pens and 1 black pen

Solution: Total blue pen=4
Total red pen= 2
total black pen= 3
total pen in a bag $= 2 + 3 + 4 = 9$
probability of drawing 2 blue pen at a random is $= \frac{{}^4C_2}{{}^9C_2} = \frac{1}{6}$
After this the pens are not replaced which reduce the number of pen in the bag to 7
So, the probability of drawing one black pen drawn at random out of 7 is $\frac{{}^3C_1}{{}^7C_1} = \frac{3}{7}$
Therefore the probability of drawing two back and one black pen is $= \frac{1}{6} \times \frac{3}{7} = \frac{1}{14}$

Example 133 (homework)

There is a bag full of coloured balls, red, blue, green and orange. Balls are picked out and replaced. John did this 1000 times and obtained the following results:

- Number of blue balls picked out: 300
- Number of red balls: 200
- Number of green balls: 450
- Number of orange balls: 50

1. What is the probability of picking a green ball
2. If there are 100 balls in the bag, how many of them are likely to be green

Solution: a) ans: total number of balls in a bag=1000

Then, n(S) = 1000

E be the event of selecting 1 balls out of 450 green balls is

$n(E) = {}^{450}C_1 = 450$

$P(E) = \frac{n(E)}{n(S)} = \frac{450}{1000}$

So the probability of picking a green ball is $\frac{9}{20}$

b) ans: a bag contains 1000 balls in which 450 are green balls
then, if a bag contain 1 balls in which $\frac{450}{1000}$ green balls
so, if bag contains 100 balls then , $\frac{450\times100}{1000}$ are green balls.
Therefore, If there are 100 balls in the bag, 45 of them are likely to be green balls.

Example 134

Consider another example where a pack contains 4 blue, 2 red and 3 black pens. If a pen is drawn at random from the pack, replaced and the process repeated 2 more times. What is the probability of drawing 2 blue pens and 1 black pen?

Probability of drawing 1 blue pen = $\frac{4}{9}$
Probability of drawing another blue pen = $\frac{4}{9}$
Probability of drawing 1 black pen = $\frac{3}{9}$
Probability of drawing 2 blue pens and 1 black pen = $\frac{4}{9} \times \frac{4}{9} \times \frac{3}{9} = \frac{48}{729} = \frac{16}{243}$

Example 135

In a box, there are 9 blue, 6 white and some black stones. A stone is randomly selected and the probability that the stone is black is $\frac{1}{4}$Find the total number of stones in the box?

Solution: We know that, Total probability = 1
Given probability of black stones = $\frac{1}{4}$
Probability of blue and white stones = $1 - \frac{1}{4} = \frac{3}{4}$
But, given blue + white stones = 9 + 6 = 15
Hence, 0.75 holds total probability out of 15 stones
1 holds total probability out of 15 stones is : $\frac{15}{0.75} = 20$
Therefore, total number of stone is 20.

Theorem 136

Probability of the impossible event is zero

Proof.

Impossible event contains no sample point and
Hence the certain event S and the impossible event φ are mutually exclusive.
Hence, $S \cup \varphi = S$
$P(S \cup \varphi) = P(S)$
$P(S) + P(\varphi) = P(S)$
$P(\varphi) = 0$
$\square$

Theorem 137

Probability of the complementary event $\bar{A}$ of A is given by $P(\bar{A}) + P(A) = 1$

Proof.

$\bar{A}$ of A are the disjoint events

Moreover, $A \cup \bar{A} = S$

$P(A \cup \bar{A}) = P(S)$

$P(A) + P(\bar{A}) = P(S)$

$P(A) + P(\bar{A}) = 1$

$\square$

Solution. (a) The word '*REGULATIONS*' consists of 11 letters. The two letters R and E can occupy ${}^{11}P_2$, *i.e.*, $11 \times 10 = 110$ positions.

The number of ways in which there will be exactly 4 letters between R and E are enumerated below:

(i) R is in the 1st place and E is in the 6th place.

(ii) R is in the 2nd place and E is in the 7th place.

... ...

... ...

... ...

(vi) R is in the 6th place and E is in the 11th place.

Since R and E can interchange their positions, the required number of favourable cases is $2 \times 6 = 12$

$\therefore$ The required probability $= \frac{12}{110} = \frac{6}{55}$

Example 138

If the letters of the word 'REGULATIONS' be arranged at random, what is'the chance that there will be exactly 4 letters between R and E.

Example 139

A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn are white and blue.

Example 140

1. Two cards are drawn at random from a well-shuffled pack of 52 cards, Show that the chance of drawing two aces is $\frac{1}{21}$
2. From a pack of 52 cards, three are drawn at random, Find the chance that they are a king, a queen and a ace.

Example 141

1. Four cards are drawn from a pack of cards, Find the probability that
    1. all are diamond.
    2. there are two spades and two hearts.
    3. there is one card of each suit.

Some more NumericalII

Example 142

A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, two officers of the sales department and 1 chartered accountant, Find the probability of forming the committee in the following manner:

1. There must be one from each category.

Example 143

From a set of raffle tickets numbered 1 to 100,three are drawn at random. What is the probability that all the tickets are odd-numbered.

Example 144

Seven cards are drawn at random from a pack of 52 cards. What is the probability that 4 will be red and 3 black

Example 145

A card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace.

Example 146

A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box ,at random. Find the probability that among the balls drawn there is at least one ball of each color.

Example 147

A problem in Statistics is given to the three students A,B and C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{4}$ respectively. What is ,the probability that the problem will be solved if all of them try independently.

Example 148

Let A and B be two events such that $P(A) = \frac{3}{4}$, and $P(B) = \frac{5}{8}$ Show that,

- $P(A \cup B) \leq \frac{3}{4}$
- $\frac{3}{8} \leq P(A \wedge B) \leq \frac{5}{8}$