MIE1622 Assignment 3 Tutorial

Credit Risk Modeling and Simulation

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March 16, 2018

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In-sample vs Out-of-sample

- 2 Monte Carlo in-sample scenarios
- 1 out-of-sample scenario (Monte Carlo) as the true distribution

Credit worthiness Index

sensitivity of CP *j* to the credit driver

$\sigma_j = \sqrt{1 - \beta_j^2}$

$$w_j = \beta_j y_{j(k)} + \sigma_j z_j$$

systemic risk
credit driver factor
correlated credit drivers

$\mathcal{N}(0,1)$

idiosyncratic
component
independent across
CPs *j*

sqrt_rho = (chol(rho))’;
% Cholesky decomp of rho (for generating correlated Normal random numbers)

Loss for each bond

- [Ltemp, CS] = max(prob, [], 2);
- exposure(:, 1) = (1-recov_rate) .* exposure(:, 1);
- CS_Bdry = norminv(cumsum(prob(:,1:C-1), 2));

Use exposure and CS_Bdry to determine losses

Var and CVar Estimation

From MC simulation:

- Given a random sample of size N, let $\lambda_{(k)}$ be the k^{th} order statistic, i.e., $\lambda_{(1)} \leq \lambda_{(2)} \leq \ldots \leq \lambda_{(N)}$
  - An estimate of VaR_α is $\mathrm{VaR}_{\alpha,N} = \ell_{(\lceil N\alpha \rceil)}$
  - An estimate of CVaR_α is

$$\mathrm{CVaR}_{\alpha,N} = \frac{1}{N(1-\alpha)} \left[(\lceil N\alpha \rceil - N\alpha)\, \ell_{(\lceil N\alpha \rceil)} + \sum_{k=\lceil N\alpha \rceil+1}^{N} \ell_{(k)} \right]$$

Var and CVar Estimation

From Normal Distribution:

VaR for Normally distributed losses:

$$\mathrm{VaR}_\alpha^{\mathcal{N}} = \mu_{\mathcal{L}} + \Phi^{-1}(\alpha) \cdot \sigma_{\mathcal{L}}$$

CVaR for Normally distributed losses:

$$\mathrm{CVaR}_\alpha^{\mathcal{N}} = \mu_{\mathcal{L}} + \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha} \cdot \sigma_{\mathcal{L}}$$

Φ is the cdf of $\mathcal{N}(0,1)$
ϕ is the pdf of $\mathcal{N}(0,1)$

Mu and sigma from MC simulation

Portfolio Loss plot