The Map Coloring Model

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Background

Map Coloring is the process of coloring different features of a map. It is an example of a constraint satisfiability problem where the goal is to find a solution to the problem satisfying all the constraints. In graph theory, a graph coloring is an assignment of labels traditionally called 'colors' to elements of a graph subject to certain constraints. A face coloring or Map Coloring assigns a color to each face of a planar graph or a region such that no two faces that share a boundary have the same color. This can be modeled to several types of scheduling problems. Other applications include register allocation in compilers, pattern matching, designing seat plans, etc.

Problem

The Map Coloring Model models a single agent that attempts to color the regions of the map of Australia using only three colors, such that no two regions sharing a boundary are of the same color. Australia has seven regions namely, Western Australia, Northern Territory, South Australia, Queensland, New South Wales, Victoria and Tasmania. The agent has information about these regions and the neighbors of each region, but it has no prior knowledge of which color to choose for each region of the map. The model when run, generates a trace of its attempt at coloring the map following the above mentioned constraints. The point of interest would be to see how many steps it takes the model to color the map by following the constraints and how the model behaves when a hint is given to it or when a new constraint is added.

The Model

The environment consists of a set of regions along with their corresponding neighboring regions and a set of colors to be used. The actions of the agent include selecting a region and assigning a color to it. The basic strategy is, the agent chooses a color for a region and checks if any constraints are violated. If a constraint is violated, it backtracks to the previous steps and chooses a different color until it finds a solution or until all possible combination of assignments are checked. The working memory of the agent stores the color assigned to each region respectively and replaces the colors which violate the constraints. The concept of Long Term Memory is used, which stores the connections between the regions of the map in the form of

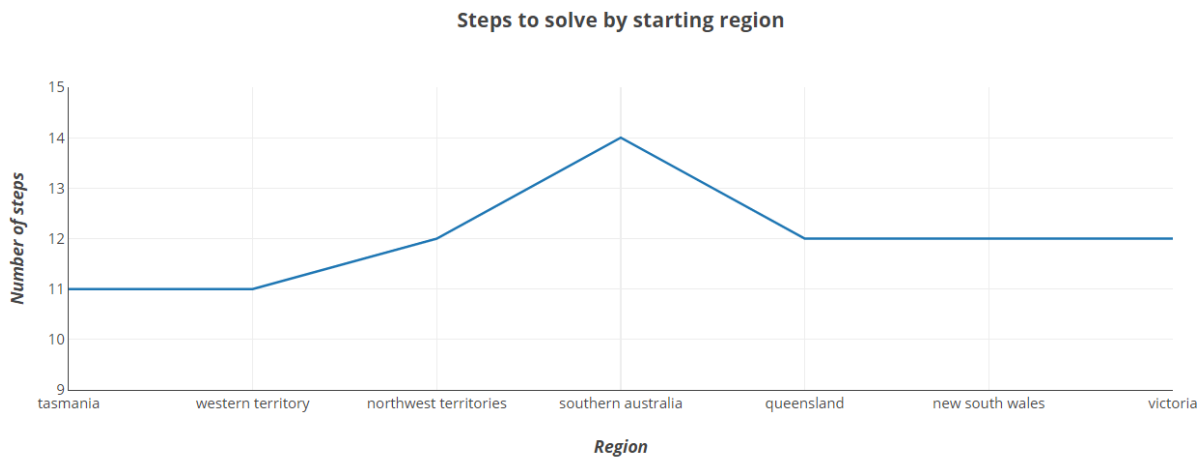
an adjacency matrix to help the agent verify the constraint that no two neighbors have the same color assigned to them.

Procedure

The Model is first given the environment to work on. It then proceeds to store the adjacency matrix in its long term memory. Colors are assigned to the regions if the constraints are not violated. If a constraint is violated i.e. if two neighboring regions end up having the same color, the Model assigns a different color and checks the constraint again. It backtracks to the previous step if no assignment is possible and assigns a different color. If there are no possible assignments left and there are still regions to be colored, the Model terminates. A solution is returned with the assignment of colors if all the regions are colored and all the constraints are satisfied.

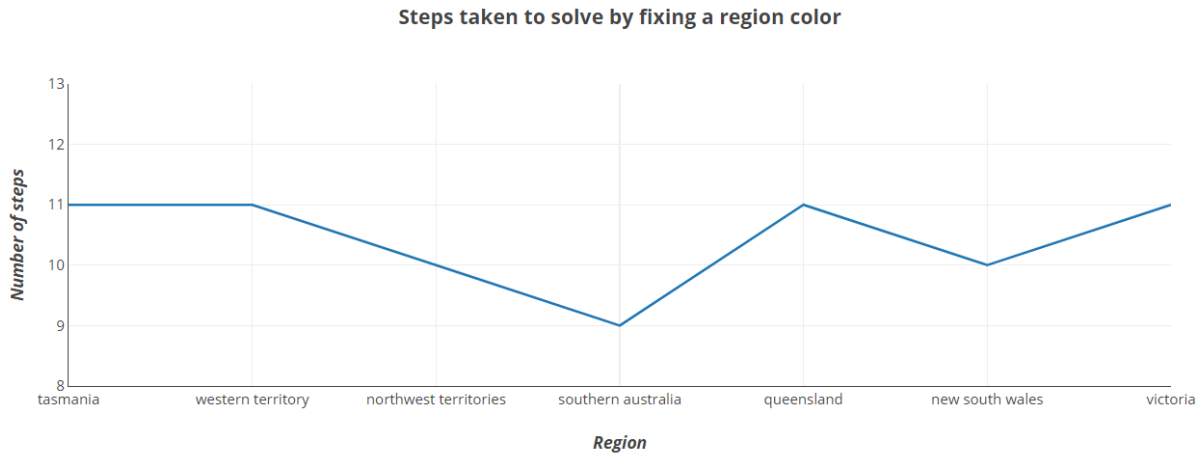
Results

The number of steps taken by the model if the starting region is fixed are as follows:



We can see from the above graph that, if the Model starts from Southern Australia it takes the most number of steps to find the solution and if it starts from Tasmania or Western Australia, it takes the least number of steps to find a solution.

The behavior of the model is also checked when a hint is given to it. Here are the number of steps taken by the Model when the color of a region is given:



Conclusion

From the above results, it can be observed that the Model takes the most number of steps to find a solution when it starts from a region having the most number of neighbors and takes the least number of steps to find a solution when it starts from the region with the least number of neighbors. From the second experiment, it can be observed that the number of steps taken by the Model to solve the problem is not greatly affected when it takes a hint about the color of any region in advance. The number only slightly reduces if the hint is about a region with the most number of neighbors.

Even with the limitations in the size of the environment, the Model gives an insight into how choosing a region to start with can greatly impact the performance. The effects of hints can however be studied further using bigger maps before generalizing their impact on the performance of the Model.