Helwan University
Faculty of Computers and Information

Assignment
convex and affine combinations

Department of Computer Science
Optimization

1. Prove that $|x_i| < \|x\|_p \ \forall p < \infty$. That is, each coordinate alone is less than L_p-norm of the vector for any p; (of course makes sense).

2. Prove that $\|x\|_p$ is monotonic in $p \ \forall x$. Said differently, $\|x\|_p \leq \|x\|_q \ \forall p > q$.

3. Consider $x = (3,4)'$. Plot $\|x\|_p$ vs $p = 1, 2, \cdots, 5$.

4. Prove that $\delta(x,y) = I_{x=y}$, where I is the indicator function, is a metric.

5. Consider the vector $v1 = (3,4)'$; then

   a) find the other unit vector $v2$ perpendicular to $v1$. Then construct the symmetric and positive definite matrix Σ with these two vectors as its eigenvectors and with $\lambda = 2, 1$ respectively.

   b) Draw a unit Euclidean ball in R^2, along with a unit ellipsoid generated by deforming this unit ball with the matrix Σ. **Hint** drawing is by free hand; however, using calculations, indicate the special values on the drawing.