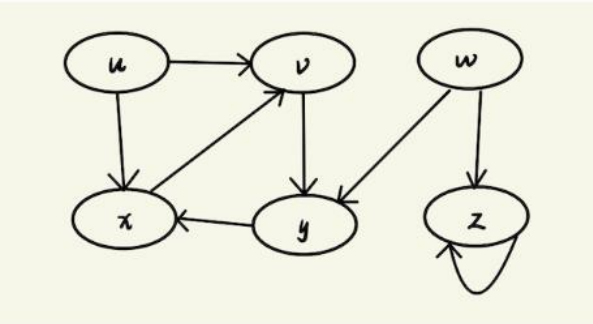
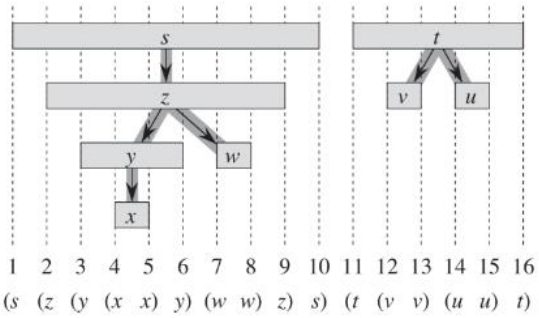
EL9343 Homework 8

Due: Nov. 4th 11:59 p.m.

1. What is the running time of DFS if the graph ($G = (V, E)$, where there are $|V|$ nodes and $|E|$ edges) is given as an adjacency list and adjacency matrix? Justify your running time.
2. Write a method that takes any two nodes u and v in a tree T whose root node is s, and quickly determines if the node u in the tree is a *descendant* or *ancestor* of node v.
3. Draw the parenthesis structure of the DFS of bottom left figure (start from u, assume that DFS considers vertices in alphabetical order) and see the example parenthesis structure as is shown in the bottom right.





4. **Bipartiteness** Given a undirected graph $G = (V, E)$, it is *bipartite* if there exist U and W such that $U \cup W = V$, $U \cap W = \emptyset$, and every edge has one endpoint each in U and W.
   (a) Prove: G is bipartite only if G has no odd cycle. (Hint: proof by contradiction)
   (b) In fact, G is bipartite if and only if G has no odd cycle. Suppose this is given, consider the Algorithm 1 and briefly describe why it is correct.
   (c) Analyze the time complexity (worst-case, big-O) of the algorithm, in terms of $|V|$ and $|E|$. (Hint: can we check that there is no edge inside any layer in $O(|E|)$? Why?)

Algorithm 1 Testing bipartiteness of graphs

Do BFS starting at any node u
Let $L_0, L_1, L_2, \ldots, L_k$ be layers in the resulting breadth-first tree ($L_0 = \{u\}$, $L_i, i = 1, 2, \ldots, k$ contains the vertices at distance i from u)
if There is no edge inside any layer L_i **then**
  Declare G to be bipartite, and $U = L_0 \cup L_2 \cup L_4 \cup \ldots, W = L_1 \cup L_3 \cup L_5 \cup \ldots$ are the bipartition
else
  Declare G to be non-bipartite
end if