NCERT Solution For Class 8 Maths Chapter 9 Algebraic Expressions and Identities

Exercise 9.1 Page No: 140

1. Identify the terms, their coefficients for each of the following expressions.

(i) $5xyz^2 - 3zy$
(ii) $1 + x + x^2$
(iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$
(iv) $3 - pq + qr - p$
(v) $(x/2) + (y/2) - xy$
(vi) $0.3a - 0.6ab + 0.5b$

Solution :

Sl. No.	Expression	Term	Coefficient
i)	$5xyz^2 - 3zy$	Term: $5xyz^2$ Term: $-3zy$	5 -3
ii)	$1 + x + x^2$	Term: 1 Term: x Term x^2	1 1 1
iii)	$4x^2y^2 - 4x^2y^2z^2 + z^2$	Term: $4x^2y^2$ Term: $-4\ x^2y^2z^2$ Term : z^2	4 -4 1
iv)	$3 - pq + qr - p$	3 -pq qr -p	3 -1 1 -1
v)	$(x/2) + (y/2) - xy$	x/2 Y/2 -xy	½ 1/2 -1
vi)	$0.3a - 0.6ab + 0.5b$	0.3a -0.6ab 0.5b	0.3 -0.6 0.5

2. Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?
$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y – 3y^2$, $2y – 3y^2 + 4y^3$, $5x – 4y + 3xy$, $4z – 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$

Solution:
Let us first define the classifications of these 3 polynomials:
Monomials, Contain only one term.
Binomials, Contain only two terms.
Trinomials, Contain only three terms.

$x + y$	two terms	Binomial
1000	one term	Monomial
$x + x^2 + x^3 + x^4$	four terms	Polynomial, and it does not fit in listed three categories
$2y – 3y^2$	two terms	Binomial
$2y – 3y^2 + 4y^3$	three terms	Trinomial
$5x – 4y + 3xy$	three terms	Trinomial
$4z – 15z^2$	two terms	Binomial
$ab + bc + cd + da$	four terms	Polynomial, and it does not fit in listed three categories
pqr	one term	Monomial
$p^2q + pq^2$	two terms	Binomial
$2p + 2q$	two terms	Binomial

3. Add the following.
(i) ab – bc, bc – ca, ca – ab
(ii) a – b + ab, b – c + bc, c – a + ac
(iii) $2p^2q^2 – 3pq + 4$, $5 + 7pq – 3p^2q^2$
(iv) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, $2lm + 2mn + 2nl$

Solution:

i) (ab - bc) + (bc - ca) + (ca-ab)
= ab - bc + bc - ca + ca - ab
= ab - ab – bc + bc – ca + ca
= 0

ii) (a - b + ab) + (b - c + bc) + (c - a + ac)
= a - b + ab + b - c + bc + c - a + ac
= a - a +b – b +c – c + ab + bc + ca
=0 + 0 + 0 + ab + bc + ca
= ab + bc + ca

iii) $2p^2q^2 – 3pq + 4, 5 + 7pq – 3p^2q^2$
$= (2p^2q^2 - 3pq + 4) + (5 + 7pq - 3p^2q^2)$
$= 2p^2q^2 - 3p^2q^2 - 3pq + 7pq + 4 + 5$
$= - p^2q^2 + 4pq + 9$

iv) $(l^2 + m^2) + (m^2 + n^2) + (n^2 + l^2) + (2lm + 2mn + 2nl)$
$= l^2 + l^2 + m^2 + m^2 + n^2 + n^2 + 2lm + 2mn + 2nl$
$= 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

4. (a) Subtract *4a – 7ab + 3b + 12* from *12a – 9ab + 5b – 3*

(b) Subtract *3xy + 5yz – 7zx* from *5xy – 2yz – 2zx + 10xyz*

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2$q

Solution:

(a) (12a - 9ab + 5b - 3) - (4a - 7ab + 3b + 12)

= 12a - 9ab + 5b - 3 - 4a + 7ab - 3b - 12

= 12a – 4a -9ab + 7ab +5b – 3b -3 -12

= 8a - 2ab + 2b - 15

b) (5xy - 2yz - 2zx + 10xyz) - (3xy + 5yz - 7zx)

= 5xy - 2yz - 2zx + 10xyz - 3xy - 5yz + 7zx

=5xy – 3xy – 2yz – 5yz – 2zx + 7zx + 10xyz

= 2xy - 7yz + 5zx + 10xyz

c) $(18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q) - (4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$

$= 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - 4p^2q + 3pq - 5pq^2 + 8p - 7q + 10$

=18+10 -3p+8p -11q – 7q + 5 pq+ 3pq- 2pq^2 – 5pq^2 + 5 p^2 q – 4p^2 q

$= 28 + 5p - 18q + 8pq - 7pq^2 + p^2q$

Exercise 9.2 Page No: 143

1. Find the product of the following pairs of monomials.

(i) 4, 7p
(ii) - 4p, 7p
(iii) – 4p, 7pq
(iv) $4p^3$, - 3p
(v) 4p, 0

Solution:

(i) $4 \times 7p = 4 \times 7 \times p = 28p$

(ii) $-4p \times 7p = (-4 \times 7) \times (p \times p) = -28p^2$

(iii) $-4p \times 7pq = (-4 \times 7)(p \times pq) = -28p^2q$

(iv) $4p^3 \times -3p = (4 \times -3)(p^3 \times p) = -12p^4$

(v) $4p \times 0 = 0$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

(p, q) ; (10m, 5n) ; ($20x^2$, $5y^2$) ; (4x, $3x^2$) ; (3mn, 4np)

Solution:

Area of rectangle = Length x breadth. So, it is multiplication of two monomials.
The results can be written in square units.

(i) $p \times q = pq$

(ii) $10m \times 5n = 50mn$

(iii) $20x^2 \times 5y^2 = 100x^2y^2$

(iv) $4x \times 3x^2 = 12x^3$

(v) $3mn \times 4np = 12mn^2p$

3. **Complete the following table of products:**

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	...	...	...	...	...
$-5y$	...	...	$-15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$-4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$-9x^2y^2$	...	...	...	...	...	...

Solution:

First monomial / Second monomial	2x	-5y	$3x^2$	-4xY	$7x^2Y$	$-9x^2y^2$
2x	$4x^2$	-10xy	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
-5y	-10xy	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
-4xy	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27\ x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

4. Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

(i) 5a, $3a^2$, $7a^4$
(ii) *2p, 4q, 8r*
(iii) xy, $2x^2y$, $2xy^2$
(iv) *a, 2b, 3c*

Solution:

Volume of rectangle = length x breadth x height. To evaluate volume of rectangular boxes, multiply all the monomials.

(i) $5a \times 3a^2 \times 7a^4 = (5 \times 3 \times 7)\ (a \times a\text{^}2 \times a\text{^}4) = 105a^7$

(ii) $2p \times 4q \times 8r = (2 \times 4 \times 8)\ (p \times q \times r) = 64pqr$

(iii) $y \times 2x^2y \times 2xy^2 = (1 \times 2 \times 2)(x \times x\text{^}2 \times x \times y \times y \times y\text{^}2) = 4x^4y^4$

(iv) a x 2b x 3c = (1 × 2 × 3) (a × b × c) = 6abc

5. Obtain the product of

(i) xy, yz, zx
(ii) a, - a^2 , a^3
(iii) 2, 4y, $8y^2$, $16y^3$
(iv) a, 2b, 3c, 6abc
(v) m, - mn, mnp

Solution:

(i) $xy \times yz \times zx = x^2 y^2 z^2$

(ii) $a \times -a^2 \times a^3 = -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3 = 1024\ y^6$

(iv) $a \times 2b \times 3c \times 6abc = 36a^2 b^2 c^2$

(v) $m \times -mn \times mnp = -m^3 n^2 p$

Exercise 9.3 Page No: 146

1. Carry out the multiplication of the expressions in each of the following pairs.
(i) 4p, q + r
(ii) ab, a – b
(iii) a + b, $7a^2b^2$
(iv) a^2 – 9, 4a
(v) pq + qr + rp, 0

Solution:

(i)$4p(q + r) = 4pq + 4pr$

(ii)$ab(a - b) = a^2 b - a b^2$

(iii)$(a + b)(7a^2b^2) = 7a^3b^2 + 7a^2b^3$

(iv) $(a^2 - 9)(4a) = 4a^3 - 36a$

(v) $(pq + qr + rp) \times 0 = 0$ (Anything multiplied by zero is zero)

2. **Complete the table.**

	First expression	Second expression	Product
(i)	a	$b + c + d$	...
(ii)	$x + y - 5$	$5xy$	...
(iii)	p	$6p^2 - 7p + 5$	...
(iv)	$4p^2q^2$	$p^2 - q^2$	...
(v)	$a + b + c$	abc	...

Solution:

	First expression	Second expression	Product
(i)	a	b + c + d	a(b+c+d) = a×b + a×c + a×d = ab + ac + ad
(ii)	x + y - 5	5xy	5 xy (x + y - 5) = 5 xy * x + 5 xy * y - 5 xy * 5 = 5 x^2y + 5 xy^2 - 25xy
(iii)	p	6p^2 - 7p + 5	p (6 p^2-7 p +5) = p× 6 p^2 - p× 7 p + p×5 = 6 p^3 - 7 p^2 + 5 p
(iv)	4 p^2 q^2	P^2 - q^2	4p^2 q^2 * (p^2 – q^2) =4 p^4 q^2 - 4p^2 q^4
(v)	a + b + c	abc	abc(a + b + c) = abc × a + abc × b + abc × c = a^2bc + ab^2c + abc^2

3. Find the product.
i) $a^2 \times (2a^{22}) \times (4a^{26})$
ii) $(2/3\ xy) \times (-9/10\ x^2y^2)$
(iii) $(-10/3\ pq^3) \times (6/5\ p^3q)$
(iv) $(x) \times (x^2) \times (x^3) \times (x^4)$

Solution:

i) $a^2 \times (2a^{22}) \times (4a^{26}) = (2 \times 4)\ (a\text{^}2 \times a\text{^}22 \times a\text{^}26) = 8 \times a\text{^}(2+22+26) = 8a^{50}$

ii) $(2xy/3) \times (-9x^2y^2/10)$
$= (2/3 \times -9/10)\ (x \times x\text{^}2 \times y \times y\text{^}2)$
$= (-3/5\ x^3y^3)$

iii) $(-10pq^3/3) \times (6p^3q/5)$
$= (-10/3 \times 6/5)\ (p \times p\text{^}3 \times q\text{^}3 \times q)$
$= (-4p^4q^4)$

iv) $(x) \times (x^2) \times (x^3) \times (x^4)$
$= x\text{^}(1+2+3+8)$
$= x^{10}$

4. (a) Simplify $3x\ (4x - 5) + 3$ and find its values for (i) x = 3 (ii) x =1/2

(b) Simplify *a* $(a^2 + a + 1) + 5$ and find its value for (i) *a = 0*, (ii) *a = 1* (iii) *a = – 1*.

Solution:

a) $3x\ (4x - 5) + 3$

$= 3x\ (4x) - 3x(5) + 3$

$= 12x^2 - 15x + 3$

(i) Putting x=3 in the equation we gets
$12x^2 - 15x + 3 = 12(3\text{^}2) - 15\ (3) + 3$
$= 108 - 45 + 3 = 66$

(ii) Putting x=1/2 in the equation we get

$12x^2 - 15x + 3 = 12\ (1/2)\text{^}2 - 15\ (1/2) + 3$

$= 12\ (1/4) - 15/2 + 3$

$= 3 - 15/2 + 3$

= 6- 15/2
= (12- 15) /2
= -3/2

b) $a(a^2 + a + 1) + 5$
$= a^* a^2 + a^* a + a^* 1 + 5$
$= a^3 + a^2 + a + 5$

(i) putting a= 0 in the equation we get
$0^3 + 0^2 + 0 + 5 = 5$

(ii) putting a=1 in the equation we get
$1^3 + 1^2 + 1 + 5 = 1 + 1 + 1 + 5 = 8$

(iii) Putting a = -1 in the equation we get
$(-1)^3 + (-1)^2 + (-1) + 5 = -1 + 1 - 1 + 5 = 4$

5. (a) Add: *p (p – q), q (q – r) and r (r – p)*
(b) Add: *2x (z – x – y)* and *2y (z – y – x)*
(c) Subtract: *3l (l – 4 m + 5 n)* from *4l (10 n – 3 m + 2 l)*
(d) Subtract: *3a (a + b + c) – 2 b (a – b + c)* from *4c (– a + b + c)*

Solution:

a) *p (p – q) + q (q – r) + r (r – p)*

$= (p^2 - pq) + (q^2 - qr) + (r^2 - pr)$
$= p^2 + q^2 + r^2 - pq - qr - pr$

b) *2x (z – x – y) + 2y (z – y – x)*

$= (2xz - 2x^2 - 2xy) + (2yz - 2y^2 - 2xy)$
$= 2xz - 4xy + 2yz - 2x^2 - 2y^2$

c) *4l (10 n – 3 m + 2 l) - 3l (l – 4 m + 5 n)*
$= (40ln - 12lm + 8l^2) - (3l^2 - 12lm + 15ln)$
$= 40ln - 12lm + 8l^2 - 3l^2 + 12lm - 15ln$
$= 25ln + 5l^2$

d) *4c (– a + b + c) - (3a (a + b + c) – 2 b (a – b + c))*
$= (-4ac + 4bc + 4c^2) - (3a^2 + 3ab + 3ac - (2ab - 2b^2 + 2bc))$
$= -4ac + 4bc + 4c^2 - (3a^2 + 3ab + 3ac - 2ab + 2b^2 - 2bc)$

$= -4ac + 4bc + 4c^2 - 3a^2 - 3ab - 3ac + 2ab - 2b^2 + 2bc$
$= -7ac + 6bc + 4c^2 - 3a^2 - ab - 2b^2$

Exercise 9.4 Page No: 148

1. Multiply the binomials.

i) (2x + 5) and (4x – 3)
(ii) (y – 8) and (3y – 4)
(iii) (2.5l – 0.5m) and (2.5l + 0.5m)
(iv) (a + 3b) and (x + 5)
(v) $(2pq + 3q^2)$ and $(3pq – 2q^2)$
(vi) (3/4 a^2 + 3b^2) and 4(a^2 - 2/3 b^2)

Solution :

i) $(2x + 5)(4x - 3)$
$= 2x * 4x - 2x * 3 + 5 * 4x - 5 * 3$
$= 8x^2 - 6x + 20x -15$
$= 8x^2 + 14x -15$

ii) (y - 8)(3y - 4)
= y * 3y - 4y - 8 * 3y + 32
= $3y^2$ - 4y - 24y + 32
= $3y^2$ - 28y + 32

(iii) (2.5l - *0.5m)(2.5l + 0.5m)*
= 2.5l * 2.5 l + 2.5l * 0.5m – 0.5m * 2.5l - 0.5m * 0.5m
= *6.25l^2 + 1.25 lm* – 1.25 lm - 0.25 m^2
= *6.25l^2* - 0.25 m^2

iv) (a + 3b) (x + 5)
= *ax + 5a + 3bx + 15b*

v) $(2pq + 3q^2)$ $(3pq – 2q^2)$
= 2pq * 3pq - 2pq * $2q^2$ + $3q^2$ * 3pq - $3q^2$ * $2q^2$
= $6p^2q^2$ - $4pq^3$ + $9pq^3$ - $6q^4$
= $6p^2q^2$ + $5pq^3$ – $6q^4$

(vi) (3/4 a^2 + 3b^2) and 4(a^2 - 2/3 b^2)

= (3/4 a^2 + 3b^2) * 4(a^2 - 2/3 b^2)
=(3/4 a^2 + 3b^2) * (4a^2 - 8/3 b^2)
=3/4 a^2 * (4a^2 - 8/3 b^2) + 3b^2 * (4a^2 - 8/3 b^2)
=3/4 a^2 * 4a^2 -3/4 a^2 * 8/3 b^2 + 3b^2 * 4a^2 - 3b^2* 8/3 b^2
=3a^4 – 2a^2b^2 + 12 a^2 b^2 - 8b^4
= 3a^4 + 10a^2 b^2 – 8b^4

2. Find the product.

(i) (5 – 2x) (3 + x)
(ii) (x + 7y) (7x – y)
(iii) (a^2+ b) (a + b^2)
(iv) (p^2 – q^2) (2p + q)

Solution:

i) (5 – 2x) (3 + x)

= 5 (3 + x) - 2x (3 + x)
=15 + 5x - 6x - $2x^2$
= 15 - x -2 x ^{2}

(ii) (x + 7y) (7x – y)

= x(7x-y) + 7y (7x-y)

=$7x^2$ - xy + 49xy - $7y^2$
= $7x^2$ - $7y^2$ + 48xy

iii) (a^2+ b) (a + b^2)
= a^2 (a + b^2) + b(a + b^2)
= a^3 + a^2b^2 + ab + b^3
= a^3 + b^3 + a^2b^2 + ab

iv) (p^2– q^2) (2p + q)

= p^2 (2p + q) - q^2 (2p + q)
=$2p^3$ + p^2q - 2pq^2 - q^3
= $2p^3$ - q^3 + p^2q - 2pq^2

3. Simplify.

(i) (x^2– 5) (x + 5) + 25
(ii) (a^2+ 5) (b^3+ 3) + 5
(iii)(t + s^2)(t^2 – s)
(iv) (a + b) (c – d) + (a – b) (c + d) + 2 (ac + bd)
(v) (x + y)(2x + y) + (x + 2y)(x – y)
(vi) (x + y)(x^2– xy + y^2)
(vii) (1.5x – 4y)(1.5x + 4y + 3) – 4.5x + 12y
(viii) (a + b + c)(a + b – c)

Solution :

i) (x^2– 5) (x + 5) + 25
= x^3 + $5x^2$ - 5x - 25 + 25
= x^3 + $5x^2$ - 5x

ii) $(a^2+ 5) (b^3+ 3) + 5$
$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$
$= a^2b^3 + 5b^3 + 3a^2 + 20$

iii) $(t + s^2)(t^2 – s)$
$= t (t^2 – s) + s^2(t^2 – s)$
$= t^3 – st + s^2t^2 – s^3$
$= t^3 – s^3 – st + s^2t^2$

iv) $(a + b) (c – d) + (a – b) (c + d) + 2 (ac + bd)$

$= (a + b) (c – d) + (a – b) (c + d) + 2 (ac + bd)$
$=(ac - ad + bc - bd) + (ac + ad - bc - bd) + (2ac + 2bd)$
$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$
$= 4ac$

v) $(x + y)(2x + y) + (x + 2y)(x – y)$

$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$
$= 3x^2 + 4xy - y^2$

vi) $(x + y)(x^2– xy + y^2)$
$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$
$= x^3 + y^3$

vii) $(1.5x – 4y)(1.5x + 4y + 3) – 4.5x + 12y$

$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$
$= 2.25x^2 - 16y^2$

viii) $(a + b + c)(a + b – c)$
$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$
$= a^2 + b^2 - c^2 + 2ab$

Exercise 9.5

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1. Use a suitable identity to get each of the following products.

(i) (x + 3) (x + 3)
(ii) (2y + 5) (2y + 5)
(iii) (2a - 7) (2a - 7)
(iv) (3a - 1/2)(3a - 1/2)
(v) (1.1m – 0.4) (1.1m + 0.4)
(vi) (a^2+ b^2) (- a^2+ b^2)
(vii) (6x - 7) (6x + 7)
(viii) (- a + c) (- a + c)
(ix) $(\frac{1}{2}x + \frac{3}{4}y)(\frac{1}{2}x + \frac{3}{4}y)$
(x) (7a - 9b) (7a - 9b)

Solution:

i) $(x + 3)(x + 3) = (x + 3)^2$
$= x^2 + 6x + 9$
Using $(a+b)^2 = a^2 + b^2 + 2ab$

ii)
$(2y + 5)(2y + 5) = (2y + 5)^2$
$= 4y^2 + 20y + 25$
Using $(a+b)^2 = a^2 + b^2 + 2ab$

iii) $(2a – 7)(2a – 7) = (2a – 7)^2$
$= 4a^2 – 28a + 49$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

iv) $(3a - 1/2)(3a - 1/2) = (3a - 1/2)^2$

$= (3a-1/2)(3a-1/2) = 9a^2 - 3a + (1/4)$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

v) $(1.1m – 0.4)(1.1m + 0.4)$

$= 1.21m^2 – 0.16$

Using $(a - b)(a + b) = a^2 - b^2$

vi) $(a^2+ b^2)(– a^2+ b^2)$
$= (b^2 + a^2)(b^2 – a^2)$
$= -a^4 + b^4$
Using $(a - b)(a + b) = a^2 - b^2$

vii)
(6x - 7) (6x + 7)
$=36x^2 - 49$
Using $(a - b)(a + b) = a^2 - b^2$

viii) $(– a + c) (– a + c) = (– a + c)^2$
$= c^2 + a^2 - 2ac$
Using $(a-b)^2 = a^2 + b^2 - 2ab$

ix) $\left(\frac{1}{2}x + \frac{3}{4}y\right)\left(\frac{1}{2}x + \frac{3}{4}y\right) = \left(\frac{1}{2}x + \frac{3}{4}y\right)^2$

$= (x^2/4) + (9y^2/16) + (3xy/4)$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

x) $(7a - 9b) (7a - 9b) = (7a - 9b)^2$

$= 49a^2 - 126ab + 81b^2$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

2. Use the identity $(x + a) (x + b) = x^2 + (a + b) x + ab$ to find the following products.
(i) (x + 3) (x + 7)
(ii) (4x + 5) (4x + 1)
(iii) (4x – 5) (4x – 1)
(iv) (4x + 5) (4x – 1)
(v) (2x + 5y) (2x + 3y)
(vi) $(2a^2 + 9) (2a^2 + 5)$
(vii) (xyz - 4) (xyz - 2)

Solution:

i) (x + 3) (x + 7)

$= x^2 + (3+7)x + 21$
$= x^2 + 10x + 21$

ii) (4x + 5) (4x + 1)
$= 16x^2 + (5 + 1)4x + 5$
$= 16x^2 + 24x + 5$

iii) (4x – 5) (4x – 1)
$= 16x^2 + (-5-1)4x + 5$
$= 16x^2 - 20x + 5$

iv) $(4x + 5)(4x - 1)$
$= 16x^2 + (5-1)4x - 5$
$= 16x^2 + 16x - 5$

v) $(2x + 5y)(2x + 3y)$
$= 4x^2 + (5y + 3y)2x + 15y^2$
$= 4x^2 + 16xy + 15y^2$

vi) $(2a^2 + 9)(2a^2 + 5)$
$= 4a^4 + (9+5)2a^2 + 45$
$= 4a^4 + 28a^2 + 45$

vii) $(xyz - 4)(xyz - 2)$
$= x^2y^2z^2 + (-4 -2)xyz + 8$
$= x^2y^2z^2 - 6xyz + 8$

3. Find the following squares by using the identities.

(i) $(b - 7)^2$
(ii) $(xy + 3z)^2$
(iii) $(6x^2 - 5y)^2$
(iv) $[(2m/3) + (3n/2)]^2$
(v) $(0.4p – 0.5q)^2$
(vi) $(2xy + 5y)^2$

Solution:

Using identities:

$(a - b)^2 = a^2 + b^2 - 2ab$
$(a + b)^2 = a^2 + b^2 + 2ab$

(i) $(b - 7)^2 = b^2 - 14b + 49$

(ii) $(xy + 3z)^2 = x^2y^2 + 6xyz + 9z^2$

(iii) $(6x^2 - 5y)^2 = 36x^4 - 60x^2y + 25y^2$

(iv) $[(2m/3\}) + (3n/2)]^2 = (4m^2/9) + (9n^2/4) + 2mn$

(v) $(0.4p – 0.5q)^2 = 0.16p^2 - 0.4pq + 0.25q^2$

(vi) $(2xy + 5y)\text{^}2 = 4x^2y^2 + 20xy^2 + 25y^2$

4. Simplify.

(i) $(a^2 – b^2)^2$
(ii) $(2x + 5)^2 - (2x - 5)^2$
(iii) $(7m – 8n)^2 + (7m + 8n)^2$
(iv) $(4m + 5n)^2 + (5m + 4n)^2$
(v) $(2.5p – 1.5q)^2 – (1.5p – 2.5q)^2$
(vi) $(ab + bc)^2 – 2ab^2c$
(vii) $(m^2 – n^2m)^2 + 2m^3n^2$

Solution:

i) $(a^2 – b^2)^2 = a^4 + b^4 - 2a^2b^2$

ii) $(2x + 5)^2 - (2x - 5)^2$
$= 4x^2 + 20x + 25 - (4x^2 - 20x + 25)$
$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25$
$= 40x$

iii) $(7m – 8n)^2 + (7m + 8n)^2$
$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$
$= 98m^2 + 128n^2$

iv) $(4m + 5n)^2 + (5m + 4n)^2$
$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$
$= 41m^2 + 80mn + 41n^2$

v) $(2.5p – 1.5q)^2 – (1.5p – 2.5q)^2$
$= 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2 = 4p^2 - 4q^2$

vi) $(ab + bc)^2 – 2ab^2c$
$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$
$= a^2b^2 + b^2c^2$

vii) $(m^2 – n^2m)^2 + 2m^3n^2$
$= m^4 - 2m^3n^2 + m^2n^4 + 2m^3n^2$
$= m^4 + m^2n^4$

5. Show that.

(i) $(3x + 7)^2 – 84x = (3x – 7)^2$
(ii) $(9p – 5q)^2 + 180pq = (9p + 5q)^2$
(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$
(iv) $(4pq + 3q)^2 – (4pq – 3q)^2 = 48pq^2$
(v) $(a – b)(a + b) + (b – c)(b + c) + (c – a)(c + a) = 0$

Solution:

i) LHS = $(3x + 7)^2 - 84x$
$= 9x^2 + 42x + 49 - 84x$
$= 9x^2 - 42x + 49$
= RHS
LHS = RHS

ii) LHS = $(9p - 5q)^2 + 180pq$
$= 81p^2 - 90pq + 25q^2 + 180pq$
$= 81p^2 + 90pq + 25q^2$
RHS = $(9p + 5q)^2$
$= 81p^2 + 90pq + 25q^2$
LHS = RHS

(iii) $LHS = \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2 - 2mn + 2mn$
$= \frac{16}{9}m^2 + \frac{9}{16}n^2$
= RHS
LHS = RHS

iv) LHS = $(4pq + 3q)^2 - (4pq - 3q)^2$

$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$
$= 48pq^2$

RHS = $48pq^2$
LHS = RHS

v) LHS = $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$
$= a^2 - b^2 + b^2 - c^2 + c^2 - a^2$
$= 0$
= RHS

6. Using identities, evaluate.

(i) 71^2
(ii) 99^2
(iii) 102^2
(iv) 998^2
(v) 5.2^2
(vi) 297 x 303
(vii) 78 x 82
(viii) 8.9^2
(ix) 10.5 x 9.5

Solution:

i) $71^2 = (70+1)^2$
$= 70^2 + 140 + 1^2$
$= 4900 + 140 + 1$
$= 5041$

ii) 99^2
$= (100 - 1)^2$
$= 100^2 - 200 + 1^2$
$= 10000 - 200 + 1$
$= 9801$

iii) $102^2 = (100 + 2)^2$
$= 100^2 + 400 + 2^2$
$= 10000 + 400 + 4$
$= 10404$

iv) $998^2 = (1000 - 2)^2$
$= 1000^2 - 4000 + 2^2$
$= 1000000 - 4000 + 4$
$= 996004$

v) $5.2^2 = (5 + 0.2)^2$
$= 5^2 + 2 + 0.2^2$
$= 25 + 2 + 0.04$
$= 27.04$

vi) 297×303
$= (300 - 3)(300 + 3)$
$= 300^2 - 3^2$
$= 90000 - 9$
$= 89991$

vii) 78×82
$= (80 - 2)(80 + 2)$
$= 80^2 - 2^2$
$= 6400 - 4$
$= 6396$

viii) $8.9^2 = (9 - 0.1)^2$
$= 9^2 - 1.8 + 0.1^2$
$= 81 - 1.8 + 0.01$
$= 79.21$

ix) 10.5 x 9.5
= (10 + 0.5)(10 - 0.5)
= 10^2 - 0.5^2
= 100 - 0.25
= 99.75

7. Using $a^2 – b^2 = (a + b) (a – b)$, find
(i) $51^2 – 49^2$
(ii) $(1.02)^2 – (0.98)^2$
(iii) $153^2 – 147^2$
(iv) $12.1^2 – 7.9^2$

Solution:

i) $51^2 – 49^2$
= (51 + 49)(51 - 49)
= 100 x 2
= 200

ii) $(1.02)^2 – (0.98)^2$
= (1.02 + 0.98)(1.02 - 0.98)
= 2 x 0.04
= 0.08

iii) $153^2 – 147^2$
= (153 + 147)(153 - 147)
= 300 x 6
= 1800

iv) $12.1^2 – 7.9^2$
= (12.1 + 7.9)(12.1 - 7.9)
= 20 x 4.2= 84

8. Using $(x + a) (x + b) = x^2 + (a + b) x + ab$, find
(i) 103 x 104
(ii) 5.1 x 5.2
(iii) 103 x 98
(iv) 9.7 x 9.8

Solution:

i) 103 x 104
= (100 + 3)(100 + 4)
= 100^2 + (3 + 4)100 + 12
= 10000 + 700 + 12
= 10712

ii) 5.1 x 5.2
= (5 + 0.1)(5 + 0.2)
= 5^2 + (0.1 + 0.2)5 + 0.1 x 0.2
= 25 + 1.5 + 0.02
= 26.52

iii) 103 x 98
= (100 + 3)(100 - 2)
= 100^2 + (3-2)100 - 6
= 10000 + 100 - 6
= 10094

iv) 9.7 x 9.8
= (9 + 0.7)(9 + 0.8)
= 9^2 + (0.7 + 0.8)9 + 0.56
= 81 + 13.5 + 0.56
= 95.06