$\boxed{\text{Thm}}$ Let $\lambda$ be a Young diagram, and let $M(q)$, $R_\lambda(q)$, and $A_\lambda(q)$ denote the generating functions of plane partitions, reverse plane partitions of shape $\lambda$, and plane partitions asymptotic to $\lambda$, respectively. Then $A_\lambda(q) = R_\lambda(q)M(q)$.

$\boxed{\text{Proof}}$ Fix $\lambda$, and let PP, $\text{RPP}_\lambda$, and $\text{APP}_\lambda$ denote the sets of plane partitions, reverse plane partitions of shape $\lambda$, and plane partitions asymptotic to $\lambda$, respectively. We seek a bijection $S : \text{RPP}_\lambda \times \text{PP} \longrightarrow \text{APP}_\lambda$, such that if $\alpha = S(\rho, \pi)$, then $w(\alpha) = w(\rho) + w(\pi)$, where $w$ denotes the weight function.

We will first construct an injection $f$ from boxes of $\rho$ to boxes of $\alpha$ that preserves hook length. Consider the infinite binary string $(a_k \mid k \in \mathbb{Z})$ defining $\lambda$, where vertical edges are denoted 0 and horizontal edges denoted 1, and the sequence increases from bottom-left to top-right. Fixing $n$, suppose the first $k \in \mathbb{Z}$ for which $a_k = 1$ satisfies $k \equiv b$ mod $n$. Enumerate the $n$ distinct $n$-cores of $\rho$ as $c_{n,0}, c_{n,1}, \ldots, c_{n,n-1}$, where $c_{n,i}$ is defined by the substring $(a_k \mid k \equiv b+i$ mod $n)$. Now in each $c_{n,i}$, inner corners correspond to $n$-hooks of $\rho$, while outer corners correspond to $n$-hooks of $\alpha$. In fact, there is a bijection between $n$-hooks of $\rho$ and all inner corners of the $c_{n,i}$, and similarly between $n$-hooks of $\alpha$ and all outer corners of the $c_{n,i}$. Now in any Young diagram, inner corners are defined by the substring 10, and outer corners by the substring 01. Thus there is at least one outer corner, and inner and outer corners alternate moving from bottom-left to top-right along the ragged edge. We may therefore enumerate the inner and outer corners of $c_{n,i}$ as

$$y_{n,i,0}, x_{n,i,1}, y_{n,i,1}, x_{n,i,2}, y_{n,i,2}, \ldots, x_{n,i,m-1}, y_{n,i,m-1}, x_{n,i,m}, y_{n,i,m}$$

for some $m \in \mathbb{N}$, possibly zero. Here, the $x$ denote inner corners and the $y$ outer corners.

Now we are prepared to define $f$. Given a box $x \in \rho$, let $n = h(x)$ be its hook length. Then $x$ corresponds to a unique inner corner in some $c_{n,i}$, say $x_{n,i,j}$. The proceeding outer corner in the sequence, $y_{n,i,j}$, corresponds to a unique box $y \in \alpha$ with $h(y) = n$. Define $f(x) = y$.

Since every box of $\rho$ corresponds to a unique inner corner of some $c_{n,i}$, $f$ is well-defined and injective, and it also preserves hook length. Importantly, the boxes in $\alpha$ that do not lie in the image of $f$ are in bijection with the bottom-left outer corners in each $c_{n,i}$. Since there are $n$ distinct $n$-cores and each contains one missed outer corner, $\alpha$ contains $n$ boxes of hook length $n$ not in image$(f)$ — specifically, those $n$ boxes are

$$y_{n,0,0}, y_{n,1,0}, \ldots, y_{n,n-1,0}.$$

We will now define a map $g$ sending boxes of $\pi$ to these $y_{n,i,0}$. Denote the $n$ distinct $n$-cores of $\pi$ as $d_{n,0}, d_{n,1}, \ldots, d_{n,n-1}$, ordered as before. Since $\pi$ has no ragged edge, its binary string is just $\cdots 000111 \cdots$, so every $d_{n,i}$ has the same string, and therefore a single outer corner, which we denote $z_{n,i}$. By the same logic as before, $\pi$ therefore contains $n$ distinct boxes with hook length $n$, and each corresponds uniquely with some $z_{n,i}$. Given a box $z \in \pi$ with $h(z) = n$, first find the corresponding $z_{n,i}$. The outer corner $y_{n,i,0}$ corresponds to a box $y \in \alpha$ with $h(y) = n$. Define $g(z) = y$.

The maps $f$ and $g$ therefore put the boxes of $\alpha$ in bijection with the union of the boxes in $\rho$ and $\pi$, preserving hook length. However, we cannot simply copy the entries of $\rho$ and $\pi$ into the corresponding locations in $\alpha$, since the resulting object will not necessarily satisfy the defining inequalities of a plane partition. Instead, we will use the Hillman-Grassl bijection (denoted HG) to convert both $\rho$ and $\pi$ into $\mathbb{N}$-tableaux of the same shape, copy those entries into an $\mathbb{N}$-tableau of the same shape as $\alpha$, and then apply $\mathrm{HG}^{-1}$ to convert the resulting object back to a plane partition.

Let $\rho' = \mathrm{HG}(\rho)$ and $\pi' = \mathrm{HG}(\pi)$. Then $\rho'$ is an $\mathbb{N}$-tableau of the same shape as $\rho$, such that $\sum_{x\in\rho'} \rho'(x)h(x) = w(\rho)$. Similarly, $\pi'$ is an $\mathbb{N}$-tableau of the same shape as $\pi$, such that $\sum_{x\in\pi'} \pi'(x)h(x) = w(\pi)$. Define an $\mathbb{N}$-tableau $\alpha'$ of the same shape as $\alpha$ such that $\rho'(x) = \alpha'(f(x))$ for all $x \in \rho'$, and $\pi'(x) = \alpha'(g(x))$ for all $x \in \pi'$. Thus $\alpha'$ is uniquely determined by $\rho'$ and $\pi'$, and therefore by $\rho$ and $\pi$, since HG is a bijection. Now define $\alpha = \mathrm{HG}^{-1}(\alpha')$. This uniquely determines $\alpha$ in terms of $\rho$ and $\pi$. Since HG is a bijection and every entry of $\alpha'$ is determined by some entry of either $\rho'$ or $\pi'$, the map $S : (\rho, \pi) \longmapsto \alpha$ is a bijection between $\mathrm{RPP}_\lambda \times \mathrm{PP}$ and $\mathrm{APP}_\lambda$, and we have

$$\begin{aligned}
w(\alpha) &= \sum_{x\in\alpha'} \alpha'(x)h(x) \\
&= \sum_{x\in\rho'} \alpha'(f(x))h(f(x)) + \sum_{x\in\pi'} \alpha'(g(x))h(g(x)) \\
&= \sum_{x\in\rho'} \rho'(x)h(x) + \sum_{x\in\pi'} \pi'(x)h(x) \\
&= w(\rho) + w(\pi),
\end{aligned}$$

as required. $\square$