



MIE1622 Assignment 2 Tutorial

Risk-Based and Robust Portfolio Selection Strategies

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Equal risk contributions



Compute Gradient Example

Initial Matrices

$$x = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}, \quad Q = \begin{bmatrix} a11 & a21 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix} \text{ and } Q \text{ is symetric}$$

$$y = x \cdot (Qx) = \begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix} = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} \cdot \begin{bmatrix} a11.x1 + a12.x2 + a13.x3 \\ a21.x1 + a22.x2 + a23.x3 \\ a31.x1 + a32.x2 + a33.x3 \end{bmatrix}$$

Objective Function

$$f = obj.function = \sum \sum (y_i - y_j)^2$$

$$f = 2 \times [(y1 - y2)^2 + (y2 - y3)^2 + (y1 - y3)^2]$$

Equal risk contributions



Compute Gradient Example

Gradient - Formula

$$\nabla f = \text{gradient } f = \frac{\partial f}{\partial x} = \begin{bmatrix} \partial f / \partial x_1 \\ \partial f / \partial x_2 \\ \partial f / \partial x_3 \end{bmatrix}$$

$$\nabla f = 2 \times 2 \times \begin{bmatrix} (y_1 - y_2) \left(\frac{\partial y_1}{\partial x_1} - \frac{\partial y_2}{\partial x_1} \right) + (y_2 - y_3) \left(\frac{\partial y_2}{\partial x_1} - \frac{\partial y_3}{\partial x_1} \right) + (y_1 - y_3) \left(\frac{\partial y_1}{\partial x_1} - \frac{\partial y_3}{\partial x_1} \right) \\ (y_1 - y_2) \left(\frac{\partial y_1}{\partial x_2} - \frac{\partial y_2}{\partial x_2} \right) + (y_2 - y_3) \left(\frac{\partial y_2}{\partial x_2} - \frac{\partial y_3}{\partial x_2} \right) + (y_1 - y_3) \left(\frac{\partial y_1}{\partial x_2} - \frac{\partial y_3}{\partial x_2} \right) \\ (y_1 - y_2) \left(\frac{\partial y_1}{\partial x_3} - \frac{\partial y_2}{\partial x_3} \right) + (y_2 - y_3) \left(\frac{\partial y_2}{\partial x_3} - \frac{\partial y_3}{\partial x_3} \right) + (y_1 - y_3) \left(\frac{\partial y_1}{\partial x_3} - \frac{\partial y_3}{\partial x_3} \right) \end{bmatrix}$$

Equal risk contributions



Gradient Validation:

Finite Difference Method/ Check Equal Risk

Leveraged Equal risk



Alternative method:

- Multiply the number of shares (x_{optimal}) by 2 only in the first period.
- The money I borrow will be equal to the initial value of portfolio but I will pay interest for each period throughout two years.



Robust MVO

- Target Risk:

E.g. from min-Var, from $1/n$ portfolio

Consider diagonal of the covariance matrix Q

- Target Return:

E.g. Risk-free rate, median of mean returns from the previous period

Justify your selection.

Other Notes



- Negative returns:

choose a strategy that makes sense if there is no feasible solution. E.g. hold the portfolio

- Disable cplex output:

```
options.Display = 'off';
```

```
Cplex.DisplayFunc = [];
```